Promotional Allowances: Loss Leading as an Incentive Device*

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Abstract

A retailer may boost the demand for a manufacturer's product through unobservable promotional efforts. Fixed fees cannot be used to freely allocate profit within the vertical structure. When manufacturers have market power, the equilibrium wholesale contract features a retail price below cost together with a rebate for incremental units bought by the retailer when effort has succeeded in boosting sales. Loss leading emerges as an incentive device in such an incomplete contracting scenario. A ban on below-cost pricing leads to higher retail prices and lower promotional efforts.

Keywords: Vertical restraints; Loss leading; Promotional allowances; Resale-below-cost laws.

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1. Introduction

Vertical relationships between producers and their distributors are governed by a variety of contractual agreements, ranging from simple linear prices to more sophisticated contractual provisions. The literature on the law and economics of vertical restraints is large, which certainly reflects the diversity of practices encountered in real-life.[1] One practice, although pervasive, has surprisingly been overlooked by the existing literature: the use of promotional allowances.

Manufacturers pay mostly two types of allowances to their retailers: slotting fees and promotional allowances. Slotting fees are fixed payments given in return for a either a prominent placement on the retailer's shelves or any space at all in the case of new products. Promotional allowances are, by contrast, variable rewards, contingent on future performance, earned when retailers boost sales through their own promotional efforts. The literature has essentially focused on the role of slotting fees along the value chain with a particular emphasis on their competitive effects. As a matter of fact, slotting fees have been shown to have either pro-competitive (Chu, 1992; Foros et al. 2009) or anti-competitive (Shaffer, 1991; Marx and Shaffer 2007) depending on contexts. However, much less is known on the role of promotional allowances, despite their importance in practice. To illustrate, some large retailers such as Wal-Mart or Cosco have never asked for slotting fees (Kuksov and Pazgal, 2007), while promotional allowances represent the bulk of vendor allowances earned by SafeWay.[2] Allen et al. (2011) argue that sales incentives and discounts paid by PepsiCo have exploded, accounting for 30% of its net revenues in 2009.

Although the economics of promotional allowances developed in this paper is simple and intuitive, our analysis delivers several new important insights. Some are related to the *raison d'être* for such restraints. Others are related to the consequences that some regulations might have on those practices and more generally on market conduct. First, below-cost pricing coupled with rebates emerges as a way to provide the retailer with incentives to boost the sales of the manufacturer's product. In our setting, below-cost pricing is thus purely an incentive device and has neither an anti-competitive nor an exploitative purpose. Second, whether a ban on below-cost pricing has a bite depends on the intensity of competition between manufacturers. Third, when effectively binding, a ban on below-cost pricing has the unexpected, and certainly undesirable, consequences to raise retail prices and reduce demand-enhancing promotional efforts.

To obtain these insights, our analysis relies on two key features of any manufacturer-retailer relationship. First, the retailer promotes the manufacturer's product through advertising campaigns, adequate disposal, and the like. Although, some dimensions of the retailing activity can be somewhat described contractually, other dimensions remain non-verifiable. Moral hazard is a key aspect of those relationships, a point forcefully made by Rey and Tirole (1986). Second, wholesale contracts cannot allocate profits freely between manufacturers and retailers, especially when fixed fees/slotting allowances are

[1] See, for instance, Rey and Vergé (2008) for a survey on vertical restraints and their legal treatment.

[2] According to SafeWay annual report to the SEC (2015, p. 28) (available at http://investor.safeway.com/phoenix.zhtml?c=64607&p=irol-sec) vendor allowances totaled to $2.5 billion in 2014, $2.4 billion in 2013 and $2.3 billion in 2012 and "[...] promotional allowances make up the vast majority of all allowances."

absent or limited in size; an incomplete contract assumption often made in the literature that echoes casual evidence. Wholesale contracts can thus rely only on a wholesale price and a retrospective rebate earned when the retailer's promotional effort has induced a boost in demand for the manufacturer's product. The incentive role of such a rebate is clear: The retailer exerts enough promotional effort to boost demand and enjoys the rebate whenever such boost realizes.

Below-Cost Pricing. The important issue is to determine how the manufacturer reconciles the twin objectives of extracting the retailer's profit and inducing promotional effort. Contractual limits, and more specifically the absence of fixed fees, certainly prevent making the retailer the residual claimant for its effort to boost sales. Our analysis unveils that an optimal wholesale contract induces below-cost pricing at the retail level by imposing a high wholesale price, raising the retailer's cost. At the same time, the retailer is rewarded for its promotional effort by means of a significant rebate earned only when demand has been boosted.

The logic of the argument is intuitive. Below-cost pricing acts as a bonding device, forcing a negative base profit, thereby obliging the retailer to exert enough promotional effort to boost demand and cover its loss with the windfall profit that comes when effort has been successful. Hence, the manufacturer plays on a *stick and carrot strategy* to induce effort and reap downstream profits. The stick is the threat of only keeping negative base profits when demand remains at its base level. The carrot is the possibility to be rewarded with a rebate when effort has been successful in boosting demand. A rough yet useful analogy is that everything happens as if the good was marketed on two different markets. On the *base market*, the loss leader, the retailer sells below cost whereas on the *extra market*, the retailer pockets rebates.

Sales-Below-Cost Laws. Such a *stick and carrot* strategy is no longer available when the market is regulated by sales-below-cost laws. Sales-below-cost laws are often promoted explicitly to protect low-volume and high-price suppliers, notably smaller producers. These laws have a long history throughout the world, with mixed views on their overall impact. Several Member States in the European Union, as well as several States in the U.S., prohibit sales below cost, although the nature and the scope of such bans vary substantially (Keirsbilck, 2012; Calvani, 2001).[3]

Leaving aside the reasons why such laws are enacted in the first place, we analyze their impact on market conduct. When the manufacturer has no bargaining power vis-à-vis its retailer, the wholesale price is set at marginal cost; a ban on below-cost pricing has thus no bite. A ban on below-cost pricing imposes a binding constraint on the retail price only when the manufacturer has some bargaining power vis-à-vis its retailer. Any shift in the wholesale price is now passed through, at least partly, to final consumers. A naive view could then argue that, when binding, such a ban forces the retail price to be equal to the wholesale price, and, thus leads to more allocative efficiency. This is an erroneous view.

[3]Ireland and France are two cases in point. The Irish "Groceries Order" banned sales below net invoice prices, thereby prohibiting retailers from passing on to customers off-invoice discounts they receive from suppliers. The Irish Competition Agency has shown that, for the 2001-2005 period, food covered by Groceries Order have risen in price while food not covered have fallen (see Irish Competition Agency, 2005). Biscourp, Boutin and Vergé (2013) report a similar inflationary effect of the so-called *Loi Galland* in France, which prevents retailers from passing on to consumers anticipated rebates that come at the end of the year.

In fact, imposing such a law can only change market conduct when, in its absence, the retail price would be below the wholesale price. As stressed above, this arises precisely when the manufacturer has a strong market power.

Banning below-cost pricing significantly limits the ability of the manufacturer to extract the downstream profit. The wholesale contract can no longer play on a 'stick' to induce effort since the base profit can no longer be negative. In order to promote effort, the manufacturer must increase the rebate and give up a *moral hazard rent* to the retailer. Echoing a basic tenet of the Theory of Incentives,[4] the manufacturer must now reduce that rent and, to do so, induces a lower level of effort from the retailer at the optimum. At the same time, a ban on below-cost pricing limits the ability of the manufacturer to capture downstream profits. Overall, under weak specifications on preferences and technologies, we show that a ban on below-cost pricing increases the retail price and decreases the retailer's effort.

Application and Extensions. In 2014, several manufacturers of household and hygiene products have been condemned for collusion by the French Competition Authority. Collusion aimed both to maintain artificially high retail prices but also to limit the amount of retailers' rebates and the levels of promotion. Since the *Loi Galland* prevented below-cost pricing, we use our analysis to sketch how the profit loss suffered by the concerned retailers should be computed. Three relevant extensions to our model are then considered: the case of manufacturers producing differentiated products; the case of competition by an integrated retailer that does not rely on the manufacturers' product; the case of complementary products sold by the retailer. We show that our insights carry over naturally to all these scenarios.

Literature Review. That retailers can increase the sales of manufacturers' products is a central tenet of the literature on vertical relationships. With simple contracts between manufacturers and retailers, and when retailers cannot fully appropriate the benefits of their efforts because of free-riding or spillovers, vertical restraints find a possible pro-competitive rationale. That argument, first made by Tesler (1960), has been extended to more general settings by Mathewson and Winter (1984), Rey and Tirole (1986), Krishnan and Winter (2007), Kastl, Piccolo and Martimort (2011), and Hunold and Muthers (2017). We do not consider externalities across retailers and focus, as in Winter (1993), on a vertical externality between the retailer and the manufacturer. Our main point of departure is that we assume that the retailer's effort has some observable, but random, impact on the demand for the manufacturers' product. Incentive payments, such as rebates conditional on performance, can thus be used.[5]

Several explanations have been pushed forward to explain below-cost pricing. Loss leading emerges as an advertising strategy in Ellison (2005): When add-on prices are unobserved firms may advertise a base good at a low price so as to sell add-ons at high unadvertised prices. Bliss (1988) views loss leading as a cross-subsidization strategy between products with different demand elasticities, an idea further developed in Beard and Stern (2008) and Ambrus and Weinstein (2008). Chen and Rey (2012, 2016) show that, with asymmetric competition between retailers, loss leading facilitates screening

[4]See Laffont and Martimort (2002, Chapter 4).

[5]Lømo and Ulsaker (2016) view promotional allowances as fixed payments used discretionarily, in addition to two-part tariffs, by manufacturers in a relational contracting framework.

of consumers according to their shopping costs. Finally, loss leading is a response to vertical opportunism in Allain and Chambolle (2011). In stark contrast with all these papers, loss leading emerges in our simpler setting (one retailer, one product, perfectly informed consumers) as a disciplinary device to solve a simple (but overlooked) moral hazard problem on the retailer's side.

ORGANIZATION OF THE PAPER. Section 2 sets up the model. Section **??** deals with the polar cases of competitive manufacturers and a monopoly manufacturer respectively. Section 5 uses our analysis to study the impact of a ban on below-cost pricing. Section **??** extends our basic set-up to more complex scenarios where the manufacturer does not control all activities of his retailer, either because it is vertically integrated itself or because it sells other products. All proof are relegated in the Appendix.

2. MODEL AND BENCHMARKS

MODEL. We consider the bilateral relationship between an upstream manufacturer M, who produces at marginal cost c, and a downstream retailer R, whose cost is normalized to 0 without loss of generality. Given a retail price p, the demand for the good is denoted by $D(p)$, with $D'(p) < 0$ for all price p such that $D(p) > 0$.

The retailer exerts an effort e that might boost demand for the manufacturer's product. For instance, the demand for standard services and products can be increased by improving promotional services. For more complex products, retailers can improve customers' information and reduce search costs. We denote by $\psi(e)$ the retailer's cost of exerting promotional effort e. We assume that $\psi(\cdot)$ is strictly increasing and sufficiently convex to ensure interior solutions to all optimization problems below ($\psi'(\cdot) > 0$, $\psi''(\cdot) > 0$) and $\psi(0) = 0$. For future reference, we also denote $\eta = \psi'^{-1}$ the inverse marginal disutility of effort (with thus $\eta' > 0$). Finally, we consider a scenario where effort e is non-verifiable so that the vertical relationship between the manufacturer and his retailer is plagued by moral hazard.

We normalize effort so that e belongs to $[0, 1]$. This normalization allows us to identify the retailer's effort with the probability that consumer demand jumps from $D(p)$ to $(1+\theta)D(p)$ where $\theta \geq 0$ is a scale parameter. With the complementary probability $1-e$, the demand remains equal to its base level $D(p)$. Since it raises revenues from sales, a positive shock on demand is of course observable by the manufacturer.

The manufacturer offers a wholesale contract, which consists of a (per-unit) wholesale price w paid by the retailer and a rebate z paid by the manufacturer on all incremental sales that might arise when demand has been boosted by the promotional effort beyond the base level. This means that the retailer receives $\theta D(p)z$ when demand goes beyond its base level. The retailer accepts or rejects that contract and, upon acceptance, exerts a promotional effort e and chooses a retail price p. When demand is high, the manufacturer pays the rebate z for all incremental units sold.[6]

THE VERTICALLY INTEGRATED STRUCTURE. Suppose that the upstream manufacturer is vertically integrated with the downstream retailer. We take the standard short-cut that

[6] An equivalent but more abstract formulation would be that the manufacturer offers two different wholesale prices, one when the demand remains at its base level $D(p)$ and another one when the demand jumps at $(1 + \theta)D(p)$. In practice, this solution may be hard to implement because wholesale prices are set before demand realizes.

integration gives access to information and facilitates control.[7] The manufacturer thus dictates the choice of promotional effort made by the retailer. The integrated outcome maximizes the overall industry profit

$$\Pi_I(p, e) = \pi(p, c)(1 + \theta e) - \psi(e),$$

where $\pi(p, c) = (p - c)D(p)$ stands as the 'base profit' absent promotional effort.

Throughout, we shall assume that the following condition holds.

ASSUMPTION 1.

$$p + \frac{D(p)}{D'(p)} \textit{ is decreasing.}$$

Assumption 1 holds for most of usual demand specifications (linear, exponential, constant elasticity, etc.). It ensures that $\pi(p, c)$ and $\Pi_I(p, e)$ are both quasi-concave in p.

The monopoly outcome (p^m, e^m) that maximizes the profit of the vertically integrated structure is thus readily obtained as follows:

$$p^m - c = -\frac{D(p^m)}{D'(p^m)}, \tag{2.1}$$

and

$$\psi'(e^m) = \theta\pi^m, \tag{2.2}$$

where $\pi^m = \pi(p^m, c)$ stands for the monopoly profit.

Since the promotional effort boosts demand multiplicatively, the monopoly price always maximizes profit whether the demand has been scaled up or not. The optimal effort thus simply trades off the marginal benefit coming from enjoying some extra monopoly profit $\theta\pi^m$ beyond the base level against the retailer's marginal disutility of effort.

COMPETITIVE MANUFACTURERS. Assume that several manufacturers produce perfect substitutes and compete in wholesale contracts for the exclusivity of the retailer's services. Head-to-head competition between manufacturers is akin to shifting all the bargaining power towards the retailer. Competition thus drives the manufacturers' profit to zero, or

$$((w - c)(1 + \theta e) - \theta e z)\, D(p) = 0. \tag{2.3}$$

It is straightforward to see that the simple wholesale contract with a wholesale price that just covers the manufacturer's marginal cost and no rebate (i.e., $(w^d = c, z^d = 0)$) ensures that manufacturers makes zero profit.[8] Moreover, this solution clearly aligns the retailer's objectives with those of the vertically integrated structure.

PROPOSITION 1. *When manufacturers compete head-to-head for the retailer's services, the industry achieves the vertically integrated outcome with a retail price and a promotional effort given by*

$$p^d = p^m \textit{ and } e^d = e^m.$$

[7]See for instance Arrow (1975) and Riordan (1990).
[8]Superscript 'd' refers to the fact that with competitive manufacturers everything happens as if the downstream retailer had all the bargaining power.

An immediate corollary is that rebates are useless in this setting. With upstream competition between manufacturers, the retailer ends up being residual claimant for the consequences of his promotional effort. From the perspective of the vertically integrated structure, the retailer has thus the right incentives to exert such an effort.[9] Importantly, this outcome is such that[10]

$$p^m = p^d > w^d = c.$$

The retail price p^d is thus always above the wholesale price w^d. Imposing a ban on below-cost pricing would have no bite when manufacturers have no bargaining power.

TWO-PART TARIFF. It is well known, at least since Dixit (1983) and Mathewson and Winter (1984), that the vertically integrated profit is also achieved when the manufacturer and the retailer are two independent units as long as the manufacturer uses a two-part tariff (w, F) to regulate the relationship with his retailer. To see why in the context of our model, suppose that the manufacturer charges a wholesale price equal to its marginal cost, $w = c$. The retailer is again the residual claimant for the choice of retail price and promotional effort. If, furthermore, the rebate is null ($z = 0$), the retailer *de facto* maximizes the industry overall profit (up to the constant fee), i.e.,

$$\Pi_I(p, e) - F.$$

The solution thus coincides with the integrated outcome (p^m, e^m). The manufacturer then optimally sets the fixed fee

$$F^m = (1 + \theta e^m)\pi^m - \psi(e^m)$$

to capture the whole downstream profit of the retailer. An immediate corollary, but an important one in view of the rest of our analysis, is that when fixed fees are available, rebates are useless.

Using the price and effort levels given by (2.1) and (2.2), the retailer's profit, gross of the fixed fee, may be rewritten as $\pi^m + R(e^m)$, where $R(e) = e\psi'(e) - \psi(e)$ is the retailer's *moral hazard rent* to use the jargon of the Theory of Incentives. This rent is the amount of profit that must be given up by the manufacturer to the retailer in order to induce a level of effort e.[11] Observe that, when failing to increase demand, the retailer only earns the base profit π^m and thus incurs a loss worth

$$\pi^m - F^m = -R(e^m) < 0. \tag{2.4}$$

The retailer's profit is thus fully extracted by the manufacturer because that loss is compensated by the positive moral hazard rent that the retailer receives following a boost in demand.

RATIONALE FOR NOT USING FIXED FEES. Suppose now that the retailer incurs a

[9]Observe that there are actually other equilibrium wholesale contracts that lead to the same retail price, effort level and allocation of surplus. Any pair (w, z) such that (2.3) is binding leads the retailer to perform effort e^m and to charge a price p^m. Introducing a small degree of risk-aversion on the retailer's side selects $(w^d = c, z^d = 0)$ as the unique optimum.

[10]Retrospective payments given at the end of the accounting year are typically not accounted for to evaluate whether the retailer sells below its cost. In terms of our model, the retailer is thus said to sell below its cost when $p - w < 0$.

[11]See Laffont and Martimort (2002, Chapter 4). From the assumptions on $\psi(\cdot)$, we immediately deduce that $R(e) \geq 0$ with $R'(e) \geq 0$ for all effort level $e \in [0, 1]$.

positive fixed cost K (which may stand for a specific investment in some contexts) whose value remains small enough so the base profit net of that cost remains positive. Optimal price and effort remain unchanged. Yet, the profit-extracting fee must now be adapted and reduced to compensate the retailer for that fixed cost. Had the fixed cost been private information for the retailer, new strategic opportunities would be opened. The retailer would like to manipulate the fixed cost so as to pay a lower fee and inflate net profits.[12] Within the framework of a full-fledged modeling of such asymmetric information, screening considerations suggest, at best, a limited role for such fixed fees. This in turn means that the lessons of our simple model, and in particular the prevalence of below-cost pricing, would carry over in such full-fledged model.

Observe also that fixed fees are necessarily paid *ex post*, i.e., once downstream demand and effort disutility are known and profits are realized, especially if the retailer is cash-constrained in the first place. In practice, various shocks, whose values may be hard to contract upon *ex ante*, may affect demand and cost. The retailer might take advantage of those contractual loopholes, behave opportunistically and renege on his earlier commitment to pay back those fees. Avoiding the use of fixed fees is thus a response to such opportunism. Our goal in the present paper is certainly not to develop full-fledged models along those lines. Nevertheless, the formal arguments just sketched would offer some strong motivation for looking at models where fixed fees are not available and wholesale contracts thereby incomplete.

In practice, and as reported in the Introduction, much evidence also suggests that fixed fees (like slotting allowances) might not be as often used as theory predicts; an argument which again calls for analyzing the case of incomplete contracts which are solely based on price instruments (wholesale prices and rebates).

3. A Simple Example

To give some flavor on several of our results in a simple setting, let us consider the following specification of our model. The base level of demand for the manufacturer's product comes from a unit mass of homogeneous buyers with valuation v, with $v > c$. When promotional effort is successful, the demand is scaled up. The mass of consumers with valuation v is now $1 + \theta > 1$. The retailer's cost of effort is quadratic and given by $\psi(e) = \frac{e^2}{2}$.

In this setting, whether the demand has been boosted by the promotional effort or not, the retailer always charges the monopoly price downstream, namely $p^u = v$.[13] In a vertically integrated structure, the overall profit of the industry would thus be

$$(v - c)(1 + \theta e) - \frac{e^2}{2}.$$

This expression is of course maximized at

$$e^m = \theta(v - c),$$

[12]This point is clearly reminiscent of an argument often found in Regulatory Economics to justify that Ramsey-Boîteux pricing might have some appeal in some regulatory settings; see Laffont and Tirole (1993, Chapter 1).

[13]Superscript 'u' stands for the fact that the manufacturer has all the bargaining power in the relationship with the retailer.

and we assume that $\theta(v-c) < 1$ to ensure that the demand-enhancing effort remains interior. The corresponding integrated profit writes as

$$v - c + \frac{1}{2}\theta^2(v-c)^2.$$

Consider now the scenario where the retailer and the manufacturer stand as two separate entities. Upon acceptance of the manufacturer's contract, the retailer also charges a retail price

$$p^u = v \tag{3.1}$$

on the downstream market; exactly as what would be done by the vertically integrated structure. The retailer's profit is then given by

$$(v-w)(1+\theta e) + \theta e z - \frac{e^2}{2}.$$

This expression is strictly concave in e and the optimal effort (when interior) is characterized by the first-order condition

$$e^u(w,z) = \theta(v - w + z). \tag{3.2}$$

On the effort's side, observe that offering a rebate on all incremental sales worth $z = w - c$ would also align the retailer's effort choice with that of the vertically integrated structure.

Expressed in terms of the wholesale contract, the retailer's profit is finally worth

$$\Pi_R(w,z) = v - w + R(e^u(w,z)),$$

where $R(e) = e\psi'(e) - \psi(e) = \frac{e^2}{2}$ stands for a moral hazard rent left to the retailer to induce promotional effort.

The manufacturer sets the wholesale price w and the rebate z so as to maximize her profit

$$\Pi_M(w,z) = (w-c)(1+\theta e(w,z)) - e(w,z)\theta z,$$

anticipating that the retailer accepts the contract if he makes a non-negative profit

$$\Pi_R(w,z) \geq 0.$$

Next proposition summarizes our findings.

PROPOSITION 2. *Assume that buyers are homogeneous with valuation $v > c$.*

1. *The manufacturer offers a wholesale contract (w^u, z^u) that maximizes the profit of the vertical structure with*

$$w^u = v + \frac{1}{2}\theta^2(v-c)^2 \text{ and } z^u = v - c + \frac{1}{2}\theta^2(v-c)^2. \tag{3.3}$$

2. *The retailer makes no profit, exerts the monopoly effort level and charges the monopoly*

retail price:

$$\Pi_R(w^u, z^u) = 0, \tag{3.4}$$

$$e^u(w^u, z^u) = e^m > 0 \text{ and } p^u = p^m = v. \tag{3.5}$$

3. *The retailer's margin is strictly negative when demand remains at its base level, and strictly positive when demand is boosted:*

$$p^u - w^u < 0 < p^u - w^u + z^u. \tag{3.6}$$

First, and foremost, the manufacturer sets a high wholesale price w^u so that the retailer's profit ends up being negative if demand is not boosted by the promotional effort as shown on the left-hand side inequality of (3.6). Simultaneously, the manufacturer pays a substantial rebate $z^u > 0$ so that the retailer makes a positive margin on all incremental sales as shown on the right-hand side of (3.6). The joint use of a wholesale price and a positive rebate creates a wedge between the retailer's profit if his effort boosts the demand and his profit if demand stays at the base level. By setting $z^u = w^u - c$, the manufacturer fully transfers her margin to the retailer when effort has successfully increased the demand. This wedge aligns the retailer's incentives to exert a demand-enhancing effort with those of the vertically integrated structure.

To grasp the retailer's surplus, it is then enough to charge a sufficiently high wholesale price. Because the retailer continues to charge the monopoly price on the downstream market, raising the wholesale price has no impact on demand and acts thus as a pure extractive tool for the manufacturer.

BAN ON BELOW-COST-PRICING. Suppose now that pricing below-cost is banned. In our setting, this implies that the retailer's margin must always remain positive, or $p - w \geq 0$. Pricing incentives of the retailer are unchanged. The retailer continues to charge the monopoly price downstream, or $p^b = v$.[14] A ban on below-cost pricing acts thus as a cap on the wholesale price that can be charged by the manufacturer, or

$$w \leq p^b = v.$$

PROPOSITION 3. *Assume that buyers on the downstream market are homogeneous with valuation $v > c$ and that there is a ban on below-cost-pricing. The optimal wholesale contract (w^b, z^b) offered by a monopoly manufacturer entails the following properties.*

1. *The constraint on the retailer's margin is binding*

$$w^b = p^b = v. \tag{3.7}$$

2. *The promotional effort is distorted downwards:*

$$e^b = \frac{1}{2}e^m < e^u. \tag{3.8}$$

3. *Industry profit is not maximized and the retailer enjoys a strictly positive profit.*

[14] Superscript 'b' stands for a ban on below-cost-pricing.

Imposing a ban of pricing-below-cost is a significant constraint borne by the manufacturer to incentivize the retailer's effort. Boosting the retailer's incentives becomes now costly because creating a wedge between the wholesale price and the rebate is now constrained by the ban on below-cost-pricing. A familiar logic from Incentive Theory bites here.[15] Because only rewards can be used to provide incentives and punishments are banned, the retailer must receive a moral hazard liability rent to exert effort. This rent can no longer be fully grasped by the manufacturer. As a result, the promotional effort is reduced. A ban on below-cost-pricing has thus both an efficiency and a redistributive impact.

Incentive vs Traditional Modes of Vertical Control. We conclude this section with some perspective on the choice of managerial strategies that might be adopted by manufacturers to control their retailers in the economic environment under scrutiny. Suppose that the manufacturer cannot rely on rebates, so that the choice of the demand-enhancing effort is solely decided by the retailer. This scenario can be referred to as being one relying on a 'traditional' mode of vertical control. In this scenario, the manufacturer has only one tool, namely the wholesale price, to simultaneously boost the retailer's incentives to exert effort and extract some of the latter's profit. In contrast, when rebates can be used, managerial control can relay much more on incentives. A first obvious remark is that such an 'incentive' mode of vertical control always dominates traditional control since the manufacturer has now more instruments to regulate the retailer's behavior.

A more subtle insight is that the gain in moving from traditional to incentive control is strongly impacted by whether below-cost-pricing is prohibited or not. We show in the Appendix that this gain increases with the value of the demand-enhancing effort (i.e., with the scale parameter θ), and it is the case whether below-cost-pricing is allowed or not. However, that gain is strictly convex when below-cost-pricing is allowed, whereas it is strictly concave when it is banned. Assuming a fixed cost of changing control modes (possibly due to various administrative and/or negotiation costs), we should thus expect manufacturers to switch towards incentive control whenever below-cost-pricing is possible and the retailer's effort has a sufficiently large impact on demand.

A key feature of the simple example developed in this section is that the retailer's price is not responsive to the manufacturer's wholesale price when demand is perfectly inelastic. As a result, the manufacturer can choose a wholesale price and a rebate to reach the two antagonistic objectives of providing incentives to boost demand and of capturing the retailer's profit, without creating any distortions on the final price. The wholesale price and the rebate act thus like non-distortionary quasi-transfers. In the sequel, we consider a more general demand function. We then show that the wholesale price and the rebate are still used as incentive devices, but they also have a distortionary impact. Likewise, a ban on below-cost-pricing will have both efficiency and distributive consequences.

4. Main Analysis

We now tackle a more general setting in which the buyers' demand for the product when the retail price is p is given by $D(p)$ with $D'(p) < 0$ for all p such that $D(p) > 0$. With an elastic demand and in contrast with Section 3 above, the retail price has both an

[15]See Laffont and Martimort (2002, Chapter 4).

allocative and a redistributive impact along the supply chain. This feature significantly complicates the impact of a ban on below-cost pricing.

The Retailer's Problem. Suppose that the retailer operates under a wholesale contract (w, z). The retail price p and promotional effort e are optimally choosen so as to maximize his expected profit that we now write as

$$(4.1)\quad (p-w)D(p)+e(p-w+z)\theta D(p)-\psi(e)\equiv(p-w)D(p)(1+\theta e)+\theta ezD(p)-\psi(e).$$

This expression makes it clear that the retail price is chosen before the scale parameter θ realizes. Implicitly, the retailer commits to that price beforehand but attracts further demand through promotional effort.

With more compact notations, the retailer's profit in (4.1) can be rewritten as

$$\pi(p, w)+\theta e\pi(p, w-z)-\psi(e).$$

When Assumption 1 holds, this latter profit function is strictly concave in (p, e). Focusing on interior solutions, the first-order optimality conditions associated to the optimal retail price p and effort e are thus respectively given by

$$(4.2)\qquad \pi_p(p, w)+\theta e\pi_p(p, w-z)=0,$$

and

$$(4.3)\qquad \theta\pi(p, w-z)=\psi'(e).$$

We may develop the first-order condition (4.2) and rewrite it as

$$(4.4)\qquad \frac{1}{1+\theta e}\frac{p-w}{p}+\frac{\theta e}{1+\theta e}\frac{p-w+z}{p}=-\frac{D(p)}{pD'(p)},$$

Condition (4.4) shows that the inverse price-elasticity of demand is actually an average of the retail price-cost markups with and without rebate. As the impact of the promotional effort e on demand increases, the retail price becomes more responsive to the rebate z earned on incremental sales and, as such, decreases.

Turning now to the optimality condition (4.3) in terms of effort. Observe that decreasing the wholesale price w and increasing the rebate z increase the retailer's profit from incremental sales (since $\pi(p, w-z)$ is decreasing in its second argument), which boosts incentives to exert effort. Simultaneously, choosing a lower retail price maximizes this profit on incremental sales and also boosts effort.

A Graphical Representation. Figure 1 offers a graphical representation of the retailer's problem when he operates under a contract (w, z) and accordingly chooses an optimal pair $(p(w, z), e(w, z))$. In this respect, the optimal retail price $p(w, z)$ trades off the distortions induced by an inefficiently high price when demand is large and the retailer's perceived marginal cost is low because of a rebate and an inefficiently low price when demand is low and the perceived marginal cost is high. Those inefficiencies of course depend on the shape of the demand function. In terms of effort provision, the retailer chooses an effort level $e(w, z)$ such that the marginal disutility of effort equals the difference in profits between a high and a low realized demand. Adopting these optimal retailing strategies, the retailer's profit can be expressed as

$$\pi(p(w,z),w) + R(e(w,z))$$

where $R(e) = e\psi'(e) - \psi(e) \geq 0$ is the retailer's liability rent.[16]

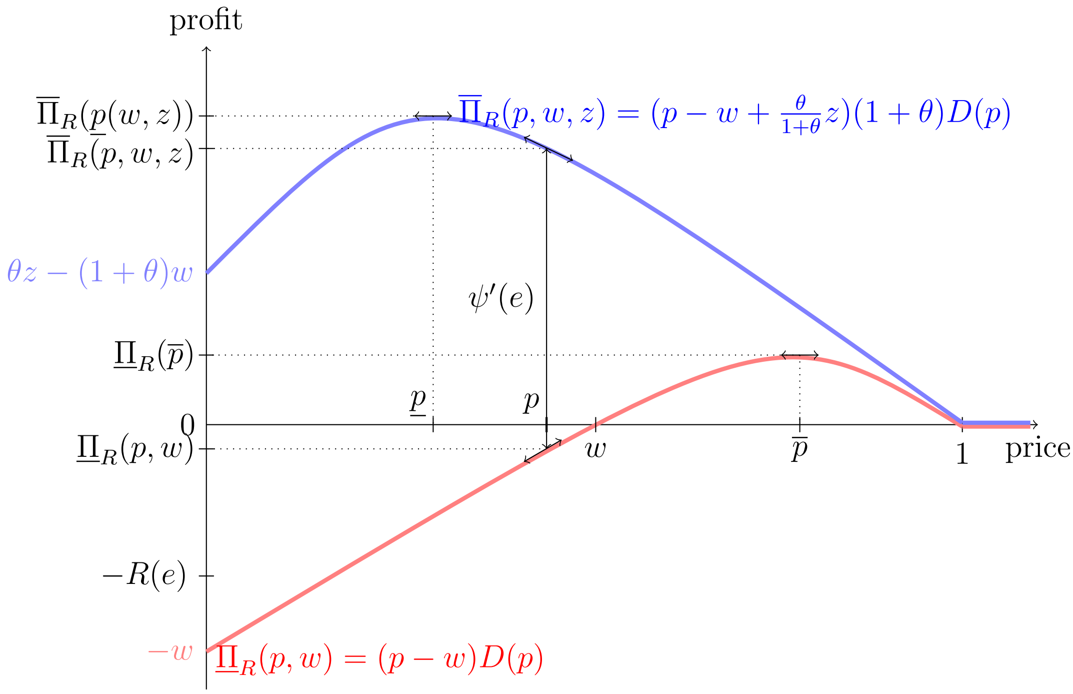


Figure 1: Illustration of the choice of price and effort level by the retailer (with $D(p) = 1 - p$).

Reformulation of the Manufacturer's Problem. The optimality conditions (4.2) and (4.3) also allow us to express the contracting variables (w, z) in terms of the pair (p, e) that the manufacturer induces from the retailer through the wholesale contract. This approach is reminiscent of the principal-agent literature where the focus is not necessarily on the contracting instruments used to implement a given effort profile but, instead, on this effort profile and on the cost for the principal of reaching it. So doing thus yields

$$w = p + \frac{D(p)}{D'(p)}(1+\theta e) + \frac{e\psi'(e)}{D(p)}, \tag{4.5}$$

and

$$z = (1+\theta e)\left(\frac{\psi'(e)}{\theta D(p)} + \frac{D(p)}{D'(p)}\right). \tag{4.6}$$

Equation (4.5) is particularly important in view of our forthcoming analysis of the

[16]See Laffont and Martimort (2002, Chapter 4) for a definition.

role of a ban on below-cost pricing. Indeed, this condition delineates the key restriction on implementable pairs (p, e) that ensures a positive margin $p - w$.

Taking stock of Equations (4.5) and (4.6), we may express the manufacturer's and the retailer's profits, still in terms of the pair (p, e) to be implemented, respectively as:[17]

$$\Pi_M(p, e) = (\pi(p, c) - \varphi(p))\,(1 + \theta e), \tag{4.7}$$

and

$$\Pi_R(p, e) = (1 + \theta e)\varphi(p) - \psi(e). \tag{4.8}$$

The function $\varphi(p) = -D^2(p)/D'(p)$ stems for the retail's base profit once the retailer has optimally chosen his retail price in response to the wholesale contract. This profit is expressed in terms of the retail price only, consistently with our approach that highlights retail decisions and not the wholesale contract that induces these decisions. When Assumption 1 holds, a higher retail price decreases retail profit $(\varphi'(\cdot) < 0)$.[18] On top, we shall further assume that $\pi(p, c) - \varphi(p)$ is quasi-concave in p which requires a slightly stronger version of Assumption 1.

ASSUMPTION 2.

$$p + \frac{D(p)}{D'(p)} - (1 - \lambda)\frac{\varphi'(p)}{D'(p)} \text{ *decreasing for any* } \lambda \in [0, 1].$$

Again, this assumption, that we suppose to hold throughout, is satisfied for most of usual demand specifications (linear, exponential, constant elasticity, etc.).

To better understand the expressions of profits (4.7) and (4.8) and as a reference point for the forthcoming analysis, consider a scenario where the manufacturer does not use rebates. The retailer would maximize

$$(p - w)D(p)(1 + \theta e) - \psi(e).$$

The optimal retail price p induced by a wholesale price w follows the familiar pass-through formula in the context of a double-marginalization scenario à la Spengler (1950), namely

$$w = p + \frac{D(p)}{D'(p)}.$$

When the manufacturer cannot use any fixed fee to reap the entirety of the retailer's profit, charging this wholesale price w would still leave to the retailer a positive profit that is worth

$$(p - w)D(p)(1 + \theta e) - \psi(e) \equiv \varphi(p)(1 + \theta e) - \psi(e). \tag{4.9}$$

The optimal retail price $\tilde{p}$ that the manufacturer would like to induce by a convenient choice of w would thus maximize the sole manufacturer's profit namely $(\pi(p, c) - \varphi(p))(1+$

[17]Notice that the profit of the vertically integrated structure is $\Pi_M(p, e) + \Pi_R(p, e) = \pi(p, c)(1 + \theta e) - \psi(e)$.

[18]Indeed, we may compute $\varphi'(p) = -D(p)\left(2 - \frac{D''(p)D(p)}{(D'(p))^2}\right) = -D(p)\frac{d}{dp}\left(p + \frac{D(p)}{D'(p)}\right) < 0$ where the right-hand side inequality follows from Assumption 1.

θe). Assuming an interior solution, such $\tilde{p}$ is thus defined as

$$\pi_p(\tilde{p}, c) = \varphi'(\tilde{p}).$$

From now on, we shall assume that the corresponding retail profit $\varphi(\tilde{p})$ does not suffice to induce the maximal effort level $e = 1$ that would then best serve the manufacturer's interest.

ASSUMPTION 3.

$$(1 + \theta)\varphi(\tilde{p}) < \psi(1).$$

When Assumption 3 holds, the manufacturer certainly wants to use a rebate to boost the retailer's incentives. Without a rebate, incentives provided by the sole share of the base profit that cannot be appropriated by the manufacturer do not suffice.

OPTIMAL WHOLESALE CONTRACT. This contract must maximize the manufacturer's profit $\Pi_M(p, e)$ subject to the retailer's participation condition

$$\Pi_R(p, e) \geq 0. \tag{4.10}$$

We shall assume that this problem is quasi-concave in (p, e) and denote by (w^u, z^u) the optimal wholesale contract for the manufacturer. Let also (p^u, e^u) be the corresponding retail price and effort level induced by such contract.

PROPOSITION 4. *The optimal wholesale contract (w^u, z^u) satisfies the following properties.*

1. *The retailer makes zero profit:*

$$\Pi_R(p^u, e^u) = (1 + \theta e^u)\varphi(p^u) - \psi(e^u) = 0. \tag{4.11}$$

2. *There is below-cost-pricing when demand remains at its base level:*

$$p^u - w^u < 0. \tag{4.12}$$

3. *Rebates are strictly positive:*

$$z^u > 0. \tag{4.13}$$

Ideally, the manufacturer would like to implement the highest possible promotional effort and induce the heavily distorted retailed price $\tilde{p}$ corresponding to the double-marginalization scenario as defined in (4.9). Unfortunately, doing so would induce the retailer to make negative profits when Assumption 5 holds. The optimal contract must instead ensure that the retailer's break-even constraint (4.10) holds. The optimal contract moves along the retailer's beak-even condition towards a point that maximizes the manufacturer's profit. The wholesale price is used to extract profit whereas the rebate serves to induce the promotional effort. More precisely, using Equation (4.5) and the binding participation constraint (4.10), the retailer's base profit can be expressed as follows:

$$(p^u - w^u)D(p^u) = -R(e^u) < 0. \tag{4.14}$$

This expression echoes the earlier formula (2.4) found in a scenario where fixed fees are available. In particular, Equation (4.14) implies below-cost pricing as stated in (4.12) and a negative base profit; the *stick* side of the mechanism. Instead, the rebate provides the moral hazard rent needed to induce effort; the *carrot* side. At the optimal wholesale contract, the retailer would not recover the loss on the base profit, $(p^u - w^u)D(p^u) < 0$ without exerting at least effort e^u. The loss on base profit thus acts as a bonding device triggering the demand-expanding effort.

We now turn to some important comparative statics.

PROPOSITION 5. *In comparison with the vertically-integrated outcome, the retail price and the promotional effort both increase:*

$$p^u > p^m \text{ and } e^u > e^m.$$

Even though fixed fees are no longer available, the manufacturer still needs to incentivize the retailer for the promotional effort on the one hand and extract the downstream profit on the other hand. Without a fixed fee, this extraction is incomplete but the residual $\varphi(p)$ left to the retailer can also play an incentive role. Increasing p beyond the monopoly outcome increases the retailer profit and thus relax the break even condition.

The effort upward distortion beyond its level in the vertically-integrated structure is more subtle. To see why, it is useful to take as granted the result of Proposition 4 and denote by $E(p)$ the decreasing function implicitly defined through the binding break-even condition (4.10).[19] It is straightforward to show that the optimality condition on effort can be rewritten in terms of p only as follows:

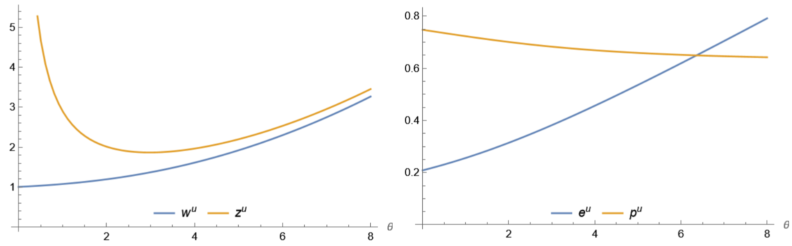
$$\psi'(e^u) = \theta\pi(p^u, c) + (1 + \theta e^u)\frac{\pi_p(p^u, c)}{E'(p^u)}.$$

There are thus two forces that determine the effort distortion. On the one hand, the first term on the right-hand side captures how increasing the retail price from p^m to p^u decreases overall profit ($\pi(p^u, c) < \pi(p^m)$) and this tends to reduce the optimal effort. On the other hand, increasing the retail price along the break-even condition (4.10) also raises effort. This effect is captured in the second term above which is negative (since $\pi_p(p^u, c) < 0$ for $p^u > p^m$ and $E'(p^u) < 0$). Proposition 5 shows that this second effect always dominates.

COMPARATIVE STATICS. We now provide some important comparative statics on the optimal contract. This exercise is generally difficult because of the significant nonlinearities of the problem. Assuming a linear demand $D(p) = 1 - p$ and a quadratic cost of effort $\psi(e) = \frac{3}{2}e^2$ helps us on this front. Indeed, we then determine the optimal contract for $\theta \in [.1, 8]$ and vary θ by an increment of .1 for a total of 80 simulations.[20] Figure 2 represents the optimal contract for the manufacturer, the corresponding price and effort level chosen by the retailer, and the associated retailer's margins (when demand is low and when it is high).

[19] See the Proof of Proposition 10 for details.

[20] Simulations are performed using Mathematica and can be found on the second author's webpage.



(a) Wholesale price and rebate. (b) Price and effort level.

Figure 2: With a linear demand and a quadratic disutility of effort ($D(p) = 1 - p$ and $\psi(e) = \frac{3}{2}e^2$), the optimal contract for the manufacturer (w^u, z^u) is represented in Panel (a) and the optimal price and effort level are depicted in Panel (b).

As the benefits of effort increases (i.e., θ increases), the wholesale price w^u and the bonus z^u becomes more similar (Figure 2 (a)). This result is intuitive. Under those circumstances, there is less need to enlarge the gap between the wholesale price and the bonus to induce effort. More subtle is the fact that the bonus z^u is non-monotonic in the scale parameter. Indeed, there are potentially two effects from raising the bonus. First, it boosts efforts and this first effect is more significant when effort is more valuable. Second, raising the bonus also decreases the retail price, depresses demand and thus reduces the residual profit left to the retailer. Figure 2 (a) shows that this second effect may dominate when effort has a relatively small marginal value.

An intuitive consequence of an increase in the benefits of effort, is that the equilibrium effort also increases (Figure 2 (b)). In turn, the fact that $E'(p) < 0$ implies that the retail price decreases under those circumstances.

5. Impact of Sales-Below-Cost Laws

We assume now that, by regulation, retail prices cannot be set below the retailer's cost. The following non-negativity constraint must thus always hold:

$$p - w \geq 0. \tag{5.1}$$

We investigate the impact of such a constraint on the retail price, the promotional effort, and profits.

Because the retail price margin satisfies the moral hazard incentive constraint (4.5), profits must at least cover the moral hazard rent needed to induce effort. The non-negativity constraint (5.1) becomes a non-negative lower bound on the retailer's profit, or

$$\Pi_R(p, e) \geq R(e), \tag{5.2}$$

where the retailer's profit $\Pi_R(p, e)$ is still given by (4.8).

Since the retailer's profit may also be written as $(p-w)D(p)+R(e)$ (as we did to derive (4.14)), a first consequence of a ban of below-cost pricing appears immediately. The base profit $(p - w)D(p)$ that accrues to the retailer must be non-negative and, therefore, can no longer be used to extract the retailer's overall profit. Put differently, the retailer can no longer run a loss if the promotional effort fails to increase demand. *Sticks* are no longer available and only *carrots* can be used. Implementing a large rebate becomes the only channel to reward effort, and this is obviously costly for the manufacturer.

The contractual constraint on retail price imposed by regulation refers to an argument familiar from the moral hazard literature.[21] When payments to the retailer cannot be negative, the manufacturer must give up a positive moral hazard rent $R(e)$ to induce a positive level of effort e from the retailer. Of course, the retailer's participation constraint (5.2) is hardened in comparison with that that prevails in the absence of a ban, namely (4.10). As a result, Assumption 3 still ensures that this participation constraint is binding at the optimum that we now characterize.

Proposition 6. *Suppose there is a ban on below-cost pricing.*

1. *The retail price is equal to the wholesale price,*

$$p^b = w^b.$$

2. *The retailer makes a strictly positive profit equal to the moral hazard rent,*

$$\Pi_R(p^b, e^b) = R(e^b). \tag{5.3}$$

With a ban on below-cost pricing, the manufacturer's ability to extract the retailer's profit through the wholesale price is reduced because the base profit can no longer be negative. A moral hazard rent must be given up to the retailer.

Satisfying constraint (5.2) is thus clearly more demanding than satisfying the break-even condition (4.10) that prevails absent such a ban. A first intuition would then go as follows. Imposing a ban on below-cost pricing is akin to replacing the cost of effort $\psi(e)$ by a *virtual* disutility of effort $e\psi'(e) = \psi(e) + R(e)$ which is more costly. This calls for lowering the promotional effort. Moreover, lowering the retail price towards the monopoly level p^m also contributes to relaxing constraint (5.2). These are the direct effects of a ban on below-cost pricing.

Indirect effects come from the fact that reducing effort affects marginal incentives to change the retail price, and vice versa. More formally, the value of the Lagrange multiplier associated to the retailer's participation constraint changes as one moves from one institutional environment to the other. A priori, replacing the break-even condition (4.10) by the more stringent condition (5.2) calls for increasing the value of the Lagrange multiplier. This intuition may be misleading, though, since the value of the multiplier depends on the equilibrium choices of price and effort. These indirect effects make the comparison between (p^u, e^u) and (p^b, e^b) difficult. Comparative statics can nevertheless be further explored in some specific environments.

Proposition 7. *Suppose that demand is given by $D(p) = (a - bp)^{\frac{1}{\delta}}$ (with $\delta \geq 1$)*[22] *and the disutility of effort is quadratic ($\psi(e) = \frac{\mu}{2}e^2$). Suppose also there is a ban on below-cost pricing.*

[21] See Laffont and Martimort (2002, Chapter 4).

[22] This class of demand, first used by Bulow and Pfleiderer (1983), has the property that the cost pass-through is constant and given by $1/(1+\delta)$.

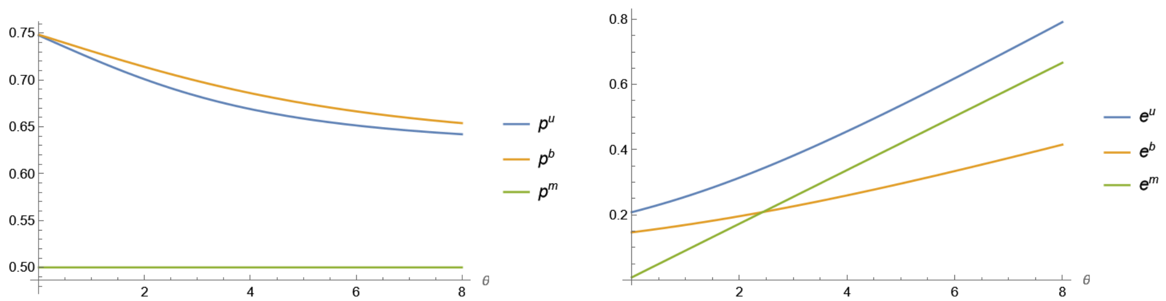
1. *The effort decreases:*
$$e^b < e^u.$$

2. *The retail price decreases:*
$$p^b > p^u.$$

The value of the Lagrange multipliers of the retailer's participation constraints is lower:
$$\lambda^b < \lambda^u.$$

Given that a linear demand ($\delta = 1$) and a quadratic disutility of effort may be viewed as first-order approximations of more complex specifications of preferences and technologies, Proposition 7 strongly suggests that the retail price should increase following a ban of below-cost pricing. Far from promoting competition in the hypothetical scenario where below-cost pricing would be used for predatory purposes, such a law may well harm consumers and reduce overall welfare.

Figure 3 provides a comparison of price and effort as function of the demand shock θ. It shows in particular that the effort level under a ban on below-cost pricing (e^b) may be below or above the efficient level (e^m). Keeping this in mind will be useful later on.



(a) Price. (b) Effort level.

Figure 3: With a linear demand and a quadratic disutility of effort ($D(p) = 1 - p$ and $\psi(e) = \frac{3}{2}e^2$), comparison of the optimal price (Panel (a)) and effort level (Panel (b)) with no ban on below-cost pricing (p^u and e^u), with a ban on below-cost pricing (p^b and e^b), and under integration (p^m and e^m).

6. APPLICATION AND EXTENSIONS

6.1. Quantifying Antitrust Damages in Upstream Collusion Cases.

In 2014, the French Competition Authority condemned several manufacturers of cleansing and hygiene products for collusion over the years 2003 to 2006. These producers were sanctioned with a fine approaching €1B.[23] Some of these manufacturers had

[23] See "Décision n° 14-D-19 du 18 décembre 2014 relative à des pratiques mises en œuvre dans le secteur des produits d'entretien et des insecticides et dans le secteur des produits d'hygiène et de soins pour le corps," available at www.autoritedelaconcurrence.fr/pdf/avis/14d19.pdf. Two sanctions have been pronounced: €345.2M for manufacturers of cleansing products, and €605.9M for manufacturers of hygiene products.

already been sanctioned, in 2011, with a fine of €367.9M for colluding on the sales of consumer detergents, thereby confirming a decision taken by the European Commission.[24]

Collusion was meant not only to maintain artificially high retail prices for these products, but also to limit payments made to retailers for their promotion activities. Within that time frame, the *Loi Galland* was in place and prevented retailers from selling below the net invoice price. That threshold may include any rebates obtained at the date of contracting between the retailer and the manufacturer, but must exclude any retrospective payments earned after the contracting date such as promotional allowances. Last, according to industry experts, fixed payments such as slotting fees are negligible for these products. Although stylized, our model fits well with that case if one is ready to approximate collusive manufacturers with a monopoly. Our model thus gives some guidance on how to evaluate the damages suffered by retailers from such collusion between manufacturers.

Absent collusion, the retailer appropriates the whole profit of the vertically integrated structure while, with collusion and a ban on below-cost pricing, it only appropriates the corresponding moral hazard rent. Formally, the retailer's loss from the collusion between manufacturers Δ is worth

$$\Delta \equiv \underbrace{(1+\theta e^m)\pi^m - \psi(e^m)}_{\substack{\text{Retailer's profit}\\ \text{with upstream competition}}} \quad - \quad \underbrace{R(e^b),}_{\substack{\text{Moral hazard rent under collusion}\\ \text{and no below-cost pricing}}}$$

which may be rewritten as (using Equation (2.2))

$$\Delta = \underbrace{\pi^m}_{\substack{\text{Monopoly}\\ \text{profit}}} + \underbrace{R(e^m) - R(e^b).}_{\substack{\text{Difference in moral hazard rents}\\ \text{with and without collusion}}}$$

Damages are thus the sum of two terms. The first one is the monopoly level of the base profit that accrues to the retailer if, absent collusion, no effort was undertaken. The second term is the incremental value of the moral hazard rent as collusion would be deterred and effort would change from e^b to e^m, which may be positive or negative.

Considering the specification $D(p) = (a - bp)^{\frac{1}{\delta}}$ and $\psi(e) = \frac{\mu}{2}e^2$, it is remarkable that damages can be expressed only in terms of values of the demand function before and after collusion has been deterred, namely $D(p^b)$ and $D(p^m)$. Indeed, the percentage of after-collusion profits that should be paid in terms of damages is worth

$$\frac{\Delta}{\pi^m} = 1 + \frac{4(\lambda^b)^2(1+\delta(1-\lambda^b))^{\delta} - (1+\delta(1-\lambda^b))^{-\delta}}{2(1+\delta-(2+\delta)\lambda^b)} \tag{6.1}$$

where the Lagrange multiplier of (5.2) λ^b is given by $(1+\delta(1-\lambda^b))^{\frac{1}{\delta}} = \frac{D(p^m)}{D(p^b)}$. In other words, the evaluation of damages, a task often viewed as informationally demanding, only

[24]See "Décision n° 11-D-17 du 8 décembre 2011 relative à des pratiques mises en œuvre dans le secteur des lessives," available at `www.autoritedelaconcurrence.fr/pdf/avis/11d17.pdf` and "Commission Decision of 13.4.2011 relating to a proceeding under Article 101 of the Treaty on the Functioning of the European Union and Article 53 of the EEA Agreement (COMP/39579 – Consumer Detergents)," available at `http://ec.europa.eu/competition/antitrust/cases/dec_docs/39579/39579_2633_5.pdf`.

requires the pre- and post-collusion demand levels if one is ready to adopt the chosen specification of the model.

Adopting the same linear-quadratic specification as in Section 4, we can quantify the damages suffered by the retailer from upstream collusion between manufacturers using (6.1). These damages are, obviously always positive, but, perhaps less intuitively, non-monotonic with the demand shock θ.

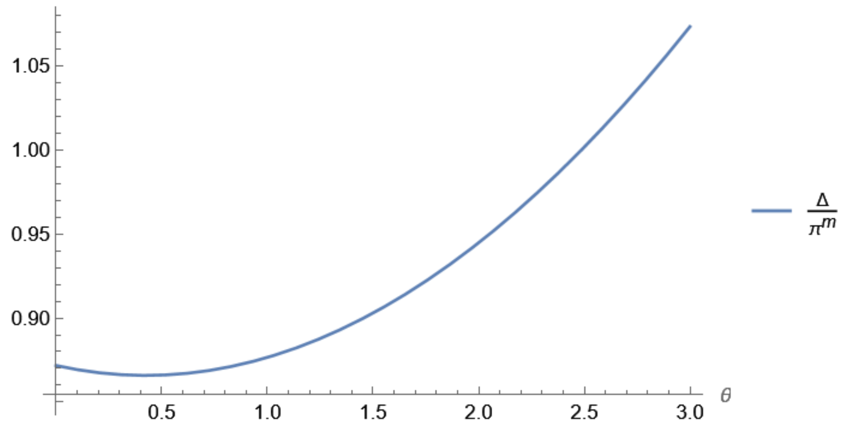


Figure 4: Damages $\frac{\Delta}{\pi^m}$ as function of the demand shock $\theta > 0$ (with $D(p) = 1 - p$, $\psi(e) = \frac{3}{2}e^2$, $c = 0$).

6.2. Traditional vs. Incentive Vertical Control

Let us consider the situation in which the manufacturer does not rely on promotional allowances, i.e., $z = 0$. Faced with a wholesale price w, the retailer maximizes its profit, which writes now as

$$\Pi_R(p, e) = \pi(p, w)(1 + \theta e) - \psi(e).$$

The corresponding first-order necessary conditions for optimality with respect to p and e write now as follows:

$$p + \frac{D(p)}{D'(p)} = w, \tag{6.2}$$

$$\theta\varphi(p) = \psi'(e). \tag{6.3}$$

The manufacturer has only one instrument, the wholesale price w, to control for the retail price and the demand-enhancing effort decided by the retailer. Using (6.2) to express the wholesale price w in terms of p, the manufacturer's profit maximization may be expressed as follows:

$$\max_{(p,e)} \left(\pi(p, c) - \varphi(p)\right)(1 + \theta e) \text{ subject to (6.3)}. \tag{6.4}$$

The condition (6.3) allows us to express effort in terms of price. Assuming quasi-concavity in p of the maximand so obtained, the first-order condition writes as follows:

$$(1 + \theta e^{\emptyset})\left(\pi_p(p^{\emptyset}, c) - \varphi'(p^{\emptyset})\right) + \theta^2 \frac{\varphi'(p^{\emptyset})}{\psi''(e^{\emptyset})}\left(\pi(p^{\emptyset}, c) - \varphi(p^{\emptyset})\right) = 0. \tag{6.5}$$

Absent any promotional effort, the manufacturer would implement the large retail price $\tilde{p}$ as defined in (4.9). When the retailer can boost the demand through its effort, the manufacturer is willing to implement a smaller price to increase the retailer's incentives, because a lower price leads to a higher effort from (6.3). This corresponds to the second term in (6.5). Overall, we have

$$\tilde{p} > p^{\emptyset} > p^m \text{ and } 0 < e^{\emptyset} < e^m.$$

6.3. Competing Against an Integrated Retailer

We now study how the competitive pressure exerted downstream by a vertically-related retailer has an impact on the incentives of a manufacturer to use below-cost pricing with his own non-integrated retailer. This case is of a particular relevance in view of the substantial development of so-called *'hard discounters'* over the last past decades, one of their characteristics being that they mostly distribute their own brands or private labels.[25]

To do so, assume the non-integrated retailer (called retailer 1) faces a vertically-integrated competitor (called retailer 2), that is, retailer 2 produces and sells his own good. Retailers 1 and 2 produce differentiated products with demands $D_i(p_i, p_{-i})$, $i = 1, 2$. For simplicity, demand functions are assumed to be symmetric:

$$D_1(p_1, p_2) = D_2(p_2, p_1) \equiv D(p_1, p_2).$$

Still to avoid unnecessary notational burden, goods 1 and 2 are produced at the same marginal cost c.

Let also assume, for simplicity, that the contract (w_1, z_1) that governs the relationship between the manufacturer and the non-integrated retailer 1 is secret. That contract has thus no commitment value to impact downstream competition. We therefore look for a Nash equilibrium in which the integrated competitor takes as given the contract offered by this vertical structure and the retail price charged by the non-integrated retailer. Denoting by p_1 this price, the integrated retailer's best response $P_2(p_1)$ is then characterized as follows:

$$P_2(p_1) - c = -\frac{D(P_2(p_1), p_1)}{\frac{\partial D}{\partial p_2}(P_2(p_1), p_1)}. \tag{6.6}$$

Under standard assumptions, which are detailed in the Appendix, the best response defined in (6.6) is upward-sloping with a slope smaller than 1.

Competitive Manufacturers. We proceed as in our base scenario, the only change being that, the vertical structure takes now the price p_2 charged by the integrated competitor as given. Mimicking our earlier findings, the best response contract offered by competitive manufacturers entails marginal cost pricing and no rebates, $w_1 = c$ and $z_1 = 0$. From this, it immediately follows that the best response in price $P_1(p_2)$ for

[25] See, for instance, Cleeren et al. (2010) for an account of the development of hard discount chains in Europe. In the U.S., the hard discount format has been relatively absent from the competitive landscape until recently, and is now quickly gaining market shares.

non-integrated retailer 1 satisfies the following condition:

$$P_1(p_2) - c = -\frac{D(P_1(p_2), p_2)}{\frac{\partial D}{\partial p_1}(P_1(p_2), p_2)}. \tag{6.7}$$

Equations (6.6) and (6.7) define the familiar upward-sloping best responses $P_2(p_1)$ and $P_1(p_2)$. Under familiar conditions, those best responses have slope less than 1. In other words, any increase in p_{-i} induces an increase in p_i along the best response that is of a lower magnitude. At a Nash equilibrium, both Equations (6.6) and (6.7) hold and the symmetric Nash equilibrium $p_1^* = p_2^* = p^*$ solves:

$$p^* - c = -\frac{D(p^*, p^*)}{\frac{\partial D}{\partial p_1}(p^*, p^*)}.$$

The optimal effort of the non-integrated retailer is then characterized as follows:

$$\psi'(e^*) = \theta\pi^*,$$

where $\pi^* \equiv (p^* - c)D(p^*, p^*)$.

MONOPOLY MANUFACTURER. Suppose now that the manufacturer has all bargaining power in dealing with the retailer and behaves as an upstream monopoly, yet facing competition from the integrated retailer on the downstream market. The analysis again replicates some of our earlier findings but in an equilibrium context.

For future reference, we denote

$$\pi(p_1, p_2, c) = (p_1 - c)D(p_1, p_2)$$

and

$$\varphi(p_1, p_2) = -\frac{D^2(p_1, p_2)}{\frac{\partial D}{\partial p_1}(p_1, p_2)}.$$

Given the price p_2 charged by the integrated competitor, the manufacturer maximizes its profit

$$\Pi_M(p_1, p_2, e_1) = (1 + \theta e_1)\left(\pi(p_1, p_2, c) - \varphi(p_1, p_2)\right),$$

subject to the retailer's participation constraint

$$\Pi_{R_1}(p_1, p_2, e_1) = (1 + \theta e_1)\varphi(p_1, p_2) - \psi(e_1) \geq 0. \tag{6.8}$$

The following assumption generalizes Assumption 5 to this competitive environment. This condition again ensures that the retailer's participation constraint (6.8) is binding at equilibrium.

ASSUMPTION 4. *For all price p_2,*

$$(1 + \theta)\varphi(\tilde{P}_1(p_2), p_2)) < \psi(1),$$

where

$$\tilde{P}_1(p_2) = \arg\max_{p_1} \pi(p_1, p_2, c) - \varphi_1(p_1, p_2).$$

The best response of the non-integrated structure $P_1^u(p_2)$ is then defined as the solution to

$$\pi_{p_1}(p_1, p_2,) - (1 - \lambda_1^u)\varphi_{p_1}(p_1, p_2) = 0, \tag{6.9}$$

where λ_1^u is the non-negative Lagrange multiplier associated to (6.8). That best response remains upward-sloping and has slope less than one, despite the addition of the third term on the right-hand side of (6.9) provided that this term and its derivative are of a limited magnitude; an assumption that we will make from now on. Because $\varphi_{p_1} \leq 0$ and $\lambda_1^u < 1$, the best response $P_1^u(p_2)$ defined by (6.9) lies above the best response $P_1(p_2)$ obtained when manufacturers are competitive. It then follows that, with a monopoly manufacturer, all retail prices raise at equilibrium in comparison with the scenario of competitive manufacturers. Yet, the price increase is stronger for the non-integrated retailer than for the vertically-integrated competitor.

PROPOSITION 8. *With a monopoly manufacturer, all retail prices increase, but more so for the price charged by a non-integrated retailer:*

$$p_1^u > p_2^u > p^*.$$

BAN ON BELOW-COST-PRICING. The reasoning here should now be familiar from our previous analysis. The constraint on non-negative margin writes now as

$$p_1 \geq w_1.$$

This constraint still implies that the non-integrated retailer must receive a positive moral hazard rent $R(e_1)$. The latter's participation constraint writes thus as follows:

$$\Pi_{R_1}(p_1, p_2, e_1) \geq R(e_1). \tag{6.10}$$

Observe that the moral hazard rent does not directly depend on the competitive pressure exerted by the integrated retailer. Denoting by λ_1^b the Lagrange multiplier of this constraint in the manufacturer's program, we can state the following proposition.

PROPOSITION 9. *Suppose there is a ban on below-cost pricing.*

1. *Provided that $\lambda_1^b \leq \lambda_1^u$, all retail prices increase, but more so for the non-integrated retailer's price:*
$$p_1^b > p_1^u > p_1^* \text{ and } p_2^b > p_2^u > p_2^*,$$
with $p_1^b > p_2^b$.
2. *Assuming linear demands and a quadratic cost of effort, we further obtain that*
$$\lambda_1^b < \lambda_1^u \text{ and } e_1^b < e_1^u.$$

6.4. Multi-Product Retailer With Demand Complementarities

In this section, we evaluate the importance of below-cost-pricing in the more traditional multi-product context where loss leading has already been proved useful for competitive reasons. To illustrate, suppose now that the retailer can also sell another good, say good 2. For notational simplicity, we assume that good 2 is also produced marginal cost c. In this setting, good 1, whose demand is still given by $D(p_1)$ as before, can be viewed as a loss leader. The non-integrated structure stands ready to make a loss on those sales if doing so sufficiently raises demand and profit on good 2. Indeed, consumers may be eager to obtain the loss leader and, when visiting the retailer, they may also express a demand for good 2. In our set-up; such complementary is captured by assuming a simple multiplicative structure; namely the demand for good 2 writes as

$$D_2(p_2, p_1) = D(p_1)\tilde{D}(p_2).$$[26]

The retailer's profit on good 2, conditional on selling good 1, writes as

$$\tilde{\pi}(p_2, c) = (p_2 - c)\tilde{D}(p_2).$$

This profit is maximized at a monopoly price, independent of p_1, which is

$$\tilde{p}^m = \arg\max_{p_2}(p_2 - c)\tilde{D}(p_2).$$

Denoting by $\tilde{\pi}^m$ the corresponding monopoly profit, we may rewrite the retailer's overall downstream profit as

$$(p_1 - w_1 + \tilde{\pi}^m)D(p_1)(1 + \theta e_1) + \theta e_1 z_1 D(p_1) - \psi(e_1). \tag{6.11}$$

Competitive Manufacturers. As in our previous analysis, the wholesale contract $w_1 = c$ and $z_1 = 0$ maximizes the retailer's profit subject to the manufacturer's break-even constraint. The optimal retail price $p^m(\tilde{\pi}^m)$ and effort $e^m(\tilde{\pi}^m)$ are readily obtained from our previous analysis as:

$$p^m(\tilde{\pi}^m) - c + \tilde{\pi}^m = -\frac{D(p^m(\tilde{\pi}^m))}{D'(p^m(\tilde{\pi}^m))}, \tag{6.12}$$

and

$$\psi'(e^m(\tilde{\pi}^m)) = \theta\pi^m(\tilde{\pi}^m), \tag{6.13}$$

where now $\pi^m(\tilde{\pi}^m) = \pi(p^m, c - \tilde{\pi}^m)$ stands for the monopoly profit on good 1.

The comparison with the retail price and effort found in the single-good scenario (see (2.1) and (2.2)) is straightforward. As before, the retailer extracts all profit from competitive manufacturers with a wholesale price equal to marginal cost. Then, conditionally on selling good 1, the retailer makes the monopoly profit on good 2. This profit acts as an implicit subsidy on the cost of producing good 1. This reduces the retail price, boosts demand, profits for good 1 and in turn raises promotional effort:

$$p^m(\tilde{\pi}^m) < p^m, \quad \pi^m(\tilde{\pi}^m) > \pi^m, \quad e^m(\tilde{\pi}^m) > e^m.$$

[26] For a model endogenizing such multiplicative demand structure, see Martimort, Pommey and Pouyet (2022) who develops a model of conditional sales. More complex demand structures could be entertained with no additional insight. The benefit of this multiplicative structure is that it allows us to import *mutatis mutandis* our previous analysis.

MONOPOLY MANUFACTURER. Consider first a scenario where there is no restriction on the wholesale price w_1 that can be charged by the manufacturer. The expression of the retailer's profits in (6.11) higlights that the retail price and effort level that are chosen by the retailer only depend on the *net* wholesale price:

$$\tilde{w}_1 = w_1 - \tilde{\pi}^m$$

which is the opportunity cost for the retailer of buying one extra unit of good 1. When not buying one extra unit of good 1, the retailer also foregoes the monopoly profit $\tilde{\pi}^m$ made on good 2. This extra benefit can *in fine* be captured by the manufacturer whose perceived cost of producing good 1 is now

$$\tilde{c} = c - \tilde{\pi}^m.$$

From there, our previous analysis can be replicated *mutatis mutandis*. First, we need to modify Assumption 3 to account for this change of cost as

ASSUMPTION 5.
$$(1 + \theta)\varphi(\tilde{p}(\tilde{\pi}^m)) < \psi(1)$$
where $\tilde{p}(\tilde{\pi}^m)$ *is now defined as*

$$\pi_p(\tilde{p}(\tilde{\pi}^m), \tilde{c}) = \varphi'(\tilde{p}(\tilde{\pi}^m)).$$

Second, while the retailer's profit, when expressed in terms of the retail price and effort (p, e) which are implemented by the retailer, is kept unchanged as in (4.8), we observe that the manufacturer's profit accounts for the perceived cost of producing good 1 and can be written as:

$$\Pi_M(p, e) = (\pi(p, \tilde{c}) - \varphi(p))\,(1 + \theta e). \tag{6.14}$$

The characterization of the optimal wholesale contract $(w^u(\tilde{\pi}^m), z^u(\tilde{\pi}^m))$ would thus be readily obtained by applying Proposition 4 for the perceived cost $\tilde{c}$. In particular, a positive rebate for expanding demand on good 1 is again warranted. More interesting are comparative statics with respect to $\tilde{\pi}^m$ which are presented in the next proposition.

PROPOSITION 10. *In comparison with the baseline scenario of a single good, the optimal wholesale contract* $(w^u(\tilde{\pi}^m), z^u(\tilde{\pi}^m))$ *in a multi-product context implements*

1. *a lower retail price*
$$p^u(\tilde{\pi}^m) < p^u; \tag{6.15}$$
2. *a higher effort level*
$$e^u(\tilde{\pi}^m) < e^u. \tag{6.16}$$

Because the perceived cost of good 1 is now reduced, the manufacturer offers a lower wholesale price which induces *in fine* a lower retail price. A lower retail price increases the retailer's profit which boosts effort.

BAN ON BELOW-COST-PRICING. The analysis here also mimics our earlier findings. A ban on below-cost pricing imposes the condition

$$p_1 - w_1 \geq 0.$$

It in turn implies that the retailer's break-even condition writes now as

$$\Pi_R(p_1, e_1) \geq \tilde{\pi}^m D(p_1) + R(e_1). \tag{6.17}$$

The monopoly manufacturer now maximizes its profit $\Pi_M(p_1, e_1)$ as given in (6.14) subject to (6.17). Again, the retailer's participation constraint (6.17) is hardened in comparison with and Assumption 5 suffices to ensure that this participation constraint is binding.

Next proposition provides some comparative statics.

PROPOSITION 11. *In comparison with the baseline scenario of a single good, the optimal wholesale contract $w^b(\tilde{\pi}^m)$ in a multi-product context with a ban on below-cost pricing implements*

1. *a lower retail price*
$$p^b(\tilde{\pi}^m) < p^b; \tag{6.18}$$
2. *a higher effort level*
$$e^b(\tilde{\pi}^m) < e^b. \tag{6.19}$$

PROOF OF PROPOSITION 11. The monopoly manufacturer's optimization problem can be written as follows:

$$\max_{(p,e)} \Pi_M(p, e) \equiv (\pi(p, \tilde{c}) - \varphi(p))\,(1 + \theta e) \text{ subject to (6.17)},$$

Let denote by λ^b the non-negative Lagrange multiplier of the retailer's break-even condition (6.17). The corresponding Lagrangian for this problem writes as follows:

$$\mathcal{L}(p, e, \lambda^u) = (\pi(p, \tilde{c}) - \varphi(p))\,(1 + \theta e) + \lambda^b\,(\varphi(p)(1 + \theta e) - \tilde{\pi}^m D(p_1) - R(e))\,.$$

Making the dependence on $\tilde{\pi}^m$ explicit, the Karush-Kühn-Tucker first-order necessary conditions for optimality with respect to p and e (for an interior solution) write respectively as

$$\pi_p(p^b(\tilde{\pi}^m), \tilde{c}) - (1 - \lambda^b(\tilde{\pi}^m))\varphi'(p^b(\tilde{\pi}^m)) - \lambda^b(\tilde{\pi}^m)\tilde{\pi}^m D'(p^b(\tilde{\pi}^m)) = 0, \tag{6.20}$$

$$\theta\left(\pi(p^b(\tilde{\pi}^m), \tilde{c}) - (1 - \lambda^b(\tilde{\pi}^m))\varphi(p^b(\tilde{\pi}^m))\right) = \lambda^b(\tilde{\pi}^m) R'(e^b(\tilde{\pi}^m)), \tag{6.21}$$

while the complementary slackness condition is

$$\lambda^b(\tilde{\pi}^m)\left((1 + \theta e^b(\tilde{\pi}^m))\varphi(p^b(\tilde{\pi}^m)) - \tilde{\pi}^m D(p^b(\tilde{\pi}^m)) - R(e^b(\tilde{\pi}^m))\right) = 0. \tag{6.22}$$

Eliminating $\lambda^b(\tilde{\pi}^m)$ from (6.20) and (6.21) yields

$$\pi_p(p^b(\tilde{\pi}^m),\tilde{c}) - \varphi'(p^b(\tilde{\pi}^m)) = -\varphi'(p^b(\tilde{\pi}^m))\frac{\left(\pi(p^b(\tilde{\pi}^m),\tilde{c}) - \varphi(p^b(\tilde{\pi}^m))\right)}{\frac{R'(e^u(\tilde{\pi}^m))}{\theta} - \varphi(p^u(\tilde{\pi}^m))}. \tag{6.23}$$

Following the same steps as in the PROOF OF PROPOSITION 4, we know from Claim 1 that (4.10) is binding when Assumption 5 holds. Because $\frac{\psi(e)}{1+\theta e}$ is increasing in e (the derivative is $\frac{1+\theta R(e)}{(1+\theta e)^2} > 0$), we may unambiguously express e in terms of p from (4.10) as a function $E(p)$ and accordingly rewrite (A.41) as

$$p^u(\tilde{\pi}^m) - \tilde{c} + \frac{D(p^u(\tilde{\pi}^m))}{D'(p^u(\tilde{\pi}^m))} - \frac{\varphi'(p^u(\tilde{\pi}^m))}{D'(p^u(\tilde{\pi}^m))} = -\frac{\varphi'(p^u(\tilde{\pi}^m))}{D'(p^u(\tilde{\pi}^m))}\frac{(\pi(p^u(\tilde{\pi}^m),\tilde{c}) - \varphi(p^u(\tilde{\pi}^m)))}{\frac{\psi'(E(p^u(\tilde{\pi}^m)))}{\theta} - \varphi(p^u(\tilde{\pi}^m))}. \tag{6.24}$$

From there, it follows that

$$p^u(\tilde{\pi}^m) - c + \frac{D(p^u(\tilde{\pi}^m))}{D'(p^u(\tilde{\pi}^m))} - \frac{\varphi'(p^u(\tilde{\pi}^m))}{D'(p^u(\tilde{\pi}^m))} < -\frac{\varphi'(p^u(\tilde{\pi}^m))}{D'(p^u(\tilde{\pi}^m))}\frac{(\pi(p^u(\tilde{\pi}^m),\tilde{c}) - \varphi(p^u(\tilde{\pi}^m)))}{\frac{\psi'(E(p^u(\tilde{\pi}^m)))}{\theta} - \varphi(p^u(\tilde{\pi}^m))}. \tag{6.25}$$

Using (4.10) when binding, we may rewrite and sign the denominator on the right-hand side as

$$\frac{\psi'(E(p^u(\tilde{\pi}^m)))}{\theta} - \frac{\psi(E(p^u(\tilde{\pi}^m)))}{1+\theta E(p^u(\tilde{\pi}^m))} = \frac{\psi'(E(p^u(\tilde{\pi}^m))) + \theta R((E(p^u(\tilde{\pi}^m)))}{\theta(1+\theta E(p^u(\tilde{\pi}^m)))} > 0.$$

Therefore, we may find an upper bound on the right-hand side of (A.43) and obtain

$$p^u(\tilde{\pi}^m) - \tilde{c} + \frac{D(p^u(\tilde{\pi}^m))}{D'(p^u(\tilde{\pi}^m))} - \frac{\varphi'(p^u(\tilde{\pi}^m))}{D'(p^u(\tilde{\pi}^m))} = -\frac{\varphi'(p^u(\tilde{\pi}^m))}{D'(p^u(\tilde{\pi}^m))}\frac{(\pi(p^u(\tilde{\pi}^m),c) - \varphi(p^u(\tilde{\pi}^m)))}{\frac{\psi'(E(p^u(\tilde{\pi}^m)))}{\theta} - \varphi(p^u(\tilde{\pi}^m))}. \tag{6.26}$$

By the same construction as above, p^u from the baseline scenario satisfies

$$p^u - c + \frac{D(p^u)}{D'(p^u)} - \frac{\varphi'(p^u)}{D'(p^u)} = \frac{\varphi'(p^u)}{D'(p^u)}\frac{(\pi(p^u,c) - \varphi(p^u))}{\frac{\psi'(E(p^u))}{\theta} - \varphi(p^u)}. \tag{6.27}$$

By quasi-concavity of the Lagrangean in the baseline scenario, (A.44) and (A.45) imply that (6.15) holds.

Finally, because $\frac{\psi(e)}{1+\theta e}$ is increasing in e and $\varphi' < 0$, the function $E(p)$ is decreasing. Condition (6.16) then follows from (6.15). □

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APPENDIX

PROOF OF PROPOSITION 1. The optimization problem with competitive manufacturers writes as follows:

$$\max_{(w,z,p,e)} \pi(p,w)(1+\theta e) + \theta e z D(p) - \psi(e) \text{ subject to (2.3), (4.5), (4.6).}$$

Inserting the values of (w, z) obtained from (2.3) into the maximand allows to restate this maximization problem as follows:

$$\max_{(p,e)} \pi(p,c)(1+\theta e) - \psi(e) \text{ subject to (4.5) and (4.6).}$$

Observe that (4.5) and (4.6) define the wholesale price and the rebate in terms of the final retail price and the effort but, the formers do not enter the maximand. The maximand is thus maximized for the monopoly outcome (p^m, e^m). Inserting into (4.5) and (4.6) then gives the value of the wholesale price and the rebate that implement this outcome, namely $(w^d = c, z^d = 0)$. Finally, thanks to Assumption 1 and our assumption on the convexity of $\psi(e)$, the maximand is quasi-concave in (p, e) so that (p^m, e^m) is a global maximum. □

PROOF OF PROPOSITION 2. With an inelastic demand, the industry profit can be written as

$$(1+\theta e)(p-c) - \psi(e),$$

where we also assume $\psi(e) = \frac{e^2}{2}$. It is thus maximized for

$$p^m = v \text{ and } e^m = \theta(v-c).$$

The profits of the retailer and of the manufacturer are respectively given by

$$(p-w)(1+\theta e) + \theta e z - \frac{e^2}{2}$$

and

$$(w-c)(1+\theta e) - \theta e z.$$

For a given wholesale contract (w, z), the retailer thus optimally chooses the effort level given by (3.2) and sets the monopoly price on the downstream market (3.1). Let us assume in first step that (i) $z > 0$, and (ii) $v - w + z > 0$ so that $e^u(w, z) > 0$. We will check later that these conditions are indeed satisfied at the optimum. For future reference (and with a slight abuse of notations), let also define the manufacturer's and the retailer's profits in terms of the components of the wholesale contract as

$$\Pi_M(w,z) \equiv (w-c)(1+\theta e^u(w,z)) - \theta e^u(w,z)z,$$

and

$$\Pi_R(w,z) \equiv (v-w)(1+\theta e^u(w,z)) + \theta e^u(w,z)z - \frac{(e^u(w,z))^2}{2}.$$

The manufacturer's maximization problem is thus given by:

$$\max_{(w,z)} \Pi_M(w,z)$$

$$\text{subject to } \Pi_R(w,z) \geq 0. \tag{A.1}$$

Denote by λ the non-negative Lagrange multiplier associated to (A.1). The Lagrangian is thus given by

$$\mathcal{L}(w,z,\lambda) = (w-c)(1+\theta e^u(w,z)) - \theta e^u(w,z)z + \lambda\left((v-w)(1+\theta e^u(w,z)) + \theta e^u(w,z)z - \frac{(e^u(w,z))^2}{2}\right).$$

The Karush-Kühn-Tucker first-order necessary conditions for optimality with respect to w and

z respectively write as follows:

$$\text{(A.2)}\quad \frac{\partial \mathcal{L}}{\partial w}(w,z,\lambda)=0$$
$$\Leftrightarrow 1+\theta e^u(w,z)+\theta^2(w-c-z)+\lambda\left(-(1+\theta e^u(w,z))-\theta^2(w-c-z-e^u(w,z))\right)=0,$$

and

$$\text{(A.3)}\quad \frac{\partial \mathcal{L}}{\partial z}(w,z,\lambda)=0$$
$$\Leftrightarrow -\theta e^u(w,z)+\theta^2(w-c)+\lambda\left(\theta e^u(w,z)+\theta^2(v-w+z)-\theta e^u(w,z)\right)=0,$$

while the complementary slackness condition is

$$\text{(A.4)}\quad \lambda\left((v-w)(1+\theta e^u(w,z))+\theta e^u(w,z)z-\frac{(e^u(w,z))^2}{2}\right)=0.$$

Solving for the system (A.2)-(A.3)-(A.4) immediately gives (3.3) and $\lambda=1$. The manufacturer's profit is given by $\Pi_M^u=v-c+\frac{1}{2}\theta^2(v-c)^2$.

Lastly, we have $\frac{\partial^2\mathcal{L}}{\partial w^2}(w,z,1)=\frac{\partial^2\mathcal{L}}{\partial z^2}(w,z,1)=-\theta^2<0$, and $\frac{\partial^2\mathcal{L}}{\partial w^2}(w,z,1)\frac{\partial^2\mathcal{L}}{\partial z^2}(w,z,1)-(\frac{\partial^2\mathcal{L}}{\partial w\partial z}(w,z,1))^2=0$, so that the Lagrangean is concave at $\lambda=1$ and necessary conditions for optimality are also sufficient. □

Proof of Proposition 3. From Proposition 2, the constraint $p-w\geq 0$ must bind at the optimum. Moreover, pricing incentives of the retailer are unchanged so that $p^b=v$. Therefore, $w^b=v$. Accordingly, the retailer earns a profit worth $e\theta z-\frac{e^2}{2}$, which is maximized for the effort level $e=\theta z$. The retailer now enjoys a positive profit equal to $R(e)=\frac{e^2}{2}$.

Using $w^b=v$ and $e^b=\theta z$, the manufacturer's profit may be rewritten as $(v-c)(1+\theta^2 z)-\theta^2z^2$, which is maximized for $z^b=\frac{1}{2}(v-c)$. At the optimum, the effort implemented by the retailer is $e^b=\frac{1}{2}\theta(v-c)=\frac{1}{2}e^m$. The retailer enjoys a profit $R(e^b)=\frac{1}{8}\theta^2(v-c)^2>0$. The manufacturer earns $\Pi_M^b=v-c+\frac{1}{4}\theta^2(v-c)^2$. The industry profit is $v-c+\frac{1}{4}\theta^2(v-c)^2$.

It then comes immediately that $\Pi_M^u-\Pi_M^b=\frac{\theta^2(v-c)^2}{4}$, which is positive, strictly increasing and strictly convex in θ. □

Proof of Propositions 4 and 5. The monopoly manufacturer's optimization problem can be written as follows:

$$\max_{(p,e)}\Pi_M(p,e)\equiv(\pi(p,c)-\varphi(p))(1+\theta e)\text{ subject to (4.10)},$$

Let denote by λ^u the non-negative Lagrange multiplier of the retailer's break-even condition (4.10). The corresponding Lagrangian for this problem writes as follows:

$$\mathcal{L}(p,e,\lambda^u)=(\pi(p,c)-\varphi(p))(1+\theta e)+\lambda^u(\varphi(p)(1+\theta e)-\psi(e)).$$

The Karush-Kühn-Tucker first-order necessary conditions for optimality with respect to p and e (for an interior solution) write respectively as

$$\text{(A.5)}\quad \pi_p(p^u,c)-(1-\lambda^u)\varphi'(p^u)=0,$$
$$\text{(A.6)}\quad \theta(\pi(p^u,c)-(1-\lambda^u)\varphi(p^u))=\lambda^u\psi'(e^u),$$

while the complementary slackness condition is

$$\lambda^u \left((1+\theta e^u)\varphi(p^u) - \psi(e^u)\right) = 0. \tag{A.7}$$

We now establish several properties of the optimum:

CLAIM 1. *$\lambda^u > 0$ and thus (4.10) is binding:*

$$(1+\theta e^u)\varphi(p^u) = \psi(e^u). \tag{A.8}$$

Proof. Suppose instead that $\lambda^u = 0$. Then (A.5) leads to $p^u = \tilde{p}$. Because $\pi(\tilde{p}, c) > \varphi(\tilde{p})$, the optimal effort choice that the manufacturer would like to implement would be $e^u = 1$. The retailer's profit would then be equal to $(1+\theta)\varphi(\tilde{p}) - \psi(1)$, which is strictly negative when Assumption 3 holds; a contradiction. Therefore, at the optimum, it must be $\lambda^u > 0$. Condition (A.8) then follows from (A.40). □

CLAIM 2.

$$\lambda^u < 1.$$

Proof. Suppose $\lambda^u = 1$. Then, from (A.5) and (A.6), we obtain $p^u = p^m$ and $e^u = e^m$. Observe now that, by definition of the monopoly price p^m, $\varphi(p^m) = \pi^m$, and e^m is such that $\pi^m = \psi'(e^m)/\theta$. Thus,

$$(1+\theta e^m)\varphi(p^m) - \psi(e^m) = (1+\theta e^m)\pi^m - \psi(e^m) = \pi^m + R(e^m) > 0. \tag{A.9}$$

(A.9) shows the retailer's participation constraint is strictly satisfied; a contradiction with Claim 1. Hence, by continuity, it must be that $\lambda^u \in (0,1)$. □

Claim 2 together with Assumption 2 implies that the Lagrangean is quasi-concave in p, and thus in (p,e) given the convexity of ψ and Claim 1.

CLAIM 3.

$$\tilde{p} > p^u > p^m.$$

Proof. Together with Claim 2, (A.5) implies

$$\pi_p(p^u, c) = (1-\lambda^u)\varphi'(p^u) < 0.$$

Because $\pi(p,c)$ is quasi-concave in p when Assumption 1 holds, we have

$$p^u > p^m.$$

Similarly, we have

$$\pi_p(p^u, c) - \varphi'(p^u) = -\lambda^u \varphi'(p^u) > 0.$$

Because $\pi(p,c) - \varphi(p)$ is also quasi-concave in p when Assumption 2 holds, we have

$$p^u < \tilde{p}.$$

□

CLAIM 4.

$$z^u > 0 \tag{A.10}$$

and

$$p^u - w^u = -R(e^u) < 0. \tag{A.11}$$

Proof. Let us rewrite the first-order condition (4.3) for the optimal solution (p^u, e^u) in a more explicit manner as

$$\theta(p^u - w^u + z^u)D(p^u) = \psi'(e^u). \tag{A.12}$$

From (A.12), we obtain:

$$p^u - w^u = \frac{\psi'(e^u)}{\theta D(p^u)} - z^u.$$

Inserting into (4.4) for the optimal solution (p^u, e^u) yields:

$$z^u \frac{D(p^u)}{1+\theta e^u} = \frac{\psi'(e^u)}{\theta} - \varphi(p^u). \tag{A.13}$$

Using (A.5) and (A.6), we get:

$$\psi'(e^u) = \theta f(p^u) \tag{A.14}$$

where

$$f(p) = \pi(p,c) + \frac{\pi_p(p,c)}{\varphi'(p) - \pi_p(p,c)}(\pi(p,c) - \varphi(p)).$$

Using (A.14), (A.13) can be rewritten as follows:

$$z^u \frac{D(p^u)}{1+\theta e^u} = \frac{\varphi'(p^u)(\pi(p^u,c) - \varphi(p^u))}{\varphi'(p^u) - \pi_p(p^u,c)}.$$

From Claim 1 and Equation (A.5), we get $\varphi'(p^u) - \pi_p(p^u,c) < 0$. Therefore, we have also $\varphi'(p^u) < \pi_p(p^u,c) < 0$ since $p^u > p^m$. Moreover, notice that $\pi(p,c) > \varphi(p) \Leftrightarrow p - c > -\frac{D(p)}{D'(p)} \Leftrightarrow p > p^m$. Since $p^u > p^m$, we have $\pi(p^u,c) > \varphi(p^u)$. Hence, the right-hand side of (A.14) is positive and (A.10) holds.

Inserting now (A.13) into (4.4), we obtain

$$(p^u - w^u)D(p^u) = (1+\theta e^u)\varphi(p^u) - e^u\psi'(e^u).$$

Using the binding participation constraint (4.10) and $R(e) = e\psi'(e) - \psi(e)$, we obtain (A.11). □

CLAIM 5.

$$e^u > e^m. \tag{A.15}$$

Proof. Observe that

$$\psi'(e^m) = \theta f(p^m). \tag{A.16}$$

Taken together with (A.14), Condition (A.15) will be proved (thanks for the convexity of ψ) if $f(p^u) > f(p^m)$. We thus compute

$$f'(p) = (\pi(p,c) - \varphi(p))\frac{d}{dp}\left(\frac{\pi_p(p,c)}{\varphi'(p) - \pi_p(p,c)}\right).$$

We may write

$$\frac{\pi_p(p,c)}{\varphi'(p) - \pi_p(p,c)} = \frac{p + \frac{D(p)}{D'(p)} - c}{-\left(p + \frac{D(p)}{D'(p)} - \frac{\varphi'(p)}{D'(p)} - c\right)}.$$

Assumption 1 (resp. Assumption 2) ensures that the numerator (resp. denominator) is non-decreasing (resp. non-increasing). Hence, $f'(p) \geq 0$ whenever $\pi(p,c) - \varphi(p) \geq 0$ which holds for $p \geq p^m$ (with a strict inequality for $p > p^m$). Since $p^u > p^m$, we then obtain $f(p^u) > f(p^m)$ and thus $e^u > e^m$. □

□

PROOF OF PROPOSITION 6. With a ban on below-cost pricing, the manufacturer's problem is to maximize $\Pi_M(p,e)$ subject to (5.2). We rewrite (5.2) as follows:

$$(1 + \theta e)\varphi(p) - e\psi'(e) \geq 0, \tag{A.17}$$

so that the manufacturer's problem may be expressed in a more compact form as follows:

$$\max_{(p,e)} \,(\pi(p,c) - \varphi(p))\,(1 + \theta e) \text{ subject to (A.17).}$$

Denote by λ^b the Lagrange multiplier for constraint (A.17). The Lagrangian writes as follows:

$$\mathcal{L}(p,e,\lambda^b) = (\pi(p,c) - \varphi(p))\,(1 + \theta e) + \lambda^b\left((1 + \theta e)\varphi(p) - e\psi'(e)\right).$$

Assuming concavity of this Lagrangian in (p,e) and optimizing with respect to p and e respectively yields the following Karush-Khün-Tucker first-order necessary conditions:

$$\pi_p(p^b) - (1 - \lambda^b)\varphi'(p^b) = 0, \tag{A.18}$$

$$\theta\left(\pi(p^b) - (1 - \lambda^b)\varphi(p^b)\right) = \lambda^b\left(\psi'(e^b) + e^b\psi''(e^b)\right). \tag{A.19}$$

We now prove that (A.17) is binding. We proceed by contradiction. Suppose not, that is, $\lambda^b = 0$. Then, we would have $p^b = \tilde{p}$ and $e^b = 1$. These values do not satisfy the break-even condition (4.10) and, *a fortiori*, the more demanding constraint (5.2) when Assumption 5 holds. Therefore, (5.2), or equivalently (5.1), is binding at the optimum.

The last item in the proposition follows from observing that, when $e^b > 0$, we have $(1 + \theta e^b)\varphi(p^b) = e^b\psi'(e^b) > \psi(e^b)$. □

PROOF OF PROPOSITION 7. With $D(p) = (a - bp)^{\frac{1}{\delta}}$, (4.5) and (4.6) can respectively be expressed as

$$w = p + e^2\mu(a - bp)^{-\frac{1}{\delta}} - \frac{(a - bp)\delta(1 + \theta e)}{b}, \tag{A.20}$$

$$z = (1 + \theta e)\left(\frac{e\mu(a - bp)^{-\frac{1}{\delta}}}{\theta} - \frac{(a - bp)\delta}{b}\right). \tag{A.21}$$

From this, we can express the manufacturer's and the retailer's profit as function of (p, e) respectively as follows:

$$\Pi_M(p, e) = \frac{(1 + \theta e)(a - bp)^{\frac{1}{\delta}}(b(-c + \delta p + p) - a\delta)}{b}, \tag{A.22}$$

$$\Pi_R(p, e) = \frac{\delta(1 + \theta e)(a - bp)^{\frac{1}{\delta}+1}}{b} - \frac{\mu}{2}e^2. \tag{A.23}$$

Optimizing the Lagrangian with respect to p and e and solving yields the expressions of the price and the effort level as function of the multiplier λ^u, which we denote by $P^u(\lambda^u)$ and $E^u(\lambda^u)$:

$$P^u(\lambda^u) = \frac{a}{b} - \frac{a - bc}{b(1 + \delta)(1 + \delta(1 - \lambda^u))}, \tag{A.24}$$

$$E^u(\lambda^u) = \frac{\delta\theta}{b\mu}\left(\frac{a - bc}{\delta + 1}\right)^{\frac{1}{\delta}+1}\frac{1}{\lambda^u}\left(\frac{1}{1 + \delta(1 - \lambda^u)}\right)^{\frac{1}{\delta}}. \tag{A.25}$$

Plugging these expressions into the binding retailer's participation constraint, we obtain that

$$\Pi_R(P^u(\lambda^u), E^u(\lambda^u)) = 0$$

amounts to

$$\frac{\delta\theta^2}{2b\mu}\left(\frac{a - bc}{\delta + 1}\right)^{\frac{1}{\delta}+1} = \frac{(\lambda^u)^2(1 + \delta(1 - \lambda^u))^{\frac{1}{\delta}}}{1 + \delta - \lambda^u(2 + \delta)}. \tag{A.26}$$

The left-hand side in (A.26) is strictly positive. It can easily be shown that, for $\lambda^u \in [0, \frac{1+\delta}{1+2\delta})$, the right-hand side is strictly increasing in λ^u and takes values in $[0, +\infty)$. Therefore, there exists a unique $\lambda^u \in [0, \frac{1+\delta}{1+2\delta})$ which satisfies (A.26).

Using (A.20) and (A.24)-(A.25), we can reconstruct the wholesale price as a function of the multiplier λ^u, which we denote by $W^u(\lambda^u)$. It is immediate to show that $\overline{p}(w^u) = \arg\max_p (p - w^u)D(p) = \frac{a\delta + bw^u}{b(1+\delta)}$, so that $D(\overline{p}(w^u)) = (\frac{a - bw^u}{1+\delta})^{\frac{1}{\delta}}$. Therefore, given the contract (w^u, z^u) offered by the manufacturer, the retailer has no incentives to deviate towards no effort ($e = 0$) and the corresponding price ($\overline{p}(w^u)$) if and only if $\overline{p}(w^u) - w^u \leq 0$ or $D(\overline{p}(w^u)) \leq 0$, which is equivalent to:

$$\frac{1}{1 + \delta(1 - \lambda^u)}\left(a - bc - \frac{\delta^2\theta^2(1 - \lambda^u)((\delta + 1)(1 + \delta(1 - \lambda^u)))^{-\frac{1}{\delta}}(a - bc)^{\frac{1}{\delta}+2}}{b(\delta + 1)(\lambda^u)^2\mu}\right) \leq 0.$$

Using (A.26) and eliminating positive terms, this can be simplified to:

$$\frac{1 - \delta(1 - \lambda^b) - 2\lambda^b}{1 + \delta(\delta + 2)(1 - \lambda^b)^2 - 2\lambda^b} \leq 0. \tag{A.27}$$

The left-hand side term in (A.27) is strictly decreasing in λ^b. Hence, a sufficient condition for

(A.27) to always hold is that this inequality is satisfied for $\lambda^b = 0$, or $\frac{1-\delta}{(\delta+1)^2} \leq 0$, or $\delta \geq 1$.

Notice that (A.27) is never satisfied for $\delta = 0$. Our demand function $D(p) = (a - bp)^{\frac{1}{\delta}}$ is defined for $\delta > 0$ and gives at the limit case $\delta \to 0$ the log-linear/exponential demand. We now consider this case and assume that demand is given by $D(p) = e^{a-bp}$.

Observe that $\overline{p}(w)(w)$ is the solution of $h(\overline{p}(w)) = w$ with $h(\overline{p}(w)) = \overline{p}(w) + \frac{D(\overline{p}(w))}{D'(\overline{p}(w))}$. Under our assumptions on $D(\cdot)$, we have $h'(\cdot) > 0$ and thus $(h^{-1})' = \frac{1}{h'} > 0$. A wholesale price w leads to a price $\overline{p}(w)$ if the retailer makes no effort characterized by $\overline{p}(w) = h^{-1}(w)$. The corresponding profit for the retailer is thus $\varphi(h^{-1}(w))$.

We can then adapt the methodology used in the proof of Propositions 4 and 5 to show that the manufacturer's problem writes as follows:

$$\max_{(p,e)} \left(\pi(p,c) - \varphi(p)\right)(1 + \theta e) \text{ subject to } (1 + \theta e)\varphi(p) - \psi(e) \geq \varphi(\overline{p}(w)(p,e)),$$

where $\overline{p}(w) = h^{-1}(w)$ and w can be expressed as function of the pair (p, e) that the manufacturer wants to implement using (4.5): $\overline{p}(w)(p,e) = h^{-1}\left(h(p) + \frac{D(p)}{D'(p)}\theta e + \frac{e\psi'(e)}{D(p)}\right)$. The price and effort level are now chosen to limit the retailer's profit if it exerts no effort.

Closed-form solutions of the optimum are not possible. We therefore rely on simulations using the following values of parameters: $a = b = 1$, $c = 0$, $\mu = 15$ and $\theta \in [.1, 8]$. We vary θ with an increment of .1 to obtain 80 simulations. For each simulation, we determine numerically the optimum using Mathematica.[27]

(w, z) as functions of (p, e) are still given by (A.20) and (A.21). Profits of the retailer and the manufacturer are still given by (A.22) and (A.23).

The Karush-Khün-Tucker first-order condition with respect to price is the same as in the case with no ban on below-cost-pricing. Therefore,

$$P^b(\lambda^b) = P^u(\lambda^b), \tag{A.28}$$

where P^u is given by (A.24). The Karush-Khün-Tucker first-order condition with respect to effort leads to $E^b(\lambda^b) = \frac{1}{2}E^u(\lambda^b)$ where E^u is given by (A.25). Replacing in (A.17), which must hold as an equality at the optimum, characterizes the Lagrange multiplier λ^b:

$$\frac{\delta\theta^2}{2b\mu}\left(\frac{a - bc}{\delta + 1}\right)^{\frac{1}{\delta}+1} = 2\frac{(\lambda^b)^2(1 + \delta(1 - \lambda^b))^{\frac{1}{\delta}}}{1 + \delta(1 - \lambda^b) - 2\lambda^b}. \tag{A.29}$$

The left-hand side in (A.29) is strictly positive. The right-hand side in (A.29) is, for $\lambda^b \in [0, \frac{1+\delta}{4+\delta})$, strictly increasing and takes values in $[0, +\infty)$. Hence, there exists a unique $\lambda^b \in (0, \frac{1+\delta}{4+\delta})$ such that (A.29) holds.

Comparing (A.26) and (A.29), it comes immediately that $\lambda^u > \lambda^b$. From (A.24), we obtain that $(P^u)'(\lambda) < 0$. Therefore, we have that: $p^b = P^b(\lambda^b) = P^u(\lambda^b) > P^u(\lambda^u) = p^u$.

It remains to compare effort levels. Remind that $e^u = E^u(\lambda^u)$ and $e^b = \frac{1}{2}E^u(\lambda^b)$, where $E^u(\cdot)$ is defined by (A.25). Since λ^u is implicitly defined by (A.26), we obtain the following simplification:

$$E^u(\lambda^u) = \frac{1}{\theta}\frac{2\lambda^u}{(1 + \delta) - (2 + \delta)\lambda^u}. \tag{A.30}$$

[27] The file used for these numerical simulations is available on the second author's webpage.

Similarly, since λ^b is implicitly defined by (A.29), we obtain:

$$E^b(\lambda^b) = \frac{1}{2}E^u(\lambda^b) = \frac{1}{\theta}\frac{2\lambda^b}{(1+\delta)-(2+\delta)\lambda^b}. \tag{A.31}$$

Since $\frac{2x}{(1+\delta)-(2+\delta)x}$ is strictly increasing in x and $\lambda^u > \lambda^b$, we have $e^u > e^b$. □

Comparative Statics with $D(p) = (a-bp)^{\frac{1}{\delta}}$ and $\psi(e) = \frac{\mu}{2}e^2$. From (A.24), (A.28), (A.30) and (A.31), let $f_p(\lambda) = \frac{a}{b} - \frac{a-bc}{b(1+\delta)(1+\delta(1-\lambda))}$ and $f_e(\lambda) = \frac{2\lambda}{(1+\delta)-(2+\delta)\lambda}$, so that we have: $p^u(\theta) = f_p(\lambda^u(\theta))$, $p^b(\theta) = f_p(\lambda^b(\theta))$, $e^u(\theta) = \frac{1}{\theta}f_e(\lambda^u(\theta))$ and $e^b(\theta) = \frac{1}{\theta}f_e(\lambda^b(\theta))$. The profit of the manufacturer is given by $(1+\theta e)(\pi(p,c)-\varphi(p))$ and depends therefore only on the Lagrange multiplier λ. That profit is denoted by $\hat{\Pi}_M(\lambda(\theta))$ where:

$$\hat{\Pi}_M(\lambda) = \frac{\delta(a-bc)^{\frac{1}{\delta}+1}}{b(1+\delta)^{\frac{1}{\delta}}}\frac{(1-\lambda)(1+\delta(1-\lambda))^{-\frac{1}{\delta}}}{(1+\delta-\lambda(2+\delta))},$$

which is positive since $\lambda \in (0, \frac{1+\delta}{2+\delta})$.

Straightforward computations show that

$$\hat{\Pi}'_M(\lambda) = \frac{\delta(a-bc)^{\frac{1}{\delta}+1}}{b(\delta+1)^{\frac{1}{\delta}}}\frac{(\delta(2-\lambda)(1-\lambda)-\lambda(3-2\lambda)+2)}{(1+\delta(1-\lambda))^{\frac{1}{\delta}+1}(1+\delta-\lambda(2+\delta))^2}.$$

The sign of $\hat{\Pi}'_M(\lambda)$ is given by the sign of $\delta(2-\lambda)(1-\lambda)-\lambda(3-2\lambda)+2$, which is strictly positive for $\lambda \in (0, \frac{1+\delta}{2+\delta})$. We have $\frac{d}{d\theta}\hat{\Pi}_M(\lambda(\theta)) = \hat{\Pi}'_M(\lambda(\theta))\dot{\lambda}(\theta)$, where $\dot{\lambda}(\theta)$ can be obtained by totally differentiating the binding participation, namely (4.10) expressed at $(e^u(\theta), p^u(\theta), \lambda^u(\theta))$ when below-cost-pricing is allowed and (5.2) expressed at $(e^b(\theta), p^b(\theta), \lambda^b(\theta))$ when there is a ban on below-cost-pricing. Doing so shows that $\dot{\lambda}^u(\theta) = f_\lambda(\lambda^u(\theta))$ and $\dot{\lambda}^b(\theta) = f_\lambda(\lambda^b(\theta))$, where

$$f_\lambda(\lambda) = \frac{2}{(1+\delta)\theta}\frac{\lambda(1+\delta(1-\lambda))(1+\delta-\lambda(2+\delta))}{\delta(2-\lambda)(1-\lambda)-\lambda(3-2\lambda)+2}.$$

We have $f_\lambda(\lambda) > 0$ for $\lambda \in (0, \frac{1+\delta}{2+\delta})$. Therefore, we deduce:

$$\frac{d}{d\theta}\hat{\Pi}_M(\lambda(\theta)) = \frac{2\delta}{\theta}\frac{(a-bc)^{\frac{1}{\delta}+1}}{b(\delta+1)^{\frac{1}{\delta}+1}}\frac{\lambda(1+\delta(1-\lambda))^{-\frac{1}{\delta}}}{1+\delta-\lambda(2+\delta)} > 0.$$

Proceeding to a further round of differentiation, we obtain:

$$\frac{d^2}{d\theta^2}\hat{\Pi}_M(\lambda(\theta)) = \frac{2\delta}{\theta^2}\frac{(a-bc)^{\frac{1}{\delta}+1}}{b(\delta+1)^{\frac{1}{\delta}+2}}\frac{\lambda^2(1+\delta(1-\lambda))^{-\frac{1}{\delta}}(\delta(5+\delta)(1-\lambda)+5+\delta-6\lambda)}{(1+\delta-\lambda(2+\delta))(\delta(2-\lambda)(1-\lambda)-\lambda(3-2\lambda)+2)},$$

which is strictly positive under our assumptions.

Summarizing, the profit of the manufacturer is strictly increasing and strictly convex in θ, both when there is no ban on below-cost-pricing and when there is a ban. Moreover, since (i) the manufacturer's profits with no ban and with a ban depend on θ only through the Lagrange multiplier associated to the respective retailer's participation constraint, (ii) $\lambda^b(\theta) < \lambda^u(\theta)$ for all θ, and (iii) the manufacturer's profit $\hat{\Pi}_M(\lambda(\theta))$ is strictly convex in θ, we get that the profit difference $\hat{\Pi}_M(\lambda^u(\theta)) - \hat{\Pi}_M(\lambda^b(\theta))$ is strictly increasing in θ.

□

QUANTIFYING ANTITRUST DAMAGES IN UPSTREAM COLLUSION CASES. Simple computations lead to the following expressions: $\pi^m = \frac{\delta}{b}(\frac{a-bc}{1+\delta})^{1+\frac{1}{\delta}}$, $D(p^m) = (\frac{a-bc}{1+\delta})^{\frac{1}{\delta}}$, $e^m = \frac{\theta}{\mu}\frac{\delta}{b}(\frac{a-bc}{1+\delta})^{1+\frac{1}{\delta}}$. When manufacturers collude under a ban on below-cost pricing, using from the proof of Proposition 7, we obtain:

$$p^b = \frac{a}{b} - \frac{a-bc}{b(1+\delta)(1+\delta(1-\lambda^b))}, D(p^b) = \left(\frac{a-bc}{(1+\delta)(1+\delta(1-\lambda^b))}\right)^{\frac{1}{\delta}},$$

and

$$e^b = \frac{\delta\theta}{2b\mu}\left(\frac{a-bc}{\delta+1}\right)^{\frac{1}{\delta}+1}\frac{1}{\lambda^b}\left(\frac{1}{1+\delta(1-\lambda^b)}\right)^{\frac{1}{\delta}},$$

where λ^b solves (A.29).

Then, the damage can be expressed as a function of λ^b only:

$$\frac{\Delta}{\pi^m} = 1 + \frac{4(\lambda^b)^2(1+\delta(1-\lambda^b))^{\delta} - (1+\delta(1-\lambda^b))^{-\delta}}{2(1+\delta-(2+\delta)\lambda^b)},$$

where λ^b depends only on the ratio of the demand with and without collusion:

$$(1+\delta(1-\lambda^b))^{\frac{1}{\delta}} = \frac{D(p^m)}{D(p^b)}.$$

This concludes the proof. □

PROOF OF PROPOSITIONS 8 AND 9. Let denote by $\pi_1(p_1,p_2) = (p-c)D_1(p_1,p_2)$ and $\varphi(p_1,p_2) = \frac{(D(p_1,p_2))^2}{-\frac{\partial D}{\partial p_1}(p_1,p_2)}$. Let us assume that, for all relevant (p_1,p_2), $\frac{\partial^2\pi}{\partial p_1^2}(p_1,p_2) < 0$ and $\frac{\partial^2\pi}{\partial p_1\partial p_2}(p_1,p_2) > 0$. This implies that the best response $P_2(p_1)$ defined in (6.7) is upward-sloping. Let us further assume that that best response has a slope smaller than 1. Observe that $\varphi_{p_1}(p_1,p_2) \leq 0$, which is a familiar condition from our analysis of the single-product scenario, follows from assuming concavity of the demand function in its own price, that is, $\frac{\partial^2 D}{\partial p_1^2}(p_1,p_2) \leq 0$. As is the main analysis, we shall assume that $\frac{\partial^2\varphi}{\partial p_1^2}(p_1,p_2) \leq 0$, and we further impose that $\frac{\partial^2\varphi}{\partial p_1\partial p_2}(p_1,p_2) \geq 0$; these assumptions are satisfied with the linear demands system that we shall use later on.

NO BAN ON BELOW-COST-PRICING. With no ban on below-cost-pricing, the manufacturer maximizes $\Pi_M(e_1,p_1,p_2) = (1+\theta e_1)(\pi_1(p_1,p_2) - \varphi(p_1,p_2))$ with respect to (e_1,p_1) and subject to (6.8). The Lagrangian associated to this problem is $(1+\theta e_1)[\pi_1(p_1,p_2) - \varphi(p_1,p_2)] + \lambda_1^u[(1+\theta e_1)\varphi(p_1,p_2) - \psi(e_1)]$. Optimizing with respect to p_1 leads to the following:

$$\pi_{p_1}(p_1,p_2) - (1-\lambda_1^u)\varphi_{p_1}(p_1,p_2) = 0.$$

The solution of that equation defines the best response in price $P_1^u(p_2)$. It is straightforward to show that, in equilibrium, $\lambda_1^u \in (0,1)$. Under our assumptions on $\varphi(\cdot)$, the best response $P_1^u(p_2)$ remains upward-sloping and lies above $P_1(p_2)$. This allows to prove Proposition 8.

BAN ON BELOW-COST-PRICING. Consider now the case of a ban on below-cost-pricing. The participation constraint of the retailer is now given by (6.10). The Lagrangian associated to this problem is $(1+\theta e_1)[\pi_1(p_1,p_2) - \varphi(p_1,p_2)] + \lambda_1^b[(1+\theta e_1)\varphi(p_1,p_2) - \psi(e_1) - R(e_1)]$. Optimizing with respect to p_1 leads to the following:

$$\pi_{p_1}(p_1,p_2) - (1-\lambda_1^b)\varphi_{p_1}(p_1,p_2) = 0.$$

The solution of that equation defines the best response in price $P_1^b(p_2)$. Assuming that $\lambda_1^b < \lambda_1^u$, the best response $P_1^b(p_2)$ lies above $P_1^u(p_2)$. This allows to prove the first item in Proposition 9.

Linear-Quadratic Specification. Let us now consider the following linear-quadratic specification of the model. Demand functions are given by $D_i(p_i,p_j) = \frac{1}{b^2-\gamma^2}[b(a-p_i)-\gamma(a-p_j)]$ for $i \neq j \in \{1,2\}$, with $b > \gamma \geq 0$. The cost of effort is given by $\psi(e_1) = \frac{\mu}{2}e_1^2$.

For a given p_2, the optimal price, effort level and Lagrange multiplier are given by

$$P_1^u(p_2,\lambda_1^u) = \frac{a(b-\gamma)(3-2\lambda_1^u)+bc+\gamma p_2(3-2\lambda_1^u)}{2b(2-\lambda_1^u)}, \tag{A.32}$$

$$E_1^u(\lambda) = \frac{2\lambda_1^u}{\theta(2-3\lambda_1^u)}, \tag{A.33}$$

$$\frac{(b(a-c)-\gamma(a-p_2))^2\theta^2}{8\mu b(b^2-\gamma^2)} = \frac{(2-\lambda_1^u)(\lambda_1^u)^2}{2-3\lambda_1^u}. \tag{A.34}$$

The best response of the integrated retailer is given by:

$$P_2(p_1) = \frac{1}{2b}(b(a+c)-\gamma(a-p_1)). \tag{A.35}$$

Combining (A.32) and (A.35) allows to obtain the prices as function of the multiplier:

$$P_1(\lambda_1^u) = \frac{a(3-2\lambda_1^u)(b-\gamma)(2b+\gamma)+bc(\gamma(3-2\lambda_1^u)+2b)}{4b^2(2-\lambda_1^u)-\gamma^2(3-2\lambda_1^u)},$$
$$P_2(\lambda_1^u) = \frac{a(b-\gamma)(4b+3\gamma-2\lambda_1^u(b+\gamma))+bc(2b(2-\lambda_1^u)+\gamma)}{4b^2(2-\lambda_1^u)-\gamma^2(3-2\lambda_1^u)}.$$

$P_2(\lambda_1^u)$ can be replaced in (A.34) to obtain the multiplier:

$$\frac{(\lambda_1^u)^2\left(4b^2(2-\lambda_1^u)-\gamma^2(3-2\lambda_1^u)\right)^2}{(2-\lambda_1^u)(2-3\lambda_1^u)} = \frac{b\theta^2(a-c)^2(b-\gamma)(2b+\gamma)^2}{2\mu(b+\gamma)}. \tag{A.36}$$

The right-hand side in (A.36) is strictly positive. Notice that $\lambda_1^u \in (0,1)$ and $b > \gamma > 0$ implies that $4b^2(2-\lambda_1^u)-\gamma^2(3-2\lambda_1^u) > 0$. It is then straightforward to show that the left-hand side in (A.36) is strictly increasing in λ_1^u for $\lambda_1^u \in (0,\frac{2}{3})$ and takes values in $(0,+\infty)$. To summarize, there exists a unique $\lambda_1^u \in (0,\frac{2}{3})$ such that the participation constraint (6.8) is binding at equilibrium.

We shall not detail the computations in that case as they are extremely similar to the ones performed in the previous case. Simply notice that the binding participation writes at equilibrium as follows:

$$\frac{(\lambda_1^b)^2\left(4b^2(2-\lambda_1^b)-\gamma^2(3-2\lambda_1^b)\right)^2}{(2-\lambda_1^b)(2-3\lambda_1^b)} = \frac{b\theta^2(a-c)^2(b-\gamma)(2b+\gamma)^2}{4\mu(b+\gamma)}. \tag{A.37}$$

Comparing (A.36) and (A.37) immediately leads to $\lambda_1^b < \lambda_1^u$. □

Proof of Proposition 10. The monopoly manufacturer's optimization problem can be written as follows:

$$\max_{(p,e)} \Pi_M(p,e) \equiv (\pi(p,\tilde{c})-\varphi(p))\,(1+\theta e) \text{ subject to (4.10)},$$

Let denote by λ^u the non-negative Lagrange multiplier of the retailer's break-even condition (4.10). The corresponding Lagrangian for this problem writes as follows:

$$\mathcal{L}(p,e,\lambda^u) = \left(\pi(p,\tilde{c}) - \varphi(p)\right)(1+\theta e) + \lambda^u \left(\varphi(p)(1+\theta e) - \psi(e)\right).$$

Making the dependence on $\tilde{\pi}^m$ explicit, the Karush-Kühn-Tucker first-order necessary conditions for optimality with respect to p and e (for an interior solution) write respectively as

$$\text{(A.38)} \qquad \pi_p(p^u(\tilde{\pi}^m),\tilde{c}) - (1-\lambda^u(\tilde{\pi}^m))\varphi'(p^u(\tilde{\pi}^m)) = 0,$$

$$\text{(A.39)} \qquad \theta\left(\pi(p^u(\tilde{\pi}^m),\tilde{c}) - (1-\lambda^u(\tilde{\pi}^m))\varphi(p^u(\tilde{\pi}^m))\right) = \lambda^u(\tilde{\pi}^m)\psi'(e^u(\tilde{\pi}^m)),$$

while the complementary slackness condition is

$$\text{(A.40)} \qquad \lambda^u(\tilde{\pi}^m)\left((1+\theta e^u(\tilde{\pi}^m))\varphi(p^u(\tilde{\pi}^m)) - \psi(e^u(\tilde{\pi}^m))\right) = 0.$$

Eliminating $\lambda^u(\tilde{\pi}^m)$ from (A.38) and (A.39) yields

$$\text{(A.41)} \qquad \pi_p(p^u(\tilde{\pi}^m),\tilde{c}) - \varphi'(p^u(\tilde{\pi}^m)) = -\varphi'(p^u(\tilde{\pi}^m))\frac{\left(\pi(p^u(\tilde{\pi}^m),\tilde{c}) - \varphi(p^u(\tilde{\pi}^m))\right)}{\frac{\psi'(e^u(\tilde{\pi}^m))}{\theta} - \varphi(p^u(\tilde{\pi}^m))}.$$

Following the same steps as in the PROOF OF PROPOSITION 4, we know from Claim 1 that (4.10) is binding when Assumption 5 holds. Because $\frac{\psi(e)}{1+\theta e}$ is increasing in e (the derivative is $\frac{1+\theta R(e)}{(1+\theta e)^2} > 0$), we may unambiguously express e in terms of p from (4.10) as a function $E(p)$ and accordingly rewrite (A.41) as

$$\text{(A.42)} \quad p^u(\tilde{\pi}^m) - \tilde{c} + \frac{D(p^u(\tilde{\pi}^m))}{D'(p^u(\tilde{\pi}^m))} - \frac{\varphi'(p^u(\tilde{\pi}^m))}{D'(p^u(\tilde{\pi}^m))} = -\frac{\varphi'(p^u(\tilde{\pi}^m))}{D'(p^u(\tilde{\pi}^m))}\frac{\left(\pi(p^u(\tilde{\pi}^m),\tilde{c}) - \varphi(p^u(\tilde{\pi}^m))\right)}{\frac{\psi'(E(p^u(\tilde{\pi}^m)))}{\theta} - \varphi(p^u(\tilde{\pi}^m))}.$$

From there, it follows that

$$\text{(A.43)} \quad p^u(\tilde{\pi}^m) - c + \frac{D(p^u(\tilde{\pi}^m))}{D'(p^u(\tilde{\pi}^m))} - \frac{\varphi'(p^u(\tilde{\pi}^m))}{D'(p^u(\tilde{\pi}^m))} < -\frac{\varphi'(p^u(\tilde{\pi}^m))}{D'(p^u(\tilde{\pi}^m))}\frac{\left(\pi(p^u(\tilde{\pi}^m),\tilde{c}) - \varphi(p^u(\tilde{\pi}^m))\right)}{\frac{\psi'(E(p^u(\tilde{\pi}^m)))}{\theta} - \varphi(p^u(\tilde{\pi}^m))}.$$

Using (4.10) when binding, we may rewrite and sign the denominator on the right-hand side as

$$\frac{\psi'(E(p^u(\tilde{\pi}^m)))}{\theta} - \frac{\psi(E(p^u(\tilde{\pi}^m)))}{1+\theta E(p^u(\tilde{\pi}^m))} = \frac{\psi'(E(p^u(\tilde{\pi}^m))) + \theta R((E(p^u(\tilde{\pi}^m)))}{\theta(1+\theta E(p^u(\tilde{\pi}^m)))} > 0.$$

Therefore, we may find an upper bound on the right-hand side of (A.43) and obtain

$$\text{(A.44)} \quad p^u(\tilde{\pi}^m) - \tilde{c} + \frac{D(p^u(\tilde{\pi}^m))}{D'(p^u(\tilde{\pi}^m))} - \frac{\varphi'(p^u(\tilde{\pi}^m))}{D'(p^u(\tilde{\pi}^m))} = -\frac{\varphi'(p^u(\tilde{\pi}^m))}{D'(p^u(\tilde{\pi}^m))}\frac{\left(\pi(p^u(\tilde{\pi}^m),c) - \varphi(p^u(\tilde{\pi}^m))\right)}{\frac{\psi'(E(p^u(\tilde{\pi}^m)))}{\theta} - \varphi(p^u(\tilde{\pi}^m))}.$$

By the same construction as above, p^u from the baseline scenario satisfies

$$\text{(A.45)} \qquad p^u - c + \frac{D(p^u)}{D'(p^u)} - \frac{\varphi'(p^u)}{D'(p^u)} = \frac{\varphi'(p^u)}{D'(p^u)}\frac{\left(\pi(p^u,c) - \varphi(p^u)\right)}{\frac{\psi'(E(p^u))}{\theta} - \varphi(p^u)}.$$

By quasi-concavity of the Lagrangean in the baseline scenario, (A.44) and (A.45) imply that (6.15) holds.

Finally, because $\frac{\psi(e)}{1+\theta e}$ is increasing in e and $\varphi' < 0$, the function $E(p)$ is decreasing. Condition (6.16) then follows from (6.15). □