Privacy spillovers and interoperability in network markets*

Alexander Rasch[1,2,†] Shiva Shekhar[3,‡] Tobias Wenzel[1,2,§]

[1]DICE, University of Düsseldorf
[2]ZEW Mannheim
[3]Tilburg University

March 2024

Abstract

This paper studies the effects of introducing interoperability in the market for messenger services. We model competition between an incumbent service that employs a data-driven business model in exchange for "free" access to the service and a privacy-preserving entrant. On the user side, there are direct network effects and users are privacy-conscious. We characterize market outcomes with and without interoperability. A key feature of the model is that interoperability may induce data or privacy spillovers as user data is transferred across different services. Mandating interoperability can hurt user welfare if these privacy spillovers are sufficiently large. Moreover, the entrant's market share also decreases when privacy spillovers are large, that is, contestability is limited.

JEL Classification: L13, L15, L96.

Keywords: Compatibility; Data; Interoperability; Messenger services; Network effect; Privacy.

*We would like to thank Flavio Pino, seminar participants at ZEW Mannheim, at the University of Passau, and participants of the Digital Economy Workshop 2023 (Lausanne) and of the ICT conference 2023 (ZEW Mannheim) for helpful comments and suggestions.

†Email: rasch@dice.hhu.de. Address: DICE, University of Düsseldorf, Universitätsstr. 1, 40225 Düsseldorf, Germany.

‡Email: shiva.shekhar.g@gmail.com, s.shekhar_1@tilburguniversity.edu. Address: Warandelaan 2, Koopmans Building, room K 945, 5037 AB Tilburg, Netherlands.

§Corresponding author. Email: wenzel@dice.hhu.de. Address: DICE, University of Düsseldorf, Universitätsstr. 1, 40225 Düsseldorf, Germany.

1 Introduction

The issue of interoperability (or compatibility/interconnection) has been an an important aspect for competition and antitrust policy in network markets for decades (see, for example, Economides & White 1994, Armstrong 1998 and Laffont et al. 1998). The aim of this paper is to analyze mandated interoperability in markets with strong network effects, where some firms are operating under a data-driven business model. In recent years, policy-makers and academics have been concerned about increasing concentration trends and rising market power of firms in digital markets in which network effects are a key characteristic. There are concerns about adverse effects that could result. For instance, users could get worse deals due to a lack of competition or the contestability of markets may be severely limited, such that new competitors are unable to enter. There are also concerns that the pace of innovation might be reduced. Several recent policy reports, such as the Stigler Report in the US or the Vestager Report in the EU, document these trends and make policy recommendations. One key proposal is to mandate interoperability between different services. Such interoperability requirements are also part of the Digital Markets Act (DMA) in the EU and of the proposed Augmenting Compatibility and Competition by Enabling Service Switching (ACCESS) Act in the US.

The goal of this paper is to analyze the effect of mandated interoperability between network firms and its implications on user surplus and on market contestability. The specific focus will be on an asymmetric market with one incumbent and one entrant, where the incumbent relies on the monetization of user data as its revenue source. Despite its relevance, relatively little is yet known about the effects of interoperability in such data-driven market environments.

An immediate application is the market for messenger services. Messenger services are widely used. For example, in the US, 81.5% of those aged between 16 and 64 use instant messengers each month (`messengerpeople.com`).[1] The range of services has grown over the years (video, group chat, etc.). Because users currently cannot communicate across different services and a larger user base increases the attractiveness of a service due to network effects, the market for these services is rather concentrated. For example, Facebook Messenger is by far the most popular service among users in the US (`messengerpeople.com`). Similar concentration levels can be observed in other countries. For instance, WhatsApp is by far the most popular service among German users.

Another key characteristic of the market is multi-homing. Because network effects are important and services lack interoperability, users often register with more than one service (multi-homing). For example, the German *BNetzA* reports that 73% of those of 16 years of age and older use more than one messenger service.

Messenger services typically come at a price of zero (or, at least, at a very low cost), and many services make profits from the data that users produce when signing up with a service and using it. This data-driven business model has gained greater attention over the past years, and (some) users have become more suspicious of leaving too big of a digital footprint. As a consequence, data

[1] This is similar in other countries. For instance, the *Bundesnetzagentur* (*BNetzA*, Federal Network Agency) in Germany reports that 88% of those of 16 years of age and older use messenger services.

protection has become an ever more important aspect for messaging services. Some messengers respond to this trend by reducing the amount of data that they collect.

The market for messenger services and the question of mandated interoperability have also seen substantial interest by policy-makers (see, for example, Bundesnetzagentur 2021) and researchers (see, for example, Brown 2020 and Bourreau et al. 2022) recently. In Europe, the regulation of messenger services is a key element in the Digital Markets Act (DMA). In its Art. 6(1)(fa), the DMA defines the obligation to allow interconnection with number-independent interpersonal communication services: "A gatekeeper has to allow providers of number-independent interpersonal communication services to interconnect with the gatekeeper's number-independent interpersonal communication service (if identified by the EC as a CPS under Art. 3(7)). The gatekeeper has to allow for a functional interaction, while guaranteeing security and personal data protection." Regulators in Europe hope to achieve several goals with the introduction of the interoperability requirement, among which are more intense (and fairer) competition and the reduction of excessive user data collection and analysis. At the same time, the effects of interoperability in data-driven markets in which privacy issues also play a prominent role are not yet well explored (see, for example, Scott Morton et al. 2021, pp. 13–14). It is not clear whether additional measures (for instance, banning monetization of non-user data) are required to achieve the policy goals.

To gain a better understanding of the effects of such a policy intervention, the aim of this paper is to provide a theoretical model of interoperability in the market for network services paying close attention to the above-mentioned market characteristics. More precisely, we are interested in the impact of mandated interoperability of services on users, the effects of mandated interoperability on the incentives of network firms to collect and monetize user data, and the contestability of the market.

We model competition between an incumbent service and a privacy-preserving entrant. The incumbent employs a data-driven business model in exchange for "free" access. The service collects and monetizes users' data. By contrast, the entrant does not collect any user data. On the demand side, users are privacy-conscious and differ in their costs of adopting the entrant's service. The adoption costs reflect the fact that an incumbent has an advantage in attracting new users which might, for instance, arise due to being a more prominent brand or having a larger number of existing users (larger installed base). Moreover, there are (direct) network effects, such that a service with more users is more attractive for users.

Within this framework, we evaluate the effects of introducing interoperability. As in standard models, interoperability allows users to interact across services. A key feature of our model is that interoperability induces *data or privacy spillovers*. As under interoperability messages (and data included in such messages) are exchanged across services, privacy concerns can spill over across services. Although the privacy-preserving service does not collect any data itself, a user of this service might nevertheless experience privacy concerns under interoperability because messages and data is exchanged with the data-collecting service.

Our results, based on the analysis of single-homing users, suggest that mandated interoperability is not necessarily always user-surplus enhancing or increasing market contestability. On the one hand, mandated interoperability offers users access to a greater mass of users to interact with, which benefits them. However, this cross-network interaction between users who use different services also creates a privacy spillover that hurts users of the privacy-preserving service. Such a spillover exists when a data-collecting service can use data from those users who are connected to its privacy-preserving competitor, but who interact with its own users. These privacy spillovers encourage the incumbent to invest in higher data collection, which also hurts the entrant's users. If the privacy spillover level is sufficiently large, we find that under mandated interoperability, users are worse off along with the entrant's market share falling, which means that contestability is reduced.

In another version of the model, we allow users to multi-home. Consumers face adoption costs when opting for the entrant's service. We find, in line with Bourreau & Krämer (2022), that interoperability reduces the incentives to multi-home. As with single-homing consumers, we find that mandating interoperability leads to larger levels of data collection and, most importantly, user surplus decreases if privacy spillovers are sufficiently large. Interestingly, we find that when users can multi-home, mandating interoperability increases the market share of the entrant, which is contrast to the findings under single-homing. The intuition is that when users multi-home, the market segment of the entrant is 'squeezed' between the multi-homing segment and the single-homing segment of the incumbent service, such that the competitive pressure on the entrant is high. This competitive pressure is eased when interoperability is introduced.

Our analysis suggests that the effects of interoperability may differ in a data-driven environment. In particular, conventional wisdom with regard to contestability may not hold (Crémer et al. 2000), and these findings may inform regulators of the unintended consequences of mandated interoperability in messenger services markets. We recommend that policy-makers should keep in mind this privacy spillover and its effect on the incentives of the incumbent to collect user data. Specifically, our analysis suggests that for any mandated interoperability regime to have a definite positive effect on users, regulators must also stipulate that the incumbent cannot exploit the data generated from cross-service interactions of users.

The rest of this paper is organized as follows. In Section 2, we discuss the related literature and our contribution. We present the model in Section 3. In Section 4, we analyze the case of no interoperability and single-homing users. We present the interoperability case in Section 5, where we also discuss the welfare consequences of mandated interoperability. In Section 6, we present the case in which consumers can multi-home under no interoperability and then compare it with the case of mandated interoperability. We discuss the policy implications from our framework and aspects of market design in Section 7. Section 8 concludes.

2 Related literature

Our contribution to the literature lies at the intersection of privacy concerns/spillovers and interoperability/compatibility between services featuring (within-group or direct) network effects, such as messenger services.

A number of contributions analyze firms' incentives to make their services compatible in markets with network effects. The article closest to ours is Doganoglu & Wright (2006). The authors analyze the relationship between multi-homing and compatibility in a linear-city model when users have a high or a low valuation for a platform's network size. The authors find that platforms always opt for costly compatibility – modeled as access to users of the competing platform – when users cannot multi-home, where incentives are excessive. By contrast, if users multi-home, such multi-homing can increase the social desirability of compatibility. Under compatibility, total transport costs are lower, but platforms have lower compatibility incentives due to less intense competition under multi-homing.[2]

In their seminal article, Crémer et al. (2000) build on the network model by Katz & Shapiro (1985) to compare the incentives of a large network and a smaller network to accept horizontal interoperability, where the large network has a larger installed user base. In their set-up, interoperability may not be perfect in the sense that not all users of the competing service can be accessed. The authors show that interoperability has two different and partly diverging effects on network profits. First, both networks benefit from an increase in user demand due to the introduction of interoperability. Because users can interact with more peers, their utility increases due to larger network benefits, and both networks see higher adoption rates. As a result, networks can set higher prices. The second effect has opposing consequences for networks' profits. Due to interoperability, users who join the larger network can access (almost) the same network of users when joining the smaller network. Hence, whereas interoperability reduces the advantage of the larger installed base for the large network, it results in a greater competitiveness for the small network. The authors thus conclude that the large network has less incentive to accept interoperability than the small network.

The key difference to both, Doganoglu & Wright (2006) and Crémer et al. (2000), is that our paper considers the effects of interoperability in markets with data-driven business models. This gives raise to privacy-spillover effects that have not been studied before, and we show that policy conclusions regarding the desirability of mandating interoperability may change if privacy spillovers are large.

Our work is closely related to a recent paper by Bourreau & Krämer (2022). The paper develops a model with multi-homing users and imperfect interoperability and focuses on user behavior in a dynamic framework. As in the present paper, interoperability can reduce the contestability of the market. Whereas we share this focus, the model itself and the mechanisms behind the results are different. Our focus is on a market in which both services are active and on the services' strategic behavior. Furthermore, the privacy spillovers in our set-up can lead to a reduced contestability of the market and adverse effects on user surplus when interoperability is mandated.

Bourreau et al. (2023) is also closely related. The paper considers interoperability decisions of two ad-financed platforms in a framework where consumers can decide to multi-home, but single-

[2] Rasch (2017) analyzes the incentives to make products compatible when firms collude on price.

homing consumers are more valuable to platforms. The papers shows when platforms are symmetric excessive or insufficient interoperability can result. When platforms differ in their installed base, the larger platform prefers lower levels of interoperability, similar to Crémer et al. (2000). Our contribution differs in that we find that with privacy spillovers this result can be overturned and the larger rival can benefit from interoperability at the expense of the smaller firm.

Another strand of the literature considers the incentives to introduce compatibility in two-sided platform markets. A first set of contributions (Viecens 2011; Maruyama & Zennyo 2013; Adner et al. 2020) consider asymmetric competing platforms that sell hardware and software to consumers and study the incentive of platforms to choose compatibility of content with the hardware of their rivals. The results in these articles are consistent and intuitively show that platforms' compatibility choices are linked to asymmetric value proposition of platforms and their revenue focus. Specifically, platforms may choose compatibility when platforms value the sales of software more than the sales of hardware. One-way compatibility, where one platform allows compatibility, can also be an outcome when one platform values hardware sales more than the other. The platform that values hardware more than its rival allows compatibility of the rival's software on its hardware as it encourages greater sales of hardware. This platform is willing to trade-off sales of software in favor of more valuable sales from hardware. Further, to defend its market share in the hardware market, the hardware-focused platform does not find it profitable to make its software compatible with the rival's hardware. Finally, when the sales from hardware are highly valued by both platforms, they both choose to be incompatible.

Rasch & Wenzel (2014) study the ambiguous welfare effects of compatibility in a symmetric platform market with endogenous content provision. The authors find that compatibility has ambiguous effects on the license fee for content providers. As a consequence, compatibility can be particularly harmful if it results in less content. By contrast, compatibility can be beneficial if content is sufficiently increased.

Maruyama & Zennyo (2015) build on Rasch & Wenzel (2014), but consider application compatibility, where interoperability is an independent decision of a firm (and not an industry standard). As in Doganoglu & Wright (2006), they assume compatibility is feasible after incurring a fixed cost. Platforms earn by charging consumers and content providers an access fee. They find that when there are no costs associated with compatibility, the platforms always choose to be compatible. This result arises directly from the fact that when there are no costs for compatibility, platforms can lower competition to attract consumers by allowing compatibility. Here, platforms do not earn on the content provider side but benefit from lower competition due to compatibility. This arises from the ex ante symmetry of platforms. For intermediate costs of compatibility, compatibility is chosen by one platform and incompatibility by the other. Finally, when costs of compatibility are large, no platform chooses to be compatible. Interestingly, consumer surplus under compatibility is lower than under incompatibility. The rationale for this is that under incompatibility, platforms compete more fiercely for each marginal consumer and set lower prices. Moreover, they employ a model of covered demand where increased content provision does not expand demand. In this case, demand expands due to compatibility consumers are expected to better-off under compatibility. Finally, they show consumer surplus is lowest under asymmetric choice of compatibility. This result is also driven by the covered demand assumption where demand cannot expand due to compatibility.

Finally, our paper is also related to the growing literature on privacy. For an early review, see Acquisti et al. (2016). For instance, Argenziano & Bonatti (2023) investigate the strategic responses of privacy-conscious users when they are aware of their data being utilized but are uncertain about the precise manner of its utilization. Fainmesser et al. (2023) study how data collection and security depends on a platform's business model. Galperti & Perego (2023) analyze the impact of data privacy laws on the value of personal data for firms and, in turn, on consumer welfare. The authors argue that redistributive effects can come into play, where the value of data from certain consumer groups increases while others might suffer. This phenomenon arises from a specific externality stemming from the methods employed by firms, such as e-commerce and matching platforms, to leverage consumer data in facilitating interactions among parties with differing interests (see also Galperti et al. 2023). In Ke & Sudhir (2023), firms gather consumer information to enable personalization and targeted pricing. Consumers weigh the benefits of personalization against the potential drawbacks of compromised privacy and discriminatory pricing when making choices related to purchasing, opting-in to data usage, requesting data erasure, and transferring data. We contribute to this literature by studying the relationship between interoperability of services and privacy considerations, in particular, we add by analyzing effects of privacy spillovers or externalities across different platforms.

3 Model

There are two competing messaging services denoted by A and B that compete to attract users who value the possibility of interacting with other users. The messaging service A is the incumbent service, and its revenue-generation business model is based on monetizing data collected from its users. Service B is an entrant that operates as a not-for-profit service. It does not collect any data and can be thought of as being funded by donations. Both services are "free" messaging services in the sense that they do not charge direct subscription fees to users.

Users. Users have a base valuation v from either service. In addition, users receive surplus from interacting with other users. In our benchmark scenario, we consider single-homing users and a covered market, so that every user chooses either service A or service B. Users affiliating with a service value interactions with other users on the service.

A user's utility from adopting messaging service A is given by

$$U_A(D_A^e, D_B^e, \psi) \triangleq v + \theta(D_A^e + g \cdot D_B^e) - \psi. \tag{1}$$

The benefit of interacting with other users is $\theta(D_A^e + g \cdot D_B^e)$. D_A^e represents the expected mass of users at messaging service i (with $i \in \{A, B\}$), and θ is the extra value users attach to interacting with one additional user. The variable $g \in \{0, 1\}$ is an indicator function that take value $(g = 1)$ when the two services are interoperable $(g = 1)$ and take value $(g = 0)$ when the services are not interoperable. Without interoperability, (single-homing) users can only interact with the expected users on the same platform (that is, D_i^e), whereas with mandated interoperability, users benefit from also being able to interact with the expected mass of users on the competing platform (that is, $D_A^e + D_B^e$).[3] Finally, ψ is the level of data collected by platform A. Because users care about their privacy, it negatively enters a user's utility function.[4]

Alternatively, users can choose service B. The utility from accessing this service is

$$U_B(D_B^e, D_A^e, \psi) \triangleq v + \theta(D_B^e + g \cdot D_A^e) - h \cdot \alpha\psi - k. \tag{2}$$

As with service A, the value of interaction depends on the interoperability regime. Whereas service B does not collect any data itself, our framework allows for *data or privacy spillovers*. This is relevant in the interoperability case in which users of service B also communicate with users of the data-collecting service A. The parameter $h \in \{0, 1\}$ is an indicator function that takes value $h = 1$ if there is a spillover or loss in privacy of users at service B and $h = 0$ if there is no spillover of privacy. Here, $\alpha\psi$ describes the privacy costs for users under mandated interoperability with $\alpha \in (0, 1]$ being the level of the *privacy spillover*. Finally, there is an *experience cost* or *adoption cost* k when choosing service B because it is a new entrant messenger service. We assume that users are heterogeneous in the extent of this cost level. For tractability we assume that the cost is uniformly distributed on the unit interval: $k \sim \mathcal{U}(0, 1)$. By contrast, there is no such adoption cost when a user adopts service A. Hence, this (lack of) adoption cost can be interpreted as the incumbency advantage of service A. Users with larger values of k face higher costs of adopting service B.

Services. We normalize the marginal cost of producing and selling either service to zero without loss of generality. Further, to stress the importance of costly data collection and analysis, we include a data collection cost function denoted by $I(\psi) = \psi^2/2$. This convex function reflects that collecting and analyzing additional data becomes increasingly costly.

The profit of service A is given as

$$\pi_A \triangleq \underbrace{r(\psi) \cdot D_A(\cdot)}_{\text{Value of data collection}} - I(\psi), \tag{3}$$

where $r(\psi) = \psi$ is the per-user margin from monetizing data. We abstract away from the data-selling market and employ a reduced-form approach, such that, all else equal, more data collected results in a higher per-user revenue.

[3] Note that we assume that interoperability is perfect. For an analysis with imperfect interoperability, see, for instance, Bourreau & Krämer (2022).

[4] For instance, users switched to Signal, a privacy-preserving service, after WhatsApp changed it privacy settings (Link).

Interoperability regimes. We consider and compare the incentive to monetize user data in two interoperability regimes.

- *No interoperability:* In this setting, users on either service can only interact with users on the same service, that is, $g = h = 0$.
- *Interoperability with privacy spillovers:* In this scenario, users on the two services can seamlessly interact with users on the other service, and, thus, their potential for interaction is all active users in the messaging services market. Thus, we have $g = h = 1$. Moreover, we assume that there are privacy spillovers due to mandated interoperability.

For each interoperability regime, we consider the following two-stage game. In the first stage, service A decides on the amount of data that is collected and monetized. Service B is a passive player and has no decisions to make. In the second stage, users form expectations about other users' adoption decisions and decide which service to join; then, all payoffs are realized. We assume that users have rational expectations in stage 2 and solve for the subgame perfect rational expectations equilibrium.

We impose the following assumption on parameter θ:

Assumption 1 $\theta < 1 - \frac{1}{\sqrt{2}}$.

This assumption ensures that an interior solution exists and that the entrant obtains a positive market share.

4 No interoperability with single-homing users

We start by considering a benchmark case in which the two services are not interoperable, and in which users choose to adopt one of the two services (single-homing users). In our notation, this implies that $g = 0$ and $h = 0$, so that users can only interact with those users who have adopted the same service and there are no privacy spillovers.

User decisions. In stage 2, users adopt messenger service B if and only if the messenger service provides higher value than service A, that is,

$$U_B(\cdot) > U_A(\cdot) \implies k < k_A(D_A^e, D_B^e, \psi) \triangleq \theta(D_B^e - D_A^e) + \psi.$$

Therefore, user demand at services A and B is

$$D_A(D_A^e, D_B^e, \psi) \triangleq 1 - k_A(D_A^e, D_B^e, \psi) \text{ and } D_B(D_B^e, D_A^e, \psi) \triangleq k_A(D_A^e, D_B^e, \psi).$$

Demand at service i increases with users' expectation of the mass of users on the same service and falls with the expectation of the mass of users on the rival service. Further, an increase in the data-collection level ψ increases demand at service B and reduces demand at service A.

Imposing rational expectations, that is, $D_A^\star = D_A^e$ and $D_B^\star = D_B^e$, yields demands for the two services as functions of the data-collection level ψ and the benefit from interaction θ:

$$D_A^\star(\psi) \triangleq \frac{1-(\psi+\theta)}{1-2\theta}$$

and

$$D_B^\star(\psi) \triangleq \frac{\psi-\theta}{1-2\theta}.$$

Intuitively, user demand for service A is falling in the data-collection level (ψ), whereas the demand for using service B is rising in the data-collection level (ψ); that is, $\partial D_A^\star(\cdot)/\partial\psi < 0$ and $\partial D_B^\star(\cdot)/\partial\psi > 0$. Note that, when $\psi > \theta$ (so that both services have a positive market share), an increase in θ reduces demand for service B and, hence, benefits service A ($\partial D_A^\star(\cdot)/\partial\theta > 0$ and $\partial D_B^\star(\cdot)/\partial\theta < 0$). This stems from the incumbency advantage of A. Specifically, an increase in θ implies greater value from interaction with other users on the service. As affiliating with service A comes at no adoption cost, more users find it valuable to affiliate with service A because they expect a larger number of interactions on this service.

Data-collection stage. In stage 1, service A sets ψ to maximize profits:

$$\max_{\psi} \pi_A(\psi) \triangleq r(\psi)\cdot D_A^\star(\psi) - I(\psi). \tag{4}$$

Differentiating with respect to ψ yields the following trade-off that service A faces when choosing ψ:

$$\underbrace{r'(\psi)D_A^\star(\psi)}_{\text{Margin effect }(+)} + r(\psi)\left(\underbrace{\underbrace{\frac{\partial D_A(\cdot)}{\partial D_A^e}}_{(+)}\underbrace{\frac{\partial D_A^\star(\cdot)}{\partial\psi}}_{(-)}}_{\text{Own demand effect}} + \underbrace{\underbrace{\frac{\partial D_A(\cdot)}{\partial D_B^e}}_{(-)}\underbrace{\frac{\partial D_B^\star(\cdot)}{\partial\psi}}_{(+)}}_{\text{Rival's demand effect}} + \underbrace{\frac{\partial D_A(\cdot)}{\partial\psi}}_{\substack{\text{Direct}\\ \text{effect }(-)}}\right) - \underbrace{I'(\psi)}_{\substack{\text{Marginal cost of}\\ \text{data collection}}} = 0. \tag{5}$$

The margin effect is straightforward and arises directly from the fact that an increase in the data-collection level increases the per-user margin on the service. Second, the volume effect can be decomposed into a direct effect and two reinforcing indirect effects that constrain the incentive of the service A to collect data. The first effect is the *own demand effect* that summarizes how an increase in the data-collection level ψ lowers demand at platform A through a reduction in user expectation on potential interactions on platform A. The second effect is the *rival's demand effect* that arises due to an increase in the rival's demand due to an increase in ψ that further lowers user demand for service A. Finally, an increase in ψ directly lowers user utility from adopting service

A, which further lowers user demand. The optimal data-collection level ψ is governed by these forces – as well by the additional cost associated with data collection – and is given as the solution to the first-order condition. Let superscript S denote the case with single-homing and without interoperability. We then have:

Lemma 1 (Data-collection level) *Under no interoperability with single-homing users, the equilibrium data-collection level is given as*

$$\psi^S \triangleq \frac{1-\theta}{3-2\theta}.$$

Lemma 1 characterizes the equilibrium level of data collection. We note that data collection is increasing in the benefits from interaction (θ). This finding that larger values of θ lead to more competitive market outcomes is in line with standard findings in the literature (see, for example, Doganoglu & Wright 2006). When network effects are stronger (larger θ), the incentives to offer better terms for users are higher. In our model with privacy-sensitive users, this leads to a lower level of data collection by service A.

Using the equilibrium level of data collection, we can calculate equilibrium market shares and profits.

Lemma 2 (Equilibrium market shares and profit) *Under no interoperability with single-homing users, in equilibrium, the market shares of services A and B are given as*

$$D_A^S \triangleq D_A^\star(\psi^S) = \frac{2(1-\theta)^2}{3-4\theta(2-\theta)} \quad \textit{and} \quad D_B^S \triangleq D_B^\star(\psi^S) = \frac{1-2\theta(2-\theta)}{3-4\theta(2-\theta)}.$$

The equilibrium profit of service A is given as

$$\pi_A^S \triangleq \pi_A(\psi^S) = \frac{(1-\theta)^2}{2(3-4\theta(2-\theta))}.$$

The lemma provides the market outcomes in terms of market shares and profits. Comparing the market shares, we find that – as expected – service A serves a larger share of users than service B, which reflects the incumbency advantage of service A. Because the market shares of services A and B move in opposite directions when θ increases, this advantage in terms of market shares is magnified when users' value from interactions increases. This is also reflected in service A's profits that are increasing in θ. However, we note that there is a countervailing effect on profits. While service A benefits from a larger market share as θ increases, competition also intensifies leading to lower levels of data monetization hurting the service (see Lemma 1). In sum, the effect of intensified competition is dominated by the positive market-share effect.

To compute user surplus under no interoperability, we first denote the equilibrium indifferent user as $k_A^S \triangleq k_A(D_A^S, D_B^S, \psi^S)$. Using this equilibrium indifferent user, total user surplus can be expressed as

$$CS^S = v + \int_0^{k_A^S} (\theta D_B^S - k)dk + \int_{k_A^S}^1 (\theta D_A^S - \psi^S)dk$$
$$= v - \frac{(1-2\theta(2-\theta))(5-2\theta(9-\theta(11-4\theta)))}{2(3-4\theta(2-\theta))^2}.$$

The users' total surplus in equilibrium is rising with an increase in θ.

5 Interoperability

When interoperability is mandated, all users can interact with each other, independent of the service they are subscribed to. We assume that interoperability is perfect in the sense that a user at a service values additional users on either network equally.

In a first step, we describe the resulting market outcomes when interoperability is mandated. In a second step, we compare the market outcomes with those under no interoperability as described in the previous section.

5.1 Market outcomes with interoperability

When users can freely interact with users on any service, utilities associated with services A and B become

$$U_A = v + \theta \cdot 1 - \psi, \quad \text{and } U_B = v + \theta \cdot 1 - \alpha\psi - k. \tag{6}$$

On the user side, introducing interoperability has two effects. First, because users can now communicate across services, adopting either service yields the same utility from interacting with other users ($\theta \cdot 1$). This standard effect of interoperability is also present in existing works (see, for example, Crémer et al. 2000 and Doganoglu & Wright 2006). The second effect is novel and relates to our focus on data collection and monetization. Because part of their communication (interaction) is with users of service A, privacy considerations now also apply to users of service B. Due to interaction with users of service A, their data is also obtained, at least partly, by service A. We refer to these effects as privacy or data spillovers the extent of which amounts to $\alpha\psi$.

User decisions. The location of the marginal user follows from equating the expression for utilities presented in equation (6) associated with both services

$$k_C(\psi) \triangleq (1-\alpha)\psi,$$

such that users with $k < k_C(\psi)$ prefer service B and those with $k > k_C(\psi)$ adopt service A. The demands on platforms A and B under interoperability are given as

$$D_A(\psi) \triangleq 1 - k_C(\cdot) \text{ and } D_B(\psi) \triangleq k_C(\cdot).$$

Note that due to interoperability, the marginal user and, hence, the services' market shares are independent of the network parameter θ. However, the location of the indifferent user is affected by the strength of data/privacy spillovers. For given ψ, a larger level of spillovers α affects the marginal user negatively, increasing the market share of service A at the expense of service B.

For later reference, we already note that under interoperability no user would have an incentive to adopt multiple services (multi-homing) if given the option do so. Because users can interact with all users by subscribing to any service, they are strictly better off with adopting only one service if this is associated with any additional, though arbitrarily small, cost.

Data-collection stage. Given user decisions, the profit of service A can be written as

$$\pi_A(\psi) = r(\psi)[D_A(\cdot) + \alpha(1-\beta)D_B(\psi)] - I(\psi), \tag{7}$$

which now also includes data monetization of *non-users* (that is, users adopting service B). The extent to which network A can monetize data depends on two factors. It depends on the additional amount of data that can be collected by service A, which is collected per user on service B denoted by the parameter α. Moreover, it depends on the degree to which this additional data can be monetized, perhaps due to privacy regulation or regulations that restrict the monetization of non-user data. We capture this via the parameter β. When $\beta = 0$, there are no restrictions, and service A can fully analyze and monetize data from non-users, whereas when $\beta = 1$, data from non-users is collected, but cannot be monetized. For $0 < \beta < 1$, service A can partly monetize data from non-users, where larger values of β refer to larger privacy protection for non-users of service A.

Differentiating the profit of service A with respect to ψ yields

$$\underbrace{r'(\psi)[D_A(\cdot) + \alpha(1-\beta)D_B(\psi)]}_{\text{Margin effect }(+)} + \underbrace{r(\psi)\left[\underbrace{\frac{\partial D_A(\cdot)}{\partial \psi}}_{\substack{\text{Own demand}\\ \text{effect }(-)}} + \alpha(1-\beta)\underbrace{\frac{\partial D_B(\psi)}{\partial \psi}}_{\substack{\text{Rival's demand}\\ \text{effect }(+)}}\right]}_{\text{Volume effect }(-)} - I'(\psi) = 0. \tag{8}$$

The optimal level of data collection ψ is determined by the trade-off of the opposing margin and volume effects together with the marginal costs associated with collecting data as presented in the first-order condition. An increase in ψ increases the margin given demand, which incentivizes the firm to raise data collection. However, this incentive to raise is countervailed by the negative effect on the total volume of collected data. Specifically, an increase in ψ directly lowers the mass of own users – users who generate high level of data. Importantly, an increase in the privacy spillover level, α, further encourages service A to collect more data because an increased data-collection level increases demand at the rival whose data can be monetized due to interoperability. To put it simply, as the price spillover level increases, service A is able to better monetize the data from non-users. Thus, user switch from users to non-users (users of service B) hurts profitability less and this encourages service A to set higher data extraction levels. Technically, it is straightforward to observe this because the direct effect of an increase in α on the first-order condition presented above is positive; that is, $\partial^2\pi_A(\cdot)/\partial\psi\partial\alpha > 0$. Thus, interoperability (with data spillovers) leads to increased data-collection levels by service A.[5]

In the following, we discuss the equilibrium market outcomes under interoperability and how they are affected by changes in our key parameters. We start by analyzing the equilibrium level of data collection (where superscript I denotes the case with interoperability).

Lemma 3 (Data-collection level) *Under interoperability the equilibrium level of data collection is*

$$\psi^I \triangleq \frac{1}{3 - 2\alpha[1 + (1-\alpha)(1-\beta)]}.$$

The equilibrium data-collection level rises with an increase in privacy spillovers and an increase in monetization opportunities; that is, $\partial\psi^I/\partial\alpha > 0$ and $\partial\psi^I/\partial\beta < 0$.

The lemma shows the equilibrium level of data collection by service A. The key property is that this level is strictly increasing in the privacy spillover α. As α becomes larger, user utility of service B also decreases when more data is collected. As a result, the incumbent finds it optimal to increase ψ. Increasing β mitigates this effect. When the incumbent is more restricted in monetizing non-users' data, there are less incentives to collect data. The strength of this effect is related to the level of privacy spillovers.

Given ψ^I, the resulting market shares and profits can be stated as follows:

Lemma 4 (Equilibrium market shares and profit) *Under interoperability equilibrium market shares of the two services are given as*

$$D_A^I = 1 - \frac{1-\alpha}{3 - 2\alpha[1 + (1-\alpha)(1-\beta)]}$$

and

$$D_B^I = \frac{1-\alpha}{3 - 2\alpha[1 + (1-\alpha)(1-\beta)]}.$$

If $\alpha < (>)\alpha_{MS} \triangleq 1 - 1/\sqrt{2(1-\beta)}$, the equilibrium market share of service A falls (rises) with an increase in α. The equilibrium market share of service B changes in the opposite direction.[6] *The equilibrium market share of service A (service B) rises (falls) with an increase in β.*

[5] This result is similar in flavor of the price-increasing effect of partial ownership among rivals.

[6] Note that $\alpha_{MS} \geq 0$ if and only if $\beta < 1/2$. If $\beta > 1/2$, an increase in α always increases the market share of service A.

Under interoperability the equilibrium profit of service A is given as

$$\pi_A^I = \frac{1}{2(3 - 2\alpha(1 + (1-\alpha)(1-\beta)))}.$$

The equilibrium profit of service A rises in α and falls with β.

The result states that the effect of the privacy spillover α on market shares is non-monotonic. The market share of service A is first decreasing and then increasing in α, whereas the market share of service B moves in opposite directions. There are two effects driving this result. First, higher levels of α induce service A to increase its data-collection level, thereby decreasing its own market share and increasing that of the rival. Second, for any given level of ψ, increasing α raises the market share of platform A at the expense of service B's market share. It turns out that for small levels of α (that is, $\alpha < \alpha_{MS}$), the first effect dominates, whereas for large values of α (that is, $\alpha > \alpha_{MS}$), the second effect is dominant. By contrast, the effect of β is clear-cut. Because increasing β reduces service A's level of data collection, it increases its market share.

Substituting the equilibrium outcome into the cut-off k_C, we define the cut-off in equilibrium as $k_C^I \triangleq k_C(\psi^I) = (1-\alpha)\psi^I$. Given this cut-off, we can derive the equilibrium user surplus as

$$\begin{aligned} CS^I = v + \int_0^{k_C^I} (\theta - \alpha\psi^I - k)dk + \int_{k_C^I}^1 (\theta - \psi^I)dk \\ = v + \theta - \frac{5 - 6\alpha + 3\alpha^2 + 4\alpha\beta(1-\alpha)}{2(3 - 2\alpha(2 - \alpha - \beta + \alpha\beta))^2}. \end{aligned} \tag{9}$$

The expression shows that every user is now receiving the full interconnection surplus θ, but is also paying the privacy cost ψ^I, although users of service B are only affected at a lower rate α.

Lemma 5 (User surplus) *Under interoperability user surplus increases with an increase in regulation on data monetization (as β increases) and it falls with an increase in privacy spillover α.*

User surplus decreases with α, but increases with β. This follows because the level of data collection decreases with lower values of α and higher values of β (see Lemma 3).

5.2 The implications of mandating interoperability

We are now in a position to evaluate the effects of mandating interoperability. The first result shows that the level of data collection increases with interoperability.

Proposition 1 (Data-collection levels) *(i) Data-collection levels are always higher under interoperability than under no interoperability. (ii) The difference $\Delta_\psi \triangleq \psi^I - \psi^S$ is increasing in the level of privacy spillovers from interoperability (α) and it falls with the level of regulation on data monetization (β).*

The first part of the result shows that mandating interoperability weakens competition and can induce firms to offer worse deals for users. This finding is in line with existing works (see, for example, Doganoglu & Wright 2006). In our setting, this implies that service A collects more data, which hurts privacy-sensitive users. The second part of the result is novel and relates to the strength of this effect. It shows that the potentially negative effect of mandating interoperability via increased data collection is particularly large when the level of data spillovers is high. However, it also points to the role privacy regulations may play. Regulations that either limit the level of data collection from non-users (decreasing α) or limit the firm's ability to monetize it (increasing β) may help reduce data collection. This finding will later be important when we discuss the effects of a mandatory interoperability on user welfare.

One key objective of mandating interoperability is to level the playing field between incumbents and new entrants. We find that mandating interoperability does not always help the entrant to build market share.

Proposition 2 (Market shares) *(i) There exists a critical level of* α_D^S*, such that* $\Delta_D \triangleq D_B^I - D_B^S > 0$ *when* $\alpha < \alpha_D^S$*, and* $\Delta_D < 0$ *when* $\alpha > \alpha_D^S$*. (ii)* Δ_D *and* α_D^S *are decreasing in* β*.*

Proposition 2 points to the importance of privacy spillovers when mandating interoperability. When such spillovers do not exist or are small, mandating interoperability leads to an increased market share for service B. This result mirrors the standard finding in the literature that a smaller competitor gains market share when interoperability is introduced because the competitive disadvantage due to direct network effects is eliminated (e.g., Crémer et al. 2000). However, the proposition also shows that this result is actually reversed when privacy spillovers are sufficiently high (α is sufficiently large). In this case, instead of increasing the market share of the smaller firm, the dominance of the incumbent is actually magnified by mandating interoperability. This key finding is novel in the literature and suggests that in markets in which firms follow a data-driven business model and users are privacy-conscious, such as in messaging services, mandating interoperability may not necessarily increase the contestability of a market, but may rather entrench the dominant position of an incumbent. When data spillovers are large, interoperability reduces the attractiveness of the privacy-preserving service B as a consequence of data leakage between these messenger services.

The finding in part (ii) that increased privacy protection for users via limiting the monetization of non-users (increasing β) can actually help the incumbent keep its dominant position can also be understood in this context. When β increases, the dominant firm decreases its data-collection level, which increases its markets share and reduces the number of users served by its smaller rival.

With regard to profits, we arrive at the following result:

Proposition 3 (Profits) *(i) There exists a critical level of* α_π*, such that* $\Delta_\pi \triangleq \pi_A^I - \pi_A^S < 0$ *when* $\alpha < \alpha_\pi$*, and* $\Delta_\pi > 0$ *when* $\alpha > \alpha_\pi$*. (ii)* Δ_π *is decreasing and* α_π *is increasing in* β*.*

The findings on profits mirror our findings with regard to market shares. The incumbent service A benefits from interoperability when privacy spillovers are sufficiently large and is hurt otherwise.

When privacy protection is tightened alongside the introduction of interoperability, the incumbent is more likely to lose profits.

The final comparison evaluates whether users can benefit from mandatory interoperability.

Proposition 4 (User surplus) *(i) There exists a critical level of* α_{CS}*, such that* $\Delta_{CS} = CS^I - CS^S > 0$ *when* $\alpha < \alpha^S_{CS}$*, and* $\Delta_{CS} < 0$ *when* $\alpha > \alpha^S_{CS}$*. (ii)* α^S_{CS} *and* Δ_{CS} *are increasing in* β*.*

The overall impact of mandated interoperability on users comes from several opposing effects. On the positive side, users benefit from interacting with users across the two services. However, because interoperability relaxes competition, users are hurt by service A's increased efforts to collect and analyze user data. This negative effect is more pronounced in our setting due to privacy spillovers. When α is large, the negative effect affects users of service A due to more data collection and the resulting privacy concerns, but also hurts the users of service B who are directly affected by the privacy spillovers. Therefore, compared to standard models, the welfare effects tend to be more negative in our framework compared to situations in which services charge direct prices and privacy spillovers play no role.

6 Multi-homing users

In this section, we consider the case in which users can multi-home between services, that is, users can register with both services. As highlighted in the introduction, multi-homing is a widespread phenomenon in the market for messenger services. The section has two aims. We first develop a multi-homing version of our model in which users sort according to learning costs. Users with low learning costs adopt both services, those with intermediate learning costs choose the entrant, and those with high learning costs stay with the incumbent. Within this setting, we show that when multi-homing occurs, interoperability does not lead to lower market shares of the entrant, which is in contrast to the single-homing case in which the market share of the entrant can also decrease. With regard to user surplus, the main finding is that users only benefit from interoperability if privacy spillovers are sufficiently small.

6.1 Market outcomes with multi-homing and no interoperability

Suppose users can multi-home and services are not interoperable. We distinguish three different types of users (see Figure 1): (i) those who exclusively use service A, (ii) those who exclusively use service B, and (iii) those who use both services. The expected mass of users who single-home on service A, single-home on B, and multi-home are denoted as $D^e_{A,E}$, $D^e_{B,E}$, and D^e_M.

A user's utility from accessing the messaging services A or B exclusively is given by

$$\tilde{U}_A(D^e_{A,E}, D^e_M, \psi) \triangleq v + \theta(D^e_{A,E} + D^e_M) - \psi \tag{10}$$

and

$$\tilde{U}_B(D^e_{B,E}, D^e_M, \psi) \triangleq v + \theta(D^e_{B,E} + D^e_M) - k. \tag{11}$$

The benefit of interacting with other users on the service is $\theta(D^e_{A,E} + D^e_M)$, that is, the user can interact with all other users (exclusive and multi-homing) who have signed up with the same service. As in the base model with single-homing users, the associated privacy cost with service A is ψ and there is the adoption cost k if the user chooses service B.

The utility of a multi-homing user is

$$\tilde{U}_M(D^e_{A,E}, D^e_{B,E}, D^e_M, \psi) \triangleq v + \theta \underbrace{(D^e_{A,E} + D^e_{B,E} + D^e_M)}_{\text{Total interactions}} - \alpha\psi - 2k. \tag{12}$$

Multi-homing enables users to interact with all possible users, so that their value from interactions is highest. However, these users face two additional costs: (i) privacy spillover costs of $\alpha\psi$, with $\alpha \in (0, 1)$, and (ii) an additional experience cost of $2k$ arising from the need to switch between services to access users.[7] We assume that when users face the same users on both platforms (multi-homers), they benefit from this interaction only once, that is, $D^e_{A,E} + D^e_{B,E} + D^e_M = 1$.[8]

Assumption 2 *We impose the following restrictions.*

- *The value of user interaction is not too high, that is,* $0 < \theta < \overline{\theta} \triangleq 0.217$.
- *The privacy spillover due to interoperability is in the following range* $\max\{0, \underline{\alpha}\} \leq \alpha \leq \min\{\overline{\alpha}_1, \overline{\alpha}_2\}$,

where $\underline{\alpha} \triangleq \frac{1-\theta-7\theta^2-\theta^3-\sqrt{(1-\theta(2+\theta))(1+\theta(12-\theta(84-\theta(32+7\theta))))}}{2\theta(3-\theta(6+\theta))}$, $\overline{\alpha}_1 \triangleq \frac{1-2\theta-3\theta^2-\sqrt{1-8\theta^5-3\theta^4+36\theta^3-10\theta^2-4\theta}}{2\theta(1-2\theta)}$, *and* $\overline{\alpha}_2 \triangleq \frac{1-\theta-7\theta^2-\theta^3+\sqrt{(1-\theta(2+\theta))(1+\theta(12-\theta(84-\theta(32+7\theta))))}}{2\theta(3-\theta(6+\theta))}$.

Assumption 2 ensures that there exists an equilibrium in which multi-homing occurs and both services attract a positive number of single-homing users.

User decisions. Market shares for the two services are derived by identifying the two marginal users (see Figure 1). There is a clear ordering associated with the experience cost k: Users with small experience costs tend to join both services, those with intermediate costs tend to join only service B, whereas those with high costs tend to join service A.

Users choose to single-home on platform A and not on platform B when

$$\tilde{U}_A(\cdot) \geq \tilde{U}_B(\cdot) \implies k > \tilde{k}_A(D^e_{A,E}, D^e_{B,E}, \psi) \triangleq \theta(D^e_{B,E} - D^e_{A,E}) + \psi.$$

Users choose multi-homing over single-homing on platform B when

$$\tilde{U}_B(\cdot) \geq \tilde{U}_M(\cdot) \implies k > \tilde{k}_M(D^e_{A,E}, \psi) \triangleq \theta(D^e_{A,E}) - \alpha\psi.$$

[7] We note that the level of the additional experience cost associated with multi-homing, $2k$, has been chosen for ease of exposition. Qualitatively similar results, but more complicated expressions, would result when considering additional costs of Δk with $\Delta > 1$.

[8] A similar assumption is made, for instance, in Bakos & Hallaburda (2020).

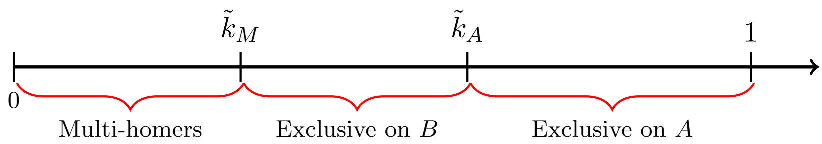


Figure 1: Distribution of user demand when multi-homing is possible.

By deriving the marginal consumers, the demands for the services and the scope of multi-homing are given as follows:

$$\begin{aligned}
D_{A,E}(D^e_{A,E}, D^e_{B,E}, \psi) &\triangleq 1 - \tilde{k}_A(\cdot) = 1 - \theta(D^e_{B,E} - D^e_{A,E}) - \psi, &(13)\\
D_M(D^e_{A,E}, \psi) &\triangleq \tilde{k}_M(\cdot) = \theta(D^e_{A,E}) - \alpha\psi, &(14)\\
D_{B,E}(D^e_{A,E}, D^e_M, \psi) &\triangleq \tilde{k}_A(\cdot) - \tilde{k}_M(\cdot) = \theta(D^e_{B,E} - 2D^e_{A,E}) - (1-\alpha)\psi. &(15)
\end{aligned}$$

In stage 2, imposing rational expectations, that is, $D^\star_M = D^e_M$, $D^\star_{A,E} = D^e_{A,E}$, and $D^\star_{B,E} = D^e_{B,E}$, and solving for the demands yields user demand as a function of privacy choice by service A:

$$\begin{aligned}
D^\star_{A,E}(\psi) &\triangleq \frac{(1 - \theta - \psi(1 + \alpha\theta))}{1 - \theta(2 + \theta)}, &(16)\\
D^\star_{B,E}(\psi) &\triangleq \frac{(1 + \alpha)\psi - \theta(2 - \psi(1 - \alpha))}{1 - \theta(2 + \theta)}, &(17)\\
D^\star_M(\psi) &\triangleq \frac{\theta(1 - \theta - \psi(1 - 2\alpha)) - \alpha\psi}{1 - \theta(2 + \theta)}. &(18)
\end{aligned}$$

Differentiating user demand at each platform with respect to ψ reveals that

$$\frac{\partial D^\star_{A,E}(\cdot)}{\partial\psi} < 0, \ \frac{\partial D^\star_M(\cdot)}{\partial\psi} < 0, \text{ and } \ \frac{\partial D^\star_{B,E}(\cdot)}{\partial\psi} > 0.$$

The above comparative statics states that, as service A increases its data-collection level, the single-homing demand on service A and the share of multi-homing users fall, whereas the single-homing demand on (the privacy-protecting) service B rises. Thus, an increase in data-collection levels hurts service A in two ways. First, an increase in ψ lowers its own exclusive demand, which is transformed into exclusive demand for service B. Second, some of the users who multi-homed earlier start using service B exclusively. This two-fold reduction in usage that translates into a gain in exclusive demand for service B restrains the incentives of service A to set data-collection levels. The following Figure 2 illustrates the changes in the user composition (in blue) due to an increase in the data-collection level.

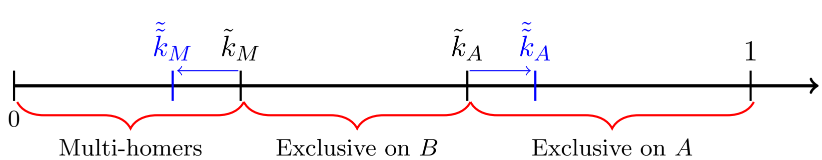


Figure 2: Changes in user demand configuration due to increased data collection.

Data-collection stage. In stage 1, service A sets ψ to maximize its profit that is given by

$$\max_{\psi} \tilde{\pi}_A(\psi) = \underbrace{r(\psi)(D^{\star}_{A,E}(\cdot) + \alpha D^{\star}_{M}(\cdot))}_{\text{Revenue from data collection}} - \underbrace{I(\psi)}_{\substack{\text{Cost of data}\\\text{collection}}}.$$

Note that an exclusive user is more valuable than a multi-homing user. Because only part of a multi-homing user's interactions are via service A, the service generates lower revenues of $\alpha r(\psi)$ from users subscribing to both services.

Differentiating the above profit expression with respect to ψ yields the following trade-off that platform A must consider when choosing ψ:

$$\underbrace{r^{'}(\psi)(D^{\star}_{A,E}(\cdot) + \alpha D^{\star}_{M}(\cdot))}_{\text{Margin effect } (+)} + \underbrace{r(\psi)\left(\underbrace{\frac{\partial D_{A,E}(\cdot)}{\partial \psi}}_{\text{Volume effect } (-)} + \underbrace{\underbrace{\frac{\partial D_{A,E}(\cdot)}{\partial D^{e}_{A,E}}}_{(+)} \underbrace{\frac{\partial D^{\star}_{A,E}(\cdot)}{\partial \psi}}_{(-)}}_{\text{Own exclusive demand effect}} + \underbrace{\underbrace{\frac{\partial D_{A,E}(\cdot)}{\partial D^{e}_{B,E}}}_{(-)} \underbrace{\frac{\partial D^{\star}_{B,E}(\cdot)}{\partial \psi}}_{(+)}}_{\text{Rival's demand effect}}\right)}_{\text{Impact of own exclusive demand due to increase in data collection}(-)}$$

$$+ \underbrace{r(\psi)\alpha\left(\underbrace{\frac{\partial D_{M}(\cdot)}{\partial D^{e}_{A}}}_{(+)} \underbrace{\frac{\partial D^{\star}_{A,E}(\cdot)}{\partial \psi}}_{(-)} - \alpha\right)}_{\text{Multi-homing demand effect } (-)} - \underbrace{I^{'}(\psi)}_{\substack{\text{Marginal cost of}\\\text{data collection}}} = 0. \tag{19}$$

The choice of the optimal data-collection level must take into account multiple opposing forces. The margin effect is straightforward and arises directly from the fact that an increase in the data-collection level increases the per-user margin on the service. By contrast, there are two reinforcing negative channels that discourage service A to increase data collection and that arise from their effect on the exclusive demand for service A and from the multi-homing demand in the market. The impact of an increase in the data-collection level on the exclusive demand on the service can be decomposed as follows. An increase in the data-collection level negatively affects the exclusive demand on service A directly by lowering users' value from using the service (*volume effect*) and indirectly by lowering user expectations of the interaction with the exclusive users on service A. Additionally, an increase in ψ increases the user's expectations on the volume of interactions with the rivals' exclusive users, which further diverts users to the rival. Secondly, there is a negative impact also on the mass of active multi-homers. An increase in ψ lowers users' expectation on the mass of exclusive users at service A, which then lowers the mass of multi-homers some of whom switch to exclusively using service B. Finally, there is an additional effect arising from the increased marginal cost of data collection, which reflects that costs are increasing when the firm collects and analyzes more data.

Solving the above first-order condition yields the equilibrium data-collection level as

$$\psi^M \triangleq \frac{(1-\theta)(1+\alpha\theta)}{3-2\theta+2\alpha^2+4\alpha\theta(1-\alpha)-\theta^2}.$$

Substituting this data-collection level back into user demand as presented in equations (16), (17), and (18) yields the following expressions for equilibrium user demand and profit for service A

$$\begin{aligned}
D^M_{A,E} = D^\star_{A,E}(\psi^M) &= \frac{(1-\theta)(2(1-\theta+\alpha\theta)-\theta^2+\alpha^2(2-4\theta-\theta^2))}{(1-2\theta-\theta^2)(3-2\theta+2\alpha^2+4\alpha\theta(1-\alpha)-\theta^2)}, \\
D^M_{B,E} = D^\star_{B,E}(\psi^M) &= \frac{1-\theta+\alpha(1+\theta(1-\alpha))}{(3-2\theta+2\alpha^2+4\alpha\theta(1-\alpha)-\theta^2)} - \frac{\theta}{1-\theta(2+\theta)}, \\
D^M_{M} = D^\star_{M}(\psi^M) &= \frac{(1-\theta)(\theta(2+\alpha(2+\alpha))-\alpha-\theta^3-\theta^2(2-\alpha(3-2\alpha)))}{(1-2\theta-\theta^2)(3-2\theta+2\alpha^2+4\alpha\theta(1-\alpha)-\theta^2)}, \\
\pi^M_A &\triangleq \tilde{\pi}_A(\psi^M) = \frac{(1-\theta)^2(1+\alpha\theta)^2}{2(1-2\theta-\theta^2)(3-2\theta+2\alpha^2+4\alpha\theta(1-\alpha)-\theta^2)}.
\end{aligned}$$

The effect of privacy spillovers on equilibrium data collection is presented in the following lemma:

Lemma 6 (Data-collection level) *The equilibrium data-collection level unambiguously falls with an increase in privacy spillover α, that is, $\frac{\partial \psi^M}{\partial \alpha} < 0$.*

An increase in the privacy spillover α lowers the incentive of the incumbent to collect data. This is because an increase in the privacy spillover level directly and negatively affects multi-homing demand. A reduction in multi-homing demand translates into a direct gain of single-homing users for service B. This gain in users on service B makes it relatively more attractive for the single-homers on service A to join platform B exclusively. To avoid losing both multi-homing and single-homing users due to an increase in the privacy spillovers, the service lowers its data-collection levels with an increase in spillover level.

We further note that data-collection levels are non-monotonic in θ and fall with an increase in user value from interactions only if the value of spillovers is low. An increase in user value from interactions has two effects on users. First, it increases utility from interactions, which encourages the service to set higher data-collection levels. Second, it also makes user demand more elastic and users are easily willing to switch between services due to an increase in ψ, thus creating a greater competitive constraint on the incumbent service. This incentivizes the service to lower data-collection levels. The positive utility effect of an increase in θ dominates when α is high; otherwise, the negative elasticity effect dominates. This is because for low $\alpha < \alpha_M$, the mass of multi-homing consumers is large and any increase in θ makes these consumers more willing to switch to service B as exclusives. An increase in θ enhances competition to retain them, and thus data collection levels falls.

Lemma 7 (Market shares) *An increase in the privacy spillover α unambiguously lowers multi-homing demand, lowers exclusive use for service A if and only if $\theta/2 > \alpha$, and lowers the exclusive use of service B if and only if $\alpha > \alpha_{BM} = \frac{\sqrt{(2+\theta)^2(5-\theta(2-\theta))}-2+\theta-\theta^2}{2}$.*

Interestingly, single-homing user participation on either service is non-monotonic in α. Specifically, if $\theta > 2\alpha$, an increase in privacy spillover lowers participation on the incumbent platform even as the data-collection level also falls. An increase in α has two opposing effects on single-homers active on service A. First, there is a reduction in the data-collection level, which enhances single-homing user participation on service A. Second, there is a reduction in multi-homing users who are transformed into single-homers on service B. This effect makes service B relatively more attractive than service A. When the value of interactions θ is large, an increase in the privacy spillover encourages a greater mass of multi-homing users to affiliate exclusively with service B. This transformation of multi-homers into single-homers at platform B then indirectly also spurs single-homers at A to also transform into single-homers at service B. This negative effect of an increase in α outweighs the positive affect arising from reduced data-collection level. Thus, an increase in α when $\theta > 2\alpha$ lowers single-homing demand at service A. On the contrary, when θ is low, users do not value interactions as much and, therefore, the utility-enhancing effect from a reduction in the data-collection level dominates the negative effect of multi-homers transforming into single-homers on service B. As a result, single-homing participation rises on service A. A similar intuition holds for the impact of an increase in α on single-homing participation on service B.

We also note that an increase in user value from interactions θ, intuitively, favors the incumbent and increases user participation on the incumbent platform (both single-homers and multi-homers) and lowers participation on service B. The effect of an increase in the privacy spillover on multi-homing user participation is monotonic but not obvious. Specifically, we observe that an increase in α lowers multi-homing participation. Recall that an increase in α unambiguously lowers the data-collection level set by service A. Therefore, at a cursory glance, one would expect an increase in single-homing participation of users on service A as well as increased participation from multi-homing consumers. However, we observe that multi-homing participation falls. This is because the positive effect on multi-homing utility from a reduction in the data-collection level is outweighed by the direct negative impact of privacy loss on the utility of multi-homing users as α increases.

User surplus. To compute user surplus in this case, we first denote the equilibrium indifferent users as $\tilde{k}_A^M \triangleq \tilde{k}_A(D_{A,E}^M(\psi^M), D_{B,E}^M(\psi^M), \psi^M)$ and $\tilde{k}_M^M \triangleq \tilde{k}_M(D_{A,E}^M(\psi^M), \psi^M)$. Total user surplus is then given by

$$CS^M \triangleq v + \int_0^{\tilde{k}_M^M} (\theta \cdot 1 - \alpha\psi^M - 2k)dk + \int_{\tilde{k}_M^M}^{\tilde{k}_A^M} (\theta(D_{B,E}^M(\psi^M) + D_M^M(\psi^M)) - k)dk + \int_{\tilde{k}_A^M}^{1} (\theta(D_{A,E}^M(\psi^M) + D_M^M(\psi^M)) - \psi)dk. \tag{20}$$

Lemma 8 (User surplus) *There exists a critical level α_L, such that user surplus falls with an increase in the privacy spillover α if and only if $\alpha < \alpha_L$.*

The impact of an increase in privacy spillover on total user surplus is non-monotonic and is *U-shaped*. Specifically, when the privacy spillover levels are low, an increase in the spillover α decreases user surplus. Note that when α is low, the mass of multi-homers is large and an increase in α directly hurts multi-homing users. In this case, an increase in α also lowers the amount of data collection. The direct negative effect on multi-homers dominates any positive impact due to lowered data collection levels arising from an increase in α. Instead when α is high, the mass of multi-homing users is low and in this case, the indirect positive effect of reduced data collection dominates and thus user surplus is higher.

6.2 The welfare implications of mandating interoperability in markets with multi-homing users

We are now in a position to evaluate the effects of mandating interoperability. Note that the market outcomes under interoperability are equal to those described in Section 5 because users have no incentive to multihome under interoperability.

The first result shows that the level of data collection increases with interoperability:

Proposition 5 (Data-collection levels) *(i) Data-collection levels are always higher under interoperability than under no interoperability with multi-homing users. (ii) The difference $\Delta_{\psi}^{M} \triangleq \psi^{I} - \psi^{M}$ is unambiguously increasing in the level of privacy spillovers from interoperability (α) and unambiguously falling with the level of regulation on data monetization (β).*

The above proposition provides meaningful results for policy. First, interoperability leads to a greater incentive for the incumbent service to collect user data than without interoperability. The intuition for this result arises directly from the fact that the service faces less fierce competitive constraints due to interoperability. Specifically under interoperability, an increase in data collection has no impact on the total value from interactions on a service. Instead, without interoperability an increase in data-collection levels has a significant effect on user value for the service offered by firm A. Additionally, any demand lost by the incumbent is gained by the rival (privacy-preserving) service B. All these factors encourage higher investments in data collection under interoperability.

Further, we observe that as the level of data spillover α increases, the difference between the data-collection levels under interoperability vis-à-vis no interoperability also increases. The rationale for this result is that as the level of spillover increases, the rival's demand is negatively impacted by data collection to a greater extent. Any demand lost by the rival is gained by the incumbent, and this spurs the incentive to increase data collection. By contrast, an increase in β lowers the ability of the incumbent to monetize the data collected from users active on the rival service. This reduced monetization ability lowers the incentive to increase data-collection levels because the marginal gains from data collection are lower.

One of the objectives of mandating interoperability is to level the playing field between incumbents and new entrants. Toward this, we define the difference in total demand under interoperability with demand under no interoperability as follows: $\Delta D_A \triangleq D_A^I(\psi^I) - (D_{A,E}^M + D_M^M)$ and $\Delta D_B \triangleq D_B^I - (D_{B,E}^M + D_M^M)$.

Proposition 6 (Market shares) *Compared to no interoperability with multi-homing users, under interoperability the market share of the privacy-preserving entrant rises, whereas the market share of the incumbent falls, that is, $\Delta D_B > 0$ and $\Delta D_A < 0$.*

Interoperability increases the market share of service B, whereas the market of service A falls vis-à-vis demand under interoperability. This finding contrasts with the effect of mandating interoperability compared to no interoperability when users single-home. The intuition is that when users multi-home, the market segment of the entrant is 'squeezed' between the multi-homing segment and the single-homing segment of the incumbent service, such that the competitive pressure on the entrant is high. As discussed earlier, a direct consequence of interoperability is that the competitive situation of the entrant is relaxed, which enhances its total market share. This finding also has policy implications. Although, interoperability can lead to higher market shares for entrants, the overall volume of sign-ups (total demand) decreases because under interoperability users do not have an incentive to sign up with multiple services.

Proposition 7 (Profits) *The profit of service A is higher under interoperability than without interoperability when $\alpha > \alpha_\pi^M$; else, the profit of service A is lower under interoperability.*

The findings on profits mirror our results from the single-homing case. As in that analysis, service A only benefits from interoperability if privacy spillovers are sufficiently large. The reasoning is as follows. As the amount of data spillover increases, service A finds it profitable to extract more data under interoperability. The positive effect of data extraction along with lower (service) competition in the market increases profit under interoperability with respect to profit without interoperability.

The final comparison evaluates whether users can benefit from mandatory interoperability when instead users are able to multi-home:

Proposition 8 (User surplus) *User surplus under interoperability is higher than under no interoperability with multi-homing if and only if $\alpha < \alpha_{CS}^M$; else, user surplus is lower under interoperability.*

The intuition for the results presented in the above proposition are as follows. User surplus under interoperability is higher than under no interoperability when the level of data spillover is low enough. Note that mandated interoperability is associated with increased user interaction, which raises user surplus. However, interoperability also encourages the messenger service to extract more data, which hurts users on both platforms. When the level of spillover is high $\alpha > \alpha_{CS}^M$, the negative impact of additional incentive to collect data under interoperability dominates any positive impact of increased user interaction.

Further, when β increases, the platform is unable to monetize the data collected from users on service B. This lowers the marginal revenue of data extraction and, hence, also the level of data extraction under interoperability. Thus, it is straightforward that as β increases, the difference in user surplus rises because user surplus under interoperability rises due to a fall in data-extraction levels.

7 Policy implications and design suggestions

Policy implications. After studying the effects of mandated interoperability across messenger services vis-à-vis the no interoperability status quo, we are now in a position to make some policy design recommendations. These recommendations are complementary to the interoperability regulation and align its welfare effects with the intended objectives. Specifically, regulators hope to achieve several goals with the introduction of the interoperability requirement. These objectives include more (and fairer) competition among messenger services,[9] the reduction or avoidance of lock-in effects, the reduction of excessive user data collection and analysis, and better privacy and data protection.

The two main policy objectives that we address are increasing competition (or contestability) in the messenger services markets and the effects on user welfare. In the following, we discuss a complementary policy that should be implemented along with interoperability to ensure that the market is more competitive and users are better off:

Policy implication 1 (User surplus goal) *Mandated interoperability accompanied with strict prohibition of data collection for inter-service user interactions unambiguously increases user surplus. Moreover, restricting monetization of non-users by the incumbent increases user surplus when* $\alpha > 0$.

Recall that user surplus is higher under mandated interoperability when the privacy spillover is sufficiently low compared to both single-homing and multi-homing without interoperability (that is, $\alpha < \alpha^S_{CS}$ and $\alpha < \alpha^M_{CS}$). In both cases, we find that the difference in user surplus under interoperability vis-à-vis no interoperability is positive and highest when $\alpha = 0$. The intuition for this result is quite straightforward. A fall in the ability to collect data from inter-service interaction of users makes it less profitable to invest in data collection and hurt the rival. This reduction in data-collection benefits users on both platforms, and, therefore, user surplus unambiguously rises. A direct consequence of this result is that if user surplus is the most important objective of regulators, then strict prohibition of inter-service interaction-data usage must be accompanied with interoperability to maximize the benefits from the regulation. Moreover, our analysis shows

[9]See, for example, https://www.europarl.europa.eu/news/de/press-room/ 20220315IPR25504/ deal-on-digital-markets-act-ensuring-fair-competition-and- more-choice-for-users.

that there are positive effects from banning the monetization of non-user data on user surplus (for $\alpha > 0$), suggesting that prohibiting monetization should also be mandated if interoperability is introduced and user surplus is the regulator's main concern.[10]

Policy implication 2 (Contestability goal) *If the ability of service A to monetize inter-service interaction data of non-users is low (that is, $\beta > 1/2$), then authorities should implement strict prohibition of data collection along with mandated interoperability, that is, $\alpha = 0$.*
Instead, when the ability of service A to monetize inter-service interaction data of non-users is high (that is, $\beta < 1/2$), then authorities should implement a less strict prohibition of inter-service data collection coupled with mandated interoperability, where data collection is tolerated up to a maximum privacy spillover of α_{MS}.

When the ability of service A to monetize data of users at service B is low ($\beta > 1/2$), data-collection levels are low because service A is unable to monetize from the spillover. As a result, its market share is high also in the case of mandated interoperability. To increase demand of service B, the authorities must resort to strict prohibition of data collection along with mandated interoperability to maximize the market share of service B. Instead, when the ability of service A to monetize data from users of service B is high ($\beta > 1/2$), the policy suggestions are more nuanced. Specifically, when service A can monetize inter-service transaction data to a high degree, its incentive to collect data is also high under interoperability. In such a case, the policy-makers should tolerate privacy spillovers up to α_{MS} because the demand-increasing effect of an increase of α on service B through increased data extraction dominates the demand-decreasing effect of privacy spillovers. Thus, interoperability can be tolerated up to α_{MS}.

Bearing in mind the two policy goals and the discussion on additional policy instruments necessary to align these goals with mandated interoperability, we note that there are cases in which the same complementary prohibition regime is sufficient to achieve these goals. Specifically, the same complementary prohibition regime is enough to maximize user surplus and increase market share of the incumbent in two main cases: (i) when the ability of the incumbent to monetize inter-service interaction data of non-users is high, and (ii) when the level of privacy spillovers is high. In all other cases, the complementary prohibition regime that maximizes user surplus does not maximize the market share of the privacy-preserving entrant.

Finally, we comment on the entrant's preferences toward interoperability. So far, we have not considered the entrant's behavior explicitly because the entrant was a passive player in our model. However, this issue becomes relevant because the DMA gives the entrant the choice as to whether interoperability should be implemented. One way to consider the entrant's preferences would be to analyze changes in its market share. However, in practice many independent messenger services also intrinsically care about privacy considerations. To take these diverse objectives into account, we think of the entrant's objective as a convex combination of market share and user surplus (which

[10]Note that when $\alpha = 0$, the incumbent does not collect any data on non-users and, hence, cannot monetize it. Therefore, in this case, all market outcomes are independent of β.

relates to privacy considerations):

$$\pi^B = \omega \cdot D_B + (1 - \omega) \cdot CS, \tag{21}$$

where weight ω (with $\omega \in (0, 1)$) measures the importance of the entrant's market share.

Policy implication 3 *Suppose the entrant's preferences are described by Equation (21). (i) If single-homing occurs in the absence of interoperability, for $\alpha \in (\alpha^S_{CS}, \alpha^S_D)$ there exists a level $\bar{\omega}^S$, such that the entrant implements user-surplus-decreasing interoperability if $\omega > \bar{\omega}^S$. (ii) If multi-homing occurs in the absence of interoperability, for $\alpha > \alpha^M_{CS}$ there exists a level $\bar{\omega}^M$, such that the entrant implements user-surplus-decreasing interoperability if $\omega > \bar{\omega}^M$.*

The finding suggests that interoperability, when viewed from a user surplus perspective, can be excessive. This holds for both cases in which single-homing or multi-homing would result in the absence of interoperability.

If compared to both single-homing and multi-homing, for small values of α, interests are aligned and both users and the entrant would benefit from interoperability. However, for intermediate or larger values of α, interests may no longer be aligned. The finding shows that in this case, there can be excessive interoperability if the entrant carries sufficient weight on its market share. In such situations, interoperability would be implemented by the entrant, but would decrease user surplus.

We note, however, that when single-homing is the alternative to interoperability, the rule of letting the entrant decide about the implementation of interoperability, rules out the implementation of interoperability in the worst cases from a user's perspective, namely when privacy spillovers are particularly large. In such situations, users' and the entrant's preferences are aligned, and both would be hurt via interoperability.

Managerial implications. Our analysis provides distinct insights that managers of companies impacted by this regulation can readily use.

Managerial implication 1 *Incumbent data-funded services may benefit from interoperability due to positive privacy spillovers that enhance data collection.*

Our study reveals that the incumbent entity employing a data-funded business model stands to gain from the introduction of interoperability, particularly when the extent of privacy spillovers is substantial. This holds true in both scenarios in which users either can single-home or multi-home. The manifestation of interoperability gives rise to privacy spillovers, enabling the data-funded service to fortify its market share through heightened data collection levels. This increased incentive for data collection presents a dual advantage: It attenuates the detriments arising from interoperability, while concurrently augmenting profitability due to the increased acquisition of data from a larger user base. Consequently, under conditions characterized by elevated privacy spillovers, the adoption of interoperability emerges as advantageous for the incumbent entity.

Managerial implication 2 *Entrant privacy preserving services that request interoperability should carefully gauge the effect of interoperability on their goals – privacy protection vs. market shares.*

Interoperability confers advantages upon entrant entities adopting privacy-preserving models by mitigating the competitive asymmetry between incumbents and entrants. This policy adjustment allows users of the entrant's service to capitalize on the unique selling proposition (USP) of enhanced privacy, while simultaneously enjoying access to a substantial network of users. Nevertheless, the implementation of interoperability is not devoid of counterproductive repercussions, exemplified by the emergence of privacy spillovers that exert detrimental influences on users' welfare. These spillovers, in turn, engender negative externalities that dissuade potential users from affiliating with the incumbent establishment, thereby lowering its initial appeal.

Furthermore, the interoperability paradigm has a financial incentive for the data-endowed incumbent to escalate its data aggregation efforts. This outcome ensues from the increased feasibility of exploiting cross-service interactions and data sharing, thereby enhancing data extraction capabilities. Consequently, notwithstanding the potential augmentation of the incumbent's market share, the overarching user benefit is compromised, particularly for those users who transitioned to the incumbent service to safeguard their privacy. Thus, entrants contemplating the decision to embrace interoperability must judiciously assess their core strategic objectives in light of these multifaceted dynamics.

In this context, entrants striving to optimize the advantages from interoperability should contemplate investing in technological solutions that curtail the propagation of data spillovers. By concurrently augmenting the quality of users' interaction experiences within their service, entrants can foster an environment that not only safeguards privacy but also sustains an enhanced level of user satisfaction, thereby contributing to the attainment of their overarching business objectives.

8 Conclusion

In this paper, we argue that the proposed interoperability between messenger services may have unintended welfare consequences. In particular, we show that interoperability between messenger services can lower user surplus and also the market share of an entrant privacy-preserving service. The intuition for this result is that interoperability creates privacy spillovers, which encourages the incumbent service to collect more data as a way to handicap the privacy-preserving entrant's business model. We find that greater privacy spillovers are accompanied by a greater incentive to collect data. As a result, when the privacy spillover due to interoperability is sufficiently large, then users are worse off because the incentives to collect data are higher and users on both services are worse off. Additionally, the data-collection level is also used as a tool to make the privacy-preserving service less attractive, and we find that the market share of the (privacy-preserving) entrant service is also lower. This implies that a mandated interoperability regulation between services may actually lower market contestability in contrast to the goals of policy-makers.

These results have several policy implications and inform policy-makers on efficient policy design. In particular, we have uncovered a positive link between privacy spillovers due to interoperability and the incentives to attract data. We find that mandated interoperability may actually lower contestability and user surplus in the messenger services market due to privacy spillovers and the supposed gains from interoperability may not be present. We further discuss what additional measures policy-makers should take to ensure that the mandated interoperability policy is aligned with their main objectives and goals.

A Appendix

Proof of Lemma 1. Solving the first order condition presented in equation (5) for ψ, we get the optimal data-collection level

$$\psi^S = \frac{1-\theta}{3-2\theta}.$$

Differentiating ψ^S with respect to θ, we observe

$$\frac{\partial \psi^S}{\partial \theta} = -\frac{1}{(3-2\theta)^2} < 0.$$

■

Proof of Lemma 2. Substituting the optimal data-collection levels in the demands, we get the equilibrium demand expressions presented in Lemma 2. Differentiating these demands with respect to θ yields

$$\frac{\partial D_A^S}{\partial \theta} = \frac{4(1-\theta)}{(3-4\theta(2-\theta))^2} > 0, \quad \frac{\partial D_B^S}{\partial \theta} = -\frac{4(1-\theta)}{(3-4\theta(2-\theta))^2} < 0.$$

Substituting the equilibrium data collection levels and the equilibrium demand levels in the profit expression of service A yields the profit expression presented in the Lemma 2. Differentiating the equilibrium profit level with respect to θ yields

$$\frac{\partial \pi_A^S}{\partial \theta} = \frac{1-\theta}{(3-4\theta(2-\theta))^2} > 0.$$

The above relation holds under Assumption 1. ■

Proof of Lemma 3. Solving the first order condition presented in equation (8) for ψ, we get the optimal data-collection level

$$\psi^I = \frac{1}{3-2\alpha[1+(1-\alpha)(1-\beta)]}.$$

Differentiating ψ^I with respect to α and β, we observe

$$\frac{\partial \psi^I}{\partial \alpha} = \frac{2(2(1-\alpha(1-\beta))-\beta)}{(3-2\alpha[1+(1-\alpha)(1-\beta)])^2} > 0, \quad \frac{\partial \psi^I}{\partial \beta} = -\frac{2\alpha(1-\alpha)}{(3-2\alpha[1+(1-\alpha)(1-\beta)])^2} < 0.$$

The above relations hold under Assumption 1. ■

Proof of Lemma 4. Substituting the optimal data-collection levels in the demands, we get the equilibrium demand expressions presented in Lemma 4. Differentiating the demand of service A with respect to α yields

$$\frac{\partial D_A^I}{\partial \alpha} = \frac{2(\beta+\alpha(2-\alpha)(1-\beta))-1}{(3-2\alpha[1+(1-\alpha)(1-\beta)])^2}.$$

The market share of A rises if and only if $\alpha > \alpha_{MS} \triangleq 1 - \frac{1}{\sqrt{2(1-\beta)}}$. Since consumers are single-homing and the market is covered, the market share of service B changes in the opposite direction.

Differentiating the demand of service A respectively with respect to β yields

$$\frac{\partial D_A^I}{\partial \beta} = -\frac{2\alpha(1-\alpha))}{(3-2\alpha[1+(1-\alpha)(1-\beta)])^2} < 0.$$

The above relations hold under Assumption 1.

Substituting the equilibrium data collection levels and the equilibrium demand levels in the profit expression of service A yields the profit expression presented in Lemma 4. Differentiating the equilibrium profit level with respect to α and β yields

$$\frac{\partial \pi_A^I}{\partial \alpha} = \frac{2(1+\alpha(1-\beta))-\beta}{(3-2\alpha[1+(1-\alpha)(1-\beta)])^2} > 0,$$

and

$$\frac{\partial \pi_A^I}{\partial \beta} = -\frac{\alpha(1-\alpha)}{(3-2\alpha[1+(1-\alpha)(1-\beta)])^2} < 0.$$

The above relations hold under Assumption 1. ■

Proof of Lemma 5. The equilibrium user surplus is presented below.

$$CS^I \triangleq v + \int_0^{k_C^I} (\theta - \alpha\psi^I - k)dk + \int_{k_C^I}^1 (\theta - \psi^I)dk$$
$$= v + \theta - \frac{5-6\alpha+3\alpha^2+4\alpha\beta(1-\alpha)}{2(3-2\alpha(2-\alpha-\beta+\alpha\beta))^2}. \quad (22)$$

Differentiating the equilibrium user surplus with respect to θ, β and α yields

$$\frac{\partial CS^I}{\partial \theta} = 1 > 0,$$

$$\frac{\partial CS^I}{\partial \beta} = \frac{2\alpha(1-\alpha)(2-\alpha-2\beta(1-\alpha))}{(3-2\alpha[1+(1-\alpha)(1-\beta)])^3} > 0,$$

and

$$\frac{\partial CS^I}{\partial \alpha} = -\frac{11-4\beta-\alpha(23-22\beta+4\beta^2-6\alpha(1-\beta)(3-2\beta)+2\alpha^2(1-\beta)(3-4\beta))}{(3-2\alpha[1+(1-\alpha)(1-\beta)])^3} < 0.$$

The above relations hold under Assumption 1. ■

Proof of Proposition 1. Comparing the data collection levels under interoperability with the data collection levels under no interoperability, we observe

$$\Delta_\psi \triangleq \psi^I - \psi^S = \frac{2\alpha(1-\theta)(2-\beta-\alpha(1-\beta))+\theta}{(3-2\theta)(3-2\alpha(2-\beta-\alpha(1-\beta)))} > 0.$$

Differentiation Δ_ψ with respect to α, θ and β yields

$$\frac{\partial \Delta_\psi}{\partial \alpha} = \frac{\partial \psi^I}{\partial \alpha} > 0,$$

$$\frac{\partial \Delta_\psi}{\partial \theta} = -\underbrace{\frac{\partial \psi^S}{\partial \theta}}_{(-)} > 0,$$

and

$$\frac{\partial \Delta_\psi}{\partial \beta} = \frac{\partial \psi^I}{\partial \beta} < 0.$$

These relations are a direct consequence of the results expressed in Lemma 1 and Lemma 3. ■

Proof of Proposition 2. Taking the difference in service B's market share yields the following expression

$$\Delta_D \triangleq D_B^I - D_B^S = \frac{2\theta(2-\theta) + 2\alpha^2(1-\beta)(1-2\theta(2-\theta)) + \alpha(1-4\theta(2-\theta) - \beta(2-4\theta(2-\theta)))}{(3-2\theta)(1-2\theta)(3-4\alpha+3\alpha^2(1-\beta)+2\alpha\beta)}.$$

The above relation is positive if and only if $\alpha < \alpha_D$ where α_D is defined as follows.

$$\begin{aligned}\alpha_D \quad &\triangleq \quad \frac{1}{2} - \frac{1}{4(1-\beta)(1-2\theta(2-\theta))} + \\ &\quad \frac{\sqrt{1-4\beta(1-2\theta(2-\theta)) + 8\theta(2-\theta)(1-2\theta(2-\theta)) + 4(2\beta\theta(2-\theta)-\beta)^2}}{4(1-\beta)(1-4\theta+2\theta^2)}.\end{aligned}$$

Differentiating Δ_D with respect to α, θ and β yields α, θ and β yields

$$\frac{\partial \Delta_D}{\partial \alpha} = \frac{\partial D_B^I}{\partial \alpha} > 0, \text{ if and only if } 0 < \alpha < \alpha_{MS}$$

Note that $\alpha_{MS} > 0$ if and only if $\beta < 1/2$.

$$\frac{\partial \Delta_D}{\partial \theta} = -\frac{\partial D_B^S}{\partial \theta} > 0,$$

and

$$\frac{\partial \Delta_D}{\partial \beta} = \frac{\partial D_B^I}{\partial \beta} > 0.$$

These relations are a direct consequence of the results expressed in Lemma 2 and Lemma 4. ■

Proof of Proposition 3. Taking the difference in service $A's$ profits yields the following expression

$$\Delta_\pi \triangleq \pi_A^I - \pi_A^S = \frac{2\alpha(1-\theta)^2((2-\beta) - \alpha(1-\beta)) - \theta(2-\theta)}{2(3-4\theta(2-\theta))(3-2\alpha(2-\beta-\alpha(1-\beta)))}.$$

The above relation is positive if and only if $\alpha > \alpha_\pi$ with α_π defined as follows

$$\alpha_\pi \triangleq \frac{1}{2}\left(1 + \frac{1-\theta-\sqrt{(6-6\beta+\beta^2)(1-\theta)^2 - 2(1-\beta)}}{(1-\beta)(1-\theta)}\right)$$

Differentiating Δ_π with respect to α, θ and β yields

$$\frac{\partial \Delta_\pi}{\partial \alpha} = \frac{\partial \pi_A^I}{\partial \alpha} > 0,$$

$$\frac{\partial \Delta_\pi}{\partial \theta} = -\underbrace{\frac{\partial \pi_A^S}{\partial \theta}}_{(+)} < 0,$$

and

$$\frac{\partial \Delta_\pi}{\partial \beta} = \frac{\partial \pi_A^I}{\partial \beta} < 0.$$

The above relations follow directly from the results in Lemma 2 and in Lemma 4. ■

Proof of Proposition 4. Taking the difference in consumer surplus under interoperability with no interoperability, yields the following expression

$$\Delta_{CS} \triangleq CS^I - CS^S = \theta - \frac{5-\alpha(6-4\beta-\alpha(3-4\beta))}{2(3-2\alpha[1+(1-\alpha)(1-\beta)])^2} + \frac{(1-2\theta(2-\theta))(5-2\theta(9-\theta(11-4\theta)))}{2(3-4\theta(2-\theta))^2}.$$

At $\alpha = 0$, the above difference in profit is given as

$$\Delta_{CS}|_{\alpha=0} = \frac{2\theta(5-4\theta)(3-\theta(16-\theta(23-9\theta)))}{9(3-4\theta(2-\theta))^2} > 0.$$

At $\alpha = 1$, the above difference in profit is given as

$$\Delta_{CS}|_{\alpha=1} = -\frac{13-4\theta(19-\theta(42-\theta(43-\theta(21-4\theta))))}{9(3-4\theta(2-\theta))^2} < 0.$$

From Lemma 5, we know that CS^I is decreasing in α . Therefore, by continuity, there exists an α_{CS} such that for $\alpha < \alpha_{CS}$ we must have $\Delta_{CS} > 0$. Else, $\Delta_{CS} < 0$.

Differentiating Δ_{CS} with respect to α, θ and β yields

$$\frac{\partial \Delta_{CS}}{\partial \alpha} = \frac{\partial CS^I}{\partial \alpha} > 0,$$

$$\frac{\partial \Delta_{CS}}{\partial \theta} = 1 - \frac{\partial CS^S}{\partial \theta},$$

and

$$\frac{\partial \Delta_{CS}}{\partial \beta} = \frac{\partial CS^I}{\partial \beta} > 0.$$

The expression for $\frac{\partial \Delta_{CS}}{\partial \theta}$ is positive for $\theta < \theta_{CS} \approx 0.201$ and negative otherwise. ■

Proof of Lemma 6. Solving the first order condition presented in Equation (19) for ψ, we get the equilibrium data-collection levels as

$$\psi^M \triangleq \frac{(1-\theta)(1+\alpha\theta)}{3-2\theta+2\alpha^2+4\alpha\theta(1-\alpha)-\theta^2}.$$

Differentiating this equilibrium data collection level with respect to α yields

$$\frac{\partial \psi^M}{\partial \alpha} = -\frac{(1-\theta)(\alpha(4-8\theta)+2\theta\alpha^2(1-2\theta)+\theta(1+\theta)^2)}{(3-2\theta+2\alpha^2+4\alpha\theta(1-\alpha)-\theta^2)^2} < 0.$$

The above relation always holds under Assumption 2.

Differentiating this equilibrium data collection level with respect to θ yields

$$\frac{\partial \psi^M}{\partial \theta} = -\frac{(1-\theta)^2+\alpha(1+3\theta(2-\theta))-\alpha^3(2-4\theta(1-\theta))+\alpha^2(2-4\theta^2)}{(3-2\theta+2\alpha^2-4\alpha\theta(1-\alpha)-\theta^2)^2}.$$

The sign of the above expression follows the sign of the numerator which is defined as $H_P = -\left((1-\theta)^2+\alpha(1+3\theta(2-\theta))-\alpha^3(2-4\theta(1-\theta))+\alpha^2(2-4\theta^2)\right)$.
Differentiating twice H_P with respect to α, yields $\frac{\partial^2 H_P}{\partial \alpha^2} = 6\alpha(4(\theta-1)\theta+2)-8\theta^2+4 > 0$. Next differentiating H_P with respect to α yields $\frac{\partial H_P}{\partial \alpha} = 6\alpha^2(2(\theta-1)\theta+1)+\alpha\left(4-8\theta^2\right)+3(\theta-2)\theta-1$. Equating the above expression to zero and and solving for α yields the following admissible solution $\alpha_P = \frac{4\theta^2+\sqrt{2}\sqrt{\theta(\theta(2(27-5\theta)\theta-47)+12)+5}-2}{12(\theta-1)\theta+6} > 0$. For any $\alpha > \alpha_P$, we must have that $\frac{\partial H_P}{\partial \alpha} > 0$ else it is $\frac{\partial H_P}{\partial \alpha} < 0$.

In addition, we find that at $\alpha = \alpha_P$, $H_P|_{\alpha=\alpha_P} < 0$ is always negative. Thus, for all $\alpha < \alpha_P$, we must have that H_P is negative. For $\alpha > \alpha_P$, we know $\frac{\partial H_P}{\partial \alpha_P} > 0$ and at $\alpha = 1$, we $H_P|_{\alpha=1} = 2-2\theta(4-\theta) > 0$. Thus, when α_P is in the relevant range of α as presented in Assumption 2 and $\alpha > \alpha_P$, there exists a threshold α_M such that for all $\alpha > \alpha_M$, $H_P > 0$ and negative otherwise. This confirms the result that $\frac{\partial \psi^M}{\partial \theta} > 0$ for $\alpha > \alpha_M$ and is negative otherwise. ■

Proof of Lemma 7. Differentiating the expression for exclusive demand for service A with respect to θ yields

$$\frac{\partial D^M_{A,E}}{\partial \theta} = \frac{\mathcal{A}}{(1-2\theta-\theta)^2(3-2\theta+4\alpha\theta-\theta^2+\alpha^2(2-4\theta))^2} > 0. \tag{23}$$

$\mathcal{A} \triangleq (1-\theta)^2(4+14\theta+\theta^2-4\theta^3-\theta^4)+2\alpha^4(2-6\theta-3\theta^2+16\theta^3-3\theta^4)-2\alpha(1-10\theta-6\theta^2+20\theta^3+\theta^4-2\theta^5)+\alpha^2(6-2\theta-33\theta^2+56\theta^3-6\theta^5-\theta^6)-4\alpha^3(1-6\theta+2\theta^2+14\theta^3-3\theta^4) > 0$. The above expression is always positive.

Differentiating the expression for multi-homing demand with respect to θ yields

$$\begin{aligned}\frac{\partial D^M_M}{\partial \theta} &= \frac{1}{2}\left(\frac{4\left(2\alpha^2+1\right)\left(3-\alpha^2-(\alpha(2\alpha-3)+3)\theta-\alpha\right)}{\left(-2\alpha^2+4(\alpha-1)\alpha\theta+\theta^2+2\theta-3\right)^2}+\frac{4-8\theta}{(\theta(\theta+2)-1)^2}\right)\\ &\quad \frac{1}{2}\left(\frac{3}{1-\theta(\theta+2)}+\frac{4\alpha^2-2\alpha+3}{3+2\alpha^2+4(1-\alpha)\alpha\theta-\theta^2-2\theta}\right).\end{aligned}$$

The above expression is positive under Assumption 2.

Differentiating the expression for exclusive demand for service B with respect to θ yields

$$\begin{aligned}\frac{\partial D^M_{B,E}}{\partial \theta} &= -\left(\frac{2\left(1-\theta\right)\left(1+2\alpha^2\right)(2-\alpha(2-\alpha))}{\left(-2\alpha^2+4(\alpha-1)\alpha\theta+\theta^2+2\theta-3\right)^2}+\frac{2(1-\theta)}{(\theta(\theta+2)-1)^2}\right)\\ &\quad +\left(\frac{1}{1-\theta(\theta+2)}-\frac{1+\alpha^2-\alpha}{3+2\alpha^2+4(1-\alpha)\alpha\theta-\theta^2-2\theta}\right) < 0.\end{aligned}$$

The above relation holds under Assumption 2.

Differentiating the expression for exclusive demand for service A with respect to α yields

$$\frac{\partial D_{A,E}^{M}(\psi^M)}{\partial \alpha} = \frac{2(1-\theta)(1+\alpha\theta)(2\alpha-\theta)}{(3-2\theta+2\alpha^2+4\alpha\theta(1-\alpha)-\theta^2)^2}. \tag{24}$$

It is straightforward to observe that the above expression is negative for $\theta > 2\alpha$ and positive otherwise.

Differentiating the expression for multi-homing demand with respect to α yields

$$\frac{\partial D_{M}^{M}}{\partial \alpha} = -\frac{(1-\theta)(3+2\alpha\theta(1-2\theta)-\theta(2-\theta)-\alpha^2(2-4\theta))}{(3-2\theta+2\alpha^2+4\alpha\theta(1-\alpha)-\theta^2)^2} < 0. \tag{25}$$

The above relation always holds under Assumption 2.

Differentiating the expression for exclusive demand for service B with respect to α yields

$$\frac{\partial D_{B,E}^{M}}{\partial \alpha} = \frac{(1-\theta)(3-2\alpha(2+\alpha)+2\alpha\theta+\theta^2(1-2\alpha))}{(3-2\theta+2\alpha^2+4\alpha\theta(1-\alpha)-\theta^2)^2}. \tag{26}$$

Note that the sign of the above comparative static depends on the sign of the term $\mathcal{M} = (3-2\alpha(2+\alpha)+2\alpha\theta+\theta^2(1-2\alpha))$. Differentiating $\mathcal{M}$ with respect to α, we note that $\frac{\partial \mathcal{M}}{\partial \alpha} < 0$. Next, equating $\mathcal{M}$ to 0 and and solving for α, we find that the only solution within the relevant parameter range is given as $\alpha_{BM} = \frac{\sqrt{(2+\theta)^2(5-\theta(2-\theta))}-2+\theta-\theta^2}{2}$.

Thus, we show that above expression is negative if and only if $\alpha > \alpha_{BM}$. ■

Proof of Lemma 8. The expression for consumer surplus as presented in equation (20) is given as

$$CS^M \quad \triangleq \quad \frac{\mathcal{B}}{(1-2\theta-\theta)^2(3-2\theta+4\alpha\theta-\theta^2+\alpha^2(2-4\theta))^2} > 0 \tag{27}$$

where $\mathcal{B} \triangleq v - 524\alpha^4\theta^7 + 49\alpha^4\theta^6 - 82\alpha^4\theta^5 - 22\alpha^4\theta^4 + 86\alpha^4\theta^3 - 47\alpha^4\theta^2 + 8\alpha^4\theta - 56\alpha^3\theta^7 - 148\alpha^3\theta^6 + 130\alpha^3\theta^5 + 152\alpha^3\theta^4 - 216\alpha^3\theta^3 + 84\alpha^3\theta^2 - 10\alpha^3\theta + 14\alpha^2\theta^8 + 94\alpha^2\theta^7 + 93\alpha^2\theta^6 - 218\alpha^2\theta^5 - 23\alpha^2\theta^4 + 214\alpha^2\theta^3 - 145\alpha^2\theta^2 + 38\alpha^2\theta - 3\alpha^2 - 16\alpha\theta^8 - 82\alpha\theta^7 - 32\alpha\theta^6 + 236\alpha\theta^5 + 8\alpha\theta^4 - 222\alpha\theta^3 + 128\alpha\theta^2 - 20\alpha\theta + 2\theta^9 + 14\theta^8 + 16\theta^7 - 54\theta^6 - 58\theta^5 + 113\theta^4 + 14\theta^3 - 80\theta^2 + 38\theta$.

Differentiating the above expression with respect to θ yields

$$\frac{\partial CS^M}{\partial \theta} = \frac{\mathcal{D}}{(1-2\theta-\theta)^3(3-2\theta+4\alpha\theta-\theta^2+\alpha^2(2-4\theta))^3} > 0, \tag{28}$$

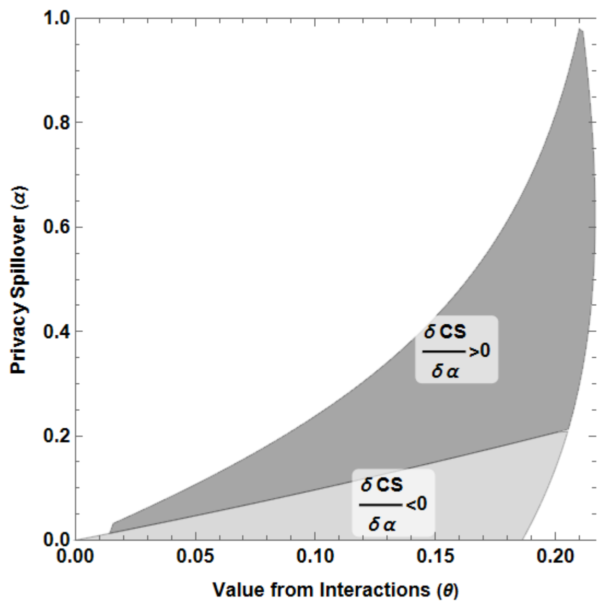
where $\mathcal{D} \triangleq 17 + 48\alpha^6\theta^9 + 216\alpha^6\theta^8 - 22\alpha^6\theta^7 - 774\alpha^6\theta^6 + 762\alpha^6\theta^5 - 116\alpha^6\theta^4 - 230\alpha^6\theta^3 + 186\alpha^6\theta^2 - 62\alpha^6\theta + 8\alpha^6 - 160\alpha^5\theta^9 - 792\alpha^5\theta^8 - 344\alpha^5\theta^7 + 2390\alpha^5\theta^6 - 1128\alpha^5\theta^5 - 710\alpha^5\theta^4 + 944\alpha^5\theta^3 - 462\alpha^5\theta^2 + 112\alpha^5\theta - 10\alpha^5 + 44\alpha^4\theta^{10} + 467\alpha^4\theta^9 + 1305\alpha^4\theta^8 - 162\alpha^4\theta^7 - 3147\alpha^4\theta^6 + 2424\alpha^4\theta^5 - 63\alpha^4\theta^4 - 874\alpha^4\theta^3 + 603\alpha^4\theta^2 - 191\alpha^4\theta + 26\alpha^4 - 92\alpha^3\theta^{10} - 732\alpha^3\theta^9 - 1509\alpha^3\theta^8 + 1164\alpha^3\theta^7 + 3850\alpha^3\theta^6 - 3564\alpha^3\theta^5 - 516\alpha^3\theta^4 + 1876\alpha^3\theta^3 - 1038\alpha^3\theta^2 + 264\alpha^3\theta - 23\alpha^3 + 12\alpha^2\theta^{11} + 159\alpha^2\theta^{10} + 659\alpha^2\theta^9 + 609\alpha^2\theta^8 - 1738\alpha^2\theta^7 - 1762\alpha^2\theta^6 + 2976\alpha^2\theta^5 - 852\alpha^2\theta^4 - 486\alpha^2\theta^3 + 519\alpha^2\theta^2 - 191\alpha^2\theta + 31\alpha^2 - 12\alpha\theta^{11} - 119\alpha\theta^{10} - 360\alpha\theta^9 + 6\alpha\theta^8 + 1436\alpha\theta^7 + 426\alpha\theta^6 - 2436\alpha\theta^5 + 620\alpha\theta^4 + 904\alpha\theta^3 - 603\alpha\theta^2 + 148\alpha\theta - 10\alpha + \theta^{12} + 12\theta^{11} + 48\theta^{10} + 30\theta^9 - 222\theta^8 - 262\theta^7 + 545\theta^6 + 366\theta^5 - 832\theta^4 + 262\theta^3 + 123\theta^2 - 88\theta > 0$.

The above relation holds under Assumption 2.

Similarly, differentiating the expression for consumer surplus with respect to α yields

$$\frac{\partial CS^M}{\partial \alpha} = \frac{\mathcal{F}}{(1-2\theta-\theta)(3-2\theta+4\alpha\theta-\theta^2+\alpha^2(2-4\theta))^3}, \tag{29}$$

where $\mathcal{F} \triangleq -16\alpha^4\theta^6 - 12\alpha^4\theta^5 + 88\alpha^4\theta^4 - 102\alpha^4\theta^3 + 52\alpha^4\theta^2 - 10\alpha^4\theta + 8\alpha^3\theta^7 + 26\alpha^3\theta^6 - 44\alpha^3\theta^5 - 18\alpha^3\theta^4 + 64\alpha^3\theta^3 - 58\alpha^3\theta^2 + 28\alpha^3\theta - 6\alpha^3 - 12\alpha^2\theta^7 - 30\alpha^2\theta^6 + 99\alpha^2\theta^5 - 108\alpha^2\theta^3 + 66\alpha^2\theta^2 - 15\alpha^2\theta + 2\alpha\theta^8 + 10\alpha\theta^7 - 21\alpha\theta^6 - 66\alpha\theta^5 + 129\alpha\theta^4 - 26\alpha\theta^3 - 67\alpha\theta^2 + 50\alpha\theta - 11\alpha + \theta^7 + 8\theta^6 - 8\theta^5 - 32\theta^4 + 61\theta^3 - 40\theta^2 + 10\theta$. It is difficult to analytically get a sign. Therefore, we provide a graphical proof of our result. The following regionplot graphically shows the region where $\frac{\partial CS^M}{\partial \alpha} > 0$ and $\frac{\partial CS^M}{\partial \alpha} < 0$.



Thus, we graphically prove our result. ■

Proof of Proposition 5. The difference in data collection level between the interoperability versus the no-interoperability case is denoted as $\Delta_\psi^M \triangleq \psi^I - \psi^M > 0$.

It is straightforward to observe that the above is positive under Assumption 2.

Differentiation Δ_ψ^M with respect to α, θ and β yields

$$\frac{\partial \Delta_\psi^M}{\partial \alpha} = \frac{\partial \psi^I}{\partial \alpha} > 0,$$

$$\frac{\partial \Delta_\psi^M}{\partial \theta} = -\underbrace{\frac{\partial \psi^M}{\partial \theta}}_{(-)} > 0, \text{ iff } \alpha > \alpha_M$$

and

$$\frac{\partial \Delta_\psi^M}{\partial \beta} = \frac{\partial \psi^I}{\partial \beta} < 0.$$

These relations are a direct consequence of the results expressed in Lemma 3 and Lemma 6.

■

Proof of Proposition 6.

Taking the difference in service B's market share yields the following expression

$$\Delta D_B \triangleq D_B^I - (D_{B,E}^M + D_M^M) = \frac{G_B}{(3-4\alpha+2\alpha(1+\beta-\alpha\beta))(1-2\theta+\theta^2)(3+2\alpha^2-2\theta+4\alpha\theta-4\alpha^\theta-\theta^2)}$$

where $G_B = -2\alpha^3 - 2(\alpha-1)\alpha\theta(2\alpha^2(\beta-1)-2\alpha\beta+4\beta-5)+\theta^4(\alpha(2\alpha(\beta-1)-2\beta+3)-2)+\theta^3(\alpha(\alpha(2\alpha(3\alpha(\beta-1)-7\beta+8)+14\beta-23)-6\beta+16)-5)+\theta^2(\alpha(\alpha(6(\alpha-3)\alpha(\beta-1)+10\beta-17)+2(\beta+3))+3)+2(\alpha-1)\alpha\beta+\alpha+4\theta$. The sign of the difference in demand follows the sign of G_B. Differentiating G_B with respect to β yields

$$\frac{\partial G_B}{\partial \beta} = -2\alpha(1-\alpha)(1-\theta(4+\alpha^2(2-3\theta(1+\theta))+\theta(1-\theta(3+\theta))-\alpha(2-2\theta(3+2\theta)))) < 0.$$

Equating G_B equal to zero and solving, there exists a unique solution given by

$$\beta_B = \frac{2\alpha + \frac{(1-\theta)\left(2+\alpha^2(2-\theta(\theta+4))+2\alpha\theta-\theta(\theta+2)\right)}{1+\theta(\alpha^2(3\theta(\theta+1)-2)+\alpha(2-2\theta(2\theta+3))+\theta(\theta(\theta+3)-1)-4)} - \frac{1}{1-\alpha} - 1}{2\alpha} > 1.$$

Since $0 < \beta < 1$, it must be that G_B is always positive. Thus, we confirm that $\Delta D_B > 0$.

Taking the difference in service A's market share yields the following expression

$$\Delta D_A \triangleq D_A^I - (D_{A,E}^M + D_M^M) = \frac{G_A}{(3-4\alpha+2\alpha(1+\beta-\alpha\beta))(1-2\theta+\theta^2)(3+2\alpha^2-2\theta+4\alpha\theta-4\alpha^\theta-\theta^2)}$$

where $G_A = \theta^3(2-(\alpha-1)\alpha(2\alpha^2(\beta-1)-2\alpha\beta+4\beta-3))+\theta^2(\alpha(2\alpha(\alpha(-6\alpha(\beta-1)+13\beta-16)-10\beta+19)+6\beta-25)+9)+(\alpha-1)\alpha\theta(2\alpha(3\alpha(\beta-1)+\beta-2)+12\beta-9)+2\alpha(-\alpha(\alpha(\beta-2)+2)+\beta+1)+(\alpha-1)\theta^4-10\theta$. The sign of the difference in demand follows the sign of G_A. Differentiating G_A with respect to β yields

$$\frac{\partial G_A}{\partial \beta} = 2\alpha(1-\alpha)((1+\alpha)-\theta((6+\alpha+3\alpha^2)-\theta(3-\alpha(7-6\alpha))-\theta^2(2\alpha(1-\alpha)))) > 0.$$

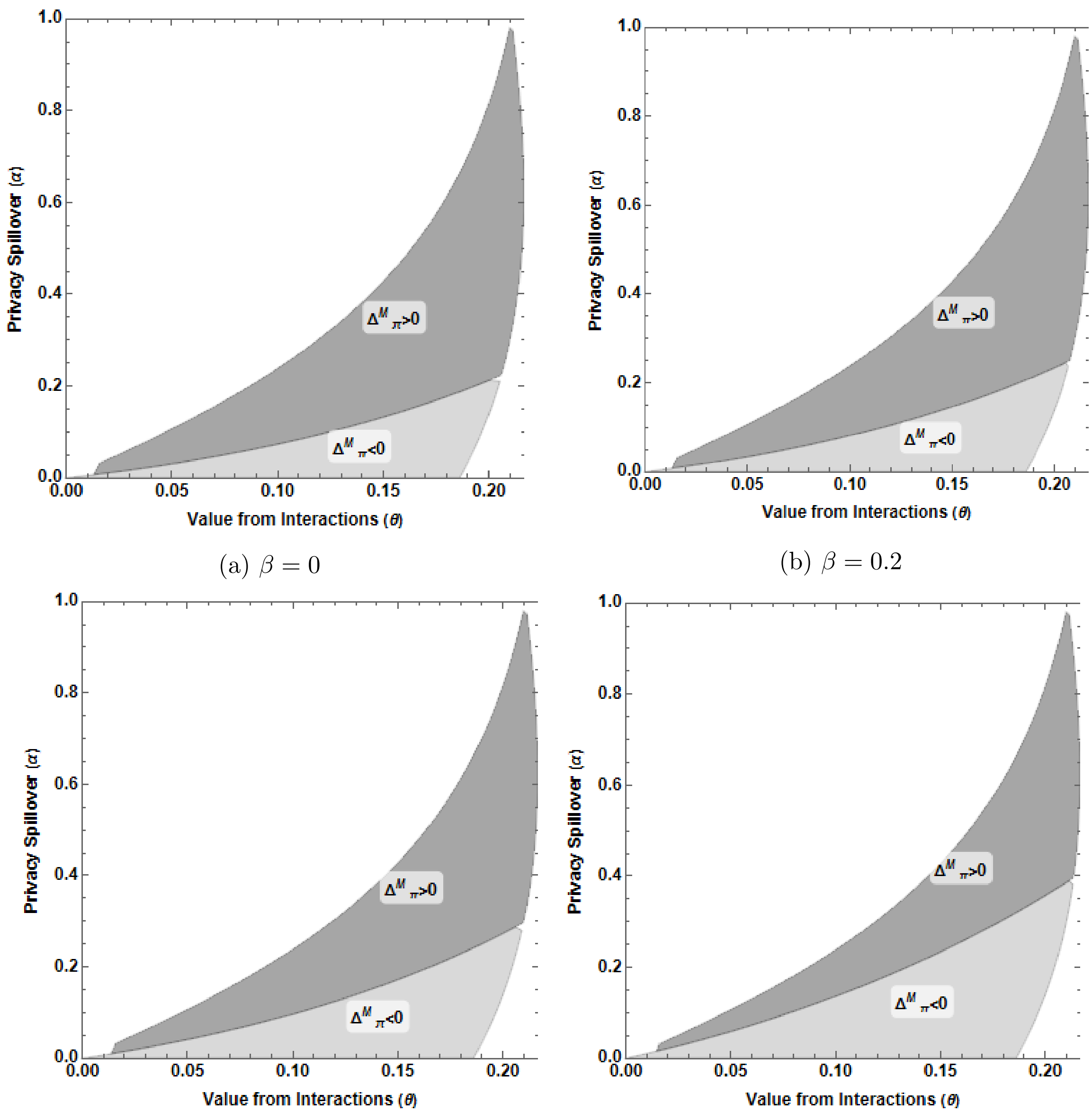
Equating G_A equal to zero and solving, there exists a unique solution given by

$$\beta_A = \frac{\begin{pmatrix} 2\alpha^4\theta(\theta(\theta+6)-3)+2\alpha^3\left(-\left((\theta+16)\theta^2\right)+\theta+2\right) \\ +\alpha^2(\theta(\theta(3\theta+38)-5)-4)+\alpha(\theta(\theta((\theta-3)\theta-25)+9)+2)-(\theta-1)\theta((\theta-1)\theta-10) \end{pmatrix}}{2(\alpha-1)\alpha\left(-\left(3\alpha^2+\alpha+6\right)\theta+((\alpha-1)\alpha+2)\theta^3+(\alpha(6\alpha-7)+3)\theta^2+\alpha+1\right)} > 1.$$

Since $0 < \beta < 1$, it must be that G_A is always negative. Thus, we confirm that $\Delta D_A < 0$.

■

Proof of Proposition 7. The difference in equilibrium profit of service A between the interoperability versus the no-interoperability case is denoted as $\Delta_\pi^M \triangleq \pi_A^I - \pi_A^M$. The conditions are quite complex and therefore, we simulate the results for the admissible parameter space.



(a) $\beta = 0$

(b) $\beta = 0.2$

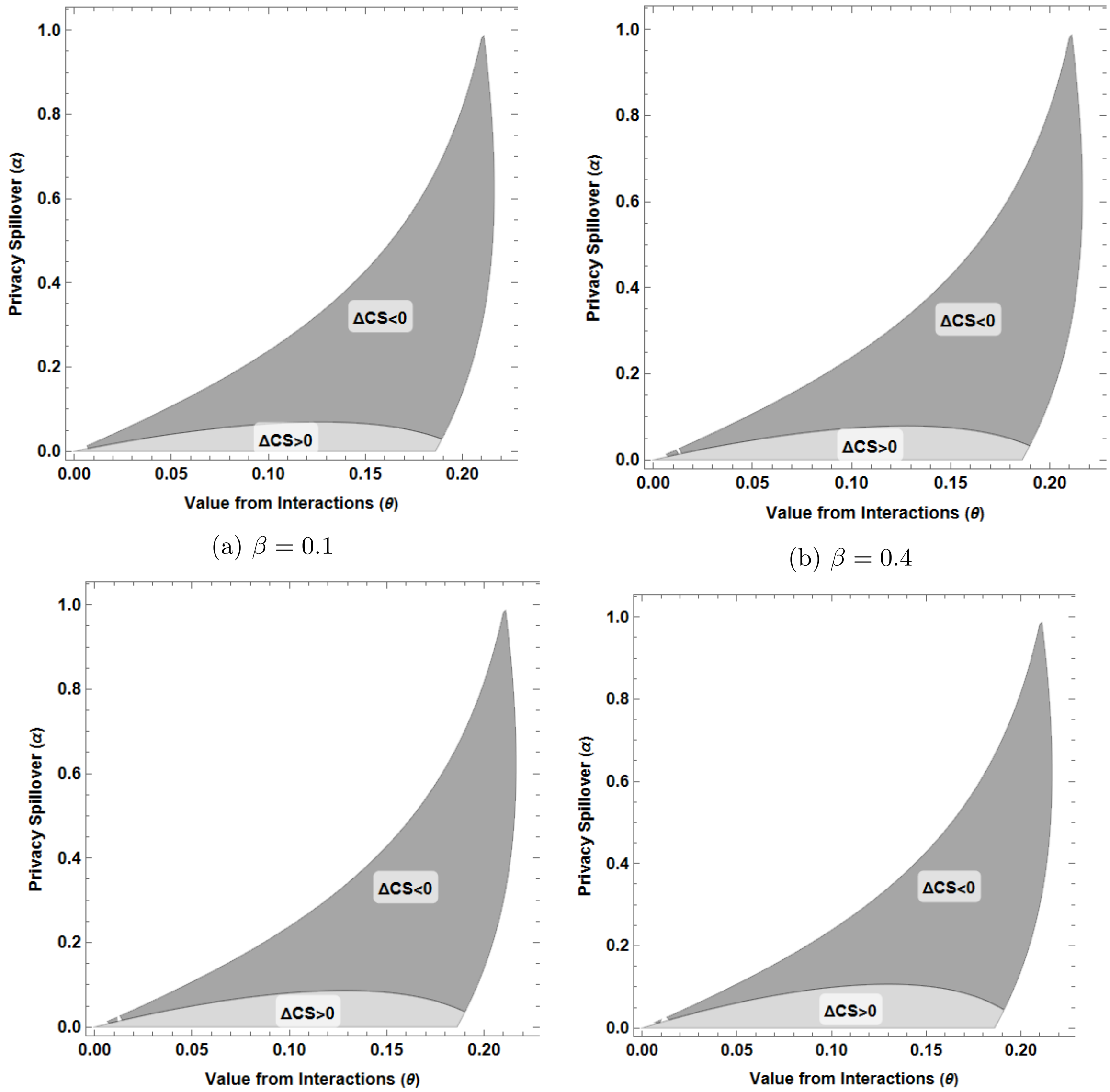
(c) $\beta = 0.5$

(d) $\beta = 1$

Figure 3: The following graph compares the profit of service A under interoperability with the profit of service A under no interoperability, that is, $\Delta_\pi^M = \pi_A^I - \pi_A^M$

■

Proof of Proposition 8. We provide a graphical proof of our results where the figure below plots the feasible region and shades the areas where $\Delta_{CS}^M = CS^I - CS^M > 0$ or $\Delta_{CS}^M = CS^I - CS^M < 0$.



(a) $\beta = 0.1$

(b) $\beta = 0.4$

(c) $\beta = 0.6$

(d) $\beta = 1$

Figure 4: The following graph compares the total consumer surplus under interoperability with the consumer surplus no interoperability, that is, $\Delta_{CS}^{M} = CS^{I} - CS^{M}$

The result on the difference in consumer surplus with respect to β is a direct consequence of the results presented in Lemma 5. ■

Proof of Proposition 1. This result is straightforward and is a direct consequence of the results discussed in Proposition 4. ■

Proof of Proposition 2. From Proposition 2, it is obvious that for $\beta > 1/2$ it is optimal to set $\alpha = 0$. Instead for $\beta < 1/2$, to ensure the highest possible market share level of service B, it is optimal to allow spillovers up to $\alpha = \alpha_{MS}$. This gives us the highest level of market share service B can achieve. ■

Proof of Proposition 3 In the following, we first discuss the single-homing case and then the multi-homing case.

Single-homing. The payoff of service B when there is no interoperability and consumers single-home is given as

$$\pi_B^S = \omega \cdot D_B^S(\psi^S) + (1-\omega) \cdot CS^S.$$

The payoff of service B under mandated interoperability is given as

$$\pi_B^I = \omega \cdot D_B^I + (1-\omega) \cdot CS^I.$$

We denote $\Delta\pi_B^S$ as the difference in profit of service B under interoperability with the case of no interoperability and is given as

$$\Delta\pi_B^S = \pi_B^I - \pi_B^S = \omega(D_B^I - D_B^S(\Psi^S)) + (1-\omega)(CS^I - CS^S).$$

Before proceeding further, it is worth noting that under our regularity Assumptions 1, $\alpha_D^S > \alpha_{CS}^S > 0$. Thus, it is straightforward that in the interval $\alpha \in (\alpha_{CS}^S, \alpha_D^S)$, user demand of service B under interoperability is higher ($D_B^I - D_B^S(\Psi^S) > 0$) but total user surplus under interoperability is lower ($CS^I - CS^S < 0$). In this case, there is a tension between these two components of the payoff of service B. Since $\Delta\pi_B^S$ is an affine combination of these two parts, by continuity we can state that for $\omega > \omega_S$, service B chooses interoperability but no interoperability will be a user surplus improving choice. In all other cases, the interoperability decision of service B is aligned with total user surplus enhancing choice.

Multi-homing. The payoff of service B under no interoperability and multi-homing consumers is given as

$$\pi_B^M = \omega \cdot (D_{B,E}^M(\psi^M) + D_M^M(\psi^M)) + (1-\omega) \cdot CS^M.$$

As before, we denote $\Delta\pi_B^M$ as the difference in payoff of service B under interoperability with the case of no interoperability (under multi-homing) and is given as

$$\Delta\pi_B^M = \pi_B^I - \pi_B^M = \omega(D_B^I - D_{B,E}^M(\psi^M) - D_M^M(\psi^M)) + (1-\omega)(CS^I - CS^M).$$

We note the following facts from previous results. From the results presented in Proposition 6, we know that $D_B^I - D_{B,E}^M(\psi^M) - D_M^M(\psi^M) > 0$ always holds. Instead, from the results on the comparison of total user surplus as presented in Proposition 8, we know that $CS^I - CS^M > 0$ if and only if $\alpha < \alpha_{CS}^M$. In addition, $\Delta\pi_B^M$ is an affine combination of $D_B^I - D_{B,E}^M(\psi^M) - D_M^M(\psi^M)$ and $CS^I - CS^M$ with coefficients ω and $(1-\omega)$ with $\Delta\pi_B^M > 0$ at $\omega = 1$.
Bearing the above facts in mind, it straightforward that when $\alpha > \alpha_{CS}^M$ for ω large enough, it must be that the service B chooses interoperability but this choice hurts total user surplus. In all other cases, the decisions of the privacy preserving service are aligned with the user surplus enhancing choice. ■

References

Acquisti, A., Taylor, C. & Wagman, L. (2016), 'The economics of privacy', *Journal of Economic Literature* **54**(2), 442–492.

Adner, R., Chen, J. & Zhu, F. (2020), 'Frenemies in platform markets: Heterogeneous profit foci as drivers of compatibility decisions', *Management Science* **66**, 2432–2451.

Argenziano, R. & Bonatti, A. (2023), 'Data markets with privacy-conscious consumers', *AEA Papers and Proceedings* **113**, 191–196.

Armstrong, M. (1998), 'Network interconnection in telecommunications', *Economic Journal* **108**(448), 545–564.

Bakos, Y. & Hallaburda, H. (2020), 'Platform competition with multi-homing on both sides: Subsidize or not?', *Management Science* **66**(12), 5599–5607.

Bourreau, M. & Krämer, J. (2022), 'Interoperability in Digital Markets: Boon or Bane for Market Contestability?', Mimeo.

Bourreau, M., Krämer, J. & Buiten, M. (2022), 'Interoperability in Digital Markets', Report. Centre on Regulation in Europe (CERRE).

Bourreau, M., Raizonville, A. & Thebaudin, G. (2023), Interoperability between ad-financed platforms with endogenous multi-homing. Unpublished workinger paper.

Brown, I. (2020), 'Interoperability as a Tool for Competition Regulation', Report. Open Forum Academy.

Bundesnetzagentur (2021), '*Interoperabilität zwischen Messengerdiensten – Überblick der Potenziale und Herausforderungen*', Report.

Crémer, J., Rey, P. & Tirole, J. (2000), 'Connectivity in the commercial internet', *Journal of Industrial Economics* **48**, 433–471.

Doganoglu, T. & Wright, J. (2006), 'Multihoming and compatibility', *International Journal of Industrial Organization* **24**, 45–67.

Economides, N. & White, L. J. (1994), 'Networks and compatibility: Implications for antitrust', *European Economic Review* **38**, 651–662.

Fainmesser, I., Galeotti, A. & Momot, R. (2023), 'Digital privacy', *Management Science* **69**(6), 3157–3173.

Galperti, S., Levkun, A. & Perego, J. (2023), The value of data records. Forthcoming *Review of Economic Studies*.

Galperti, S. & Perego, J. (2023), 'Privacy and the value of data', *AEA Papers and Proceedings* **113**, 197–203.

Katz, M. L. & Shapiro, C. (1985), 'Network externalities, competition, and compatibility', *American Economic Review* **75**, 424–440.

Ke, T. T. & Sudhir, K. (2023), 'Privacy rights and data security: GDPR and personal data markets', *Management Science* **69**(8), 4389–4412.

Laffont, J. J., Tirole, J. & Rey, P. (1998), 'Network competition: I. overview and nondiscriminatory pricing', *RAND Journal of Economics* **29**(1), 1–37.

Maruyama, M. & Zennyo, Y. (2013), 'Compatibility and the product life cycle in two-sided markets', *Review of Network Economics* **12**, 131–155.

Maruyama, M. & Zennyo, Y. (2015), 'Application compatibility and affiliation in two-sided markets', *Economics Letters* **130**, 39–42.

Rasch, A. (2017), 'Compatibility, network effects, and collusion', *Economics Letters* **151**, 39–43.

Rasch, A. & Wenzel, T. (2014), 'Content provision and compatibility in platform markets', *Economics Letters* **124**, 478–481.

Scott Morton, F. M., Crawford, G. S., Crémer, J., Dinielli, D., Fletcher, A., Heidhues, P. & Seim, K. (2021), 'Equitable Interoperability: The "Super Tool" of Digital Platform Governance', Policy Discussion Paper No. 4, Digital Regulation Project. Yale Tobin Center for Economic Policy.

Viecens, M. (2011), 'Compatibility with firm dominance', *Review of Network Economics* **10**, Article 4.