Vertical Contracting with Price Caps*

Jan Boone[†] Florian Schuett[‡] Emanuele Tarantino[§]

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Abstract

Self-imposed price caps have recently been proposed as a solution to competition problems arising, in particular, in the context of mergers, patent pools, or standardization. A common feature of many of the relevant settings is that they involve vertical relationships. Using a standard vertical-contracting framework, we show that price caps can be used by upstream firms to solve the opportunism problem of Hart and Tirole (1990) and thus restrict the quantity supplied to the downstream market. Our results imply that price caps can be anticompetitive in vertically related industries.

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JEL classification: D42, D43, K21, L12, L13, L41, L42

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[†]Tilburg University. E-mail: `j.boone@tilburguniversity.edu`.

[‡]KU Leuven and Tilburg University. E-mail: `florian.schuett@kuleuven.be`.

[§]Luiss University and EIEF. E-mail: `etaranti@gmail.com`

1 Introduction

Cooperative arrangements among firms often have ambiguous welfare effects. Mergers and joint marketing agreements, such as patent pools, reduce prices and raise welfare if products are complements but have the opposite effect if products are substitutes. Standardization can lead to enhanced network effects and reduced duplication of R&D investments but, by selecting one particular technology as the standard and eliminating alternatives, can also create artificial monopoly power for the holders of standard-essential patents (SEPs). Determining whether products are complements or subsitutes, and thus whether a merger or patent pool should be allowed, tends to be a difficult exercise for competition authorities. And although commitments to license on "fair, reasonable, and non-discriminatory" (FRAND) terms, if interpreted as an incremental-value rule, could in principle curb the monopoly power of SEP holders, it is hard for outsiders, such as courts, to assess what the incremental value of a technology is.

Self-imposed price caps have recently been proposed as a regulatory instrument to address these problems. Rey and Tirole (2019) make the case for price-cap agreements as an alternative to a merger or joint marketing agreement. According to their proposal firms would agree on price caps for their various products but otherwise retain control, in particular over pricing. Rey and Tirole show that, under quite general conditions, price-cap agreements solve the Cournot complements problem without harming competition when products are substitutes. In the standard-setting context, Lerner and Tirole (2015) advocate structured price commitments prior to standardization, whereby patent holders commit to the maximum royalties they would practice if selected into the standard. Lerner and Tirole show that price commitments restore the competitive benchmark of royalties. In both cases, self-imposed price caps place low informational requirements on public authorities, which do not have to determine caps themselves.[1]

Although mandating price caps is not currently part of the standard toolbox of competition authorities, in practice, price caps have surfaced under a number of guises. Patent pools with rules that require independent licensing and unbundling (rules which the European Commission's guidelines on technology transfer agreements specify as a safe harbor) essentially create the possibility for firms to commit to price caps.[2] Some standard-setting organizations (SSOs) already require ex ante price commitments (e.g., VITA, an SSO setting standards for computer architecture). And in the Rambus case, responding to the

[1] Rey and Tirole (2019) suggest that competition authorities could approve price-cap agreements through business review letters.

[2] Unbundling refers to the requirement that the pool offer to license patents not only as a bundle, but also each patent individually, and that the sum of prices of individual patents not exceed the price of the bundle. Therefore the pool's prices put a cap on the prices patent holders can charge under independent licensing.

European Commission's statement of objections, the company proposed to put a cap on its royalties.

A common feature of these examples is that they involve vertical contracting: the firms using self-imposed price caps do not sell directly to consumers, but instead supply inputs to other firms. Vertical contracting differs from direct-to-consumer sales in two important ways that matter for the analysis of price caps. First, unlike a consumer, a buyer of an input cares about the price at which other buyers obtain the good. The strategic considerations that this entails imply, in particular, that an upstream monopolist may not be able to obtain monopoly profits (Hart and Tirole, 1990). Second, firms in a vertical relationship typically use pricing schemes that are more complex than simple linear pricing, such as two-part tariffs. This raises the question which component of the tariff the price cap applies to.

In this paper, we investigate whether it is always a good idea, from a welfare perspective, to allow firms in a vertical relationship to commit to price caps. Specifically, we study how the introduction of a self-imposed price cap influences the equilibrium of an otherwise standard vertical-contracting game in which contracts take the form of two-part tariffs. We examine how the results depend on whether the cap applies to the fixed or variable component of the tariff and how they are affected by the level of upstream competition.

We use the workhorse model based on Hart and Tirole (1990) and later widely employed in the modern vertical relationship literature (see, among many others, O'Brien and Shaffer, 1992; McAfee and Schwartz, 1994; Rey and Tirole, 2007; Arya and Mittendorf, 2011; Reisinger and Tarantino, 2015; Nocke and Rey, 2018; Gaudin, 2019; Pagnozzi, Piccolo, and Reisinger, 2019). In the simplest version of the model, an upstream monopolist offers two-part tariffs to two downstream firms competing in quantities. Importantly, contract offers are *bilateral and secret*, i.e., the contract offered to downstream firm i is only observed by i. The upstream firm would like to set the variable component of the tariff above marginal cost in order to soften competition between the downstream firms. As is well known, however, absent price caps, an opportunism problem arises: because bilateral profits are decreasing in the unit price, the upstream firm cannot commit to above marginal-cost pricing and thus cannot implement the monopoly outcome.

We then study the effect of allowing the upstream firm to commit to (and publicly announce) a price cap prior to making contract offers. We show that, while a cap on the variable price has no effect, a cap on the *fixed fee* can support equilibrium unit prices above marginal cost and allow the upstream firm to implement the monopoly outcome. Intuitively, although a deviation to marginal-cost pricing would raise bilateral profits, the cap takes away the upstream firm's ability to extract these profits from the downstream firm and thus eliminates the incentive to deviate.

We go on to analyze how this result depends on the presence of upstream competition. To do so, we introduce a less efficient upstream competitor. In the absence of caps, the outcome is similar as before – in particular, it involves marginal-cost pricing – but the existence of an outside option forces the efficient upstream firm to leave rents to the downstream firms. Again, the introduction of a cap on the fixed fee allows the upstream firm to restrict output below Cournot level. The choice of the optimal cap involves a tradeoff between the size of the pie and the share of the pie going to the efficient upstream firm. Although the monopoly quantity may be implementable, it is not necessarily the quantity that maximizes upstream profits; if the upstream competitor is sufficiently efficient, it is better to implement a quantity above the monopoly level. This reduces the value of the outside option and thus the rent that needs to be left to the downstream firms.

Our analysis suggests that price caps can be anticompetitive in vertically related industries. Thus, policy makers should exercise caution with regard to proposals that would give firms the ability to commit to price caps. These include price commitments in the standard-setting context and unbundling requirements for patent pools. Nevertheless, a distinction needs to be made between caps on variable and fixed components of vertical contracts. While our analysis identifies caps on fixed fees as potentially problematic, we have not found reasons to be concerned about caps on variable components.

The remainder of the paper is organized as follows. Section 2 introduces the basic model with a single upstream firm. Section 3 derives the equilibrium of the game in the benchmark without caps, using, as the solution concept, Perfect Bayesian Equilibrium with the familiar passive-beliefs refinement. The section then analyzes how price caps on the variable and fixed components of a two-part tariff affect the equilibrium. Section 4 studies upstream competition with a less efficient competitor. Finally, Section 5 concludes.

2 The Model with Upstream Monopoly

An upstream firm, U, is a monopoly producer of an intermediate good with constant marginal cost which, without loss of generality, we normalize to 0. U supplies two downstream firms, D_1 and D_2, that are Cournot rivals in a downstream market. The downstream firms transform the intermediate good into a homogeneous final product on a one-to-one basis at zero marginal cost of production.

Each downstream firm produces a quantity of q_i, $i = 1, 2$, resulting in an aggregate retail output of $Q = q_1 + q_2$. The (inverse) demand function for the final good is $p =$

$P(Q)$. It is strictly decreasing and thrice continuously differentiable whenever $P(Q) > 0$. Moreover, we employ the standard assumption that $P'(Q) + q_i P''(Q) < 0$, which guarantees that the profit functions are (strictly) concave in q_i and that the Cournot game exhibits strategic substitutability (Vives, 1999).

When contracting with downstream firm D_i, $i = 1, 2$, the upstream monopolist makes a take-it-or-leave-it offer that takes the form of a two-part tariff consisting of a fixed component, F_i, and a unit price, w_i. If it accepts, downstream firm D_i's total marginal cost is w_i.

The game proceeds as follows:

1. U secretly offers to each downstream firm D_i a two-part tariff $(w_i, F_i) \equiv T_i$.

2. Downstream firms simultaneously accept or reject the contract offer.

3. Downstream firms order a quantity of the intermediate good, q_i, and pay the tariff. Then, they transform the intermediate good into the final good and bring output to the market.

Afterwards, retail purchases are made, and profits are realized.

We solve for the perfect Bayesian Nash equilibrium that satisfies the standard "passive beliefs" refinement (Hart and Tirole, 1990; O'Brien and Shaffer, 1992; McAfee and Schwartz, 1994; Rey and Tirole, 2007; Arya and Mittendorf, 2011; Reisinger and Tarantino, 2015). With passive beliefs, a downstream firm's conjecture about the contract offered to the rival is not influenced by an out-of-equilibrium contract offer it receives. This is a natural restriction on the potential equilibria of a game with secret offers and supply to order because, from the perspective of the upstream monopolist, under these two assumptions downstream firms D_1 and D_2 form two separate markets (Rey and Tirole, 2007).

When introducing price commitments in the form of caps on the tariffs paid by downstream firms, the model changes in two ways. First, we will solve an augmented game in which, in a stage *0*, the supplier U decides on the value of the cap so as to maximize its total profits. Second, the presence of price commitments implies that any contract that violates the cap can be invalidated in court. In our game with secret contracts, then, price caps constrain the content of supplier's offers both on and off equilibrium.

Our model relies on the assumption that the upstream firm can commit to the price caps announced at stage 0, even though it cannot commit to more specific contract offers or to other constraints on its pricing, such as price floors. The justification for this assumption comes from the motivating examples, where there is an external source of commitment that allows for price caps but not for other constraints. For example, when a patent holder agrees not to license its technology above a certain maximum price if selected into a standard, it is the contractual relationship with the standard-setting organization that provides commitment. If the patent holder violates the announced cap in an offer to a potential licensee, contract law allows the licensee to enforce the cap in court.[3] In section 4, we consider an additional source of commitment arising in a context with upstream competition, namely, the possibility for the upstream firm to enter into a price-cap agreement with a rival.

Below, we denote by q^m half the monopoly quantity, i.e.,

$$q^m \equiv \arg\max_q \; P(2q)q. \tag{1}$$

Similarly, π^m denotes downstream firm D_i's profit when both produce q^m:

$$\pi^m \equiv \max_q \; P(2q)q.$$

The best response of downstream firm $i = 1, 2$ when facing unit price w_i and its rival produces q_{-i} is to produce

$$\hat{q}(w_i, q_{-i}) \equiv \arg\max_q \; (P(q + q_{-i}) - w_i)q, \tag{2}$$

and is unique given the assumed properties of inverse demand. For convenience, we denote the value of q solving $q = \hat{q}(0, q)$ (i.e., the Cournot quantity) by q^c. Let

$$\pi^c \;\equiv\; P(2q^c)q^c \tag{3}$$

denote D_i's equilibrium profit when both firms produce the Cournot quantity.

Before proceeding, we remark two properties of our game. The first concerns firms' profit functions. Letting $\hat{\pi}(w_i, q_{-i})$ denote D_i's profit when it faces unit price w_i and the rival produces q_{-i}, we have

$$\hat{\pi}(w_i, q_{-i}) \equiv \max_q (P(q + q_{-i}) - w_i)q. \tag{4}$$

The envelope theorem implies that D_i's profits are decreasing in w_i:

$$\frac{\partial \hat{\pi}(w_i, q_{-i})}{\partial w_i} = -\hat{q}(w_i, q_{-i}) < 0. \tag{5}$$

Note that if D_i expects U to offer a unit price w_i^* and its rival to produce q_{-i}^* in equilibrium, receiving an unexpected offer $w_i \neq w_i^*$ would not lead it to expect its rival to produce a different quantity; thus D_i believes that its profit from the unexpected offer (gross of the fixed fee) is $\hat{\pi}(w_i, q_{-i}^*)$. This is because (i) secret offers imply that the rival D_{-i} cannot react to the change in D_i's unit price, and (ii) passive beliefs imply that D_i does

[3] In future research, we plan to consider an explicit renegotiation game in the shadow of litigation.

not expect D_{-i} to receive an offer that differs from the equilibrium one when it receives such an offer itself. Further, $\partial^2 \hat{\pi}(w_i, q_{-i})/\partial w_i^2 = -\partial \hat{q}(w_i, q_{-i})/\partial w_i > 0$: a firm's profit is convex in the unit price it pays.

The second follows from the aggregate nature of Cournot games: the sum of the product-market first-order conditions of D_1 and D_2 is given by

$$2P(Q) + P'(Q)Q = w_1 + w_2. \tag{6}$$

Therefore, the industry quantity ($Q = q_1 + q_2$) is uniquely determined by the sum of the unit prices paid by downstream firms. Since the left-hand side of (6) is decreasing in Q, this property implies that an increase in $w_1 + w_2$ causes a reduction of the industry quantity, and thus commands a fall in consumer surplus.[4]

3 Equilibrium Analysis: Upstream Monopoly

Below, we first solve the (benchmark) game without price commitments. Then, we study how the presence of commitments in the form of caps on (i) the unit price w and (ii) the fixed component of the tariff F change the equilibrium outcome.

3.1 Benchmark

Let $q^*(w)$ and $\pi^*(w)$ denote equilibrium quantity and downstream profit when both firms face the same unit price, $w_i = w$ for $i = 1, 2$. That is, $q^*(w)$ is the value of q that solves

$$P(2q) + P'(2q)q = w,$$

and $\pi^*(w) = [P(2q^*(w)) - w]q^*(w)$. Ideally, U would like to monopolize the product market by inducing D_i and D_{-i} to jointly sell the monopoly output ($2q^m$). Since for $w = 0$ downstream firms produce the Cournot quantity, $q^*(0) = q^c > q^m$, U would thus like to soften competition by setting $w > 0$. Specifically, if U could commit, it would choose (w, F) to maximize $wq^*(w) + F$ subject to $F \leq \pi^*(w)$, or, since the constraint must be binding at the optimum, to choose w to maximize

$$\Pi^*(w) \equiv wq^*(w) + \pi^*(w).$$

The optimal unit price is w^m such that $q^*(w^m) = q^m$. The corresponding fixed fee is $F^m = \pi^*(w^m)$. For the case of linear demand, figure 1 depicts the profits U would obtain through sales, through the fixed fee, and in total, as a function of w, if it could commit to a contract.

[4]Moreover, since a price increase also implies a reduction in total surplus, the conclusions on welfare do not depend on the specific standard employed.

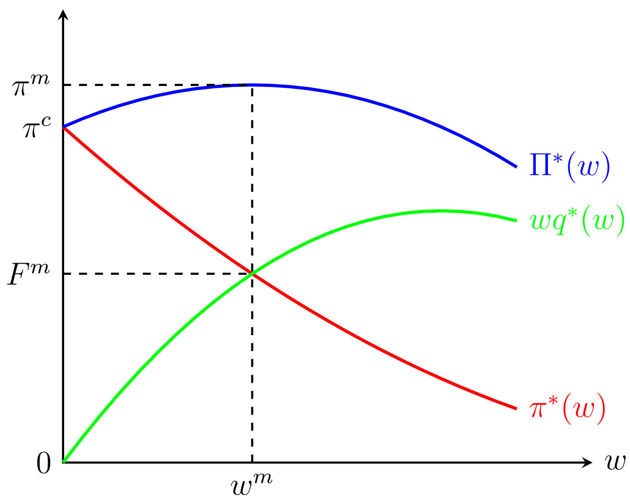

Figure 1: Profits under commitment as a function of a common unit price w

When contract offers are secret, however, U cannot commit to a contract. Consider a candidate equilibrium in which U chooses $(w_i, F_i) = (w^m, F^m)$, $i = 1, 2$. For this to be an equilibrium, it must maximize U's *bilateral profits* with each downstream firm. That is, (w^m, F^m) must be the solution to the problem:

$$\max_{w_i, F_i} w_i \hat{q}(w_i, q^m) + F_i \text{ subject to } F_i \leq \hat{\pi}(w_i, q^m).$$

Since the constraint will bind at the optimum, this means that $w_i = w^m$ must maximize $\hat{\Pi}(w_i, q^m)$, where $\hat{\Pi}(w_i, q_{-i}) \equiv \hat{\pi}_i(w_i, q_{-i}) + w_i \hat{q}(w_i, q_{-i})$. As we show below, however, $\hat{\Pi}(w_i, q_{-i})$ is decreasing in w_i: bilateral profits are maximized by setting $w_i = 0$. Figure 2 illustrates this for the case of linear demand. It follows that the best that U can achieve is the Cournot outcome.[5] The following lemma formalizes this result.

LEMMA 1. *The upstream monopoly (U) offers, in equilibrium, each downstream firm a two-part tariff with the unit price equal to U's marginal cost and the fixed component equal to the downstream firm's resulting profit in the ensuing Cournot competition; that is, the equilibrium tariff offered to downstream firm D_i is $T_i = (0, \pi^c)$.*

[5] Another way of seeing U's opportunism problem is as follows. U would like to have each downstream firm produce half of the monopoly quantity. However, D_i understands that, because offers are secret, U has an incentive to sell an additional amount to D_{-i}. This result follows from the observation that, if U and D_2 agree to produce $q_2 = q^m$, then U and D_1 would have an incentive to agree on a quantity q_1 that maximizes their joint profit:

$$q_1 = \arg\max_q \; P(q + q^m)q > q^m,$$

where the inequality derives from a standard revealed preference argument (see Rey and Tirole, 2007). In other words, given $T^m = (w^m, \pi^m)$, when secretly renegotiating with D_1, the upstream monopolist maximizes the value of the contractual relationship with this downstream firm, and, given q_2, the profits of this relationship can be increased by raising q_1 above q^m. So D_2 would incur a loss if it accepts, therefore D_2 turns T^m down.

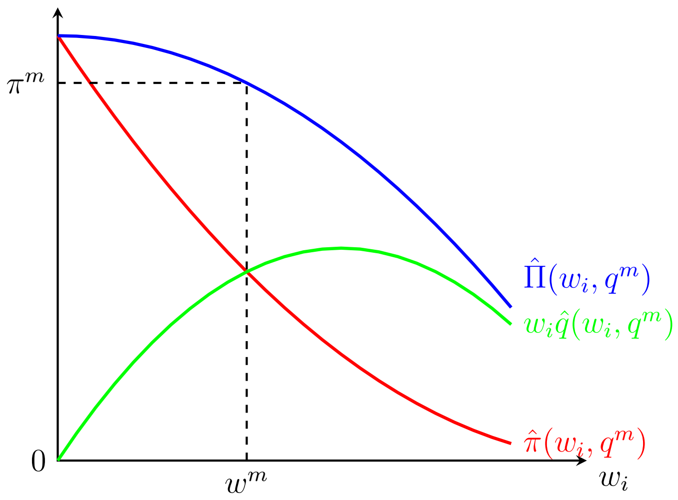

Figure 2: U and D_i's bilateral profits under secret offers as a function of w_i when $q_{-i} = q^m$

Proof. We solve the game by backward induction. In the last stage, downstream firm D_i produces $\hat{q}(w_i, q_{-i})$ as defined by (2). Accordingly, one-to-one production technology implies that D_i orders $\hat{q}(w_i, q_{-i})$ from the monopoly producer U.

We now determine U's tariffs. With passive beliefs, the equilibrium contract offered by U to each downstream firm D_i must maximize their joint profits (McAfee and Schwartz, 1994). Therefore, U's first-stage maximization problem can be written as

$$\max_{w_i} \ \hat{q}(w_i, q_{-i})w_i + \left(P(\hat{q}(w_i, q_{-i}) + q_{-i}) - w_i\right)\hat{q}(w_i, q_{-i}).$$

Taking the first-order condition with respect to w_i and invoking the Envelope theorem and passive beliefs, we obtain

$$w_i \frac{\partial \hat{q}(w_i, q_{-i})}{\partial w_i} + \hat{q}(w_i, q_{-i}) - \hat{q}(w_i, q_{-i}) = w_i \frac{\partial \hat{q}(w_i, q_{-i})}{\partial w_i} \leq 0,$$

where the inequality follows from the fact that $\partial \hat{q}(w_i, q_{-i})/\partial w_i < 0$. Hence, in equilibrium, $w_i = 0$. At this unit price, both downstream firms are active and produce the symmetric Cournot quantity, q^c, to obtain Cournot profits of π^c. In turn, the monopoly producer fully extracts downstream firms' Cournot profits by setting the fixed component of the two-part tariff equal to $F_i = \pi^c$, $i = 1, 2$. □

This result is well-known. Intuitively, a downstream firm's decisions cannot change if U deviates in its offer to the rival downstream firm. Therefore, when the monopoly producer contracts with each downstream firm, it acts as if the two are integrated. This pairwise maximization problem requires that the contractual arrangements between U and D_i maximize bilateral profits. This entails a unit price equal to the monopoly producer's marginal cost (0). Consequently, each downstream firm produces its Cournot quantity. The upstream monopolist cannot obtain monopoly profits but only Cournot profits ($2\pi^c$).

3.2 Cap on the unit price

We first consider the impact on the benchmark equilibrium of a cap on w, denoted by $\overline{w}$. To set its contract offer to D_i, then, U solves the following problem:

$$\max_{w_i \leq \overline{w}, F_i \in \mathbb{R}_+} \hat{q}(w_i, q_{-i})w_i + F_i, \text{ for all } i = 1, 2,$$

subject to

$$F_i \leq \max_{q_i} \left(P(q_i + \hat{q}(w_{-i}, q_i)) - w_i\right) q_i.$$

Given that no cap is imposed on the fixed component of the tariff, U's full bargaining power implies that the last constraint is binding. Then, the maximization problem can be rewritten as

$$\max_{w_i \leq \overline{w}} \hat{q}(w_i, q_{-i})w_i + \left(P(\hat{q}(w_i, q_{-i}) + q_{-i}) - w_i\right) \hat{q}(w_i, q_{-i}),$$

subject to $w_i \in \mathbb{R}_+$, for all $i = 1, 2$.

When ignoring these constraints, the problem is equivalent to the one solved in the benchmark (see the proof of Lemma 1); thus, it yields the same solution ($w_i = 0$). Then, if $0 \leq \overline{w}_i$, the equilibrium tariffs will be $T_i = (0, \pi^c)$, with $i = 1, 2$. Otherwise, $w_i = \overline{w}$ for $i = 1, 2$ and $F_i = \pi^*(\overline{w})$. Moving back to the cap-setting stage, since $\Pi^*(w)$ is increasing in w for $w \leq w^m$, the upstream monopolist will set $\overline{w} \geq 0$. The next lemma summarizes these results.

LEMMA 2. *The upstream monopoly (U) sets, in equilibrium, a cap $\overline{w}$ above its marginal cost (0), with $i = 1, 2$. The equilibrium unit price and fixed component of the tariff will be the same as in the benchmark, that is, $T_i = (0, \pi^c)$.*

The lemma shows that the presence of a cap on w_i does not allow the supplier to raise more profits than in the benchmark. The intuition is straightforward: the opportunism problem is rooted in the supplier's incentive to bilaterally renegotiate D_i's contract offer by proposing a cheaper deal to D_{-i}. The presence of a cap does not prevent the supplier from such a deviation, and thus, under passive beliefs, it yields the same equilibrium outcome as in the benchmark.

3.3 Cap on the fixed component of the tariff

Consider now a cap on F, denoted by $\overline{F}$. Then, U's maximization problem is:

$$\max_{w_i, F_i} \hat{q}(w_i, q_{-i})w_i + F_i, \text{ for all } i = 1, 2,$$

subject to $w_i \in \mathbb{R}_+$, and

$$F_i \leq \min\{\max_q \left(P(q + \hat{q}(w_{-i}, q)) - w_i\right) q, \overline{F}_i\} \equiv \Phi.$$

As the following proposition shows, by appropriately choosing the cap, the upstream firm can implement the monopoly outcome.

PROPOSITION 1. *Suppose $w\hat{q}(w, q^m)$ is quasi-concave in w. Then, U can make monopoly profits by setting $\overline{F} = F^m$.*

Proof. Suppose a cap $\overline{F} = F^m$ is in place, and consider a candidate equilibrium in which U chooses $(w_i, F_i) = (w^m, F^m)$, $i = 1, 2$ at the contract-offer stage. For this to be an equilibrium, it must maximize U's bilateral profits with each downstream firm. That is, (w^m, F^m) must solve

$$\max_{w_i, F_i} w_i \hat{q}(w_i, q^m) + F_i \text{ subject to } F_i \leq \min\{\hat{\pi}(w_i, q^m), \overline{F}\},$$

or, given that the constraint must bind at the optimum, w^m must solve

$$\Pi(w_i, q^m) = \max_{w_i} w_i \hat{q}(w_i, q^m) + \min\{\hat{\pi}(w_i, q^m), \overline{F}\}. \tag{7}$$

First note that a deviation to $w > w^m$ reduces bilateral profits, and can thus not be profitable. Moreover, although setting $w < w^m$ increases bilateral profits, U cannot extract them through the fixed fee because of the cap. However, such a deviation could be profitable if it raised U's profits by generating higher revenue from input sales. So, to establish the result in the claim we need to check that U's variable profits are increasing in w over $[0, w^m]$. If this is the case, then no deviation from w^m is profitable.

□

The proposition implies that, by optimally choosing the cap on the fixed fee, the upstream supplier can obtain monopoly profits. The fact that U can raise the unit price above marginal cost, and raise profits above $2\pi^c$, is probably not a particularly surprising result. Thanks to the presence of the cap, the monopolist *de facto* constrains itself with respect to a deviation down to $w = 0$, as it would require an increase in the fixed component of the tariff above the cap. What is striking is that the cap allows the supplier to obtain the unconstrained monopoly profits. Rey and Tirole (2008) obtain the same result under the assumption of symmetric beliefs, or public offers. In this context, we only require that the upper limit to the fixed fee is public to achieve the same result.

Intuitively, U's (deviation) profits come from the variable profit w and from the fixed fee F. Given that U cannot extract more than F^m from a downstream firm (because $\overline{F} = F^m$), the question is whether a deviation can raise its profits from input sales

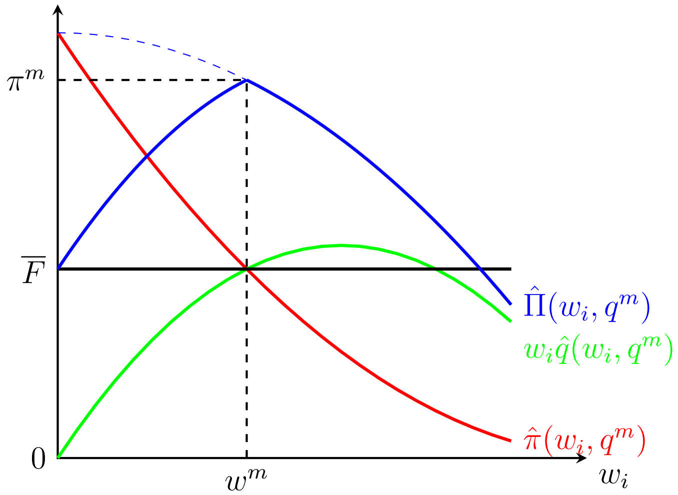

Figure 3: U and D_i's bilateral profits in the presence of a cap $\overline{F} = F^m$ when $q_{-i} = q^m$

(through the unit price w). As we show in the proof, our assumptions that $w\hat{q}(w, q^m)$ is quasi-concave and that marginal revenue is decreasing ($P'(Q) + q_i P''(Q) < 0$) imply that sales revenue is increasing in w at w^m, and hence that such a deviation is not profitable. Figure 3 illustrates this, again, for the linear-demand case.

4 Upstream competition

Consider a setting with two upstream firms $U_{a,b}$ and two downstream firms $D_{1,2}$. In line with the notation used thus far, we let the downstream firms be indexed by i and the upstream firms by j. Assume U_a is more efficient than U_b: $c_a = 0$ while $c_b = c \geq 0$. As before, $D_{1,2}$ bear no marginal cost of production.

The upstream firms set two-part tariffs (w_{ij}, F_{ij}) denoting the offer of U_j to D_i. The timing is the following:

1. $U_{a,b}$ coordinate on, and publicly announce, caps $\overline{F}_a \geq 0$ and $\overline{F}_b \geq 0$.

2. $U_{a,b}$ simultaneously and independently offer two-part tariffs (w_{ij}, F_{ij}) to the two downstream firms; with $F_{ij} \leq \overline{F}_j$. Each downstream firm observes only its own contract offers.

3. Downstream firms accept/reject offers. If they accept, they pay the fixed fee.

4. Downstream firms choose quantities. Afterwards, retail purchases are made, and profits are realized.

4.1 Pricing without caps

Assuming caps cannot be set, we find the following result:

LEMMA 3. *The efficient producer (U_a) offers a contract $T = \big(0, \pi^c - \pi^b\big)$ to D_i with $i = 1, 2$, where*

$$\pi^b = \max_q (P(q + q^c) - c)q. \tag{8}$$

The result in Lemma 3 is a direct implication of Bertrand competition between U_a and U_b, implying that U_b sets a per-unit price of c and a fixed payment of zero. The proof follows the same steps as in Hart and Tirole (1990). The only difference with the model without upstream competition is that U cannot extract the full Cournot profit via the fixed fee, because it is constrained by the presence of the inefficient source.

4.2 Pricing with caps on the fixed fees

We now assume that U_a and U_b can sign a price-cap agreement prior to the contract-offer stage. We focus on caps on the fixed component of the tariff and assume that the upstream firms coordinate their caps to maximize upstream profits. For any $c > 0$, it is more efficient that both downstream firms purchase from U_a. But importantly, the caps cannot prevent U_b from offering downstream firms an outside option at a tariff $(w_b, F_b) = (c, 0)$. Given the presence of the outside option, the question is how the upstream firms want to set the caps. We saw in the monopoly case that the cap on F can influence the equilibrium w, and thus downstream quantities. We therefore ask: if the upstream firms could induce any symmetric quantity $\tilde{q}$ (or equivalently, any symmetric $\tilde{w}$), which $\tilde{q}$ would they want to induce? It might seem that they would want to implement monopoly, but this turns out not to be the case.

In an equilibrium in which both downstream firms receive the same offer from U_a, the equilibrium quantity $\tilde{q}$ induced by a symmetric unit price $\tilde{w}$ is implicitly defined by $\tilde{q} = \hat{q}(\tilde{w}, \tilde{q})$. Thus, we must have

$$P'(q + \tilde{q})q + P(q + \tilde{q}) - \tilde{w} = 0$$

at $q = \tilde{q}$, or

$$\tilde{w} = P'(2\tilde{q})\tilde{q} + P(2\tilde{q}).$$

The fixed fee U_a can charge to each downstream firm for a given $\tilde{q}$ is

$$F_{ia} \leq (P(2\tilde{q}) - \tilde{w})\tilde{q} - \pi^b, \tag{9}$$

where

$$\pi^b = \max_q (P(q + \tilde{q}) - c)q,$$

i.e., π^b is the profit a downstream firm could obtain by purchasing from U_b. The upstream firms' problem is to maximize $F_{ia} + \tilde{w}\tilde{q}$ subject to (9). Notice that, for $c \geq P(\tilde{q})$, $\pi^b = 0$. Using our previous results, it follows that, for $c \geq P(q^m)$, U_a can obtain the monopoly profit by setting $\overline{F}_a = (P(2q^m) - w^m)q^m$ and offering $T^m = (w^m, F^m)$.

For $c < P(q^m)$, given that the constraint will be binding, the optimal quantity solves

$$\max_{\tilde{q}} P(2\tilde{q})\tilde{q} - \max_{q}(P(q + \tilde{q}) - c)q. \tag{10}$$

The first term in this expression is maximized at q^m. The second term (i.e., π^b) decreases with $\tilde{q}$, however: by the envelope theorem,

$$\frac{\partial \pi^b}{\partial \tilde{q}} = P'(\hat{q}(c, \tilde{q}) + \tilde{q})\hat{q}(c, \tilde{q}) \leq 0.$$

Thus, by increasing $\tilde{q}$ above q^m, U^a can reduce the value of the outside option for the downstream firms. Recall that in equilibrium the outside option is not used and hence does not add to upstream profits.

The implications of this are reflected in the following lemma, which characterizes the optimal choice of $\tilde{q}$ as a function of c.

LEMMA 4. *For any $c \in (0, P(q^m))$, the quantity $\tilde{q}^*$ maximizing upstream profits solves*

$$2P'(2\tilde{q})\tilde{q} + P(2\tilde{q}) = c - P(\hat{q}(c, \tilde{q}) + \tilde{q}). \tag{11}$$

It is strictly greater than q^m and strictly less than q^c. As c increases, $\tilde{q}^*$ decreases.

Proof. The first-order condition of problem (10) is

$$2P'(2\tilde{q})\tilde{q} + P(2\tilde{q}) = P'(\hat{q}(c, \tilde{q}) + \tilde{q})\hat{q}(c, \tilde{q}). \tag{12}$$

Using the definition of $\hat{q}(\cdot)$, according to which

$$P'(\hat{q}(w_i, q_{-i}) + q_{-i})\hat{q}(w_i, q_{-i}) + P(\hat{q}(w_i, q_{-i}) + q_{-i}) - w_i = 0 \tag{13}$$

we can rewrite this as (11) noting that $w_i = c$ in this case.

We now prove the three remaining claims in the lemma.

Claim 1: The quantity the upstream firms want to implement is greater than the monopoly quantity: for any $c < P(q^m)$, $\tilde{q}^* > q^m$.

Any $\tilde{q} < q^m$ is dominated since industry profit is increasing and π^b is decreasing as we marginally increase $\tilde{q}$. Suppose $\tilde{q}^* = q^m$. Then, the left-hand side of (11) is zero because q^m is determined by equation (1). Using equation (13) with $w = c, q = q^m$, the right-hand side can be written as

$$c - P(\hat{q}(c, q^m) + q^m) = P'(\hat{q}(c, q^m) + q^m)\hat{q}(c, q^m) \tag{14}$$

Hence, the right-hand side of (11) at $\tilde{q} = q^m$ equals 0 if and only if $\hat{q}(c, q^m) = 0$ which happens if $c \geq P(q^m)$. If $c < P(q^m)$, $\hat{q}(c, q^m) > 0$ and the right-hand side of (11) is negative. As its left-hand side is zero, this is a contradiction. Consequently, $\tilde{q}^* > q^m$.

Claim 2: The quantity the upstream firms want to implement is smaller than the Cournot quantity: for any $c > 0$, $\tilde{q}^* < q^c$.

Consider equation (12) and suppose –by contradiction– that $\tilde{q} \geq q^c$. We write this equation as follows:

$$P'(2\tilde{q})\tilde{q} + P(2\tilde{q}) = P'(\hat{q}(c,\tilde{q}) + \tilde{q})\hat{q}(c,\tilde{q}) - P'(2\tilde{q})\tilde{q}. \tag{15}$$

The left-hand side of this equation is less than or equal to zero for $\tilde{q} \geq q^c$ by the definition of q^c. The right-hand side equals 0 if $\tilde{q} = q^c$ and $c = 0$. Now consider two other possibilities: (i) $c > 0$ and (ii) $\tilde{q} > q^c$. In each case, we get a contradiction due to the following inequality:

$$\frac{\partial(P'(q+\tilde{q})q)}{\partial q} = P''(q+\tilde{q})q + P'(q+\tilde{q}) < 0 \tag{16}$$

by assumption.

(i) As c increases, $\hat{q}$ falls and hence the right-hand side of (15) becomes positive. As the left-hand side is less than or equal to zero, this is a contradiction.

(ii) Similarly, as $\tilde{q}$ increases above q^c, we have $\hat{q}(c,\tilde{q}) < \tilde{q}$ (strategic substitutes). Equation (16) again implies that the right-hand side of (15) turns positive. This is a contradiction.

Claim 3: The value of $\tilde{q}$ decreases monotonically in c, $d\tilde{q}/dc < 0$.

By the implicit function theorem, equation (12) yields

$$\frac{d\tilde{q}}{dc} = \frac{\left[(P''(\hat{q}(c,\tilde{q}) + \tilde{q})\hat{q}(c,\tilde{q}) + P'(\hat{q}(c,\tilde{q}) + \tilde{q}))\,\frac{\partial \hat{q}(c,\tilde{q})}{\partial c}\right]}{\left[4\left(P''(2\tilde{q})\tilde{q} + P'(2\tilde{q})\right) + P'(\hat{q}(c,\tilde{q}) + \tilde{q})\left(1 + \frac{\partial \hat{q}(c,\tilde{q})}{\partial \tilde{q}}\right)\right]}. \tag{17}$$

Our assumptions on $P(\cdot)$, together with

$$\frac{\partial \hat{q}(w_i, q_{-i})}{\partial w_i} = \frac{1}{P''(\hat{q}(w_i, q_{-i}) + q_{-i})\hat{q}(w_i, q_{-i}) + P'(\hat{q}(w_i, q_{-i}) + q_{-i})} < 0,$$

imply that the numerator at the RHS of equation (17) is positive. At the denominator, the first term is negative by the assumption on $P(\cdot)$, and the second term is negative because $P'(\cdot) \leq 0$ and $(1 + \partial\hat{q}(c,\tilde{q})/\partial\tilde{q}) > 0$ (by a standard regularity condition embedded in strategic substitutability). All this means that $d\tilde{q}/dc < 0$. □

Lemma 4 shows, first, that raising $\tilde{q}$ above q^m increases upstream profits. By doing so, the upstream firms reduce the downstream firms' outside option and thus the amount of surplus that needs to be left to them; they get *a larger share of a smaller pie*. Intuitively, starting from q^m, raising $\tilde{q}$ leads to a second order loss in the total pie but to a first order reduction in the downstream firms' outside option. Second, if the upstream firms can choose which quantity to implement, they can do better than the Cournot quantity. Third, the optimal $\tilde{q}$ decreases with c: as U^b becomes less of a competitive threat to U^a, the optimal quantity moves closer to monopoly output.

The question now is whether the upstream firms can implement their preferred $\tilde{q}$ through a price cap. Proposition 2 shows that quasi-concavity of sales revenue and the decreasing marginal revenue property ensure that this is indeed the case.

PROPOSITION 2. *Suppose $w\hat{q}(w,q)$ is quasi-concave in w for $q \in [q^m, q^c]$. For $c \geq P(q^m)$, the upstream firms can obtain monopoly profits by setting $\overline{F} = F^m$. For $c \in (0, P(q^m))$, they can do strictly better than without caps by setting*

$$\overline{F} = -P'(2\tilde{q}^*)(\tilde{q}^*)^2 - [P(\hat{q}(c,\tilde{q}^*) + \tilde{q}^*) - c]\hat{q}(c,\tilde{q}^*), \tag{18}$$

which implements $\tilde{q}^*$.

Proof. The first claim follows from the fact that $\pi^b = \max_q (P(q + q^m) - c)q = 0$ for $c \geq P(q^m)$. For $c < P(q^m)$, consider a candidate equilibrium in which U_a offers a tariff $(w_{ia}, F_{ia}) = (\tilde{w}^*, \tilde{F}^*)$ to both downstream firms firms, where $\tilde{w}^*$ implements $\tilde{q}^*$ defined in (11), i.e.,

$$\tilde{w}^* = P'(2\tilde{q}^*)\tilde{q}^* + P(2\tilde{q}^*),$$

and

$$\begin{aligned} \tilde{F}^* &= \pi^*(\tilde{w}^*) - \max_q (P(q + \tilde{q}^*) - c)q \\ &= [P(2\tilde{q}^*) - \tilde{w}^*]\tilde{q}^* - (P(\hat{q}(c,\tilde{q}^*) + \tilde{q}^*) - c)\hat{q}(c,\tilde{q}^*), \end{aligned}$$

which is equal to $\overline{F}$ in (18). As before, deviations to $w > \tilde{w}^*$ reduce bilateral profits and can thus be ruled out.

To rule out deviations to $w < \tilde{w}^*$, we need to show that the sales-revenue maximizing w is greater than $\tilde{w}$. Recall that $\tilde{w}$ can be defined as

$$\hat{q}(\tilde{w}, \tilde{q}) = \tilde{q}.$$

Similarly, letting $\hat{w}$ denote the sales-revenue maximizing w, using the FOC of the sales revenue maximization problem, $\max_w w\hat{q}(w, q_{-i})$, for $q_{-i} = \tilde{q}$, we can implicitly define $\hat{w}$ as the solution to

$$\hat{q}(\hat{w}, \tilde{q}) = -\hat{w}\frac{\partial \hat{q}(\hat{w}, \tilde{q})}{\partial w_i}.$$

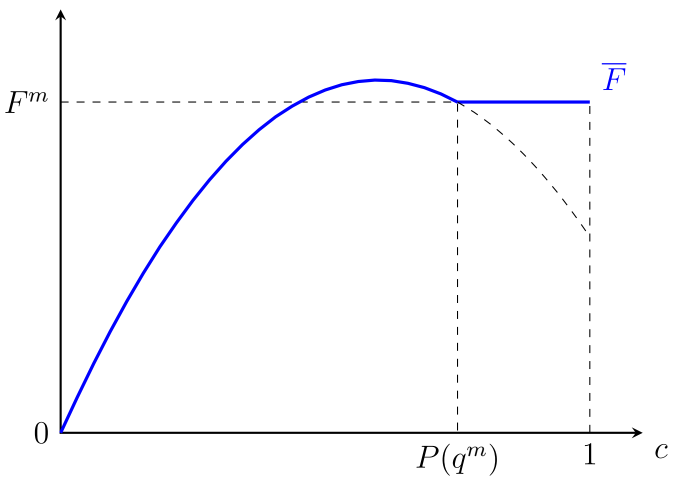

Figure 4: The optimal cap as a function of c

Since $\hat{q}$ is decreasing in w_i and we are assuming quasi-concavity of $w\hat{q}(w,q)$ in w for $q \in [q^m, q^c]$, a sufficient condition for $\tilde{w} \leq \hat{w}$ is

$$-\tilde{w}\frac{\partial \hat{q}(\tilde{w},\tilde{q})}{\partial w_i} \leq \tilde{q}. \tag{19}$$

We have

$$\frac{\partial \hat{q}(w_i,q_{-i})}{\partial w_i} = \frac{1}{P''(\hat{q}(w_i,q_{-i})+q_{-i})\hat{q}(w_i,q_{-i}) + P'(\hat{q}(w_i,q_{-i})+q_{-i})}.$$

Using $\tilde{w} = P'(2\tilde{q})\tilde{q} + P(2\tilde{q})$ and $\hat{q}(\tilde{w},\tilde{q}) = \tilde{q}$, (19) then becomes

$$P'(2\tilde{q})\tilde{q} + P(2\tilde{q}) \leq -\tilde{q}[P''(2\tilde{q})\tilde{q} + 2P'(2\tilde{q})], \tag{20}$$

where the sign of the inequality follows from $P''(2q)q + 2P'(2q)$ being negative. From the FOC of the joint-profit maximization problem, we have $P'(2q^m)q^m + P(2q^m) = -P'(2q^m)q^m$ and hence $P'(2q)q + P(2q) \leq -P'(2q)q$ for $q \geq q^m$.[6] Therefore, a sufficient condition for (20) to hold for all $\tilde{q} \geq q^m$ is

$$-P'(2\tilde{q})\tilde{q} \leq -\tilde{q}[P''(2\tilde{q})\tilde{q} + 2P'(2\tilde{q})],$$

which is satisfied if $P''(2q)q + P'(2q) \leq 0$. □

Proposition 2 shows that, even in the presence of competition, price caps allow the upstream firms to restrict output below the Cournot level. For $c \geq P(q^m)$, the cost disadvantage of the less efficient upstream rival is so large that the outside option becomes irrelevant; as a result, U_a can implement the monopoly outcome and reap monopoly profit. For $c < P(q^m)$, the less efficient rival imposes an actual constraint on the outcome. Yet

[6]This relies on concavity of $P(2q)q$, which is implied by $P''(2q)q + P'(2q) \leq 0$.

price caps continue to allow the efficient upstream firm to implement its preferred outcome conditional on the presence of an outside option for downstream firms. Because the cap cannot eliminate the outside option, this preferred quantity exceeds the monopoly level. The logic of how the cap helps the efficient upstream firm solve its opportunism problem is the same as in the case of upstream monopoly: setting the cap at the downstream firm's profit given $\tilde{q}^*$ ensures that increased bilateral profits from deviations to $w < \tilde{w}^*$ cannot be extracted through the fixed fee. The proof again consists in showing that, under some conditions, such deviations decrease sales revenue, and are thus unprofitable.

For the case of linear demand, Figure 4 depicts how the optimal cap $\overline{F}$ depends on c. As the figure shows, the optimal cap exhibits an interesting non-monotonicity, which we have yet to explore further.

5 Conclusion

We have studied how self-imposed price caps affect the equilibrium of a standard vertical-contracting game with secret offers à la Hart and Tirole (1990). Absent price caps, the upstream firm faces an opportunism problem which prevents it from implementing the monopoly outcome. We show that the possibility to commit to a cap on the fixed fee prior to contracting allows the upstream firm to obtain monopoly profits. When there is upstream competition, caps on the fixed fees allow the upstream firms to restrict output below the Cournot level and raise their profits. Our results suggest that price caps can be anti-competitive in vertically related industries.

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