Price and design comparability*

Nicolas de Roos†
University of Liverpool

December 10, 2023

Abstract

We decompose product comparability into a price component and a design component relating to preference matches, and examine the incentives for price-setting firms to manipulate each component. First, we analyse price competition for given product comparability. Improved price (design) comparability leads to more (less) intense price competition. We then examine message and price competition. If messages are impactful on equilibrium comparability, firms associate higher relative prices with messages that increase design comparability and decrease price comparability.

JEL Classification: L13, L15
Keywords: Limited comparability; price competition.

1 Introduction

Among other challenges, a consumer's attempt to obtain the best price for a product can be compromised by unanticipated sales (Varian, 1980; de Roos and Smirnov, 2020), add-on pricing (Ellison, 2005), manipulation of the framing of relative prices (Piccione and Spiegler, 2012; Chioveanu and Zhou, 2013; de Roos, 2018), and prediction errors in her own consumption patterns (Grubb, 2015b). As a result, it is not unusual for consumers to fail to secure the best price among the set of available products (Grubb, 2015a). A growing literature in behavioural industrial organisation has established that firms have an incentive to engage in obfuscation practices that make price comparison more challenging for consumers (see surveys by Heidhues and Kőszegi (2018), Spiegler (2016), and Grubb (2015a)).

*I am grateful to Murali Agastya, Mark Armstrong, Simon Loertscher, Vladimir Smirnov, Andrew Wait and seminar participants at the University of Sydney and the Melbourne Industrial Organisation Workshop 2021.

†School of Management, University of Liverpool, nicderoos@gmail.com.

A focus of this literature is the incentive of firms to influence the comparability of the price of competing products. Obfuscation has the following logic. Suppose price is the primary strategic variable available to firms, and hold fixed the remaining product characteristics of all firms. If the prices of competing products are difficult to compare, consumers may make suboptimal purchase decisions because they incompletely account for price differences. Firms therefore have less incentive to compete aggressively on price, leading to higher equilibrium prices and profits. As Spiegler (2016) illustrates, an equivalent analysis applies if firms compete by choosing quality rather than price.

In this paper, we ask whether this logic extends to the strategic manipulation of information about other product characteristics. We allow a consumer's product comparison exercise to depend both on her propensity to perceive relative prices and to understand her product match quality. If she is perfectly informed in both respects, her purchase decision will weight information about both prices and other product characteristics. However, if she is imperfectly informed about prices, she will attach more weight to other product information; and if she is ignorant about other product characteristics, she will focus on price information.

In our model, introduced in Section 2, two firms produce differentiated products at either extremes of a Hotelling line, and a single consumer is located at an intermediate position along the line that is unknown to both firms. Firms compete by simultaneously choosing prices and messages. Influenced by firm messages, consumer information takes a two-dimensional truth-or-noise form. First, firm messages influence the probability with which consumers can perfectly perceive the price vector, which we label the *price* component of the product comparability structure. Second, messages also influence the probability with which the consumer understands her own location, and therefore has well defined preferences over the non-price characteristics of the rival products. We label this the *design* component of the comparability structure.

We provide two main contributions to the literature. First, we provide a more nuanced understanding of the influence of product-market transparency on competition. We begin our analysis in Section 3 by investigating the impact of price and design transparency on pricing, taking message strategies as given. If design comparability is limited, then a positive measure of consumers perceive the products as identical, and equilibrium always involves price dispersion. This feature contrasts with findings for homogeneous product markets, in which price dispersion follows from limited price comparability (Chioveanu and Zhou, 2013). Regarding mean prices, if it is only price comparability that is limited, an improvement in transparency increases the intensity of price competition, leading to lower equilibrium prices. Conversely, if only design comparability is limited, an improvement in transparency clarifies the differentiation in products to consumers, increasing the market power of firms, and leading to higher prices. We trace out the comparative static implications of variation in price and design comparability on the mean and dispersion of prices.

Second, in Section 4, we examine the incentives for firms to influence price and design comparability through their message choices. Equilibrium analysis is simplified if the comparability structure satisfies the property of Enforceable Comparability (EC) (Spiegler, 2016). If this property holds, each firm can unilaterally enforce a given level of comparability. In this case, in equilibrium this level of comparability is indeed enforced and our earlier analysis of price competition with given messages applies directly. If EC does not hold, then firms have an incentive to tailor their message choice to their pricing strategy: firms combine high prices with messages that limit price comparability and enhance design comparability, and adopt the opposite messaging tactics when setting low prices. We also perform comparative static analysis, and find that the relationship between the comparability structure and the price distribution is complicated by message composition effects. This has the potential to frustrate policy interventions aimed at fostering price competition. For example, policies aimed at increasing parameters associated with price comparability could perversely lead to decreases in equilibrium price comparability.

An extensive literature in marketing has long recognised the importance of consumer confusion related to price and other product characteristics. Surveys are contained in, for example, Mitchell and Papavassiliou (1999), Walsh et al. (2007), and Kasabov (2015). Confusion can be multi-dimensional, relating to brand similarity, complexity and incompatible standards, product proliferation, as well as product pricing (Mitchell and Papavassiliou, 1999). Our theory offers testable implications. We would expect obfuscation devices such as randomisation, complex or heterogeneous product framing, and hidden information to be employed to obscure price or quality-related product information. At the same time, we would expect firms to take steps to clarify aspects of product design that relate to match quality. Anecdotal evidence seems consistent with this prediction. Price is not an intrinsically complicated product characteristic, while design is often multi-dimensional and subjective. However, price comparison is often a challenging task in practice because of the pricing strategies employed by firms, including: add-on pricing for hotel rooms, personal computers, car rentals, airfares (Ellison, 2005), negotiated discounts for mortgages, telecommunications and energy (Byrne et al., 2022), product proliferation and multi-dimensional pricing in mobile phone plans (Genakos et al., 2023), and intertemporal price dispersion in retail petrol markets (de Roos and Smirnov, 2020; Byrne and de Roos, 2017). Some sales strategies both complicate price comparison and enable consumers to express preferences over product characteristics. For example, when a consumer purchases a car, personal computer, or health insurance plan, or when she seeks financial advice, she will be given the opportunity to customise the product, thus allowing her to better understand and influence her product match, while complicating the process of price discovery.

In our model, price is a *vertical* product characteristic in the sense that all consumers agree on a ranking of products on the basis of price, while design is a *horizontal* characteristic over which consumers do not agree on a ranking. Armstrong (2008, Appendix) considers limited comparability over two vertical characteristics, price and quality. In the symmetric equilibrium, competitors have equal quality, and quality comparability has no impact on price competition. Moving beyond the symmetric equilibrium considered by Armstrong (2008), any increase in market power for a higher quality firm associated with improved quality comparability will be counterbalanced by a decrease in market power for a lower quality firm. By contrast, in our model, an improvement in design comparability leads to an increase in market power for all firms.

Ivanov (2013) and Hwang et al. (2019) study a related problem in which price-setting competitors decide how much information to reveal about their own product. The incentive to reveal information changes with market structure. If no information is revealed, a consumer attributes the ex ante expected valuation to a seller. With more competitors, it is more likely that at least one rival has a valuation above the expected value, and this encourages firms to reveal information about their own product. Armstrong and Zhou (2022) study a related setting in which information release is instead centralised. The authors find that the optimal signal structure from the firms' (consumers') perspective accentuates (dampens) perceived product differentiation. Relative to this literature, our model admits limitations in comparability over both prices and product design, and we find qualitative differences in the equilibrium price distribution when both dimensions of comparability are limited. Further, in our model, the comparability of a product pair for a consumer depends on the information transmitted by both firms involved, while firms unilaterally determine information about their own products in Ivanov (2013) and Hwang et al. (2019), and this information is chosen by a planner in Armstrong and Zhou (2022). Thus, our setup admits the possibility that a firm could educate or confuse consumers on aspects of product design related to competing products.

As in our study, Anderson and Renault (2009) study the incentive for firms to reveal information about horizontally differentiated product characteristics. In their model, before price competition takes place, two firms each have an opportunity to perfectly reveal information about both their own product and that of their rival. Our setting differs in several dimensions: firms have opportunities to influence information regarding price as well as product characteristics; price setting and information transmission occur simultaneously rather than sequentially (we also study the sequential case in Online Appendix B); and the information perceived by consumers depends jointly on the choices of competing firms, while firms can unilaterally reveal complete product information in Anderson and Renault (2009). While we focus on symmetric firms, Anderson and Renault (2009) also allow for heterogenous qualities across firms. If firms are sufficiently similar in terms of product quality then, as in our study, they have an incentive to reveal product information to accentuate perceived product differentiation. If qualities differ sufficiently, the lower quality firm has a greater incentive to advertise information about both its own product and its rival's product.

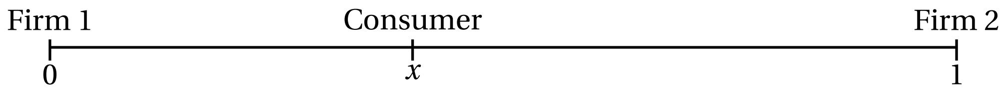
In complementary work, Hefti et al. (2022) also investigate the incentives of firms to confuse or educate consumers in a differentiated product environment. Applying their model to a market setting, two firms first choose communication strategies, and then prices. Prices are perfectly observed by consumers, and communication strategies can influence consumer perceptions of their relative valuation of the two products. Firm communication strategies may accentuate or ameliorate heterogeneity in preferences across consumers, with implications for the intensity of price competition. They find that firms seek to educate consumers if consumer preferences are initially polarized, in the sense that indifferent consumers are rare, while firms seek to confuse if indifferent consumers are more prevalent.

Both our study and that of Hefti et al. (2022) suggest firm incentives for obfuscation or education are context dependent. However, the mechanisms at play are quite different. In Hefti et al. (2022), the distribution of match values determines the incentive to obfuscate. Consumers take perceived matches into account in purchasing decisions, despite the possibility of preference manipulation. Thus, it is possible that firm communication strategies could lead to an increase or decrease in the dispersion of perceived preferences, and thereby soften or intensify price competition. By contrast, in our model, consumers who are confused about product matches disregard product matches and instead focus on price. As a result, confusion on this horizontal dimension almost always leads to more intense price competition. Further, our model admits a second, vertical, dimension of potential consumer confusion. If consumers are unable to evaluate relative prices, this generally softens price competition. In sum, communication strategies are influenced by the consumer preference distribution in Hefti et al. (2022), and by the relative strengths of the price and design dimensions of the comparability structure in our article.

All proofs are contained in Appendix A. In Online Appendix B, we discuss a two stage game in which firms simultaneously choose messages, and then simultaneously choose prices. Online Appendix C contains additional analysis to supplement our main results.

2 The model

Two firms compete in a linear city of length 1. Firm 1 is located at $y_1 = 0$, and Firm 2 is at the other extremity, $y_2 = 1$. A single consumer is located at $x \in [0,1]$, along the city. The location x is initially unknown to all parties, and is distributed uniformly along the city. Costs, including the costs of choosing different messages, are normalised to zero. A possible realisation of the city is illustrated below.



In a single period, each firm $j \in \{1,2\}$ simultaneously chooses a message $m_j \in M$ and a price $p_j \in \mathbb{R}_+$.[1] The message vector $\mathbf{m} = (m_1, m_2)$ determines the consumer's ability to compare the prices of the products $\mathbf{p} = (p_1, p_2)$, and learn about her location. Based on the information she has accumulated, the consumer chooses exactly one of the two products. If the consumer purchases product j, she obtains indirect utility

$$u(p_j, x) = \overline{u} - t|y_j - x| - p_j,$$

where $\overline{u}$ indicates her intrinsic value of buying, t indicates travel costs, and p_j and y_j are the price and location of Firm j.

A natural interpretation of $\overline{u}$ is the intrinsic value of purchasing a product, relative to the value of the outside good. In the manner of Bénabou and Tirole (2016), the consumer may purchase an outside good with value normalised to zero by travelling to the nearest end of the city. With this specification, Firm j cannot make any sales by setting $p_j > \overline{u}$. Henceforth, we presume each firm sets a price no higher than $\overline{u}$, and this ensures the market is always covered.[2]

Let $v(\mathbf{m})$ and $h(\mathbf{m})$ be the probability the consumer is able to compare the two prices and learn her location, respectively.[3] For exposition, we assume that v and h are independent. The analysis does not rely on this assumption. If the consumer learns her location and the price vector, her perspective matches that of a consumer in the Hotelling linear city. Because the consumer's location is unknown to the firm at the time of price setting, Firm j then anticipates a market share of

$$\tilde{s}_j(\mathbf{p}) = \max\{0, \min\{1, s_j(\mathbf{p})\}\}, \qquad s_j(\mathbf{p}) = \frac{p_{-j} - p_j + t}{2t},$$

when choosing a price $p_j \leq \overline{u}$. If instead the consumer is able to compare prices, but does not learn her location, she will purchase the cheapest option, with ties broken uniformly. In this case, Firm j receives a market share of

$$q_j(\mathbf{p}) = I\{p_j < p_{-j}\} + I\{p_j = p_{-j}\}/2,$$

where $I\{z\}$ is an indicator function evaluating to 1 if expression z is true and 0 otherwise. Finally, if the consumer is unable to compare product prices, each firm will expect a market share of one

[1] In Appendix B, we consider an alternative specification in which firms commit to messages prior to choosing price.

[2] For additional discussion of the role of the outside good, refer to Appendix C.1.

[3] To recall this notation, the reader may wish consider v as comparability of vertical product characteristics such as quality or price (over which consumers have an agreed ranking), and h as comparability of horizontal characteristics such as product design (over which consumers do not have an agreed ranking).

half at the time of price setting. Note that this does not depend on whether the consumer learns her location. If the consumer does not learn her location, she will have no information on which to make a decision, and she chooses each firm with equal probability. Even if she does learn her location, because the distribution of x is symmetric around 0.5, the expected market share from the perspective of each firm at the time of price setting is also one half.

In a symmetric equilibrium, let $F(p)$ be the marginal price distribution with support P, and $\lambda(m|p)$ be the conditional density of message m given price p. The marginal density of message m is then $\lambda(m) = \int_{p\in P} \lambda(m|p) dF(p)$. Let $F_m(p)$ denote the price distribution conditional on message m, and let P_m be its support. The mean and lower and upper bound of the price distribution are denoted μ, $\underline{p}$, and $\overline{p}$; and μ_m is the mean of the price distribution, conditional on message m.

Given a price and message vector $\mathbf{p}$ and $\mathbf{m}$, Firm j earns expected profits of

$$\pi_j(\mathbf{p},\mathbf{m}) = p_j \left(h(\mathbf{m})\, v(\mathbf{m})\, \tilde{s}_j(\mathbf{p}) + (1-h(\mathbf{m}))\, v(\mathbf{m})\, q_j(\mathbf{p}) + (1-v(\mathbf{m}))/2\right).$$

Let $\pi(p,m)$ indicate the expected profit to a firm when setting price p and message m, taking her rival's strategies to be well specified according to equilibrium play. Then, the expected profit function for Firm j when setting price p_j and message m_j is

$$\pi(p_j, m_j) = \int_{p_{-j}\in P} \int_{m_{-j}\in M} \pi_j(\mathbf{p},\mathbf{m}) \lambda\left(m_{-j}|p_{-j}\right) dF\left(p_{-j}\right). \tag{1}$$

Given prices and messages $\mathbf{p}$ and $\mathbf{m}$, a consumer located at x receives expected surplus

$$CS(\mathbf{p},\mathbf{m},x) = \left(\overline{u} - t(1-x) - p_2\right) + s_1(\mathbf{p},\mathbf{m},x)\left(p_2 - p_1 + t(1-2x)\right),$$

where $s_1(\mathbf{p},\mathbf{m},x)$ indicates the probability that this consumer purchases from Firm 1. The bracketed term on the left describes her expected surplus if the consumer purchases from Firm 2, and the remaining terms measure the improvement in surplus if she chooses optimally between the products.

For future reference, we collect the parameters relevant for pricing into the vector $\theta(\mathbf{m}) = (\overline{u}, t, h, v) \in \Theta = \mathbb{R}^2 \times [0,1]^2$. Where the context is clear, we omit the dependence of the endogenous parameters v and h on the message vector.

3 Transparency and prices

In this section, we consider price competition between the two firms under the assumption that the message vector is fixed or predetermined. This analysis is useful in its own right for an understanding of the relationship between price and design transparency and market outcomes. It also serves as a stepping stone for analysis of simultaneous price and message competition

(Section 4), and a related game in which firms commit to messages before choosing price (Appendix B). We consider the special case of limited price comparability in Section 3.1, and we discuss limited design comparability in Section 3.2, including the general case in which both price and design comparability are limited.

3.1 Limited price comparability

Suppose that $v \leq 1$ and $h = 1$. That is, given the message vector, the consumer learns her location for sure, but may not be able to compare prices. In this case, profits depend on rival prices only through their expected value, and the profit function for Firm j simplifies to

$$\pi_j(\mathbf{p}, \mathbf{m}) = p_j\left(v s_j(p_j, \mu) + (1-v)/2\right).$$

Proposition 1. *If $h = 1$ and $\overline{u} \leq \frac{t}{v(1-v)}$, in the symmetric Nash equilibrium, prices and profits for each firm are*

$$p^*(\mathbf{m}) = \min\left\{\frac{t}{v}, \overline{u}\right\}, \qquad \pi^*(\mathbf{m}) = \frac{p^*(\mathbf{m})}{2}. \tag{2}$$

The result follows directly from the first order conditions for profit maximisation. As long as $\overline{u}$ is not too large, there is no incentive to set a high price and focus purely on exploiting uninformed consumers, and equilibrium is in pure strategies. An increase in price comparability shifts the reaction function of each firm inwards, and equilibrium prices and profits are decreasing in price comparability. The role of price comparability is evident from the above profit function. An increase in v raises the weighting in the profit function on the Hotelling market share term associated with informed consumers, and decreases the weight on the price-insensitive component associated with consumers who are unable to compare prices. Thus, an increase in v leads to an increase in the intensity of price competition.

Expected consumer surplus is given by

$$\mathbb{E}_x CS(\mathbf{p}, \mathbf{m}, x) = \overline{u} - p^*(\mathbf{m}) - t/4,$$

where $\mathbb{E}$ denotes the expectations operator. Under perfect design comparability, given any price comparability, consumers receive the best possible product matches. With no price dispersion, consumers who are either informed or uninformed about prices will select products solely on the basis of design preference, leading to minimised average product-match efficiency costs of $t/4$. Hence, imperfect price comparability influences consumer surplus solely through its impact on market power. By increasing the intensity of price competition, an increase in price comparability directly increases expected consumer surplus.

3.2 Limited design comparability

Suppose instead that $h < 1$. In this case, the consumer may not learn her location. Given message vector $\mathbf{m}$, the expected profits of Firm j when setting price p_j are given by

$$\pi(p_j;\mathbf{m}) = p_j\left(hv\mathbb{E}_{p_{-j}}(\tilde{s}_j(\mathbf{p})) + (1-h)v(1-F(p_j)) + (1-v)/2\right). \tag{3}$$

The profit function provides firms with two contrasting incentives. With probability hv, the consumer is aware of her preferences and perfectly informed about prices. This provides firms with an incentive to extract rents due to observable product differentiation. The rent-extraction incentive is accentuated if, with probability $1-v$, the consumer is unable to compare product prices. By contrast, with probability $(1-h)v$, the consumer is unaware of her preferences, and acts as if the products are identical. Because she can compare prices, this encourages firms to marginally undercut their rival to attract her business. The combination of these contrasting incentives gives rise to an equilibrium in mixed pricing strategies, which we formalise with Lemma 1.

Lemma 1. *Suppose $v > 0$, $h < 1$. Then the symmetric equilibrium price distribution is continuous with non-degenerate, connected support.*

In a mixed strategy pricing equilibrium, the price distribution F satisfies the following properties: every price in the support of F generates the same profit:

$$\pi(p_a;\mathbf{m}) = \pi(p_b;\mathbf{m}), \text{ for all } p_a, p_b \in P; \tag{4}$$

the lower bound of the price distribution is determined by

$$F(\underline{p}) = 0; \tag{5}$$

the intrinsic taste parameter restricts the upper bound of the price distribution:

$$\overline{p} \leq \overline{u}; \tag{6}$$

and, if the inequality (6) is strict,

$$\lim_{\epsilon\to 0+} \frac{\pi(\overline{p}+\epsilon, m) - \pi(\overline{p}, m)}{\epsilon} = 0. \tag{7}$$

The final condition ensures that there is no incentive to incrementally raise price beyond the upper bound of the price distribution.

Our next result describes the equilibrium price distribution. To set up the result, consider a market in which $\overline{u}$ is sufficiently high that the outside good does not constrain pricing, and define the expression

$$J(h,v) = x(x+1)^2 \ln\left(\frac{2-h}{h}\right) - (x+1)^2 - \frac{h}{1-h} + \frac{1-v}{hv}, \qquad x = \frac{h}{2(1-h)}.$$

When $J(h,v) = 0$ for such a market, the equilibrium price distribution satisfies $\overline{p} - \underline{p} = t$. This boundary condition ensures that both firms will obtain a strictly positive share of perfectly informed consumers if they each set prices within the pricing bounds. For given h, define $V(h)$ as the solution to $J(h, V(h)) = 0$.

Proposition 2. *Suppose $h < 1$ and $v \geq V(h)$. Then, the symmetric equilibrium price distribution F is given by*

$$F\left(p;\mathbf{m}\right) = 1 - A(\mathbf{m})\frac{\overline{p}-p}{p}\left(\mu + \frac{1-v}{hv}t + t - \overline{p} - p\right), \qquad A(\mathbf{m}) = \frac{h}{2t(1-h)}. \tag{8}$$

Proposition 2 follows from the system (4) - (7). $V(h)$ describes a negative relationship between v and h, which we illustrate in Online Appendix C.2. The condition $v \geq V(h)$ is satisfied when either $h = 1$ or $v = 1$, or when v is sufficiently large for given h. When the condition is satisfied, equilibrium price dispersion is moderate in the sense that $\overline{p} - \underline{p} \leq t$, whether or not prices are constrained by the presence of an outside good. In this case, each firm competes with the entire support of the price distribution set by their rival. We focus on this case in the analysis below. By contrast, when the condition fails, it is possible that $\overline{p} - \underline{p} > t$. As a result, each firm effectively competes with a subset of the price distribution, and this reduces the cost to raising price. In this case, the constraint (6) always binds. We provide a detailed derivation for this case in Online Appendix C.3.

When the outside good does not constrain pricing (the constraint (6) is slack), it follows from (7) that the mean of the price distribution is given by

$$\mu = 2\overline{p} - t - \frac{1-v}{hv}t, \tag{9}$$

and (8) simplifies to

$$F\left(p;\mathbf{m}\right) = 1 - A(\mathbf{m})\frac{(\overline{p}-p)^2}{p}. \tag{10}$$

The support of F is then determined by (5) and (9). Firm profits simplify to

$$\pi^*(\mathbf{m}) = \overline{p}^2\frac{hv}{2t}.$$

Alternatively, if the constraint (6) binds, the mean of the price distribution is given by

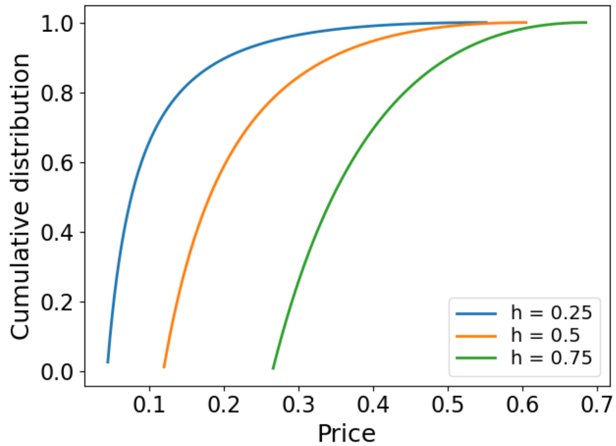
$$\mu = \left(A(\mathbf{m})\overline{p}\left(t + \frac{1-v}{hv}t - \overline{p}\right)\ln\left(\frac{\overline{p}}{\underline{p}}\right) - \frac{A(\mathbf{m})}{2}\left(\overline{p}^2 - \underline{p}^2\right)\right)\left(1 - A(\mathbf{m})\overline{p}\ln\left(\frac{\overline{p}}{\underline{p}}\right)\right)^{-1}, \tag{11}$$

and the support of F is determined by (5) and (11), with $\overline{p} = \overline{u}$.

3.2.1 Comparative statics

We now turn to comparative static analysis under imperfect design comparability. We first consider the special case of perfect price comparability, before examining the general case.

Figure 1: Cumulative distribution, $t = 1$, $v = 1$



Perfect price comparability

In the standard Hotelling model, firms set a price margin above marginal cost given by the transport cost t. Our next result ensures that the Hotelling price represents an upper bound on pricing in the case of perfect price comparability.

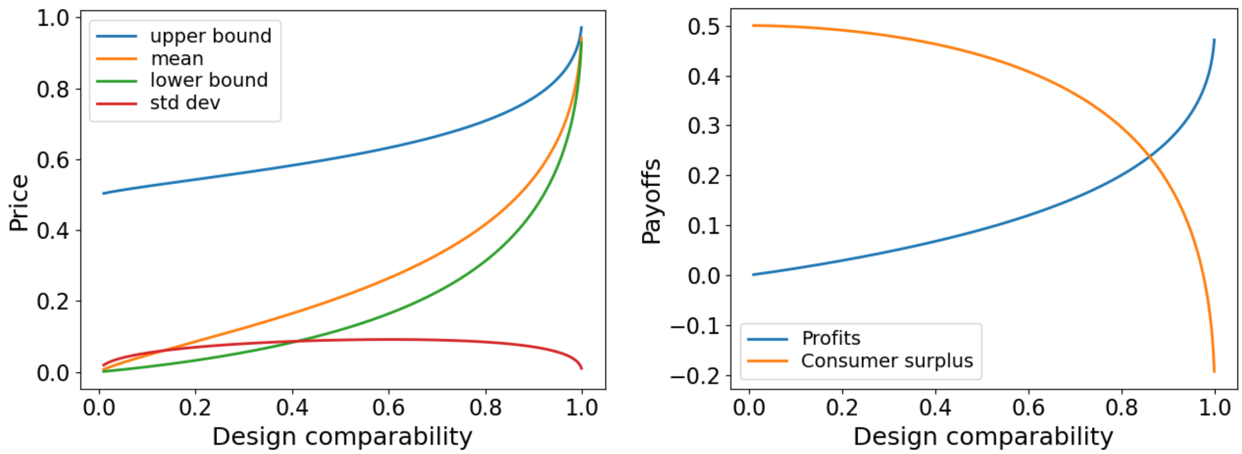
Lemma 2. *If price comparability is perfect, then* $\overline{p} \leq t$.

If $v = 1$, then each firm's profit function is a weighted average of the standard Hotelling and Bertrand profit functions. The incentive to marginally undercut one's rival, as in the Bertrand model, leads to equilibrium prices that tend to be below those of the Hotelling equilibrium. In the comparative static examples analysed below, we also focus on the case $\overline{p} < \overline{u}$, in which the outside good plays no role in price determination. In this case, equilibrium prices are determined by (10).

Figure 1 depicts the cumulative distribution of prices for $h \in \{0.25, 0.5, 0.75\}$ when $v = 1$. For higher values of h, the incentive to extract rents becomes more important relative to the incentive to undercut, and equilibrium price distributions with greater design comparability first-order stochastically dominate those with lower design comparability.

Figure 2 describes the pricing equilibrium for $h \in (0, 1)$. The left panel presents statistics on the price distribution as a function of comparability, and the right panel shows profits and expected consumer surplus as a function of comparability. The mean, lower bound and upper bound of the price distribution all increase with design comparability. There is also a non-monotonic relationship between comparability and price dispersion, as measured by the standard deviation of prices. Under perfect comparability, both firms set the Hotelling equilibrium price. When design comparability is imperfect, firms have an incentive to marginally undercut their rival's price to capture consumers who are unsure which product they prefer. Firms

Figure 2: Prices and profits as a function of design comparability, $t = 1$, $v = 1$, $\overline{u} = 1$



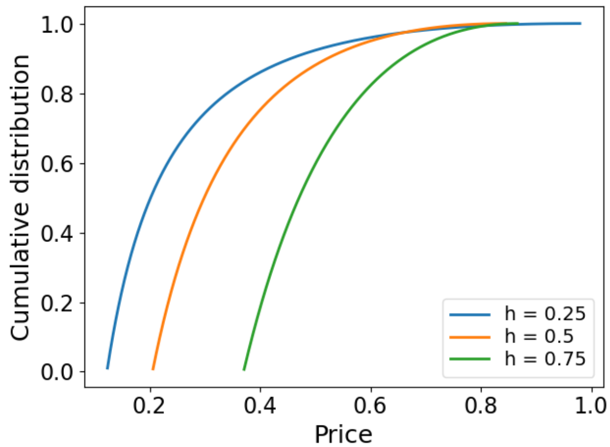
also have an incentive to exploit their market power with respect to consumers who understand their preferences. Price dispersion is greatest when these contrasting incentives are balanced. For low levels of comparability, the incentive to undercut dominates, most of the probability mass is near the lower support of the price distribution, and price dispersion diminishes. By contrast, for high levels of comparability, the incentive to exploit market power approaches that of the Hotelling model, and price dispersion also diminishes. The limiting case with $h(\mathbf{m}) = 0$ corresponds to the Bertrand model, and each firm sets price equal to marginal costs and earns no profits. At the other extreme with $h(\mathbf{m}) = 1$, we have the Hotelling model. F becomes a point distribution with $\underline{p} = \overline{p} = t$, and each firm earns profits of $t/2$.

The right panel illustrates a similar relationship between profits and comparability. Given message vector $\mathbf{m}$, expected consumer surplus is defined as

$$CS(\mathbf{m}) = \int_{\underline{p}}^{\overline{p}} \int_{\underline{p}}^{\overline{p}} \int_0^1 CS(\mathbf{p}, \mathbf{m}, x)\, dx\, dF(p_1)\, dF(p_2).$$

With $\overline{u} = 1$, if product matching were perfect, there would be a total surplus of 1 to be split between consumers and firms. Imperfect design comparability introduces a trade-off for consumers. Higher comparability improves the product matches obtained by consumers, but also lends market power to firms, leading to higher average prices. At the extreme of no design comparability, consumers perceive the products as identical, firms have no market power, and consumers extract all of the surplus from trade. However, half of this surplus is dissipated due to poor product matching, leading to consumer surplus of $t/2$. At the other extreme, with perfect design comparability, consumers minimise the costs of poor product matches, but firms receive all of the surplus from trade, and consumer surplus is $-t/4$. On balance, we observe a negative relationship between design comparability and consumer surplus, indicating that the market power effect dominates.

Figure 3: Cumulative distribution, $t = 1$, $v = 0.85$

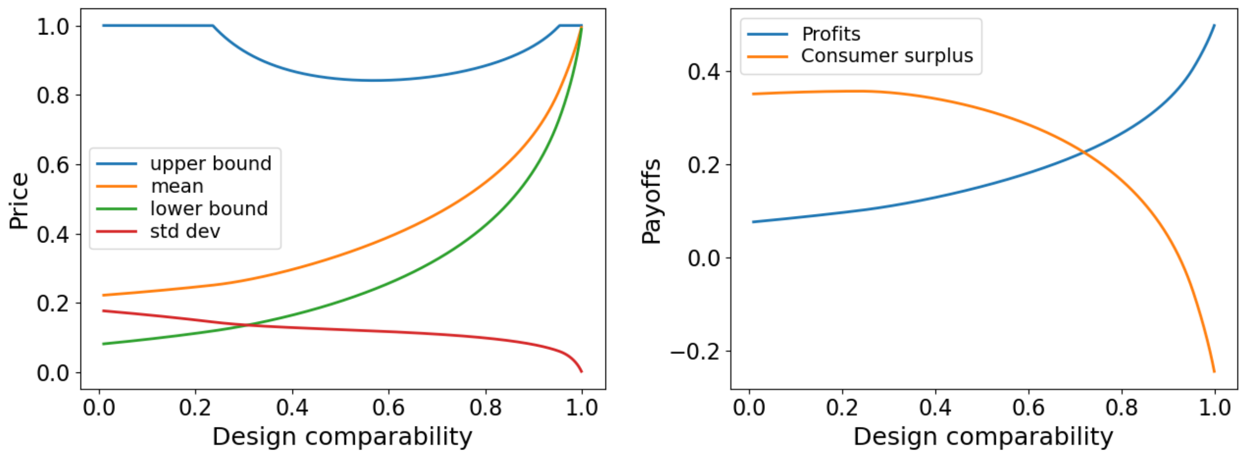


Limited design and price comparability

We next consider the general case in which $v < 1$ and $h < 1$. Thus, the consumer may not be able to compare product prices and may not learn her location. Figure 3 illustrates the cumulative distribution for the case $t = 1$, $v = 0.85$. As with the case of perfect price comparability illustrated in Figure 1, an increase in design comparability leads to an upward shift in the price distribution. By contrast, comparing the two figures, we can see that an increase in price comparability leads to a downward shift in the price distribution. We examine comparative statics more systematically below.

If both design and price comparability are limited, this can lead to qualitative differences in the equilibrium price distribution. Figure 4 illustrates comparative statics over design comparability when $v = 0.85$, $t = 1$, and $\overline{u} = 1$. The horizontal axis indexes design comparability. The left panel illustrates the price distribution, depicting the upper and lower bounds and mean of the distribution; and the right panel shows the relationship between profits and consumer surplus and design comparability. Recall that Figure 2 illustrates the special case $v = 1$. Comparison with Figure 2 suggests that profits, and the lower bound and mean of the price distribution all increase with a reduction in price comparability, while consumer surplus decreases. As before, improved design comparability is associated with higher prices and profits and lower consumer surplus. However, for low levels of design comparability, the price distribution disperses, and the constraint $\overline{p} \leq \overline{u}$ becomes binding. If design comparability is low, and price comparability is high but not perfect, a substantial measure of consumers behave as if they do not have strong preferences over products, leading to intense price competition. However, a small measure of consumers are unable to compare prices. To profitably exploit this segment of the market, the upper bound of the price distribution must be high. For high levels of design comparability, product differentiation is apparent to a greater measure of consumers, and firms have a greater

Figure 4: Prices and profits as a function of design comparability, $t = 1$, $v = 0.85$, $\overline{u} = 1$



incentive to exploit their market power. This leads to a more concentrated price distribution with higher prices, and for h sufficiently high, the constraint $\overline{p} \leq \overline{u}$ again becomes binding.

Figure 5 illustrates comparative statics over price comparability when $h = 0.85$ and $t = 1$, with $\overline{u} = 3.5$. The left panel illustrates the price distribution, and the right panel depicts profits and consumer surplus. The horizontal axis indexes price comparability in both panels. Common with the case of perfect design comparability, prices and profits decrease with price comparability. However, imperfect design comparability yields an equilibrium in mixed strategies. The support of the price distribution declines monotonically with price comparability, while the range and standard deviation of the distribution are not substantially affected. For low levels of price comparability, the constraint $\overline{p} \leq \overline{u}$ is binding.

Several factors determine the relationship between price comparability and consumer surplus. First, average prices decrease with price comparability, leading to an increase in consumer surplus. Second, an increase in the variance of the price distribution increases the expected benefit to the consumer from selecting the cheapest product in the event that she understands both relative prices and her product match. This effect is partially mitigated because the quality of product matches is compromised by selecting products partially based on price. Finally, in the event that the consumer understands relative prices, but not her product match, she selects purely on the basis of price, and her surplus is influenced by the distribution of relative prices. The first of these effects dominates, and we observe a positive relationship between consumer surplus and price comparability.

Figure 6 depicts isoprofit lines for the set of taste parameters $\overline{u} \in \{1.2, 1.4, 1.6\}$. Let $\Pi(v, h)$ indicate profits as a function of price and design comparability, with $\pi^*(\mathbf{m}) = \Pi(v(\mathbf{m}), h(\mathbf{m}))$. Each line expresses, for a given taste parameter, the combinations of price and design comparability (v, h) for which $\Pi(v, h) = \Pi(1, 1) = t/2$, which corresponds to the perfectly transparent

Figure 5: Prices and profits as a function of price comparability, $t = 1$, $h = 0.85$, $\overline{u} = 3.5$

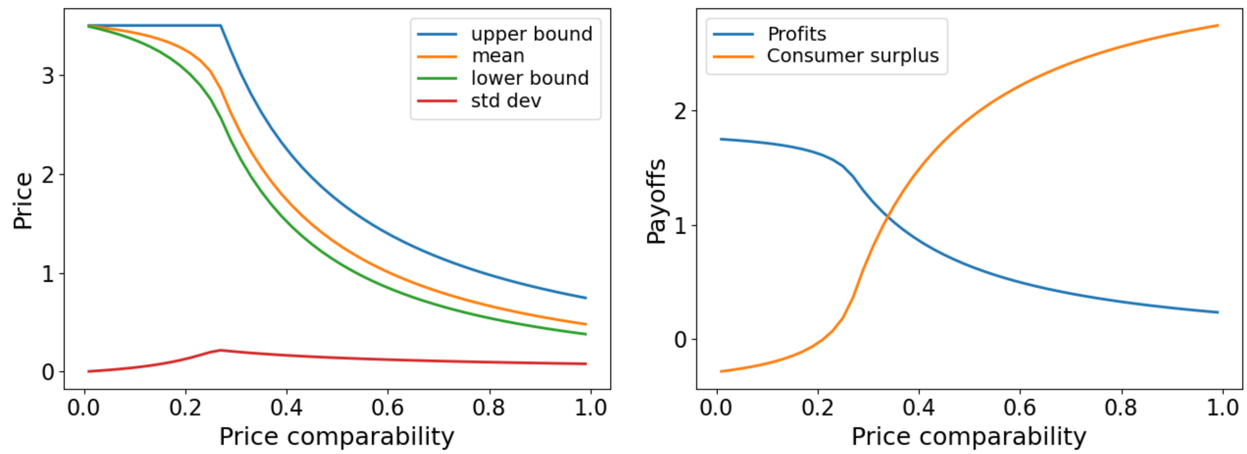
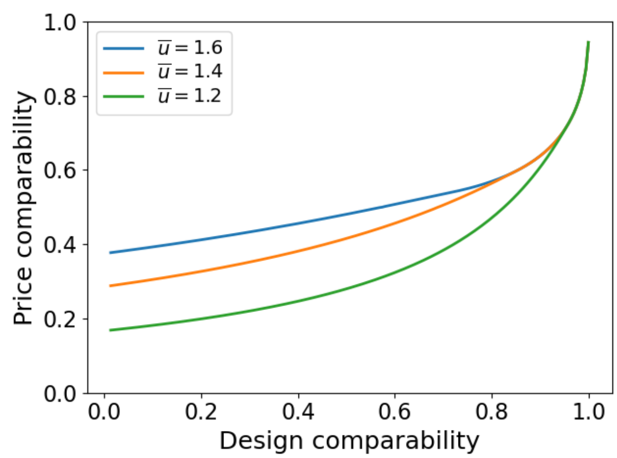


Figure 6: Isoprofit lines, $t = 1$, $\overline{u} \in \{1.2, 1.4, 1.6\}$



case. Above and to the left of each line, profits are below the perfectly transparent case, and below and to the right of each line, profits are higher. An increase in design comparability leads to an increase in profits, while an increase in price comparability reduces profits, leading to the upward slope of the isoprofit lines. The lines coincide when the constraint $\overline{p} \leq \overline{u}$ is slack for each line.

For high levels of comparability, a small decrease in design comparability leads to a large proportionate increase in the measure of consumers who are aware of prices, but not their preferences over products. These consumers are extremely price sensitive, leading to an intensification in price competition and a decrease in profits. Therefore, in this range of the parameter space, the isoprofit line is relatively steep, indicating that substantial changes in price comparability are traded off for relatively small changes in design comparability. For lower levels of design comparability, the rate of proportionate change in price sensitive consumers is moderated, leading to curvature in the isoprofit line. For lower levels of the taste parameter, the restriction on the upper bound of prices has a bigger impact on profitablity, and a greater reduction in price comparability is needed to compensate for a reduction in design comparability.

4 Message and price competition

In this section, we analyse market competition in which firms simultaneously choose both prices and messages. We discuss comparability structures and present general results in Section 4.1, before specialising our analysis to an extension of the frame competition model of Chioveanu and Zhou (2013) in the remaining sections.

4.1 Comparability structures

We begin by distinguishing between two types of comparability structures. According to Definition 1, if messages can be compartmentalised, distinct elements of the message strategy can be targeted towards price or design comparability. If instead messages are entangled, the impact of messages on price and design comparability cannot be separated.

Definition 1. A comparability structure (M, v, h) is

1. *compartmentalised* if $M = M_v \times M_h$ and price and design comparability can be expressed as $v(\mathbf{m}_v)$ and $h(\mathbf{m}_h)$ for $\mathbf{m}_v \in M_v \times M_v$ and $\mathbf{m}_h \in M_h \times M_h$; and
2. *entangled* otherwise.

We next introduce the property of Enforceable Comparability (EC), which extends the concept introduced by Spiegler (2016) to two dimensions of comparability.

Definition 2. A comparability structure (M, v, h) satisfies *Enforceable Comparability* (EC) if there exists $\lambda \in \Delta(M)$ and $(v^*, h^*) \in [0,1]^2$ such that, for any $m' \in M$,

$$\sum_{m \in M} \lambda(m) v(m, m') = v^*, \qquad \sum_{m \in M} \lambda(m) h(m, m') = h^*.$$

(M, v, h) satisfies Enforceable Price (Design) Comparability if the first (second) of the two conditions is satisfied for any $m' \in M$.

If EC holds, then each firm can unilaterally enforce a particular comparability outcome (v^*, h^*) by choosing a message strategy λ. This property holds trivially if messages are fixed or determined before price competition, or if messages are ineffective at influencing comparability. Alternatively, EC holds if a single firm is able to unilaterally educate (or confuse) consumers. As Proposition 3 shows, if EC holds, this permits a simplified analysis.

Proposition 3.

1. *If the comparability structure is compartmentalised and satisfies enforceable price (design) comparability with price (design) message strategy* λ_v^* (λ_h^*)*, then firms employ marginal price (design) message strategy* λ_v^* (λ_h^*) *in a symmetric equilibrium.*
2. *If the comparability structure satisfies Enforceable Comparability with message strategy* λ^* *and comparability* (v^*, h^*)*, then in the unique symmetric equilibrium:*
   (a) *firms employ marginal message strategy* λ^**;*
   (b) *if* $h^* = 1$*, firms choose price* $p^*(\lambda^*, \lambda^*)$ *as in (2);*
   (c) *if* $h^* < 1$ *and* $v^* \geq V(h^*)$*, firms choose pricing strategy* $F\left(p; (\lambda^*, \lambda^*)\right)$*, as in (8).*

The proof follows Piccione and Spiegler (2012). The logic of this result is that, conditional on any realised price vector, rival firms have opposed incentives to foster comparability. A low-priced firm has an incentive to improve price comparability and limit design comparability, while her rival has the opposite incentive. If comparability departs from (v^*, h^*), then at least one firm will have an incentive to enforce (v^*, h^*).

Our next result establishes the important role that EC plays for the nature of messaging strategies. To set up the result, we introduce the following notation. For any given change in messages from $\mathbf{m}$ to $\mathbf{m}'$, define $\Delta(x) = x(\mathbf{m}') - x(\mathbf{m})$ to be the change in variable x resulting from the message change.

Lemma 3.

1. *If the comparability structure satisfies* $\frac{\Delta(v)-\Delta(hv)}{\Delta(v)} \geq 0$ *for any message change and does not satisfy enforceable price comparability, then there is no equilibrium in which both firms play pure message strategies.*
2. *If the comparability structure satisfies* $\frac{\Delta(v)-\Delta(hv)}{\Delta(hv)} \geq 0$ *for any message change and does not satisfy enforceable design comparability, and* $v \geq V(h)$ *for all* $\mathbf{m} \in M^2$*, then there is no equilibrium in which both firms play pure message strategies.*

The above conditions on $\Delta(v)$ and $\Delta(hv)$ are automatically satisfied if messages are compartmentalised. If messages are entangled, the conditions are satisfied as long as message changes do not induce changes in price and design comparability that are highly positively correlated. The logic of the proof of this result is that the incentives of each firm to foster comparability are different at the upper and lower bounds of the price distribution. Therefore any pure message strategy allows a profitable deviation by either adjusting messages when setting price at the lower bound or the upper bound. This logic could break down if the condition $v \leq V(h)$ is not satisfied and prices are sufficiently dispersed. In this case, marginal changes in design comparability have little bearing on purchase decisions for products priced at the upper and lower bounds of the price distribution.

For the remainder of Section 4, we employ the framework of Chioveanu and Zhou (2013), with finite message space M. We analyse limited price comparability in Section 4.2, limited design comparability in Section 4.3, and we consider limitations in both dimensions in Section 4.4. In the analysis of Sections 4.4, we restrict attention to markets in which design and price comparability are sufficiently high that the constraint $v \geq V(h)$ is satisfied. In the earlier sections, this constraint is automatically satisfied. For all comparative static analysis, we consider examples in which the taste parameter $\overline{u}$ is sufficiently high that the constraint $\overline{p} \leq \overline{u}$ does not bind.

4.2 Limited price comparability

Suppose that design comparability is perfect and price comparability is limited; and the set of messages is binary, with $M = \{a, b\}$. Message a is considered simple and message b complex. Let λ be the probability of message a in a symmetric Nash equilibrium with mixed message strategies. Price comparability is summarised by $v(\mathbf{m}) = v_0(\mathbf{m}; \alpha)$, as specified in the table below, where $\alpha = (\alpha_0, \alpha_1, \alpha_2)$ is a vector of parameters. The rows and columns of the tables correspond to the messages of the respective players.

Price comparability, $v_0(\mathbf{m}; \alpha)$

	a	b
a	α_0	α_1
b	α_1	α_2

The parameter $\alpha_0 = (0, 1]$ indicates latent comparability when both firms choose the simple message. The parameters $\alpha_1, \alpha_2 \in (0, \alpha_0)$ indicate comparability when messages are differentiated and complex, respectively. If rival firms adopt different messages, price comparability is limited with $v = \alpha_1$, and if both firms use the complex message, price comparability is given by $v = \alpha_2$.

Define $\lambda^*(\alpha)$ as the solution to $v(a, \lambda) = v(b, \lambda)$, and $v^*(\alpha)$ as the associated vertical comparability:

$$\lambda^*(\alpha) = \frac{\alpha_2 - \alpha_1}{\alpha_0 + \alpha_2 - 2\alpha_1}, \qquad v^*(\alpha) = \frac{\alpha_0 \alpha_2 - \alpha_1^2}{\alpha_0 + \alpha_2 - 2\alpha_1}. \tag{12}$$

In the event that the comparability structure satisfies EC, by playing $\lambda^*(\alpha)$, a firm is able to unilaterally enforce vertical comparability at the level $v^*(\alpha)$.

If EC is not satisfied, the equilibrium message strategy for Firm j balances the incentive to maximise price comparability when they have a low price with the incentive to minimise comparability when they have a high price. Define $\hat{\lambda}$ as the solution to

$$\frac{x}{y} = \left(\frac{x + \alpha_1}{y + \alpha_1}\right)^2, \qquad x = \lambda\alpha_0 + (1 - \lambda)\alpha_1, \qquad y = \lambda\alpha_1 + (1 - \lambda)\alpha_2,$$

and define the prices

$$p_a = \frac{y+\alpha_1}{xy+\alpha_1\left(\hat{\lambda}x+(1-\hat{\lambda})y\right)}t, \qquad p_b = \frac{x+\alpha_1}{xy+\alpha_1\left(\hat{\lambda}x+(1-\hat{\lambda})y\right)}t.$$

Proposition 4. *Suppose design comparability is perfect.*

1. *If $\alpha_2 > \alpha_1$, equilibrium messages are given by $\mathbf{m} = (\lambda^*, \lambda^*)$, and equilibrium prices are $p_1 = p_2 = p^*(\mathbf{m})$, as in (2).*
2. *If $\alpha_1 > \alpha_2$, there is a symmetric equilibrium in which each firm chooses $(m, p) = (a, p_a)$ with probability $\hat{\lambda}$ and $(m, p) = (b, p_b)$ with probability $1-\hat{\lambda}$.*

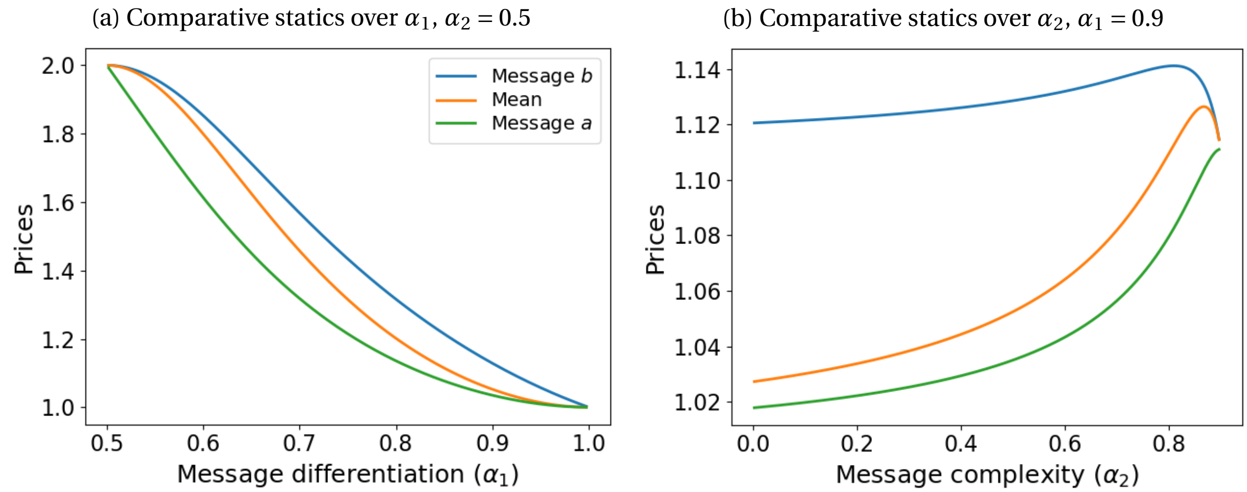
If message complexity is more confusing than message differentiation ($\alpha_2 > \alpha_1$), then the comparability structure satisfies Enforceable Comparability, and in the symmetric mixed strategy equilibrium each firm enforces comparability v^*. With comparability fixed, equilibrium is in pure price strategies, and prices decrease with each of the α parameters that determine price comparability.

If instead message differentiation is more confusing than message complexity ($\alpha_1 > \alpha_2$), Enforceable Comparability is not satisfied. Because messages influence price comparability, the equilibrium exhibits price dispersion. When setting low prices, firms have an incentive to induce a higher level of price comparability by choosing message a.

Figure 7 illustrates comparative statics over comparability with respect to the impact of message differentiation (panel a) and message complexity (panel b) when $\alpha_1 > \alpha_2$. In each panel, we fix $t = 1$ and $\alpha_0 = 1$. Panel (a) depicts the prices associated with each message and the mean price for $\alpha_2 = 0.5$ and $\alpha_1 \in (\alpha_2, 1)$. If α_1 is close to α_2, firms predominantly set message b. The incentive to undercut by switching to message a and price p_a is slight because this leads to only a marginal impact on comparability. As α_1 diverges from α_2, the incentive to undercut increases, leading to a downward shift in the price distribution, and an increase in the prevalence of low prices. Price dispersion is maximised for values of α_1 intermediate between α_2 and full transparency.

Panel (b) shows the same price information for $\alpha_1 = 0.9$ and $\alpha_2 \in (0, \alpha_1)$. This figure demonstrates that, due to message composition effects, an increase in the comparability parameter α_2 can lead to a decrease in price comparability and an associated increase in average prices. Consider first the extreme case when α_2 approaches zero. If both firms were to use message b, then consumers would be completely unable to compare the product prices. Because of this possibility, there is a much greater incentive to set high prices when choosing message b, leading to substantial price dispersion. To understand why the mean level of prices is relatively low, recall that in the mixed strategy equilibrium, the propensity to use each message type is determined by the equal profit condition. For low α_2, firms mostly use message a, so prices are usually comparable, leading to equilibrium prices that are relatively low. Higher levels of α_2 are associated

Figure 7: Prices as a function of price comparability, $t = 1$, $\alpha_0 = 1$



with a greater propensity to use message b. While α_2 is low, this reduces average comparability, leading to higher average prices. For sufficiently high levels of α_2, the effect of the increase in the level of α_2 becomes more important than the change in composition of messages, and average comparability rises with α_2 leading to a decrease in average prices.

4.3 Limited design comparability

Suppose instead that price comparability is perfect and design comparability is limited. As before, $M = \{a, b\}$, where a is a simple message and b is a complex message. Design comparability is summarised by $h(\mathbf{m}) = h_0(\mathbf{m}; \beta)$, depicted below, where $\beta = (\beta_0, \beta_1, \beta_2)$ and $1 > \beta_0 > \max\{\beta_1, \beta_2\} > 0$.[4]

Design comparability, $h_0(\mathbf{m}; \beta)$

	a	b
a	β_0	β_1
b	β_1	β_2

Define $\lambda^*(\beta)$ as the solution to $h(a, \lambda) = h(b, \lambda)$, and $h^*(\beta)$ as the associated price comparability:

$$\lambda^*(\beta) = \frac{\beta_2 - \beta_1}{\beta_0 + \beta_2 - 2\beta_1}, \qquad h^*(\beta) = \frac{\beta_0\beta_2 - \beta_1^2}{\beta_0 + \beta_2 - 2\beta_1}. \tag{13}$$

Mirroring the case of limited price comparability, if EC is satisfied, firms can enforce comparability h^* by playing message strategy λ^*.

[4] By imposing $\beta_0 < 1$, we ensure that design comparability is always imperfect. In the limiting case in which $\beta_0 = 1$ and $\beta_1 > \beta_2$, the equilibrium described in item 2 of Proposition 5 no longer exists.

Proposition 5. *Suppose price comparability is perfect.*

1. *If $\beta_2 > \beta_1$, then in the symmetric equilibrium each firm chooses message a with probability λ^*, and the equilibrium price distribution, given by (8), does not depend on message choice.*
2. *If $\beta_1 > \beta_2$, then there exists $\hat{p} \in (\underline{p}, \overline{p}]$ such that, in the symmetric equilibrium, F_b has support $P_b = [\underline{p}, \hat{p}]$ and F_a has support $P_a = [\hat{p}, \overline{p}]$.*

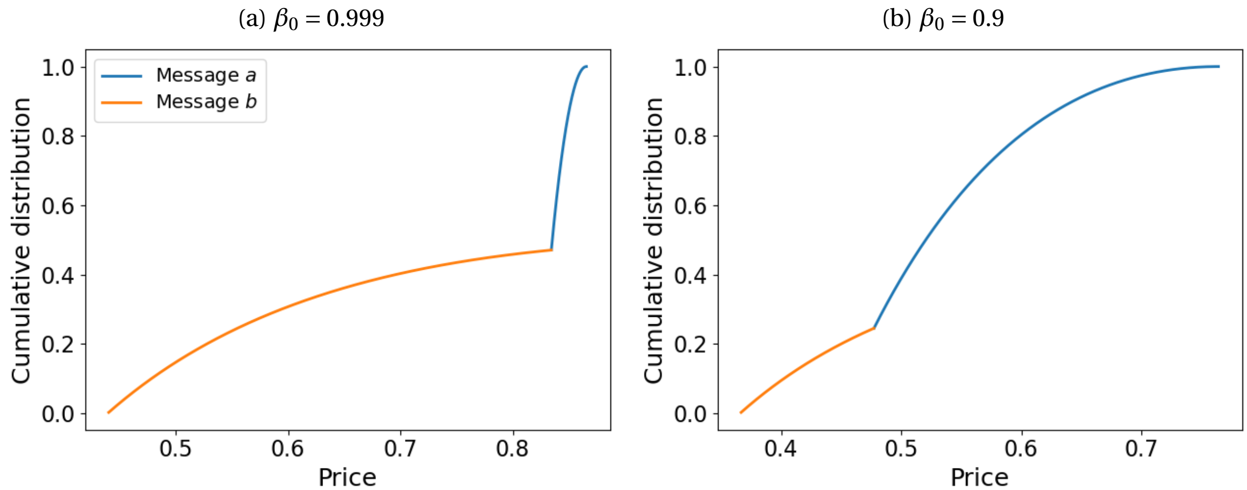
If $\beta_2 > \beta_1$, the comparability structure satisfies Enforceable Comparability and, by choosing message a with probability λ^*, each firm can enforce comparability h^*. The analysis of Section 3 can then be applied with message strategies (λ^*, λ^*). With comparability fixed, the equilibrium price distribution shifts with the β parameters that determine design comparability. An increase in any of the β parameters increases h^* and shifts up the equilibrium price distribution, as illustrated in Figure 2.

If instead message differentiation is more confusing than message complexity, Enforceable Comparability does not hold. For any given rival message, choosing message a increases design comparability. This increases perceived differentiation for the average consumer, thereby encouraging firms to set higher prices when choosing message a. Conditional on either message, the price distribution balances the incentive to extract rents from consumers who are aware of their preferences with the incentive to undercut one's rival's price to poach consumers who are not.

Figure 8 illustrates the symmetric equilibrium price distribution for $\beta_1 = 0.8$ and $\beta_2 = 0.5$. In panel (a), with $\beta_0 = 0.999$, design comparability is almost perfect if both firms use message a. In this case, there is little incentive to marginally undercut one's rival's price, and in the limit as β_0 approaches 1, the price distribution collapses towards a single point, conditional on message a. By contrast, in panel (b), with $\beta_0 = 0.9$, the price distribution is more dispersed, conditional on message a.

Figure 9 illustrates comparative statics over message differentiation (panel (a)) and message complexity (panel (b)), holding fixed the latent comparability parameter $\beta_0 = 0.999$. Consider first panel (a). The plot fixes the design message complexity parameter at $\beta_2 = 0.5$, while varying design message differentiation in the range $\beta_1 \in (\beta_2, 1)$. The horizontal axis indexes message differentiation, and the vertical axis illustrates elements of the price distribution. The coloured lines illustrate means of the price distribution, and the grey lines illustrate pricing limits. The top and bottom grey lines illustrate the upper and lower bounds of the price distribution, and the middle grey line depicts the price $\hat{p}$ at which firms switch from b to a messages. Of the coloured lines, the top (blue) line illustrates that the mean of the price distribution, conditional on message a, increases with β_1. There are two reasons for this. First, an increase in β_1 increases design transparency whenever the message vector contains a single a message. Second, with higher levels of β_1, firms are more likely to employ a messages. This can be seen by examining the next two coloured lines. The second (orange) line illustrates the unconditional mean

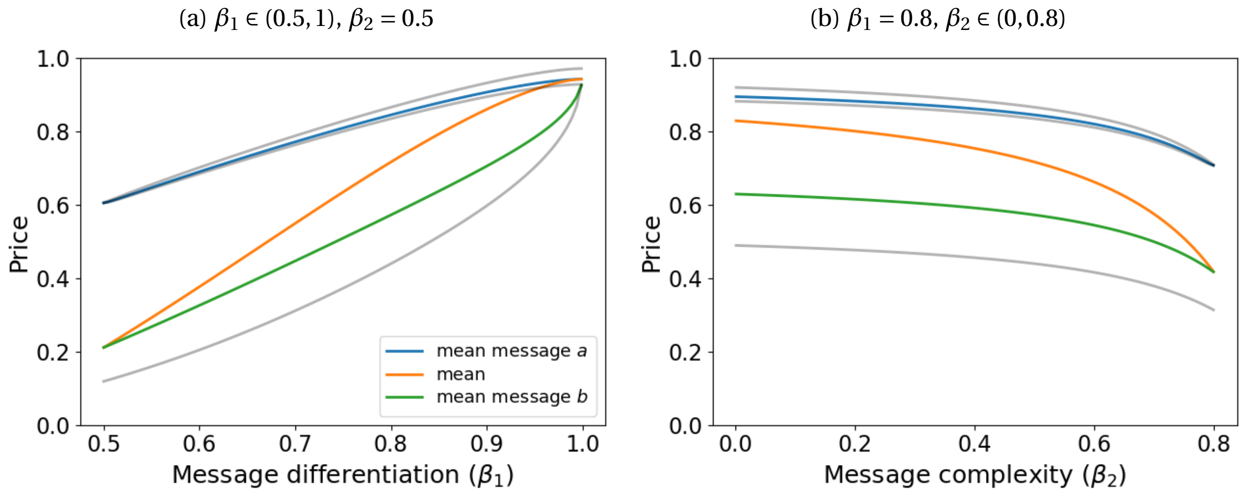
Figure 8: Price distribution, $t = 1$, $\beta_1 = 0.8$, $\beta_2 = 0.5$



of the price distribution, and the third (green) line depicts the mean, conditional on message b. For β_1 near β_2, firms predominantly choose message b. Switching to message a does little to change comparability and, because approximately half of the consumers perceive the products to be identical, there is a strong incentive to undercut and maintain message b. Accordingly, the conditional and unconditional means converge. For higher β_1, firms become more likely to set message a, and average prices increase, both because comparability increases directly with β_1, and because of the change in mix of messages. Finally, the bottom (grey) line shows the lower bound of the support of the price distribution. As β_1 approaches β_0, the mean of the conditional price distribution approaches the lower bound. This is because, for high β_1 firms predominantly choose message a. Therefore, even when setting message b, firms anticipate a high level of comparability. The incentive to set prices is then mainly determined by consumers who perceive the products as differentiated, rather than highly substitutable, and the distribution converges toward the lower bound of the support.

In panel (b) of the figure, we fix the message differentiation parameter $\beta_1 = 0.8$, and vary the message complexity parameter in the range $\beta_2 \in (0, \beta_1)$. By contrast with panel (a), an increase in message complexity leads to a downward shift in the price distribution. Two opposing forces determine comparative statics over the price distribution. First, as β_2 increases, comparability is improved whenever both firms set message b, leading to an increase in perceived differentiation and increasing the incentive to set higher prices. Second, for low levels of β_2, there is a strong incentive to set message a to avoid the possibility of very low comparability and more intense price competition. However, as β_2 increases, this incentive diminishes, message a is chosen less frequently, and perceived differentiation decreases. The latter effect dominates, leading to a decrease in average prices as the message complexity parameter increases.

Figure 9: Comparative statics over message differentiation and complexity, $t = 1$, $\beta_0 = 0.999$



4.4 Limited price and design comparability

We now examine the general case in which both price and design comparability are limited. We consider two cases. First, we consider compartmentalised message structures, in which each firm is able to target elements of the message vector specifically to influence either price or design comparability. Second, we consider the case where messages related to design and price elements cannot be disentangled in this manner. Throughout, we restrict attention to situations in which comparability is sufficiently high that $v \geq V(h)$.

Compartmentalised messages

Suppose that messages are compartmentalised, and that the set of messages is given by $M = M_v \times M_h$, where $M_v = \{a_v, b_v\}$ and $M_h = \{a_h, b_h\}$. Message components with a v subscript influence price comparability, components with an h subscript influence design comparability, a messages are simple, and b messages are complex. Message vector $\mathbf{m} \in M \times M$ can be decomposed into $\mathbf{m} = (\mathbf{m}_v; \mathbf{m}_h)$, where $\mathbf{m}_v \in M_v \times M_v$ describes the price component of the message vector and $\mathbf{m}_h \in M_h \times M_h$ the design component. Comparability is summarised below.

Price comparability, $v_0(\mathbf{m}_v; \alpha)$

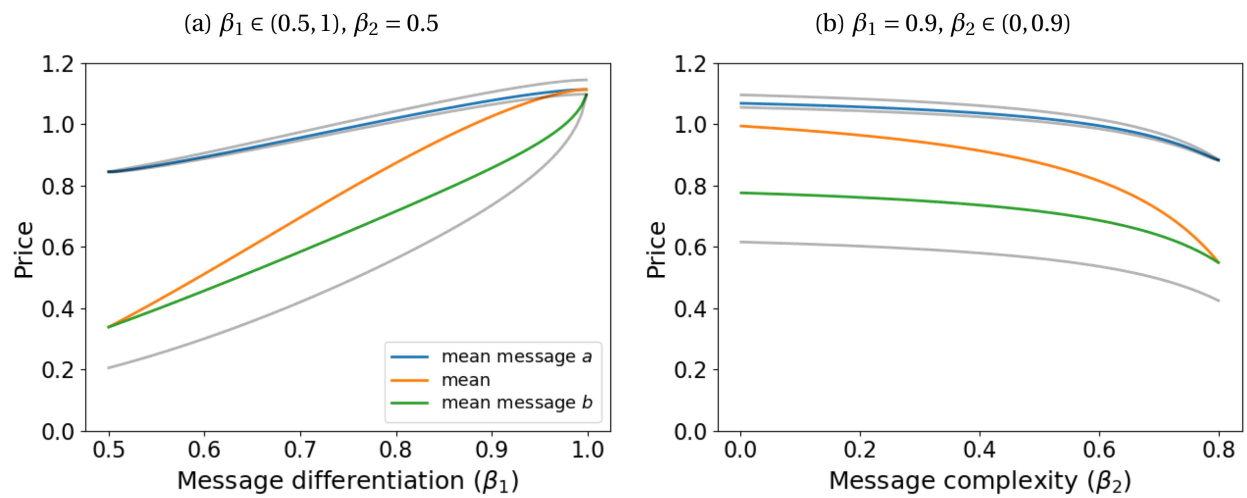
	a_v	b_v
a_v	α_0	α_1
b_v	α_1	α_2

Design comparability, $h_0(\mathbf{m}_h; \beta)$

	a_h	b_h
a_h	β_0	β_1
b_h	β_1	β_2

The parameter vectors $\alpha = (\alpha_0, \alpha_1, \alpha_2)$ and $\beta = (\beta_0, \beta_1, \beta_2)$ influence price and design comparability, respectively, where $1 > \beta_0 > \max\{\beta_1, \beta_2\} > 0$ and $1 \geq \alpha_0 > \max\{\alpha_1, \alpha_2\} > 0$.

Figure 10: Comparative statics, $t = 1$, $\beta_0 = 0.999$, $v^* = 0.85$



Define $\lambda_v^*(\alpha)$ as the solution to $v(a_v, \lambda) = v(b_v, \lambda)$, and $v^*(\alpha)$ as the associated price comparability, as specified in (12). Similarly, let $\lambda_h^*(\beta)$ be the solution to $h(a_h, \lambda) = h(b_h, \lambda)$, and $h^*(\beta)$ as the associated design comparability, as in (13).

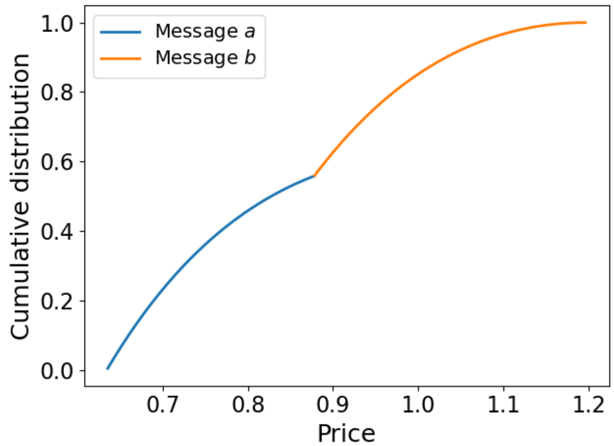
Proposition 6. *Suppose messages are compartmentalised and $\alpha_2 > \alpha_1$.*

1. *If $\beta_2 > \beta_1$, then in the symmetric equilibrium, each firm plays message strategy $\lambda^* = (\lambda_v^*, \lambda_h^*)$, and the equilibrium price distribution, given by (8), does not depend on message choice.*
2. *If $\beta_1 > \beta_2$, then there exists $\hat{p} \in (\underline{p}, \overline{p})$ such that, in the symmetric equilibrium:*
   - *(a) each firm plays price message strategy λ_v^*;*
   - *(b) F_{b_h} has support $P_{b_h} = [\underline{p}, \hat{p}]$ and F_{a_h} has support $P_{a_h} = [\hat{p}, \overline{p}]$.*

If $\alpha_2 > \alpha_1$ and $\beta_2 > \beta_1$, then the comparability structure satisfies EC with message vector λ^*, and part 1 of the proposition follows from Propositions 2 and 3. If $\alpha_2 > \alpha_1$ and $\beta_1 > \beta_2$, the comparability structure satisfies enforceable price comparability, but not enforceable design comparability. The symmetric equilibrium exhibits enforced price comparability at v^*, while design comparability varies with prices. The equilibrium exhibits similar properties to the case of perfect price comparability analysed in Proposition 5. In particular, firms seek to clarify product differentiation for the consumer by setting the simple design message when setting high prices, and obscure product differentiation by setting the complex design message when setting low prices.

Figure 10 illustrates comparative statics over design message differentiation (panel a) and design message complexity (panel b) with $t = 1$, $\beta_0 = 0.999$, and enforced price comparability $v^* = 0.85$. Aside from v^*, the parameters are chosen to match those of Figure 9 in which price comparability was perfect. The plot in panel (a) fixes design message complexity at $\beta_2 = 0.5$,

Figure 11: Price distribution, $t = 1$, $\alpha = (1, 0.8, 0.5)$, $h^* = 0.85$



while varying design message differentiation in the range $\beta_1 \in (\beta_2, 1)$. The plot in panel (b) imposes $\beta_1 = 0.9$ while allowing $\beta_2 \in (0, \beta_1)$. Relative to Figure 9, because prices are more difficult to compare, price competition is less intense, leading to higher price levels. Otherwise, the qualitative elements of comparative statics in the two examples are similar.

Proposition 7. *Suppose $\alpha_1 > \alpha_2$ and $\beta_2 > \beta_1$. Then, there exists $\hat{p} \in (\underline{p}, \overline{p})$ such that, in the symmetric equilibrium:*

1. *Each firm plays design message strategy λ_h^*.*
2. *F_{a_v} has support $P_{a_v} = [\underline{p}, \hat{p}]$ and F_{b_v} has support $P_{b_v} = [\hat{p}, \overline{p}]$.*

If $\alpha_1 > \alpha_2$ and $\beta_2 > \beta_1$, the comparability structure satisfies enforceable design comparability, but not enforceable price comparability. In the symmetric equilibrium, firms enforce design comparability at h^*, while price comparability varies with prices. Firms use the complex price message b_v to restrict price comparison when setting prices above the threshold $\hat{p}$, and the simple message a_v when setting prices below the threshold.

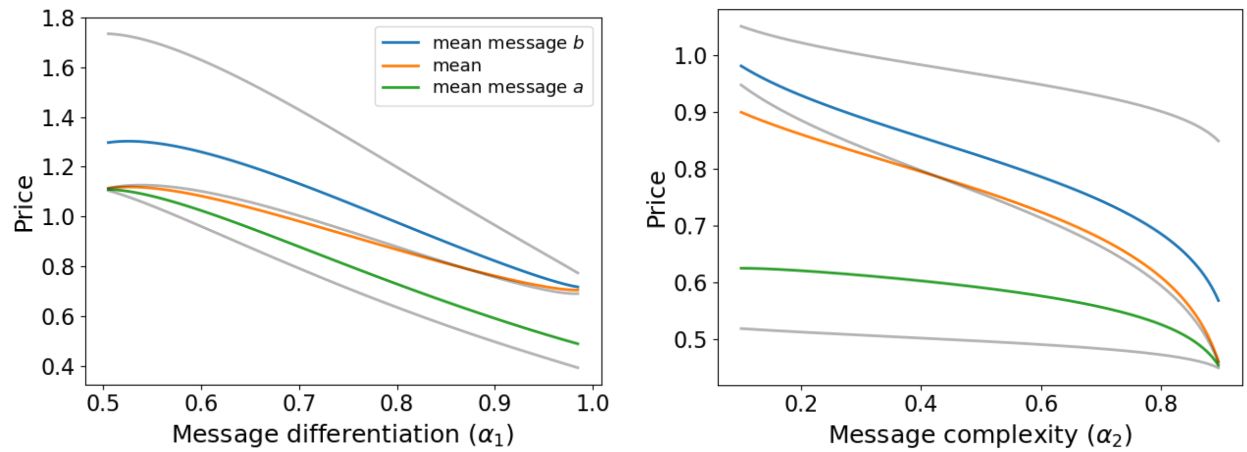
Figure 11 illustrates the symmetric equilibrium price distribution for $t = 1$, $\alpha = (1, 0.8, 0.5)$, and $h^* = 0.85$. The price distribution has lower bound $\underline{p} \approx 0.63$ and upper bound $\overline{p} \approx 1.20$. With probability $\lambda_v^* \approx 0.44$, firms use price message a_v and set price below the threshold $\hat{p} \approx 0.88$. With probability $1 - \lambda^*$, firms choose message b_v and a price above $\hat{p}$.

Figure 12 illustrates comparative statics over price message differentiation (panel a) and price message complexity (panel b) for the case $t = 1$, $\alpha_0 = 1$, and $h^* = 0.85$. Each panel illustrates the mean of prices, both conditional on each message, and overall. The pricing bounds $\underline{p}$ and $\overline{p}$ and the threshold price $\hat{p}$ are illustrated in greyscale.

Consider panel (a). At the left of the picture, when α_1 is close to α_2, firms predominantly use the simple message a_v and price comparability is almost perfect. With $h^* < 1$, some consumers also perceive no product differentiation, providing an incentive for firms to undercut

Figure 12: Comparative statics, $t = 1$, $\alpha_0 = 1$, $h^* = 0.85$

(a) $\alpha_1 \in (0.5, 1)$, $\alpha_2 = 0.5$ (b) $\alpha_1 = 0.9$, $\alpha_2 \in (0, 0.9)$



their rival and use message a_v. If a firm were to use message b_v, rival message choice would have little effect on price comparability, providing little incentive for her rival to also use message b_v. For higher levels of price message differentiation, α_1, whenever a rival uses message b_v, there is an increased incentive to set a high price and also use message b_v. This leads to an increased prevalence of b_v messages for higher values of α_1. As in our earlier examples, the mean price level shifts due to the direct effect of a change in price comparability, and due to shifts in message composition. Higher price comparability is associated with more intense price competition, and over most of the parameter range, this effect dominates.

It is instructive also to compare panel (a) of Figure 12 with panel (a) of Figure 7, in which we analysed the perfect design comparability case. There are two substantive differences. First, with perfect design comparability, no consumers perceive the products to be identical, and there is no incentive to marginally undercut. This leads to an equilibrium with no price dispersion, conditional on message choice. By contrast, with $h^* < 1$, equilibrium price dispersion arises, conditional on message choice, due to the measure of consumers who perceive the products to be identical. Second, the comparative statics of message choice are inverted. Under perfect design comparability, a greater disparity between α_1 and α_2 leads to an increased prevalence of simple messages as firms have a greater incentive to set low prices and increase the comparability of prices. When design comparability is limited, the incentive to attract price sensitive consumers drives a wedge between the mean prices conditional on each message. When $\alpha_1 - \alpha_2$ is also large, by choosing the complex price message, firms can ensure price comparability is limited, and limited price comparability becomes a more powerful influence on pricing than limited design comparability. This encourages firms to set high prices and complex price messages.

Next, consider Panel (b) of Figure 12. At the left of the figure, for low levels of α_2, firms have a strong incentive to choose the complex price message and set a high price. As we move to the right in the figure, as α_2 approaches α_1, firms predominantly set the simple message, and there is very little price dispersion. There is little incentive to switch to the complex message because this has little effect on comparability. Comparison with Figure 7 again reveals that the comparative statics with respect to message choice are inverted compared to the case of perfect design comparability.

Proposition 8. *Suppose $\alpha_1 > \alpha_2$ and $\beta_1 > \beta_2$. Then, in the symmetric equilibrium:*

1. *For any m_h, there exists price $\hat{p}(m_h) \in (\underline{p}, \overline{p})$ such that firms use price message a_v when setting $p < \hat{p}(m_h)$ and price message b_v when setting $p > \hat{p}(m_h)$.*
2. *For any m_v, there exists price $\hat{p}(m_v) \in (\underline{p}, \overline{p})$ such that firms use design message a_h when setting $p > \hat{p}(m_h)$ and design message b_v when setting $p < \hat{p}(m_h)$.*

If $\alpha_1 > \alpha_2$ and $\beta_1 > \beta_2$, then the comparability structure does not satisfy enforceable price or design comparability. Proposition 8 builds on Propositions 6 and 7. When firms set high prices, they adopt complex price messages in order to limit price comparison, and simple design messages in order to clarify product differentiation. When setting low prices, firms instead seek to facilitate price comparison and disguise product differentiation.

Entangled messages

Suppose instead that messages are entangled and the set of messages is given by $M = \{a, b\}$. The firm's message influences both price and design comparability, as summarised below. As before, we impose the following restrictions on the design and price comparability parameters: $1 > \beta_0 > \max\{\beta_1, \beta_2\} > 0$, and $1 \geq \alpha_0 > \max\{\alpha_1, \alpha_2\} > 0$.

Price comparability, $v_0(\mathbf{m}; \alpha)$

	a	b
a	α_0	α_1
b	α_1	α_2

Design comparability, $h_0(\mathbf{m}; \beta)$

	a	b
a	β_0	β_1
b	β_1	β_2

Proposition 9. *Suppose that $\alpha_1 > \alpha_2$ and $\beta_1 > \beta_2$.*

1. *Firms use message a when setting price $\overline{p}$ and message b when setting price $\underline{p}$ if α_1 and α_2 are sufficiently high.*
2. *Firms use message b when setting price $\overline{p}$ and message a when setting price $\underline{p}$ if β_1 and β_2 are sufficiently high or α_1 and α_2 are sufficiently low.*

If messages are entangled, then firms are unable to target messages towards either price or design comparability. If $\alpha_1 > \alpha_2$ and $\beta_1 > \beta_2$, Enforceable Comparability is not satisfied, and firms face a clear conflict between price and design comparability. When setting high prices, firms would like to choose the complex message to limit price comparison, but they would like to choose the simple message to clarify product differentiation. According to Proposition 9, if the α parameters that determine price comparability are sufficiently high, then messages are ineffective at limiting price comparability. Firms therefore employ the simple message when setting high prices in order to emphasise design differences. By contrast, if the β parameters associated with design comparability are high, then design comparability is not greatly influenced by messages, and firms use complex messages when setting high prices in order to frustrate price comparability.

5 Conclusion

In this paper, we develop a model of limited product comparability that distinguishes between price and design dimensions of comparability. Firms choose messages that influence both the ability of consumers to compare rival product prices, and the ability of consumers to understand their product preferences.

We first analyse a game of price competition, taking as given the comparability environment. We find that limitations on the ability of consumers to compare product prices soften price competition, leading to higher equilibrium prices and profits, and lower consumer welfare. By contrast, if consumers have a limited understanding of their product preferences, then products become more similar in their eyes, and this degrades the quality of product matches but intensifies price competition, leading to lower average prices and lower profits and, on balance, higher consumer welfare. Further, a mixture of consumers with clear and imperfect knowledge of their preferences leads to equilibrium price dispersion.

Next, we analyse a game of simultaneous price and message choice, with the following findings. Under relatively general conditions, equilibrium is in mixed strategies over both messages and prices. If the comparability structure satisfies the property of Enforceable Comparability (Spiegler, 2016), then firms choose message strategies to enforce a fixed level of comparability, and equilibrium prices are independent of messages. In this case, our earlier consumer welfare analysis continues to apply. If Enforceable Comparability does not hold then, when setting high prices, firms tend to adopt messages that add complexity to price comparison and provide clarity for product preferences. Comparative static analysis suggests that the relationship between market outcomes and the parameters that determine comparability is complicated by message composition effects.

Appendix

A Proofs

Proof of Lemma 1

Proof. In equilibrium, each price in P must yield the same profit. Suppose that there is a mass point at $p \in P$. By (3), it follows that there is a profitable deviation to set a price $p - \epsilon$ for ϵ arbitrarily small and positive, a contradiction. Therefore, P must be non-degenerate and F must be continuous.

To establish that the support is connected, suppose otherwise that there is a gap in the support $\tilde{P} = (p_a, p_b)$ with $p_a < p_b$ and $p_a, p_b \in P$. Consider the perspective of Firm j. For $p_j \in [p_a, p_b]$, $F(p_j)$ is fixed and $\pi(p_j; \mathbf{m})$ is a concave function of p_j. Consider $p_c \in \tilde{P}$. Because $p_c \notin P$, $\pi(p_c; \mathbf{m}) < \pi(p_a; \mathbf{m})$. But, by concavity, $\pi(p_a; \mathbf{m}) = \pi(p_b; \mathbf{m}) < \pi(p_c; \mathbf{m})$, a contradiction. □

Proof of Lemma 2

Proof. It follows from (7) that

$$\begin{aligned}(2\overline{p} - t)(1 - F(\overline{p} - t)) &= \int_{\overline{p}-t}^{\overline{p}} p\, dF(p) \\ &\leq \overline{p}(1 - F(\overline{p} - t)),\end{aligned}$$

and therefore $\overline{p} \leq t$, as required. □

Proof of Lemma 3

Proof. First, suppose horizontal comparability is perfect for all $\mathbf{m}$. In this case, the conditions on $\Delta(v)$ and $\Delta(hv)$ are trivially satisfied. Rewrite the profit function for Firm j as

$$\pi_j(\mathbf{p}, \mathbf{m}) = p_j \left(\frac{p_{-j} - p_j}{2t} v + \frac{1}{2} \right),$$

define the pricing reaction function for Firm j as

$$r_j(p_{-j}, \mathbf{m}) = \frac{p_{-j}}{2} + \frac{t}{2v},$$

and observe that equilibrium prices must satisfy $p_1 = p_2 = p^*(\mathbf{m}) = t/v$.

We now show that equilibrium cannot be in pure message strategies. Suppose otherwise that the equilibrium $(\mathbf{p}, \mathbf{m})$ is in pure message strategies. It follows that

$$\pi_j(\mathbf{p}, \mathbf{m}) = \pi_j(\mathbf{p}, \mathbf{m}') < \pi_j(\mathbf{p}', \mathbf{m}'),$$

where $\mathbf{m}' = (m'_j, m_{-j})$, $m'_j \neq m_j$, $\mathbf{p}' = (p'_j, p_{-j})$, and $p'_j = r_j(p_{-j}, \mathbf{m}')$. The equality follows because comparability does not impact profits if prices are equal, and the inequality follows because the reaction function indicates that it is profitable to adjust prices if the message vector changes. This leads to a contradiction.

Next, suppose horizontal comparability is imperfect, and suppose further that messages are pure. Consider the perspective of Firm j. In equilibrium, profits must be the same at the upper and lower bounds of the support of the price distribution. Rewrite these profits as follows.

$$\pi(\overline{p}, m_j) = \overline{p}\left(hv \max\left\{s_1(\overline{p}, \mu), 0\right\} + \frac{1-v}{2}\right),$$
$$\pi(\underline{p}, m_j) = \underline{p}\left(hv \min\left\{s_1(\underline{p}, \mu) - 1, 0\right\} + \frac{1+v}{2}\right).$$

Suppose further that messages are compartmentalised. This again implies that the conditions on $\Delta(v)$ and $\Delta(hv)$ are satisfied. Profits are strictly decreasing in v at the upper bound, $\overline{p}$, and strictly increasing in v at the lower bound, $\underline{p}$. If $\mathbf{m}$ is pure and enforceable price comparability is not satisfied, then Firm j can adjust m_j and either increase or decrease v. If v increases, there is a profitable deviation to switch the price component of m_j and set price $\underline{p}$. If instead v decreases, there is a profitable deviation to switch the price component of m_j and set price $\overline{p}$. Profits are increasing in h at $\overline{p}$ and decreasing in h at $\underline{h}$; and these inequalities are strict if the condition $v \geq V(h)$ is satisfied. Thus, if this condition is satisfied, messages are pure, and enforceable design comparability is not satisfied, then there must also be a profitable deviation involving a switch in the design component of messages.

Finally, suppose that messages are entangled, and suppose again that messages are pure. Consider the impact of a change in messages on profits at the top of the price distribution. If $\overline{p} > \mu + t$, then

$$\Delta(\pi(\overline{p}, m_j)) = -\overline{p}\Delta(v)/2. \tag{14}$$

If instead $\overline{p} \leq \mu + t$, then

$$\frac{2t}{\overline{p}}\Delta(\pi(\overline{p}, m_j)) = \Delta(v)(\mu - \overline{p}) + (\mu + t - \overline{p})(\Delta(hv) - \Delta(v)) \tag{15}$$
$$= \Delta(hv)(\mu - \overline{p}) + t(\Delta(hv) - \Delta(v)). \tag{16}$$

Next, consider the impact of a change in messages on profits at the bottom of the price distribution. If $\underline{p} < \mu - t$, then

$$\Delta(\pi(\underline{p}, m_j)) = \overline{p}\Delta(v)/2. \tag{17}$$

If instead $\underline{p} \geq \mu - t$, then

$$\frac{2t}{\underline{p}}\Delta(\pi(\underline{p}, m_j)) = \Delta(v)(\mu - \underline{p}) + (\mu - t - \underline{p})(\Delta(hv) - \Delta(v)) \tag{18}$$
$$= \Delta(hv)(\mu - \underline{p}) + t(\Delta(v) - \Delta(hv)). \tag{19}$$

Suppose that enforceable price comparability does not hold. If adjusting messages leads to a decrease in v, then by (14) and (15), there is a profitable deviation. If adjusting messages leads to an increase in v, then by (17) and (18), there is a profitable deviation. Suppose instead that enforceable design comparability does not hold and $v \geq V(h)$. If adjusting messages leads to an increase in hv, then by (16), there is a profitable deviation. If adjusting messages leads to a decrease in hv, then by (19), there is a profitable deviation. □

Proof of Proposition 2

Proof. Given h, suppose that $v \geq V(h)$. The condition (4) leads to (8). Let $\hat{p}$ be a solution to (7). First, consider the case $\hat{p} \leq \overline{u}$. In this case, $\overline{p} = \hat{p}$, (7) reduces to (9), and (8) simplifies to (10). By (9), taking rival pricing strategies as given, observe that an increase in v leads to a decrease in $\overline{p}$. By (10), the price distribution shifts down, and $\underline{p}$ also decreases. Using (5), we obtain $\underline{p} = A(\overline{p} - \underline{p})^2$, and it follows that $\overline{p} - \underline{p}$ also decreases. It follows that $\overline{p} - \underline{p} \leq t$ for any $v \geq V(h)$.

Next, consider the case $\hat{p} > \overline{u}$. In this case, $\overline{p} = \overline{u}$. Using (8) and (5), it follows that $\underline{p}$ is increasing in $\overline{u}$. Therefore, if $\overline{p}$ is constrained by $\overline{u}$, price dispersion will be lower than in the unconstrained case, and it again follows that $\overline{p} - \underline{p} \leq t$ for any $v \geq V(h)$. □

Proof of Proposition 3

Proof. Suppose that the comparability structure is compartmentalised and satisfies enforceable vertical comparability. Adopt the perspective of Firm 1 and suppose that Firm 2 plays $(\lambda_2, \{F_2^m\}_{m\in M})$, where λ_2 can be split into $\lambda_2 = (\lambda_{2v}, \lambda_{2h})$. Let $s(p, m_h)$ be Firm 1's maximum market share when setting price p and choosing horizontal message m_h:

$$s(p,m_h) = \max_{m_v \in M_v} \left(\frac{1 - v(m_v, \lambda_v)}{2} + \int_{m_2} \lambda(m_2) h(m_h, m_{2h}) v(m_v, m_{2v}) \frac{\mu_{m_2} - p + t}{2t} dm_2 \right.$$
$$\left. + \int_{m_2} \lambda(m_2)(1 - h(m_h, m_{2h})) v(m_v, m_{2v})(1 - F_2^{m_2}(p)) dm_2 \right),$$

where μ_{m_2} is the equilibrium mean price, conditional on message m_2. Using vertical message strategy λ_v^* ensures Firm 1 a market share of

$$s_v^*(p, m_h) = \frac{1 - v^*}{2} + v^* \int_{m_{2h}} \lambda(m_{2h}) h(m_h, m_{2h}) \frac{\mu_{m_{2h}} - p + t}{2t} dm_{2h}$$
$$+ v^* \int_{m_{2h}} \lambda(m_{2h})(1 - h(m_h, m_{2h}))(1 - F_2^{m_{2h}}(p)) dm_{2h}$$

when setting price p and horizontal message m_h, and an ex ante market share of

$$\int_{m_h}\int_{\underline{p}}^{\overline{p}} \lambda(m_h) s_v^*(p, m_h) dF(p|m_h) dm_h$$
$$= \frac{1 - v^* + h^* v^*}{2} + v^* \int_{m_h}\int_{m_{2h}} \lambda(m_h)\lambda(m_{2h})(1 - h(m_h, m_{2h})) \int_{\underline{p}}^{\overline{p}} (1 - F_2^{m_{2h}}(p)) dF(p|m_h) dm_{2h} dm_h$$
$$= \frac{1 - v^* + h^* v^*}{2} + v^* \int_{m_h}\int_{m_{2h}} \frac{\lambda(m_h)\lambda(m_{2h})(1 - h(m_h, m_{2h}))}{2} dm_{2h} dm_h = \frac{1}{2},$$

where

$$h^* = \int_{m_h}\int_{m_{2h}} \lambda(m_h)\lambda(m_{2h}) h(m_h, m_{2h}) dm_{2h} dm_h$$

is the equilibrium expected horizontal comparability. In a symmetric equilibrium, each firm can do no better than a market share of one half. Thus, λ_v^* is an equilibrium vertical message strategy.

To see that λ_v^* is the unique symmetric equilibrium vertical message strategy, suppose otherwise that Firm 1 does not play λ_v^*. In particular, suppose there exists p and m_h such that $s(p, m_h) > s_v^*(p, m_h)$. But, then there must exist p' and m_h' such that $s(p', m_h') < s^*(p', m_h')$. This implies that Firm 1 could employ λ_v^* when playing p' and m_h' and increase $s(p', m_h')$. This is a profitable deviation, a contradiction. This establishes Item 1 for vertical message strategy λ_v^*.

An equivalent argument establishes Item 1 for horizontal message strategy λ_h^*, and Item 2a. Items 2b and 2c follow from the analysis of Section 3. □

Proof of Proposition 4

Proof. 1. If $\alpha_2 > \alpha_1$, the comparability structure satistfies Enforceable Comparability, and with message vector $\mathbf{m} = (\lambda^*, \lambda^*)$, neither firm is able to unilaterally influence comparability. Both firms operate on their pricing reaction function, leading to an equilibrium in pure pricing strategies.

2. Suppose instead that $\alpha_1 > \alpha_2$, and suppose Firm 2 plays the following strategy: with probability λ, play message a and price distribution F_a, and with probability $1 - \lambda$, play message b and price distribution F_b. Then, by playing a and p_a, Firm 1 earns profits of

$$\pi(p_a, a) = p_a \left(\lambda\alpha_0 \frac{\mu_a - p_a}{2t} + (1 - \lambda)\alpha_1 \frac{\mu_b - p_a}{2t} + \frac{1}{2} \right),$$

where μ_a and μ_b are the means of the distributions F_a and F_b, respectively. The first order conditions for p_a lead to the reaction function

$$p_a = \frac{\lambda\alpha_0\mu_a + (1 - \lambda)\alpha_1\mu_b + t}{2(\lambda\alpha_0 + (1 - \lambda)\alpha_1)}.$$

If instead Firm 1 plays b and p_b, this leads to profits

$$\pi(p_b, b) = p_b\left(\lambda\alpha_1\frac{\mu_a - p_{1b}}{2t} + (1-\lambda)\alpha_2\frac{\mu_b - p_{1b}}{2t} + \frac{1}{2}\right).$$

The first order conditions for p_b lead to the reaction function

$$p_b = \frac{\lambda\alpha_1\mu_a + (1-\lambda)\alpha_2\mu_b + t}{2(\lambda\alpha_1 + (1-\lambda)\alpha_2)}.$$

Equilibrium in mixed strategies requires $\pi_1(a, p_a) = \pi_1(b, p_b)$, and in the symmetric equilibrium, $p_a = \mu_a$ and $p_b = \mu_b$. Imposing these conditions and employing the above reaction functions leads to the desired result.

□

Proof of Proposition 5

Proof. 1. Suppose $\beta_2 > \beta_1$. By choosing message λ^*, each firm can fix comparability at h^*. The result then follows from Proposition 2.

2. Suppose instead $\beta_1 > \beta_2$. Let the symmetric equilibrium strategies be described by (F_a, F_b, λ), where λ indicates the probability of message a, and F_m indicates the price distribution conditional on message m. Then, if Firm 2 plays according to (F_a, F_b, λ), and Firm 1 uses message m and price p, then her profits are:

$$\begin{aligned}\pi(p,m) = p\lambda\int_{\underline{p}}^{\overline{p}} h(m,a)s_1(p,p_2) + (1-h(m,a))q_1(p,p_2)dF_a(p_2)\\ + p(1-\lambda)\int_{\underline{p}}^{\overline{p}} h(m,b)s_1(p,p_2) + (1-h(m,b))q_1(p,p_2)dF_b(p_2).\end{aligned}$$

For each message, profits simplify to:

$$\begin{aligned}\pi(p,a) &= p\lambda\left(\beta_0 s_1(p,\mu_a) + (1-\beta_0)(1-F_a(p))\right) + p(1-\lambda)\left(\beta_1 s_1(p,\mu_b) + (1-\beta_1)(1-F_b(p))\right),\\ \pi(p,b) &= p\lambda\left(\beta_1 s_1(p,\mu_a) + (1-\beta_1)(1-F_a(p))\right) + p(1-\lambda)\left(\beta_2 s_1(p,\mu_b) + (1-\beta_2)(1-F_b(p))\right).\end{aligned}$$

Define the function $J(p) = \pi(p,a) - \pi(p,b)$ such that

$$J(p)/p = \lambda(\beta_0 - \beta_1)\left(s_1(p,\mu_a) - (1-F_a(p))\right) + (1-\lambda)(\beta_1-\beta_2)\left(s_1(p,\mu_b) - (1-F_b(p))\right). \quad (20)$$

Let the equilibrium profit level be π.

(a) Firms use message a when setting price $\overline{p}$ and message b when setting price $\underline{p}$. Observe that $J(\overline{p}) > 0 > J(\underline{p})$, and the result follows directly. Further, the price $\overline{p}$ is determined by (7) with message a.

(b) There exists $\hat{p}$ such that $J(\hat{p}) = 0$. By Lemma 1, there can be no mass points in F_a or F_b. By (20), $J(p)$ is continuous in p, and the result follows.

(c) $\hat{p}$ is the unique solution to $J(p) = 0$. Suppose otherwise that there exists $\tilde{p} > \hat{p}$ such that $J(\tilde{p}) = 0$. The same argument will apply for $\tilde{p} < \hat{p}$. Setting $J(p) = 0$ implies that

$$\gamma(1 - F_a(p)) + (1-\gamma)(1 - F_b(p)) = \gamma s_1(p, \mu_a) + (1-\gamma) s_1(p, \mu_b), \tag{21}$$

where $\gamma = \frac{\lambda(\beta_0 - \beta_1)}{\lambda(\beta_0 - \beta_1) + (1-\lambda)(\beta_1 - \beta_2)}$. Observe that the right hand side is linearly decreasing in p. By Lemma 1, the support of the price distribution is connected. Then, either both messages, just a messages, or just b messages are employed in the interval $[\hat{p}, \tilde{p}]$.

Suppose first that both a and b messages are used in the interval. Therefore $\pi(p, a) = \pi(p, b) = \pi$ for $p \in [\hat{p}, \tilde{p}]$. This implies

$$\lambda(1-\beta_0)(1 - F_a(p)) + (1-\lambda)(1-\beta_1)(1 - F_b(p)) = \lambda\beta_0 s_1(p, \mu_a) + (1-\lambda)\beta_1 s_1(p, \mu_b) + \pi/p,$$
$$\lambda(1-\beta_1)(1 - F_a(p)) + (1-\lambda)(1-\beta_2)(1 - F_b(p)) = \lambda\beta_1 s_1(p, \mu_a) + (1-\lambda)\beta_2 s_1(p, \mu_b) + \pi/p.$$

In each expression, the right hand side is convex in p. It follows that both F_a and F_b must be concave functions of p in the interval, a contradiction.

Suppose instead that a messages are used in the interval. Then, $F_b(p)$ is fixed on the interval, and $\pi(p, a) = \pi$ implies that F_a is concave, a contradiction. Similarly, if we suppose that b messages are used in the interval, this also leads to a contradiction.

(d) There are no profitable deviations. By items 2a and 2c, $\pi(p, b) < \pi(p, a)$ for $p > \hat{p}$ and $\pi(p, a) < \pi(p, b)$ for $p < \hat{p}$. With $\overline{p}$ determined by (7), $\pi(p, a) < \pi$ for $p > \overline{p}$. Finally, $\pi(p, b) < \pi(\underline{p}, b)$ for $p < \underline{p}$.

□

Proof of Proposition 6

Proof. 1. If $\alpha_2 > \alpha_1$ and $\beta_2 > \beta_1$, then the comparability structure satisfies Enforceable Comparability. The result then follows from Propositions 2 and 3.

2. Suppose instead $\alpha_2 > \alpha_1$ and $\beta_1 > \beta_2$. The comparability structure satisfies enforceable price comparability, and part (a) follows directly from Proposition 3.

   Consider part (b). Let the symmetric equilibrium strategies be described by $(F_{a_h}, F_{b_h}, \lambda_h, \lambda_{v^*})$, where λ_{v^*} indicates the probability of price message a_v, λ_h indicates the probability of design message a_h, and F_{m_h} indicates the price distribution conditional on design message m_h. Then, if Firm 2 plays according to $(F_{a_h}, F_{b_h}, \lambda_h, \lambda_{v^*})$, and Firm 1 uses design message m_h and price p, then her profits are:

$$\pi(p, m_h) = p\frac{1-v^*}{2} + pv^*\lambda_h \int_{\underline{p}}^{\overline{p}} h(m_h, a_h)s_1(p, p_2) + (1-h(m_h, a_h))q_1(p, p_2)dF_{a_h}(p_2)$$
$$+ pv^*(1-\lambda_h)\int_{\underline{p}}^{\overline{p}} h(m_h, b_h)s_1(p, p_2) + (1-h(m_h, b_h))q_1(p, p_2)dF_{b_h}(p_2).$$

For each message, profits simplify to:

$$\begin{aligned}\pi(p, a_h) &= p\frac{1-v^*}{2} + pv^*\lambda_h\left(\beta_0 s_1(p, \mu_{a_h}) + (1-\beta_0)(1-F_{a_h}(p))\right)\\ &\quad + pv^*(1-\lambda_h)\left(\beta_1 s_1(p, \mu_{b_h}) + (1-\beta_1)(1-F_{b_h}(p))\right),\\ \pi(p, b_h) &= p\frac{1-v^*}{2} + pv^*\lambda_h\left(\beta_1 s_1(p, \mu_{a_h}) + (1-\beta_1)(1-F_{a_h}(p))\right)\\ &\quad + pv^*(1-\lambda_h)\left(\beta_2 s_1(p, \mu_{b_h}) + (1-\beta_2)(1-F_{b_h}(p))\right).\end{aligned}$$

Define the function $J(p) = \pi(p, a_h) - \pi(p, b_h)$ such that

$$\frac{J(p)}{v^* p} = \lambda_h(\beta_0 - \beta_1)\left(s_1(p, \mu_{a_h}) - (1-F_{a_h}(p))\right) + (1-\lambda_h)(\beta_1-\beta_2)\left(s_1(p, \mu_{b_h}) - (1-F_{b_h}(p))\right).$$

The remainder of the proof develops in the same fashion as Proposition 5.

□

Proof of Proposition 7

Proof. If $\beta_2 > \beta_1$, then the comparability structure satisfies enforceable design comparability. By Proposition 3, each firm plays horizontal message strategy λ_h^*, leading to design comparability h^*. Then, if Firm 2 plays according to (F_a, F_b, λ_v), and Firm 1 uses design message m_v and price p, then her profits are:

$$\pi(p, m_v) = p\lambda_v \int_{\underline{p}}^{\overline{p}} h^* v(m_v, a_v)s_1(p, p_2) + (1-h^*)v(m_v, a_v))q_1(p, p_2) + \frac{1-v(m_v, a_v)}{2}dF_{a_v}(p_2)$$
$$+ p(1-\lambda_v)\int_{\underline{p}}^{\overline{p}} h^* v(m_v, b_v)s_1(p, p_2) + (1-h^*)v(m_v, b_v))q_1(p, p_2) + \frac{1-v(m_v, b_v)}{2}dF_{b_v}(p_2).$$

For each message, profits simplify to:

$$\begin{aligned}\pi(p, a_v) &= p\lambda_v\alpha_0\left(h^* s_1(p, \mu_{a_v}) + (1-h^*)(1-F_{a_v}(p))\right) + p\lambda_v\frac{1-\alpha_0}{2}\\ &\quad + p(1-\lambda_v)\alpha_1\left(h^* s_1(p, \mu_{b_v}) + (1-h^*)(1-F_{b_v}(p))\right) + p(1-\lambda_v)\frac{1-\alpha_1}{2},\\ \pi(p, b_v) &= p\lambda_v\alpha_1\left(h^* s_1(p, \mu_{a_v}) + (1-h^*)(1-F_{a_v}(p))\right) + p\lambda_v\frac{1-\alpha_1}{2}\\ &\quad + p(1-\lambda_v)\alpha_2\left(h^* s_1(p, \mu_{b_v}) + (1-h^*)(1-F_{b_v}(p))\right) + p(1-\lambda_v)\frac{1-\alpha_2}{2}.\end{aligned}$$

Define the function $J(p) = (\pi(p, a_v) - \pi(p, b_v))/p$:

$$J(p) = \lambda_v(\alpha_0 - \alpha_1)\left(h^* s_1(p, \mu_{a_v}) + (1-h^*)(1 - F_{a_v}(p)) - 0.5\right) \\ + (1-\lambda_v)(\alpha_1 - \alpha_2)\left(h^* s_1(p, \mu_{b_v}) + (1-h^*)(1 - F_{b_v}(p)) - 0.5\right).$$

1. Firms use message b_v at the top of the price distribution and message a_v at the bottom.

   This can be seen by evaluating $J(p)$ at $\overline{p}$ and $\underline{p}$:

$$J(\overline{p}) = \lambda_v(\alpha_0 - \alpha_1)\left(h^* s_1(\overline{p}, \mu_{a_v}) - 0.5\right) + (1-\lambda_v)(\alpha_1 - \alpha_2)\left(h^* s_1(\overline{p}, \mu_{b_v}) - 0.5\right) < 0,$$
$$J(\underline{p}) = \lambda_v(\alpha_0 - \alpha_1)\left(h^*\left(s_1(\underline{p}, \mu_{a_v}) - 1\right) + 0.5\right) + (1-\lambda_v)(\alpha_1 - \alpha_2)\left(h^*\left(s_1(\underline{p}, \mu_{b_v}) - 1\right) + 0.5\right) > 0.$$

2. $J(p)$ is strictly decreasing in p:

$$J'(p) = -\lambda_v(\alpha_0 - \alpha_1)\left(\frac{h^*}{2t} + (1-h^*) f_{a_v}(p)\right) - (1-\lambda_v)(\alpha_1 - \alpha_2)\left(\frac{h^*}{2t} + (1-h^*) f_{b_v}(p)\right) < 0.$$

It follows directly that there is a unique price $\hat{p}$ such that firms employ price message a_v below $\hat{p}$ and message b_v above $\hat{p}$. □

Proof of Proposition 8

Proof. If Firm 1 sets price p and chooses messages (m_h, m_v), her expected profits are:

$$\pi(p, m_h, m_v) = \sum_{m_{2h} \in M_h} \sum_{m_{2v} \in M_v} \lambda(m_{2h}, m_{2v}) \pi(p, m_h, m_v | m_{2h}, m_{2v}),$$

where $\lambda(m_{2h}, m_{2v})$ is the probability that Firm 2 chooses design message m_{2h} and price message m_{2v}.

Define the functions

$$J_v(p, m_h) = \pi(p, m_h, b_v) - \pi(p, m_h, a_v),$$
$$J_h(p, m_v) = \pi(p, b_h, m_v) - \pi(p, a_h, m_v).$$

1. First, consider pricing strategies, conditional on the design message m_h.

   (a) Observe that $J_v(\overline{p}, m_h) > 0 > J_v(\underline{p}, m_h)$ for any m_h. It follows that firms prefer price message b_v when setting price $\overline{p}$ and price message a_v when setting price $\underline{p}$.

   (b) Next, observe that, for any m_h,

$$\frac{d}{dp}\left(J_v(p, m_h)/p\right) > 0.$$

   It follows directly that there is a unique price $\hat{p}(m_h)$ such that firms employ price message a_v below $\hat{p}(m_h)$ and message b_v above $\hat{p}(m_h)$.

2. Next, consider pricing strategies, conditional on the price message m_v.

   (a) Observe that $J_h(\overline{p}, m_v) < 0 < J_h(\underline{p}, m_v)$ for any m_v. It follows that firms prefer design message a_h when setting price $\overline{p}$ and design message b_h when setting price $\underline{p}$.

   Steps (b)-(d) mirror those in part 2 of Proposition 5.

□

Proof of Proposition 9

Proof. If Firm 2 plays according to (F_a, F_b, λ), and Firm 1 uses message m and price p, then her expected profits are:

$$\pi(p,m) = p\lambda \int_{\underline{p}}^{\overline{p}} h(m,a)v(m,a)s_1(p,p_2) + (1-h(m,a))v(m,a))q_1(p,p_2) + \frac{1-v(m,a)}{2} dF_a(p_2)$$
$$+ p(1-\lambda)\int_{\underline{p}}^{\overline{p}} h(m,b)v(m,b)s_1(p,p_2) + (1-h(m,b))v(m,b))q_1(p,p_2) + \frac{1-v(m,b)}{2} dF_b(p_2).$$

For each message, profits simplify to:

$$\begin{aligned}
\pi(p,a) &= p\lambda\left(\beta_0\alpha_0 s_1(p,\mu_a) + (1-\beta_0)\alpha_0(1-F_a(p)) + \frac{1-\alpha_0}{2}\right)\\
&\quad + p(1-\lambda)\left(\beta_1\alpha_1 s_1(p,\mu_b) + (1-\beta_1)\alpha_1(1-F_b(p)) + \frac{1-\alpha_1}{2}\right),\\
\pi(p,b) &= p\lambda\left(\beta_1\alpha_1 s_1(p,\mu_a) + (1-\beta_1)\alpha_1(1-F_a(p)) + \frac{1-\alpha_1}{2}\right)\\
&\quad + p(1-\lambda)\left(\beta_2\alpha_2 s_1(p,\mu_b) + (1-\beta_2)\alpha_2(1-F_b(p)) + \frac{1-\alpha_2}{2}\right).
\end{aligned}$$

Define the function $J(p) = (\pi(p,a) - \pi(p,b))/p$:

$$\begin{aligned}
J(p) &= \lambda\left((\beta_0\alpha_0 - \beta_1\alpha_1)s_1(p,\mu_a) + ((1-\beta_0)\alpha_0 - (1-\beta_1)\alpha_1)(1-F_a(p)) - \frac{\alpha_0-\alpha_1}{2}\right)\\
&+ (1-\lambda)\left((\beta_1\alpha_1 - \beta_2\alpha_2)s_1(p,\mu_b) + ((1-\beta_1)\alpha_1 - (1-\beta_2)\alpha_2)(1-F_b(p)) - \frac{\alpha_1-\alpha_2}{2}\right).
\end{aligned}$$

Evaluating at the pricing limits $\overline{p}$ and $\underline{p}$ gives

$$\begin{aligned}
J(\overline{p}) &= \lambda\left(\alpha_1(0.5-\beta_1 s_1(\overline{p},\mu_a)) - \alpha_0(0.5-\beta_0 s_1(\overline{p},\mu_a))\right)\\
&\quad + (1-\lambda)\left(\alpha_2(0.5-\beta_2 s_1(\overline{p},\mu_b)) - \alpha_1(0.5-\beta_1 s_1(\overline{p},\mu_b))\right),\\
J(\underline{p}) &= \lambda\left(\alpha_0(0.5-\beta_0(1-s_1(\underline{p},\mu_a))) - \alpha_1(0.5-\beta_1(1-s_1(\underline{p},\mu_a)))\right)\\
&\quad + (1-\lambda)\left(\alpha_1(0.5-\beta_1(1-s_1(\overline{p},\mu_b))) - \alpha_2(0.5-\beta_2(1-s_1(\overline{p},\mu_b)))\right).
\end{aligned}$$

Observe that both $J(\overline{p})$ and $J(\underline{p})$ are continuous in all parameters.

1. Observe that $J(\overline{p}) > 0$ and $J(\underline{p}) < 0$ in the limit as α_1 and α_2 approach α_0.

2. Observe that $J(\overline{p}) < 0$ and $J(\underline{p}) > 0$ in the limit as either α_1 and α_2 approach 0 or β_1 and β_2 approach β_0.

□

References

Simon P. Anderson and Régis Renault. Comparative advertising: Disclosing horizontal match information. *The RAND Journal of Economics*, 40(3):558–581, 2009.

Mark Armstrong. Interactions between competition and consumer policy. *Competition Policy International*, 4:97–147, 2008.

Mark Armstrong and Jidong Zhou. Consumer information and the limits to competition. *American Economic Review*, 112(2):534–77, February 2022.

Roland Bénabou and Jean Tirole. Bonus culture: Competitive pay, screening, and multitasking. *Journal of Political Economy*, 124:325–270, 2016.

David P. Byrne and Nicolas de Roos. Consumer search in retail gasoline markets. *The Journal of Industrial Economics*, 65(1):183–193, 2017.

David P Byrne, Leslie A Martin, and Jia Sheen Nah. Price Discrimination by Negotiation: a Field Experiment in Retail Electricity*. *The Quarterly Journal of Economics*, 137(4):2499–2537, 04 2022.

Ioana Chioveanu and Jidong Zhou. Price competition with consumer confusion. *Management Science*, 59:2450–2469, 2013.

Nicolas de Roos. Collusion with limited product comparability. *RAND Journal of Economics*, 49: 481–503, 2018.

Nicolas de Roos and Vladimir Smirnov. Collusion with intertemporal price dispersion. *RAND Journal of Economics*, 51:158–188, 2020.

Glenn Ellison. A model of add-on pricing. *Quarterly Journal of Economics*, 120:585–637, 2005.

Christos Genakos, Tobias Kretschmer, and Ambre Nicolle. Strategic confusopoly: Evidence from the uk mobile market. Working paper, 2023.

Michael D. Grubb. Failing to choose the best price: Theory, evidence, and policy. *Review of Industrial Organization*, 47:303–340, 2015a.

Michael D. Grubb. Consumer inattention and bill-shock regulation. *Review of Economic Studies*, 82:219–257, 2015b.

Andreas Hefti, Shuo Liu, and Armin Schmutzler. Preferences, Confusion and Competition. *The Economic Journal*, 132(645):1852–1881, 01 2022.

Paul Heidhues and Botond Kőszegi. Behavioral industrial organization. In B. Douglas Bernheim, Stefano DellaVigna, and David Laibson, editors, *Handbook of Behavioral Economics - Foundations and Applications 1*, volume 1, pages 517–612. Elsevier, 1st edition, 2018.

Ilwoo Hwang, Kyungmin Kim, and Raphael Boleslavsky. Competitive advertising and pricing. Unpublished, 2019.

Maxim Ivanov. Information revelation in competitive markets. *Economic Theory*, 52:337–365, 2013.

Edward Kasabov. What we know, don't know, and should know about confusion marketing. *European Journal of Marketing*, 49:1777–1808, 2015.

Vincent-Wayne Mitchell and Vassilios Papavassiliou. Marketing causes and implications of consumer confusion. *Journal of Product & Brand Management*, 8:319–342, 1999.

Michele Piccione and Ran Spiegler. Price competition under limited comparability. *Quarterly Journal of Economics*, 127:97–135, 2012.

Ran Spiegler. Choice complexity and market competition. *Annual Review of Economics*, 8:1–25, 2016.

Hal Varian. A model of sales. *American Economic Review*, 70:651–659, 1980.

Gianfranco Walsh, Thorsten Hennig-Thurau, and Vincent-Wayne Mitchell. Consumer confusion proneness: Scale development, validation, and application. *Journal of Marketing Management*, 23(7-8):697–721, 2007.

Online Appendix

B Message commitment

In this section, we analyse a two-stage game in which firms first simultaneously choose a message, and then simultaneously choose a price. We solved the pricing game for a given message vector in Section 3. Firms simultaneously choose messages, based on an understanding of the price competition that will follow, as described in Section 3. Throughout, we employ the frame competition framework of Chioveanu and Zhou (2013), with finite message space M.

In the following sections, we consider limited price comparability and perfect design comparability (Section B.1), limited design comparability and perfect price comparability (Section B.2), and limited comparability in both dimensions (Section B.3).

B.1 Limited price comparability

Suppose first that design comparability is perfect and price comparability is limited. Let the set of messages be binary, $M = \{a, b\}$. Message a is considered "simple" and message b "complex". As in Section 4.2, price comparability is summarised by $v(\mathbf{m}) = v_0(\mathbf{m}; \alpha)$, as specified in the table below.

Price comparability, $v_0(\mathbf{m}; \alpha)$

	a	b
a	α_0	α_1
b	α_1	α_2

Let λ be the probability of message a in a symmetric mixed message Nash equilibrium. Define $\lambda^*(\alpha)$ as the solution to $v(a, \lambda) = v(b, \lambda)$, and $v^*(\alpha)$ as the associated comparability, as specified in (12). By playing λ^*, a firm is able to unilaterally enforce price comparability v^*.

Proposition 10. *Suppose design comparability is perfect.*

1. *If $\alpha_2 > \alpha_1$, there are three message equilibria: $\mathbf{m} = (\lambda^*, \lambda^*)$, $\mathbf{m} = (a, b)$, and $\mathbf{m} = (b, a)$.*
2. *If $\alpha_1 > \alpha_2$, there is a message equilibrium with $\mathbf{m} = (b, b)$. If $\alpha_1 < \alpha_0$, it is unique. If $\alpha_1 = \alpha_0$, $\mathbf{m} = (a, a)$ is also a message equilibrium.*
3. *In equilibrium, $p_1 = p_2 = p^*(\mathbf{m})$ as described by (2).*

Proof. Rewrite the profit function for Firm j as

$$\pi_j(\mathbf{p}, \mathbf{m}) = p_j \left(\frac{p_{-j} - p_j}{2t} v + \frac{1}{2} \right),$$

define the pricing reaction function for firm j

$$r_j(p_{-j}) = \frac{p_{-j}}{2} + \frac{t}{2v}.$$

Equilibrium prices must satisfy $p_1 = p_2 = p^*(\mathbf{m}) = t/v$, and profits are given by $\pi(\mathbf{m}) = t/(2v)$. Profits are monotonically decreasing in v, leading directly to the message equilibria. □

If $\alpha_2 > \alpha_1$, the comparability structure satisfies Enforceable Comparability, and there exists an equilibrium in mixed message strategies with vertical comparability given by v^*. In all but the pathological equilibrium in which $\mathbf{m} = (a, a)$, prices and profits are strictly decreasing in the vertical comparability parameters (α_1 and/or α_2).

B.2 Limited design comparability

Suppose instead that price comparability is perfect, and the set of messages is binary, $M = \{a, b\}$. As in Section 4.3, design comparability is summarised by $h(\mathbf{m}) = h_0(\mathbf{m}; \beta)$, depicted below.

Design comparability, $h_0(\mathbf{m}; \beta)$

	a	b
a	β_0	β_1
b	β_1	β_2

Define $\lambda^*(\beta)$ as the solution to $h(a, \lambda) = h(b, \lambda)$, and $h^*(\beta)$ as the associated comparability, as specified in (13). By playing λ^*, a firm is able to unilaterally enforce price comparability h^*.

Proposition 11. *Suppose price comparability is perfect.*

1. *If $\beta_2 > \beta_1$, there are three message equilibria: $\mathbf{m} = (\lambda^*, \lambda^*)$, $\mathbf{m} = (a, a)$, and $\mathbf{m} = (b, b)$.*
2. *If $\beta_1 > \beta_2$, there is a unique message equilibrium with $\mathbf{m} = (a, a)$.*
3. *The symmetric equilibrium price distribution is given by $F(p; \theta(\mathbf{m}))$ as specified in (10).*

Proof. It follows from Proposition 2 that profits are monotonically increasing in design comparability. The message equilibria follow directly. □

B.3 Limited design and price comparability

Suppose that both price and design comparability are limited. We consider two cases. First, we allow each firm to compartmentalise messages to separately influence price and design comparability. Second, we consider the case where messages related to design and price elements cannot be disentangled.

Compartmentalised messages

Suppose that messages are compartmentalised, and that the set of messages is given by $M = \{a_v, b_v\} \times \{a_h, b_h\}$, where the messages with v subscripts influence price comparability, and those with h subscripts influence design comparability. For message vector $\mathbf{m} \in M \times M$, let $\mathbf{m}_v$ describe the price component of the message vector and $\mathbf{m}_h$ the design component. Comparability is summarised below.

Price comparability, $v_0(\mathbf{m};\alpha)$

	a_v	b_v
a_v	α_0	α_1
b_v	α_1	α_2

Design comparability, $h_0(\mathbf{m};\beta)$

	a_h	b_h
a_h	β_0	β_1
b_h	β_1	β_2

Proposition 12. *Suppose messages are compartmentalised. In the equilibrium message vector, $\mathbf{m}_v$ is described by Proposition 10, and $\mathbf{m}_h$ is described by Proposition 11. The symmetric equilibrium price distribution is given by $F(p;\mathbf{m})$, as specified in (8).*

Proof. Profits are decreasing in price comparability and increasing in design comparability. Equilibrium messages follow directly. □

Entangled messages

Suppose instead that messages are entangled and each firm chooses a single message from $M = \{a, b\}$. The firm's message influences both price and design comparability, as summarised below.

Price comparability, $v_0(\mathbf{m};\alpha)$

	a	b
a	α_0	α_1
b	α_1	α_2

Design comparability, $h_0(\mathbf{m};\beta)$

	a	b
a	β_0	β_1
b	β_1	β_2

Let $\Pi(v,h)$ indicate profits as a function of price and design comparability, such that $\pi^*(\mathbf{m}) = \Pi(v(\mathbf{m}), h(\mathbf{m}))$. Define λ^* as the solution to $\pi^*(a,\lambda) = \pi^*(b,\lambda)$, where λ indicates the probability of message a:

$$\lambda^* = \frac{\Pi(\alpha_2,\beta_2) - \Pi(\alpha_1,\beta_1)}{\Pi(\alpha_2,\beta_2) + \Pi(\alpha_0,\beta_0) - 2\Pi(\alpha_1,\beta_1)}.$$

Proposition 13. *Suppose messages are entangled.*

1. *If $\lambda^* \in (0,1)$, there is an equilibrium in mixed messages with $\mathbf{m} = (\lambda^*, \lambda^*)$.*
2. *If $\Pi_1(\alpha_1,\beta_1) \le \Pi_1(\alpha_0,\beta_0)$, there is an equilibrium with $\mathbf{m} = (a,a)$.*
3. *If $\Pi(\alpha_2,\beta_2) \ge \Pi(\alpha_1,\beta_1)$, there is an equilibrium with $\mathbf{m} = (b,b)$.*

4. *If* $\Pi(\alpha_1, \beta_1) \geq \max\{\Pi_1(\alpha_0, \beta_0), \Pi(\alpha_2, \beta_2)\}$*, there are two asymmetric equilibria:* $\mathbf{m} = (a, b)$ *and* $\mathbf{m} = (b, a)$.
5. *The symmetric equilibrium price distribution is given by* $F(p; \mathbf{m})$*, as specified in (8).*

Proof. Items 1-4 follow directly from the definition of $\Pi(\alpha, \beta)$ and λ^*. □

The conditions in Items 2-4 exhaust the parameter space. Thus, there exists at least one equilibrium in pure message strategies for all feasible parameter values. If the condition in Item 2 is satisfied, equilibrium with maximum transparency is possible if the profits that follow from the message vector $\mathbf{m} = (a, b)$ are less than the maximum transparency case. This condition depends on the relative sizes of the α_1 and β_1 parameters, as well as the curvature of the profit function with respect to price and design comparability, as illustrated in Figure 6. In the equilibria described in Items 3 and 4, comparative statics of prices and profits with respect to the comparability parameters depend on the relative sizes of the α and β parameters and the curvature of the profit function. Finally, in the mixed strategy equilibrium of Item 1, comparative statics depend on all of the comparability parameters as well as the curvature of the profit function with respect to price and design comparability.

C Additional results

In this section, we provide additional background results. In Section C.1 we explain the role of the outside good for the equilibrium price distribution. In Section C.2, we illustrate the role of the comparability parameters v and h for the extent of equilibrium price dispersion. In Section C.3, we solve for the equilibrium price distribution when price dispersion is substantial in the sense that $\overline{p} - \underline{p} > t$.

C.1 The role of the outside good

In this section, we discuss the impact of the outside good specification of Bénabou and Tirole (2016). We make the following assumptions and normalisations. When purchasing the outside good, a consumer located at x obtains indirect utility

$$u_0 = -t \min\{x, 1-x\}.$$

With this normalisation, the consumer purchases the outside good at the closest end of the city. In the event that the consumer does not understand her location, she must commit to an end of the city to obtain the outside good before learning her location. If she is unable to evaluate the relative prices of the inside goods, she can still evaluate the price of an inside good relative to the outside good at the time of checkout. Ties between an outside and inside good are resolved in favour of the inside good.

Lemma 4. *For all comparability environments:*

1. *if $p_{-j} \leq \overline{u}$, then $p_j \leq \overline{u}$;*
2. *if $p_1, p_2 \leq \overline{u}$, then the market is covered.*

Proof. To establish item 1, suppose $p_2 \leq \overline{u}$. We will consider each information environment in turn. First, suppose the consumer is perfectly informed about both her location and relative prices. Suppose that $p_1 > \overline{u}$. All consumers with locations $x < 0.5$ purchase the outside good from the same location as Firm 1, and therefore these consumers strictly prefer the outside good. Let the location of the consumer who is indifferent between j and $-j$ be $x^* = \frac{p_{-j} - p_j + t}{2t} < \frac{1}{2}$. Therefore, all consumers strictly prefer either Firm 2 or the outside good to Firm 1, and Firm 1 has an incentive to set $p_1 \leq \overline{u}$.

Next, suppose the consumer is perfectly informed about relative prices, but uninformed about her location. Her expected value of product 1 is $\mathbb{E}_x(\overline{u} - p_1 - t|y_1 - x|) = \overline{u} - p_1 - t/2$. Her expected value of the outside good is $-t\min\{\mathbb{E}_x(x), \mathbb{E}_x(1-x)\} = -t/2$. Therefore, Firm 1 must set $p_1 \leq \overline{u}$ to make sales.

Next, suppose the consumer understands her location, but cannot compare prices. Because she is unable to compare prices, she prefers Firm 1 to 2 only if $x \leq 0.5$. Suppose $x \leq 0.5$. Then, the consumer prefers Firm 1 to the outside good only if $p_1 \leq \overline{u}$.

Finally, suppose the consumer is uninformed about both her location and relative prices. Because she does not know her location, she must pay an expected travel cost of $t/2$ when purchasing either product 1 or the outside good. She evaluates p_1 at checkout and therefore buys product 1 only if $p_1 \leq \overline{u}$. Thus, no matter the information environment, Firm 1 must set price $p_1 \leq \overline{u}$ in order to make sales.

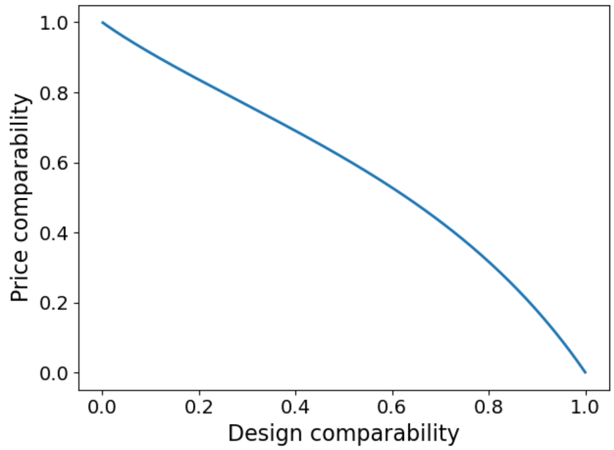
To establish item 2 it follows directly that, for all comparison environments, if both firms set price less than $\overline{u}$, at least one of the two products will be purchased. □

The immediate implication of Lemma 4 is that $\overline{u}$ places an upper bound on the upper end of the price distribution, $\overline{p}$, as specified in (6).

C.2 The price dispersion boundary

According to Proposition 2, if $v \geq V(h)$, then price dispersion is moderate in the sense that $\overline{p} - \underline{p} \leq t$, and the solution for the equilibrium price distribution is simplified. Figure 13 illustrates the price dispersion boundary $\overline{p} - \underline{p} = t$. In particular, the figure illustrates combinations of h and v for which solution of the system (4), (5) and (7) yields an equilibrium price distribution that satisfies $\overline{p} - \underline{p} = t$, $\{h, V(h) : h \in [0,1]\}$. The horizontal axis indexes design comparability, and the vertical axis indexes price comparability. Above and to the right of the line, price dispersion is "moderate", and below and to the left of the line, price dispersion is "substantial". Price

Figure 13: The boundary condition, $\overline{p} - \underline{p} = t$



dispersion is accentuated when there are strong incentives for both undercutting and raising prices. From (3), for low values of h, there is a strong incentive to marginally undercut one's rival's price to attract consumers who are uninformed about their location but informed about relative prices. For low values of v, there is a strong incentive to raise price to exploit consumers who are unable to compare prices. When both h and v are low, there are incentives to both undercut and raise prices, leading to substantial price dispersion.

For combinations of h and v above the line, the equilibrium price distribution is given by (8). For h and v below the line, the outside good constraint $\overline{p} \leq \overline{u}$ determines the upper bound of the price distribution, and the extent of price dispersion depends on $\overline{u}$. We solve for the case of substantial price dispersion, $\overline{p} - \underline{p} > t$, in the following section.

C.3 Equilibrium prices with substantial price dispersion

In this section, we solve for the equilibrium price distribution in the case where equilibrium price dispersion is substantial, in the sense that $\overline{p} - \underline{p} \geq t$. This case applies when h and v are sufficiently low to be below the boundary illustrated in Figure 13, and the taste parameter $\overline{u}$ is sufficiently high. The equilibrium price distribution is determined by the conditions (4), (5), and the constraint $\overline{p} = \overline{u}$. For exposition and simplicity, we focus on the case $\overline{p} - \underline{p} \in [t, 2t]$. For exposition, we omit the dependence of profits and prices on the message vector. Throughout, we focus on the perspective of Firm 1, taking the strategies of Firm 2 to be well specified.

Setting $\pi(p) = \pi(\overline{p})$ for Firm 1, we have

$$1 - F(p) = \frac{\overline{p}}{p}\frac{h}{1-h}\mathbb{E}_{p_2}(\tilde{s}_1(\overline{p})) - \frac{h}{1-h}\mathbb{E}_{p_2}(\tilde{s}_1(p)) + \frac{\overline{p}-p}{p}\frac{1-v}{2(1-h)v},$$

where Firm 1's expected market share is given by

$$\mathbb{E}_{p_2}(\tilde{s}_1(p)) = \begin{cases} \frac{t-p}{2t}(F(p+t)) + \frac{1}{2t}\int_{\underline{p}}^{p+t} p_2 dF(p_2) + 1 - F(p+t), & \text{for } p \leq \overline{p} - t, \\ \frac{t-p+\mu}{2t}, & \text{for } p \in [\overline{p}-t, \underline{p}+t], \\ \frac{t-p}{2t}(1 - F(p-t)) + \frac{1}{2t}\int_{p-t}^{\overline{p}} p_2 dF(p_2), & \text{for } p \geq \underline{p} + t, \end{cases}$$

and μ is the mean of the symmetric equilibrium price distribution. This leads to an implicit solution for the cdf,

$$1 - F(p) = \begin{cases} \frac{R}{p} - A\int_{\underline{p}}^{p+t} 1 - F(p_2) dp_2 - A(\underline{p} + t - p) - \frac{1-v}{2(1-h)v}, & \text{for } p \leq \overline{p} - t, \\ \frac{R}{p} + A(p - t - \mu) - \frac{1-v}{2(1-h)v}, & \text{for } p \in [\overline{p}-t, \underline{p}+t], \\ \frac{R}{p} - A\int_{p-t}^{\overline{p}} 1 - F(p_2) dp_2 - \frac{1-v}{2(1-h)v}, & \text{for } p \geq \underline{p} + t, \end{cases}$$

where

$$R = A\overline{p}\left(\int_{\overline{p}-t}^{\overline{p}} 1 - F(p)dp + \frac{(1-v)t}{vh}\right), \qquad A = \frac{h}{2t(1-h)}.$$

Evaluating $1 - F(\overline{p} - t)$ leads to

$$R = (\overline{p} - t)\left(1 - F\left(\overline{p} - t\right) - A\left(\overline{p} - 2t - \mu\right) + \frac{1-v}{2(1-h)v}\right).$$

Differentiating the cdf, we obtain the pdf

$$f(p) = \frac{R}{p^2} - A(F(p+t) - F(p-t)).$$

Given knowledge of μ, $\underline{p}$, and $\overline{p} = \overline{u}$, solution for the case $p \in [\overline{p} - t, \underline{p} + t]$ is immediate. To solve for the remaining support of p, define

$$G(p) = F(p-t), \text{ for } p \geq \underline{p} + t,$$

so that F and f describe the price distribution for prices above $\underline{p} + t$, and G and g describe the distribution for prices below $\overline{p} - t$. For $p \geq \underline{p} + t$, we can now rewrite the system as

$$f(p) - AG(p) = Rp^{-2} - A,$$
$$g(p) + AF(p) = R(p-t)^{-2}.$$

Defining D as the difference operator, rewrite the system as

$$DF - AG = Rp^{-2} - A,$$
$$DG + AF = R(p-t)^{-2},$$

where we omit the price arguments for the distributions. Solving, we obtain

$$(D^2 + A^2)F = AR(p-t)^{-2} - 2Rp^{-3},$$
$$(D^2 + A^2)G = -ARp^{-2} - 2R(p-t)^{-3} + A^2.$$

We can now solve each differential equation separately.

First, consider the reduced system

$$(D^2 + A^2)F_0 = 0, \qquad (D^2 + A^2)G_0 = 0.$$

For constants K_1 and K_2, this has general solution for F_0:

$$F_0 = K_1 e^{Api} + K_2 e^{-Api}.$$

Using the identities

$$\cos(x) = \frac{e^{ix} + e^{-ix}}{2}, \qquad \sin(x) = \frac{e^{ix} - e^{-ix}}{2},$$

yields

$$F_0 = (K_1 + K_2)\cos(Ap) + (K_1 - K_2)\sin(Ap).$$

Substituting for G gives

$$G_0 = (K_1 - K_2)\cos(Ap) - (K_1 + K_2)\sin(Ap).$$

Define $C_1 = K_1 + K_2$ and $C_2 = K_1 - K_2$, and write the system

$$F_0 = C_1\cos(Ap) + C_2\sin(Ap),$$
$$G_0 = C_2\cos(Ap) - C_1\sin(Ap).$$

Next consider the particular solution, F_a, to

$$(D^2 + A^2)F_a = AR(p-t)^{-2} \equiv q(p).$$

Using the method of variation of parameters, $F_a(p)$ must satisfy

$$F_a(p) = F_1 v_1 + F_2 v_2, \qquad F_1(p) = \cos(Ap), \qquad F_2(p) = \sin(Ap),$$

where

$$v_1 = -\int \frac{F_2(p)q(p)}{W(p)}dp, \qquad v_2 = \int \frac{F_1(p)q(p)}{W(p)}dp,$$

and

$$W(p) = F_1(p)F_2'(p) - F_2(p)F_1'(p).$$

Solving, we obtain

$$F_a(p) = -RA\big(\mathrm{Ci}(A(p-t))\cos(A(p-t)) + \mathrm{Si}(A(p-t))\sin(A(p-t))\big),$$

where $\mathrm{Ci}(x)$ and $\mathrm{Si}(x)$ refer to the cosine and sign integrals evaluated at x, respectively.

Next consider the particular solution, F_b, to

$$(D^2 + A^2)F_b = -2Rp^{-3}.$$

Solving, we obtain

$$F_b(p) = RA\big(\mathrm{Ci}(Ap)\sin(Ap) - \mathrm{Si}(Ap)\cos(Ap)\big) - \frac{Z}{p}.$$

Combining particular and reduced solutions, we obtain

$$\begin{aligned} F(p) &= S_1(Ap) - RAS_2(A(p-t)) + RAS_3(Ap) - \frac{R}{p}, \\ S_1(Ap) &= C_1\cos(Ap) + C_2\sin(Ap), \\ S_2(Ap) &= \cos(Ap)\mathrm{Ci}(Ap) + \sin(Ap)\mathrm{Si}(Ap), \\ S_3(Ap) &= \sin(Ap)\mathrm{Ci}(Ap) - \cos(Ap)\mathrm{Si}(Ap). \end{aligned}$$

Using the same method to solve for $G(p)$, we obtain

$$\begin{aligned} G(p) &= 1 + T_1(Ap) + RAS_2(Ap) + RAS_3(A(p-t)) - \frac{R}{p-t}, \\ T_1(Ap) &= C_2\cos(Ap) - C_1\sin(Ap). \end{aligned}$$

By the definition of G, for $p \le \overline{p} - t$, $F(p) = G(p+t)$. Combining this information, we have a solution for F up to the pricing bound $\underline{p}$, the integration constants, C_1 and C_2, and the variables μ and R:

$$1 - F(p) = \begin{cases} \frac{R}{p} - T_1(A(p+t)) - RAS_2(A(p+t)) - RAS_3(Ap), & \text{for } p \le \overline{p} - t, \\ \frac{R}{p} + A(p - t - \mu) - \frac{1-\nu}{2(1-h)\nu}, & \text{for } p \in [\overline{p} - t, \underline{p} + t], \\ \frac{R}{p} - S_1(Ap) + RAS_2(A(p-t)) - RAS_3(Ap) + 1, & \text{for } p \ge \underline{p} + t. \end{cases} \tag{22}$$

Equation (22) defines the symmetric equilibrium price distribution, given knowledge of the pricing bound $\underline{p}$, the mean of the price distribution μ, the intermediate parameter R, and the integration constants C_1 and C_2. To solve for these five variables, we employ five conditions: the two end points of the price distribution, $F(\overline{p}) = 1$ and $F(\underline{p}) = 0$; evaluation of (22) either side of the thresholds $\overline{p} - t$ and $\underline{p} + t$; and the definition of the mean of the price distribution.

Consider first, the extremeties of the price distribution. Imposing the conditions $F(\underline{p}) = 0$ and $F(\overline{p}) = 1$, leads to the system

$$WC = V_R R + V_1,$$

where

$$W = \begin{pmatrix} \cos(A\overline{p}) & \sin(A\overline{p}) \\ -\sin(A(\underline{p}+t)) & \cos(A(\underline{p}+t)) \end{pmatrix}, \qquad C = \begin{pmatrix} C_1 \\ C_2 \end{pmatrix},$$

and

$$V_R = A \begin{pmatrix} \frac{1}{\overline{p}} + AS_2(A(\overline{p}-t)) - AS_3(A\overline{p}) \\ \frac{1}{\underline{p}} - AS_2(A(\underline{p}+t)) - AS_3(A\underline{p}) \end{pmatrix}, \qquad V_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

The integration constants are given by $C = W^{-1}(V_R R + V_1)$. Thus,

$$\begin{aligned} C_1 &= m_1 R + n_1, \\ C_2 &= m_2 R + n_2, \end{aligned}$$

where

$$\begin{pmatrix} m_1 \\ m_2 \end{pmatrix} = W^{-1} V_R, \qquad \begin{pmatrix} n_1 \\ n_2 \end{pmatrix} = W^{-1} V_1.$$

Next, observe that the mean of the price distribution is given by

$$\mu = \int_{\underline{p}}^{\overline{p}} p f(p) dp = \underline{p} + \int_{\underline{p}}^{\overline{p}} 1 - F(p) dp.$$

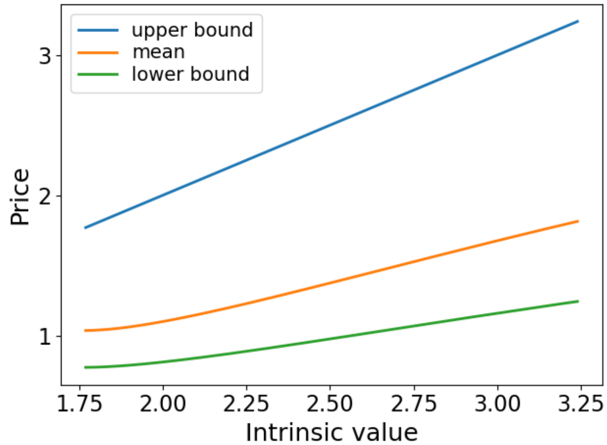
Evaluating this expression and incorporating the integration constants, we can solve for the parameter R in terms of the pricing variables $\overline{p}$, $\underline{p}$, and μ. In particular,

$$\begin{aligned} R &= \frac{\beta\mu + \gamma + d_1 n_1 + d_2 n_2}{\alpha - d_1 m_1 - d_2 m_2}, \\ \alpha &= \left[S_3(Ap) - S_3(A(p+t))\right]_{\underline{p}}^{\overline{p}-t} + \left[S_2(Ap) + S_2(A(p+t))\right]_{\underline{p}}^{\overline{p}-t} + \ln\left(\frac{\underline{p}+t}{\overline{p}-t}\right), \\ \beta &= 1 + A\left(2t + \underline{p} - \overline{p}\right), \\ \gamma &= t - \overline{p} - A\left((\underline{p}+t)^2 - (\overline{p}-t)^2\right)/2 + \left(2t + \underline{p} - \overline{p}\right)\left(At + \frac{1-v}{2(1-h)v}\right), \\ d_1 &= A^{-1}\left[\cos(Ap) + \sin(Ap)\right]_{\underline{p}+t}^{\overline{p}}, \\ d_2 &= A^{-1}\left[\sin(Ap) - \cos(Ap)\right]_{\underline{p}+t}^{\overline{p}}. \end{aligned}$$

The integration constants are then determined by R. Finally, evaluating (22) either side of the thresholds $\overline{p} - t$ and $\underline{p} + t$ leads to a system of two equations in the remaining two unknowns, μ and $\underline{p}$. We solve this system numerically.

Figure 14 illustrates the relationship between the intrinsic value parameter $\overline{u}$ and the price distribution for the parameters $t = 1$, $h = 0.6$, $v = 0.52$. These parameters lie below the boundary

Figure 14: Prices as a function of intrinsic value, $t = 1$, $h = 0.6$, $v = 0.52$



line $\overline{p} - \underline{p} = t$ illustrated in Figure 13, and the upper bound of the price distribution is determined by $\overline{u}$. The mean and lower bound of the price distribution are both increasing in $\overline{u}$, and the range of the price distribution also increases with $\overline{u}$.

Figure 15 illustrates prices and profits as a function of design comparability for $v = 0.5$ and $\overline{u} = 2.5$. For $h > 0.630$, $v \geq V(h)$ and $\overline{p} - \underline{p} < t$. In this case, the upper bound of the price distribution is determined by the first order condition (7). Above this critical value, consistent with our earlier analysis, the mean and lower bound of the price distribution increase with h, while price dispersion decreases with h. Below the critical value of h, the intrinsic value parameter $\overline{u}$ determines the upper bound of the price distribution, leading to a jump in the price distribution. Below the critical value, the mean and lower bound of the price distribution increase with h, and there is a non-monotonic relationship between price dispersion and h.

Figure 16 illustrates prices and profits as a function of price comparability for $h = 0.5$ and $\overline{u} = 2.5$. For $v > V(h) = 0.613$, $\overline{p} - \underline{p} < t$, and the first order condition (7) determines the pricing upper bound. As before, there is a jump in the price distribution at the critical value of v. Prices decline with price comparability either side of the discontinuity.

Figure 15: Prices and profits as a function of design comparability, $t = 1$, $v = 0.5$, $\overline{u} = 2.5$

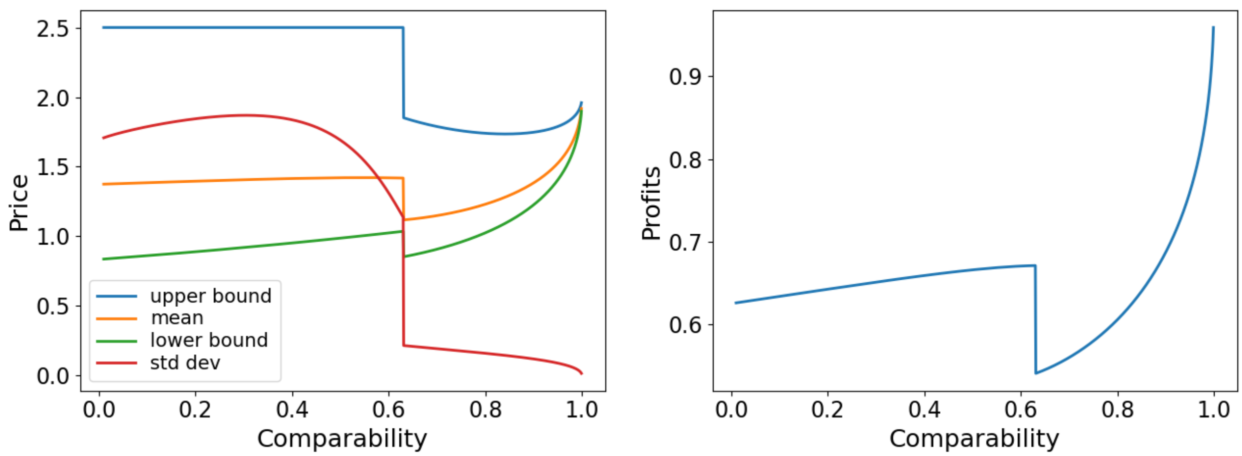


Figure 16: Prices and profits as a function of price comparability, $t = 1$, $h = 0.5$, $\overline{u} = 2.5$