# Politicians Competition in Persuading Voters

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**Abstract**

Politicians must navigate the diverse preferences of the electorate. Some voters might be concerned about environmental issues whereas others wonder about the impacts of immigrants on their lives. Using a symmetric information voting model, we explore how politicians design policy experiments to sway voters, who have limited attention. The key restriction that we impose is on the focus of these experiments that they can be informative about at most one issue of interest. We find that even with competition, the least informative outcome persists, especially as politicians align in *persuasive advantage*. This sheds light on the incomplete information disclosure in politics. Furthermore, the paper demonstrates that the competition in persuasion leads to a specific signal structure for each politician in which they only tell lies when the state is in favor of their opponent. This underscores the importance of understanding how competition and heterogeneous preferences shape political dynamics and media reporting, revealing insights into why we see media slant in practice.

**Keywords: Information, competition, media, heterogeneous preferences**

## 1 Introduction

In the intricate landscape of democratic politics, politicians find themselves at a crossroads. The diverse preferences of the electorate weave a complex network, where each thread represents a distinct issue or concern. As they navigate this multifaceted terrain, politicians

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grapple with the challenge of effectively conveying their platforms and messages. With limited opportunities for direct voter interaction, they must strategically select which topics to emphasize. The art lies in identifying the key issues that resonate most with their target audience, those pivotal points that can sway voter opinion. In this context, understanding the interplay between policy priorities and voter preferences becomes crucial. How can politicians skillfully tailor their communication to cut through the noise? This paper delves into the strategies employed by politicians as they endeavor to engage and persuade an electorate with diverse tastes and priorities.

Just as politicians face constraints, so do voters. In an era of information overload, attention is a scarce resource. Voters cannot collect information from an infinite array of sources; they must allocate their cognitive bandwidth judiciously. As a result, the media landscape becomes a battleground for capturing attention. How can politicians design their messages to stand out amidst the noise? This paper investigates the delicate balance between informativeness and focus. Our research question centers on unraveling this puzzle: How can politicians optimize the design of their communication channels to effectively sway two distinct groups of voters, each with its unique set of preferences?

In our model, two politicians compete to provide information to a mass of Bayesian voters about the uncertain prospects of their proposed policies on two different issues. The politicians decide how to signal about the effectiveness of their policies before knowing it themselves and they commit to it. Each politician is free to choose how informative her signal can be, but faces a constraint on the focus of the signal: it can be informative about at most one issue. To illustrate, imagine one of the issues is related to immigration and voters want to know how immigrants have an impact on their lives. They know that one politician is going to build a wall at the border and impose strict border security whereas another is going to facilitate the pathway to citizenship and have a welcoming policy for 'talented' immigrants in case of being elected. Another issue can be related to the environment where voters wonder if climate change is real or not, and depending on the answer, they prefer a different politician to be in charge. Each politician is in charge of a medium and faces a constraint on the coverage of the issues. For instance, her medium can ask an expert to come and provide arguments about the impact of immigrants as well as choosing which kind of expert to ask. Voters observe the choice of experts and the focus of the media, and have to decide which medium to consult. In this model, voters are Bayesian and have no limitation on comprehending the news they watch, but they cannot collect all the information there is. Finally, voters have different preferences over the issues. Some care more about climate change whereas others are more concerned with immigration status.

The equilibrium of our model demonstrates how the focus of politicians is determined by their popularity among different groups of voters. If a significant amount of voters are already in favor of a specific politician on one issue that they care about, then this politician would take the support of this group for granted and focus on the other issue to persuade the remaining voters. On the contrary, the other politician is indifferent to appeal to the supporters of her opponent or to focus on the remaining voters and compete for them with her opponent. Nevertheless, the group who gave 'power' to the first politician by believing in her is not better off in the equilibrium than the case in which they receive no information. Moreover, as the support intensity of the voters becomes similar for both politicians, e.g. a large group of voters having moderate beliefs toward one politician whereas the remaining part has extreme beliefs toward another politician, which we denote politicians having less advantage toward each other in persuading voters, less information is revealed in the equilibrium. In the extreme case, the support of each group is taken as granted and all voters are indifferent with the case that they receive no information.

The equilibrium analysis leads to insights about the slant of the media chosen by the politicians. Each politician commits to be truthful only when the state of the world is in her favor. As a result, if voters have moderate beliefs and politicians choose similar levels of informativeness, then upon consulting the media of a specific politician, the voters would observe a signal in her favor with a higher probability. Note that each voter is Bayesian so the knowing of the media cannot systematically bias his belief, and the slant in the media mainly refers to the frequency of a signal favoring one politician.

The rest of the paper is organized as follows. The next subsection discusses the related literature. Section 2 introduces the model, while section 3 simplifies the politicians' problem and discusses some benchmarks that a social planner would want. Our main result is in section 4 in which we characterize the equilibrium, and section 5 concludes the paper.

## 1.1 Literature review

The paper is related to the literature on Bayesian Persuasion with multi senders. Gentzkow and Kamenica (2016, 2017) define a Blackwell-connected environment if for any profile of others' strategies, each sender has a signal available that allows her to unilaterally deviate to any feasible outcome that is more informative. If the information environment is Blackwell-connected, any individual sender can generate as much information as all senders can do jointly. They show that every pure-strategy equilibrium outcome is no less informative than the collusive outcome (regardless of preferences) if and only if the information environment is Blackwell-connected. Our environment is not Blackwell-connected since each sender can be informative only about one issue.

Au and Kawai (2020) analyze a model of competition in Bayesian persuasion in which multi-senders persuade a receiver. Each sender privately observes his own type and can disclose information about his type (so the information environment is not Blackwell-connected). In our setting, both senders observe the two-dimensional state, and can disclose only one of them (in particular they can disclose information about the same dimension). Another difference is that in our setting there are two receivers. If we had one receiver, then both senders would focus on the same dimension. In this special case, the information environment is Blackwell-connected, and truth-telling (full disclosure) becomes the unique equilibrium. Au and Kawai (2021) consider a similar setting with only two senders. However, types are correlated.[1]

Another relevant literature pertains to papers that explore sender-receiver games where the receiver's attention is limited. A closely related paper is Knoepfle et al. (2020) in which there are many partially informed senders who dynamically compete for a decision maker's attention. The senders are assumed to choose their experiments (commitment for each period) before observing their signals. The receiver wants to match his action to the state, and senders only care about how many times the receiver paid attention to them. There are two main differences. First in our paper senders care about the final action of the receivers. Senders do not care if a receiver listens (experiments) to another sender, as long as the receiver chooses the favorable action. Second, in our paper, there are multiple receivers with heterogeneous preferences for different issues. Another paper that studies media competition with limited attention is Innocenti (2022). In Innocenti (2022) receivers have heterogeneous prior beliefs about the state and can devote attention only to one media (sender). There are two main differences. First receivers have heterogeneous prior beliefs, as opposed to ours where receivers have common prior but heterogeneous preferences. Second, in Innocenti (2022) senders choose their experiments and at the same time (as opposed to ours) as receivers choose to devote attention to which sender. The paper argues that if the allocation of attention is chosen after persuasion takes place truth-telling is the equilibrium policy. Papers such as Che and Mierendorff (2019), and Leung (2020) consider an exogenous information environment (senders do not design an information structure), and study the receiver's problem with limited attention.

Another relevant literature focuses on models that examine voters with diverse preferences, considering the heterogeneity among individual voters. Maskin and Tirole (2019) develop a model of pork-barrel politics in which a government official tries to improve her reelection chances by spending on targeted interest groups. There are two main differences. First

[1] Finally, the common differences between this literature and our paper are multiple receivers with heterogeneous preferences and the fact that receivers are inattentive, and can listen to only one media.

there is no explicit competition between politician and second there is no media competition. In Perego and Yuksel (2022) a finite identical number of firms compete to provide information to a finite number of Bayesian agents about a newly proposed policy with uncertain prospects. The probability of implementing the new policy depends on approval rate. Much like the present study, each politician (referred to as a "firm" in their paper) encounters limitations on how much information they can provide regarding various aspects of the state. The key distinction between the present paper and Perego and Yuksel (2022) is that the politicians are indistinguishable. They exhibit no bias and possess no advantage in implementing particular policies.

# 2 Model

There is a two-dimensional state of the world $\omega \in \Omega$. Each dimension corresponds to one issue of interest: Environmental and Immigration. The state of the world with respect to any dimesnsion can take two variables: $\{\ell, r\}$. For instance, people might wonder if climate change is real or not, and if it is real, the state is $\ell$. The state of the world regarding each issue is the answer to the question: Which policy is better? For instance, the question can be whether it is better to build a wall to stop immigration or have better social security for immigrants. Denote two dimensions as $\omega = (\omega_{\mathbf{I}}, \omega_{\mathbf{E}}) \in \Omega = \Omega_{\mathbf{I}} \times \Omega_{\mathbf{E}} = \{\ell, r\}^2$.

There are two politicians $L$ and $R$ indexed by $p$ who share a common prior $\mu = (\mu_{\mathbf{I}}, \mu_{\mathbf{E}})$ with $\mu_f = Pr(\omega_f = r)$. Politicians play a simultaneous move game by designing and committing to an experiment that is informative about at most one dimension of $\omega$.[2] Formally, consider the set of available signal spaces on each issue $f \in \{\mathbf{I}, \mathbf{E}\}$ by $S_f$. Then each politician $p$ chooses an experiment $\pi_f^p \in \Pi_f$ consisting of a finite realization space $S_f$, which determines the focus of the experiment, and a family of likelihood functions over $S_f$, $\{\pi^p(\cdot|\omega_f)\}_{\omega_f \in \Omega_f}$, with $\pi^p(\cdot|\omega_f) \in \Delta(S_f)$. We allow for mixing, and denote each politician's action by $a_p \in \Delta(\Pi_{\mathbf{I}} \cup \Pi_{\mathbf{E}})$. Throughout the paper, we restrict our attention to signal spaces that coincide with the state spaces, i.e. $S_f = \Omega_f$. Each politician after getting elected, implements fixed policies specific to her. $L$ would implement left-wing policies on both issues and $R$ would implement right-wing policies.

There is mass 1 of voters who share the same common prior $\mu$ as politicians. Each voter chooses to learn the outcome of only one experiment, and then decides which vote for one politician. Formally, voter $k$ chooses $\pi \in \{\pi^L, \pi^R\}$ to consult where $\pi^p$ is the realization of action of politicians. After receiving a signal, then he votes $v_k \in \{\ell, r\}$.

[2]This captures the fact that newspapers or online news agencies can have only one main headline each day. Or news agencies can only cover one main topic each day.

Voters are heterogeneous in their preferences. Specifically, there are two groups of voters: $G_{\mathbf{E}}$ with share $m_{\mathbf{E}}$ of the population and group $G_{\mathbf{I}}$ with share $m_{\mathbf{I}} = 1 - m_{\mathbf{I}}$. Every voter in group $G_f$ only cares about issue $f$ and wants to match his vote to the state of the world in dimension $\omega_f$.[3] Ex-post utility of a voter from $G_f$ when the state is $\omega$ and his vote is $v_f$ is:

$$u_f(\omega, v_f) = \begin{cases} u > 0,^{4} & \text{if } v_f = \omega_f \\ 0, & \text{otherwise.} \end{cases}$$

Politicians are vote-seekers, i.e. they want to maximize their expected share of votes. Ex-post payoff of politician $R$ facing the vector of votes $V$ is:

$$U_R(V) = \int \mathbb{1}_{v_k = r}\, dv, \text{ where } V = (v_k)_{k \in \{G_{\mathbf{I}} \cup G_{\mathbf{E}}\}}$$

As all voters vote in this model, politicians are playing a constant-sum game and the ex-post payoff of politician $L$ is simply $U_L(D) = 1 - U_R(D)$.

The timing of the game is depicted in figure 1. First politicians simultaneously choose a distribution over the set of all available experiments. Then, their choices are realized, and each voter observes the focus of each experiment as well as its signal structure. The voter then decides to consult one experiment, and for all the voters who consulted that experiment, one signal is going to be realized. Voters update their posteriors according to the Bayes rule and then vote for one politician. One politician would get elected according to the majority rule and would implement the fixed policies associated with her. Finally, the state of the world and payoffs are realized.

We assume a favorable tie-breaking rule wherever possible. Specifically, when a voter is indifferent between who to vote for after consulting an experiment, he chooses to vote for the designer of the experiment. If the voter is indifferent between which experiment to choose, he chooses the experiment of the designer whom his prior is closest to on the issue that he cares about.

We impose a notion of *sequentially rationality* on voters' behavior in any information set of the game. Formally, we assume that voters always choose an experiment that yields higher expected utility to them in all the infinitely many information sets available to them. Moreover, they always vote according to their posterior.

Fixing the voters' behavior as above, we focus on the Bayes-Nash equilibria of the politicians' game. Any equilibrium is then characterized by a tuple $(a_L, a_R)$ in which $a_p$ is politician

[3]For instance one politician wants to build a dam and another wants to build a wind turbine and voters who care about environmental issues wants to know to which policy is better.

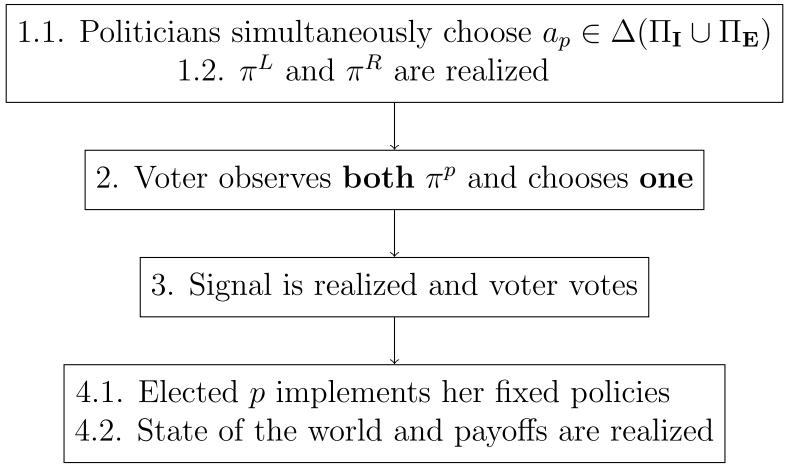


Figure 1: Timing

$p$'s strategy.

# 3 Politicians Problem

## 3.1 Experiment design

In this subsection, we plan to simplify the politician's problem. Note that every experiment $\pi_f \in (\Pi_{\mathbf{I}} \cup \Pi_{\mathbf{E}})$ can be characterized by a tuple $(\alpha_f, \beta_f)$ shown in table 1 where $\alpha_f, \beta_f \in [0, 1]$.

| state/signal | $s_\ell$ | $s_r$ |
|---|---|---|
| $\ell$ | $\alpha_f$ | $1 - \alpha_f$ |
| $r$ | $1 - \beta_f$ | $\beta_f$ |

Table 1: an arbitrary experiment $\pi_f \in (\Pi_{\mathbf{I}} \cup \Pi_{\mathbf{E}})$

The following lemma helps us to characterize each experiment by only one variable.

**Lemma 1** ***(Outcome equivalence)***
*For any equilibrium of the game $(a_L^*, a_R^*)$, there exists an equilibrium $(b_L^*, b_R^*)$ such that the outcome of the game, i.e. utilities and voters choices, is the same and every experiment $\pi_f^p \in supp(b_p^*)$, with $f \in \{\mathbf{I}, \mathbf{E}\}$ and $p \in \{L, R\}$ has the signal structure in the form of table 2.*

To get the overview of the proof, consider two distinct cases in which in one, the politician prefers to talk about an issue that is not chosen by her opponent, and in the second one,

(a) L-biased experiment source: $\pi_f^L$

| state/signal | $s_\ell$ | $s_r$ |
|---|---|---|
| $\ell$ | 1 | 0 |
| $r$ | $1-\lambda_f^L$ | $\lambda_f^L$ |

(b) R-biased experiment: $\pi_f^R$

| state/signal | $s_\ell$ | $s_r$ |
|---|---|---|
| $\ell$ | $\lambda_f^R$ | $1-\lambda_f^R$ |
| $r$ | 0 | 1 |

Table 2: Signal structure of the experiments on issue $f$

she prefers to compete on the same issue. The first case is akin to Kamenica and Gentzkow (2011) and results in the same signal structure in table 2. See appendix A for full proof.

*lemma* 1 allows us to restrict the choice set of politicians to experiments in the form of table 2. Politician $L$ chooses only from the $L$-biased[5] experiments $\pi^L$ and $R$ chooses from the $R$-biased experiments $\pi^R$. With an abuse in notation, we show each experiment $\pi_f^p$ by $\lambda_f^p \in [0,1]$ that uniquely characterizes that experiment.

From now on, to make things more interesting assume that the prior on each issue is closer to a distinct politician. More specifically consider the case that $\mu_{\mathbf{I}} < 0.5 < \mu_{\mathbf{E}}$. We denote issue **E** a *favored* issue for politician $R$ and the issue **I** an *unfavored* issue for her.

To be able to compare politicians' choices, for any $\lambda_f$ that $L$ chooses for her $\pi_f^L$, consider the dagger version of the variable, $\lambda_f^\dagger$ as the choice for $R$ that characterizes her $\pi_f^R$ such that voters from group $G_f$ are indifferent between choosing $\pi^L$ and $\pi^R$. The relation between these two variables are:

$$\lambda_f^\dagger = 1 - \frac{\mu_f}{1-\mu_f} + \frac{\mu_f}{1-\mu_f}\lambda_f, \qquad \lambda_f = 2 - \frac{1}{\mu_f} + \frac{1-\mu_f}{\mu_f}\lambda_f^\dagger, \qquad f \in \{\mathbf{I}, \mathbf{E}\}$$

To ease the notation even more, denote $\lambda_{\mathbf{I}}^p = i_p$ and $\lambda_{\mathbf{E}}^p = e_p$. As an example the expected payoff for $L$ when the strategy profile is $(e_L, e_R)$ is as follows:

$$\mathbb{E}U_L(e_L, e_R) = (1 - m_{\mathbf{E}}) + m \times \Big((1-\mu_{\mathbf{E}})e_R + \big[1 - \mu_{\mathbf{E}} e_L - (1-\mu_{\mathbf{E}})e_R\big]\mathbb{1}_{e_L \geq e_{\min}, e_L^\dagger > e_R}\Big)$$

where the first part of the RHS, $(1 - m_{\mathbf{E}})$, is the share of people that care about issue **I** and vote for $L$ in the absence of the information. The second part depends on the competition on issue **E**. If $L$ does not provide enough information to persuade voters, i.e. $e_L < e_{\min}$, or if he does not provide more information than his opponent, $e_L^\dagger < e_R$, then every voter would choose the media of the right politician and the share of $m(1-\mu_{\mathbf{E}})e_R$ would vote for $L$. Otherwise, they would listen to $L$'s media and the expected share of votes for him would become $m(1-\mu_{\mathbf{E}} e_L)$.

[5]Note that we assume that the voter is Bayesian, so the experiment cannot systematically bias her belief. The bias in the platforms mainly refers to the frequency of a signal favoring one state.

### 3.2 Benchmarks

Before we characterize the equilibrium, we discuss three benchmarks and compare voters' welfare.

#### 3.2.1 Fully informative outcome

Consider the outcome that voters get to know the true state of the world with probability one. One way to achieve this outcome is to take $i_L = e_R = 1$ as politicians' choices. This is the voters' most preferred outcome as their expected utilities are maximized.

#### 3.2.2 No information

The polar opposite case to the previous outcome is when there is no information revelation on any issue. It can be achieved by putting $i_L = e_R = 0$. This is the least desired outcome for the voters as they are voting according to their priors. Share $m_{\mathbf{E}}$ of voters are voting for the right-wing politician and $m_{\mathbf{I}}$ are voting for the left-wing one.

#### 3.2.3 Least persuasive outcome

Assume each politician specializes on her unfavored issue and each tries to maximize the probability of persuading voters on that issue. More specifically, they choose their experiments according to table 3 where $e_{\min} = 2 - 1/\mu_{\mathbf{E}}$ and $i_{\min} = 1 - \mu_{\mathbf{I}}/(1 - \mu_{\mathbf{I}})$:

(a) L's experiment : $\pi^L_{\mathbf{E}}$

| state/signal | $s_\ell$ | $s_r$ |
|---|---|---|
| $\ell$ | 1 | 0 |
| $r$ | $1 - e_{\min}$ | $e_{\min}$ |

(b) R's experiment: $\pi^R_{\mathbf{I}}$

| state/signal | $s_\ell$ | $s_r$ |
|---|---|---|
| $\ell$ | $i_{\min}$ | $1 - i_{\min}$ |
| $r$ | 0 | 1 |

Table 3: The experiments resulting in the least persuasive outcome

This outcome again is the least desired outcome for the voters as they are indifferent between voting according to their priors or by consulting the experiments.

## 4 Equilibrium

To characterize the equilibrium, first we define a measure of persuasive advantage based on the parameteres describing the environment and then discuss how the equilibrium and the interpretation solely depends on this measure.

Denote $m(1-\mu_{\mathbf{E}})/(1-m_{\mathbf{E}})\mu_{\mathbf{I}} =: Q$ as a *measure of persuasive disadvantage* for politician $R$. For $Q < 1$, $R$ prefers the imaginary case of truth-telling on both issues than the case with the absence of any information since: $Q < 1 \Leftrightarrow m\mu_{\mathbf{E}} + (1-m_{\mathbf{E}})\mu_{\mathbf{I}} > m$. In some sense, this means that information is more beneficial for $R$. For $Q > 1$, $L$ would have more persuasive advantage and with $Q = 1$, they both are indifferent between the truth-telling case and the one with no information (no persuasive advantage).

**Proposition 1 (Equilibrium Characterization)**

*The Equilibrium is unique with respect to measure zero sets and:*

*1. The Only case in which the equilibrium exists in pure strategies is when $Q = 1$, i.e. when the politicians are the same in terms of persuasive advantage. More specifically, each politician focuses on her unfavored issue and chooses the least persuasion:*

$$a_L^* = e_{\min} := 2 - \frac{1}{\mu_E}, \qquad a_R^* = i_{\min} := 1 - \frac{\mu_I}{1-\mu_I}.$$

*This coincides with the least persuasive outcome which is the least desired outcome for the voters.*

*2. When $R$ has persuasive advantage, i.e. $Q < 1$, then again she only focuses on her unfavored issue. However, $L$ mixes between the two issues. She chooses $\mathbf{I}$ w.p. $(1-Q)$ and chooses her unfavored issue $\mathbf{E}$ w.p. $Q$.*

- ***Issue* $\mathbf{E}$** : *Whenever $L$ chooses to focus on environmental issue, she chooses the least persuasion by playing $e_{\min}$*
- ***Issue* $\mathbf{I}$** : *Both politicians play distributions: $R$ plays CDF $G(\cdot)$ with atom at $i_{\min}$ w.p. $Q$ and $L$ plays an atomless CDF $F(\cdot)$ such that:*

$$supp(G) = [i_{\min}, \frac{\mu_{\mathbf{I}}}{1-\mu_{\mathbf{I}}}Q^2], \qquad G(i) = \frac{Q\sqrt{\mu_{\mathbf{I}}/(1-\mu_{\mathbf{I}})}}{\sqrt{1-i}},$$
$$supp(F) = [0, 1-Q^2], \qquad F(i) = \frac{Q}{1-Q}\Big(\frac{1}{\sqrt{1-i}} - 1\Big).$$

**Proof.** See appendix B. ■

The equilibrium in proposition proposition 1 is robust to any tie-breaking rules for the voters when choosing the experiment. More specifically, for any other tie-breaking rule that we had assumed, $L$ plays the same distribution but with the open support at the minimum level.

When $Q < 1$, that is when $R$ has more persuasive power, then the group that is already in his favor, $G_{\mathbf{E}}$, does not get better off than the case with the absence of information. In some sense, $R$ takes their support for granted and focuses solely on the issue in which she is not favored. Moreover, the expected share that politician $R$ gets in equilibrium is greater than both the fully truthful case and the case with no information. By the constant-sum nature of the game, $L$ is getting less than the share she could get in the two extreme cases.

In equilibrium, the minimum levels of information (least persuasion: $\{i_{\min}, e_{\min}\}$) are played with positive probability. This probability ($= Q^2$) increases as the politicians become more similar to each other in terms of their persuasive advantage.

Playing truthfully is not in the support of any politician's equilibrium strategy. This result would fail if we relax the restriction on politicians. Intuitively, now that they cannot be truthful on both issues, they choose to *lie* more and be the least informative as possible.

## 4.1 Office-seeking

In this subsection, we briefly discuss what would happen if the politicians were office-seekers. As these preferences do not usually result in the same equilibrium outcome as the case where they only care about the share of votes, it is good practice to study both. Since politicians only care about the probability of winning in this case and since all voters receive the same signal if they consult the same media, only the group with the highest share of voters matters. Normalizing the utility of winning to 1:

$$U_L^{\text{office}}(a_L, a_R) = \begin{cases} 0, & \text{if } U_L(a_L, a_R) < 1/2 \\ 1, & \text{otherwise} \end{cases}$$

**Observation 1** *If the politicians only care about the probability of winning, in the unique equilibrium, they choose the issue $f$ with the highest share of the voters and they will provide the maximum amount of information: $\lambda_f^L = \lambda_f^R = 1$ where $m_f > 1/2$.*

# 5 Conclusion

In many important cases, voters have to rely on the information generated by a public experiment, such as media, as acquiring information is not feasible otherwise. We consider a case in which two politicians are competing in persuading voters with heterogeneous preferences. There are multiple issues to cover and politicians have to decide which topic they want to convey information about. This key restriction gives rise to novel behavior in politicians' competition. We observe in the equilibrium, a politician with a more persuasive advantage takes the support of a group of voters close to her for granted. Moreover, as politicians lose their persuasive advantage with respect to another, we see the competition is more relaxed in a sense that less information is going to be revealed to the voters. Finally, each politician commits to a specific signal structure that only lies when the state is not in their favor. We believe such observation provides more insights on why we see media slant in practice.

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# Appendix

## A Proof of lemma 1

Fix an equilibrium $(a_L^*, a_R^*)$. In the equilibrium, politician $p \in \{L, R\}$ is indifferent between choosing any experiment $\pi_f^p \in supp(a_p^*)$ with $f \in \{\mathbf{I}, \mathbf{E}\}$ fixing $a_{k \neq p}^*$. Consider the expected utility of politician $p$ in equilibrium which is the same when the strategy profile is $(\pi_f^p, a_{k \neq p}^*)$. If the experiment $\pi_f^p$ is chosen with positive probability by group $G_f$ of voters, it provides an expected utility of $u$ for them. There are two cases:

1. $u > \max\{\mu_f, 1 - \mu_f\}$: That is consulting the experiment is beneficial for the voters than voting without consultation. Then the experiment can change the action of voters relative to the case that they were on their own. For any fixed $u$ that satisfies this condition, there exists an experiment in the form of table 2 that provides the same $u$ for the voters, but makes politician $j$ strictly better off (same as Kamenica and Gentzkow (2011)).

2. $u = \max\{\mu_f, 1 - \mu_f\}$: That is the case where the voters do not benefit from experimentation. Here the politician is indifferent between experiments that do not change the action of the voters, or in other words, any experiment that has zero value of information for the voters. One can assume that the politician chooses an uninformative experiment with the signal structure according to table 2.

Now consider the case in which $\pi_f^p$ is chosen with zero probability by group $G_i$, that is the voters expected utilities are $u' > u$ in which $u$ is their expected utility had they chosen to consult $\pi_f^p$. Similar to the second case above, the politician could choose an experiment in the form of table 2 that provides $u$ for the voters.

Thus, for any equilibrium $(a_L^*, a_R^*)$, we can build an equilibrium $(b_L^*, b_R^*)$ with every $\pi_f^p \in supp(b_p^*)$ having the same signal structure as table 2, such that the expected utility of the players and the outcomes are the same in the two equilibria.

## B Proof of proposition 1

**Proof.** First, we prove that in any equilibrium when $Q \neq 1$, only one politician mixes between the issues. Then, we show that the issue that is on the path of the play of both politicians, they play distribution with support that starts from the minimum level of persuasion specific to each politician. Finally, we construct the equilibrium.

**Lemma 2**

*No equilibrium exists in pure strategies when $Q \neq 1$.*

**Proof.** We go through the candidate pure equilibria and show that there exists an incentive for deviation in each one of them.

1. Assume both choose the same issue, e.g. **E**. Then in equilibrium, they both should choose $e_L = e_R = 1$. This way, the politician that is not getting any votes from $G_{\mathbf{I}}$ has an incentive to deviate and be active on issue $I$.

2. Assume each chooses the issue with the like minded group, that is politician $R$ chooses **E** and politician $L$ chooses **I**. Then they should provide zero information as they already have the votes of the people. This will provide an incentive for them to change their focus and compete on the other issue.

3. Assume that $R$ chooses **I** now and $L$ chooses **E**. $R$ provides just enough information to change the posterior of the voters in his favor if they receive the inconclusive signal. $L$ does the same and the candidate equilibrium becomes:

$$a_R^* = i_{\min}, a_L^* = e_{\min}$$

   The expected share of voters for each politician is:

$$\mathbb{E}U_L(a_L^*, a_R^*) = 2(1-\mu_{\mathbf{E}})m_{\mathbf{E}} + (1-2\mu_{\mathbf{I}})(1-m_{\mathbf{E}}),$$
$$\mathbb{E}U_R(a_L^*, a_R^*,) = (2\mu_{\mathbf{E}}-1)m_{\mathbf{E}} + 2\mu_{\mathbf{I}}(1-m_{\mathbf{E}})$$

Note that if any politician deviates, let's say $L$, for any $i_L > 0$, the voters in $G_{\mathbf{I}}$ would choose her experiment, so he can get arbitrarily close to having $m_{\mathbf{I}} = 1 - m_{\mathbf{E}}$ of expected share of votes. $R$ can deviate to get close to the share of $m_{\mathbf{E}}$. Writing the conditions, the degenerative equilibrium happens iff $Q = 1$. ■

A direct observation of the previous lemma is that no equilibrium exists in which politicians choose to focus on the same issue and play mixed on the informativeness of their experiments as the only equilibrium candidate is to be truthful and according to the previous lemma, one politician has a deviation.

**Lemma 3** *In any equilibrium, only one politician mixes between the issues. Moreover, the support of the politicians' strategies on both issues, in which they focus on with positive probability contains* $i_{\min}$ *and* $e_{\min}$.

**Proof.** For the proof of the first part of the lemma, we just need to prove no equilibrium exists in which both politicians play mixed between both issues. Assume by the contrary that it exists. Choose the minimum $i_L$ in the support of $L$'s strategy on issue $\mathbf{I}$ by $i_L^{\min}$ and the minimum $e_L$ in the support of $L$'s strategy on issue $\mathbf{E}$ by $e_L^{\min}$. In equilibrium, $L$ should be indifferent playing each pure strategy in the support of her equilibrium strategy, i.e. $\mathbb{E}U_L(i_L^{\min}, a_R^*) = \mathbb{E}U_L(e_L^{\min}, a_R^*)$. When $L$ plays $i_L^{\min}$, she should win the attention of a positive share of voters $G_{\mathbf{I}}$ when $R$ is present on issue $\mathbf{I}$, otherwise, $L$ has a deviation to $e_L^{\min}$. This means that $R$ has to lose the attention of the voters when she is playing the minimum $i_R$ in her support. Accordingly, $L$ has to win the attention of a positive share of voters when playing $e_L^{\min}$, otherwise, she has deviation to play $i_L^{\min}$ and win the attention of voters on issue $\mathbf{I}$ with positive probability. Thus, $R$ has to lose the attention of the voters when she is playing the minimum $e_R$ in her support as well. With similar reasoning for $R$, she should win the attention of some group of voters with positive probability when playing the minimums in her support which is a contradiction. Thus, Only one politician can mixes between issues in any equilibrium.

The proof of the second part of the lemma is an immediate based on the proof of the first part. Consider the case that only $L$ mixes between issues. In equilibrium, $R$ cannot focus on $\mathbf{E}$ as she is losing the attention of the voters when choosing $e_R^{\min}$, so she has a trivial deviation to $i_R = 1$. When focusing on $\mathbf{I}$, she is losing the attention of voters whenever she is choosing the minimum in her support, i.e. $i_R^{\min}$ and $L$ is present on issue $\mathbf{I}$. Thus, the best that $R$ can do is to choose $i_R^{\min} = i_{\min}$ to get the highest utility when $L$ is not present there. Moreover, as $L$ faces no competition when she is present on $\mathbf{E}$, she always chooses $e_{\min}$ there. The same reasoning applies when $R$ mixes between the two issues and $L$ focuses only on one. ■

An observation resulting from the proof of the previous lemma is that the politicians cannot have non measure zero holes on the support of their strategies regarding the same issue. Similar reasoning to the case of choosing minimum persuasive levels applies. Thus, we can focus on the cases that the politicians choose distributions with connected supports. More specifically, with the help of previous lemmas, we need to search for all the equilibria in a special class of strategies such that when $Q < 1$:

- $R$'s strategy is $a_R = G(\cdot)$ where $G(\cdot)$ is a CDF with support $[i_{\min}, i_M^\dagger]$, atom at $i_{\min}$, and differentiable on $(i_{\min}, i_M^\dagger]$. Denote the pdf by $G(\cdot) > 0$.
- $L$'s strategy $a_L$ is to play $e_{\min}$ with probability $r$ and $F(\cdot)$ with probability $1-r$ where $F(\cdot)$ is a CDF with support $[0, i_M]$, and differentiable on its support. Denote the pdf by $F(\cdot) > 0$.

**Lemma 4**

*There exists a unique equilibrium in the aforementioned class of strategies.*

**Proof.** We prove by construction. Consider equilibrium $(a_L, a_R)$. Throughout the proof, assume $I_R \sim G(\cdot)$ and $I_L \sim F(\cdot)$. To find the $G(i_{\min})$, note that $L$ should be indifferent between playing $e_{\min}$ and $0_{\mathbf{I}}$ in equilibrium where $0_{\mathbf{I}}$ denotes $i = 0$:

$$\mathbb{E}U_L(0_{\mathbf{I}}, a_R) = \mathbb{E}U_L(e_{\min}, a_R)$$

$$\Leftrightarrow (1 - m_{\mathbf{E}})\Big[G(i_{\min}) + (1 - \mu_{\mathbf{I}})(1 - G(i_{\min}))\mathbb{E}[I_R | I_R > i_{\min}]\Big] = (1 - m_{\mathbf{E}})(1 - \mu_{\mathbf{I}})\mathbb{E}[I_R] + 2m_{\mathbf{E}}(1 - \mu_{\mathbf{E}})$$

$$\Leftrightarrow (1 - m_{\mathbf{E}})G(i_{\min}) = (1 - m_{\mathbf{E}})(1 - \mu_{\mathbf{I}})i_{\min}G(i_{\min}) + 2m_{\mathbf{E}}(1 - \mu_{\mathbf{E}})$$

$$\Leftrightarrow 2(1 - m_{\mathbf{E}})\mu_{\mathbf{I}}G(i_{\min}) = 2m_{\mathbf{E}}(1 - \mu_{\mathbf{E}}) \Leftrightarrow G(i_{\min}) = \frac{m_{\mathbf{E}}(1 - \mu_{\mathbf{E}})}{(1 - m_{\mathbf{E}})\mu_{\mathbf{I}}} = Q.$$

Note that if $G(i_{\min}) = 0$ then $\mathbb{E}U_L(0_{\mathbf{I}}, a_R) < \mathbb{E}U_L(e_{\min}, a_R)$. Now, to pin down the distribution that $R$ plays, i.e. $G$, we check the indifference conditions for every $\lambda^L_{\mathbf{I}} = i$ that $L$ plays on issue $\mathbf{I}$. Using $\lambda^R_{\mathbf{I}} = i^\dagger$ for $R$ that make voters indifferent between choosing $i$ and $i^\dagger$:

$$\mathbb{E}U_L(0_{\mathbf{I}}, a_R) = \mathbb{E}U_L(i, a_R) \Leftrightarrow$$

$$(1 - m_{\mathbf{E}})\left[G(i_{\min}) + (1 - \mu_{\mathbf{I}})\int_{i^+_{\min}}^{i^\dagger_M} i_R dG\right] = (1 - m_{\mathbf{E}})\left[G(i^\dagger)(1 - i\mu_{\mathbf{I}}) + (1 - \mu_{\mathbf{I}})\int_{i^\dagger}^{i^\dagger_M} i_R dG]\right]$$

Where I use the following definition for $i^+_{\min}$: $\int_{i_{\min}}^{x^\dagger} i_R dG = G(i_{\min})i_{\min} + \int_{i^+_{\min}}^{x^\dagger} i_R dG$. Recall from before $\lambda_f = 2 - \frac{1}{\mu_f} + \frac{1 - \mu_f}{\mu_f}\lambda^\dagger_f$, substituting $i$ with $i^\dagger$, we would have $1 - i\mu_{\mathbf{I}} = (1 - \mu_{\mathbf{I}})(2 - i^\dagger)$:

$$\Leftrightarrow (1 - \mu_{\mathbf{I}})\int_{i^+_{\min}}^{i^\dagger} i_R dG = G(i^\dagger)(2 - i^\dagger)(1 - \mu_{\mathbf{I}}) - Q \xrightarrow{\frac{d}{di^\dagger}} G(i^\dagger) = 2g(i^\dagger)(1 - i^\dagger)$$

With initial condition $G(i_{\min}) = Q$ and denote $c := \frac{\mu_{\mathbf{I}}}{1 - \mu_{\mathbf{I}}}$, solving for the ODE:

$$G(i^\dagger) = \frac{Q\sqrt{c}}{\sqrt{1 - i^\dagger}}$$

Since $G$ is a CDF with support $[i_{\min}, i^\dagger_M]$, we have:

$$G(i^\dagger_M) = 1 \Leftrightarrow i^\dagger_M = 1 - cQ^2 \Leftrightarrow i_M = 1 - Q^2$$

So $i_M, i^\dagger_M$, and the distribution $G$ have been uniquely determined by necessary conditions.

To find a relation between the maximum in the support $F$, i.e $i_M$ and the probability that $L$ plays mixed between the issues, we use the zero-sum nature of the game. In equilibrium, $R$ is indifferent between playing $i_M^\dagger$ and $i_R$ with $i_R$ in $G$'s support, so his equilibrium share is:

$$\mathbb{E}U_R^* = \mathbb{E}U_R(a_L^*, i_M^\dagger) = (1-m_{\mathbf{E}})(1-i_M^\dagger(1-\mu_{\mathbf{I}})) + m_{\mathbf{E}}[1-2r(1-\mu_{\mathbf{E}})]$$

Similarly, $L$'s equilibrium share of votes $\mathbb{E}U_L^*$ is equal to $\mathbb{E}U_L(i_M, a_R^*) = (1-m_{\mathbf{E}})(1-i_M\mu_{\mathbf{I}})$. Note that the share of voters who vote for politicians should be equal to 1:

$$\mathbb{E}U_R^* + \mathbb{E}U_L^* = 1 \Leftrightarrow (1-m_{\mathbf{E}})(1-i_M^\dagger(1-\mu_{\mathbf{I}})) + m_{\mathbf{E}}[1-2r(1-\mu_{\mathbf{E}})] = 1-(1-m_{\mathbf{E}})(1-i_M\mu_{\mathbf{I}})$$
$$\Leftrightarrow (1-m_{\mathbf{E}})\mu_{\mathbf{I}}(2-i_M) + m_{\mathbf{E}}[1-2r(1-\mu_{\mathbf{E}})] = 1-(1-m_{\mathbf{E}})(1-i_M\mu_{\mathbf{I}}) \Leftrightarrow$$
$$2(1-m_{\mathbf{E}})\mu_{\mathbf{I}}(1-i_M) = 2rm_{\mathbf{E}}(1-\mu_{\mathbf{E}}) \Leftrightarrow i_M = 1 - \frac{m_{\mathbf{E}}(1-\mu_{\mathbf{E}})}{(1-m_{\mathbf{E}})\mu_{\mathbf{I}}} r = 1-Qr = 1-Q^2 \Rightarrow r = Q$$

To pin down the distribution for $L$, we check the indifference conditions in for every value that $R$ plays in the support of $G$. Assume again that for $i$ that $L$ plays, $i^\dagger$ is the counterpart that $R$ has to play to make voters indifferent:

$$\mathbb{E}U_R(a_L, i^\dagger) = \mathbb{E}U_R(a_L, i_M^\dagger) \Leftrightarrow r\Bigg[(1-m_{\mathbf{E}})(1-(1-\mu_{\mathbf{I}})i^\dagger)$$
$$+m_{\mathbf{E}}(2\mu_{\mathbf{E}}-1)\Bigg] + (1-r)\Bigg[(1-m_{\mathbf{E}})\big[F(i)(1-(1-\mu_{\mathbf{I}})i^\dagger + \mu_{\mathbf{I}}\int_i^{i_M} i_L dF\big] + m_{\mathbf{E}}\Bigg]$$
$$= r\Bigg[(1-m_{\mathbf{E}})(1-(1-\mu_{\mathbf{I}})i_M^\dagger) + m_{\mathbf{E}}(2\mu_{\mathbf{E}}-1)\Bigg] + (1-r)\Bigg[(1-m_{\mathbf{E}})(1-(1-\mu_{\mathbf{I}})i_M^\dagger + m_{\mathbf{E}}\Bigg]$$

Recall $\lambda_f^\dagger = 1 - \frac{\mu_f}{1-\mu_f} + \frac{\mu_f}{1-\mu_f}\lambda_f$, substituting $i^\dagger$ with $i$, we would have $1-(1-\mu_{\mathbf{I}})i^\dagger = \mu_{\mathbf{I}}(2-i)$:

$$\Leftrightarrow r(2-i) + (1-r)\Bigg[F(i)(2-i) + \int_i^{i_M} i_L dF\Bigg] = 2 - i_M$$
$$\Leftrightarrow i_M - i = (1-r)\Bigg[(1-F(i))(2-i) - \int_i^{i_M} i_L dF\Bigg]$$

Now we substitute for $i_M$ and $r$ with the equalities that we derived before: $i_M = 1-Q^2$,

$r = Q$:

$$\Leftrightarrow \frac{i_M}{1-r} = 1 + Q = \left[(1 - F(i))(2 - i) - \int_i^{i_M} i_L dF\right] + \frac{i}{1-Q}$$

$$\xrightarrow{\frac{d}{di}} 1 - F(i) + 2f(i)(1 - i) = \frac{1}{1-Q}$$

Solving for the above ODE with initial condition $F(i_M = 1 - Q^2) = 1$ results in:

$$F(i) = \frac{Q}{1-Q}\left[\frac{1}{\sqrt{1-i}} - 1\right]$$

Note that when searching for $F(\cdot)$ we did not assume that it is atomless at 0, but this is the only solution given the unique $G(\cdot)$ that we have determined earlier.

Now we check if the politicians have incentives to deviate or not. First, $L$ does not want to deviate to $e$ where $e \in (e_{\min}, 1]$ since $e_{\min}$ is the best he can do when choosing to talk about issue **E**:

$$\mathbb{E}U_L(a_L^*, a_R^*) = \mathbb{E}U_L(e_{\min}, a_R^*) > \mathbb{E}U_L(e, a_R^*) \Leftrightarrow$$
$$(1 - m_{\mathbf{E}})(1 - \mu_{\mathbf{I}})\mathbb{E}[I_R] + 2m_{\mathbf{E}}(1 - \mu_{\mathbf{E}}) > (1 - m_{\mathbf{E}})(1 - \mu_{\mathbf{I}})\mathbb{E}[I_R] + m_{\mathbf{E}}(1 - e\mu_{\mathbf{E}}) \Leftrightarrow 1 > \mu_{\mathbf{E}}(2 - e)$$

Moreover, for $e < e_{\min}$, voters of $G_{\mathbf{E}}$ would always vote for $R$. Another possible deviation for $L$ is to choose $i$ where $i > i_M = Q$. Again, he does not have an incentive to deviate this way since conditional on winning the voters' attention, a politician wants to provide minimum information possible:

$$\mathbb{E}U_L(a_L^*, a_R^*) = \mathbb{E}U_L(i_M, a_R^*) > \mathbb{E}U_L(i, a_R^*)$$
$$\Leftrightarrow (1 - m_{\mathbf{E}})(1 - i_M\mu_{\mathbf{I}}) > (1 - m_{\mathbf{E}})(1 - i\mu_{\mathbf{I}}) \Leftrightarrow i > i_M$$

With the same reasoning, $R$ does not have an incentive to deviate to $i^\dagger$ where $i^\dagger > i_M^\dagger$. His best deviation is to play $0_{\mathbf{E}}$. The reason for that is $L$ is only playing $e_{\min}$ on issue **E**, so if $R$ plays $0_{\mathbf{E}}$, every voter in group $G_{\mathbf{E}}$ would choose his media and all of them would eventually vote for him. Thus any deviation of $e_R > 0$ is strictly dominated by giving no information when talking about issue $\mathbf{E}$. Now checking for this best deviation:

$$\mathbb{E}U_R(a_L^*, a_R^*) = \mathbb{E}U_L(a_L^*, i_{\min}) > \mathbb{E}U_R(a_L^*, 0_{\mathbf{E}}) \Leftrightarrow$$
$$r\Big[2(1-m_{\mathbf{E}})\mu_{\mathbf{I}} + m_{\mathbf{E}}(2\mu_{\mathbf{E}}-1)\Big] + (1-r)\Big[(1-m_{\mathbf{E}})\mu_{\mathbf{I}}\mathbb{E}(I_L) + m_{\mathbf{E}}\Big] > m_{\mathbf{E}} + (1-r)(1-m_{\mathbf{E}})\mu_{\mathbf{I}}\mathbb{E}(I_L)$$
$$\Leftrightarrow 2(1-m_{\mathbf{E}})\mu_{\mathbf{I}} + m_{\mathbf{E}}(2\mu_{\mathbf{E}}-1) > m_{\mathbf{E}} \Leftrightarrow (1-m_{\mathbf{E}})\mu_{\mathbf{I}} > m_{\mathbf{E}}(1-\mu_{\mathbf{E}}) \Leftrightarrow Q < 1.$$

With similar reasoning as above it is easy to see why $(e_{\min}, i_{\min})$ is an equilibrium in the degenerate case $Q = 1$. Since politicians are talking about separate issues, the best they can do is to be the least informative. Moreover, if they deviate, the best deviation is again to deviate to talking about the other issue and provide the minimum level of information. The condition $Q = 1$ makes each politician indifferent between equilibrium play and his best deviation. ■

Lemma 2 to lemma 4 show the equilibrium is unique in the class of all strategies available to the politicians. ■