Price bubbles in markets with inattentive consumers and imperfectly informed firms

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Abstract

We study a model of price competition in a homogeneous good market where consumers may be fully rational or inattentive to small price differences. At the beginning, firms are pricing at marginal cost, and receive a stochastic signal concerning consumers' rationality. They then compete for two periods, observing the market outcome at the end of the first. We characterize an equilibrium in which, when consumers are effectively inattentive (and at least one firm receives the correct signal), the market price jumps to the monopoly level by the second period. This is achieved after a first period in which the informed firms (those that received the correct signal) raise their prices just a little: through this mild price increase, these firms forward their signal to the uninformed firms, and they do so in a credible way, as this makes the actual consumers' status common knowledge. Our model adds insights on the dynamics through which firms may be able to exploit consumers' inattention to prices.

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1 Introduction

Upon purchasing a new car, we usually visit (and maybe revisit) several car dealers, and we carefully evaluate the various quotations before coming to a final decision. Admittedly, this is not what we do for most of the dozens small-value products that we purchase at almost daily frequence. Consider, for example, a consumer who needs to buy a case of beer of a certain brand for a barbecue. How does she choose where to shop it? Most likely, she will have a broad idea of a normal price for that product, and an imperfect memory of the approximate prices at which that product was sold in the shops she visited recently. Then, she will probably enter the shop where she remembers to have found the best price. Only if she discovers that the actual price is significantly higher than the price she had in mind, she will refrain from purchasing there to move to another shop.

Hence, it is realistic to think that, in many situations, even when products are essentially identical, consumers do not always choose the best price. Whether this follows from consumers' limited search or lack of attention, the consequence is clear: if firms are aware of consumers' inability to find the best deal, they have less of an incentive to undercut each other. Competition will thus be softened, and, in equilibrium, firms will be able to charge higher prices, which, in the most extreme case, may converge to the monopoly level.

However, what happens if only a few firms know or believe that consumers are partially insensitive to price differences? Are firms still able to coordinate on high prices? In this paper, we show that, under certain conditions, the answer is positive.

We consider a simple model of price competition in a homogeneous good market where consumers may potentially be subject to price *inattention*, i.e. they may be unable to detect small price differences, so that they may be indifferent between two firms that sell at different but sufficiently close prices.[1] The crucial element of our model, which distinguishes it from most of the related literature, is that firms are unaware of the actual state of consumers – whether they are fully rational or inattentive in the above sense. Ex ante, they deem consumers' inattention as an extremely unlikely event; however, they receive a randomly generated private signal that provides imperfect information on it. Firms then compete on prices for two consecutive periods, observing the market outcome at the end of the first period.

We identify a Perfect Bayesian equilibrium in which, when consumers are effectively inattentive (and at least one firm receives the correct signal), firms coordinate on the monopoly price by the end of the second period. The interesting feature of this equilibrium is that this final outcome is achieved after a transitory first period in which prices are raised just a little above marginal cost. Specifically, in the first period, the informed firms (those that received the correct signal, i.e. the signal that is more consistent with consumers being inattentive) increase their prices by a little amount, so that consumers would remain indifferent between buying from them or from the uninformed firms (that instead stick to the initial price level). The purpose of this period of mild price increase is twofold. On the one hand, informed firms forward to the other firms their information about the bounded rationality of consumers. On the other hand, this mild price increase prevents uninformed firms from mimicking informed ones: in fact, the resulting price differences are small enough that consumers, by purchasing indifferently from one firm or the other, reveal their actual status. As a result, any attempt by an uninformed firm to cheat the other firms (in order to convince them to increase the price in the second period) is fruitless. In contrast, we show that an immediate jump to the monopoly price already in the first period can hardly be sustained as an equilibrium, exactly because this would generate large price differences in the first period and, thus, the consumers' status would not become common knowledge.

The separating equilibrium described above exists under certain conditions on the model's parameters that can be broadly summarized as follows. First, the event that consumers are inattentive should be, ex ante, deemed as very unlikely: this condition ensures that, if there was no signal or if all firms receive the signal that is more consistent with consumers being fully

[1] This way of modelling price inattention is common to other papers in the literature, and can be thought of as a reduced form for different hypothesis on the consumer's cognitive decision process (see the discussion of the literature at the end of this section).

rational, firms have no incentive to deviate from marginal cost pricing. Second, when consumers are inattentive, firms' incentive to undercut each other should be sufficiently low. This condition holds when consumers are sufficiently insensitive to price differences, and is easier to be satisfied in a market where firms are producing close to their capacities.

Our model is related to the literature in behavioral industrial organization that explores how rational firms may take profits from consumers' limited attention. In this field, attention is a broad concept that can be defined as the extent to which an agent's cognitive process is able to make use of all available data. In the context of consumer choice, in particular, a lack of attention translates in a difficulty to make correct value comparisons between the various alternatives, and, in a homogeneous good market, in the inability to identify the lowest price.[2] There may be several reasons why, even with homogeneous goods, consumers may not always choose the best price. For example, consumers may see imaginary quality differences in products that are actually homogeneous (maybe because of adverting strategies), ending up paying a higher price for the product that they falsely judge to be superior (Bronnenberg et al., 2015). Also, consumers may fail to take into account additional costs beyond the base price, as shipping costs (Hossain and Morgan, 2006), taxes (Chetty et al., 2009), or the price of shrouded add-ons (see, e.g., Ellison and Ellison, 2009). In general, consumers may be confused when prices are complex, and in particular are vectors rather than scalars. Finally, consumers may devote most of their attention to other attributes beyond price, so that, once they have identified their preferred variety, they do no pay enough attention to the price differences among the suppliers of the chosen variety (see the discussion in Bachi, 2016).

Whichever the reason behind it, when firms are aware of consumers' limited attention to price differences, they will try to take advantage of it in order to raise markups. Assuming, as we do, that consumers can perfectly distinguish prices that are significantly different but choose randomly between similar prices, Bachi (2016) shows that, with two firms and homogeneous goods, the Nash equilibrium is in mixed strategies and entails strictly positive profits for the firms. Shilony (1977) considers horizontally differentiated products, in which each consumer is assumed to buy her preferred product unless the difference between its price and the price of the least expensive product exceeds a certain value. Like Bachi (2016), he shows that the equilibrium is in mixed strategies with positive markups. In Allen and Thisse (1992), instead, consumers are heterogenous in the threshold above which price differences matter. They show that, under certain conditions, there is a symmetric Nash equilibrium in pure strategies, with firms pricing at the monopoly price. In a model in which attention is endogenized – consumers bear an attention cost to refine their prior beliefs about the distribution of price and qualities in the market – Matějka and McKay (2012) show that, in equilibrium, firms have positive markups that increase with the cost of attention.[3] Overall, these results provide a theoretical foundation for the high markups that are often empirically observed even in industries with quite undifferentiated products and many competing firms (see, among others, Ausubel, 1991 for the credit card market).

These results are reminiscent of those arising from models with search cost. After all, when looking at their effects on the market, the difference between search costs and inattention is largely immaterial: in the former case, consumers do not possess all information and have to bear a cost to acquire it; in the latter, the information is available, but individuals do not fully elaborate it. Diamond (1971) already showed that it is enough that consumers have to bear a small search cost to learn prices to move the equilibrium to the monopoly price even with homogeneous goods. Stahl (1989) shows that, in a market with some consumers that are informed about all prices (the shoppers), and others who must pay a cost to obtain a nw quote (the non-shoppers), the equilibrium of the price competition game is in mixed strategies, giving rise to price dispersion. As the fraction of non-shoppers increase, the model converges to the Diamond (1971)'s model.

Given that consumers' inattention or search costs allow higher markups, other papers studied

[2]We refer to Grubb (2015) for an excellent review of the literature on this topic. See also Gabaix (2019) for a more general survey on behavioral inattention.

[3]In this respect, Matějka and McKay (2012) is a model of rational inattention: the consumer weighs benefits and costs of processing all available information to determine the optimal level of attention to dedicate to the decision problem. For a recent survey of the growing literature on rational inattention see Maćkowiak et al. (2023).

how firms can adopt strategies to increase search costs or to complicate product comparisons. These strategies may involve multi-dimensional pricing (Spiegler, 2006), price framing (Piccione and Spiegler, 2012, Chioveanu and Zhou, 2013), or obfuscation (Ellison and Wolitzky, 2012, Gu and Wenzel, 2014).

The common feature of the papers cited above is that they all assume that sellers are all equally informed about the behavior of the consumers in the market, which they then try to exploit. Instead, we explicitly allow asymmetric information across firms, in the form of a stochastic signal received at the beginning of the price competition game. Hence, the novelty of our model is to investigate whether and how firms may still be able to exploit consumers' inattention to coordinate on high prices, even if the information about consumer misbehavior is dispersed.

The rest of the article is organized as follows. Section 2 presents the model. The equilibrium of the price competition game, with and without signal, is characterized in Section 3. Section 4 discusses some issues related with the equilibrium. Section 5 concludes.

2 The model

The market for a homogeneous good is populated by a unit mass of identical consumers, each with unit demand and willingness to pay v for the good; and by $N \geq 3$ identical firms, each with constant marginal and average cost normalized to 0 and with product capacity k. We let $\hat{n}$ denote the integer such that $(\hat{n}-1)k < 1$ and $\hat{n}k \geq 1$: in words, to cover the whole market at least $\hat{n}$ firms are needed. We assume that $\hat{n} \leq N-2$, which implies that $k \geq 1/(N-2)$.[4]

Concerning the behavior of consumers, there are two possible states of the world. In state $s=0$, all consumers are fully rational and buy the good at the lowest available price (provided it does not exceed v); clearly, if, at that price, the supply of the good has been exhausted, they buy at the second lowest price; and so on. As usual, if two or more firms make the same price, consumers spread equally among them. In state $s=1$, instead, all consumers are *inattentive* in the sense that they have troubles in comparing prices when these are not too far apart, so that they may end up buying at the higher price. Specifically, if the two lowest prices available differ by no more than $\Delta > 0$, consumers behave as if these two prices were identical and spread equally among the firms setting those prices. We assume that $\Delta < v/2$. Initially, firms do not know the state of the world s and hold prior beliefs that $s=1$ with probability μ (so that $1-\mu$ is the probability that $s=0$).

Firms compete on prices for two consecutive periods, denoted $t=1$ and $t=2$, discounting second-period profits at the discount rate $\beta \in (0,1]$. We denote by p_t a generic price in period t (and we add the superscript i when we want to refer to the price set by a specific firm i). At the beginning of the game (say period $t=0$), the market is already open and firms are currently pricing at marginal cost (i.e. $p_0^i = 0$ for all i). Firms then simultaneously set their prices in period $t=1$, and, after oberving the outcome of the competition − namely, all firms' prices and sales −, re-set prices in period $t=2$. We assume that, whenever a firm changes her price from one period to the next, she has to bear a (small) menu cost ω. It is worth remarking that firms, at the end of period $t=1$, observe not only all firms' prices but also their sales: this may sometimes allow them to perfectly infer the state of the world. In fact, suppose that two or more firms set prices that differ by no more than Δ: if these firms sell the same (strictly positive) amount (lower than k), this is clean evidence that the state of the world is $s=1$.

Before simultaneously choosing their prices in $t=1$, each firm receives a private signal θ, that can take one of two values: either 0 or 1. The signals received by the firms are independent realizations of either of the following signal technologies:

$$\Pr(\theta=0|s=0)=1, \qquad \Pr(\theta=1|s=1)=\alpha.$$

The signal observed by a firm is her *type*. Notice that, when consumers are rational ($s=0$), firms always receive the correct signal ($\theta=0$), whereas, when consumers are inattentive, any firm

[4]For example, if there are 10 firms in the market, then a firm's capacity must be greater than or equal to 1/8, so that 8 firms are certainly able to serve all consumers (and, possibly, less than 8 are enough).

receive the correct signal ($\theta = 1$) with probability α, which is the precision of the signal in state $s = 1$.

The signal allows a firm to update her belief on the true state of the world. It is immediate to see that, upon receiving a signal $\theta = 1$, a firm becomes certain that $s = 1$. On the other hand, a firm that receives a signal $\theta = 0$ believes that:

$$\Pr(s = 1|\theta = 0) = \frac{(1-\alpha)\mu}{1-\alpha\mu} \equiv z. \tag{1}$$

Notice also that, after observing the outcome of the first-period competition, firms can further update their beliefs on the other firms' types and, thereby, on the state of the world.

We also make the following assumptions on the model parameters:

(A1) $\mu\,\Delta/(N-1) - \omega \leq 0$;

(A2) $v/N - \omega \geq \Delta k$;

(A3) $v/N \geq (v - \Delta)k$;

(A4) $\mu\,[\Delta/N + \beta\,(v/N - \omega)] - \omega \leq 0$.

Before moving to the equilibrium analysis, it is worth discussing the assumptions of our model. With respect to a standard textbook Bertrand competition game, our model involves a capacity constraint k (which, however, is not binding when all firms set the same price) and a (small) menu cost ω associated with any price change. These assumptions allow us to avoid the unrealistic implications of a standard price competition game with homogeneous goods, while retaining its simplicity. In particular, the fact that firms are capacity constrained reduces their incentive to undercut each other. The presence of a menu cost implies that pricing at marginal cost − the equilibrium of the standard Bertrand game − is no longer a weakly dominated strategy and introduces some inertia in prices: firms cut or increase prices only if they expect a non-negligible increase in their gross profits. These assumptions – together with the additional assumptions (A1)-(A4) – guarantee that, even with possibly inattentive consumers, the initial market configuration where firms price at marginal cost is a stable one, as we show in the next section.

3 Equilibrium analysis

In this section, we characterize the equilibrium of the game described in the previuos section, both with and without signal. We restrict our analysis to symmetric Perfect Bayesian equilibria in pure strategies.

The first result that we show is that, if there was no signal (or, equivalently, if the signal were totally uninformative), the initial market configuration where firms price at marginal cost would be a stable situation, in the sense that firms would go on pricing at marginal cost also in period $t = 1$ and $t = 2$.

PROPOSITION 1. *With no signal, there is an equilibrium in which all firms' prices are equal to marginal cost in both periods.*

Proof. As a first step, we characterize firms' equilibrium behavior in $t = 2$. Clearly, it is enough to concentrate on the information sets that are reachable after at most one deviation occurred in $t = 1$.

(i) In $t = 1$, all firms chose $p_1 = 0$ (we are on the equilibrium path). If all firms follow the equilibrium strategies (i.e. $p_2^i = 0$ for all i), each firm's second period profit is equal to 0. Now, suppose firm d deviates. There are two types of deviations to be considered: (a) if firm d deviates to $p_2^d > \Delta$, she will sell nothing (even inattentive consumers would rather buy from the other $N-1$ firms, that are indeed able to serve all the market), so her second period profit would be $-\omega < 0$. Hence, this deviation is unprofitable; (b) if firm d deviates to $p_2^d \leq \Delta$, she will sell nothing if consumers

are rational, but she will sell $1/N$ if consumers are inattentive. Clearly, within this second type of deviations, the most profitable one is to set p_2^d exactly equal to Δ, in which case, firm d's expected profit in period $t = 2$ would be

$$\pi_2^d(\Delta) = \mu \frac{\Delta}{N} - \omega.$$

Assumption (A1) ensures that also such a deviation is unprofitable.

(ii) *In $t = 1$, firm d deviated to $p_1^d > \Delta$ (while all other firms $i \neq d$ set $p_1^i = 0$).* Our claim is that, in $t = 2$, firms will not change their prices, i.e. $p_2^d = p_1^d$, $p_2^i = p_1^i = 0$ for $i \neq d$. To see this, consider firm d: by choosing $p_2^d = p_1^d$, her second period profit is equal to 0. The most profitable alternative would be to set $p_2^d = \Delta$, with second period profit $\pi_2^d(\Delta) = \mu\,\Delta/N - \omega$. Again, assumption (A1) ensures that such a deviation is unprofitable. Consider now any firm $i \neq d$. Again, the most profitable alternative to sticking to the first period price is $p_2^i = \Delta$, that would yield firm i a second period profit equal to $\pi_2^i(\Delta) = \mu\,\Delta/(N-1) - \omega$. Assumption (A1) ensures that also such a deviation is unprofitable.

(iii) *In $t = 1$, firm d deviated to $p_1^d = \Delta$ (while all other firms $i \neq d$ set $p_1^i = 0$).* In this case, at the end of the first period the state of the world will become common knowledge: if firm d sells $1/N$, it has to be the case that consumers are inattentive, if she does not sell anything, then it means that consumers are rational. Now, if $s = 0$, it is straightforward to see that firms will stick to the first period prices. If, instead, $s = 1$, we claim that all firms will charge the monopoly price v. Doing so, each firm's second period profit would be $\pi_2(v) = v/N - \omega$. For firms $i \neq d$, this is clearly better than sticking to the first period price $p_1^i = 0$ (as is trivially implied by (A2)). For firm d, assumption (A2) guarantees that this is better than sticking to the first period price $p_1^d = \Delta$, that would yield a second period profit equal to $\pi_2^d(\Delta) = \Delta k$. For each firm j, we also have to consider $p_2^j = v - \Delta - \varepsilon$ as a potentially attractive alternative (when all other firms set $p_2 = v$): this is the highest price such that a firm is able to sell up to her capacity. The associated second period profit would be $\pi_2^j(v - \Delta - \varepsilon) = (v - \Delta - \varepsilon)k - \omega$. Assumption (A3) ensures that this deviation is unprofitable. Notice, finally, that any other first period deviation $p_1^d \in (0, \Delta)$ is dominated by $p_1^d = \Delta$.

As a second and final step, we check that, in $t = 1$, and taking into account the equilibrium behavior in $t = 2$ just characterized, deviations from the equilibrium strategies are not profitable. Now, suppose firm d deviates to $p_1^d > \Delta$: in this case, her profit is equal to $-\omega$ (she sells nothing in both periods, but she bears the menu cost in the first). If, instead, firm d deviates to $p_1^d = \Delta$ (again, any deviation $p_1^d < \Delta$ is dominated), her total profit would be

$$\pi^d(\Delta) = \mu \left[\frac{\Delta}{N} + \beta\left(\frac{v}{N} - \omega\right)\right] - \omega.$$

Assumption (A4) ensures that such a deviation is unprofitable. □

The intuition behind Proposition 1 is the following. Suppose a firm considers slightly increasing the price above marginal cost. This would involve a *certain* cost (the menu cost ω), and a *potential* gain, but only if consumers are inattentive. In this case, in fact, the firm would still sell a positive amount ($1/N$) in the first period (at a higher price); moreover, at the end of the first period, the mere fact that this firm sold a positive amount will reveal to all firms that consumers are indeed inattentive, which will then lead firms to coordinate on the monopoly price v in period $t = 2$ (assumptions (A2) and (A3) guarantee that this is the optimal thing to do when it is known that consumers are inattentive). However, the prior probability that consumers are indeed inattentive is so low that it does not pay for a firm to make such a price increase in the first place (see assumptions (A1) and (A4)).

Things are different when firms receive a signal at $t = 0$. In this case, not only they can immediately update their beliefs on the underlying state of the world, but can also transmit their private information through their first period choices. This may allow them to converge to the monopoly price v by the second period, as the following Proposition, which is the main result of our paper, shows.

Proposition 2. *If firms receive a signal and if*

$$\beta\,(1-\alpha)^{N-1}\left(\frac{v}{N} - \omega\right) + \frac{\Delta}{N} - \omega \geq 0, \tag{2}$$

then there is an equilibrium in which:

- *in the first period, firms that receive signal $\theta = 1$ choose $p_1 = \Delta$, whereas firms that receive signal $\theta = 0$ choose $p_1 = 0$;*
- *if at least one firm chose $p_1 = \Delta$ in the first period, all firms choose $p_2 = v$ in the second; if all firms chose $p_1 = 0$ in the first period, all firms choose $p_2 = 0$ in the second.*

To sustain this equilibrium, when, in $t = 1$, there is one firm d that deviates to a price $p_1^d > \Delta$, whereas all other firms choose $p_1 = 0$, the latter firms must believe that firm d's type is $\theta = 1$ with probability $\gamma(p_1^d) \le \mu\left[1 - (1-\alpha)^N\right]$.

Proof. As a first step, we characterize firms' equilibrium behavior in $t = 2$ for all the information sets that are reachable after at most one deviation in $t = 1$.

(i) *In $t = 1$, all firms chose $p_1 = 0$.* Consider one such firm, say i, and suppose this firm received the signal $\theta = 0$ (hence, this firm has followed her equilibrium strategy in $t = 1$). Belief consistency requires that firm i believes that all other firms got $\theta = 0$, and updates her belief on the state of the world accordingly. Clearly, firm i's updated belief that $s = 1$ is certainly lower than the prior belief μ.[5] Hence, given that choosing $p_2 = 0$ was optimal with beliefs μ (see point *(i)* in the proof of Proposition 1), it is a fortiori optimal now.
Suppose now that firm i actually received signal $\theta = 1$, i.e. firm i deviated in $t = 1$ mimicking a type-0 firm. This deviation is undetectable by other firms, that will go on playing $p_2 = 0$ in $t = 2$. What will firm i then do? She knows that the true state is $s = 1$, so she may also consider choosing $p_2^i = \Delta$ to get second-period profit $\pi_2^i(\Delta) = \Delta/N - \omega$. Hence, such firm will choose $p_2^i = \Delta$ or $p_2^i = 0$ depending on whether $\Delta/N - \omega$ is positive or negative.

(ii) *In $t = 1$, at least one firm chose $p_1 = 0$, and at least one firm chose $p_1 = \Delta$.* In this case, the true state of the world will become common knowledge at the end of the first period. If it is $s = 1$, then the second-period equilibrium is for all firms to choose $p_2 = v$, with second-period profits

$$\pi_2(v) = \frac{v}{N} - \omega.$$

Assumption (A2) guarantees that a unilateral deviation by a certain firm d to $p_2^d = 0$ is unprofitable (the corresponding profit would be 0 if firm d chose $p_1^d = 0$ and $-\omega$ if firm d chose $p_1^d = \Delta$). Assumption (A2) guarantees also that a unilateral deviation to $p_2^d = \Delta$ is unprofitable (the corresponding profit would be Δk if firm d chose $p_1^d = \Delta$ and $\Delta k - \omega$ if firm d chose $p_1^d = 0$). Finally, we have to consider the deviation $p_2^d = v - \Delta - \varepsilon$ (the highest price such that a firm is able to sell up to capacity). Assumption (A3) guarantees that also this deviation is unprofitable.
If, instead, the state of the world is $s = 0$ (which means that all firms got signal $\theta = 0$, but one of such firms deviated to $p_1 = \Delta$), then it is clear that firms will stick to the first-period prices.

(iii) *In $t = 1$, all firms chose $p_1 = \Delta$.* Consider one such firm, say i, and suppose this firm received the signal $\theta = 1$ (hence, this firm has followed her equilibrium strategy in $t = 1$). Firm i knows that the true state is $s = 1$, and believes that all other firms believe that $s = 1$ (from belief consistency). Hence, firm i expects all other firms to choose $p_2 = v$, and, accordingly, will find it optimal to choose $p_2 = v$ as well (the argument is the same as in point *(ii)* above).
Suppose, instead, that firm i received the signal $\theta = 0$ i.e. firm i deviated in $t = 1$ mimicking a type-1 firm. From belief consistency, firm i believes that $s = 1$, and believes that all other firms believe that $s = 1$. Hence, firm i expects the other firms to choose $p_2 = v$, and, accordingly, will find it optimal to choose $p_2 = v$ as well (the argument is the same as in point *(ii)* above).

(iv) *In $t = 1$, there is a firm, say firm d, that chose $p_1^d \in (0, \Delta)$.* In this case, the true state of the world will certainly become common knowledge. If it is $s = 1$, then all firms, including firm d will choose $p_2 = v$, if it is $s = 0$ (which means that all firms, including d, got signal $\theta = 0$), then it is clear that firms will stick to the first-period prices (the argument is the same as in point *(ii)* above).

(v) *In $t = 1$, there is a firm, say firm d, that chose $p_1^d > \Delta$, while all other firms $i \neq d$ chose $p_1^i = 0$.* In this case, the true state of the world does not become common knowledge at the end of period $t = 1$: hence, all firms $i \neq d$ form beliefs about firm d's type (and, thereby, on the state of the

[5] Specifically, firm i believes that the state of the world is $s = 1$ with probability $\mu(1-\alpha)^N / \left[\mu(1-\alpha)^N + 1 - \mu\right]$, which is strictly lower than μ.

world). Let $\gamma(p_1^d)$ denote the (common) belief of firms $i \neq d$ that the deviating firm d is of type $\theta = 1$. Our claim is that, if $\gamma(p_1^d)$ is not too high, in equilibrium:

– firms $i \neq d$ stick to the first period price ($p_2^i = p_1^i = 0$);

– firm d will stick to the first period price ($p_2^d = p_1^d$) if her type is $\theta = 0$,

– firm d will choose $p_2^d = p_1^d$ or $p_2^d = \Delta$, depending on whether $\Delta/N - \omega$ is negative or positive, if her type is $\theta = 1$.

The argument that confirms that the one just described is the optimal behavior of firm d (when all other firms choose $p_2^i = 0$) is essentially the same as in point *(i)* above. If firm d is of type-0, she can't do any better than sticking to the first-period price p_1^d (charging the price $p_2 = \Delta$ hoping that the true state is $s = 1$ would not be worth paying the menu cost ω). If firm d is of type-1, she knows that $s = 1$, so she may considers setting $p_2 = \Delta$ to get the second-period profit $\pi_2 = \Delta/N - \omega$: if this profit is positive, this is preferable to sticking to the first-period price p_1^d.
Consider then any firm $i \neq d$. The only viable alternative to sticking to the first-period price is to set $p_2^i = \Delta$. This alternative would yield firm i an expected second-period profit that is *at most*

$$\pi_2^i(\Delta) \leq \left[\gamma(p_1^d) + \left(1 - \gamma(p_1^d)\right) \frac{\mu(1-\alpha)^N}{\mu(1-\alpha)^N + (1-\mu)}\right] \frac{\Delta}{N-1} - \omega,$$

where the expression within square brackets is the probability that firm i attaches to the state of the world being $s = 1$, conditional on her information available and on her belief regarding firm d's type. Assumption (A1) guarantees that, for sufficiently low $\gamma(p_1^d)$, the expression above is negative, so that firms $i \neq d$ actually find it optimal to stick to the first-period price. Specifically, if $\gamma(p_1^d) \leq \mu\left[1 - (1-\alpha)^N\right]$, then assumption (A1) implies that the expression above is negative.

(vi) *In $t = 1$, there is a firm, say firm d, that chose $p_1^d > \Delta$, and at least one firm $i \neq d$ that chose $p_1^i = \Delta$.* In this case, all firms believe that the state is $s = 1$ and the equilibrium behavior in the second period depends on the actual value of p_1^d. However, it is certainly the case that the second-period profit of the deviating firm d is at most v/N (this is achieved if firm d chose v in the first period, in which case all firms, including d, find it optimal to choose v in the second), which is the maximum per-period profit a firm can get when the state is known to be $s = 1$.

As a second and final step, we check that, in $t = 1$, for both firms' types, deviations from the equilibrium strategy are not profitable, taking into account the equilibrium behavior in $t = 2$ just characterized.

- Consider a firm, say i, of type $\theta = 1$. This firm knows that the true state is $s = 1$. If she obeys to her equilibrium strategy $p_1 = \Delta$, her expected profit is:

$$\pi_i(\Delta; \theta = 1) = \frac{\Delta}{N} - \omega + \beta\left(\frac{v}{N} - \omega\right).$$

If she deviates and play $p_1 = 0$ (i.e. she mimicks a firm of type $\theta = 0$), her profit is equal to 0 in $t = 1$; in $t = 2$, her profit will be equal to $\max\{\Delta/N - \omega, 0\}$ if all other firms chose $p_1 = 0$ (see point *(i)* above), to $v/N - \omega$ if at least another firm chose $p_1 = \Delta$ (see point *(ii)* above). Thus, firm i's expected profit is

$$\pi_i(0; \theta = 1) = \beta\left[(1-\alpha)^{N-1}\max\left\{\frac{\Delta}{N} - \omega, 0\right\} + \left[1 - (1-\alpha)^{N-1}\right]\left(\frac{v}{N} - \omega\right)\right].$$

It is immediate to see that a deviation to any price $p_1 \in (0, \Delta)$ is dominated by choosing exactly $p_1 = \Delta$. As far as prices $p_1 > \Delta$ is concerned, it is enough to concentrate on $p_1 = v$: in fact, all prices $p_1 > \Delta$ yield a null profit in $t = 1$, and $p_1 = v$ is the one that leads to the highest profit in $t = 2$ (see point *(v)* and *(vi)* above). Thus, firm i's expected profit $\pi_i(p_1 > \Delta; \theta = 1)$ is no greater than

$$\pi_i(v; \theta = 1) = -\omega + \beta\left[(1-\alpha)^{N-1}\max\left\{\frac{\Delta}{N} - \omega, 0\right\} + \left[1 - (1-\alpha)^{N-1}\right]\frac{v}{N}\right].$$

Notice that $\pi_i(p_1 > \Delta; \theta = 1) \leq \pi_i(v; \theta = 1) \leq \pi_i(0; \theta = 1)$. Hence, firm i, type $\theta = 1$, has no incentive to deviate from her equilibrium first period strategy if $\pi_i(\Delta; \theta = 1) \geq \pi_i(0; \theta = 1)$, or

$$\frac{\Delta}{N} - \omega \geq \beta\,(1-\alpha)^{N-1}\left[\max\left\{\frac{\Delta}{N} - \omega, 0\right\} - \left(\frac{v}{N} - \omega\right)\right],$$

which is certainly true under (2).

- Consider a firm, say i, of type $\theta = 0$. This firm's updated belief on the state of the world are given by (1). If she obeys to her equilibrium strategy $p_1 = 0$, her expected profit is:

$$\pi_i(0; \theta = 0) = \beta z \left[1 - (1-\alpha)^{N-1}\right] \left(\frac{v}{N} - \omega\right).$$

In the above expression, $z[1 - (1-\alpha)^{N-1}]$ is the probability that firm i, type $\theta = 0$, attaches to the event that at least one firm got signal $\theta = 1$. If, instead, firm i deviates and play $p_1 = \Delta$ (i.e. she mimicks a firm of type $\theta = 1$), her profit in the first period, gross of the menu cost ω, will be equal to Δ/N if the true state is $s = 1$, to 0 if the true state is $s = 0$. In the second period, the true state will become common knowledge (or, if all firms choose $p_1 = \Delta$, it will be commonly believed that the state is $s = 1$). Hence, firm i's expected profit is

$$\pi_i(\Delta; \theta = 0) = -\omega + z\left[\frac{\Delta}{N} + \beta\left(\frac{v}{N} - \omega\right)\right].$$

It is immediate to see that a deviation to any price $p_1 \in (0, \Delta)$ is dominated by choosing exactly $p_1 = \Delta$. As far as prices $p_1 > \Delta$ is concerned, it is enough to concentrate on $p_1 = v$: in fact, all prices $p_1 > \Delta$ yield a null profit (gross of the menu cost) in $t = 1$, and $p_1 = v$ is the one that leads to the highest profit in $t = 2$ (see point *(v)* and *(vi)* above). Thus, firm i's expected profit $\pi_i(p_1 > \Delta; \theta = 0)$ is no greater than

$$\pi_i(v; \theta = 0) = -\omega + \beta z\left[1 - (1-\alpha)^{N-1}\right]\frac{v}{N}.$$

Notice that $\pi_i(p_1 > \Delta; \theta = 0) \leq \pi_i(v; \theta = 0) \leq \pi_i(0; \theta = 0)$. Hence, firm i, type $\theta = 0$, has no incentive to deviate from her equilibrium first-period strategy if $\pi_i(0; \theta = 0) \geq \pi_i(\Delta; \theta = 0)$, or

$$z\left[\frac{\Delta}{N} + \beta(1-\alpha)^{N-1}\left(\frac{v}{N} - \omega\right)\right] - \omega \leq 0,$$

which is implied by assumption (A4) because $z < \mu$.

□

The separating equilibrium in Proposition 2 implies that, when consumers are indeed inattentive and at least one firm receives the signal $\theta = 1$ – these two events jointly occur with probability $\mu \times \left[1 - (1-\alpha)^N\right]$ – then the outcome of the market is that the price will reach the monopoly level in period $t = 2$. This occurs after a transitory period ($t = 1$) in which some firms (those that received the signal $\theta = 1$) increase their prices only slightly, by an amount Δ. Through this mild price increase in $t = 1$, firms that receive the signal $\theta = 1$ forward their private information to the other firms; and they do so in a credible way because, at the end of the first period, all firms will typically be able to objectively learn what the true state of the world is. As a matter of fact, when at least one firm sets $p_1 = \Delta$ and at least one firm sticks to $p_1 = 0$, the state of the world becomes common knowledge: if the sales of the firm(s) that chose $p_1 = \Delta$ are equal to the sales of the firm(s) that chose $p_1 = 0$, then this means that consumers are indeed inattentive. This phase of mild price increase is crucial as it represents a strong disincentive for a type $\theta = 0$ firm to mimick a type $\theta = 1$, thus allowing to sustain such a separating equilibrium. To see this, consider a firm, say firm i, that receives the signal $\theta = 0$. This firm may consider mimicking a type-1 firm, setting the price $p_1 = \Delta$, in order to convince the other firms that the true state is $s = 1$, leading them to set the monopoly price, and taking advantage of this second period of high prices. However, if the true state of the world is $s = 0$ (so that all other firms received the signal $\theta = 0$ and, thereby, set price $p_1 = 0$ in the first period), firm i's attempt to cheat on the others would fail, because, by observing the first-period's outcome, with firm i selling nothing, the other firms will be able to infer that the state of the world is $s = 0$, and will therefore keep low prices also in the second period.

4 Discussion

In this section, we tackle a number of issues related to the equilibrium presented in the previous section. We start by examining the conditions of existence of such an equilibrium. We then ask ourselves whether there can be another equilibrium in which firms still coordinate on the monopoly price but without passing through a transitory period of low prices. We finally address the question of the persistence of such high prices, by informally discussing what could happen in this market if we extended the time horizon beyond the two-periods considered so far.

4.1 Existence of the equilibrium

The equilibrium characterized in Proposition 2 exists under certain (sufficient) conditions, namely assumptions (A1)-(A4) and condition (2). What is the meaning of these conditions and how restrictive are they? Assumptions (A1) and (A4) are needed to guarantee that, when firms only know that that there is a probability μ that consumers are inattentive, they find it optimal to keep their prices at their initial level (equal to marginal cost). Clearly, these conditions are satisfied if μ, the ex-ante probability that consumers are inattentive, is sufficiently low. In other words, the event that consumers are inattentive should be deemed as a rare one. Assumptions (A2) and (A3), instead, are needed to ensure that, if firms get to know that consumers are inattentive, it is an equilibrium to charge the monopoly price. In particular, each firm must not find it profitable to undercut the other firms below the monopoly price in order to increase her sale up to her capacity k. Notice that, with inattentive consumers, undercutting means to make a discount of at least Δ (otherwise consumers would not see the price difference). For these conditions to be satisfied, k, the capacity of each firm, should be sufficiently low. In this respect, our model fits a market in which capacity constraints are not binding, but firms produce close to their capacities. Moreover, assumption (A3) requires Δ, which captures the degree of inattentiveness by consumers, to be sufficiently large.[6] Finally, condition (2) ensures that a firm that receives the signal $\theta = 1$ finds it optimal to increase her price to Δ in the first period to signal the state of the world to the other firms, instead of keeping her first-period price fixed at the initial level ($p_1 = 0$). Keeping the price fixed can be thought of as a free riding behavior: doing so, the firm saves on the menu cost, hoping that at least another firm got $\theta = 1$ and, accordingly, increases her price to signal that consumers are inattentive, leading the market to the high price in the second period anyway. Clearly, for such a free riding strategy to be unattractive, the probability that at least another firm gets the signal $\theta = 1$ must be sufficiently low, i.e. α, the precision of the signal in state $s = 1$, must not be too high. As an illustration, the following parameters satisfy all the assumptions (A1)-(A4) and condition (2) (and also condition (3) below): $v = 1$, $N = 10$, $k = 1/8$, $\beta = 1$, $\omega = 0.02$, $\Delta = 0.2$, $\mu = 0.05$, $\alpha = 0.6$.

The equilibrium characterized in Proposition 2, to be sustained, involves some beliefs. Specifically, upon observing, at the end of the first period, that one firm, say d, set a price $p_1^d > \Delta$, while all other firms $i \neq d$ chose $p_1^i = 0$, the equilibrium requires that each firm i believes that firm d is of type $\theta = 1$ with probability $\gamma(p_1^d) \leq \mu\left[1 - (1-\alpha)^N\right]$. Notice that $\mu\left[1 - (1-\alpha)^N\right]$ is lower than μ, which, in turn, should be a sufficiently low number (see the discussion above). Hence, each firm i must believe that firm d's type is very likely to be $\theta = 0$. Are these beliefs reasonable? To address this point, let's apply the following logic: upon observing a deviation off-the-equilibrium, it is reasonable to think that such a deviation came from the type that had more to gain from it.[7] To this end, let's compute the maximum profit that each firm's type can obtain from such deviations, and compare it with the equilibrium profit.

Now, the equilibrium profit of a type-1 firm is

$$\pi(\Delta; \theta = 1) = \frac{\Delta}{N} - \omega + \beta\left(\frac{v}{N} - \omega\right),$$

[6]More precisely, Δ must sufficiently high to satisfy assumption (A3), but not too high to also satisfy (A2). Notice however that, realistically, it is (A3) the condition that is harder to satisfy.

[7]This logic is at the heart of the intuitive criterion by Cho and Kreps (1987).

whereas the equilibrium profit of a type-0 firm is

$$\pi(0;\theta=0)=\beta z\left[1-(1-\alpha)^{N-1}\right]\left(\frac{v}{N}-\omega\right).$$

It is immediate to see that, under condition (2), $\pi(\Delta;\theta=1)>\pi(0;\theta=0)$.

What is, instead, the maximum profit a firm can hope to gain by deviating to a price $p_1^d>\Delta$? It is quite clear that such maximal profit is obtained when, in the second period, all other firms set the monopoly price v (and, of course, the deviating firm chooses her second-period price optimally).

Now, suppose the deviating firm d is of type $\theta=1$. If firm d chose $p_1^d=v$ in the first period (and supposing that all other firms will choose $p_2=v$ in the second period), the optimal second-period price for firm d is $p_2^d=v$. Hence, the maximum profit firm d can hope to gain by deviating to $p_1^d=v$ in the first period is

$$\pi^{\max}(p_1^d=v;\theta=1)=-\omega+\beta\,\frac{v}{N}.$$

If $v-\Delta\le p_1^d<v$, the optimal second-period price for firm d is either $p_2^d=v$ or $p_2^d=p_1^d$, depending on which price yields the higher profit. Hence, the maximum profit firm d can hope to gain by deviating to $v-\Delta\le p_1^d<v$ in the first period is

$$\pi^{\max}(v-\Delta\le p_1^d<v;\theta=1)=\max\left\{-\omega+\beta\left(\frac{v}{N}-\omega\right),-\omega+\beta\,\frac{p_1^d}{N}\right\}.$$

If $\Delta<p_1^d<v-\Delta$, the optimal second-period price for firm d is either $p_2^d=v$ or $p_2^d=p_1^d$, depending on which price yields the higher profit. Hence, the maximum profit firm d can hope to gain by deviating to $\Delta<p_1^d<v-\Delta$ in the first period is

$$\pi^{\max}(\Delta<p_1^d<v-\Delta;\theta=1)=\max\left\{-\omega+\beta\left(\frac{v}{N}-\omega\right),-\omega+\beta\,p_1^d\,k\right\}.$$

Suppose then that the deviating firm d is of type $\theta=0$. If firm d chose $p_1^d=v$ in the first period (and supposing that all other firms will choose $p_2=v$ in the second period), the optimal second-period price for firm d is either $p_2^d=v$ or $p_2^d=v-\varepsilon$, depending on which price yields the higher profit. Hence, the maximum profit firm d can hope to gain by deviating to $p_1^d=v$ in the first period is

$$\pi^{\max}(p_1^d=v;\theta=0)=\max\left\{-\omega+\beta\,\frac{v}{N},-\omega+\beta\left[(v-\varepsilon)\left(z\cdot\frac{1}{N}+(1-z)\cdot k\right)-\omega\right]\right\}.$$

If $v-\Delta\le p_1^d<v$, the optimal second-period price for firm d is either $p_2^d=v$ or $p_2^d=p_1^d$ or $p_2^d=v-\varepsilon$, depending on which price yields the highest profit. Hence, the maximum profit firm d can hope to gain by deviating to $v-\Delta\le p_1^d<v$ in the first period is

$$\pi^{\max}(v-\Delta\le p_1^d<v;\theta=0)=$$
$$\max\left\{-\omega+\beta\left(\frac{v}{N}-\omega\right),-\omega+\beta p_1^d\left(z\frac{1}{N}+(1-z)k\right),-\omega+\beta\left[(v-\varepsilon)\left(z\frac{1}{N}+(1-z)k\right)-\omega\right]\right\}.$$

If $\Delta<p_1^d<v-\Delta$, the optimal second-period price for firm d is either $p_2^d=v$ or $p_2^d=p_1^d$ or $p_2^d=v-\varepsilon$, depending on which price yields the highest profit. Hence, the maximum profit firm d can hope to gain by deviating to $v-\Delta\le p_1^d<v$ in the first period is

$$\pi^{\max}(v-\Delta\le p_1^d<v;\theta=0)=$$
$$\max\left\{-\omega+\beta\left(\frac{v}{N}-\omega\right),-\omega+\beta\,p_1^d\,k,-\omega+\beta\left[(v-\varepsilon)\left(z\cdot\frac{1}{N}+(1-z)\cdot k\right)-\omega\right]\right\}.$$

It is straightforward to see that, for all $p_1^d>\Delta$, $\pi^{\max}(p_1^d;\theta=0)\ge\pi^{\max}(p_1^d;\theta=1)$. Hence, $\pi^{\max}(p_1^d;\theta=0)-\pi(0;\theta=0)>\pi^{\max}(p_1^d;\theta=1)-\pi(\Delta;\theta=1)$: a type-0 has strictly more to gain from deviating to p_1^d than a type-1 firm. Hence, the beliefs required to sustain the equilibrium in Proposition 2 seem reasonable.

4.2 Why don't firm jump to the monopoly price immediately?

The equilibrium characterized in Proposition 2 involves a transitory period in which firms that receive the signal that consumers are inattentive increase their prices by a little amount. Then, in the second period, all firms increase their prices to the monopoly level. One could wonder why firms do not jump immediately (i.e. in period $t = 1$) to the monopoly price: this seems attractive as it would involve a single price change (avoiding to pay the menu cost twice), and possibly a higher profit already in the first period (if enough firms set the price v). Specifically, the question is whether, beyond the equilibrium of Proposition 2, there exists also an equilibrium in which firms that receive the signal $\theta = 1$ set the monopoly price v already in $t = 1$. It turns out that the answer is (essentially) negative, as the following Proposition shows.

PROPOSITION 3. *If firms receive a signal and if*

$$\beta\frac{v}{N} - \omega > 0, \tag{3}$$

then it is not an equilibrium for firms to choose, in the first period, $p_1 = v$ when they receive the signal $\theta = 1$, and $p_1 = 0$ when they receive the signal $\theta = 0$.

Proof. Suppose, by contradiction, that there is an equilibrium E in which, in $t = 1$, firms choose $p_1 = v$ when they receive the signal $\theta = 1$, and $p_1 = 0$ when they receive the signal $\theta = 0$.

Consider a certain firm, say i, of type $\theta = 1$. If this firm obeys to the equilibrium (choosing $p_1^i = v$) and all other firms also do, her expected profit is

$$\pi_i(v;\theta = 1) = \sum_{j=0}^{\hat{n}-1} \Pr(n_0 = j|s = 1)\frac{1 - jk}{N - j}v - \omega + \beta\frac{v}{N},$$

where n_0 denotes the number of (other) firms that received the signal $\theta = 0$ (notice that n_0 is a random variable with binomial distribution with parameters $N - 1$ and $1 - \alpha$). Notice that, in the second period, all firms believe that the state of the world is $s = 1$ (this follows from belief consistency) and therefore will set $p_2 = v$ (the argument is essentially the same as in point *(ii)* in the proof of Proposition 2). If, instead, firm i deviates mimicking a type $\theta = 0$ firm (i.e. she chooses $p_1^i = 0$), then, in the second period: if all firms other than i chose $p_1 = 0$, then these firms will set $p_2 = 0$, and firm i, that knows that $s = 1$, will either set $p_2^i = \Delta$ or stick to the first-period price $p_2^i = p_1^i$, depending on whether $\Delta/N - \omega$ is greater or lower than 0 (see point *(i)* in the proof of Proposition 2); if, instead, at least one firm other than i chose $p_1 = v$, then all firms believe that the state of the world is $s = 1$ (this follows from belief consistency) and therefore will set $p_2 = v$ (the argument is essentially the same as in point *(ii)* in the proof of Proposition 2). Therefore, firm i's expected profit is

$$\pi_i(0;\theta = 1) = \beta\left[(1 - \alpha)^{N-1}\max\left\{\frac{\Delta}{N} - \omega, 0\right\} + \left[1 - (1 - \alpha)^{N-1}\right]\left(\frac{v}{N} - \omega\right)\right].$$

Hence, for E to be an equilibrium, it must be that $\pi_i(v;\theta = 1) \geq \pi_i(0;\theta = 1)$, which implies that, necessarily,

$$\beta\frac{v}{N} - \omega \geq \beta\left[1 - (1 - \alpha)^{N-1}\right]\left(\frac{v}{N} - \omega\right) - \sum_{j=0}^{\hat{n}-1} \Pr(n_0 = j|s = 1)\frac{1 - jk}{N - j}v. \tag{4}$$

Consider now a certain firm, say i, of type $\theta = 0$. If this firm obeys to the equilibrium (and all other firms also do), her expected profit is (see the last bullet in the proof of Proposition 2)

$$\pi_i(0;\theta = 0) = \beta z\left[1 - (1 - \alpha)^{N-1}\right]\left(\frac{v}{N} - \omega\right).$$

If, instead, firm i deviates mimicking a type $\theta = 1$ firm (i.e. she chooses $p_1^i = v$), then, in the second period, all firms other than i will set $p_2 = v$ (from belief consistency, they are led to believe that the state is $s = 1$). What about firm i? If at least one firm other than i chose $p_1 = v$, then firm i herself believe that the state is $s = 1$ and will thus choose $p_2^i = v$. If, instead, all firms other than i chose $p_1 = 0$, then firm i's updates her belief on the state of the world, and, knowing that all other firms will choose $p_2 = v$, will choose either $p_2 = v$ (in this case, she will certainly sell $1/N$) or $p_2 = v - \varepsilon$ (in this case, she will sell

$1/N$ if the state is $s = 1$, k if the state is $s = 0$), depending on which alternative yields the higher profit. Therefore, firm i's expected profit is

$$\pi_i(v; \theta = 0) = \sum_{j=0}^{\hat{n}-1} \Pr(n_0 = j|\theta = 0) \frac{1 - jk}{N - j} v - \omega + \beta \left[(1 - \Pr(n_0 = N - 1|\theta = 0)) \frac{v}{N} + \right.$$

$$\left. \Pr(n_0 = N - 1|\theta = 0) \max \left\{ \frac{v}{N}, (v - \varepsilon - \omega) \left(\frac{\mu (1 - \alpha)^N}{\mu (1 - \alpha)^N + (1 - \mu)} \frac{1}{N} + \frac{1 - \mu}{\mu (1 - \alpha)^N + (1 - \mu)} k \right) \right\} \right].$$

Notice that, in the above expression, from the point of view of firm i, n_0 – the number of (other) firms that received the signal $\theta = 0$ – is a random variable with binomial distribution with parameters $N - 1$ and $\Pr(\theta_j = 0|\theta_i = 0)$. Notice, in particular, that, if the state of the world is $s = 0$, then $\Pr(n_0 = j) = 0$ for all $j \leq \hat{n} - 1$, because, with $s = 0$, all $N - 1$ firms necessarily receive the signal $\theta = 0$ (and $\hat{n} < N$). Hence, for all $j \leq \hat{n} - 1$, it is $\Pr(n_0 = j|\theta = 0) = z \cdot \Pr(n_0 = j|s = 1)$.
For E to be an equilibrium, it must be that $\pi_i(0; \theta = 0) \geq \pi_i(v; \theta = 0)$, which implies that, necessarily,

$$\beta \frac{v}{N} - \omega \leq z \cdot \left[\beta \left[1 - (1 - \alpha)^{N-1} \right] \left(\frac{v}{N} - \omega \right) - \sum_{j=0}^{\hat{n}-1} \Pr(n_0 = j|s = 1) \frac{1 - jk}{N - j} v \right]. \tag{5}$$

Under (3), and given that $z \in (0, 1)$, conditions (4) and (5) cannot hold simultaneously, which contradicts the fact that E is an equilibrium. We thus conclude that there is no equilibrium in which, in $t = 1$, firms that receive the signal $\theta = 1$ choose $p_1 = v$, whereas firms that receive the signal $\theta = 0$ choose $p_1 = 0$. □

The intuition behind Proposition 3 is the following. If firms chose $p_1 = v$ when they receive the signal $\theta = 1$ and $p_1 = 0$ when they receive the signal $\theta = 0$, then a price equal to v in the first period would be interpreted as a signal that the state is $s = 1$. Given this, a type-0 firm would have a strong incentive to mimick a type-1 firm (choosing $p_1 = v$), because such a deviation would be undetectable by the other firms, that will be then induced to match the monopoly price in $t = 2$, even if the true state is actually $s = 0$. Notice that, on the contrary, in the equilibrium of Proposition 2, an attempt of a type-0 firm to cheat on the others by mimicking a type-1 firm would be fruitless, as the other firms are able to detect this deviation by observing the outcome of the market at the end of the first period. Hence, the transitory period of mild price increase, though costly (or at least only a little profitable) for the firms that do increase their prices[8] is necessary to transmit in a credible way the information that consumers are inattentive.

4.3 Persistence of high prices

Our model shows how, when consumers are inattentive to small price differences (and at least one firm is informed about that), the market price will increase to the monopoly level by the second period. However, having only two-periods, the model does not say anything on the persistence of this high price. What should we expect to happen if we extend the time horizon?

It is quite clear that, as long as consumers are inattentive (and firms know it) and, in addition, the number of active firms is unchanged, firms have no reason to reduce their prices. In other words, for prices to start diminishing, it has to be the case that either firms' expectation concerning consumers' rationality change; or that new firms, attracted by the possibility of high profits, enter the market.

Suppose, for example, that, at the beginning of an hypothetical third period, for some reasons, firms are no longer sure that consumers are still inattentive, but beliefs that it is possible that consumers have become rational again. Clearly, this increases the potential gain from undercutting, so that, if firms attach a sufficiently high probability that consumers are rational, prices will start diminishing.

Similarly, suppose that, at the beginning of an hypothetical third period, new firms, attracted by the high prices (and therefore profits) achievable in that market, start entering. With more

[8]These firms get $\Delta/N - \omega$ in that period, which is, at best, a small profit, and possibly negative.

firms in the market, sharing the monopoly profits becomes less attractive, whereas the profits obtainable by undercutting is unchanged. Hence, when enough firms enter the market, prices will start go down, which, in turn, will undo the incentive of further entries.

5 Concluding remarks

This paper provides a signaling behavioral model showing how non-colluding firms may coordinate on high prices, taking advantage of consumers' inattention to small price differences. The novelty of our model with respect to the existing literature is that consumers' inattention is only a possibility and firms receive a binary signal which provides stochastic information on consumers' rationality, so that they may be asymmetrically informed on it.[9] We showed that, under certain conditions, when consumers are indeed inattentive, all firms can still be able to coordinate on the monopoly prices by the second period of competition. This is achieved through a separating equilibrium where, in the first period, firms that receive the signal that is more consistent with consumers being inattentive slightly increase their prices above marginal cost. Doing so, these firms forward their signal to the others, and they do so in a credible way: in fact, since these firms increase their prices only a little, all firms, by observing the first period market outcome, are able to objectively infer the actual degree of rationality of consumers, and can confidently coordinate on the monopoly price in the second period.

That consumers' inattention may be a factor in explaining why prices may deviate from marginal costs even in markets with quite undifferentited products is testified by the vast literature on this topic. Our paper adds insights on the mechanics that may lead to such price bubbles. In particular, our results predict that, so long as consumers are fully rational (or firms believe they are so), prices remain stable and low. However, if, at some point, consumers become (temporarily) insensitive to small price differences,[10] it is necessary and sufficient that some (even only one) firms realize it to trigger a (gradual) transition towards high prices.

In the model, we assumed that the signal is generated exogenously and costlessly. In the real world, this need not be the case. A firm may possibly get information on consumers' inattention from various sources: observation of consumers behavior in other markets where the firm operates, marketing research, consultants, even newspapers. Collecting this information is likely to be a deliberate choice by the firm, which may involve effort and monetary costs, as well as expertise to interpret this information. In this respect, we expect that endogenizing the signal will reduce the likelihood to observe such price bubbles. In fact, given that the information that consumers are inattentive is like a public good – all firms benefit from it and, in the equilibrium, it is enough that one firm knows it to spread this information to the others –, when it is costly to obtain such information, firms will be tempted to free ride, leading to an underinvestment in information collection. We leave this question open for future research.

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[9]In the model, by simplicity, we assumed that a firm that observes the signal $\theta = 1$ becomes certain that consumers are inattentive. This assumption can be relaxed without affecting the qualitative features of the results.

[10]The transition from full rationality to partial price inattention may be triggered by some exogenous shock. For example, when a change of currency occurs, it takes time and effort for consumers to get used to the new unit, so that recognizing price differences may be subject to errors. More generally, to the extent that consumers base their purchasing decisions also on memory and possibly reference prices, price instability in a market (maybe due to volatility in the cost of some relevant input) may jeopardize consumers' ability to effectively identify the best deal.

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