Advertising, Price Discrimination and Content Development

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Abstract

We analyze the effects of advertising and price discrimination in content development in media markets. We show that if firms compete only on prices (i.e. they do not compete on advertising, one-sided market), firms do not diversify content (i.e. firms only offer one variety of content, single-content), and therefore firms also do not price discriminate. In this case, firms locate at the extremes of the line (maximum differentiation). If instead firms compete on both prices and advertising (i.e. two-sided market), firms diversify content (multi-content) and price discriminate between consumers. In this case, with a large advertising market, maximum differentiation arises, but in a small advertising market minimum differentiation takes place. Furthermore, in a large advertising market, firms opt for a freemium strategy: zero pricing for consumers that do not get their ideal variety, and premium pricing (price discrimination) for consumers that get their ideal variety. As the size of the advertising market decreases, prices for consumers that do not get their ideal variety become positive, and firms continue to charge a premium to consumers that get their ideal variety.

Keywords: Advertising; Price Discrimination; Content Development, Freemium.

JEL Classification: D43, L11, L13, L82, L86, M37.

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1 Introduction

In this paper, we study the effects of advertising and price discrimination in content development in two-sided media markets (on two-sided markets see for instance, Armstrong, 2006b, Rochet and Tirole, 2006). The two-sidedness in media markets arises because from one side media firms sell content to consumers, and from the other side they sell advertising space to advertisers. Advertisers prefer media firms that reach a larger audience. Therefore, media firms that attract more consumers also attract more advertisers, and therefore ad revenues. As a result, media firms have incentives to increase their consumer base, via for instance content development.

The Internet has revolutionized the media market at least in two ways. First, it opened a new channel for media firms to explore advertising (see Anderson and Gabszewicz, 2006; Bagwell, 2007; Crampes et al., 2009). If media firms manage to tap on online advertising, this extra revenue source can be used to finance content development and to be bolder in choosing more advanced pricing strategies, such as price discrimination.

Second, the Internet makes it easier for firms to develop content that fits better consumers' preferences (see Armstrong, 2006a and Levin, 2013). For example, with linear TV, consumers can only watch what is being broadcasted when they sit in front of the TV. With online TV, consumers can pick up the programs that they prefer when it suits them best. Similarly, with printed newspapers there is just one edition of a newspaper. In turn, online newspapers can constantly update news and supply consumers with a broader set of news than newspapers limited to a printed edition. In this way, since the Internet allows firms more easily to serve consumers with their preferred varieties, it is also easier for media firms to price discriminate between consumers (see Balasubramanian, 1998; Bernhardt et al., 2006; Chen, 2006; Dewan et al., 2003; Gal-Or and Gal-Or, 2005 and Syam et al., 2005).

To face this new Internet paradigm, media firms have been mainly experimenting with two online business strategies: scale versus premium (Marín and Gayo, 2009). In the scale strategy, firms do not charge for online content, and they typically target a mainstream audience with a limited assortment of content (single-content). The idea behind this strategy is that it is difficult for firms to be paid for content on the Internet, since they offer a product that can be potentially accessed for free somewhere else. Therefore, with a free content strategy, firms try to maximize the number of consumers to attract more advertising. In turn, in the premium strategy, firms charge for online content, and they target specialized audiences (in terms of preferences and tastes) with different content (multi-content). The rationale for this strategy is that online advertising might not be sufficient to keep a firm profitable and that content differentiation allows firms to charge extra to consumers. In this last case, media firms give consumers something that they value, in order to follow more complex price strategies, like price discrimination, and face the very competitive environment of the Internet.

In practice, firms many times try to combine the scale and the premium strategies. This strategy is usually called "freemium". With freemium (free plus premium), a firm offers a basic digital product for free, while charging a premium for advanced or special content (see Marín and Gayo, 2009). With the free part (single-content), the aim is to generate large circulation on the firm's website and therefore increase the advertising revenues. With the premium part (multi-content), the objective is to satisfy the demand of some consumers for specialized content at a profitable price, which can include price discrimination. Depending on which strategy media firms follow, single-content or multi-content, (online) content can either increase or decrease in a two-side market. Accordingly, a single-content strategy reduces content diversity, while a multi-content strategy increases content diversity.

Our starting point to study advertising, price discrimination and content development is the work-horse model of the media competition literature: the Hotelling (1929) ideal variety model. As it is well-known, in this model, consumers' preferences are distributed on a line of length one (i.e. from zero to one). Consumers experience a decrease in utility when they consume products that do not match their ideal variety. Therefore, with the Hotelling set-up, we introduce a force that pushes for maximum differentiation (D'Aspremont et al., 1979). Accordingly, since consumer preferences are uniformly distributed and firms compete on prices, firms try to soften price competition by locating away from the center (maximum differentiation). Firms want to locate far apart from each other to soften price competition, and in this way increase profits by having higher prices.

We differ from the standard media competition literature based on Hotelling (1929) in two ways. First, like Gabszewicz et al. (2001, 2006), we depart from the one-sided market framework, where firms derive income only from selling content, to a two-sided market framework, where firms' profits come from both selling content and advertising space (see also Peitz and Valletti, 2008). In this set-up, advertisers prefer to buy ads in firms that have a larger consumer base. As shown by Gabszewicz et al. (2001), the two-sided market brings in a force that contributes for minimum differentiation. Accordingly, in a two-sided market firms want to attract more consumers, because more consumers mean more advertisers, and therefore ad revenues. The way to attract more consumers is to reduce the transport costs consumers have to pay, and this is done by moving to the center of the line (minimum differentiation). More advertisement revenues, in turn, can compensate for fiercer price competition that ensues by moving closer to the center.

Second, we consider that firms can choose to follow a multiple-content strategy, instead of just a single-content strategy. Single content strategy (i.e. where firms choose just a point in the line) is the usual assumption in standard Hotelling models. If firms choose a multi-content strategy (i.e. a line segment instead of just one point), they can price discriminate between consumers that are offered their ideal variety. The main idea, as mentioned above, is that the multi-content strategy can increase content provision since more content is offered in the market. For the effect, we follow Dewan et al.'s (2003) modeling framework of multi-content firms and price discrimination in consumer markets. Accordingly, while in the standard Hotelling model, firms choose a point in the line (single-content) and a single price, in our set-up firms can choose a line segment (multi-content) and price discriminate between consumers inside the multi-content segment. The optimal price discrimination strategy is to set a premium over the standard content price for consumers inside the multi-content segment. This premium equals the transport costs that consumers would have to pay if they could only acquire a standard content that does not match exactly their preferred variety.

When a firm chooses a multi-content strategy, it must weigh the costs and the benefits of this strategy. The costs of a multiple-content strategy are related with the development of new content. The potential benefits are price discrimination (see Armstrong, 2006a, 2008). Accordingly, firms can price discriminate because some consumers get their ideal variety and therefore firms can charge these consumers a premium above the price of the standard content. The introduction of the multi-content strategy then adds a force that may contribute for larger content provision, once when firms follow this strategy it increases the diversity of content in the market.

Summing up, our model encompasses three forces that can affect content provision in different directions, i.e.: consumers' preferences, advertising, and a multi-content strategy. As discussed above, consumers' preferences can contribute to maximum differentiation, advertising to minimum differentiation and a multi-content strategy to higher content provision and price discrimination. Accordingly, consumers' preference is related with price competition, since price competition is softened by maximum differentiation. In turn, advertising promotes sales maximization, because by attracting more consumers, firms also attract more advertisers, and this is done by minimum differentiation. A multi-content strategy allows firms to diversify content and charge a premium to consumers that are offered their ideal variety.

We find that in a one-sided market, firms always follow a single-content strategy, i.e.: with no advertising, content provision is not increased. In turn, in a two-sided market, firms always follow a multi-content strategy, i.e.: with advertising, content provision is increased. Accordingly, in a two-sided market, firms offer some consumers their ideal variety. The differences between the one-sided and the two-sided market arise because without the advertising revenues, firms are not able to profitably face the fierce price competition that follows under a multi-content strategy and to finance the costs associated with diversifying content.

In what concerns prices and location of the firms, we have that in a one-sided market firms opt for maximum differentiation and charge positive prices. In turn, in a two-sided market, when the advertising market is very large, maximum differentiation also arises, but firms charge a zero price to consumers that do not have their ideal content offered in the market[1]. In turn, consumers that are served with their ideal variety pay a premium, i.e. freemium strategy. Accordingly, when the adverting market is very large, the ad revenues are sufficiently for firms to face fiercer price competition that can even lead prices to zero in the standard segment.

Prices for the standard content, however, go from zero to positive as the size of the ad market decreases (consumers in the standard segment continue to pay a premium), i.e. firms go from a freemium to a premium strategy. Furthermore, only when the advertising market is significantly small, the maximum differentiation result is weakened, i.e. firms locate inside the line. This is so, since when the advertising market is small, firms need to compensate for lower ad revenues not only by charging for content in the standard segment, but also by moving in the direction of the center of the line (i.e. minimum differentiation) in order to attract more audience and therefore increase advertising revenues. In turn, when the advertising market is large, firms do not care so much about triggering a more intense price competition, since the extra advertising revenues are sufficient to make up for a less mainstream appeal to advertisers.

In this sense, content diversification gives firms a more nuance way to play with maximum and minimum differentiation and price discrimination. A firm can choose maximum differentiation (to locate at the extreme of the line), but still move in the direction of the center by choosing a multi-content strategy, and price discriminate between the consumers that are

[1] Our model can then encompass the zero-price economy predicted by Anderson (2009). In his book "Free: The Future of a Radical Price", Anderson (2009) argues that the Internet is pressuring the price of content towards zero. The reasons are the Internet culture (which is based on a "free" mentality), the low costs of operating online (i.e.: the marginal cost of reproducing intangible digital goods is practically zero) and the large potential of online advertising revenues that can compensate for lost revenues from selling content.

offered their ideal variety. The advertising market is central here since revenues from the ad market allows firms to stand fiercer price competition that arises when they move in the direction of the center (i.e. extra revenues from the ad market compensates for losses in revenues from the news market that come from higher price competition). It is also the revenues from the advertising market that allows firms to charge zero pricing to consumers in the standard segment when the ad market is large. In the case of zero pricing in the standard segment, firms can follow a freemium strategy (zero price in the standard segment and a premium price in the multi-content segment). However, as the advertising market becomes smaller, firms might need to abandon the zero pricing, and charge positive prices to consumers in the standard segment, since advertising revenues being smaller in a small ad market are not enough to cover the costs. In this last case, firms need to also charge a positive price to consumers in the standard segment to generate revenues from the consumer side. When this occurs, firms follow a premium strategy: a positive price for consumers in the standard segment, and the price from the standard segment plus a premium for consumers in the multi-content segment.

The rest of the paper is organized as follows. Next, we make a review of the related literature. After, we introduce the base model. In the fourth section we look to the relation between the multi-content strategy and price discrimination. In the fifth and sixth sections, we analyze the non-advertising game (one-sided market) and the advertising game (two-sided market), respectively. Then we look at social welfare. In section 8, we check the robustness of our results to different assumptions. We conclude by discussing our main results.

2 Literature Review

As can be seen from the introduction, our paper is related, with three strands of literature: (1) platform competition; (2) two-sided markets and content provision; (3) and two-sided markets and price discrimination.

As mentioned above, the literature on platform competition usually relies on the Hotelling model. For instance, Adner et al. (2020) study the compatibility decisions of two competing platforms. They show that compatibility choices depend on the profit asymmetries between platforms. Bernstein et al. (2020) in turn look at platform competition when there are congestion effects. They show that consumers are better off when platforms can adapt prices to congestion. Chatterjee and Zhou (2021) analyze the impact of sponsor advertising (advertising that takes the format of publisher's original content) on platform competition. They show that even when consumers dislike this type of ads, consumers can be better off, while firms might be caught in a prisoner's dilemma. Chiang and Jhang-Li (2020) look to the supply of exclusive content between cable networks and streaming providers. They analyze conditions where content owners can gain more by dividing content between cable networks and streaming services. Wu et al. (2022) consider the interaction between software technology platforms (such as Google) and hardware devices (such as smartphones producers). They show that the benefits of having pre-installation of software in a hardware device depends on if the hardware producer is dominant or not.

As discussed in the Introduction, we differ from this literature in that we consider price discrimination and content diversification (single-content versus multi-content) by media firms. This allows us to give some new insights for the literature on platform competition in what respects firms' online prices and content diversification strategies.

We now discuss the literature on content provision in two-sided markets. Gabszewicz et al. (2001) show that when the ad market is large, Hotelling's maximum differentiation result can be weakened. This is so, since when firms compete for advertising, in a large advertisement market, firms choose to locate close to the center of the line to attract a larger audience (i.e. minimum differentiation), and in this way increase advertising revenues. In turn, when the advertising market is small, firms go for maximum differentiation to reduce price competition and as such to increase prices to compensate for small advertising revenues. Consequently, Gabszewicz et al. (2001) argue that content provision in the market can shrink in the presence of advertising. Accordingly, in a one-sided market, we have two types of content available in the market (maximum differentiation), while in a two-sided market, we can have just one type of content (minimum differentiation).

In a similar vein, Dukes and Gal-Or (2003) show that advertising can conduce to minimum differentiation in media markets. The mechanism is however different from Gabszewicz et al. (2001). In particular, what drives this result in Dukes and Gal-Or (2003) is the role of advertising as information and the fact that advertising is a nuisance to the audience. Accordingly, if consumers dislike ads very much, media firms choose to minimum differentiate their offers so that advertisers choose lower levels of advertising. This turns to be positive to advertisers, since they can charge higher margins to consumers, given that consumers are less informed. Media firms also gain, once they can charge advertisers higher prices for advertising space.

The above results of Gabszewicz et al. (2001) and Dukes and Gal-Or (2003) differ from ours. In particular, our paper shows that, contrary to Gabszewicz et al. (2001) and Dukes and Gal-Or (2003), advertising does not necessarily always reduce content, since advertising can financially support a multi-content strategy that increases consumers' choice. When this occurs, advertisement can contribute to an increase in content provision in the market. The main reason for the different results in our paper is that both Gabszewicz et al. (2001) Dukes and Gal-Or (2003) consider only single-content firms, while we also open for multi-content firms. When firms are single-content they can only choose between minimum and maximum differentiation. However, when firms are multi-content, they can also choose how much content to offer in the market. The implication of this is that when the ad market is large, ad revenues can finance a multi-content strategy, which reduces the need for minimum differentiation, since with a multi-content strategy, firms can cover a line segment that is closer to the center. If the ad market is small, in turn, firms need to capture a larger audience (i.e.: move to minimum differentiation) to increase advertising revenues, so they can finance the multi-content strategy.

Turn now to the literature on price discrimination in two-sided markets[2]. Böhme (2016) and Jeon et al. (2022) look at price discrimination in a monopolist setting. Jeon et al. (2022) show that in terms of social welfare, the effects of price discrimination depend on if there are conflicts between the two sides of the market. Böhme (2016), in turn, shows that the level of information (incomplete versus complete) is central to the profitability of price discrimination strategy of the monopolist. Liu and Shuai (2016) also highlight the role of information quality on profits, consumer surplus and social welfare. Accordingly, better information tends to increase profits at the expenses of consumer surplus and social welfare. Kodera (2015) analyses the effects of price discrimination on the side of advertisers and show that the profitability of price discrimination between advertisers depends on aversion to advertisement by consumers. Lin (2020), in turn, look to a common business model in online markets: the provision of premium content with fewer ads. To study this, Lin (2020) develops a model with two types of consumers where firms can price discriminate between them. It is shown that allocation of ads between different consumer types depend on the nuisance costs of advertising.

The paper on price discrimination in two-sided markets closest to ours is, however, Liu and Serfes (2012), since they also consider the Hotelling framework. In this set-up, they show that price discrimination can soften price competition. As already discussed, the opposite can occur in our model. Accordingly, we find that in a one-sided market, firms always charge a positive price, while in a two-sided market firms can charge a zero-price to the consumers that are not offered their ideal variety. The difference between our results and those of Liu and Serfes (2012) is again due to the fact that they have firms that are single-content and firms' location is fixed at the extremes of the line. We instead have multi-content firms and location is endogenous. Accordingly, when firms' location is not fixed, firms can increase or decrease price competition (locating closer or further away from the center of the line) in response to the size of the advertising market. Similarly, when firms are multi-content, they can choose a

[2] For a review of price discrimination in one-sided markets, see Varian (1985, 1989).

multi-content strategy by providing more content along a line segment (multi-content segment). Such flexibility in firms' strategies is absent from Liu and Serfes (2012), and therefore our results should be seen as complementing theirs.

3 The Model

In this section, we present the theoretical model in this paper.

Consumers' Preferences. As in Hotelling (1929), we consider a line of length one, $[0, 1]$, and a duopoly market structure. The two firms are labeled as $i = L, R$. Firm L is the firm located in the left segment of the line; and firm R is the firm located in the right segment, i.e.: firm L is located at $d_L = x_L$ and firm R at $d_R = 1 - x_R$.

The line also represents consumers' preferences. As in Hotelling it is assumed that consumers are uniformly distributed on the line and that consumers' preferences are ordered from left to right. Consumers have an ideal variety, and they incur a disutility t from being exposed to varieties that differ from their own ideal variety. In this way, t represents the intensity of the consumers' preferences (i.e.: transport costs in Hotelling). Consumers either buy from firm L or from firm R, i.e.: consumers are single-homing[3]. Then, there is an indifferent consumer, x^*, from buying from L or from R.

Most models that use the Hotelling framework assume that firms can only supply one type of content (x_L and x_R, for L and R, respectively). Accordingly, firms are located in only one location (i.e. single-content firms). Like in Dewan et al. (2003), we differ from this by opening up for firms to offer different varieties (i.e. multi-content firms) in order to satisfy better consumers' preferences and potentially to price discriminate between them. Hence, in

[3]Single-homing is the standard assumption in Hotelling models. If we introduce multi-homing consumers (i.e. consumers that buy from both firm L and firm R), firms have lower incentives to follow a multi-content strategy. Accordingly, if consumers buy from both firms, competition is weakened, and as a result, firms have less need to diversify content to compete for customers. The fact that multi-homing can weaken competition is a well-known result from the multi-homing literature (see Kim and Serfes, 2006; Doganoglu and Wright, 2006; Athey et al. 2018; Cennamo et al., 2018; Jiang et al., 2019; Bakos and Halaburda, 2020; Wu and Chamnisampan, 2021; Wu and Chiu, 2023; Wu et al., 2023).

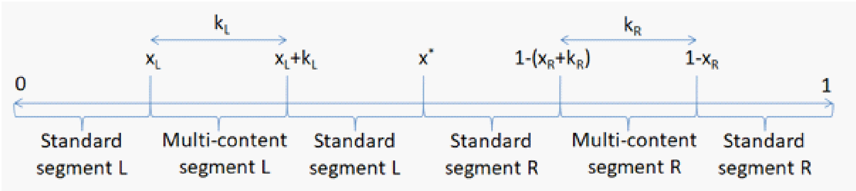

Figure 1: Multi-Content: L located at x_L and R at x_R; k_L and k_R content provision by L and R; x^* indifferent consumer

our model, firms can choose to follow a multi-content strategy by covering different locations.

We then denote by k_i the firm's multi-content scope, which equals the length of the Hotelling line cover by the firm, i.e.: $0 \leq k_i \leq 1$. Firms can decide to adopt a single-content strategy or a multi-content strategy. A single-content strategy corresponds to a single point on the line (x_L and x_R, with $k_L = 0$ and $k_R = 0$), while a multi-content strategy corresponds to a line segment ($[x_L, x_L + k_L]$ and $[1 - (x_R + k_R), 1 - x_R]$, with $k_L > 0$ and $k_R > 0$).

If a firm chooses a single-content strategy, it offers only one type of content (i.e. the content in the firm's location) to consumers with different content preferences. If a firm chooses a multi-content strategy, a firm offers different content that matches the preferred content of the consumers located inside the line segment covered by the firm. In turn, consumers outside the multi-content segment get content that does not match their preferred ideal content. Following Dewan et al. (2003), we label this last type of content as standard content.

Figure 1 and 2 represent the two possible à priori location equilibriums that can arise in the model. Figure 1 represents the equilibrium with $x_L > 0$ and $1 - x_R < 1$. Figure 2 represents the equilibrium with $x_L = 0$ and $1 - x_R = 1$. We are going to see that these two possible à priori equilibriums arise in fact à posteriori after solving the model. Note also that, if firms decide to not diversify content, then $k_L = 0$ and $k_R = 0$. In this case, the two figures need to be changed accordingly, i.e. there is no multi-content segment. When solving the model, we are going to see that both cases can also arise, i.e. with $k_L = 0$ and $k_R = 0$ and and $k_L > 0$ and $k_R > 0$.

To simplify exposition, abstract for now from price discrimination. Consider a consumer, x, that is located left of the center in the Hotelling line and is outside of the multi-content segment

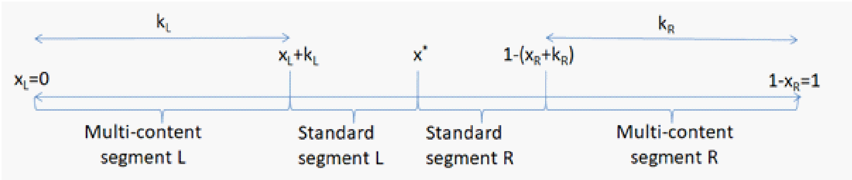

Figure 2: Multi-Content: L located at 0 and R at 1; k_L and k_R content provision by L and R; x^* indifferent consumer

of firm L (i.e.: his ideal variety is not offered). If firm L does not locate at the extreme of the line (i.e. at point 0), from figure 1, this consumer can then be located to the left or to the right of the segment $[x_L + k_L]$. The utility of this consumer will then depend on the location of firm L on the line. We can show that utility for consumer x can then be written as[4]:

$$
\begin{aligned}
U &= v - p_L - t\,(x - (x_L + k_L))^2 \text{, with } x > x_L + k_L, \\
U &= v - p_L - t\,(x_L - x)^2 \text{, with } 0 < x < x_L, \qquad (1)
\end{aligned}
$$

where v is a positive constant (that captures the reservation price of consumers) and p_L is the price of the firm L. A similar equation applies for a consumer located right of the center. We assume, as is usual in the Hotelling literature, that the parameter v is sufficiently large to ensure complete market coverage. It can also be seen that if a consumer is located inside the multi-content segment his utility is just: $U = v - p_i$, since $t = 0$ (i.e.: his ideal content is offered).

Technology: Content Development. Firms are profit-maximizing organizations, which produce with constant marginal costs (zero without loss of generality). The decision to follow a multi-content strategy depends on the costs and the benefits of this strategy *vis-à-vis* a single-content strategy. The costs of a multi-content strategy include the adaptation costs to fit content to different consumers' preferences. In turn, the benefits accrue through the possibility to price

[4] Following D'Aspremont et al. (1979), to have a location equilibrium, we assume quadratic transport costs.

discriminate amongst consumers.

Like in Dewan (2003), we assume that in order to follow a multi-content strategy, firms have to incur in costs to supply different content (C). These costs are positively related to the scope of the multi-content strategy, i.e.: if a firm offers more content, it bears higher costs. The idea is that, since consumers are uniformly distributed on the line, the amount of flexibility needed for adapting content to consumers' preferences increases with the size of the multi-content scope[5]. Like in Alexandrov (2008), we assume that the multi-content technology involves quadratic costs (in the next section, we discuss the multi-content strategy in more detail):

$$C_i = \frac{\gamma k_i^2}{2}, \tag{2}$$

where γ represents the flexibility costs pertaining to adapting to consumers' preferences, and $i = L, R$. In this sense, the costs to follow a multi-content strategy increase, as we have just said, with the diversity of content offered.

Price discrimination and consumers. One advantage of a multi-content strategy for firms is price discrimination. In the standard Hotelling (1929) model, price discrimination is not possible because firms only supply one type of content. However, in our model, firms supply different types of content which allows them to price discriminate for the different content varieties.

In particular, if a consumer is not offered his preferred variety (as is the case for all consumers when a firm follows a single-content strategy or for consumers in the standard segments when a firm follows a multi-content strategy), a firm cannot price discriminate him, i.e., charge this consumer a higher price. As a result, a firm can only charge this consumer the standard price p_i. On the contrary, if a consumer is offered his preferred variety (as it is the case for consumers

[5] Dukes and Xu (2019), in turn, present a model where firms can choose to offer different products. In particular, they show that firms can use aggregate consumer data to explore consumers perceptual errors in their intrinsic preferences.

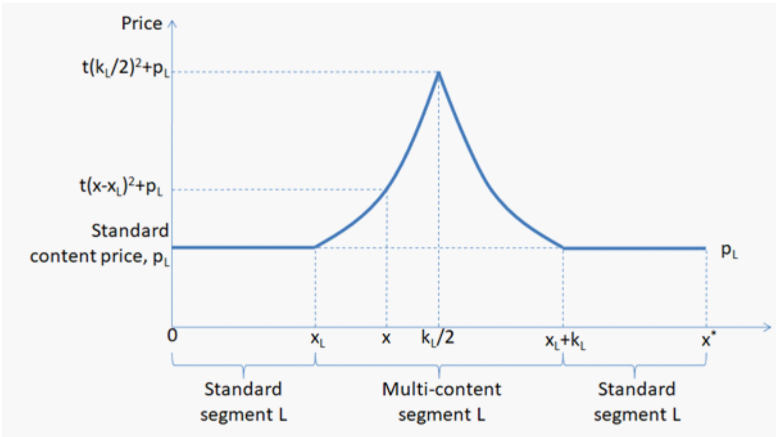

Figure 3: Price discrimination: Firm L located at x_L

in the multi-content segment when a firm follows a multi-content strategy), a firm can price discriminate, since the consumer's ideal variety is offered. Accordingly, in the multi-content segment, a firm can charge the consumer the price of the standard content (p_i) plus a premium. The premium equals the distance to the closest standard content times transport costs (t), once firms under a multi-content strategy are able to extract the full surplus from this consumer[6]. For proof see next section (and also Dewan et al., 2003).

Figures 3 and 4 exemplify the price discrimination scheme for the two possible à priori location equilibriums in the model. As we have said above, these two possible à priori locations equilibriums are: (1) $x_L > 0$ and $1 - x_R < 1$; and $x_L = 0$ and $1 - x_R = 1$. Naturally, figures 3 and 4 only apply when firms offer a multi-content segment, i.e. $k_L > 0$ and $k_R > 0$. Figure 3 represents the price discrimination scheme when $x_L > 0$ and $1 - x_R < 1$. Figure 2 represents the price discrimination scheme when $x_L = 0$ and $1 - x_R = 1$.

We can then show that profits in the customized segment for firm L equals (and symmetrically for firm R):

[6] Our paper then also is close to the spatial price discrimination literature of Beckman (1976) and Thisse and Vives (1988). This literature envisions a basic product that satisfies consumers' diverse tastes, with the marginal cost of redesign increasing with the distance between the basic product and the buyer's ideal variety.

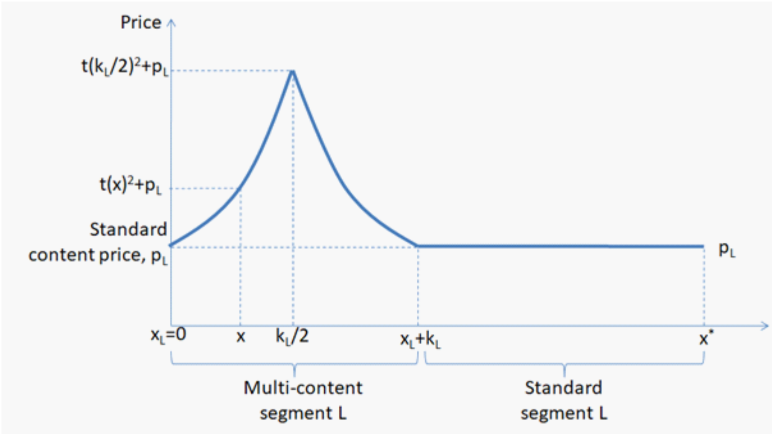

Figure 4: Price discrimination: Firm L located at 0

$$\int_{x_L}^{x_L+\frac{k_L}{2}} \left(p_L + t\left(x - x_L\right)^2\right) dx + \int_{x_L+\frac{k_L}{2}}^{x_L+k_L} \left(p_L + t\left(x_L + k_L - x\right)^2\right) dx$$

$$= 2\int_{x_L}^{x_L+\frac{k_L}{2}} \left(p_L + t\left(x - x_L\right)^2\right) dx = 2\int_{0}^{\frac{k_L}{2}} \left(p_L + tx^2\right) dx. \tag{3}$$

One interpretation for this price strategy, following the discussion in the introduction, is that when a firm follows a multi-content strategy, the firm goes after a premium strategy, since it price discriminates between consumers. In turn, when a firm follows a single-content strategy, it pursues a mass-scale strategy, once price discrimination is absent.

The main benefits for consumers of a multi-content strategy are twofold. First, price competition can become fiercer since firms move in the direction of the center of the line. Second, consumers (at least some of them) can save in transport costs, because firms offer more content varieties. Note that this can also be the case for consumers that have to pay a premium to acquire their ideal variety (see proof in the next section).

Advertising. In addition to the consumer market, firms can also explore revenues from the advertising market. We follow Anderson and Coate (2005) and Peitz and Valletti (2008) in assuming that demand for ads for firm i equals:

$$r_i = \alpha - \beta a_i, \tag{4}$$

where r_i is the price of advertising per consumer, a_i is the advertising volume, and $i = L, R,$. The parameters α and β reflect the size of the advertising market. Accordingly, a high α and a low β represent a large advertising market[7]. Then as in Anderson and Coate (2005), media firms extract all surplus from advertisers.

Gross advertising income is then:

$$A_i = (\alpha - \beta a_i)\, a_i D_i, \tag{5}$$

where D_i is the demand for the firm i, with $i = L, R,$. Accordingly, $D_L = x^*$ and $D_R = 1 - x^*$ (remember that x^* is the consumer who is indifferent between buying from firm L or firm R).

As will be seen more clearly below, in this set-up, advertising demand depends on the firm's consumer base, i.e. how many consumer it serves. Accordingly, more consumers mean more advertising; and more advertising means more revenues to invest in content. This feature gives our model a two-sided market framework, since there are positive externalities between the consumer and the advertising market.

Profits. From the above, we have that the profits for firm i equal:

$$\Pi_i = p_i\,(D_i - k_i) + 2\int_0^{\frac{k_i}{2}} \left(p_i + tx^2\right) dx - C_i + A_i, \tag{6}$$

[7] In this sense, we do not introduce targeting of advertising as in Esteban et al. (2001) and Esteves and Resende (2016, 2019). We also do not consider skippable and non-skippable adds as in Chakraborty et al. (2021). In addition, we also abstract from disutility of advertising. For a model of disutility from advertising, see Kind et al. (2007). We could introduce disutility of advertising by adding a term $-\eta a_i$ to consumers' utility function (with η the disutility of advertising and a_i, as in our model, the advertising volume of firm $i = L, R$). We can then see that if consumers dislike advertising, this would mean a smaller advertising market. This is exactly what the parameters α and β capture in our model: a large α and a small β represent a large ad market and the reverse the opposite. Then, although we do not consider the disutility costs of advertising, we can still see the effects of a small ad market by looking to α and β. In any case, note that the empirical evidence shows that consumers do not suffer from disutility of advertising. See for instance Gentzkow (2007), Fan (2013) and Gentzkow et al. (2014).

with $i = L, R$. The first and the second terms represent the revenues from the consumer market for the standard and multi-content segments, respectively. The third term stands for the costs of the multi-content strategy. The fourth term captures the revenues from the ad market.

Timing of the Game. In order to assess the effects of consumers' preferences, advertising and multi-content strategy on content provision, we consider two games. The first is a benchmark case with no advertising (i.e.: one-sided market) and the second introduces advertising (i.e.: two-sided market). In the advertising game the timing is the following: in the first stage, firms select the scope of the multi-content strategy k_i and the location of the firm x_i. In the second stage, firms decide on advertising levels a_i and the prices for the standardized content p_i, with $i = L, R$. In the benchmark case (non-advertising game, one-sided market), there is obviously no choice of advertising levels.

4 Multi-Content Strategy and Price Discrimination

In this section, we explain in more detail the relation between the multi content strategy and price discrimination. We first look at the multi-content strategy, then to price discrimination and after to the relation between the multi-content segment and transport costs.

4.1 Multi-Content Strategy

In what concerns the multi-content strategy, we have to highlight three things. Take firm L as example (by symmetry, the intuition is similar to firm R). First, when $x_L > 0$, firm L has no incentives to offer content to the left of point x_L. This is so because consumers on the left of x_L belong to its "hinterland" (see figure 1). In other words, when $x_L > 0$, like in Hotelling (1929), consumers located to the left of point x_L are captured by firm L, since due to transport costs it is always more costly for them to consume from firm R. The same occurs for firm R, which has no incentives to offer content to the right of point $1 - x_R$ (when $x_R > 0$).

Second, a firm can only have two types of content consumed in the standard segments (see figures 1 and 2): 1) the duopolist location, x_L and x_R; and 2), in the case of a multi-content strategy, the end point of the multi-content scope, $x_L + k_L$ and $1-(x_R + k_R)$. The core location of a firm always represents a standard content, since independently of following or not a multi-content strategy, the firm always delivers the content mirrored by its location on the line.

Third, given that consumers in the Hotelling (1929) model buy at most one variety (i.e. single-homing), we need to restrict the multi-content segments of the two firms to not overlap. In other words, we have a consumer x^* that is indifferent between buying from firm L or firm R (see figures 1 and 2).

4.2 Multi-Content Strategy and Price Discrimination

We now analyze the relation between the multi-content strategy and price discrimination. Take the example of firm L. Firm L can have at most two standard types of content: x_L and $x_L + k_L$. Content x_L and $x_L + k_L$ are consumed in the standard segments. Here consumers do not get their ideal variety. Firm L also has a series of content on the line segment $[x_L, x_L + k_L]$ consumed in the multi-content segment. Here consumers have access to their exact ideal variety. Suppose that consumer x is located inside the multi-content segment of firm L and that the closest standard content is x_L (the location of firm L). We then have that $p_L + t\,(x - x_L)^2$ is the price charged by firm L to consumer x. More generally:

$$
\begin{gathered}
\text{If } x_L \leq x \leq \left(x_L + \tfrac{k_L}{2}\right) \Rightarrow p_L + t\,(x - x_L)^2 \\
\text{If } \left(x_L + \tfrac{k_L}{2}\right) \leq x \leq (x_L + k_L) \Rightarrow p_L + t\,(x_L + k_L - x)^2 \\
\text{If } 1 - (x_R + k_R) \leq x \leq 1 - \left(x_R + \tfrac{k_R}{2}\right) \Rightarrow p_R + t\,(x - (1 - (x_R + k_R)))^2 \\
\text{If } 1 - \left(x_R + \tfrac{k_R}{2}\right) \leq x \leq 1 - x_R \Rightarrow p_R + t\,(1 - x_R - x)^2 .
\end{gathered} \tag{7}
$$

Note that the computation of the revenues from the multi-content segment can be simplified with the aid of symmetry. As discussed above, if firm L follows a multi-content, it has two standard types of content. Therefore, the multi-content segment can be divided into two equally sized line segments ($\left[x_L, x_L + \frac{k_L}{2}\right]$ and $\left[x_L + \frac{k_L}{2}, x_L + k_L\right]$). In this sense, in the multi-content segment, we have two symmetric consumers in terms of distance to the closest standard content offered. To see this more clearly, consider once again the example above of a consumer x that is located inside the multi-content segment of firm L. However, suppose that now the closest standard content is $x_L + k_L$ (instead of x_L). The price of the content bought by this consumer is then $p_L + t(x_L + k_L - x)^2$. Given the symmetry, however, for two different consumers in the multi-content segment of firm L, but located equally distant from the two standard contents of firm L (x_L and $x_L + k_L$), the price is the same; i.e.: if $x - x_L = x_L + k_L - x$, then $p_L + t(x - x_L)^2 = p_L + t(x_L + k_L - x)^2$.

Furthermore, as shown by Dewan et al. (2003), the above pricing scheme is optimal. To see this, suppose again that consumer x is located in the multi-content segment $[x_L, x_L + k_L]$ and that the closest standard content is x_L (the location of firm L). If firm L charges a price higher than $p_L + t(x - x_L)^2$, the price discrimination scheme collapses. In turn, if the price is lower than $p_L + t(x - x_L)^2$, firm L is not extracting the full rent from consumers. If, however, the price equals $p_L + t(x - x_L)^2$, consumers in the standard segments $[0, x_L[$ and $]x_L + k_L, x^*]$ choose the standard types of content x_L and $x_L + k_L$, respectively. In turn, consumers in the multi-content segment buy the content that is tailored for them. In this sense, the pricing scheme above is optimal and prevents arbitrage among buyers.

4.3 Multi-Content Strategy and Transport Costs

As mentioned above, the multi-content strategy can reduce transport costs that consumers incur. To show this, we have to consider two cases depending on the location of the firms on the line. In the first case, firms locate at the extremes of the line. In the second case, firms locate inside the line.

Start with the case where firms locate at the extremes of the line (firm L at 0, and firm R at 1). In this case, we can demonstrate that all consumers in the standard segment save in transport costs. To see this consider a consumer $x_1^{'}$ that is located in the standard segment between $x_L + k_L < x_1^{'} < x^*$, with $x_L = 0$ and x^* the indifferent consumer. Consumer $x_1^{'}$ under a multi-content strategy pays then transport costs $t\left(x^{'} - k_L\right)$. However, under a single-content strategy he would have to pay $tx_1^{'} > t\left(x_1^{'} - k_L\right)$, which means that he saves in transport costs under the multi-content strategy relatively to the single-content strategy.

Inside the multi-content segment, some consumers pay a premium that is inferior to the transport costs that they would have to pay under a single-content strategy (i.e. they save in transport costs). Other consumers, however, pay a premium that equals the transport costs that they would have to pay under a single-content strategy. Accordingly, consumers located between 0 and $\frac{k_L}{2}$ pay a premium under the multi-content strategy that equals the transport costs under the single-content strategy. To see this consider consumer $x^{'2}$ that is located between $0 < x_2^{'} < \frac{k_L}{2}$. In this case, under the single-content strategy the transport costs he pays are $tx_2^{'}$, and this equals the premium that firm L asks from consumer $x_2^{'}$.

Consumers located between $\frac{k_L}{2}$ and $x_L + k_L$ (with $x_L = 0$), however, pay a lower premium under the multi-content strategy than the transport costs that they would have to pay under the single-content strategy. To see this consider a consumer $x_3^{'}$ that is located between $\frac{k_L}{2} < x_3^{'} < k_L$. Under the single-content strategy the transport costs he pays are $tx_3^{'}$. However, under the multi-content strategy, he pays $tk_L < tx_3^{'}$. This is so because now for this consumer, content k_L is closer than content $x_L = 0$, and as shown above, firm L therefore cannot ask a higher premium. This occurs because under a multi-content strategy, firms offer two standard types of content in the multi-content segment: x_L and $x_L + k_L$ (and similarly, $1 - x_R$ and $1 - (x_R + k_R)$ for firm R).

If firms locate inside the line, the reasoning is similar. Take once more the example of consumers that buy from firm L. Consider first consumers in the standard segments. It can be shown that consumers on the right-hand side standard segment of firm L save on transport costs when firms offer a multi-content segment. However, those on the left-hand side standard segment pay the same as when a firm does not offer a multi-content segment. To see this consider a consumer $x_4^{'}$ that is located in the right-hand side standard segment between $x_L + \frac{k_L}{2} < x_4^{'} < x_L + k_L$. With a single-content strategy this consumer would have to incur transport costs $t\left(x_4^{'} - x_L\right)$, but with a multi-content strategy it only pays $t\left((x_L + k_L) - x_4^{'}\right) < t\left(x_4^{'} - x_L\right)$. However, for a consumer $x_5^{'}$ that is located in the left-hand side standard segment between $0 < x_5^{'} < x_L$, under the single-content strategy he incurs transport costs $t\left(x_L - x_5^{'}\right)$, which is exactly the same as he pays under a multi-content strategy. For consumers that are inside the multi-content segment of firm L, there is no change from when firm L locates at 0, and therefore the same results as for the case when firms locate at the extremes of the line apply.

5 Benchmark: Non-Advertising Game

In this section, we analyze the equilibrium choices of firms in the non-advertising game. As usual, the model is solved by backward induction. We start with prices p_i, and continue with location x_i and multi-content strategy k_i, with $i = L, R$. Before that, however, we need to find the consumer that is indifferent between buying from firm L and firm R, x^*.

Indifferent consumer. The indifferent consumer, x^*, is the one that makes:

$$v - p_L - t\left(x^* - (x_L + k_L)\right)^2 = v - p_R - t\left(1 - (x_R + k_R) - x^*\right)^2. \tag{8}$$

Solving for x^*, and noting that $D_L = x^*$ and $D_R = 1 - x^*$, we get that D_i equals (with $i, j = L, R$ and $i \neq j$):

$$D_i = \frac{p_j - p_i - t(x_i + k_i)^2 + t(1-(x_j + k_j))^2}{2t(1-(x_i + x_j + k_i + k_j))}. \tag{9}$$

Stage 2: Prices. In the second stage, firms choose prices for the standard content p_i, with $i = L, R$. Prices can be found by substituting for D_i (equation 9) in the profit expressions (equation 6) and then computing the first order condition (FOC) for p_i, $i, j = L, R$ and $i \neq j$ (all first order conditions, FOCs, and second order conditions, SOCs, are in appendix):

$$p_i = \frac{t(1-(x_i+x_j+k_i+k_j))(3+(x_i+k_i)-(x_j+k_j))}{3}. \tag{10}$$

Stage 1: Location. In the first stage, firms choose location and multi-content strategy. We start with location x_i, with $i = L, R$. In the next subsection, we pass on to the multi-content strategy. The FOC for x_i (with $i, j = L, R$ and $i \neq j$) is:

$$\frac{\partial \Pi_i}{\partial x_i} = p_i \left(\frac{\partial D_i}{\partial x_i} + \frac{\partial D_i}{\partial p_j} \frac{dp_j}{dx_i} \right). \tag{11}$$

The first and second terms inside the bracket on the right-hand side of equation 11 are usually labeled in the Hotelling literature as the direct effect and the strategic effect of location on revenues, respectively. The term $\frac{\partial D_i}{\partial x_i}$ captures the direct effect of the location of firm i (x_i) on its own demand (D_i). The term $\frac{\partial D_i}{\partial p_j}\frac{dp_j}{dx_i}$ refers to the indirect (strategic) effect of firm i's location (x_i) on its own demand (D_i), via the impact on the price of the rival firm j (p_j). In other words, when a firm chooses its location, it has to consider the effects on price competition, and not only on own demand.

As shown in appendix, as in the standard Hotelling model (see D'Aspremont et al., 1979) the direct effect is positive ($\frac{\partial D_i}{\partial x_i} > 0$), while the strategic effect is negative ($\frac{\partial D_i}{\partial p_j}\frac{dp_j}{dx_i} < 0$). The direct effect is positive, given that a firm increases its demand by moving to the center of the line. The indirect (strategic) effect is negative since, as the two firms locate closer together, price competition becomes fiercer, depressing profits. The indirect (strategic) effect however dominates the direct effect. In the absence of advertising, therefore, maximum differentiation is promoted, since locating closer to rivals unambiguously depresses profits.

Stage 1: Multi-Content Strategy. We turn now to multi-content strategy k_i, with $i = L, R$. The FOC for k_i equals (with $i, j = L, R$ and $i \neq j$):

$$\frac{\partial \Pi_i}{\partial k_i} = p_i \left(\frac{\partial D_i}{\partial k_i} + \frac{\partial D_i}{\partial p_j} \frac{dp_j}{dk_i} \right) + \frac{tk_i^2}{4} - \gamma k_i. \tag{12}$$

In equation 12, the first term is the effect of the multi-content strategy on the firm's demand, the second term is the effect of the multi-content strategy on price discrimination and the third term is the effect of the multi-content strategy on costs. Note that, as for location, the effects of the multi-content strategy on the firm's demand (the first term in equation 12) can be divided into a direct effect ($\frac{\partial D_i}{\partial k_i}$) and an indirect (strategic) effect ($\frac{\partial D_i}{\partial p_j} \frac{dp_j}{dk_i}$). The term $\frac{\partial D_i}{\partial k_i}$ captures the direct effect of firm i's multi-content strategy (k_i) on its own demand (D_i). The term $\frac{\partial D_i}{\partial p_j} \frac{dp_j}{dk_i}$ refers to the indirect effect of firm i's multi-content strategy (k_i) on its own demand (D_i), via the impact on the price of the rival (p_j). Therefore, when a firm chooses a multi-content strategy it also has to consider the effects of the multi-content strategy on price competition, and not only on demand.

Like for location, while the direct effect of the multi-content strategy on profits is positive, the indirect (strategic) effect is negative. The direct effect is positive, since with the multi-content strategy, firms move in the direction of the center of the line, therefore increasing demand. In turn, the indirect (strategic) effect is negative because a multi-content strategy increases price competition and consequently it also reduces the profits from price discrimination in the multi-content segment. Remember that the price in the multi-content segment equals the price of the standard segment plus the premium. Therefore, if the price of the standard segment is low, the total price charged in the multi-content segment is also low.

As shown in appendix, the direct effect is smaller than the indirect effect. A multi-content strategy can therefore reduce profits via an increase in price competition. A multi-content strategy hence depresses profits through fierce price competition and higher costs (the first and the third terms in equation 46, respectively), but increases profits through price discrimination in the multi-content segment (the second term). Next, we investigate which effect dominates.

Solution of the Model. The solution of the model is found by solving $\frac{\partial \Pi_i}{\partial k_i}$, $\frac{\partial \Pi_i}{\partial x_i}$, $\frac{\partial \Pi_j}{\partial k_j}$ and $\frac{\partial \Pi_j}{\partial x_j}$ simultaneously for k_i, x_i, k_j and x_j, with $i,j = L,R$ and $i \neq j$ (equations 41 and 46). We obtain four solutions, but only the following one satisfies all SOCs (see appendix), with $i = L,R$:

$$\begin{aligned} k_i &= 0 \\ x_i &= 0. \end{aligned} \tag{13}$$

To find prices, we just need to substitute in equation 10 for k_i and x_i from equation 13 to obtain (with $i = L,R$):

$$p_i = t. \tag{14}$$

In this sense, in the absence of an ad market (one-sided market), a duopolist locates at the extremes of the line (maximum differentiation) and it does not follow a multi-content strategy. In other words, the negative effects of the multi-content strategy shown in equation 46 (the first and the third terms) dominate the positive effects (the second term), i.e.: the possibility to price discriminate with the multi-content strategy does not compensate for the increase in price competition and the costs of the multi-content strategy. Hence, without advertising, firms do not follow a multi-content strategy. The following proposition summarizes the above results.

Proposition 1 *In a duopolist one-sided market with endogenous choice of location, firms locate at the opposite extremes of the line, they do not follow a multi-content strategy, and they do not price discriminate.*

6 Advertising Game

In this section, we analyze the equilibrium of the advertising game. Like in the non-advertising game, we need first to define the indifferent consumer (D_i, with $i = L, R$ and $D_L = x^*$ and $D_R = 1 - x^*$). We find that in the advertising game, the indifferent consumer is the same as in the non-advertising game. Therefore equations 8 and 9 continue to apply. We can then go forward to solve the model by backward induction. We start with prices p_i and advertising a_i, and then location x_i and multi-content strategy k_i, with $i = L, R$.

Stage 2: Prices. In the second stage, firms choose the price for the standard content p_i and advertising levels a_i, $i = L, R$. Start with prices. Prices are found by solving $\frac{d\Pi_i}{dp_i}$ and $\frac{d\Pi_j}{dp_j}$ simultaneously for p_i and p_j (with $i, j = L, R$ and $i \neq j$):

$$p_i = \frac{(2a_i(\beta a_i - \alpha) + a_j(\beta a_j - \alpha)) + t(1 - (x_i + x_j + k_i + k_j))(3 - (k_j + x_j) + (k_i + x_i))}{3}. \tag{15}$$

Stage 2: Advertising. Turn now to advertising levels a_i, with $i = L, R$. To find advertising levels, we solve $\frac{d\Pi_i}{da_i}$ and $\frac{d\Pi_j}{da_j}$ simultaneously for a_i and a_j. After substituting for for p_i and p_j, we obtain:

$$a_i = \frac{\alpha}{2\beta}. \tag{16}$$

Gross advertising income (A_i) can be found by substituting for a_i from equation 16 in equation 5 (with $i = L, R$):

$$A_i = \frac{\alpha^2}{4\beta} D_i. \tag{17}$$

Advertising income then increases with demand (D_i). This shows the two-sided nature of the market since there are positive externalities between the consumer and the advertising market. In other words, a firm with higher sales is more attractive for advertisers and, as such, firms have incentives to increase the demand to augment the demand for ads.

Stage 1: Location. In the first stage, firms choose location and the multi-content strategy. We start with the choice of location x_i ($i = L, R$). The FOC for x_i equals (with $i, j = L, R$ and $i \neq j$):

$$\frac{\partial \Pi_i}{\partial x_i} = p_i \left(\frac{\partial D_i}{\partial x_i} + \frac{\partial D_i}{\partial p_j} \frac{dp_j}{dx_i} \right) + \frac{\partial A_i}{\partial D_i} \frac{dD_i}{dx_i}. \tag{18}$$

Relatively to the non-advertising game (equation 11), equation 18 has a new term: the second one ($\frac{\partial A_i}{\partial D_i} \frac{dD_i}{dx_i}$). This term is related with the advertising market. Now the choice of location affects not only revenues in the consumer market (the first term in equation 18) but also revenues in the advertising market (the second term).

The first term in equation 18 ($p_i \left(\frac{\partial D_i}{\partial x_i} + \frac{\partial D_i}{\partial p_j} \frac{dp_j}{dx_i} \right)$) has the same sign as above for the non-advertising game. In other words, the direct effect is positive, the indirect (strategic) effect is negative, and the indirect effect dominates the direct effect. In this sense, in the advertising game, we also have that the positive direct effect of locating closer to the center of the line in order to capture more demand ($\frac{\partial D_i}{\partial x_i}$) is smaller than the negative indirect effect of tougher price competition ($\frac{\partial D_i}{\partial p_j} \frac{dp_j}{dx_i}$). As such, competition in the content market continues to promote maximum differentiation.

It can be shown that the second term ($\frac{\partial A_i}{\partial D_i} \frac{dD_i}{dx_i}$), the effect of advertising on location, is positive. In other words, the advertising market contributes to minimum differentiation. This result is similar to Gabszewicz et al. (2001). The intuition is that firms by locating closer to the center of the market can attract more demand (D_i), which in turn increases demand for ads and therefore augments advertising revenues (equation 17). Despite this similarity to Gabszewicz et al. (2001), we are going to see that this result has different implications in our model in what concerns the multi-content strategy.

Summing up, with advertising, location affects the profits of firms in two opposing ways. The first term in equation 18 (the effect of the firm's location on the firm's demand) contributes to maximum differentiation, while the second term (the effect of the firm's location on the firm's ad revenues) gives support to minimum differentiation. Below, we will analyze which effect dominates.

Stage 1: Multi-Content Strategy. We now turn to the multi-content strategy k_i, with $i = L, R$. The FOC for k_i equals (with $i, j = L, R$ and $i \neq j$):

$$\frac{d\Pi_i}{dk_i} = p_i \left(\frac{\partial D_i}{\partial k_i} + \frac{\partial D_i}{\partial p_j} \frac{dp_j}{dk_i} \right) + \frac{\partial A_i}{\partial D_i} \frac{dD_i}{dk_i} + \frac{tk_i^2}{4} - \gamma k_i. \tag{19}$$

Relatively to the non-advertising game (equation 12), equation 19 has a new term: the second one ($\frac{\partial A_i}{\partial D_i} \frac{dD_i}{dk_i}$) that, as above for location, concerns the ad market. In particular, now the multi-content strategy affects not only the firm's demand (the first term in equation 19), price discrimination (the third term) and the costs of the multi-content strategy (the fourth term), but also the revenues from the advertising market (the second term).

The first term in equation 19 ($p_i \left(\frac{\partial D_i}{\partial k_i} + \frac{\partial D_i}{\partial p_j} \frac{dp_j}{dk_i} \right)$) has the same sign as above for the non-advertising game. In other words, the direct effect ($\frac{\partial D_i}{\partial k_i}$) is positive, the indirect (strategic) effect ($\frac{\partial D_i}{\partial p_j} \frac{dp_j}{dk_i}$) is negative, and the indirect effect dominates the direct effect. In this sense, in the advertising game, the direct positive effect of the multi-content strategy on demand is smaller than the indirect negative effect of fierce price competition. Consequently, the multi-content strategy can reduce profits in the consumer market, given that it contributes to lower prices in the standard segment and in the multi-content segment. Accordingly, as said above, a lower price in the standard segment implies a lower price also in the multi-content segment, given that the price in the multi-content segment equals the price in the standard segment plus the premium.

In turn, the second term in equation 19 ($\frac{\partial A_i}{\partial D_i} \frac{dD_i}{dk_i}$) is positive. The rationale is that higher demand increases demand for ads (two-sided market). In addition, advertising can also contribute positively to a multi-content strategy, because higher advertising revenues can make it possible for firms to finance the costs of the multi-content strategy.

Summing up, in the presence of advertising, the multi-content strategy affects the profits of firms in four opposing ways. The first and the fourth terms in equation 19 (the effect of the firm's multi-content levels on the firm's demand and on the firm's costs, respectively) contribute negatively to the multi-content strategy. In turn, the second and the third terms in equation 19 (the effect of the firm's multi-content levels on ad revenues and on price discrimination, respectively) have a positive impact on the multi-content strategy. Next, we analyze which effect dominates.

Solution of the Model. The solution of the model is found by solving $\frac{\partial \Pi_i}{\partial k_i}$, $\frac{\partial \Pi_i}{\partial x_i}$, $\frac{\partial \Pi_j}{\partial k_j}$ and $\frac{\partial \Pi_j}{\partial x_j}$ simultaneously for k_i, x_i, k_j and x_j, with $i, j = L, R$ and $i \neq j$ (equations 41 and 46). The advertising game gives four solutions (two asymmetric and two symmetric, see appendix). However, only the following symmetric solution satisfies all SOCs (with with $i = L, R$):

$$\begin{aligned} k_i &= \tfrac{4\gamma}{t} \\ x_i &= \tfrac{\alpha^2 - 4\beta(32\gamma - t) + \sqrt{\alpha^4 + 24t\beta(6t\beta - 5\alpha^2)}}{32\beta t} \\ a_i &= \tfrac{\alpha}{2\beta}. \end{aligned} \tag{20}$$

From equation 20 we have that $k_i > 0$ and $a_i > 0$. In this sense, given that firms always follow a multi-content strategy, in the advertising game the positive effects of the multi-content strategy (the second and the third terms in equation 19) dominate the negative ones (the first and the fourth terms).

In what concerns location, x_i can either be positive or negative[8]. In other words, in the

[8] Since $x_i \in [0, 1]$, when $x_i < 0$ it follows that $x_i = 0$ (see appendix).

advertising game the forces for maximum differentiation (the first term in equation 18) can be weakened by those for minimum differentiation (the second term in equation 18), i.e.: if $x_i \leq 0 \Rightarrow x_i = 0$, firms choose maximum differentiation; and if $x_i > 0$ firms move in the direction of minimum differentiation. As shown in appendix, the sign of x_i depends on the threshold level $\beta' = \frac{\alpha^2(2\gamma-t)}{(t+16\gamma)(8\gamma-t)}$ (i.e.: β' makes $x_i = 0$). Remember that a low β represents a large advertising market, while a high β means a small advertising market (equation 4). Furthermore, as shown in the appendix, in order to the multi-content segments of the two firms do not overlap, we need to impose that $t > 8\gamma$. The game is then only valid for $t > 8\gamma$, i.e.: when trade costs are significantly larger than the costs associated with the multi-content strategy[9]. As a result of the previous observations, and as in Gabszewicz et al. (2001), we have that the solution of the advertising game has two cases that depend on the size of the advertising market.

A) Equilibrium of the game for $0 < \beta \leq \frac{\alpha^2(2\gamma-t)}{(t+16\gamma)(8\gamma-t)}$ and $t > 8\gamma$ (large advertising market): In this case, we get (with $i = L, R$):

$$\begin{aligned} k_i &= \tfrac{4\gamma}{t} \\ x_i &= 0 \\ a_i &= \tfrac{\alpha}{2\beta}. \end{aligned} \tag{21}$$

We then have that when the advertising market is relatively large, firms choose maximum differentiation. Substituting for k_i, x_i and a_i (equation 21) into p_i (equation 15), we obtain ($i = L, R$):

[9] As mentioned in the introduction, the standard assumption in the Hotelling model is that consumers just buy one variety of content from one firm, i.e. there is an indifferent consumer between buying from L or R. Because of this, we need to impose that the two segments of content do not overlap (see appendix). The intuition for this condition is very straightforward. If the transport costs are not sufficiently larger than the costs of the multi-content strategy, nothing stops the firms from overlapping their multi-content segments.

$$p_i = \frac{4\beta(t-8\gamma)-\alpha^2}{4\beta}. \tag{22}$$

Again the sign of p_i depends on the value of β, now with threshold level $\beta^{''} = \frac{\alpha^2}{4(t-8\gamma)}$ (i.e.: β that makes $p_i = 0$). In particular, $p_i > 0$ for $\beta > \frac{\alpha^2}{4(t-8\gamma)}$. Since, $\beta^{'} - \beta^{''} = \frac{3\alpha^2}{4(t+16\gamma)} > 0$, we have (with $i = L, R$):

$$\begin{aligned} \text{If } 0 \quad &< \quad \beta \leq \frac{\alpha^2}{4(t-8\gamma)} \Rightarrow p_i = 0 \\ \text{If } \frac{\alpha^2}{4(t-8\gamma)} \quad &< \quad \beta < \frac{\alpha^2(2\gamma-t)}{(t+16\gamma)(8\gamma-t)} \Rightarrow p_i = \frac{4\beta(t-8\gamma)-\alpha^2}{4\beta} > 0. \end{aligned} \tag{23}$$

When the advertising market is extremely large ($0 < \beta \leq \frac{\alpha^2}{4(t-8\gamma)}$), the price in the standard segment is zero. However, when the advertising market is large, but not extremely large ($\frac{\alpha^2}{4(t-8\gamma)} < \beta < \frac{\alpha^2(2\gamma-t)}{(t+16\gamma)(8\gamma-t)}$), prices in the standard segment become positive.

B) Equilibrium of the game for $\beta > \frac{\alpha^2(2\gamma-t)}{(t+16\gamma)(8\gamma-t)}$ and $t > 8\gamma$ (small advertising market): In this case, we obtain (with $i = L, R$):

$$\begin{aligned} k_i &= \frac{4\gamma}{t} \\ x_i &= \frac{\alpha^2-4\beta(32\gamma-t)+\sqrt{\alpha^4+24t\beta(6t\beta-5\alpha^2)}}{32\beta t} \\ a_i &= \frac{\alpha}{2\beta}. \end{aligned} \tag{24}$$

We then have that when the advertising market is relatively small, firms do not opt anymore for maximum differentiation, since $x_i > 0$. Still, firms continue to choose a multi-content segment.

Substituting for k_i, x_i and a_i (equation 24) into p_i (equation 15), we have (with $i = L, R$):

$$p_i = \frac{12t\beta - 5\alpha^2 - \sqrt{\alpha^4 + 24t\beta(6t\beta - 5\alpha^2)}}{16\beta} > 0. \tag{25}$$

It can be easily checked that $p_i > 0$. As a result, when the advertising market is small, firms charge positive prices. The next proposition summarizes the results of the advertising game.

Proposition 2 *In a duopolist two-sided market with endogenous choice of location, firms' advertising and multi-content scope are always positive and equal to* $a_i = \frac{\alpha}{2\beta}$ *and* $k_i = \frac{4\gamma}{t}$. *In terms of location, two equilibriums arise. For* $0 < \beta \leq \frac{\alpha^2(2\gamma - t)}{(t+16\gamma)(8\gamma - t)}$ *and* $t > 8\gamma$, *firms locate at the extremes of the line. Also, for* $0 < \beta \leq \frac{\alpha^2}{4(t-8\gamma)}$, *firms charge zero price, while for* $\frac{\alpha^2}{4(t-8\gamma)} < \beta < \frac{\alpha^2(2\gamma - t)}{(t+16\gamma)(8\gamma - t)}$ *prices are positive. In turn, for* $\beta > \frac{\alpha^2(2\gamma - t)}{(t+16\gamma)(8\gamma - t)}$ *and* $t > 8\gamma$, *firms do not locate at the extremes of the line. In this case, firms charge positive prices.*

7 Social Welfare: No Advertising Game versus Advertising Game

In this section, we analyze social welfare under the advertising and the no advertising game. To do this, we have to look at profits and consumer surplus (the expressions for profits, consumer surplus and social welfare are in appendix)[10]. In this regard, as we have seen above, it can also be instructive to study transport costs and prices in the two games, since these determine social welfare, consumer surplus and profits. We do this in Appendix.

To analyze social welfare, we must consider that the equilibrium of the advertising game depends on the size of the advertising market. Therefore, we analyze the differences between the two games taking this into account. In particular, we look at three cases: large advertising market, medium advertising market and small advertising market. In the following, we denote the no advertising game with superscript N and the advertising game with A.

[10] As already mentioned, as in Anderson and Coate (2005), media firms extract all surplus from advertisers, and therefore we do not have to take them into account in the social welfare analysis.

1) Social Welfare for $0 < \beta \leq \frac{\alpha^2}{4(t-8\gamma)}$ (large advertising market): Start with profits. The difference in profits between the no advertising game and the advertising game, when the advertising market is large is:

$$\left(\Pi_L^N + \Pi_R^N\right) - \left(\Pi_L^A + \Pi_R^A\right) = t - \frac{\left(3t^2\alpha^2 - 64\beta\gamma^3\right)}{12t^2\beta}. \tag{26}$$

The first thing to see is that the sign of the previous expression can either be positive or negative. Note that the advertising game tends to have higher profits than the no advertising game, when the advertising market is large (i.e. the larger α is relatively to β). In addition, high transport costs (high t) is only good to the no advertising game, if the costs of the multi-content strategy are low (low γ), since $d\left(\left(\Pi_L^N + \Pi_R^N\right) - \left(\Pi_L^A + \Pi_R^A\right)\right)/dt = \frac{3t^3-32\gamma^3}{3t^3} > 0$. This is so because if γ is small, firms invest in a larger multi-content segment, which increases price competition, reducing therefore profits.

In turn, the difference in consumer surplus between the no advertising game and the advertising game, when the advertising market is large, equals:

$$\left(CS^N\right) - \left(CS^A\right) = -\frac{64\gamma^3 + t(t-4\gamma)(t+8\gamma)}{t^2}. \tag{27}$$

Since $t > 8\gamma$, then consumer surplus is higher under the advertising game than under the no advertising game. The reasons for this are that under the multi-content strategy consumers save in transport costs and pay lower prices (see appendix).

In what respects social welfare, the difference between the no advertising game and the advertising game, when the advertising market is large equals:

$$\left(W^N\right) - \left(W^A\right) = -\frac{704\beta\gamma^3 + 3t\left(t\alpha^2 + 16\beta\gamma(t-8\gamma)\right)}{12t^2\beta}. \tag{28}$$

Since $t > 8\gamma$, then social welfare is higher under the advertising game. The reasons for this are that a large advertising market gives extra revenues for firms to finance a multi-content strategy and a multi-content strategy allows consumers to pay less to acquire content (transport costs and price plus premium) than under a single-content strategy.

2) Social Welfare for $\frac{\alpha^2}{4(t-8\gamma)} < \beta < \frac{\alpha^2(2\gamma-t)}{(t+16\gamma)(8\gamma-t)}$ (medium advertising market): Now we consider a medium size adverting market. In this case, the difference between the no advertising game and the advertising game is:

$$\left(\Pi_L^N + \Pi_R^N\right) - \left(\Pi_L^A + \Pi_R^A\right) = \frac{8\gamma}{3}\left(\frac{2\gamma^2\left(3t^3-16\gamma^6\right)}{t^5} + 3\right). \tag{29}$$

The sign of the previous expression can again either be positive or negative. In particular, the sign depends on the relation between t (transport costs) and γ (costs of multi-content strategy). We can see that high transport costs (high t) are always good for firms' profits under the no advertising game. This is so because prices in the no advertising game equals transport costs, and therefore high t means higher prices. In turn, high costs of multi-content strategy (high γ) are positive for profits under the advertising game, because they reduce the size of the multi-content segment and therefore, they also reduce price competition.

We turn now to consumer surplus. The difference between the no advertising game and the advertising game, when the advertising market is medium, equals:

$$\left(CS^N\right) - \left(CS^A\right) = -\frac{256\beta\gamma^3 + t\left(t\alpha^2 + 16\beta\gamma(3t-8\gamma)\right)}{4t^2\beta}. \tag{30}$$

Since $t > 8\gamma$, then consumer surplus is again higher under the advertising game than under the no advertising game. The reasons for this to occur are the same as for a large advertising market. First, a multi-content strategy relatively to a single-content strategy, reduces the transport costs that consumers have to pay to acquire their ideal variety (see proof in appendix). Second, a multi-content strategy reduces prices because it increases price competition (see proof in appendix).

In terms of social welfare, the difference between the no advertising game and the advertising game, when the advertising market is medium, equals:

$$\left(W^{N}\right)-\left(W^{A}\right)=-\frac{1024\beta\gamma^{9}+3t^{3}\left(t^{2}\alpha^{2}+16\beta\gamma(t-6\gamma)(t-2\gamma)\right)}{12t^{5}\beta}. \tag{31}$$

As for the large advertising market case, under a medium advertising market, social welfare is always higher under the advertising game. The reasons are the same as above for a large advertising market. A medium advertising market provides revenues for firms to finance a multi-content strategy; and a multi-content strategy allows consumers to pay less to acquire content (transport costs and price plus premium) than under a single-content strategy (see appendix).

3) Social Welfare for $\beta > \frac{\alpha^{2}(2\gamma-t)}{(t+16\gamma)(8\gamma-t)}$ (small advertising market): We look now to the small adverting market case. Remember that with a small advertising market, firms locate inside of the line. This is important for two reasons. First, because when firms locate inside the line, as we have seen above (Section 4), fewer consumers benefit with a multi-content strategy in terms of transport costs savings (see also appendix). Accordingly, fewer consumers pay less under the multi-content strategy (transport costs or premium) than under the single-content strategy (transport costs). Second, when firms locate inside the line, price competition is fiercer than when they locate at the extremes of the line, because firms compete closer to the center of the line. As we have discussed previously, this is triggered by a small advertising market: large ad revenues, turns firms more dependent on revenues from the consumer market. This makes firms to fight more fiercely for the marginal consumer.

Start again with profits. We have that the difference between the no advertising game and the advertising game is:

$$\left(\Pi_{L}^{N}+\Pi_{R}^{N}\right)-\left(\Pi_{L}^{A}+\Pi_{R}^{A}\right)=\frac{3t^{2}\left(4t\beta+\alpha^{2}\right)+256\beta\gamma^{3}+3t^{2}\sqrt{144t^{2}\beta^{2}+\alpha^{2}(\alpha^{2}-120t\beta)}}{48t^{2}\beta}. \tag{32}$$

When the ad market is small, profits are unambiguously smaller under the advertisement game. As we have just mentioned, the reason for this result is that now firms do not locate anymore at the extreme of the line. Since firms continue to follow a multi-content strategy, this means that price competition is very tough, reducing therefore profits.

In what respects consumer surplus, we have that the difference between the no advertising game and the advertising game, when the advertising market is small equals:

$$\left(CS^N\right) - \left(CS^A\right) = \frac{\left(\begin{array}{c} \left(3t\left(\alpha^2 - 4\beta\left(9t + 8\gamma\right)\right) - 24\gamma\left(\alpha^2 - 64\beta\gamma\right)\right)\sqrt{\alpha^4 + 24t\beta\left(6t\beta - 5\alpha^2\right)} \\ -t\left(16t\beta\left(\beta\left(31t + 120\gamma\right) + 42\alpha^2\right) - 3\left(\alpha^4 + 64\beta\gamma\left(7\alpha^2 + 32\beta\gamma\right)\right)\right) - 24\gamma\left(\alpha^4 - 64\beta\gamma\left(\alpha^2 - 32\beta\gamma\right)\right) \end{array}\right)}{1536t^2\beta^2}. \tag{33}$$

This expression can be positive or negative. Then contrary to the previous cases above (large and medium advertisement markets), it is not anymore sure that consumer surplus is higher under the advertising game. The main reason for this is that, as shown above (Section 4), when firms locate inside the line, fewer consumers save on transport costs relatively to when firms locate at the extremes of the line. The savings in transport costs is the determining factor for consumer surplus to not continue to be higher under the multi-content strategy, because as we have seen above, prices continue to be lower under the advertising game.

Finally, for social welfare, we have that the difference between the no advertising game and the advertising game, when the advertising market is small, amounts to:

$$\left(W^N\right) - \left(W^A\right) = \frac{\left(\begin{array}{c} \left(3\left(t-8\gamma\right)\left(\alpha^2-4\beta\left(t+16\gamma\right)\right)\right)\sqrt{\alpha^4+24t\beta\left(6t\beta-5\alpha^2\right)} \\ -t\left(16t\beta\left(36\alpha^2+\beta\left(7t+120\gamma\right)\right)-3\left(\alpha^4+64\beta\gamma\left(7\alpha^2+32\beta\gamma\right)\right)\right)-8\gamma\left(3\alpha^4-64\beta\gamma\left(3\alpha^2-80\beta\gamma\right)\right) \end{array}\right)}{1536t^2\beta^2}. \tag{34}$$

Social welfare is not anymore unambiguously larger under the advertising game than under the no advertising game, as it occurred for a large and a medium advertising market. This is so because now under the advertising game, profits are always smaller and consumer surplus can also be smaller under the advertising game comparatively to the no advertising game. The reasons for this are the same as discussed above for profits and consumer surplus: fiercer price competition that is not compensate with advertising revenues, and smaller savings for consumers in transport costs from the multi-content strategy (see also appendix).

8 Robustness of Results

In this section, we discuss two extensions of the model to check the robustness of the results. First, we allow for consumers to self-select out of the multi-content segment. Second, we open for firms to choose a discontinuous multi-content segment, instead of a continuous multi-content segment.

8.1 Self-Selection out of the Multi-Content Segment

A question that can arise is if consumers in the premium multi-content segment have incentives to self-select out of the premium plan in order to not pay the premium.

We can start by saying, as we have seen before, that the consumers that are offered their ideal variety, what firms do is to price discriminate between them by charging them the transport costs that they would have to pay if their ideal variety was not offered (as in Dewan et al., 2003). In other words, premium consumers do not have to incur in the transport costs because they are offered their ideal variety, but firms charge them these transports costs with the premium pricing.

In this sense, if consumers in the premium segment opted out of this plan, they would have anyway to incur the transport costs, because the firm would just offer them the standard product (i.e. they would not get their ideal variety). In fact, as we have seen above (Section 4), in the premium segment, some consumers (those between $x_L + k_L/2$ and $x_L + k_L$) pay a premium that is lower than the transport costs that they would have to pay if they would opt out of the premium plan. These consumers then do not have incentives to go out of the premium plan. The other consumers in the premium segment (those between x_L and $x_L + k_L/2$) pay a premium that equals exactly the transport costs. These consumers then are indifferent between the premium and the standard segment. Then, introducing self-selection out of the premium plan would not change the results in the paper.

8.2 Discontinuous Multi-Content Segment

Now we look to the case where firms can choose to have a discontinuous multi-content segment away from its location. To illustrate this, consider firm L (the same applies by symmetry for firm R). Consider then that firm L is located at x_L, and it can choose to have a multi-content segment at $x_L^{'} + k_L$, with $x_L^{'} > x_L$.

To make the case clearer, we assume way that it is more costly to provide content discontinuously from the firm's location, and that a firm has to pay one cost for the content in its core location and another cost for the content in the discontinuous multi-content segment. We believe that it would have been reasonable to assume that these two costs are higher in the discontinuous case. Accordingly, in the continuous case a firm could reap higher economies of scope, since content are more closely related than in the discontinuous case. In other words,

x_L versus $]x_L + k_L]$ are closers substitutes than x_L versus $[x_L^{'} + k_L]$, and therefore firms can reap higher economies of scope with the continuous case, i.e. x_L versus $]x_L + k_L]$. Also in the continuous case, a firm needs to develop less content. In the continuous case, a firm has to develop the following content, $[x_L + k_L]$, while in the discontinuous case it has to develop x_L plus $[x_L^{'} + k_L]$.

We abstract however from these costs, because they would tilt the firms' choice for the continuous case. We want to show that the discontinuous case is not attractive for a firm even when the costs in the continuous and discontinuous case are the same.

In the case of a (discontinuous) multi-content segment at $x_L^{'} + k_L$, with $x_L^{'} > x_L$, consumer utility is:

$$\begin{aligned} U &= v - p_L - t\left((x_L - x)\right)^2, \text{ with } 0 < x < x_L \\ U &= v - p_L - t\left((x - x_L)\right)^2, \text{ with } x_L < x < \frac{x_L + x_L'}{2} \\ U &= v - p_L - t\left((x_L' - x)\right)^2, \text{ with } \frac{x_L + x_L'}{2} < x < x_L' \\ U &= v - p_L, \text{ with } x_L' < x < x_L' + k_L, \\ U &= v - p_L - t\left(x - \left(x_L' + k_L\right)\right)^2, \text{ with } x > x_L' + k_L, \end{aligned} \tag{35}$$

The equations for costs of the multi-content segment, price discrimination, advertising market, and profits are as in the main model. It can be easily seen that the indifferent consumer is the same as in the main model with x_L substituted for $x_L^{'}$. Since the expression for the indifferent consumer is the same, this means that the other equations also do not change, the only thing again being that we have to substitute x_L for $x_L^{'}$. This also applies for the FOC for location (x_L). Moreover, when solving the FOC for $x_L^{'}$, we get that the FOC for x_L is the same as the FOC for $x_L^{'}$, which means that $x_L = x_L^{'}$. This shows that in the context of the Hotelling model a firm has no incentives to provide a multi-content segment away from its location.

The reason for this to be the case is price competition. We know from the Hotelling model (see also discussion above) that when firms move in the direction of the center, price competition increases. Then by moving discontinuously in the direction of the center, since $x_L^{'}+k_L > x_L+k_L$, a firm is increasing price competition more than it would by just moving continuously in the direction of the center[11].

9 Discussion

In this paper, we have shown that price discrimination can affect content provision in two-sided markets (markets where firms compete on both prices and advertising). First, if firms do not compete on advertising (i.e. they compete only on prices, one-sided market), firms do not diversify content, i.e. firms follow a single-content strategy. If firms compete on both prices and advertising (i.e. two-sided market), firms diversify content, i.e. firms follow a multi-content strategy. This is so independently of the size of the advertising market.

Second, firms only price discriminate between consumers, when they compete on prices and advertising (i.e. two-sided market), since only then firms follow a multi-content strategy. In particular, firms set different prices to the consumers which are offered their ideal variety, i.e. consumers located in the multi-content segment of the firms. For the other consumers (i.e. the ones that are not offered their ideal variety but just a standard type of content), firms do not price discriminate. The price discrimination strategy consists in setting a premium on the top of the price for the standard segment. The premium equals the transports costs that a consumer would have to pay if his ideal variety was not offered. Furthermore, a multi-content strategy triggers fierce price competition. In fact, when the advertising market is large, firms set zero prices in the standard segment. This strategy is similar to the freemium strategy followed in online markets (free plus premium, free to some consumers and premium to others that

[11] Here we have looked only at the case with one discontinuous multi-content segment. It can however be seen that with more than one multi-content segment, price competition would be even stronger, and therefore, more than one discontinuous multi-content segments would also not arise in equilibrium.

subscribe, for instance, extra services or content).

The reason why price competition is fiercer under the advertising game is that when firms diversify content (i.e. they follow a multi-content segment), firms compete closer to each other in the center of the line to conquer the marginal consumer. As a result of this fiercer price competition in the advertising game, the multi-content strategy (with price discrimination) conduces to lower prices than in the no advertising game where firms follow a single-content strategy. This is so independently of the size of the advertising market.

In terms of social welfare, we have seen that social welfare is bigger under the advertising game relatively to the no advertising game, when the advertising game is medium to large. This is so because when the advertising market is medium to large consumers benefit in terms of lower prices and lower transport costs to consume their ideal variety (resulting from the multi-content strategy), and firms get extra revenues from advertising. This might not be any longer the case when the advertising market is small. In this case, social welfare can be larger under the no advertising game. The reasons for this are the following. First, when the advertising game is small, the savings in transport costs that consumers experience due to the multi-content strategy of firms are not as large as when the advertising market is medium to large. Second, for firms, the revenues from advertising are smaller and price competition is fiercer.

In this way, we see the importance of advertising on firms' strategies: both in terms of content diversification strategies and price strategies, like price discrimination. Accordingly, advertising give extra revenues that can compensate for fiercer price competition that ensues by competing closer to the rival's market and allow therefore firms to finance a multi-content strategy and to price discriminate between consumers.

10 Appendix

10.1 Non-Advertising Game

FOCs and SOCs: Non-Advertising Game. FOC for prices, p_i ($i, j = L, R$ and $i \neq j$):

$$\frac{\partial \Pi_i}{\partial p_i} = \frac{p_j - 2p_i + t(1-(x_i+x_j+k_i+k_j))((x_i+k_i)-(x_j+k_j)+1)}{2(1-(x_i+x_j+k_i+k_j))t}. \tag{36}$$

SOC for prices (with $i, j = L, R$ and $i \neq j$):

$$\frac{\partial^2 \Pi_i}{\partial p_i^2} = -\frac{1}{t(1-(x_i+x_j+k_i+k_j))}. \tag{37}$$

The SOC for prices requires that $(1 - (x_i + x_j + k_i + k_j)) > 0$. This is a intuitive SOC, since it implies that the sum of the firms' location and multi-content levels cannot be bigger than the length of the line.

FOC for location, x_i (with $i, j = L, R$ and $i \neq j$). We can show that equation 11 equals:

$$\begin{aligned} \frac{\partial D_i}{\partial x_i} &= \frac{p_j - p_i + t(1-(x_i+x_j+k_i+k_j))^2}{2t(1-(x_i+x_j+k_i+k_j))^2} \\ \frac{\partial D_i}{\partial p_j} &= \frac{1}{2t(1-(x_i+x_j+k_i+k_j))} > 0 \\ \frac{dp_j}{dx_i} &= -\frac{2t(2-(x_i+k_i))}{3} < 0. \end{aligned} \tag{38}$$

Substituting for p_i and p_j from equation 10 in $\frac{\partial D_i}{\partial x_i}$ (with $i, j = L, R$ and $i \neq j$), we have:

$$\frac{\partial D_i}{\partial x_i} = \frac{3-5(x_i+k_i)-(x_j+k_j)}{6(1-(x_i+x_j+k_i+k_j))}. \tag{39}$$

It can be shown that in the symmetric equilibrium (i.e.: $x_i = x_j$ and $k_i = k_j$), $\left(\frac{\partial D_i}{\partial x_i}\right)_{Sym} = \frac{1}{2} > 0$. Then, as in the standard Hotelling model (see D'Aspremont et al., 1979) the direct effect is positive, while the strategic effect is negative (i.e.: $\frac{\partial D_i}{\partial p_j}\frac{dp_j}{dx_i} < 0$). The net effect equals (with

$i, j = L, R$ and $i \neq j$):

$$\left(\frac{\partial D_i}{\partial x_i} + \frac{\partial D_i}{\partial p_j}\frac{dp_j}{dx_i}\right) = -\frac{(1+3(x_i+k_i)+(x_j+k_j))}{6(1-(x_i+x_j+k_i+k_j))} < 0. \tag{40}$$

In the non-advertising case, then, similar to standard Hotelling models, the strategic effect dominates the direct effect. Substituting in equation 11 for equation 40 and p_i from equation 10 , we have (with $i, j = L, R$ and $i \neq j$):

$$\frac{\partial \Pi_i}{\partial x_i} = -\frac{t(3+(x_i+k_i)-(x_j+k_j))(1+3(x_i+k_i)+(x_j+k_j))}{18} < 0. \tag{41}$$

SOC for location (with $i, j = L, R$ and $i \neq j$):

$$\frac{\partial^2 \Pi_i}{\partial x_i^2} = -\frac{t(1-(k_j+x_j)+2(k_i+x_i))(3-(k_j+x_j)+(k_i+x_i))}{9(1-(x_i+x_j+k_i+k_j))}. \tag{42}$$

FOC for content provision, k_i ($i, j = L, R$ and $i \neq j$). It can be demonstrated that the first term in equation 12 equals:

$$\begin{aligned}
\frac{\partial D_i}{\partial k_i} &= \frac{p_j-p_i+t(1-(x_i+x_j+k_i+k_j))^2}{2t(1-(x_i+x_j+k_i+k_j))^2} \\
\frac{\partial D_i}{\partial p_j} &= \frac{1}{2t(1-(x_i+x_j+k_i+k_j))} > 0 \\
\frac{dp_j}{dk_i} &= -\frac{2t(2-(x_i+k_i))}{3} < 0.
\end{aligned} \tag{43}$$

Substituting for p_i and p_j from equation 10 in $\frac{\partial D_i}{\partial k_i}$(with $i, j = L, R$ and $i \neq j$), we obtain:

$$\frac{\partial D_i}{\partial k_i} = \frac{3-(5(x_i+k_i)+(x_j+k_j))}{6(1-(x_i+x_j+k_i+k_j))}. \tag{44}$$

At the symmetric equilibrium (i.e.: $x_i = x_j$ and $k_i = k_j$), we have that $\left(\frac{\partial D_i}{\partial k_i}\right)_{Sym} = \frac{1}{2} > 0$. Like for the location choices, while the direct effect of the multi-content strategy on profits is positive, the strategic effect is negative.

We can show that the first term in equation 12 simplifies to (with $i, j = L, R$ and $i \neq j$):

$$\left(\frac{\partial D_i}{\partial k_i} + \frac{\partial D_i}{\partial p_j}\frac{dp_j}{dk_i}\right) = -\frac{1+3(x_i+k_i)+(x_j+k_j)}{6(1-(x_i+x_j+k_i+k_j))} < 0. \tag{45}$$

The direct effect, as such, is smaller than the indirect effect. A multi-content strategy can therefore reduce profits via an increase in price competition.

Substituting in equation 12 for equation 45 and for p_i from equation 10, we obtain the following FOC for the multi-content scope (with $i, j = L, R$ and $i \neq j$):

$$\frac{\partial \Pi_i}{\partial k_i} = -\frac{(1+3(k_i+x_i)+(k_j+x_j))(3-(k_j+x_j)+(k_i+x_i))t}{18} + \frac{tk_i^2}{4} - \gamma k_i. \tag{46}$$

SOC for multi-content strategy (with $i, j = L, R$ and $i \neq j$):

$$\frac{\partial^2 \Pi_i}{\partial k_i^2} = -\frac{(1-(k_j+x_j)+2(k_i+x_i))t(3-(k_j+x_j)+(k_i+x_i))}{9(1-(x_i+x_j+k_i+k_j))} + \frac{tk_i-2\gamma}{2}. \tag{47}$$

Cross SOC (with $i, j = L, R$ and $i \neq j$):

$$\frac{d^2\Pi_i}{dx_i^2}\frac{d^2\Pi_i}{dk_i^2} - \left(\frac{\partial^2 \Pi_i}{\partial x_i \partial k_i}\right)^2 =$$

$$\frac{(2\gamma-tk_i)(1-(k_j+x_j)+2(k_i+x_i))(3-(k_j+x_j)+(k_i+x_i))t}{18(1-(x_i+x_j+k_i+k_j))} > 0. \tag{48}$$

Solution: Non-Advertising Game. Solution non-advertising game (with $i, j = L, R$ and $i \neq j$):

$$(1)\ \ k_i = \tfrac{4\gamma}{t}, k_j = 0,\ x_i = -\tfrac{(t+16\gamma)}{4t} < 0 \text{ and } x_j = -\tfrac{1}{4} < 0 \Rightarrow x_i = x_j = 0$$

$$(2)\ k_i = 0,\ k_j = \tfrac{4\gamma}{t},\ x_i = -\tfrac{1}{4} < 0 \text{ and } x_j = -\tfrac{(t+16\gamma)}{4t} < 0 \Rightarrow x_i = x_j = 0$$

$$(3)\ k_i = k_j = \tfrac{4\gamma}{t} \text{ and } x_i = x_j = -\tfrac{(t+16\gamma)}{4t} < 0 \Rightarrow x_i = x_j = 0$$

$$(4)\ k_i = k_j = 0 \text{ and } x_i = x_j = -\tfrac{1}{4} < 0 \Rightarrow x_i = x_j = 0. \tag{49}$$

Since $\frac{\partial \Pi_i}{\partial x_i} < 0$, then also $x_i = 0$ under all the previous solutions. The asymmetric solutions (1) and (2) fail to satisfy simultaneously all SOCs. The symmetric solution (3) satisfies the SOC for prices, location and multi-content strategy if $t > 8\gamma$. However for $t > 8\gamma$ the cross SOC is not satisfied (i.e.: $\frac{d^2\Pi_i}{dx_i^2}\frac{d^2\Pi_i}{dk_i^2} - \left(\frac{\partial^2 \Pi_i}{\partial x_i \partial k_i}\right)^2 < 0$). Only solution (4) satisfies all SOCs (i.e.: $\frac{\partial^2 \Pi_i}{\partial p_i^2} < 0$, $\frac{\partial^2 \Pi_i}{\partial x_i^2} < 0$, $\frac{\partial^2 \Pi_i}{\partial k_i^2} < 0$ and $\frac{d^2\Pi_i}{dx_i^2}\frac{d^2\Pi_i}{dk_i^2} - \left(\frac{\partial^2 \Pi_i}{\partial x_i \partial k_i}\right)^2 > 0$).

10.2 Advertising Game

SOCs: Advertising Game. The FOC for p_i equals ($i, j = L, R$ and $i \neq j$):

$$\frac{d\Pi_i}{dp_i} = \frac{t(1-(x_i+x_j+k_i+k_j))((k_i+x_i)-(k_j+x_j)+1)-2p_i+p_j+a_i(\beta a_i-\alpha)}{2(1-(x_j+k_j+x_i+k_i))t}. \tag{50}$$

The SOC for prices in the advertising game is the same as for the non-advertising game.

The FOC for a_i is (with $i, j = L, R$ and $i \neq j$):

$$\frac{d\Pi_i}{da_i} = -\frac{(p_i-p_j-t(1+(k_i+x_i)-(k_j+x_j))(1-(x_i+x_j+k_i+k_j)))(\alpha-2\beta a_i)}{2(1-(x_i+x_j+k_i+k_j))t}. \tag{51}$$

Substituting for p_i and p_j from equation 15, we can simplify $\frac{d\Pi_i}{da_i}$ to ($i, j = L, R$ and $i \neq j$):

$$\frac{d\Pi_i}{da_i} = -\left(2\beta a_i - \alpha\right)\frac{(a_j-a_i)(\beta(a_i+a_j)-\alpha)+t(3-(k_j+x_j)+(k_i+x_i))(1-(k_i+k_j+x_i+x_j))}{6(1-(x_i+x_j+k_i+k_j))t}. \tag{52}$$

SOC for advertising (with $i, j = L, R$ and $i \neq j$):

$$\frac{d^2\Pi_i}{da_i^2} =$$

$$-\frac{((a_j - a_i)(\beta(a_i + a_j) - \alpha) + t(3 - (k_j + x_j) + (k_i + x_i))(1 - (x_i + x_j + k_i + k_j)))\beta}{3(1 - (x_i + x_j + k_i + k_j))t}. \tag{53}$$

Note that the SOC for advertising is always satisfied if $a_i = a_j$.

FOC for location, x_i (equation 18). We find that the first term in equation 18 ($p_i \frac{\partial D_i}{\partial x_i} + \frac{\partial D_i}{\partial p_j} \frac{dp_j}{dx_i}$, i.e.: the effects of location in the consumer market sales) is exactly the same as in the non-advertising case. Then equations 38 to 40 continue to apply. In what respects the second term in the FOC for location ($\frac{\partial A_i}{\partial D_i} \frac{dD_i}{dx_i}$), however, we have (with $i = L, R$):

$$\frac{\partial A_i}{\partial D_i} = a_i (\alpha - \beta a_i) = \frac{\alpha^2}{4\beta} > 0. \tag{54}$$

And (with $i, j = L, R$ and $i \neq j$):

$$\frac{dD_i}{dx_i} = \frac{p_j - p_i + t(1 - (x_i + x_j + k_i + k_j))^2}{2t(1 - (x_i + x_j + k_i + k_j))^2}. \tag{55}$$

Substituting for p_i, p_j, a_i and a_j (equations 15 and 16) in equation 55, we obtain (with $i, j = L, R$ and $i \neq j$):

$$\frac{\partial D_i}{\partial x_i} = \frac{3 - 5(x_i + k_i) - (x_j + k_j)}{6(1 - (x_i + x_j + k_i + k_j))}. \tag{56}$$

Since at the symmetric equilibrium $\left(\frac{\partial D_i}{\partial x_i}\right)_{Sym} = \frac{1}{2} > 0$, then, advertising contributes to minimum differentiation.

SOC for location (with $i, j = L, R$ and $i \neq j$):

$$\frac{d^2\Pi_i}{dx_i^2} = \frac{3\alpha^2(k_j+x_j+1)}{36\beta(1-(x_i+x_j+k_i+k_j))^2}$$

$$-\frac{4\beta t(3-(k_j+x_j)+(k_i+x_i))(1-(k_j+x_j)+2(k_i+x_i))}{36\beta(1-(x_i+x_j+k_i+k_j))}. \quad (57)$$

FOC for content provision, k_i. We have that the first term in equation 19 ($p_i\left(\frac{\partial D_i}{\partial k_i}+\frac{\partial D_i}{\partial p_j}\frac{dp_j}{dk_i}\right)$, i.e.: the effect of the multi-content strategy on demand) is exactly the same as in the non-advertising case. Equations 43 to 45 therefore continue to apply.

Turning now to the second term ($\frac{\partial A_i}{\partial D_i}\frac{dD_i}{dk_i}$), we have that (with $i = L, R$):

$$\frac{\partial A_i}{\partial D_i} = (\alpha-\beta a_i)\, a_i = \frac{\alpha^2}{4\beta} > 0. \quad (58)$$

And also (with $i, j = L, R$ and $i \neq j$):

$$\frac{dD_i}{dk_i} = \frac{p_j-p_i+t(1-(x_i+x_j+k_i+k_j))^2}{2(1-(x_i+x_j+k_i+k_j))^2 t}. \quad (59)$$

Substituting for p_i, p_j, a_i and a_j (equations 15 and 16), equation 59 can be simplified to (with $i, j = L, R$ and $i \neq j$):

$$\frac{dD_i}{dk_i} = \frac{3-5(x_i+k_i)-(x_j+k_j)}{6(1-(x_i+x_j+k_i+k_j))}. \quad (60)$$

Given that at the symmetric equilibrium $\left(\frac{\partial D_i}{\partial k_i}\right)_{Sym} = \frac{1}{2} > 0$, the multi-content strategy affects positively ad revenues via the positive effect on sales.

SOC for multi-content strategy (with $i, j = L, R$ and $i \neq j$):

$$\frac{d^2\Pi_i}{dk_i^2} = \frac{3\alpha^2(k_j+x_j+1)-4\beta t(1-(x_i+x_j+k_i+k_j))(3-(k_j+x_j)+(x_i+k_i))(1-(k_j+x_j)+2(k_i+x_i))}{36\beta(1-(x_i+x_j+k_i+k_j))^2} + \frac{1}{2}tk_i - \gamma. \tag{61}$$

Cross SOC (with $i, j = L, R$ and $i \neq j$):

$$\frac{d^2\Pi}{dx_i^2}\frac{d^2\Pi}{dk_i^2} - \left(\frac{\partial^2\Pi}{\partial x_i \partial k_i}\right)^2 = (2\gamma - tk_i) \left(\frac{4\beta t(3-(k_j+x_j)+(k_i+x_i))(1-(k_j+x_j)+2(k_i+x_i))}{72(1-(x_i+x_j+k_i+k_j))\beta} - \frac{3\alpha^2(k_j+x_j+1)}{72(1-(x_i+x_j+k_i+k_j))^2\beta}\right) > 0. \tag{62}$$

Solution: Advertising Game. Solution advertising game (with $i, j = L, R$ and $i \neq j$):

$$\begin{aligned} 1)\ k_i &= \frac{4\gamma}{t};\ k_j = 0 \\ x_i &= \frac{\alpha^2 - 4\beta(32\gamma - t) + \sqrt{\alpha^4 + 24t\beta(6t\beta - 5\alpha^2)}}{32\beta t} \\ x_j &= \frac{4t\beta + \alpha^2 + \sqrt{\alpha^4 + 24t\beta(6t\beta - 5\alpha^2)}}{32\beta t}, \end{aligned}$$

$$\begin{aligned} 2)\ k_i &= 0 \\ x_i &= \frac{4t\beta + \alpha^2 + \sqrt{\alpha^4 + 24t\beta(6t\beta - 5\alpha^2)}}{32\beta t}, \end{aligned}$$

$$\begin{aligned} 3)\ k_i &= \frac{4\gamma}{t} \\ x_i &= \frac{\alpha^2 - 4\beta(32\gamma - t) + \sqrt{\alpha^4 + 24t\beta(6t\beta - 5\alpha^2)}}{32\beta t}, \end{aligned}$$

$$\begin{aligned}
4)\ k_i &= 0;\ k_j = \tfrac{4\gamma}{t} \\
x_i &= \tfrac{4t\beta+\alpha^2+\sqrt{\alpha^4+24t\beta(6t\beta-5\alpha^2)}}{32\beta t} \\
x_j &= \tfrac{\alpha^2-4\beta(32\gamma-t)+\sqrt{\alpha^4+24t\beta(6t\beta-5\alpha^2)}}{32\beta t}. \qquad (63)
\end{aligned}$$

All solutions satisfy the SOC for prices, advertising and location. However, only solution (3) satisfies the Cross SOC. In turn, the SOC for multi-content for solution (3) is (with $i = L, R$):

$$\frac{d^2\Pi_i}{dk_i^2} = \frac{\alpha^4(t+2\gamma)+8\beta t\left(\alpha^2(t-18\gamma)+6\beta t(6\gamma-t)\right)+\left(\alpha^2(t+2\gamma)+4\beta t(t-6\gamma)\right)\sqrt{\alpha^4+24t\beta(6t\beta-5\alpha^2)}}{2\left(\alpha^4+72\beta t(2t\beta-\alpha^2)+(\alpha^2-12t\beta)\sqrt{\alpha^4+24t\beta(6t\beta-5\alpha^2)}\right)} < 0. \qquad (64)$$

It can be easily checked that the denominator is always negative. Then, the SOC is only satisfied if the numerator is also negative. The numerator has two solutions: $\beta_1 = 0$ and $\beta_2 = -\frac{\alpha^2(t+2\gamma)}{6(2\gamma-t)(6\gamma-t)}$. Also, the second derivative of the numerator in relation to β, equals $-1536\alpha^4t^2\,(2\gamma-t)\,(6\gamma-t)$. Three cases can arise. First, if $t < 2\gamma$, $\beta_2 = -\frac{\alpha^2(t+2\gamma)}{6(2\gamma-t)(6\gamma-t)} < 0$, and $-1536\alpha^4t^2\,(2\gamma-t)\,(6\gamma-t) < 0$ (concave inverse-U shaped). The SOC for the multi-content strategy is then satisfied for $\beta > 0$. Second, if $2\gamma < t < 6\gamma$, $\beta_2 = -\frac{\alpha^2(t+2\gamma)}{6(2\gamma-t)(6\gamma-t)} > 0$, and $-1536\alpha^4t^2\,(2\gamma-t)\,(6\gamma-t) > 0$ (convex U shaped). Therefore, the SOC for the multi-content strategy is satisfied for $0 < \beta < -\frac{\alpha^2(t+2\gamma)}{6(2\gamma-t)(6\gamma-t)}$. Third, if $t > 6\gamma$, $\beta_2 = -\frac{\alpha^2(t+2\gamma)}{6(2\gamma-t)(6\gamma-t)} < 0$, and $-1536\alpha^4t^2\,(2\gamma-t)\,(6\gamma-t) < 0$ (concave inverse-U shaped). As a consequence, the SOC for the multi-content strategy is satisfied for $\beta > 0$. Summing up, the advertising game holds: (1) if $t < 2\gamma$ and/or $t > 6\gamma$ and $\beta > 0$; (2) if $2\gamma < t < 6\gamma$ and $0 < \beta < -\frac{\alpha^2(t+2\gamma)}{6(2\gamma-t)(6\gamma-t)}$.

Advertising Game: Sign of x_i. The numerator of x_i has two solutions $\beta_1^{'} = 0$ and $\beta_2^{'} = \frac{\alpha^2(2\gamma-t)}{(t+16\gamma)(8\gamma-t)}$. Also the second derivative of the numerator in relation to β equals:

$-256\,(8\gamma-t)\,(t+16\gamma)$. Then three cases arise. First, if $t<2\gamma$, $\beta_2^{'}=\frac{\alpha^2(2\gamma-t)}{(t+16\gamma)(8\gamma-t)}>0$ and $-256\,(8\gamma-t)\,(t+16\gamma)<0$ (concave inverse-U shaped). Therefore: $x_i>0$ for $0<\beta<\frac{\alpha^2(2\gamma-t)}{(t+16\gamma)(8\gamma-t)}$ and $x_i<0$ for $\beta>\frac{\alpha^2(2\gamma-t)}{(t+16\gamma)(8\gamma-t)}$. Second, if $2\gamma<t<8\gamma$, $\beta_2^{'}=\frac{\alpha^2(2\gamma-t)}{(t+16\gamma)(8\gamma-t)}<0$ and $-256\,(8\gamma-t)\,(t+16\gamma)<0$ (concave inverse-U shaped). We then have $x_i<0$ for $\beta>0$. Third, if $t>8\gamma$, $\beta_2^{'}=\frac{\alpha^2(2\gamma-t)}{(t+16\gamma)(8\gamma-t)}>0$ and $-256\,(8\gamma-t)\,(t+16\gamma)>0$ (convex U shaped). As such, $x_i<0$ for $0<\beta<\frac{\alpha^2(2\gamma-t)}{(t+16\gamma)(8\gamma-t)}$ and $x_i>0$ for $\beta>\frac{\alpha^2(2\gamma-t)}{(t+16\gamma)(8\gamma-t)}$.

Summing up, from equation 38, the sign of x_i has three cases. First, if $t<2\gamma$, $x_i>0$ for $0<\beta<\frac{\alpha^2(2\gamma-t)}{(t+16\gamma)(8\gamma-t)}$; $x_i<0\Rightarrow x_i=0$ for $\beta>\frac{\alpha^2(2\gamma-t)}{(t+16\gamma)(8\gamma-t)}$; and $x_i=0$ for $\beta=\frac{\alpha^2(2\gamma-t)}{(t+16\gamma)(8\gamma-t)}$. Second, if $2\gamma<t<8\gamma$, $x_i<0\Rightarrow x_i=0$ for $\beta>0$. Third, if $t>8\gamma$, $x_i<0\Rightarrow x_i=0$ for $0<\beta<\frac{\alpha^2(2\gamma-t)}{(t+16\gamma)(8\gamma-t)}$; $x_i>0$ for $\beta>\frac{\alpha^2(2\gamma-t)}{(t+16\gamma)(8\gamma-t)}$; and $x_i=0$ for $\beta=\frac{\alpha^2(2\gamma-t)}{(t+16\gamma)(8\gamma-t)}$.

In the Hotelling model, we must have, $x_i\in[0,1]$. However, as we have seen, x_i can either be negative or positive. Then, like is usual in Hotelling models, when $x_i<0$, we make $x_i=0$. But the SOC for prices only assures that the multi-content strategy segments of the two firms never intercept (i.e.: $x_i+x_j+k_i+k_j<1$, $i,j=L,R$ and $i\neq j$) when x_i equals the value in equation 38, but not necessarily when $x_i=0$. In this sense, for $x_i<0\Rightarrow x_i=0$, the SOC for prices is not sufficient to assure that the two multi-content segments do not overlap and therefore we need to introduce an extra condition to satisfy this restriction. It can be shown that with $x_i<0\Rightarrow x_i=0$, $k_i<\frac{1}{2}$ only for $t>8\gamma$. Putting all this together, it results that only the third solution above can be considered in the advertising game.

10.3 Transport costs and Prices

Here we analyze the differences in transport costs and prices that consumers pay under the no advertising and the advertising game.

1) Transport costs and prices for $0<\beta\leq\frac{\alpha^2}{4(t-8\gamma)}$ (large advertising market): Start with transport costs. As we have mentioned in Section 3 (Model), with a multi-content strategy some consumers are offered their ideal variety and firms have at most two standard types of content. Due to this, it can be shown that total transport costs paid by consumers under the single-content strategy (T^N) are higher than under the multi-content strategy (T^A):

$$\left(T^N\right) - \left(T^A\right) = 2\gamma \tfrac{3t(t-8\gamma)+64\gamma^2}{t^2} > 0. \tag{65}$$

In what concerns prices, with a multi-content strategy, price competition is fiercer since firms compete more head-to-head close to the center of the line. As we have seen, when the advertising market is large, the price in the standard segment is zero, which is smaller than the price paid under the single-content strategy, t. Note that this occurs in spite of the fact that with a multi-content strategy firms price discriminate and some consumers pay a premium to acquire their ideal variety. We can then see that, when the advertising market is large, the price competition that ensues with a multi-content strategy is beneficial to consumers, even with price discrimination.

2) Transport costs and prices for $\frac{\alpha^2}{4(t-8\gamma)} < \beta < \frac{\alpha^2(2\gamma-t)}{(t+16\gamma)(8\gamma-t)}$ (medium advertising market): It can be shown that the savings in transport costs for a medium advertising market equal the savings in transport costs under the large advertising market. In other words, equation 65 above continues to apply. This occurs because in both cases (large and medium advertising market), firms locate at the extremes of the line and the size of the multi-content segments of the two firms are the same (i.e. $k_L = k_R = \frac{4\gamma}{t}$).

Furthermore, as for a large advertising market, the price in the standard segment under the multi-content strategy is lower than the price under the single-content strategy (with $i = L, R$):

$$p_i^N - p_i^A = \tfrac{\alpha^2+32\beta\gamma}{4\beta} > 0. \tag{66}$$

We can then see once more that a multi-content strategy conduces to fiercer price competition, which benefits consumers. This is so since with a multi-content strategy some consumers save in transport costs relatively to the single-content strategy case (see above Section 3: Model). Note again that this occurs although with price discrimination some consumers pay a premium to acquire their ideal variety.

3) Transport costs and prices for $\beta > \frac{\alpha^2(2\gamma - t)}{(t+16\gamma)(8\gamma - t)}$ **(small advertising market):** When the advertising market is small, the difference in total transport costs that consumers paid under the advertising and no advertising game equals:

$$
\left(T^N\right) - \left(T^A\right) = \frac{\left(\begin{array}{c} 24\alpha^4\gamma - 512\beta\gamma^2\left(3\alpha^2 - 128\beta\gamma\right) + t\left(16t\beta\left(7t\beta + 12\alpha^2 + 120\beta\gamma\right) - 3\alpha^4 - 192\beta\gamma\left(7\alpha^2 + 32\beta\gamma\right)\right) \\ + \left(24\gamma\left(\alpha^2 - 64\beta\gamma\right) + 3t\left(4\beta\left(t + 8\gamma\right) - \alpha^2\right)\right)\sqrt{\alpha^4 + 24t\beta\left(6t\beta - 5\alpha^2\right)} \end{array}\right)}{1536t^2\beta^2}. \tag{67}
$$

This equation can be positive or negative. Then in a small advertising market, it is no longer certain that total transport costs paid by consumers are smaller under the advertising game relatively to the no advertising game. The reason for this is that when the advertising market is small, firms locate inside of the line. As discussed above, when this occurs, fewer consumers save on transport costs than when firms locate at the extreme of the line.

In what respects prices, we have (with $i = L, R$):

$$
p_i^N - p_i^A = \frac{4t\beta + 5\alpha^2 + \sqrt{\alpha^4 + 24t\beta(6t\beta - 5\alpha^2)}}{16\beta} > 0. \tag{68}
$$

Then, when the advertising market is small, prices continue to be lower with the multi-content strategy, as it was also the case for a large and a medium advertising market. This shows once again that, independently of the size of the advertising market, the multi-content strategy conduces always to fiercer price competition.

10.4 Consumer Surplus and Social Welfare.

Consumer surplus in the model equal:

$$
\begin{aligned}
CS = & \int_0^{x_L} (v - p_L)\, dx + \int_{(x_L+k_L)}^{x^*} (v - p_L)\, dx + 2 \int_{x_L}^{x_L+\frac{k_L}{2}} \left(v - \left(p_L + t\,(x - x_L)^2\right)\right) dx \\
& + \int_{x^*}^{1-(x_R+k_R)} (v - p_R)\, dx + \int_{1-x_R}^{1} (v - p_R)\, dx \\
& + 2 \int_{1-(x_R+k_R)}^{1-\left(x_R+\frac{k_R}{2}\right)} \left(v - \left(p_R + t\,((1-x) - (x_R+k_R))^2\right)\right) dx \\
& - t \int_0^{x_L} (x_L - x)^2\, dx - t \int_{(x_L+k_L)}^{x^*} (x - (x_L+k_L))^2\, dx \\
& - t \int_{x^*}^{1-(x_R+k_R)} ((1-x) - (x_R+k_R))^2\, dx - t \int_{1-x_R}^{1} ((1-x) - x_R)^2\, dx. \qquad (69)
\end{aligned}
$$

We measure social welfare in the standard way, in that it equals consumer surplus plus profits of the firms. Social welfare then is:

$$W = \Pi_L + \Pi_R + CS. \qquad (70)$$

Social Welfare: No Advertising Game. In the no advertising game, we have that profits per firm equal (with $i = L, R$):

$$\Pi_i = \tfrac{1}{2}t. \qquad (71)$$

In this way, total profits in the market are:

$$\Pi_L + \Pi_R = t. \qquad (72)$$

In what respects consumer surplus, we have:

$$CS = v - \tfrac{5t}{4}. \tag{73}$$

As a result, social welfare in the no advertising game is:

$$W = v - \tfrac{t}{4}. \tag{74}$$

Social Welfare: Advertising Game. We turn now to the advertising game.

1) Social Welfare for $0 < \beta \leq \frac{\alpha^2}{4(t-8\gamma)}$ (large advertising market): When the advertising market is very large (i.e. $0 < \beta \leq \frac{\alpha^2}{4(t-8\gamma)}$), profits per firm equal (with $i = L, R$):

$$\Pi_i = \tfrac{3t^2\alpha^2 - 64\beta\gamma^3}{24t^2\beta}. \tag{75}$$

As a result, total profits in the market equal:

$$\Pi_L + \Pi_R = \tfrac{3t^2\alpha^2 - 64\beta\gamma^3}{12t^2\beta}. \tag{76}$$

Turning now to consumer surplus, we have:

$$CS = v + \tfrac{16\gamma(t-4\gamma)^2 - t^3}{4t^2}. \tag{77}$$

From the above, it results that social welfare amounts to:

$$W = v + \tfrac{16\beta\gamma\left(3t^2 - 4\gamma(6t-11\gamma)\right) + 3t^2\left(\alpha^2 - t\beta\right)}{12t^2\beta}. \tag{78}$$

2) Social Welfare for $\frac{\alpha^2}{4(t-8\gamma)} < \beta < \frac{\alpha^2(2\gamma-t)}{(t+16\gamma)(8\gamma-t)}$ (medium advertising market): When the advertising market is medium (i.e. $\frac{\alpha^2}{4(t-8\gamma)} < \beta < \frac{\alpha^2(2\gamma-t)}{(t+16\gamma)(8\gamma-t)}$), profits per firm equal (with $i = L, R$):

$$\Pi_i = \frac{3t^3\left(t^3-8\gamma\left(2\gamma^2+t^2\right)\right)+256\gamma^9}{6t^5}. \tag{79}$$

Total profits in the market then are:

$$\Pi_L + \Pi_R = \frac{3t^3\left(t^3-8\gamma\left(2\gamma^2+t^2\right)\right)+256\gamma^9}{3t^5}. \tag{80}$$

In what respects consumer surplus, we have:

$$CS = v + \frac{t\left(t\left(\alpha^2-5t\beta\right)+16\beta\gamma(3t-8\gamma)\right)+256\beta\gamma^3}{4t^2\beta}. \tag{81}$$

Social welfare then follows:

$$W = v + \frac{3t^3\left(16\beta\gamma(t-6\gamma)(t-2\gamma)+t^2\left(\alpha^2-t\beta\right)\right)+1024\beta\gamma^9}{12t^5\beta} \tag{82}$$

3) Social Welfare for $\beta > \frac{\alpha^2(2\gamma-t)}{(t+16\gamma)(8\gamma-t)}$ (small advertising market): When the advertising market is small (i.e. $\beta > \frac{\alpha^2(2\gamma-t)}{(t+16\gamma)(8\gamma-t)}$), profits per firm are (with $i = L, R$):

$$\Pi_i = \frac{3t^2\left(12t\beta-\alpha^2\right)-256\beta\gamma^3-3t^2\sqrt{144t^2\beta^2+\alpha^2(\alpha^2-120t\beta)}}{96t^2\beta}. \tag{83}$$

It results that total profits amount to:

$$\Pi_L + \Pi_R = \frac{3t^2\left(12t\beta-\alpha^2\right)-256\beta\gamma^3-3t^2\sqrt{144t^2\beta^2+\alpha^2(\alpha^2-120t\beta)}}{48t^2\beta}. \tag{84}$$

For consumer surplus, we have:

$$CS = v + \frac{\left(\begin{array}{c} 24\gamma\left(\alpha^4 - 64\beta\gamma\left(\alpha^2 - 32\beta\gamma\right)\right) - t\left(3\left(\alpha^4 + 64\beta\gamma\left(7\alpha^2 + 32\beta\gamma\right)\right) - 16t\beta\left(42\alpha^2 - \beta\left(89t - 120\gamma\right)\right)\right) \\ -\left(3t\left(\alpha^2 - 4\beta\left(9t + 8\gamma\right)\right) - 24\gamma\left(\alpha^2 - 64\beta\gamma\right)\right)\sqrt{\alpha^4 + 24t\beta\left(6t\beta - 5\alpha^2\right)} \end{array}\right)}{1536t^2\beta^2}. \tag{85}$$

It then results that social welfare is:

$$W = v + \frac{\left(\begin{array}{c} 8\gamma\left(3\alpha^4 - 64\beta\gamma\left(3\alpha^2 - 80\beta\gamma\right)\right) - t\left(3\left(\alpha^4 + 64\beta\gamma\left(7\alpha^2 + 32\beta\gamma\right)\right) - 16t\beta\left(36\alpha^2 - \beta\left(17t - 120\gamma\right)\right)\right) \\ -\left(3\left(t - 8\gamma\right)\left(\alpha^2 - 4\beta\left(t + 16\gamma\right)\right)\right)\sqrt{\alpha^4 + 24t\beta\left(6t\beta - 5\alpha^2\right)} \end{array}\right)}{1536t^2\beta^2}. \tag{86}$$

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