Vertical Foreclosure: A Dynamic Perspective

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Abstract

The paper shows that capability and incentives to vertically foreclose rivals can be affected by intertemporal linkages. By raising the costs of the foreclosed firm(s), the unintegrated supplier alters its customer's competitive position and reduces its own ability to extract rents in future periods. This, in turn, mitigates the supplier's incentive to exploit its market power vis-à-vis its customers. However, intertemporal linkages also magnify the impact of cost increases. As a consequence, intertemporal linkages can either increase or decrease the scope for foreclosure. I show that this scope is increased if there is no threat of a counter-merger or if intertemporal linkages are not too large. When a counter-merger is possible, for high enough intertemporal linkages, there is no foreclosure whatsoever.

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1 Introduction

The emergence of large digital platforms has prompted a renewed interest for vertical and conglomerate mergers. It is well accepted that, by bringing together complementary activities, such mergers can help resolve coordination problems and bring efficiency benefits, such as the elimination of double marginalization. However, they also trigger anti-competitive foreclosure concerns, particularly when one of the merging firms has substantial market power. The risk is that the integrated firm may then leverage its market power to foreclose competitors in adjacent markets – foreclosure can be complete or partial, and can take multiple forms such as denial of proper access to an essential input or degraded interoperability.

While the early literature on foreclosure focused on *ability*, the Chicago critique has stressed the need to account for *incentives*. In this paper, I highlight the need to account also for industry dynamics. Firms operate in an ever-changing environment, and their actions have long-term consequences. This is particularly important in the digital economy, where industry dynamics are on a new scale, due not only to rapid technological and organizational developments, but also to economies of scale and scope, network effects, multi-sidedness, switching costs and lock-in effects, and the – not yet fully understood – role of data. These features – together with the emergence of large platforms – have exacerbated competition concerns and triggered calls for strengthened policy intervention; however, they may also call for revisiting the traditional theories of harm from a dynamic perspective.

For example, the best-known post-Chicago theories of *vertical* foreclosure relies on *raising rival's costs* (Salinger, 1988; Ordover, Saloner and Salop, 1990 – OSS hereafter), which has had a remarkable impact on antitrust theory and practice.[1] This theory of harm, which focuses on input foreclosure, shows that an integrated supplier has an incentive to deny access to downstream rivals, so as to increase the market power of alternative suppliers over these rivals; this, in turn, raises the costs of the rivals, to the benefit of the downstream subsidiary. This theory, however, is eminently static. From a dynamic perspective, an alternative supplier may refrain from charging excessive input prices to its customers, so as to preserve their ability to compete effectively in the downstream market. This is particularly relevant where intertemporal linkages are important, due, e.g., to reputation, advertising, learning by doing, data accumulation, habit formation or word of mouth. The future profits of the supplier and of its customer are then not only closely linked, but also depend on the customer's current position in the downstream market. This, in turn, mitigates the supplier's incentive to exploit its market power vis-à-vis

[1] An alternative approach relies on *supplier opportunism* (Hart and Tirole, 1990; O'Brien and Shaffer, 1992; Mc Afee and Schwartz, 1994). More recently, Allain *et al.* (2016) have stressed instead the risk of *hold-up*.

its customers, as it would be shooting itself in the foot: degrading the competitive position of the downstream customers weakens their ability to compete effectively against the integrated firm in subsequent periods, and thus reduces the supplier's future profit. At the same time, introducing intertemporal linkages can also damage the ability to compete of the foreclosed firm. The reason is that intertemporal linkages magnify inefficiencies given a cost asymmetry. For instance, if a firm accumulates less data today because is less efficient, it will be even less able to compete in the future because its competitors will have learned more about consumers. Therefore, it is not clear ex-ante which is the effect that dominates.

The recent decision of the European Commission to prohibit the implemented acquisition of GRAIL by Illumina provides an example of a case in which foreclosure concerns and dynamics have been central. The Commission found that the above-mentioned merger would have affected competition and innovation and that Illumina would have had the ability and the incentives to foreclose downstream rivals.[2] In this industry, the ability of each firm to compete and innovate depends on past performance. However, the interplay between industry dynamics and foreclosure theories has never been investigated.

As a whole, this suggests that accounting for industry dynamics calls for revisiting well-established foreclosure theories. This paper aims at providing a first step in that direction, studying the ability and incentives to raise rivals' cost in a dynamic setting.

Specifically, I consider a two-period version of the successive duopoly model of OSS, in which I introduce an intertemporal linkage in the downstream market: the more a firm sells in the first period, the better its ability to compete in the second period. This can for instance be due to learning-by-doing or word of mouth, or through the collection of valuable data. In the absence of such intertemporal linkages, the classic logic of OSS would readily apply. If the integrated firm were to refuse to supply, the market power conferred to the independent supplier would induce it to charge a high input price, thereby impeding the downstream rival's ability to compete. However, this, in turn, gives the rivals an incentive to merge as well, thereby eliminating the benefit of foreclosure. To prevent such a counter-merger, the integrated firm's optimally offers to supply at an appropriately chosen price: low enough to discourage the independent firms from merging, but high enough to raise rival's costs (partial foreclosure). Against this background, I find that introducing an intertemporal linkage creates a cooperation incentive among the independent firms. This, in turn, tends to reduce the scope for foreclosure through three different channels. First, it acts as a disciplining device that limits the exercise

[2] See European Commission Press Release of 6 September 2022 at https://ec.europa.eu/competition/elojade/isef/case_details.cfm?proc_code=2_M_10188

of market power by the alternative supplier. Second, it limits integrated firm's ability to discourage a counter-merger. Third, even when the integrated firm can prevent its rivals from merging, it must offer a lower price, which further reduces the market power conferred to the independent supplier. Yet, intertemporal linkages also tend to exacerbate the consequences of any cost asymmetry. Therefore, it is sometimes the case that even though the wholesale prices faced by the unintegrated downstream firm is smaller than in the static case, the resulting foreclosure has more severe effects.

I find that intertemporal linkages can either increase or decrease the scope of foreclosure. It turns out that it is increased if there is no threat of a counter- merger or intertemporal linkages are small. When a counter-merger is possible, for intermediate levels of intertemporal linkages, foreclosure is reduced, and for high levels, there is no foreclosure at all. The extent to which the scope for foreclosure is affected by the magnitude of the intertemporal linkages is magnified by the degree of product substitution of the firms.

Related literature. To be completed.

2 Model

The setting is a two-period of the classic successive duopoly model of OSS: the upstream firms (U_A and U_B) produce an homogeneous good, which the downstream firms (D_1 and D_2) transform, on a one-to-one basis, into differentiated products; at each stage of the vertical chain, firms are symmetric and compete in linear prices. Let $w_{A,t}$ and $w_{B,t}$ denote the upstream firms' wholesale prices, and $p_{1,t}$ and $p_{2,t}$ denote the downstream firms' prices.[3]

All firms are supposed to have constant unit costs, which without loss of generality are normalized to zero. Downstream firms face a linear demand, characterized by a substitution parameter s: specifically, the demand for D_i's product in period t is given by $q_{i,t} = D(p_{i,t}, p_{j,t})$, where $j \neq i \in \{1, 2\}$, $t \in \{1, 2\}$ and

$$D(p_{1,t}, p_{2,t}) \equiv 1 - p_{1,t} + s(p_{2,t} - p_{1,t}).$$

To introduce, intertemporal linkages, suppose that each D_i collects valuable data from its first-period customers, so that selling a quantity q_i in period 1 generates an additional revenue λq_i per customer in the second period; hence, the more customers D_i attracts at $t = 1$, the more data it collects, and the more revenue it obtains from each customer at $t = 2$. It follows that

[3] Whether a supplier can offer different prices to D_1 and D_2 does not affect the analysis.

D_i's intertemporal profit is equal to $\Pi_i = \Pi(p_{i,1}, p_{i,2}, p_{j,1}, p_{j,2})$, where:

$$\Pi(p_{1,1}, p_{1,2}, p_{2,1}, p_{2,2}) \equiv (p_{1,1} - \tilde{w}_{1,1})D(p_{1,1}, p_{2,1}) + (p_{1,2} + \lambda D(p_{1,1}, p_{2,1}) - \tilde{w}_{1,2})D(p_{1,2}, p_{2,2}),$$

with $\tilde{w}_{i,t}$ denoting the input price faced by D_i in period t, which is equal to 0 if D_i is vertically integrated, and to the offer of the upstream firm that leads D_i with highest continuation profit.

Remark: intertemporal linkages. While the focus is here on data collection, a similar analysis would apply to learning-by-doing (e.g., if selling a quantity q in period 1 would reduce the cost in period 2, from some $c > 0$ to $c - \lambda q$) or to word-of-mouth fostering the demand (e.g., increasing the demand intercept by λq).

The timing and information structure are as follows. At the beginning of the first period, U_A and D_1 first decide whether to merge; if they do, they also set the wholesale prices $w_{A,1}$ and $w_{A,2}$ at which they are willing to supply to firm D_2 in each period, and U_B and D_2 then decide whether to merge. Finally, there is an upstream competition stage, in which independent suppliers (if any) simultaneously choose their long-term wholesale prices, followed by a downstream competition stage, in which D_1 and D_2 simultaneously set their own long-term prices. All decisions are publicly observable.

I assume away any frictions in merger decisions and their implementation; hence, firms merge whenever it is profitable to do so, that is, whenever the sum of their profits is higher under integration. Furthermore, I follow OSS and assume that, if U_A and D_1 merge, the integrated supplier's offer is firm and final; that is, U_A can commit itself not to enter again the competition to supply. I will look for the subgame perfect equilibria of the above game.

The outcome of the integration stages can generate three types of continuation subgames. Let Γ_S denote the subgame in which both downstream firms remain independent (with the subscript S standing for full *separation*), by Γ_I the subgame in which both are integrated (with I for full *integration*), and by $\Gamma_F(w_{A,1}, w_{A,2})$ the subgame in which only D_1 is integrated (with F for *foreclosure*). In the last subgame, $w_{A,1}$ and $w_{A,2}$ denote the input price offered by the integrated U_A. To characterize the subgame perfect equilibria, I proceed by backward induction and thus start with the price competition stages.

3 Price competition

3.1 Downstream prices

Consider the last stage of the game, then for any vector of input prices faced by downstream firms, $\tilde{w} = (\tilde{w}_{1,1}, \tilde{w}_{1,2}, \tilde{w}_{2,1}, \tilde{w}_{2,2})$, D_i's best-response is characterized by the following first-order conditions:[4]

$$\begin{aligned}
0 &= \partial_1 \Pi \left(p_{i,1}, p_{i,2}, p_{j,1}, p_{j,2}; \tilde{w}\right) \\
&= D(p_{1,1}, p_{2,1}) + (p_{1,1} - w_1) D_1(p_{1,1}, p_{2,1}) + \lambda D_1(p_{1,1}, p_{2,1}) D(p_{1,2}, p_{2,2}), \\
0 &= \partial_2 \Pi \left(p_{i,1}, p_{i,2}, p_{j,1}, p_{j,2}; \tilde{w}\right) \\
&= D(p_{1,2}, p_{2,2}) + (p_{1,2} + \lambda D(p_{1,1}, p_{2,1}) - w_2) D_1(p_{1,2}, p_{2,2}),
\end{aligned}$$

where $\partial_h g$ denotes the partial derivative of function g with respect to its h^{h} argument. To ensure that best-responses remain well-behaved, I will assume that the intertemporal linkages are not too large, namely:

$$\lambda < \bar{\lambda}(s) \equiv \frac{2 + 3s}{1 + 3s + 2s^2}. \tag{A1}$$

This makes sure that downstream firms' profits remain concave in their own prices, and that prices remain strategic complements.

Under full integration, both downstream firms are supplied at cost by their in-house suppliers. Likewise, under full separation, Bertrand-like competition among the suppliers leads them to supply both downstream firms at cost. When instead only D_1 is integrated, it is internally supplied at cost, whereas D_2 faces wholesale prices equal to the offer of whatever upstream firm that leads D_2 with highest continuation payoff. Thus, without loss of generality, we can restrict attention to situations in which D_1 faces $\tilde{w}_{1,t} = 0$ whereas D_2 faces some wholesale prices $\tilde{w}_{2,t}$ where $t \in \{1, 2\}$. It can be checked that this yields a unique downstream price equilibrium, described by the following lemma:

Lemma 1 (Downstream prices) *In the downstream competition stage, for any given wholesale prices faced by D_2, there is a unique price equilibrium, $\left(p^e_{1,1}(\tilde{w}), p^e_{1,2}(\tilde{w}), p^e_{2,1}(\tilde{w}), p^e_{2,2}(\tilde{w})\right)$, where:*

[4] The revenue λq_i acts as a subsidy, which induces D_i to price below cost, normalized here to zero; for the sake of exposition, I thus allow for negative prices, which should then be interpreted as negative "gross price-cost margins" – the total margin, taking into account the revenue λq_i, is always positive.

(i) $p^e_{1,t}$ *is increasing in* $\tilde{w}_{2,t}$ *if intertemporal linkages,* λ*, are not too large, and strictly decreasing in* $\tilde{w}_{2,t'}$ *if* $\lambda > 0$*, where* $t \neq t' \in \{1,2\}$*.*

(ii) $p^e_{2,t}$ *is weakly increasing in* $\tilde{w}_{B,t'}$*, where* $t, t' \in \{1,2\}$ *and strictly increasing whenever* $t = t'$ *or* $\lambda > 0$*.*

(iii) and such that $p^e_{1,t}(\tilde{w}) \leq p^e_{2,t}(\tilde{w})$ *as long as* $(2+3s)\tilde{w}_{2,1} + (1+s)(1+2s)\tilde{w}_{2,2}\lambda \geq 0$*, which is always the case in equilibrium as it will be shown in the next section.*

The proof of lemma 1, as well as of other lemmas and propositions, is to be found in the appendix.

The previous lemma states formally the continuation downstream price equilibrium and how it depends on the input prices faced by downstream firm D_2. Namely, $p^e_{1,t}$ are (generally) increasing in the rival's input price, and weakly decreasing in the non-contemporaneous ones, and no response to non-contemporaneous costs if there are not intertemporal linkages in the industry. In addition, $p^e_{2,t}$ is weakly increasing in the wholesale prices it faces, and the prices of D_2 are larger than that those of D_1 as long as the former faces "sufficiently" higher input costs than the latter.

For $\lambda = 0$, the equilibrium prices are the same as in the static game with no intertemporal linkages. It can moreover be checked that introducing intertemporal linkages induces firms to lower their price responses. The intuition is that the quest for data intensifies competition in order to boost their margins and to enhance their ability to compete in the second period.

3.2 Upstream prices

As already noted, under full separation and full integration, both downstream firms end up being supplied at cost. Let $\bar{\Pi}$ denote the joint profit generated by each downstream firm in these two subgames.

If instead only U_A and D_1 are integrated, U_B supplies D_2 at wholesale prices that maximize its intertemporal profit subject to this offer being more attractive than the offer by the integrated U_A, $w_{A,t}$. If we abstract from this constraint and we consider the unconstrained problem, U_B would charge:

$$(w^*_{B,1}, w^*_{B,2}) = \underset{w_1,w_2}{\operatorname{argmax}}\, w_1 D(p^e_{2,1}(w_1,w_2), p^e_{1,1}(w_1,w_2)) + w_2 D(p^e_{2,2}(w_1,w_2), p^e_{1,2}(w_1,w_2)).$$

In the absence of intertemporal linkages, this problem is the same as in the classic static model of OSS. By contrast, introducing intertemporal linkages induces U_B to moderate the exercise of its market power, so as to preserve D_2's ability to compete in the downstream market:

Proposition 1 (Unconstrained wholesale prices) *Suppose that the integrated firm refuses to supply the downstream rival. As the size of the intertemporal linkages (measured by λ) increases, the alternative supplier (U_B) reduces its input prices ($w^*_{B,1}, w^*_{B,2}$), but not to an extent sufficient to offset the resulting foreclosure (measured by the ratio of the intertemporal market share, ϕ^*) . All effects are exacerbated with the degree of substitutability (s). Formally:*

$$\frac{\partial w^*_{B,t}}{\partial \lambda} < 0, \quad \frac{\partial^2 w^*_{B,t}}{\partial \lambda \partial s} < 0,$$

define

$$\phi^*(\lambda, s) \equiv \frac{D\left(p_{2,1}, p_{1,1}\right) + D\left(p_{2,2}, p_{1,2}\right)}{D\left(p_{1,1}, p_{2,1}\right) + D\left(p_{1,2}, p_{2,2}\right)},$$

then,

$$\frac{\partial \phi^*}{\partial \lambda} < 0, \quad \frac{\partial^2 \phi^*}{\partial \lambda \partial s} < 0.$$

Proposition 1 provides the first insight of this paper: intertemporal linkages act as a disciplining device that limits the exercise of market power by the alternative supplier. Because of industry dynamics, it must now preserve its customer's ability to compete in the future; as a result, it charges a lower wholesale price. However, intertemporal linkages incorporate a second effect. Namely, their presence magnifies the consequences on competition coming from any cost asymmetry. Indeed, if for some reason a counter-merger were infeasible (e.g., due to large costs of integration or to merger control) and the integrated firm were to refuse to supply the downstream rival, then $w^*_{B,t}$ would be the input prices eventually charged to D_2.[5] In such a scenario, while it is the case that this cost difference generated by the integration shrinks due to intertemporal linkages (lower wholesale prices faced by D_2), interestingly, the second effect dominates and foreclosure is aggravated even though U_B's exercise of market power has been decreased.

Consider now the case were the integrated firm does not deny to supply the unintegrated rival but sets an offer to supply $w_A = (w_{A,1}, w_{A,2})$. Then, in the foreclosure subgame $\Gamma_F(w_{A,1}, w_{A,2})$, the unintegrated upstream firm maximizes its profits subject to its offer being more attractive to firm D_2 than the offer of the integrated firm. This is, firm U_B faces the following maximization problem (in vector notation).

[5]When a counter-merger is not feasible, it is indeed optimal for the integrated firm to deny access to the unintegrated downstream firm. The rival upstream firm would undercut any offer made by the integrated firm, and denial to supply is actually what maximizes the costs that the unintegrated downstream firm faces.

$$(\mathcal{P}_B) \quad w_B^R \in \underset{w_B}{\operatorname{argmax}}\, \Pi_B(w_B)$$
$$\text{s.t.} \quad \Pi_2(w_B) \geq \Pi_2(w_A)$$

If the continuation profits that D_2 would make by accepting the offer of the integrated firm are lower than those that could achieve by accepting the unconstrained offer of U_B, then the unintegrated firm is not constrained and would set prices $w^*_{B,t}$; otherwise, it will set the constrained wholesale prices $w_B \equiv \{w_{B,1}(w_{A,1}, w_{A,2}), w_{B,2}(w_{A,1}, w_{A,2})\}$. In the following sections, the paper will endogenize a counter-merger and will analyize the optimization problems of the different firms. Namely, the optimal supply offer of the integrated firm subject to the rivals remaining separated and taking into account that the best response of the unintegrated upstream firm is governed by $(\mathcal{P}_B)$. This problem can be difficult to solve with analytical solutions due their non-linear and multivariate nature. It is therefore convenient to introduce an important result in the following Lemma that will allow us to simplify the analysis and provide closed form solutions in the subsequent sections.

Lemma 2 (Stationary constrained wholesale prices) *Given a target* Π_2 *that the unintegrated upstream firm has to leave to firm* D_2*, the solution to problem* $(\mathcal{P}_B)$ *is stationary (i.e.* $w_{B,1} = w_{B,2}$*).*

Notice that w_A is never played in equilibrium, its only role is to influence the equilibrium w_B through affecting the target profit $\Pi_2(w_A)$ the unintegrated upstream firm, U_B, has to leave to its customer. There are infinite w_A that lead to the same target profit, and the integrated firm is indifferent among any w_A that would trigger the same response, therefore w_A cannot be unique. Yet, for any offer from the integrated firm that leads some target profit to P_2, B's response is stationary (i.e. $w_{B,1} = w_{B,2}$) and unique, and this response can be triggered by a stationary w_A. Therefore, the previous Lemma implies that, without loss of generality, we can restrict attention to stationary equilibria. Thus, I will focus on stationary prices for the remaining of the paper on vertical mergers.[6] With a slight abuse of notation, I will denote by w_A and w_B the stationary offers, which now are scalars and not vectors.

[6]It is easy to see now that we can further restrict attention to situations in which D_1 faces $\tilde{w}_{1,t} = 0$ whereas D_2 faces some stationary wholesale prices $\tilde{w}_2$. U_B will never undercut $w_{A,t}$ with a negative price, as this would trigger a loss. Thus, if $w_{A,t} \geq 0$, then $w_{B,t} \geq 0$ as well. Furthermore, if $w_{A,t} < 0$, then U_A would make a loss – and, if integrated, would moreover face a more aggressive D_2. Hence, without loss of generality we can restrict attention to the case where $\min\{w_{A,t}, w_{B,t}\} \geq 0$.

Before moving to the following section where the paper endogeneizes the mergers, let $\Pi_B^F(w_B)$, $\Pi_2^F(w_B)$ and $\Pi_{A-1}^F(w_B)$ respectively denote the resulting profits for the two independent firms and for the integrated one.

4 Merger decisions

4.1 Counter-merger

Suppose that U_A and D_1 decided to merge and offered to supply at given prices w_A, and consider the counter-merger decision. U_B and D_2 prefer to merge whenever their joint profit is lower in the foreclosure subgame than under integration, i.e., if $\Delta\left(w_B^R(w_A)\right) < 0$, where:

$$\Delta\left(w_B^R(w_A)\right) \equiv \sum_{i\in\{B,2\}} \Pi_i^F(w_B^R(w_A)) - \bar{\Pi}.$$

By construction, $\Delta(0) = 0$, as both downstream firms are then supplied at cost. Furthermore, as shown by OSS, in the absence of intertemporal linkages, $\Delta(w_B) > 0$ for w_B positive but small enough. This is because, starting from $w_B = 0$, a slight increase in the wholesale price creates only a second-order distortion in the independent firms' ability to coordinate their pricing decisions, but a first-order strategic benefit, as the integrated firm, anticipating a higher price from D_2, raises its own price. However, as w_B further increases, double marginalization becomes a more severe problem for the independent firms and, for w_B high enough, their joint profit becomes lower than under integration. Indeed, if the integrated firm were to refuse to supply (i.e., set $w_A = \infty$), thus leading U_B to charge the unconstrained wholesale price w_B^* a counter-merger is then always profitable, which makes the first merger inconsequential. The next proposition shows that the same patterns arise in a dynamic setting: to foreclose its downstream rival, the integrated firm must limit the market power conferred to U_B, and must do so to a sufficient extent, in order to prevent the rivals from merging as well. Moreover, this proposition introduces and important result on the inability to foreclose when intertemporal linkages are sufficiently high.

Proposition 2 (Foreclosure Domain) *There exists $w^F(\lambda)$ which is decreasing in λ, such that U_B and D_2 strictly prefer to merge if and only if $w_B > w^F(\lambda)$. Furthermore,*

(i) If the unintegrated downstream firm is not constrained, wholesale prices are such that a counter-merger is always optimal.

(ii) *(No Foreclosure) When intertemporal linkages are larger than $\frac{1}{1+s}$, there is no ability to foreclose at all.*

Therefore, the previous proposition shows that if the integrated firm does not constrain the market power of the separated upstream firm, it will be optimal for the unintegrated firms to merge. Since separation is a necessary condition for raising rival´s costs, it will be in the interest of firm $U_A - D_1$ to set an offer such that when observed by the unintegrated upstream firm, it offers to D_2 wholesale prices which are elements of the set of positive real numbers weakly smaller than $w^F(\lambda)$. This set is decreasing in λ, because when intertemporal linkages are present, the efficiency losses stemming from double marginalization become exacerbated due to the dynamic considerations and outweigh more quickly the strategic benefit of inducing the integrated firm to raise its own price. This, in turn, limits the set of input prices that can prevent a counter-merger, to the point that this set becomes a singleton with wholesale prices equal to marginal cost when intertemporal linkages become large enough. When intertemporal linkages are higher than $1/(1+s)$, even the smallest presence of a coordination problem would put the separated structure to such a competitive disadvantage that they would be better off by integrating right away. At this point, either if they integrate back or if they remain separated at this sole offer that keeps them unintegrated, competition downstream is symmetric and there is no ability to foreclose.

4.2 First merger

Once it has been shown that for intertemporal linkages high enough foreclosure is not possible, in the following I restrict attention to cases where the effect is not clear-cut (i.e. $\lambda < \frac{1}{1+s}$). If firms U_A and D_1 decide to merge, the integrated firm wants to maximize its profits with respect to the offer it can make to the unintegrated downstream firm, subject to this offer being successful at preventing the second merger from happening. It then needs to take into account that the unintegrated upstream firm will best respond to this offer and maximize accordingly. Thus, the maximization problem that the integrated firm faces is given by:

$$
\begin{aligned}
(\mathcal{P}_A) \quad \max_{w_A} \quad & \Pi^F_{A-1}(w^R_B(w_A)) \\
\text{s.t.} \quad & \sum_{i \in \{B,2\}} \Pi^F_i(w^R_B(w_A)) \geq \bar{\Pi}
\end{aligned}
$$

$$
\begin{aligned}
(\mathcal{P}_B) \quad & w^R_B \in \operatorname*{argmax}_{w_B} \Pi_B(w_B) \\
& \text{s.t.} \quad \Pi_2(w_B) \geq \Pi_2(w_A)
\end{aligned}
$$

The following Lemma summarizes the equilibrium of the first stages of the game.

Lemma 3 (Equilibrium of the game) *In the first stage of the game, U_A and D_1 merge and offer to supply the unintegrated downstream firm at the highest stationary offer such that the rivals remain separated ($w_A = w^F(\lambda)$). The unintegrated upstream firm observes this offer and matches it in order to supply D_2.*

The following Proposition shows the impact of intertemporal linkages on the ability and the consequences of foreclosure when both mergers are endogenized.

Proposition 3 (Foreclosure) *As intertemporal linkages become more important, (i) the ability to raise rival's costs is reduced, (ii) the scope for foreclosure is reduced for high enough λ or degree of substitutability:*

(i)
$$\frac{dw^F}{d\lambda} < 0$$

(ii) There exists $\lambda^*(s)$ such that if $\lambda > \lambda^*(s)$ then the relative intertemporal market share $(q_{2,1}+q_{2,2})/(q_{1,1}+q_{1,2})$ is an increasing function of λ. In addition, if downstream products are sufficiently close substitutes ($s > \sqrt{2}$), then $\lambda^*(s) = 0$.

As already noted, Proposition 1 shows that intertemporal linkages increase the scope for foreclosure when firms U_B and D_2 cannot merge. However, Proposition 3 goes further and shows that the scope for foreclosure is reduced when a counter-merger is feasible. This is because intertemporal linkages magnify the consequences of double marginalization, which, if possible, urges separated firms to merge back. Absent this possibility, the cooperation incentive of the unintegrated firms (lower wholesale prices) is not enough as to outweigh the fact that intertemporal linkages also exacerbate cost asymmetries. Yet, if a counter-merger is indeed possible, this is no longer always the case. The fact that the second effect dominates is directly reflected in the fact that firms would be better off by integrating for lower levels of double marginalization. Therefore, even though the integrated firm can keep them separated (most of the time) by limiting the market power to U_B, it becomes more difficult for the integrated firm to discourage such a counter-merger. Then, the previous two propositions have shown that for low levels of intertemporal linkages the scope for foreclosure is increased, for intermediate levels it is decreased, and for high levels there is no foreclosure at all. More specifically, the change of sign as intertemporal linkages increase is due to the fact that the decrease in the wholesale price that the integrated firm can impose on the separated firms becomes larger than by how much asymmetries are magnified.

5 Conglomerate mergers

To be completed.

6 Conclusion

The literature on vertical foreclosure has been eminently static. Yet, Firms operate in an ever-changing environment, and their actions have long-term consequences. This is particularly so in the digital economy, where industry dynamics are on a new scale. These features – together with the emergence of large platforms – have exacerbated competition concerns and triggered calls for strengthened policy intervention. In this paper I highlight the need to account also for industry dynamics when assessing ability and incentives to foreclose. In this context, the future profits of the supplier and of its customer are then not only closely linked, but also depend on the customer's current position in the downstream market. This, in turn, mitigates the supplier's incentive to exploit its market power vis-à-vis its customers.

The model in this paper delivers two predictions that show that the scope for vertical foreclosure depends crucially on the extend of intertemporal linkages in the industry, and also on whether a counter-merger is possible. More specifically, if a counter-merger is not feasible (e.g. due to large costs of integration or to merger control), I show that intertemporal linkages only worsen the situation. Even though they limit the unintegrated firm's exercise of market power, the inefficiencies are magnified in the dynamic context and foreclosure is exacerbated. It is when the integrated firm fears a counter-merger, that intertemporal-linkages provide a form of a disciplining device and then, for large enough intertemporal linkages, the scope of foreclosure is reduced. This is so because intertemporal linkages magnify the consequences of any inefficiencies resulting from double marginalization, which urges separated firms to eliminate those inefficiencies by integrating. This makes harder the task of discouraging the second merger, which leads the unintegrated downstream firm facing lower wholesale prices than absent dynamics.

A Proof of Lemma 1

The downstream prices as a function of the wholesale prices $(\tilde{w}_{2,1}, \tilde{w}_{2,2})$ faced by D_2, are given by:

$$
\begin{aligned}
p_{1,1}^{e}(\tilde{w}_{2,1}, \tilde{w}_{2,2}) &\equiv \frac{1}{4}\left(4-\psi_1(-4+\tilde{w}_{2,1}+\tilde{w}_{2,2})+\psi_2(\tilde{w}_{2,1}-\tilde{w}_{2,2})+\psi_3(\tilde{w}_{2,1}+\tilde{w}_{2,2})-\psi_4(\tilde{w}_{2,1}-\tilde{w}_{2,2})\right),\\
p_{2,1}^{e}(\tilde{w}_{2,1}, \tilde{w}_{2,2}) &\equiv \frac{1}{4}\left(4-\psi_1(-4+\tilde{w}_{2,1}+\tilde{w}_{2,2})+\psi_2(\tilde{w}_{2,1}-\tilde{w}_{2,2})-\psi_3(\tilde{w}_{2,1}+\tilde{w}_{2,2})+\psi_4(\tilde{w}_{2,1}-\tilde{w}_{2,2})\right),\\
p_{1,2}^{e}(\tilde{w}_{2,1}, \tilde{w}_{2,2}) &\equiv \frac{1}{4}\left(4-\psi_1(-4+\tilde{w}_{2,1}+\tilde{w}_{2,2})-\psi_2(\tilde{w}_{2,1}-\tilde{w}_{2,2})+\psi_3(\tilde{w}_{2,1}+\tilde{w}_{2,2})+\psi_4(\tilde{w}_{2,1}-\tilde{w}_{2,2})\right),\\
p_{2,2}^{e}(\tilde{w}_{2,1}, \tilde{w}_{2,2}) &\equiv \frac{1}{4}\left(4-\psi_1(-4+\tilde{w}_{2,1}+\tilde{w}_{2,2})-\psi_2(\tilde{w}_{2,1}-\tilde{w}_{2,2})-\psi_3(\tilde{w}_{2,1}+\tilde{w}_{2,2})-\psi_4(\tilde{w}_{2,1}-\tilde{w}_{2,2})\right),
\end{aligned}
$$

where

$$
\psi_1 \equiv \frac{(1+s)}{-2+s(\lambda-1)+\lambda}, \qquad \psi_2 \equiv \frac{(1+s)}{2+s+\lambda+s\lambda},
$$

$$
\psi_3 \equiv \frac{(1+s)}{-2+\lambda+s(-3+(3+2s)\lambda)}, \qquad \psi_4 \equiv \frac{(1+s)}{s((2s+3)\lambda+3)+\lambda+2},
$$

Then, given the assumption on the upper-bound of λ, one can see that

$$
\begin{aligned}
\frac{\partial p_{1,t}^{e}(\tilde{w}_{2,1}, \tilde{w}_{2,2})}{\partial \tilde{w}_{2,t}} &= -\psi_1+\psi_2+\psi_3-\psi_4 \quad > \quad 0\\
&\iff (2+s)(2+3s) > (1+s)^2(3+2s(3+s))\lambda^2 \quad > \quad 0\\
&\iff \lambda < \sqrt{\frac{4+8s+3s^2}{3+12s+17s^2+10s^3+2s^4}},\\
\frac{\partial p_{1,t}^{e}(\tilde{w}_{2,1}, \tilde{w}_{2,2})}{\partial \tilde{w}_{2,t'}} &= -\psi_1-\psi_2+\psi_3+\psi_4 \quad \leq \quad 0 \quad \forall \quad \lambda \in \left[0, \bar{\lambda}(s)\right)\\
&\quad \text{(with strict inequality whenever } \lambda > 0)\\
\frac{\partial p_{2,t}^{e}(\tilde{w}_{2,1}, \tilde{w}_{2,2})}{\partial \tilde{w}_{2,t}} &= -\psi_1+\psi_2-\psi_3+\psi_4 \quad > \quad 0 \quad \forall \quad \lambda \in \left[0, \bar{\lambda}(s)\right)\\
\frac{\partial p_{2,t}^{e}(\tilde{w}_{2,1}, \tilde{w}_{2,2})}{\partial \tilde{w}_{2,t'}} &= -\psi_1-\psi_2-\psi_3-\psi_4 \quad \geq \quad 0 \quad \forall \quad \lambda \in \left[0, \bar{\lambda}(s)\right)\\
&\quad \text{(with strict inequality whenever } \lambda > 0)
\end{aligned}
$$

The last part of Lemma 1 is easy to verify, and it will be shown shortly that it is always true as wholesale prices are non-negative in equilibrium.

$$\begin{aligned} p^e_{2,1}(\tilde{w}_{2,1},\tilde{w}_{2,2}) - p^e_{1,1}(\tilde{w}_{2,1},\tilde{w}_{2,2}) &= \frac{1}{2}\left(-\psi_3(\tilde{w}_{2,1}+\tilde{w}_{2,2}) + \psi_4(\tilde{w}_{2,1}-\tilde{w}_{2,2})\right) \geq 0 \\ &\iff (2+3s)\tilde{w}_{2,1} + (1+s)(1+2s)\lambda\tilde{w}_{2,2} \geq 0 \\ p^e_{2,2}(\tilde{w}_{2,1},\tilde{w}_{2,2}) - p^e_{1,2}(\tilde{w}_{2,1},\tilde{w}_{2,2}) &= \frac{1}{2}\left(-\psi_3(\tilde{w}_{2,1}+\tilde{w}_{2,2}) - \psi_4(\tilde{w}_{2,1}-\tilde{w}_{2,2})\right) \geq 0 \\ &\iff (2+3s)\tilde{w}_{2,2} + (1+s)(1+2s)\lambda\tilde{w}_{2,1} \geq 0 \end{aligned}$$

B Proof of Proposition 1

The unconstrained wholesale prices that the unintegrated downstream firm will face, are given by:

$$w^*_{B,t} \equiv \frac{1}{2}\left(1 + \frac{s(1+s)}{-2+\lambda+s(-4+3\lambda+s(-1+2\lambda)}\right)$$

Then, it is easy to verify than the derivative with respect to λ (the intertemporal linkages), and the second-order cross partial derivative are negatives given our assumption on λ.

$$\begin{aligned} \partial_1 w^*_{B,t}(\lambda,s) &= -\frac{s(1+s)^2(1+2s)}{2\left(-2+\lambda+s^2(-1+2)+s(-4+3\lambda)\right)^2} < 0 \quad \forall \quad \lambda \in \left[0,\bar{\lambda}(s)\right) \\ \frac{\partial^2 w^*_{B,t}(\lambda,s)}{\partial\lambda\partial s} &= -\frac{(1+s)\left(-2+\lambda+s^3(-11+2\lambda)+2s(-5+2\lambda)+s^2(-17+5\lambda)\right)}{2\left(-2+\lambda+s^2(-1+2)+s(-4+3\lambda)\right)^3} < 0 \\ &\iff \lambda < \frac{2+s(10+s\,(17+11s))}{(1+s)^2(1+2s)} > \bar{\lambda}(s) \end{aligned}$$

The extend of foreclosure is captured with the ratio of the sum of intertemporal demands as follows:

$$\phi^*(\lambda,s) \equiv \frac{D(p_{2,1},p_{1,1}) + D(p_{2,2},p_{1,2})}{D(p_{1,1},p_{2,1}) + D(p_{1,2},p_{2,2})} = \frac{-2+\lambda+s^2(-1+2\lambda)+s(-4+3\lambda)}{2(-2+\lambda)+s^2(-3+4\lambda)+s(-9+6\lambda)}$$

Similarly, the first-order derivative is negative for all value of λ, and one can see that the second-order cross partial derivative is negative for lower than the one provided, but this is always the case under the assumptions that ensure strategic complementarity of prices and well-behaved maximization problems.

$$
\begin{aligned}
\partial_1\phi^*(\lambda,s) &= -\frac{s(1+s)^2(1+2s)}{\left(2(-2+\lambda)+s(-9+6\lambda+s(-3+4\lambda))\right)^2} < 0 \quad \forall \quad \lambda\in\left[0,\bar{\lambda}(s)\right)\\
\frac{\partial^2\phi^*(\lambda,s)}{\partial\lambda\partial s} &= -\frac{(1+s)\left(2(-2+\lambda)+s^3(-21+4\lambda)+s(-19+8\lambda)+2s^2(-16+5\lambda)\right)}{\left(2(-2+\lambda)+s(-9+6\lambda)+s^2(-3+4\lambda)\right)^3} < 0\\
&\iff \lambda < \frac{4+s(19+s(32+21s))}{2(1+s)^2(1+2s)}
\end{aligned}
$$

C Proof of Lemma 2

The maximization problem of the unintegrated upstream firm is given by:

$$
\begin{aligned}
(\mathcal{P}_B) \qquad w_B^R \in \underset{w_B}{\operatorname{argmax}} \qquad & \Pi_B(w_B)\\
s.t. \qquad & \Pi_2(w_B)\geq \Pi_2(w_A)
\end{aligned}
$$

where w_A is the offer to supply that the unintegrated downstream firm receives from the integrated firm. Notice that w_A is never played in equilibrium, its only role is to influence the equilibrium w_B through affecting the target $\Pi_2(w_A)$ firm B has to leave to firm 2. There are infinite w_A that lead to the same target profit of 2, therefore w_A cannot be unique. What we want to see next is that for any offer from the integrated firm that leads to some target profit P_2, the B's response is stationary (i.e. $w_{B,1}=w_{B,2}$), and this response can be triggered by a stationary w_A. Again, the integrated firm is indifferent for any w_A that would trigger the same response. And therefore I will focus on stationary prices for the remaining of the section.

If we look again at $(\mathcal{P}_B)$, we can see that both profit functions are quadratic symmetric polynomials. The profits of the upstream firm are concave in wholesale prices and the profits of the downstream firm are convex.

Rewrite the profit functions:

$$\begin{aligned}\Pi_B(w_B) &= \alpha_B(w_{B,1}+w_{B,2})-\beta_B(w_{B,1}+w_{B,2})^2-\gamma_B(w_{B,1}-w_{B,2})^2,\\ \Pi_2(w_B) &= C-\alpha_2(w_{B,1}+w_{B,2})+\beta_2(w_{B,1}+w_{B,2})^2+\gamma_2(w_{B,1}-w_{B,2})^2,\end{aligned}$$

where

$$\alpha_B \equiv -\frac{1+s}{-2+s(-1+\lambda)+\lambda},$$

$$\beta_B \equiv -\frac{1}{4}(1+s)\left(\frac{1}{-2+s(-1+\lambda)+\lambda}+\frac{1+2s}{-2+\lambda+s(-3+(3+2s)\lambda)}\right),$$

$$\gamma_B \equiv -\frac{1}{4}(1+s)\left(-\frac{1}{2+s+\lambda+s\lambda}+\frac{-1-2s}{2+\lambda+s(3+(3+2s)\lambda)}\right),$$

$$\alpha_2 \equiv -\frac{(1+s)((-2+\lambda)^2+s^3\lambda(-1+2\lambda)+4s(2-3\lambda+\lambda^2)+s^2(2-9\lambda+5\lambda^2))}{(-2+s(-1+\lambda)+\lambda)^2(-2+3s(-1+\lambda)+\lambda+2s^2\lambda)},$$

$$\beta_2 \equiv -\frac{(1+s)(-2+\lambda+s\lambda)(-2+\lambda+s^2(-1+2\lambda)+s(-4+3\lambda))^2}{4(-2+s(-1+\lambda)+\lambda)^2(-2+3s(-1+\lambda)+\lambda+2s^2\lambda)^2},$$

$$\gamma_2 \equiv \frac{(1+s)(2+\lambda+s\lambda)(2+\lambda+s^2(1+2\lambda)+s(4+3\lambda))^2}{4(2+s+\lambda+s\lambda)^2(2+\lambda+2s^2\lambda+3s(1+\lambda))^2},$$

$$C \equiv -\frac{(1+s)(-2+\lambda+s\lambda)}{(-2+s(-1+\lambda)+\lambda)^2},$$

Define:

$$\sigma = w_{B,1}+w_{B,2}, \quad \delta = w_{B,1}-w_{B,2}$$

And reconsider $(\mathcal{P}_B)$ where the unintegrated upstream firm has to match a given Π_2. Then, the profits are given by the following expressions where all coefficients previously defined are positive.

$$\begin{aligned}\Pi_B(\sigma,\delta) &= \alpha_B\sigma-\beta_B\sigma^2-\gamma_B\delta^2,\\ \Pi_2(\sigma,\delta) &= C-\alpha_2\sigma+\beta_2\sigma^2+\gamma_2\delta^2,\end{aligned}$$

The lagrangian is given by:

$$\mathcal{L}(\sigma, \delta, \mu) \;=\; (\alpha_B - \mu\alpha_2)\sigma - (\beta_B - \mu\beta_2)\sigma^2 - (\gamma_B - \mu\gamma_2)\delta^2$$

For this problem to be concave on sigma, the term accompanying sigma square has to be negative. This implies that $\mu < \frac{\beta_B}{\beta_2}$, otherwise the solution would be sigma equal infinity. Given this, one can check that $(\gamma_B - \mu\gamma_2) > 0$ always as $\frac{\beta_B}{\beta_2} < \frac{\gamma_B}{\gamma_2}$, which means that the optimal solution is always delta equal to zero (i.e. the solution is always stationary).

D Proof of Proposition 2

Consider again the profit expressions used in the previous proof.

$$\begin{aligned}
\Pi_B(w_B) &= \alpha_B(w_{B,1} + w_{B,2}) - \beta_B(w_{B,1} + w_{B,2})^2 - \gamma_B(w_{B,1} - w_{B,2})^2, \\
\Pi_2(w_B) &= C - \alpha_2(w_{B,1} + w_{B,2}) + \beta_2(w_{B,1} + w_{B,2})^2 + \gamma_2(w_{B,1} - w_{B,2})^2,
\end{aligned}$$

Now, We can rewrite the function that will determine whether firms counter-merge taking into account the result introduced in Lemma 2. Recall that the intercept of the sum of profits under separation equals $\bar{\Pi}$.

$$\begin{aligned}
\Delta(w_B) &\equiv \sum_{i\in\{B,2\}} \Pi_i^F(w_B) - \bar{\Pi} \\
&= 2(\alpha_B - \alpha_2)w_B - 4(\beta_B - \beta_2)w_B^2 \\
&= w_B\left(2(\alpha_B - \alpha_2) - 4(\beta_B - \beta_2)w_B\right)
\end{aligned}$$

Observe that:

$$\left.\frac{d\Delta(w_B)}{dw_B}\right|_{w_B=0} = 2(\alpha_B - \alpha_2) > 0 \iff \lambda < \frac{1}{1+s}$$

Which shows that when intertemporal linkages are high enough, the sum of profits under separation is smaller than the sum of profits under integration for any infinitesimally small increase in the wholesale price. This means that for this range of intertemporal linkages, there is no ability to successfully foreclose the rival since the only offer that would prevent the counter-merger (a necessary condition for foreclosure) would be cost-based pricing. which in turn would still be inconsequential.

Now, if we restric attention to cases where there is still scope for foreclosure, i.e. $\lambda < \frac{1}{1+s}$, one can see that the function is always concave.

$$\frac{d^2\Delta(w_B)}{dw_B^2} = -4(\beta_B - \beta_2) < 0 \quad \forall\lambda < \frac{1}{1+s}$$

It is a second order polynomial with an obvious root at wholesale price equal marginal cost, as already argued in the main text. The second root is given by the following expression, which in turn is easy to check that its derivative with respect to λ is always negative in the relevant region.

$$w^F(\lambda) \equiv \frac{2(\alpha_B - \alpha_2)}{4(\beta_B - \beta_2)} > 0 \quad \forall\lambda < \frac{1}{1+s}$$

E Proof of Proposition 3

E.1 Part i

$$\begin{aligned}
\sum_{i\in\{B,2\}} \Pi_i^F(\bar{w}_B) &= \frac{1}{16}\Big(\kappa_1\left(-4+\bar{w}_{B,1}+\bar{w}_{B,2}\right)^2+\kappa_2\left(\bar{w}_{B,1}-\bar{w}_{B,2}\right)^2+\kappa_3\left(\bar{w}_{B,1}-\bar{w}_{B,2}\right)^2+\kappa_4\left(\bar{w}_{B,1}+\bar{w}_{B,2}\right) \\
&\quad +\kappa_5\left(\bar{w}_{B,1}+\bar{w}_{B,2}\right)\left(4+(2+3s)\left(\bar{w}_{B,1}+\bar{w}_{B,2}\right)\right)+\kappa_6\left(\bar{w}_{B,1}-\bar{w}_{B,2}\right)^2+\kappa_7\left(\bar{w}_{B,1}-\bar{w}_{B,2}\right)^2 \\
&\quad +\kappa_8\left(-4+\bar{w}_{B,1}+\bar{w}_{B,2}\right)\left(4\left(1+s\right)+\left(2+s\right)\left(\bar{w}_{B,1}+\bar{w}_{B,2}\right)\right)\Big)
\end{aligned}$$

$$\bar{\Pi} = \frac{(1+s)\left(2-(1+s)\lambda\right)}{\left(-2+s\left(\lambda-1\right)+\lambda\right)^2}$$

$$\Pi_{A-1}^F(\bar{w}_B) = \chi_1+\chi_2\left(\bar{w}_{B,1}+\bar{w}_{B,2}\right)-\chi_3\bar{w}_{B,1}\bar{w}_{B,2}+\chi_4\left(\bar{w}_{B,1}+\bar{w}_{B,2}\right)^2$$

where

$$\kappa_1 \equiv -\frac{s\left(1+s\right)}{\left(-2+s\left(\lambda-1\right)+\lambda\right)^2}, \qquad \kappa_2 \equiv -\frac{s\left(1+s\right)}{\left(2+s+\lambda+s\lambda\right)^2},$$

$$\kappa_3 \equiv -\frac{2+s}{2+s+\lambda+s\lambda}, \qquad \kappa_4 \equiv \frac{s\left(1+s\right)\left(1+2s\right)}{\left(-2+\lambda+s\left(-3+\left(3+2s\right)\lambda\right)\right)^2},$$

$$\kappa_5 \equiv -\frac{1+2s}{-2+\lambda+s\left(-3+\left(3+2s\right)\lambda\right)}, \qquad \kappa_6 \equiv \frac{s\left(1+s\right)\left(1+2s\right)}{\left(2+\lambda+s\left(3+\left(3+2s\right)\lambda\right)\right)^2},$$

$$\kappa_7 \equiv -\frac{\left(1+2s\right)\left(2+3s\right)}{2+\lambda+s\left(3+\left(3+2s\right)\lambda\right)}, \qquad \kappa_8 \equiv \frac{1}{-2+s\left(\lambda-1\right)+\lambda},$$

and

$$\chi_1 \equiv -\frac{(1+s)(-2+\lambda+s\lambda)}{(-2+s(-1+\lambda)+\lambda)^2},$$

$$\chi_2 \equiv \frac{s(1+s)^2(-2+\lambda+s\lambda)}{(-2+s(-1+\lambda)+\lambda)^2(-2+\lambda+s(-3+(3+2s)\lambda))},$$

$$\chi_3 \equiv \frac{s^2(1+s)^3(2+\lambda+s\lambda)}{(2+s+\lambda+s\lambda)^2(2+\lambda+s(3+(3+2s)\lambda))^2},$$

$$\chi_4 \equiv \frac{\left(4+8s+3s^2\right)^2-2(1+s)^2(-4+s(-16+d(-17+(-2+s)s)))\lambda^2-(1+s)^4(1+2s)^2(3+2s(3+s))\lambda^4}{\chi_3^{-1}(-2+s(-1+\lambda)+\lambda)^2(2+\lambda+s\lambda)(-2+\lambda+s(-3+(3+2s)\lambda))^2}.$$

Then, we have that both profit functions are symmetric polynomials in the wholesale prices.

Write problem, write KKT conditions, try symmetric guess.

The integrated firm wants to maximize its profits with respect to the offer it can make to the unintegrated downstream firm, subject to this offer being successful at preventing the second merger from happening. It then needs to take into account that the unintegrated upstream firm will best respond to this offer and maximize accordingly. Thus, the maximization problem that the integrated firm faces is given by:

$$(\mathcal{P}_1) \qquad \max_{w_A} \qquad \Pi^F_{A-1}(w^R_B(w_A))$$

$$s.t. \qquad \sum_{i\in\{B,2\}} \Pi^F_i(w^R_B(w_A)) - \bar{\Pi} \geq 0$$

where

$$(\mathcal{P}_2) \qquad w^R_B \in \underset{w_B}{\operatorname{argmax}} \qquad \Pi_B(w_B)$$

$$s.t. \qquad \Pi_2(w_B) \geq \Pi_2(w)$$

Consider also the First Best (FB) problem of the integrated firm, where it can dictate at which price U_B will supply to D_2. Then, assuming that the best response of firm B will be to offer the same wholesale prices and then be the supplier of the unintegrated downstream firm.

Along this preliminary assumption, I will discard from the analysis negative wholesale prices as it would never be in the interest of the unintegrated upstream firm to undercut any negative offer. The FB problem is then:

$$(\mathcal{P}_3) \qquad \max_{w} \qquad \Pi^F_{A-1}(w)$$

$$s.t. \qquad \sum_{i\in\{B,2\}} \Pi^F_i(w) - \bar{\Pi} \quad \geq \quad 0$$

Then, $w^* = \{w_1^*, w_2^*\}$ solves the problem $(\mathcal{P}_3)$ if and only if

1. $\sum_{i\in\{B,2\}} \Pi^F_i(w_1^*, w_2^*) - \bar{\Pi} \geq 0$

2. $\exists \quad \mu \geq 0$

3. $\mu \left(\sum_{i\in\{B,2\}} \Pi^F_i(w_1^*, w_2^*) - \bar{\Pi}\right) = 0$

4. $\nabla\Pi^F_{A-1}(w_1^*, w_2^*) + \mu \left(\sum_{i\in\{B,2\}} \nabla\Pi^F_i(w_1^*, w_2^*) - \bar{\Pi}\right) = 0$

Evaluating the last two conditions at a symmetric point $w_1^* = w_2^* = w^*$, one get three potential solutions $(w^*_{C1}, w^*_{C2}, w^*_{C3})$:

$$w^*_{C1} = -\frac{2s^2\left(-1+\lambda+s\lambda\right)\left(-2+\lambda+s\left(-3+(3+2s)\lambda\right)\right)}{(1+s)\left((-2+\lambda)^2+s^2\lambda\left(-3+2\lambda\right)+s\left(-2+\lambda\right)\left(-2+3\lambda\right)\right)\left(-2+\lambda+s\left(-4+3\lambda+s\left(-1+2\lambda\right)\right)\right)},$$

$$w^*_{C2} = \frac{-2+3s(-1+\lambda)+\lambda+2s^2\lambda}{s(1+s)},$$

$$w^*_{C3} = 0.$$

Where the second candidate equilibrium is discarded as it consists of a negative wholesale price. Then, the solution of the optimization problem that satisfy all the four conditions is given by:

$$w^{FB}_{A1} = \begin{cases} \{w^*_{C1}, w^*_{C1}\} & \text{if } \lambda \leq \frac{1}{1+s} \\ \{w^*_{C3}, w^*_{C3}\} & \text{otherwise.} \end{cases}$$

Therefore, the solution of the First Best problem is characterized by a symmetric offer. Notice that this offer is positive for $\lambda < 1/(1+s)$ and zero when intertemporal linkages are too large.

In order to show that this solution is also the solution of the problem $(\mathcal{P}_1)$, next I show that it is implementable in the constrained problem. In these lines, we want to show that the best response of the unintegrated upstream firm when it observes a symmetric offer by the integrated firm (more specifically, the symmetric offer than maximizes the FB problem), is to match this offer. Therefore, if we consider again $(\mathcal{P}_2)$ for the specific case that w is symmetric, the system of equations characterized by the last two corresponding conditions for optimality is:

$$\begin{cases} \mu\left(\Pi_2(w_B(w)) - \Pi_2(w)\right) = 0 \\ \nabla\Pi^F_{A-1}(w_1^*, w_2^*) + \mu\left(\sum_{i\in\{B,2\}} \nabla\Pi_i^F(w_1^*, w_2^*) - \bar{\Pi}\right) = 0 \end{cases}$$

Which has only one solution that satisfies the other conditions and that leads to positive demands in the continuation game. This solution consists in matching the symmetric offer made by the integrated firm.

E.2 Part ii

If we further investigate w^*_{C1}, one can see the following:

$$w^*_{C1} = \begin{cases} \frac{s^2(2+3s)}{2(1+s)^2(2+s(4+s))} & \text{if } \lambda = 0 \\ 0 & \text{if } \lambda = \frac{1}{1+s} \end{cases}$$

$$\begin{aligned} \frac{\partial w^*_{C1}}{\partial\lambda} = & \frac{s^2\left(-2\left(4+9s+6s^2\right)+(1+s)\left(4+s\left(11+9s\right)\right)\lambda\right)}{(1+s)\left((-2+\lambda)^2+s^2\lambda\left(-3+2\lambda\right)+s\left(-2+\lambda\right)\left(-2+3\lambda\right)\right)^2} \\ & + \frac{(1+3s)\,s}{(-2+\lambda)^2+s^2\lambda\left(-3+2\lambda\right)+s\left(-2+\lambda\right)\left(-2+3\lambda\right)} \\ & - \frac{(1+s)^2\left(1+2s\right)s}{\left(-2+\lambda+s\left(-4+3\lambda+s\left(-1+2\lambda\right)\right)\right)^2} \end{aligned}$$

Where the derivative of the wholesale price with respect to the intertemporal linkages is negative in the relevant domain. Then, we have that the optimal wholesale price for $\lambda \in \left[0, \frac{1}{1+s}\right]$

is a continuous quadratic function in this domain, positive for $\lambda = 0$, strictly decreasing, and equal to zero for $\lambda = \frac{1}{1+s}$.

Intuitively, given that the profits of the integrated firm are increasing and convex in the wholesale price that D_2 faces, it is clear than given the behavior of the optimal wholesale price as intertemporal linkages increase, the profits of the integrated firm are going to decrease. Yet, to see it formally, first notice that the profits of the integrated firm evaluated at a symmetric point can be written as:

$$\Pi^F_{A-1}(w,w) = -\frac{(1+s)\left(-2+\lambda+s\lambda\right)\left(2-\lambda+s\left(3+w+sw-3\lambda-2s\lambda\right)\right)^2}{\left(-2+s(-1+\lambda)+\lambda\right)^2\left(-2+\lambda+s\left(-3+(3+2s)\lambda\right)\right)^2}$$

E.3 Part iii

Next, I investigate the resulting ratio of intertemporal market shares. It is straightforward to see that when the wholesale prices that maximize the problem of the integrated firm are zero (i.e for $\lambda > 1/(1+s)$, then the ratio of the intertemporal market shares is one as both downstream firms are symmetric. Therefore, the subsequent analysis focuses on the case where the wholesale price that D_2 faces is positive and decreasing in the extend of intertemporal linkages. In that case, the

$$\phi^*(\lambda,s) \equiv \frac{D\left(p_{2,1},p_{1,1}\right)+D\left(p_{2,2},p_{1,2}\right)}{D\left(p_{1,1},p_{2,1}\right)+D\left(p_{1,2},p_{2,2}\right)} = \frac{1}{1+s}\frac{\Phi_1}{\Phi_2}$$

where

$$\Phi_1 \equiv (-2+\lambda+s\lambda)(-2+\lambda+s(-4+3\lambda+s(-1+2\lambda)))^2,$$
$$\Phi_2 \equiv (-2+\lambda)^3+6s(-2+\lambda)^2(-1+\lambda)+6s^3(-1+\lambda)(1+2(-2+\lambda)\lambda)+s^4\lambda(5+4(-2+\lambda)\lambda)$$
$$+\,s^2(-2+\lambda)(10+13(-2+\lambda)\lambda).$$

Given our assumption on the upper-bound of λ, the ratio of intertemporal demands is positive if the denominator is negative. Due to the complexity to check, and leveraging from the fact that the object is a ratio of market shares, the previous condition is taken as granted.

Furthermore,

$$\partial_1\phi^*(\lambda,s) = -\frac{2s^2\left(1+2s\right)\left(-2+\lambda+s\left(-4+3\lambda+s\left(-1+2\lambda\right)\right)\right)}{1+s}\frac{\Phi_3}{\Phi_2^2}$$

where

$$\Phi_3 \equiv s\left(-2+s^2\right)+(1+s)\,\lambda\,\underbrace{\left[4+s\,(13+6s)+(1+s)\,\lambda\,\underbrace{\left[(1+s)\,(1+2s)\,\lambda-(4+s\,(10+3s))\right]}_{(1)}\right]}_{(2)}$$

Notice that:

$$-2+\lambda+s\,(-4+3\lambda+s\,(-1+2\lambda)) \quad < \quad 0 \quad \forall \lambda \in \left[0, \frac{1}{1+d}\right]$$

Then, given our on Φ_2, we have that:

$$\partial_1 \phi^*(\lambda, s) \quad > \quad 0 \iff \Phi_3 > 0$$

Notice that the term (1) is negative as long as $\lambda < \frac{4+s(10+3s)}{(1+s)(1+2s)}$, which is always true given the domain of interest.

E.3.1 ***Case 1:*** $s^2 \geq 2$

I shall investigate the sign of (2), which is positive iff:

$$\lambda \quad < \quad \frac{4+10d+3d^2}{2(1+d)(1+2d)} - \frac{1}{2}\sqrt{\frac{-4d-4d^2+12d^3+9d^4}{(1+d)^2(1+2d)^2}}$$

$$\lambda \quad > \quad \frac{4+10d+3d^2}{2(1+d)(1+2d)} + \frac{1}{2}\sqrt{\frac{-4d-4d^2+12d^3+9d^4}{(1+d)^2(1+2d)^2}}$$

Given our condition on λ, only the first one is relevant. One can check that the radicand is positive since $s \geq \sqrt{2}$.

Now, we need to check whether the condition on λ is more restrictive or not than our upperbound that we already consider. So if the following is true, for all lambdas in our domain, the sign of Φ_3 is clearly positive.

$$\frac{1}{1+d} \quad < \quad \frac{4+10d+3d^2}{2(1+d)(1+2d)} - \frac{1}{2}\sqrt{\frac{-4d-4d^2+12d^3+9d^4}{(1+d)^2(1+2d)^2}}$$

Whcih is indeed true. So the condition is true for all the lambdas that I consider. Therefore, when $s\sqrt{2}$, the extend of foreclosure is decreasing in the size of intertemporal linkages.

E.3.2 ***Case 2:*** $s^2 < 2$

Now, we should show the following:

$$\{\exists \lambda^*(s) \in \mathbb{R} : \partial_1 \phi^*(\lambda, s) > 0 \quad \iff \quad \lambda > \lambda^*(s)\}$$

As a preliminary observation, we can evaluate Φ_3 at the extremes values of λ.

$$\begin{aligned} \Phi_3 (\lambda = 0) &= s\left(-2 + s^2\right) < 0 \\ \Phi_3 \left(\lambda = \frac{1}{1+s}\right) &= (1+s)^3 > 0 \end{aligned}$$

Then, since Φ_3 is a continuous function on a closed interval $\lambda \in \left[0, \frac{1}{1+s}\right]$, it is negative fot $\lambda = 0$ and positive for $\lambda = \frac{1}{1+s}$, by the Bolzano's theorem, then the function has a root in the interval. Therefore, since Φ_3 is positive for lambdas higher than this threshold, the derivative of ϕ is positive with respect to lambda for lambdas in such domain.