Redistribution in the context of political regime change

Laura Araújo de Freitas

Faculty of Economics, University of Cambridge

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Abstract

The model explains transitions between democracy and autocracy as the result of conflicts of interest over redistributive policies between poor citizens, an incumbent elite in charge of the autocracy, and a non-governing elite. The model merges two strands of literature in that it combines conflict between the elite and the poor, and the conflict within the elites. There are two novel findings. First, democratization occurs only at intermediate equality levels and stability of democracy is sensitive to small changes in equality. Second, the size of government is not necessarily larger in democracies than in autocracies.

1 Introduction

Why do countries experience political instability? Constant change in political institutions is bad for the economy: it decreases investment and productivity, and increases uncertainty and inflation (Barro, 1991; Alesina & Perotti, 1996; Carmigani, 2003; Dupas & Robinson, 2010; Aisen & Viega, 2013). These results have remained robust to different datasets, empirical strategies and variables over the past 30 years. Political instability is also a widespread situation, which affected multiple countries recently: from the wave of unrest sweeping former colonies in the second half of the 20th century, to the instability characterising the former soviet nations in the 1990s, to the 2010s Arab Springs which sparked conflicts still ongoing today. The persistence of political instability across countries and time makes a case for understanding what triggers regime change.

Parallel to this question, runs another: do democracies and autocracies differ in terms of policy? There is consensus that political institutions strongly impact economic outcomes, but there is no agreement on which institutions have an impact on which outcomes. Citizens from different income levels are bound to have conflicts of interest over taxation. Consider an autocracy is a political system where only a rich minority elite is enfranchised. Then redistributive conflicts are inherently different between democracies and autocracies. Hence, redistributive policy is a benchmark for understanding whether electorate size and institutions significantly impact policies. Empirical evidence on the relation is mixed (Acemoglu et al., 2015), calling for a theory to make sense of the patterns of redistribution and democracy.

This paper highlights pre-tax income inequality as the fundamental driver of the relationship between democracy, regime stability and government size. The model comprises a game with three agents: the poor, and two mutually exclusive elites. Initially, only one of these elites is enfranchised so they are the only ones that set the tax policy. The non-enfranchised groups may threaten to overthrow the incumbent elite. The incumbent elite has two tools to prevent this: franchise extension and redistributive policy. Clearly, the groups have different preferences over tax policy. The threats are transitory, determined by a stochastic process, so the enfranchised cannot permanently commit to a single policy. There are therefore three possible political regimes: an autocracy, where only one elite is enfranchised; a post-revolution, where only the poor are enfranchised; and a democracy, where all agents are enfranchised. The model is therefore an extension on Acemoglu & Robinson (2001), modelling two elites instead of one and allowing for group-specific tax discrimination.

The model makes two contributions which set it apart from the literature. First, it develops a model of regime change that merges two strands of literature: the conflict between rich and poor, and the conflict between elites. The only other paper in this spirit is Gilli & Li (2015), but they do not focus on redistribution polices. Second, the paper conciliates the theoretical and empirical literature on the relation between democracy and government size. While theoretical intuition claims a democracy should support more redistributive policies due to poor citizens having the power to sway policy in their favour, empirical evidence has been mixed. I argue this discrepancy occurs because modes of democracy and redistribution treat the elite as a single unified group. In this model, the inclusion of a second elite allows an autocratic regime to charge different tax rates to the poor and the concurrent elite, allowing for the possibility of a large government size.

There are three main findings from the model. First, tax rates in democracies should decrease with equality, with very equal democracies eventually suffering a coup and reverting to autocracy. Second, very unequal autocracies suffer coups: the two elites keep overthrowing each other almost every period so that the there is no redistributive taxation. This occurs because inequality is so high that neither democratization nor redistribution can please the poor, who will stage a revolution if given the chance. Because the poor can only stage a revolution if the elite foregoes the chance of a coup, the concurrent elite throws a coup to not give them a chance. Third, tax rates in autocracy decrease with equality.

Employing minimalist assumptions, I discuss a case study which explores the long-term dynamics of political instability. This case study is characterized by three traits. First, very unequal societies are characterized by autocratic regimes which keep overthrowing each other, and where there is no redistributive taxation. Second, very equal societies are home to a single stable autocracy. Total tax revenue in this instance might be larger than under democracy. In this case, the incumbent elite keeps the poor from staging a revolution by redistributing from themselves and the concurrent elite to the poor. Third, democracy arises at intermediate levels of inequality. Whether democracy persists forever or eventually reverts to autocracy due to a coup is sensitive to small changes in inequality. That is, there are instances where the inequality rate is simultaneously too small and too large to allow for a stable democracy.

The remainder of the paper is structured as follows. Section 2 discusses a literature review. Section 3 introduces the model, and Section 4 showcases the main results. Section 5 concludes.

2 Literature review

The closest papers to this one are Acemoglu & Robinson (2001, 2006) and Gilli & Li (2015). Acemoglu & Robinson (2001, 2006) build a model where the poor may threaten an autocracy with overthrowal and the elite can credibly threaten to overthrow a democracy. The enfranchised group can prevent overthrowal by setting tax policy favorable to the other group. However, the papers model a single unified elite, failing to replicate how coups are more common against autocracies than democracies (Belkin & Schoefer, 2003). The absence of inter-elite conflict implies the model cannot account for tax discrimination - where the poor are charged one tax rate, and each elite another, as occurs in this paper.

Gilli & Li (2015) create a model of an autocracy facing threat of both coup and revolution. There are three differences between their model and this paper. First, they do not consider redistribution a tool to prevent overthrowal. They instead focus on economic efficiency policies and how these impact coup and revolution threats. Nevertheless, they briefly mention how these interact with redistribution. Second, their model focuses on autocracy and does not allow for transitions to democracy. Third, they model elite conflict with a selectorate which can oust an autocratic leader. In contrast, this paper's approach is less refined, modelling two elites as two different agents[1].

The model in Section 3 contributes to two strands of literature. First, the literature on the link between franchise extension and redistribution. The link between franchise extension, inequality and tax policy was first formalized by Meltzer & Richard (1981). In this model, only wealthier agents are allowed to vote on a universal tax rate, using a median voter theorem. Enfranchising poorer agents causes the tax rate to increase as the median voter becomes more open to redistribution. Grossman (1991), Roemer (1985) and Verdier & Ades (1996) explore these relations as strategic games where non-enfranchised agents credibly threaten the elite with overthrowal. Acemoglu & Robinson (2000) apply this "threat of revolution" idea to the context of 19th century European franchise extensions. Their model portrays franchise extension and redistribution as tools used by the ruling elite to prevent poor citizens from overthrowing them. Their main result is threat of revolution triggers the elite to change redistributive polices in equal societies, and to democratize in unequal societies. Boix (2003) and Conley & Temimi (2001) develop similar models.

Second, the literature on inter-elite conflict. Here, the focus is on coups d'etat - "overt attempts by other elites within the state apparatus to unseat the sitting head of state using unconstitutional means" (Powell & Thyne, 2011). Note "the sitting head of state" can be either a democracy or an autocracy. Models focus on elites' conflict of interests over policy and the competence of the autocratic leaders (Bueno de Mesquita et al., 2003; Besley & Kudamatsu, 2008), with a particular focus on military coups (Acemoglu et al., 2010; Besley & Robinson, 2010; Leon, 2014)[2].

2.1 Empirical literature

This section presents the empirical evidence for the relations highlighted by the model in Section 3. First, I focus on the relationship between equality and democratization. Most empirical works find no evidence of a significant relation between equality and democratization (Przeworksi et al., 2006; Acemoglu et al., 2015) and highlight how reverse causality is likely to bias estimates. Dorsch & Maarek (2020) claim previous papers estimate an unconditional relation, when the focus should be on estimating an effect conditional on the threat of revolution being present. They do this by using economic recession as a proxy variable for threat of revolution and find more unequal societies have a higher probability of experiencing democratic improvements following an economic downturn, for countries in the late 20th century.

[1] For a model of complex regime dynamics, see Carvalho & Dippel (2020).

[2] There is a tangent strand of literature which is not modelled in this paper. The literature on collective action (Olson, 1971) explores how individuals coordinate their actions to overthrow the government. They present an information problem, where government overthrow is a public good but agents have an incentive to free ride (Chwe, 2000; Lohmann, 1994b; Kuran, 1989; Battagini, 2017). Ellis & Fender (2010) combine Acemoglu & Robinson (2000; 2006) with Lohmann (1994a,b). Global games portray the government as an agent who manipulates information about its strength. Citizens update their beliefs about the regime and decide whether to overthrow it (Ginkell and Smith, 1999; Li et al., 2022). Kiss et al. (2017), Edmond (2013) and Guriev & Treisman (2019, 2020) highlight the role of media in this type of information manipulation.

A concern with this approach is the reverse causality between democracy, equality and growth. While a recession might cause citizens to be dissatisfied with the government and thus push for regime change, a change in regime might also affect growth by proposing new institutions and policies (Acemoglu et al., 2019). Burke & Leigh (2010) and Bruckner & Ciccone (2011) deal with reverse causality by using temperature, rainfall and commodity export princes as instrumental variables for economic growth, but caution against weather shocks as weak instruments. They find faster economic growth reduces the likelihood of democracy. In a similar vein, Kim (2016) argues that using temperature and rain fall as instruments for GDP corrects for the simultaneous causality between coups and growth. Kim finds an inverse positive correlation between the two, contrasting the insignificant results found by Powell (2012).

There are alternate measures of threat of revolution and coup in addition to recession. For instance, Aidt & Jensen (2014) measure threat of revolution in one country with the occurrence of revolutionary events abroad in 19th century Europe. In their seminal paper, Belkin and Schoefer (2003) attribute threat of a coup to a weighted index of strength of civil society, regime legitimacy and occurrence of past coups. The latter element of the index is in keeping with Londrengard and Poole (1990)'s hypothesis of a coup trap whereby, once a coup against autocracy occurs, it is likely more will follow. This pattern is most common in poorer countries. Gassebner et al. (2016) use extreme bounds analysis to analyse the veracity of these results.

Second, the relationship between government size and franchise extension. Acemoglu and Robinson (2000)'s model implies democracies endorse a larger government, specifically a larger proportion of taxes . This view is shared by the literature on fiscal capacity investment (Besley & Person, 2008, 2009, 2010) who predict taxation to be lower in autocracies and in the presence of internal conflict. Acemoglu et al. (2015) provide a summary of the literature on the empirical evidence which is mixed, ranging from insignificant results (Gil et al., 2004), to results in favour of franchise extension (Persson & Tabelini, 2003; Aidt & Jensen, 2013), to evidence of a U-shaped relation (Aidt et al., 2010). Acemoglu et al. (2015) and Kammas & Sarantides (2019) propose that democracy is associated with higher spending on public goods rather than direct redistributive taxes, which are more common in autocracies.

Third, the relation between regime duration and democracy. Here, the primary estimation strategy is survival analysis (Quiroz Flores, 2022; Metzger & Jones, 2016; Maeda, 2010). These papers share key insights on whether certain covariates of democracy are proportional. Specifically, Quiroz Flores (2022) finds that ethnic fragmentation is non-proportional, only playing a role in the early stages of democracy. He also shows the risk of autocratic transition begins to accumulate quickly late in a democratic period for most countries which recently transitioned into democracies.

The paper focuses on pre-tax income inequality as the main driver of different regime paths across countries. However, there is a literature pointing at other factors which are, when possible, included in the empirical estimation in Section 5. These factors include: education and human capital (Glaeser et al., 2007; Bourguignon & Verdier, 2000; Acemoglu et al., 2015), opennes to trade (Dorsch & Maarek, 2020; Burke & Leigh, 2010), culture (Alesina & Giuliano, 2015; Gorodnichenko & Roland, 2021; Ang et al., 2021), ethnic diversity (Galor & Klemp, 2017) and history of democratic institutions and geography (Giuliano & Nunn, 2013; Bentzen et al., 2017; Aghion et al., 2004).

Despite not explicitly modelled in this paper, ethnic diversity is present if we interpret the two elites in the model as two rich groups from different ethnicities. According to this interpretation, the elite originally in power is the religious or racial group dominating the country, which must keep the other elite in check. In this context, if the incumbent elite imposes a tax on the concurrent elite but not on themselves, this could be interpreted as a tax on identity, as in the case of Saleh & Tirale (2021).

3 Model

This section presents the main theoretical model and discusses its main results. Before that, however, it presents a simplified version of the main model, with only the poor and a single unified elite. This model is essentially the same as Acemoglu & Robinson (2001) with only two minor modifications: the tax rate is imposed only on the elite, instead of on all agents; and the stochastic process does not affect asset productivity. The aim of including this model is to build up intuition behind the two competing elites extension which makes up the main model in Section 3.2., thus showcasing the main contribution of the paper and what exactly am I doing differently.

I employ a Pure Strategy Markov Equilibrium concept. This concept is the standard used in the literature, as history independence simplifies computation and allows for straightforward comparative statics. The focus on pure strategies steams from mixed strategies not having any clear cut implications for real life political instability.

3.1 Simplified model

Consider an infinitely repeated game with complete information. The economy is populated with a continuum 1 of agents. A fraction $\lambda > \frac{1}{2}$ of agents are poor, in that they collectively own a fraction $\theta < \lambda$ of assets in the economy. Therefore each poor agent is endowed with $\frac{\theta}{\lambda}$. The remaining agents belong to an elite and collectively own the remaining assets, so that each elite agent is endowed with $\frac{1-\theta}{1-\lambda}$. I denote the poor and the elite with subscripts $i = \{p, e\}$.

Assets can be invested into one of two sectors. The market sector A turns assets into final consumption goods given function $y_A(h) = h$ and these final goods can be taxed. The private sector B turns assets into final goods at function $y_B(h) = bh$. By setting $b < 1$, I insure agents invest all their assets in the more productive market sector A, unless the tax rate is superior to $1 - b$, at which point the agents are better off switching to the less productive but tax free private sector. The sole purpose of this feature is to establish an upper limit to the tax rate, $1 - b$, ensuring that no regime can set the tax rate to 1 and tax agents' entire income. The expression $1 - b$ is fiscal capacity, in the sense that it is the maximum tax rate that can be charged.

Agents' utility is equal to their after tax and transfers income, and they discount future utility at rate β. There is no saving allowed in this model, so that agents have to convert their assets to final goods and consume them within the same period. In the next period, the endowments are recreated and redistributed. In each period, enfranchised citizens are the only ones allowed to choose a tax rate on the elite's income, using a median voter theorem. Here, I assume the poor's income cannot be taxed. One can interpret this assumption as the poor's assets being in such a small amount the elites have no interest in them, or the poor switching to the private sector immediately if any kind of taxation is attempted on them.

At the beginning of each period, nature determines the realization of the stochastic Markov process:

$$\{\mu, \rho\} = \begin{cases} \{\mu^h, 0\} & \text{with prob. } q \\ \{0, \rho^h\} & \text{with prob. } 1 - q \end{cases} \tag{1}$$

Where $1 > \mu^h, \rho^h > 0$. Here, μ is the fraction of assets in the whole economy surviving a revolution if one is staged and ρ is the fraction surviving a coup. The form of the process serves two functions. First, The Markov nature ensures the threat of revolution to an autocratic regime, and of a coup to a democracy, is transitory, so that future threat of instability might play a role in agents' actions.

Second, it ensures that periods which are conductive to revolution are not conductive to coups and vice versa. In this model, a credible threat to revolt allows non-enfranchised groups to have de-facto power. That is, although the groups are not enfranchised they can convince enfranchised groups to sway redistribution their way by threatening to revolt -in this sense, we can also interpret μ and rho as the ease with which agents can organise themselves and overcome the collective action problem to convincingly pose a threat of revolt. The process ensures that in some periods the poor have de facto power, whereas in others the rich have de facto power. Hence, non-governing groups can exert pressure on the incumbent rulers to steer policies their way.

The state of the economy is therefore summarized by the value of mu and by the initial political regime s. Below, we see there are three potential political regimes: autocracy E, democracy D and post-revolution R. Agent i's utility is therefore given by $V^i(\mu, S)$.

In the first period of the game, the elite are the only ones enfranchised. I call this political regime an autocracy, $s = E$, as only a minority of citizens are enfranchised. Nature determines μ and all agents, the poor and the elite, observe it. The elite chooses one of two actions: they can extend the franchise to the poor, or they can propose a tax schedule to try to remain in power. After observing the elite's action, the poor choose whether or not to stage a revolution.

If the poor stage a revolution, they destroy all but a fraction μ of the assets in the economy, and distribute the surviving assets among themselves. The destruction in assets is permanent - that is, $1 - \mu$ assets are destroyed in every subsequent period and the surviving assets are always redistributed only among poor agents. When a revolution takes place, the political regime transitions to a post-revolution, $s = R$, forever. Here, we can say that a post-revolution is an absorbing state. This implies that payoff from staging a revolution is $\frac{\mu}{\lambda(1-\beta)}$ for the poor and 0 for the elite. Here the value of μ from the poor's payoff is the value of μ from the period where they staged the revolution.

Note that, when $\mu = \mu^l$, the poor too earn 0 payoff from a revolution. This implies the poor only pose a credible threat of revolution, and thus only have de facto power, when $\mu = \mu^h$. Hence, whenever $\mu = \mu^l$ and $s = E$, the elite has no incentive to redistribute towards the poor and will choose to not tax their own income.

If the poor do not stage a revolution, there are two outcomes. If the elite initially proposed a tax rate on their own income, τ^e, then taxation is realized, and the period ends. Autocracy persists into the next period and the game repeats. If the elite initially proposed franchise extension, all agents, poor and elite, become enfranchised. As the poor are the majority, $\lambda > \frac{1}{2}$, the median voter is a poor agent. As this is the last action of the period, the poor set the maximum tax rate $1 - b$. In the next period, the political regime becomes a democracy, $s = D$.

In a democracy, nature determines μ and all agents observe it. If $\mu = \mu^h$, I assume the elite cannot stage a coup. As such, the poor will set the maximum tax rate, $1 - b$. Payoffs in a democracy when $\mu = \mu^h$ are the same as in the case of franchise extension above $V^i(\mu^h, D) = V^i_{FE}(\mu^h, E)$. If $\mu = \mu^l$, the poor set a tax rate τ^d on the elite's income. After observing the poor's choice of a tax rate, the elite decides whether to stage a coup against democracy.

If they stage a coup against democracy, all but ρ^h of assets in the economy are destroyed and the remaining assets are redistributed across the elite members. The destruction in assets is temporary

- that is, assets or destroyed only for this period. Staging a coup does not lead to an absorbing state, as a revolution does. Instead, in the next period, all assets are reinstated and the economy transitions to an autocracy, $s = E$.

In summary, the timing of the model is given below and summarized in Figure 1.

1. Nature determines μ. All agents observe it.
2. In an autocracy, $s = E$, the elite chooses to either propose a tax rate $\tau^e(\mu)$ or to extend the franchise to the poor. In a democracy, $s = D$, the poor propose a tax rate $\tau^p(\mu)$.
3. In an autocracy, the poor decide whether to stage a revolution against the elite. If they stage a revolution, they destroy but a fraction of assets and redistribute them among themselves. In the next period, the economy becomes a post-revolution, $s = R$. If they do not stage a revolution one of two things happens. If the elite initially proposed a tax schedule, the payoffs are realized, the period ends, and the autocracy persists in the next peirod. If the elite extended the franchise, the poor impose the maximum tax rate, $1 - b$ on the elite's payoffs. In the next period, the political regime is a democracy, $s = D$. In a democracy, the elite observe $\tau^p(\mu)$ and decide whether to stage a coup. If they do not stage a coup, payoffs are realized and a democracy persists into the next period. If they do stage a coup, they destroy a fraction of assets in the economy and redistribute the remains among themselves. In the next period, we return to autocracy, $s = E$.

3.2 Main model

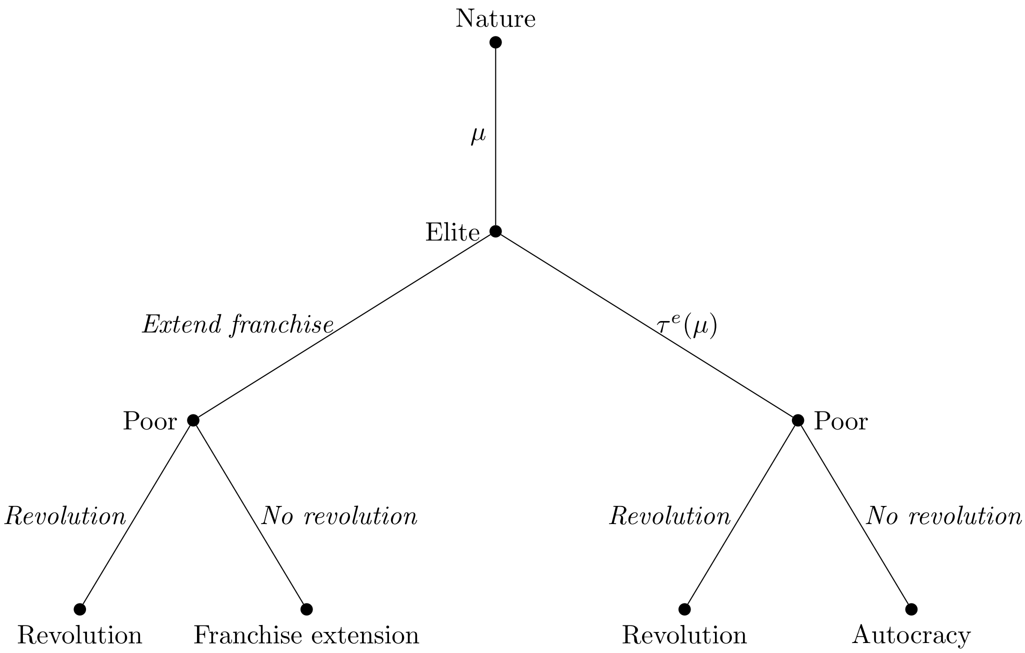
As before, consider an infinitely repeated game with complete information. The economy is still populated by a fraction $\lambda > \frac{1}{2}$ of poor agents and a fraction $1 - \lambda$ of elite agents. Each poor and elite agent own $\frac{\theta}{\lambda}$ and $\frac{1-\theta}{1-\lambda}$ respectively. Now, consider that elite agents are divided into two homogeneous groups such that each elite group has $\frac{1-\lambda}{2}$ members. I denote the poor and the two elites with subscripts $i = \{p, e^1, e^2\}$.

Most primitives of the model remain unchanged: fiscal capacity is $1 - b$, the discount rate is β, utility is given by $V^i(\mu, S)$, and the stochastic Markov Process for μ is the same as in (1). However, now enfranchised citizens no longer choose a single tax rate. Instead, they choose a tax schedule consisting of tax rates for each of the two elites, and a transfer for the poor and for each of the elites. There are now four potential political regimes: autocracy of the elite e^1, E^1; autocracy of the elite e^2, E^2; democracy, D; and post-revolution, R.

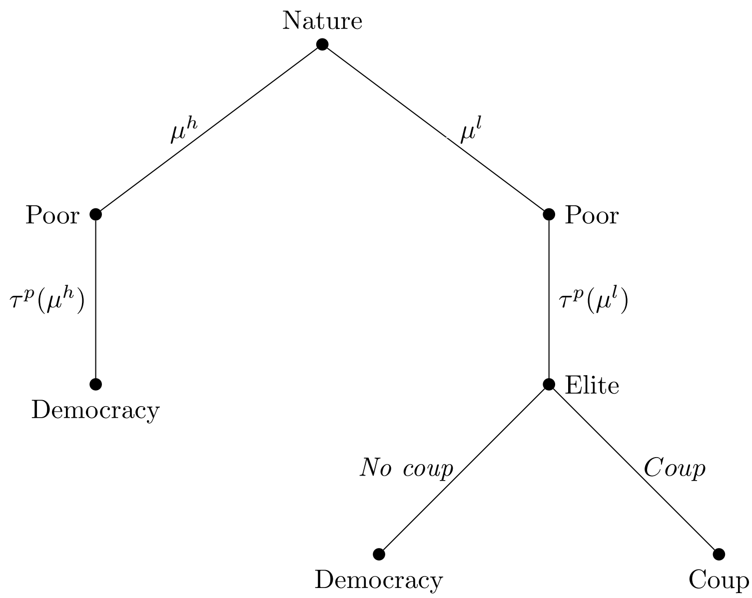
In the first period, only the members of elite e^1 are enfranchised, so the poor and elite e^2 have no direct power over the tax policy. This political regime, where only one of the elites is enfranchised, is an autocracy. Nature realizes μ and all agents observe it. The elite e^1 chooses one of two actions: they can extend the franchise the rest of population (elite e^2 and the poor), or they can attempt to remain in an autocracy by setting a tax schedule $\tau^e(\mu^l)$.

After observing the elite e^1's action, the elite e^2 decides whether to stage a coup against them. If they stage a coup, they destroy all but a fraction ρ of assets in the economy for that period and distribute the remaining assets among themselves. Hence the elite e^2 receives a period payoff of $\frac{2\rho}{1-\lambda}$. The poor and elite e^1's payoffs are 0. In the next period, assets are reinstated and the political regime becomes an autocracy of the elite e^2. Note that the destruction of assets by a coup is only temporary and assets are reinstated in the next period. Hence, even in periods where $\rho = 0$, the

Figure 1: Timing in simplified model



(a) Timing in an autocracy



(b) Timing in a democracy

concurrent elite might still find it optimal to stage a coup in order to be in power in the following period.

If the elite e^2 forsakes the opportunity of a coup, the poor decide whether to stage a revolution. If they stage a revolution, they destroy all but a fraction μ of assets in every period forever, as in Section 3.A. The payoff of revolution is $\frac{\mu}{(1-\beta)\lambda}$ to the poor and 0 to both elites. As before,the poor only pose a credible threat of revolution at periods when $\mu = \mu^h$ and it is only in these periods the incumbent elite finds it beneficial to swing policies in favour of the poor.

If the poor forsake the opportunity of a revolution, one of two things happens. If the elite e^1 initially proposed a tax schedule $\tau^e(\mu)$ to maintain their autocracy, their autocracy survives and, in the next period, the game repeats itself. To simplify computation, I set $\tau^e(\mu) = \{\tau^a(\mu), \tau^r(\mu), \tau^c(\mu)\}$. $\tau^a(\mu)$ distributes from the incumbent elite, in this case elite e^1 to the poor; $\tau^r(\mu)$ redistributes from the concurrent to the incubent elite, in this case from elite e^2 to elite e^1; and $\tau^c(\mu)$ redistributes from the concurrent elite, e^2 to the poor.

If the elite e^1 initially proposed franchise extension, the franchise is extended. Since all agents are now enfranchised and the poor make up the majority of the population, a poor agent sets the tax schedule. In the next period, the economy transitions to a democracy.

In a democracy, the period begins with nature setting μ. All agents observe μ and the poor propose tax policy $\tau^p(\mu)$. Because the poor's income cannot be taxed and the elites' incomes are identical, the tax policy can be represented as a single tax rate on the elite's income, redistributing from them to the poor. If $\mu = \mu^h$, neither elite can stage a coup against democracy. Hence, the median poor agent sets the maximum possible tax rate, $1 - b$, on the elites' incomes. If $\mu = \mu^l$, the poor set a tax policy. The elites observe the tax policy and decide whether to stage a coup. The elites observe the tax policy and commit to staging a coup, knowing each of them has a probability $\frac{1}{2}$ of being the one to stage a coup. After a coup is staged, all but a fraction ρ^h of assets are destroyed, and the remains are distributed across the elite members. In the next period, there is an autocracy of the rebelling elite.

The agents' payoffs for each situation are reported in Table 1. To summarize, the timing of the model in an autocracy is given below, as well as in panel (a) of Figure 2:

1. Nature determines μ. All agents observe it.
2. The incumbent elite chooses to either propose a tax schedule $\tau^e(\mu)$ or extend the franchise to all agents.
3. The concurrent elite observes the incubent's decision and chooses whether to stage a coup. If they perform a coup, they destroy a fraction of assets in the economy and redistribute them among themselves.
4. If the concurrent elite did not stage a coup, the poor decide whether to stage a revolution. If they stage a revolution, they destroy all but a fraction of assets in the economy and redistribute them among themselves. The period ends and the next period becomes a post-revolution. If they do not stage a revolution and the incubent elite originally proposed a tax schedule, the tax schedule is realized and the period ends, with autocracy persisting into the next period. If the poor do not stage a revolution and the incumbent elite originally extended the franchise, the poor impose the maximum tax rate $1 - b$ on the elites' income. The period ends and the next period becomes a democracy.

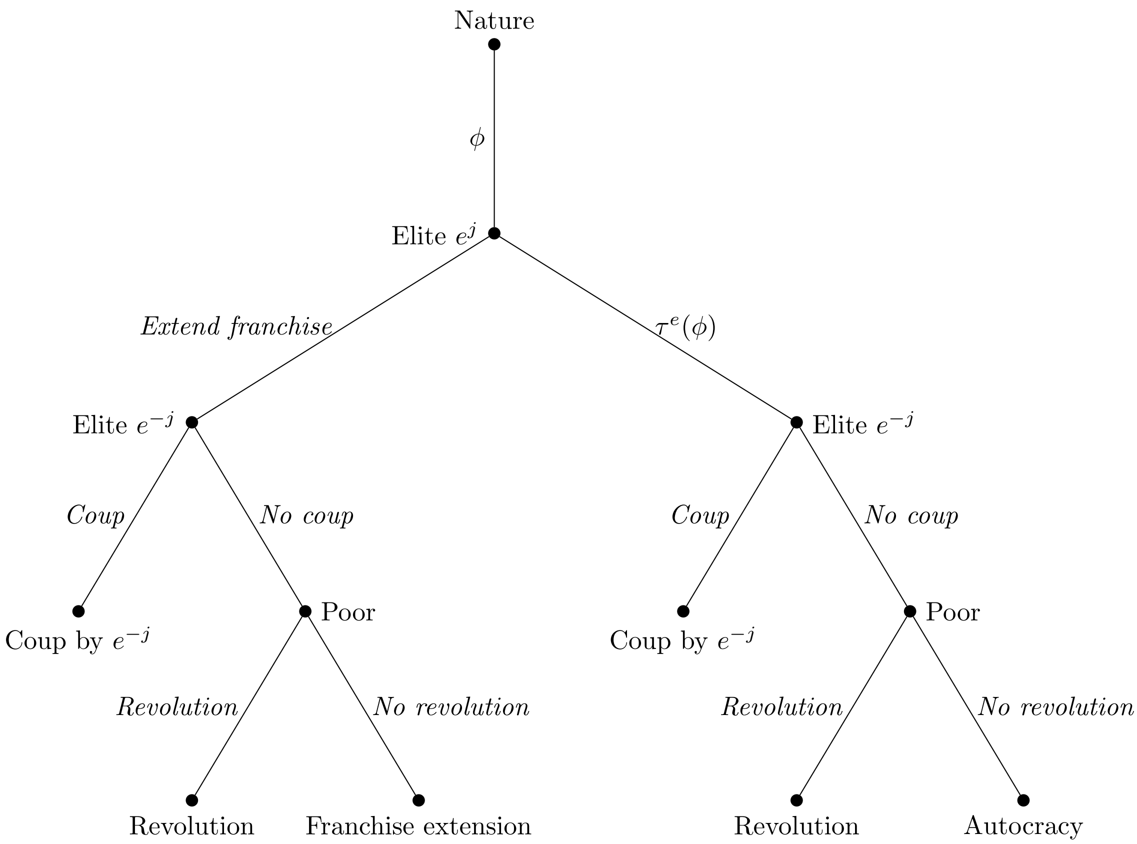
In a democracy, timing is given as below and in panel (b) of Figure 2.

1. Nature determines μ. All agents observe it.
2. The poor propose a tax schedule $\tau^p(\mu)$.
3. The elites decide whether to accept the tax schedule or commit to a coup. If they accept the schedule, redistribution is realized and democracy persists into the next period. If they commit to a coup, each elite knows they will be picked with probability $\frac{1}{2}$.
4. If the elites committed to a coup, nature determines which elite stages a coup. The rebelling elite destroys a fraction of assets and redistribute the remaining among themselves. The period ends and the next period becomes an autocracy of the rebelling elite.

4 Results

In this section, I analyse the main results. Step-by-step proofs are included in the Appendix. Before beginning, it is worth outlining the main assumption of the model.

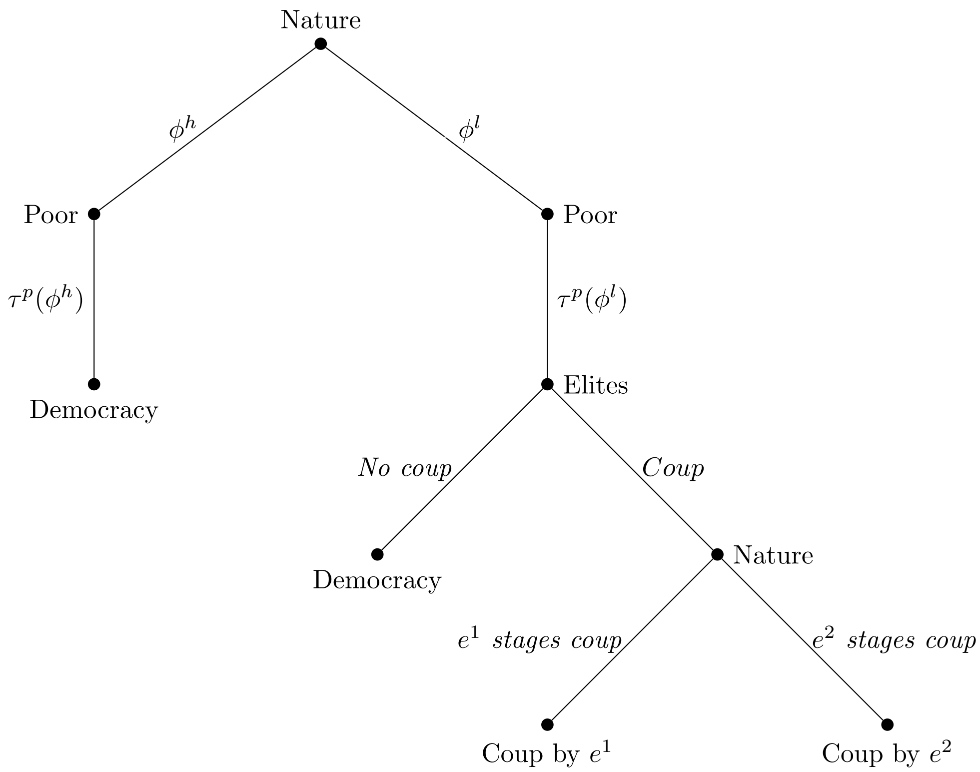
Figure 2: Timing in the main model



(a) Timing in an autocracy

Table 1: Payoffs for the Main Model

Outcome	Incumbent elite, e^j
Revolution	0
Coup	$\beta q V^{e^j}(\mu^h, E^{-j}) + \beta(1-a)V^{e^j}(\mu^l, E^{-j})$
Autocracy (redistribution)	$[1-\tau^a(\mu)+\tau^r(\mu)]\frac{1-\theta}{1-\lambda} + \beta q V^{e^j}(\mu^h, E^j) + \beta(1-q)V^{e^j}(\mu^l, E^j)$
Franchise extension	$b\frac{1-\theta}{1-\lambda} + \beta q V^{e^j}(\mu^h, D) + \beta(1-q)V^{e^j}(\mu^l, D)$
Democracy (no coup)	$[1-\tau^p(\mu)]\frac{1-\theta}{1-\lambda} + \beta q V^{e^j}(\mu^h, D) + \beta(1-q)V^{e^j}(\mu^l, D)$
	Concurrent elite, e^{-j}
Revolution	0
Coup	$\frac{2\rho}{1-\lambda} + \beta q V^{e^{-j}}(\mu^h, E^{-j}) + \beta(1-a)V^{e^{-j}}(\mu^l, E^{-j})$
Autocracy (redistribution)	$[1-\tau^c(\mu)-\tau^r(\mu)]\frac{1-\theta}{1-\lambda} + \beta q V^{e^{-j}}(\mu^h, E^j) + \beta(1-q)V^{e^{-j}}(\mu^l, E^j)$
Franchise extension	$b\frac{1-\theta}{1-\lambda} + \beta q V^{e^{-j}}(\mu^h, D) + \beta(1-q)V^{e^{-j}}(\mu^l, D)$
Democracy (no coup)	$[1-\tau^p(\mu)]\frac{1-\theta}{1-\lambda} + \beta q V^{e^{-j}}(\mu^h, D) + \beta(1-q)V^{e^{-j}}(\mu^l, D)$
	Poor, p
Revolution	$\frac{\mu}{1-\beta}$
Coup	$\beta q V^p(\mu^h, E^{-j}) + \beta(1-a)V^p(\mu^l, E^{-j})$
Autocracy (redistribution)	$\frac{\theta}{\lambda} + \frac{\tau^a(\mu)+\tau^c(\mu)}{2}\frac{1-\theta}{\lambda} + \beta q V^p(\mu^h, E^j) + \beta(1-q)V^p(\mu^l, E^j)$
Franchise extension	$\frac{\theta+(1-b)(1-\theta)}{\lambda} + \beta q V^p(\mu^h, D) + \beta(1-q)V^p(\mu^l, D)$
Democracy (no coup)	$\frac{\theta+\tau^p(\mu)(1-\theta)}{\lambda} + \beta q V^p(\mu^h, D) + \beta(1-q)V^p(\mu^l, D)$



(a) Timing in a democracy

4.1 Redistribution in democracy

In a democracy, the poor set a tax rate on the elites' income. $\tau^p(\mu, \theta)$. As stated before, when $\mu = \mu^h$ and there is no threat of a coup, the poor simply impose the highest possible tax rate such that $\tau^p(\theta, \mu^h) = 1 - b$. The more interesting case occurs when $\mu = \mu^l$ so that the elites credibly threaten a coup. The poor may then impose a lower tax rate, to prevent a coup. Here, a key consideration of the elites when deciding to stage a coup is their payoff if they transition to autocracy. If they expect a coup to be staged against autocracy, their payoff will be different than if they expect no coup. There are thus two possible scenarios, depending on whether a coup will be staged against autocracy, summarized in Lemmas 1 and 2, as well as in Figure 3. Note also Assumption 2:

ASSUMPTION 1: $\rho^h > 1 - \lambda$

The assumption states the fraction of assets surviving a coup must be higher than the fraction of elite members in the economy. The assumption ensures that a coup against democracy is attractive to the elites for at least some values of θ, so that in very equal economies, threat of a coup is credible.

LEMMA 1: Assume Assumption 1 holds. The current state of the world is $s = D$ and $\mu = \mu^l$ so that the political regime is a democracy under the threat of a coup. The poor fight the threat of a coup by proposing a tax rate $\tau^p(\mu^l, \theta)$. Consider also that if a coup is staged against democracy, there is no threat of a coup against autocracy. The unique Perfect Markov Equilibrium is:

- If $\theta < 1-\rho^h$, the elite's income is taxed at $\tau^p(\mu^l,\theta) = min\{1-b, [1-\beta(1-q)]\left[1-\frac{\rho^h}{1-\theta}\right]\}$ and democracy survives;
- If $\theta > 1-\rho^h$, the elite stages a coup.

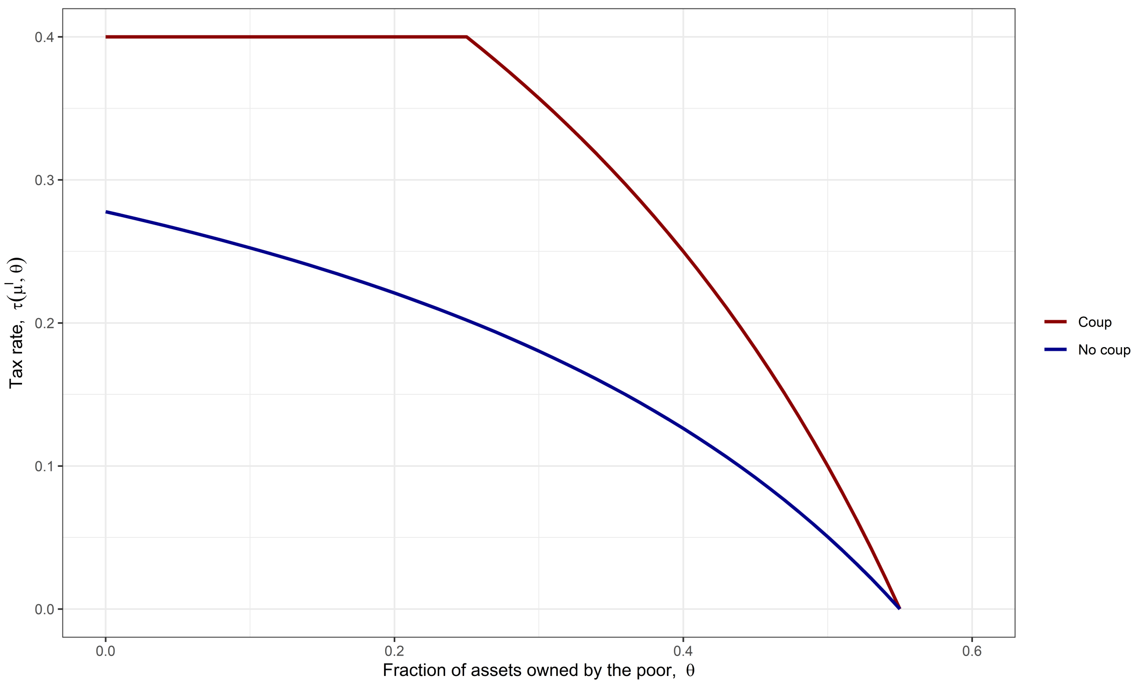
LEMMA 2: Assume Assumption 1 holds. The current state of the world is $s = E$ and $\mu = \mu^l$ so that the political regime is a democracy under the threat of a coup. The poor fight the threat of a coup by proposing a tax rate $\tau^p(\mu^l,\theta)$. Consider also that if a coup is staged against democracy, a coup is staged against autocracy. The unique Perfect Markov Equilibrium is:

- If $\theta < 1-\rho^h$, the elite's income is taxed at $\tau^p(\mu^l,\theta) = min\{1-b, 1-\frac{\rho^h}{1-\theta}\}$ and democracy survives;
- If $\theta > 1-\rho^h$, the elite stages a coup.

Three things are worth noting here. First, the tax rate is decreasing in θ. This is unsurprising. As the fraction of assets owned by the poor θ grows, the poor become relatively better off, so that the elites become relatively worse off. The elites become less and less willing to sacrifice part of their income and a coup becomes more and more attractive. Consequentially, to avoid a coup, the poor must lower the tax rate.

Second, the tax rate charged in a democracy is always at least as high when a coup is expected against autocracy. If a coup is staged against autocracy, then one of the elites will get 0 payoff in all the periods when $\mu = \mu^l$ until the economy transitions to a democracy again. Consequentially, the

Figure 3: Tax rates under democracy



Note: The figure is computed using $\beta = 0.99$, $\lambda = 0.6$, $b = 0.6$, $\rho^h = 0.45$, $\mu^h = 0.3$ and $q = 0.5$.

elites are willing to bear a higher tax rate under democracy than if they expected a secure autocracy with no coups. Note, however, that as θ increases, the gap between the two tax rates decreases.

Third, note these two observations are not dependent on Assumption 1 - the only role of Assumption 1 here is guaranteeing that there is a risk of a coup for high enough values of θ. Specifically, in very equal democracies, where $\theta > 1 - \rho^h$, the elites choose to stay a coup regardless of what they expect to happen in an autocracy. That is, the decision of staging a coup is purely dependent on taxation under democracy and not on what happens in the next period when there is an autocracy. This result steams, of course, from uncertainty when staging a coup against democracy: the elites are not sure which one of them will be the autocrat in the next period.

4.2 Redistribution in autocracy

In an autocracy, the incumbent elite choose between proposing a tax schedule $\tau^e(\mu, \theta)$ or extending the franchise. When $\mu = \mu^l$ and there is no threat of revolution, the elite has no reason to extend the franchise, so they choose to propose a tax schedule instead. As there is no threat of revolution, there is no need to redistribute towards the poor so that $\tau^a(\mu^l, \theta) = \tau^a(\mu^l, \theta) = 0$. So the incumbent elite only sets one tax, $\tau^r(\mu^l, \theta)$ which redistributes from the concurrent elite to the incumbent elite. After observing $\tau^r(\mu^l, \theta)$, the concurrent elite decides whether to accept it or stage a coup, so that they become the incumbent next period.

Here, the concurrent elite's decision depends on what they expect to happen in the next period, when $\mu = \mu^h$. Three things can happen in an autocracy under the threat of revolution: the franchise can be extended so we transition to a democracy in the following period; the concurrent elite may stage a coup; or the incumbent elite can propose a tax rate $\tau^e(\mu^h, \theta)$ which is accepted by the remaining agents. These cases are not interesting in on themselves. There are no straightforward comparisons between the cases that can be made without additional assumptions. The key takeaway, summarized in Lemma 3, is that $\tau^r(\mu^l, \theta)$ is decreasing in equality and, if equality is above a certain threshold θ^*, the concurrent elite stages a coup.

LEMMA 3: Suppose the current state of the world is $s = E^1$and $\mu = \mu^l$ so the political regime is an autocracy under no threat of a revolution. The incubent elite fights the threat of a coup by proposing a tax rate $\tau^r(\mu^l, \theta)$. Then, the unique Perfect Markov Equilibrium is such that there is a unique function $f(\rho^h, \theta)$ and a unique threshold θ^* such that:

- If $\theta* \in (0, \lambda)$ and $\theta < \theta^*$, the concurrent elite e^2's income is taxed at $\tau^r(\mu^l, \theta) = f(\rho^h, \theta)$ and autocracy survives;
- If $\theta^* \in (0, \lambda)$ and $\theta > \theta^*$, the concurrent elite e^2 stages a coup;
- If $\theta^* < 0$, the concurrent elite e^2 stages a coup.
- If $\theta^* > \lambda$, the concurrent elite e^2's income is taxed at $\tau^r(\mu^l, \theta) = f(\rho^h, \theta)$ and autocracy survives.

Instead, the most interesting case emerges when $\mu = \mu^h$ and the incumbent elite decides to respond to the threat of revolution by proposing a tax schedule $\tau^e(\mu^h, \theta)$. The concurrent elite observes the tax schedule and decides whether or not to stage a coup. If they do not stage a coup, then the poor must decide whether or not to stage a revolution. Once again, the poor and the concurrent elite's decision depends on what they expect to happen at $\mu = \mu^l$ - that is, it depends on whether there is a coup or not.

First, let us focus on the case where a coup is expected at $\mu = \mu^l$. Here, the outcome is summarized in Lemma 4 and Figure 4. Lemma 4 requires Assumption 2 below. The lower bound on

μ^h guarantees there are low enough values of θ for which the poor will stage a revolution if given a chance. The upper bound guarantees that there are high enough values of θ for which the incumbent elite will not have to tax their own income to stop a revolution.

ASSUMPTION 2:

LEMMA 4: Suppose $\frac{2\lambda}{1+\beta(1-q)+\lambda[1-\beta(1-q)]} > \mu^h > 1-\beta(1-q)$. The current state of the world is $s = E^1$ so that there is an autocracy of elite e^1. Consider the elite fights off the threat of revolution at $\mu = \mu^h$ by proposing tax schedule $\tau^e(\mu^h, \theta) = \{\tau^a(\mu^h, \theta), \tau^c(\mu^h, \theta)\}$. Consider also that agents expect a coup against autocracy when $\mu = \mu^l$. Then, the unique Perfect Markov Equilibrium is:

- if $\theta < \frac{\mu^h-(1-b)[1-\beta(1-q)]}{b[1-\beta(1-q)]}$, the elite e^2 stages a coup;
- if $\theta \geq \frac{\mu^h-(1-b)[1-\beta(1-q)]}{b[1-\beta(1-q)]}$, the poor and the concurrent elite e^2 accept the tax schedule and autocracy survives. The tax schedule is given by:

$$\tau^a(\mu^h, \theta) = \begin{cases} \frac{2}{1-\theta}\left[\frac{\mu^h}{1-\beta(1-q)} - \theta\right] - 1 + b & \text{if } \theta < \frac{1}{1+b}\left[\frac{2\mu^h}{1-\beta(1-q)} - (1-b)\right] \\ 0 & \text{if } \theta > \frac{1}{1+b}\left[\frac{2\mu^h}{1-\beta(1-q)} - (1-b)\right] \end{cases}$$

$$\tau^c(\mu^h, \theta) = \begin{cases} 1-b & \text{if } \theta < \frac{1}{1+b}\left[\frac{2\mu^h}{1-\beta(1-q)} - (1-b)\right] \\ max\{\frac{2}{1-\theta}\left[\frac{\mu^h}{1-\beta(1-q)} - \theta\right], 0\} & \text{if } \theta > \frac{1}{1+b}\left[\frac{2\mu^h}{1-\beta(1-q)} - (1-b)\right] \end{cases}$$

$$\tau^r(\mu^h, \theta) = \begin{cases} 0 & \text{if } \theta < \frac{1}{1+b}\left[\frac{2\mu^h}{1-\beta(1-q)} - (1-b)\right] \\ 1-b-\tau^c(\mu^h, \theta) & \text{if } \theta > \frac{1}{1+b}\left[\frac{2\mu^h}{1-\beta(1-q)} - (1-b)\right] \end{cases}$$

Three things are worth noting here. First, note that the concurrent elite only stages a coup for low values of θ. Here, the concurrent elite stage a coup not because they are discontent with the proposed tax schedule but rather because they know that, if they do not, the poor will stage a revolution. Recall from the timing of the model in panel (a) of Figure 2 that the poor can only stage a revolution if the elite forego the opportunity to stage a coup. Hence, a coup is a fail-safe mechanism to prevent the worst possible outcome for the elites - a revolution - from ever happening. This feature of the model potentially explains why coups tend to take place following protests and demonstrations of civil unrest.

Second, note that other than staging a coup to prevent a revolution, the concurrent elite is willing to bear the maximum tax rate $1-b$ and never stages a coup because they are discontent with the tax schedule themselves. This result is independent of Assumption 2. Instead, the concurrent elite knows they will stage a coup at the next realization of $\mu = \mu^l$, so that they will someday become the incumbent elite themselves and benefit from the tax schedule. So they are willing to bear the tax burden now.

Third, while the tax burden on the concurrent elite remains constant, its composition is different. Figure 4 displays this best. The solid blue line displays the tax rate on concurrent elite, while the dashed line shows the tax that is redistributed from the concurrent elite to the poor and the dotted line shows the tax that redistributes towards the incumbent elite. —For low values of θ the solid and dashed lines overlap, so that all of the tax collected from the elite is redistributed towards the poor to prevent a revolution. As θ rises, the poor become relatively wealthier, so that a revolution becomes less and less appealing: the poor would rather benefit from the tax schedule than stage a revolution. So redistribution from the concurrent elite to the poor decreases. Instead, the incumbent redistribute from the concurrent elite to themselves, explaining the upward slope of the dotted blue curve.

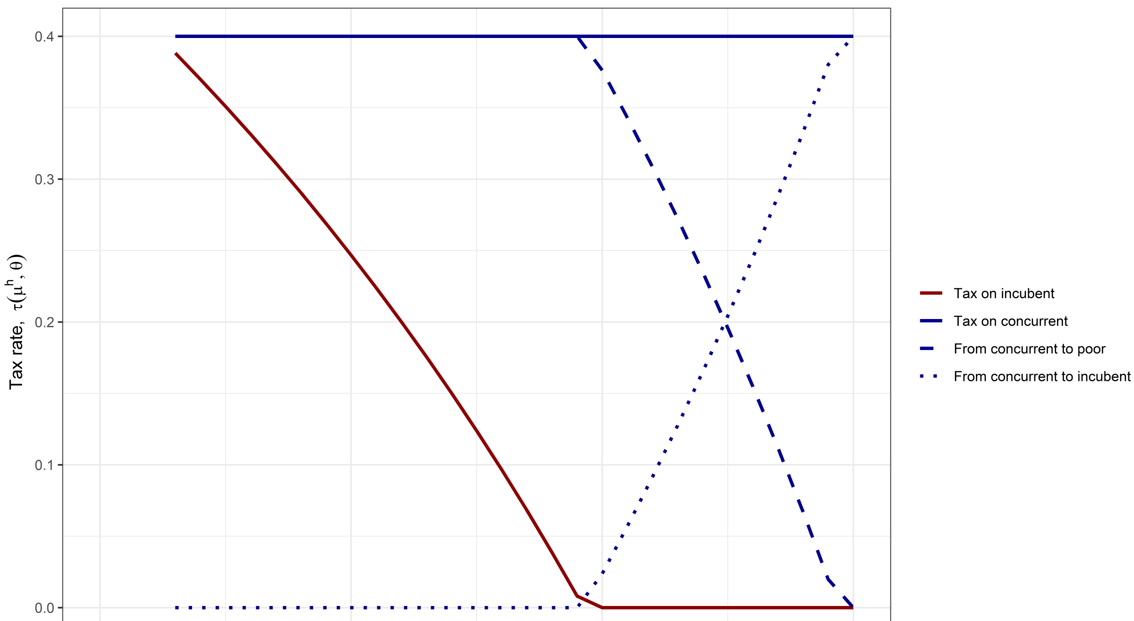
Fourth, also in Figure 4, the solid red line portrays the tax rate on the incumbent elite's income. For low values of θ, the threat of revolution can only be pacified by redistribution from both elites to the poor - redistribution from the concurrent elite alone does not suffice. Consequentially, the incubent elite taxes their own income. As θ increases, however, the redistribution necessary to pacify the poor lessens. The incumbent elite needs to forego less and less of their own income, until they no longer need to forego anything.

Let us now focus on the case where no coup is expected at $\mu = \mu^l$. Note Assumption 3 below. It has an equivalent interpretation to Assumption 2. The first inequality ensuring there are high enough values of θ such that a democracy for which the incumbent elite will not have to tax their own income. The second ensures there are low enough values of θ at which the poor will stage a revolution if given a chance.

ASSUMPTION 3: $\lambda > \frac{\mu^h - [1-\beta(1-q)][\frac{1}{2} - \tau^r(\mu^l,\theta)\frac{\beta(1-q)}{1-\beta+2\beta q}]}{1-[1-\beta(1-q)][\frac{1}{2} - \tau^r(\mu^l,\theta)\frac{\beta(1-q)}{1-\beta+2\beta q}]}, \frac{\mu^h - [1-\beta(1-q)][1-\frac{b}{2} - \tau^r(\mu^l,\theta)\frac{\beta(1-q)}{1-\beta+2\beta q}]}{1-[1-\beta(1-q)][1-\frac{b}{2} - \tau^r(\mu^l,\theta)\frac{\beta(1-q)}{1-\beta+2\beta q}]} > 0$

LEMMA 5: Suppose Assumption 3 holds. The current state of the world is $s = E^1$ so that there is an autocracy of elite e^1. Consider the elite fights off the threat of revolution at $\mu = \mu^h$ by proposing tax schedule $\tau^e(\mu^h, \theta) = \{\tau^a(\mu^h, \theta), \tau^c(\mu^h, \theta), \tau^r(\mu^h, \theta)\}$. Consider also that agents expect

Figure 4: Tax rates under autocracy



Note: The figure is computed using $\beta = 0.99$, $\lambda = 0.6$, $b = 0.6$, $\rho^h = 0.45$, $\mu^h = 0.3$ and $q = 0.5$.

no coup against autocracy when $\mu = \mu^l$, such that there is a tax on the concurrent elite's income $\tau^r(\mu^l, \theta)$. Then, the unique Perfect Markov Equilibrium is:

- If $\tau^r(\mu^l, \theta) > \frac{(2-b)(1-\beta+2\beta q)}{2\beta(1-q)}$, the concurrent elite e^2 stages a coup;
- If $\tau^r(\mu^l, \theta) \leq \frac{(2-b)(1-\beta+2\beta q)}{2\beta(1-q)}$ and $\theta < \frac{\mu^h - [1-\beta(1-q)][1-\frac{b}{2}-\tau^r(\mu^l,\theta)\frac{\beta(1-q)}{1-\beta+2\beta q}]}{1-[1-\beta(1-q)][1-\frac{b}{2}-\tau^r(\mu^l,\theta)\frac{\beta(1-q)}{1-\beta+2\beta q}]}$, the concurrent elite e^2 stages a coup;
- If $\tau^r(\mu^l, \theta) \leq \frac{(2-b)(1-\beta+2\beta q)}{2\beta(1-q)}$ and $\theta \geq \frac{\mu^h - [1-\beta(1-q)][1-\frac{b}{2}-\tau^r(\mu^l,\theta)\frac{\beta(1-q)}{1-\beta+2\beta q}]}{1-[1-\beta(1-q)][1-\frac{b}{2}-\tau^r(\mu^l,\theta)\frac{\beta(1-q)}{1-\beta+2\beta q}]}$, the poor and the concurrent elite e^2 accept the tax schedule and autocracy survives. The tax schedule is given by:

$$\tau^a(\mu^h, \theta) = \begin{cases} \frac{2}{1-\beta(1-q)}\frac{\mu^h-\theta}{1-\theta} - 1 + \tau^r(\mu^l,\theta)\frac{2\beta(1-q)}{1-\beta+2\beta q} & \text{if } \theta < \frac{\mu^h - [1-\beta(1-q)][\frac{1}{2}-\tau^r(\mu^l,\theta)\frac{\beta(1-q)}{1-\beta+2\beta q}]}{1-[1-\beta(1-q)][\frac{1}{2}-\tau^r(\mu^l,\theta)\frac{\beta(1-q)}{1-\beta+2\beta q}]} \\ 0 & \text{otherwise} \end{cases}$$

$$\tau^c(\mu^h, \theta) = \begin{cases} 1 - \tau^r(\mu^l,\theta)\frac{2\beta(1-q)}{1-\beta+2\beta q} & \text{if } \theta < \frac{\mu^h - [1-\beta(1-q)][\frac{1}{2}-\tau^r(\mu^l,\theta)\frac{\beta(1-q)}{1-\beta+2\beta q}]}{1-[1-\beta(1-q)][\frac{1}{2}-\tau^r(\mu^l,\theta)\frac{\beta(1-q)}{1-\beta+2\beta q}]} \\ max\{\frac{2}{1-\beta(1-q)}\frac{\mu^h-\theta}{1-\theta}, 0\} & \text{otherwise} \end{cases}$$

$$\tau^r(\mu^h, \theta) = \begin{cases} 0 & \text{if } \theta < \frac{\mu^h - [1-\beta(1-q)][\frac{1}{2}-\tau^r(\mu^l,\theta)\frac{\beta(1-q)}{1-\beta+2\beta q}]}{1-[1-\beta(1-q)][\frac{1}{2}-\tau^r(\mu^l,\theta)\frac{\beta(1-q)}{1-\beta+2\beta q}]} \\ 1 - \tau^r(\mu^l,\theta)\frac{2\beta(1-q)}{1-\beta+2\beta q} - \tau^c(\mu^h,\theta) & \text{otherwise} \end{cases}$$

According to Lemma 5, the outcome depends on the tax rate at $\mu = \mu^l$, $\tau^r(\mu^l, \theta)$. We know from Lemma 3 that it is a decreasing function of θ and ρ^h. Two things are worth noting here.

First, as before, the concurrent elite may stage a coup when θ is low enough to prevent a revolution. However, in this case they may also stage a coup if the tax rate they are charged at $\mu = \mu^l$ is high enough. Here, the intuition is that the concurrent elite will stage a coup at $\mu = \mu^h$ if they expect a high tax on their income at the next realization of $\mu = \mu^l$. However, note from the expressions that if $\tau^r(\mu^l, \theta) < frac(2-b)(1-\beta+2\beta q)2\beta(1-q)$, the elite will never stage a coup. Moreover, if $\tau^r(\mu^l, \theta) < \frac{1-\beta+2\beta q}{2\beta(1-q)}$, then concurrent elite e^2 is willing to be taxed the highest possible tax rate, $1-b$ at all levels of θ.

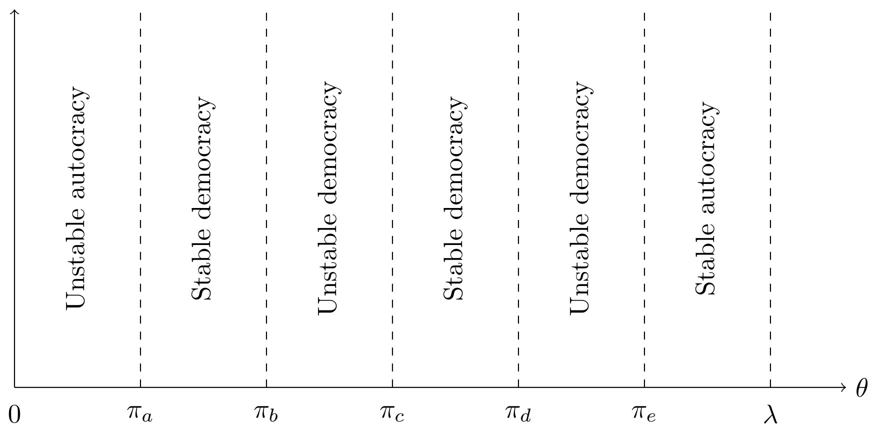
Second, the tax schedule follows the same pattern as in Lemma 4: for low enough θ both the incumbent and concurrent elites must tax their own incomes to prevent a revolution. As θ rises, however, the incumbent elite contributes less and less, until they contribute nothing at all. Simultaneously, a larger and larger fraction of the concurrent elite's taxed income is allocated towards the incumbent elite rather than the poor.

4.3 Political instability

To get tractable results in terms of political instability, I must make assumptions on the parameters b, μ^h and ρ^h. In this section, I present a case study, the proof and assumptions for which lie in the appendix. The case study makes as little assumptions as possible. Most of the assumptions impose an upper threshold for μ^h and lower bounds for rho^h. These assumptions are "weak" if we rely on the intuition that the fraction of assets surviving a revolution, μ^h, is low, and the fraction of assets surviving a coup, ρ^h, is high.

Figure 5 summarizes the outcomes for the political regime as a function of θ. Figure 6 details the equilibrium outcome at each period for each value of θ, s and μ. Essentially, the two figures

Figure 5: Pure Strategy Markov Perfect Equilibrium as a function of θ



are the same, but while Figure 5 summarizes the results, Figure 6 highlights the mechanisms. The relevant thresholds are given by:

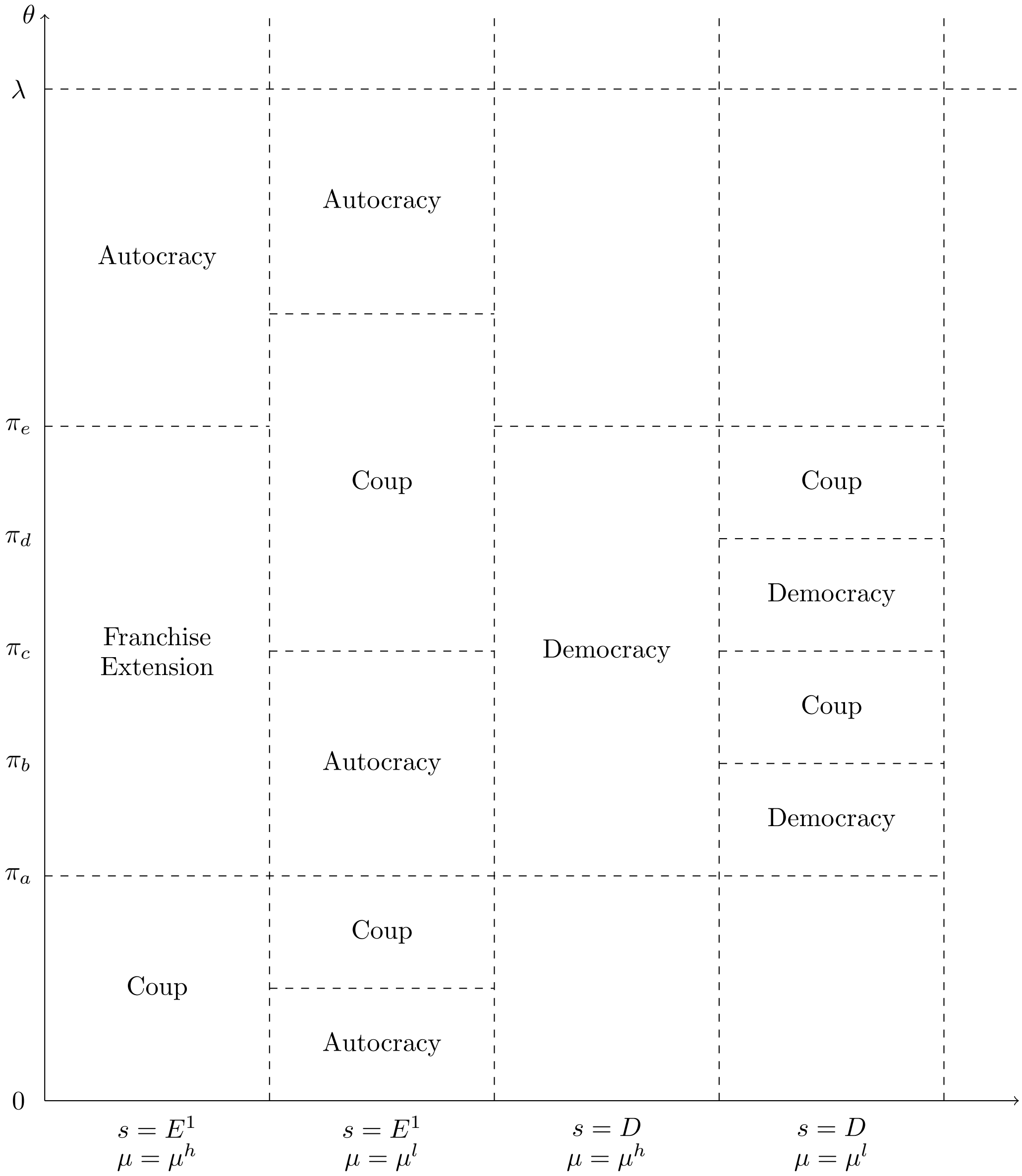
$$
\begin{aligned}
\pi_a &= \frac{\mu^h - (1-b)(1-\beta(1-q)) - \beta(1-q)[1-2\rho^h k_\rho - 2\beta(1-q) + k_b b]}{1-(1-b)(1-\beta(1-q)) - \beta(1-q)[1-2\rho^h k_\rho - 2\beta(1-q) + k_b b]} \\
\pi_b &= \frac{2\rho[1-\beta(1-q)][1+\frac{\beta(1-q)\alpha}{1-\beta+\beta^3 q(1-q)^2}]}{1-2\beta(1-q)+\beta^2 q(1-q)[1-\beta(1-q)]\left(\frac{1+\beta(1-q)}{1-\beta q}\alpha - 1\right)b} \\
\pi_c &= 1 - \frac{2\alpha\rho}{\frac{1-\beta+\beta^3 q(1-q)^2}{1-\beta(1-q)} + b\beta q\left(1-\frac{1+\beta(1-q)}{1-\beta q}\alpha\right)} \\
\pi_d &= 1 - \frac{1-\beta}{1-\beta q - \beta^2 q(1-q)} \\
\pi_e &= \frac{1}{b}\left[\frac{\mu^h}{1-\beta(1-q)} - (1-b)\right]
\end{aligned}
\tag{2}
$$

Four things stand out about the figures. First, when θ is low, so that inequality is high, the economy never democratizes. Instead, there is a series of "unstable" autocracies, with the two elites staging coups against one another frequently. In this scenario, the concurrent elite stages a coup not because they are unhappy with the incubent's policy, but rather to avoid a revolution. Recall from the timing of the model, that, if the concurrent elite stages a coup, the poor no longer have the chance to stage a revolution. Hence, when θ is so low that the poor are bound to stage a revolution, the concurrent elite prevents this by staging a coup themselves. One can interpret this as the elites creating political instability which prevents the poor from successfully organizing themselves to stage a revolution.

Second, when $\theta > \pi_e$, so that inequality is low, the elite originally in power e^1 remains in power forever. This occurs because the poor are relatively well-off and thus less likely to stage a revolution. The incubent elite offers the poor tax benefits in periods where the poor have de facto power $\mu = \mu^h$ and the poor prefer these occasional benefits to transitioning to a post-revolutionary regime forever[3]

[3] At a first glance, this result seems to be contradicting Lemma 3 above which states that, if $\theta$is above a certain

Figure 6: Pure Strategy Markov Equilibrium, as a function of θ, s and μ



Third, the elite only extend the franchise at intermediate levels of equality, when $\theta \in (\pi_a, \pi_e)$. Here, the elite extends the franchise and the concurrent elite and the poor accept because there is a balance of their interests. The poor are wealthy enough that they will not stage a revolution and thus accept democracy. The elites are also wealthy enough that they can withstand the taxation imposed by the poor and still prefer it to staging a coup. The interval $(\pi_a, \pi_e$ corresponds to democratic window of opportunity - the interval at which democratization is possible. Note that:

$$\begin{aligned} \frac{\partial \pi_a}{\partial b} &= [1-\mu^h]\frac{1-\beta(1-q)-k_b\beta(1-q)}{[1-(1-b)(1-\beta(1-q))-\beta(1-q)[1-2\rho^h k_\rho-2\beta(1-q)+k_b b]]^2} \\ \frac{\partial \pi_e}{\partial b} &= \frac{1}{b^2}\left[\frac{\mu^h}{1-\beta(1-q)}-1\right] \end{aligned} \tag{3}$$

Both of which are greater than 0. So that both thresholds are decreasing in fiscal capacity, $1-b$. Without further assumptions, we cannot tell whether the democratic window of opportunity tightens or widens given a change in b.

Fourth, whether democracy is "stable" (lasts forever) or "unstable" (eventually reverts to autocracy when an elite stages a coup) is sensitive to small changes in θ. Figure 6 highlights the reason why. If $\theta < \pi_c$ and an elite stages a coup against democracy, the autocracy they revert back to is not overthrown by a coup. That is, if an elite stages a coup, this elite will remain in power until the next realization of $\mu = \mu^h$ when they next democratize. If $\theta > \pi_c$ instead, if an elite stages a coup against democracy, the two elites will stage coups against one another until they democratize once again. Hence, the discrepancy in the pattern of stability in democracy steams from the elite's expectations as to what will happen in the future.

5 Conclusion

The paper develops a game theoretic model of regime dynamics.The model is based on Acemoglu and Robinson (2001, 2006) but adds to the literature on regime change in two ways. First, it allows rich agents to be divided into groups with conflicting interests, rather than modelling a single united elite as previous papers did. Second, the model permits group-based tax discrimination, allowing for complex tax schedules, and for large tax revenues in autocracies as well as democracies.

There are four main results. First, very unequal societies tend not to democratize. This result contradicts the main conclusion of some papers in the literature, but this finding results from the modelling of a fragmented elite. Second, in keeping with the results from other works, very equal societies tend not to democratize at all. Instead, autocracy persists and the incumbent elite staves off threat of revolution or coup through redistribution. Third, two types of tax schedules emerge from stable autocracies. In those autocracies where the poor are relatively well off, the ruling elite manages to stave off the threat of revolution by redistributing solely from the concurrent elite to the poor. In autocracies where the poor are relatively worse off, the first elite foregoes their own tax benefits and tax their own income to remain in power. Fourth, democracy emerges at intermediate levels of inequality, and its stability is highly elastic with respect to inequality.

Despite its contributions, the current paper falls short in two ways. First, the theoretical model requires a large number of assumptions in order to gain tractable results. Second, the model is yet to be empirically tested.

threshold θ^*, then the concurrent elite will stage a coup. However, the assumptions made for this case study essentially assume that $\theta^* > \lambda$ when the concurrent elite expects autocracy to survive at $\mu = \mu^h$. For more details, please refer to the Appendix.

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7 Appendix: Reditribution in democracy

7.1 Step 1

Consider that $s = E^1$ and $\mu = \mu^l$, so that the current regime is an autocracy under no threat of revolution. Because there is no threat of a revolution, the elite e^1 has no incentive to democratize. Instead, they set tax schedule $\tau^e(\mu^l, \theta)$. There is no need to redistribute towards the poor so $\tau^a(\mu^l, \theta) = \tau^c(\mu^l, \theta) = 0$. The only relevant tax rate is $\tau^r(\mu^l, \theta)$ which redistributes from the concurrent elite (in this case e^2) to the incumbent. $\tau^r(\mu^l, \theta) = [b-1, 1-b]$ so that redistribution can go either way.

The concurrent elite e^2 observes $\tau^r(\mu^l,\theta)$ and decides whether or not to stage a coup. If the concurrent elite does not stage a coup, the incubent elite e^1 remains in power until the next period. I denote the payoffs in this case by the subscript NC.

$$\begin{aligned}
V_{NC}^{e^1}(\mu^l,E^1) &= \frac{1}{1-\beta(1-q)}\left[(1+\tau^r(\mu^l,\theta)\frac{1-\theta}{1-\lambda}+\beta q V^{e^1}(\mu^h,E^1)\right]\\
V_{NC}^{e^2}(\mu^l,E^1) &= \frac{1}{1-\beta(1-q)}\left[(1-\tau^r(\mu^l,\theta)\frac{1-\theta}{1-\lambda}+\beta q V^{e^2}(\mu^h,E^1)\right]\\
V_{NC}^{p}(\mu^l,E^1) &= \frac{1}{1-\beta(1-q)}\left[\frac{\theta}{\lambda}+\beta q V^{p}(\mu^h,E^1)\right]
\end{aligned}$$

If the concurrent elite e^2 instead stages a coup, then they destroy all but a fraction ρ^h of assets in the economy and redistribute these among themselves. In the next period, the political regime becomes E^2 - an autocracy ruled by the elite who staged a coup in the previous period. If we have another realization of $\mu=\mu^l$, then the new concurrent elite (e^2) will find it optimal to stage a coup. I denote the payoffs in this case by subscript C.

$$\begin{aligned}
V_C^{e^1}(\mu^l,E^1) &= \frac{\beta}{1-\beta^2(1-q)^2}\left[qV^{e^1}(\mu^h,E^2)+(1-q)\frac{2\rho^h}{1-\lambda}+\beta q(1-q)V^{e^1}(\mu^h,E^1)\right]\\
V_C^{e^2}(\mu^l,E^1) &= \frac{1}{1-\beta^2(1-q)^2}\left[\frac{2\rho^h}{1-\lambda}+\beta q(1-q)V^{e^2}(\mu^h,E^2)+\beta^2 q(1-q)V^{e^2}(\mu^h,E^1)\right]\\
V_C^{p}(\mu^l,E^1) &= \frac{\beta q}{1-\beta(1-q)}V^p(\mu^h,E)
\end{aligned}$$

Clearly the payoffs depends on what happens at $\mu=\mu^h$ and $s=\{E^1,E^2\}$. That is, what happens in an autocracy when the threat of revolution is credible.

7.2 Step 2

Consider $s=D$ and $\mu=\mu^h$, so the political regime is a democracy and there is no treat of a coup. Because they do not fear a coup, the poor charge the highest possible tax rate $\tau^p(\mu^h,\theta)=1-b$ to the two elites.

$$\begin{aligned}
V^{e^1}(\mu^h,D) &= \frac{1}{1-\beta q}\left[b\frac{1-\theta}{1-\lambda}+\beta(1-q)V^{e^1}(\mu^l,D)\right]\\
V^{e^2}(\mu^h,D) &= \frac{1}{1-\beta q}\left[b\frac{1-\theta}{1-\lambda}+\beta(1-q)V^{e^2}(\mu^l,D)\right]\\
V^{p}(\mu^h,D) &= \frac{1}{1-\beta q}\left[\frac{\theta+(1-b)(1-\theta)}{\lambda}+\beta(1-q)V^{p}(\mu^l,D)\right]
\end{aligned}$$

Clearly, the payoffs depend on what happens at $\mu=\mu^l$ and $s=D$. That is, what happens in a democracy under the threat of a coup.

7.3 Step 3: Proof of Lemmas 1 and 2

Consider $s=D$ and $\mu=\mu^l$, so the political regime is a democracy under the threat of a coup. The poor propose tax rate $\tau^p(\mu^l,\theta)$ to the two elites. The elites decide whether to accept the tax rate or to stage a coup. They know that if they do decide to stage a coup, each elite will only have a 0.5 chance of being the one to successfully stage the coup.

If the elites do not stage a coup, democracy persists into the next period. Hence, democracy persists forever. We can obtain the payoffs by plugging in the functions derived in Step 2. I denote the payoffs in this case with subscript NC.

$$V_{NC}^{e^1}(\mu^l, D) = V_{NC}^{e^2}(\mu^l, D) = \frac{1}{1-\beta}\frac{1-\theta}{1-\lambda}\left[(1-\beta q)(1-\tau^p(\mu^l,\theta)) + \beta q b\right]$$
$$V_{NC}^{p}(\mu^l, D) = \frac{\theta}{\lambda(1-\beta)} + \frac{1-\theta}{\lambda(1-\beta)}\left[(1-\beta q)\tau^p(\mu^l,\theta) + \beta q(1-b)\right]$$

Suppose instead the elites decide to stage a coup. Each elite knows they have a 0.5 probability of being the one to carry out the coup. Also recall that, if the elites are staging a coup against democracy, this means the franchise was extended previously. That is, if we have another realization of $\mu = \mu^h$ and $s = E^1, E^2$, the franchise must be extended. Taking this into account and plugging in the functions from Step 2, I obtain the following payoff. I denote the payoff by subscript C.

$$\begin{aligned}
V_{C}^{e^1}(\mu^l, D) =& \frac{1}{([1-\lambda][1-\beta q - \beta^2 q(1-q)]}\left[(1-\beta q)\rho^h + b\beta q(1-\theta)\right] + \\
& \frac{\beta(1-q)(1-\beta q)}{1-\beta q-\beta^2 q(1-q)}\frac{V^{e^1}(\mu^l, E^1) + V^{e^1}(\mu^l, E^2)}{2} \\
V_{C}^{e^2}(\mu^l, D) =& \frac{1}{([1-\lambda][1-\beta q - \beta^2 q(1-q)]}\left[(1-\beta q)\rho^h + b\beta q(1-\theta)\right] + \\
& \frac{\beta(1-q)(1-\beta q)}{1-\beta q-\beta^2 q(1-q)}\frac{V^{e^2}(\mu^l, E^1) + V^{e^2}(\mu^l, E^2)}{2}
\end{aligned}$$

Clearly, the payoffs depend on what happens at $s = E^1, E^2$ and $\mu = \mu^l$. That is, what happens in an autocracy under no threat of revolution. Recall the payoffs derived in Step 1. I showed that, under these circumstances, the concurrent elite can either stage a coup against autocracy or not. Let us refer to these cases as A and B.

CASE A

Consider the concurrent elite does not stage a coup under autocracy at $s = E^1, E^2$ and $\mu = \mu^l$. Plug in the values of $V_{NC}^{e^1}(\mu^l, E^1)$ and $V_{NC}^{e^2}(\mu^l, E^1)$ from Step 1 and plugging them into the expressions above for $V_C^{e^1}(\mu^l, D)$ and $V_C^{e^2}(\mu^l, D)$. The expressions simplify to:

$$\begin{aligned}
V_C^{e^1}&(\mu^l, D) = V_C^{e^2}(\mu^l, D) \\
&= \frac{[1-\beta(1-q)][1-\beta q]}{1-\beta}\frac{\rho^h}{1-\lambda} + \frac{\beta q b}{1-\beta}\frac{1-\theta}{1-\lambda} + \frac{\beta(1-q)(1-\beta q)}{1-\beta}\frac{1-\theta}{1-\lambda}
\end{aligned}$$

The elites will stage a coup as long as their payoff from doing so is greater than their payoff from accepting the continuation of democracy. That is, a coup is avoided as long as $V_{NC}^{e^1}(\mu^l, D) \geq V_C^{e^1}(\mu^l, D)$. Rearranging these terms, I obtain the following expression.

$$[1-\beta(1-q)]\left[1 - \frac{\rho^h}{1-\theta}\right] \geq \tau^p(\mu^l, \theta)$$

This expression has a straightforward interpretation. To prevent a coup, the taxation imposed on the elites must be below a certain threshold - given by the LHS of the expression. Too high a tax rate will incentivise the elites to rebel. Recall that $\tau^p(\mu^l,\theta) \in [0, 1-b]$. If the maximum tax burden the rich can bear - the LHS - is smaller than 0, this implies that the elites would require income to be redistributed from the poor to themselves in order to prevent a coup. As we are assuming the poor's income cannot be taxed, this corresponds to the case where the elites stage a coup regardless of the poor's decision. Rearranging, we can see a coup occurs when:

$$\theta > 1 - \rho^h \tag{1}$$

By assumption, we require that θ lie in $[0, \lambda]$. Hence, for the threshold in Expression (1) to be relevant, we also require it to lie on that interval. That is, I require that $\lambda > 1 - \rho^h > 0$. The second inequality holds by constructions. For the first inequality, I must make our first assumption.

ASSUMPTION A: $\lambda > 1 - \rho^h$

The tax rate under democracy is given by the following expression:

$$\tau^p_A(\mu^l,\theta) = \begin{cases} \text{Coup occurs} & \text{if } \theta > 1-\rho^h \\ min\left[[1-\beta(1-q)]\left[1-\frac{\rho^h}{1-\theta}\right], 1-b\right] & \text{if } \theta < 1-\rho^h \end{cases}$$

CASE B

Consider the concurrent elite does stage a coup under autocracy at $s = E^1, E^2$ and $\mu = \mu^l$. Plug in the values of $V^{e^1}_C(\mu^l, E^1)$ and $V^{e^2}_C(\mu^l, E^1)$ from Step 1 and plugging them into the expressions above for $V^{e^1}_C(\mu^l, D)$ and $V^{e^2}_C(\mu^l, D)$. The expressions simplify to:

$$V^{e^1}_C(\mu^l, D) = V^{e^2}_C(\mu^l, D) = \frac{1}{1-\beta}\left[(1-\beta q)\frac{\rho^h}{1-\lambda} + \beta q b\frac{1-\theta}{1-\lambda}\right]$$

The elites will stage a coup as long as their payoff from doing so is greater than their payoff from accepting the continuation of democracy. That is, a coup is avoided as long as $V^{e^1}_{NC}(\mu^l, D) \geq V^{e^1}_C(\mu^l, D)$. Rearranging these terms, I obtain the following expression.

$$1 - \frac{\rho^h}{1-\lambda} > \tau^p(\mu^l,\theta)$$

This expression has a straightforward interpretation. To prevent a coup, the taxation imposed on the elites must be below a certain threshold - given by the LHS of the expression. Too high a tax rate will incentivise the elites to rebel. Recall that $\tau^p(\mu^l,\theta) \in [0, 1-b]$. If the maximum tax burden the rich can bear - the LHS - is smaller than 0, this implies that the elites would require income to be redistributed from the poor to themselves in order to prevent a coup. As we are assuming the poor's income cannot be taxed, this corresponds to the case where the elites stage a coup regardless of the poor's decision. Rearranging, we can see a coup occurs when $\theta > 1 - \rho^h$ just as in Case A.

The tax rate under democracy is given by the following expression:

$$\tau^p_B(\mu^l,\theta) = \begin{cases} \text{Coup occurs} & \text{if } \theta > 1-\rho^h \\ min\left[1-\frac{\rho^h}{1-\theta}, 1-b\right] & \text{if } \theta < 1-\rho^h \end{cases}$$

8 Appendix: Redistribution in autocracy

8.1 Step 1

Consider that $s = E^1$ and $\mu = \mu^l$, so that the current regime is an autocracy under no threat of revolution. Because there is no threat of a revolution, the elite e^1 has no incentive to democratize. Instead, they set tax schedule $\tau^e(\mu^l, \theta)$. There is no need to redistribute towards the poor so $\tau^a(\mu^l, \theta) = \tau^c(\mu^l, \theta) = 0$. The only relevant tax rate is $\tau^r(\mu^l, \theta)$ which redistributes from the concurrent elite (in this case e^2) to the incumbent. $\tau^r(\mu^l, \theta) = [b - 1, 1 - b]$ so that redistribution can go either way.

The concurrent elite e^2 observes $\tau^r(\mu^l, \theta)$ and decides whether or not to stage a coup. If the concurrent elite does not stage a coup, the incubent elite e^1 remains in power until the next period. I denote the payoffs in this case by the subscript NC.

$$
\begin{aligned}
V_{NC}^{e^1}(\mu^l, E^1) &= \frac{1}{1 - \beta(1 - q)} \left[(1 + \tau^r(\mu^l, \theta) \frac{1 - \theta}{1 - \lambda} + \beta q V^{e^1}(\mu^h, E^1) \right] \\
V_{NC}^{e^2}(\mu^l, E^1) &= \frac{1}{1 - \beta(1 - q)} \left[(1 - \tau^r(\mu^l, \theta) \frac{1 - \theta}{1 - \lambda} + \beta q V^{e^2}(\mu^h, E^1) \right] \\
V_{NC}^{p}(\mu^l, E^1) &= \frac{1}{1 - \beta(1 - q)} \left[\frac{\theta}{\lambda} + \beta q V^p(\mu^h, E^1) \right]
\end{aligned}
$$

If the concurrent elite e^2 instead stages a coup, then they destroy all but a fraction ρ^h of assets in the economy and redistribute these among themselves. In the next period, the political regime becomes E^2 - an autocracy ruled by the elite who staged a coup in the previous period. If we have another realization of $\mu = \mu^l$, then the new concurrent elite (e^2) will find it optimal to stage a coup. I denote the payoffs in this case by subscript C.

$$
\begin{aligned}
V_C^{e^1}(\mu^l, E^1) &= \frac{\beta}{1 - \beta^2(1 - q)^2} \left[q V^{e^1}(\mu^h, E^2) + (1 - q) \frac{2\rho^h}{1 - \lambda} + \beta q (1 - q) V^{e^1}(\mu^h, E^1) \right] \\
V_C^{e^2}(\mu^l, E^1) &= \frac{1}{1 - \beta^2(1 - q)^2} \left[\frac{2\rho^h}{1 - \lambda} + \beta q (1 - q) V^{e^2}(\mu^h, E^2) + \beta^2 q (1 - q) V^{e^2}(\mu^h, E^1) \right] \\
V_C^{p}(\mu^l, E^1) &= \frac{\beta q}{1 - \beta(1 - q)} V^p(\mu^h, E)
\end{aligned}
$$

Clearly the payoffs depends on what happens at $\mu = \mu^h$ and $s = \{E^1, E^2\}$. That is, what happens in an autocracy when the threat of revolution is credible.

8.2 Step 2: Proof of Lemma 4

Consider that $s = E^1$ and $\mu = \mu^h$ so that the political regime is an autocracy under the threat of revolution. Consider that the incumbent elite e^1 proposes a tax schedule $\tau^e(\mu^h, \theta)$. As detailed in Step 1, payoffs in this case depend on what happens at $\mu = \mu^l$ - that is in an autocracy where there is no threat of revolution. For now, let us focus on what happens if the concurrent elite e^2 does stage a coup against autocracy at $\mu = \mu^l$.

The incumbent elite e^1 proposes a tax schedule $\tau^e(\mu^h, \theta) = \{\tau^a(\mu^h, \theta), \tau^c(\mu^h, \theta), \tau^r(\mu^h, \theta)\}$. Recall that $\tau^a(\mu^h, \theta)$ redistributes from the incumbent elite to the poor, $\tau^c(\mu^h, \theta)$ redistributes from the concurrent elite to the poor, and $\tau^r(\mu^h, \theta)$ redistributes from the concurrent elite to the incumbent elite. After observing the tax schedule, the concurrent elite e^2 decide whether or not to stage a coup. If they do not, the poor decide whether or not to stage a revolution. Recall that a coup occurs whenever $\mu = \mu^l$ so that the two elites effectively swap places at such time periods.

If there is neither a coup nor a revolution, I denote payoffs by subscript E. If there is a coup, I denote the subscript by C. If there is a revolution, I denote the subscript by R.

$$
\begin{aligned}
V_E^{e^2}(\mu^h, E^1) &= \frac{\beta(1-q)(1-\beta q)}{(1-\beta)(1+\beta-2\beta q)}\frac{2\rho^h}{1-\lambda} + \frac{1-\theta}{1-\lambda}\frac{(1-\tau^c-\tau^e)(1-\beta q-\beta^2(1-q)^2)+\beta^2 q(1-q)(1+\tau^e-\tau^a)}{(1-\beta)(1+\beta-2\beta q)} \\
V_E^p(\mu^h, E^1) &= \frac{1-\beta(1-q)}{1-\beta}\left[\frac{\theta}{\lambda} + \frac{1-\theta}{1-\lambda}\frac{\tau^a(\mu^h,\theta)+\tau^c(\mu^h,\theta)}{2}\right] \\
V_C^{e^2}(\mu^h, E^1) &= \frac{\beta^2(1-q)}{1-\beta^2}\frac{2\rho^h}{1-\lambda} \\
V_C^p(\mu^h, E^1) &= 0 \\
V_R^{e^2}(\mu^h, E^1) &= 0 \\
V_R^p(\mu^h, E^1) &= \frac{\mu^h}{\lambda(1-\beta)}
\end{aligned}
$$

Suppose that the incumbent elite e^1 proposes the tax rate and the concurrent elite e^2 does not stage a coup. The poor must now decide whether to stage a revolution. The poor will stage a revolution as long as their payoff from doing so is greater than their payoff from accepting the tax schedule. So, a revolution is avoided as long as $V_E^p(\mu^h, E^1) \geq V_E^p(\mu^h, R)$. Rearranging the terms:

$$\frac{\tau^a+\tau^c}{2} \geq \frac{1}{1-\theta}\left[\frac{\mu^h}{1-\beta(1-q)} - \theta\right]$$

This expression has a straightforward interpretation. To prevent a revolution, redistribution towards the poor must be above a certain threshold - given by the RHS of the expression. If redistribution is too low, the poor will rebel.

Now suppose the incumbent elite e^1 proposes a tax rate. The concurrent elite e^2 must now decide whether to stage a coup. The elite e^2 will stage a coup as long as their payoff of doing so is greater than their payoff from accepting the schedule. So a coup is avoided as long as $V_E^{e^2}(\mu^h, E^1) \geq V_C^{e^2}(\mu^h, E^1)$. Rearranging the terms:

$$1+\beta(1-q)\frac{2\rho^h}{1-\theta} + \frac{\beta^2 q(1-q)}{1-\beta q-\beta^2(1-q)^2}(1+\tau^e-\tau^a) \geq \tau^c+\tau^e$$

This expression has a straightforward interpretation. To prevent a coup, the taxation imposed on the concurrent elite must be below a certain threshold - given by the LHS of the expression. Too high a tax rate will incentivise the elite to rebel. Note the LHS of the expression is greater than 1 and, by construction, we know that the maximum tax rate that can be imposed on the concurrent elite is $1-b$. This implies that the tax burden on the concurrent elite $\tau^e(\mu^h,\theta)+\tau^c(\mu^h,\theta) = 1-b$. In other words, the concurrent elite always withstands the maximum possible tax rate and does not stage a coup.

In order for autocracy to survive to next period, two conditions must be met:

$$\tau^e(\mu^h, \theta) + \tau^c(\mu^h, \theta) = 1 - b$$
$$\frac{\tau^a(\mu^h, \theta) + \tau^c(\mu^h, \theta)}{2} \geq \frac{1}{1-\theta}\left[\frac{\mu^h}{1-\beta(1-q)} - \theta\right]$$

Three cases emerge. First, there is no possible redistribution that can prevent the poor from staging a revolution. This occurs when the minimum redistribution required by the poor is above the tax rate that can be imposed on the two elites. That is, when $\frac{\tau^a(\mu^h,\theta)+\tau^c(\mu^h,\theta)}{2} > 1-b$. Rearranging the terms:

$$1 - \frac{1}{b}\left[1 - \frac{\mu^h}{1-\beta(1-q)}\right] > \theta \tag{2}$$

In this case, a revolution is unavoidable. To prevent a revolution - the worst , the concurrent elite stages a coup.

Second, a revolution can be avoided but only if both elites contribute. The incubent elite e^1 places the maximum possible tax rate on the concurrent elite e^2 without instigating a coup so that $\tau^c(\mu^h, \theta) = 1 - b$. By doing this, the incubent elite e^1 foregoes transferring from the concurrent elite to themselves so that $\tau^e(\mu^h, \theta) = 0$. The incubent elite taxes their own income. So that:

$$\tau^c(\mu^h, \theta) = 1 - b$$
$$\tau^a(\mu^h, \theta) = \frac{1}{1-\theta}\left[\frac{\mu^h}{1-\beta(1-q)} - \theta\right] - 1 + b$$
$$\tau^e(\mu^h, \theta) = 0$$

This situation occurs when taxation required by the poor lies somewhere on $\left[\frac{1-b}{2}, 1-b\right]$. Rearranging the terms:

$$\frac{1}{1+b}\left[\frac{2\mu^h}{1-\beta(1-q)} - (1-b)\right] > \theta > 1 - \frac{1}{b}\left[1 - \frac{\mu^h}{1-\beta(1-q)}\right] \tag{3}$$

Third, the incumbent elite e^1 sets no tax burden on themselves so that $\tau^a(\mu^h, \theta) = 0$ and instead requires the concurrent elite to bear the full tax burden. The concurrent elite redistributes as much of their income as necessary to prevent a revolution. Any remaining taxation that the concurrent elite can withstand is redistributed towards the incubent. So that:

$$\tau^c(\mu^h, \theta) = max\{0, \frac{1}{1-\theta}\left[\frac{\mu^h}{1-\beta(1-q)} - \theta\right]$$
$$\tau^a(\mu^h, \theta) = 0$$
$$\tau^e(\mu^h, \theta) = 1 - b - \tau^c(\mu^h, \theta)$$

This occurs whenever the redistribution required by the poor is below $\frac{1-b}{2}$. Rearranging the terms:

$$\theta > \frac{1}{1+b}\left[\frac{2\mu^h}{1-\beta(1-q)} - (1-b)\right] \tag{4}$$

By construction, we require that θ lie in $[0, \lambda]$. Hence, for the thresholds to be relevant, we require they also lie in that interval. To ensure this, the lowest threshold, from Expression (1),

$1-\frac{1}{b}\left[1-\frac{\mu^h}{1-\beta(1-q)}\right]>0$, and the highest threshold, from Expression (3), $\frac{1}{1+b}\left[\frac{2\mu^h}{1-\beta(1-q)}-(1-b)\right]<\lambda$. Rearramging the terms:

$$\frac{1+\lambda}{1-\lambda}-\frac{2}{1-\lambda}\frac{\mu^h}{1-\beta(1-q)}>b>1-\frac{\mu^h}{1-\beta(1-q)}$$

By construction, $b>0$. Recall also, Assumption B, which states $b<1-\mu^h$. So for the assumption above to always hold I require only that the upper threshold is larger than $1-\mu^h$, and the lower threshold is smaller than 0. This yields Assumption C.

ASSUMPTION C: $\frac{2\lambda}{1+\beta(1-q)+\lambda[1-\beta(1-q)]}>\mu^h>1-\beta(1-q)$

Taxation under an autocracy is given as:

$$\tau^a(\mu^h,\theta)=\begin{cases}\text{Coup occurs} & \text{if } \theta<1-\frac{1}{b}\left[1-\frac{\mu^h}{1-\beta(1-q)}\right]\\ \frac{1}{1-\theta}\left[\frac{\mu^h}{1-\beta(1-q)}-\theta\right]-1+b & \text{if } \theta\in\left(1-\frac{1}{b}\left[1-\frac{\mu^h}{1-\beta(1-q)}\right],\frac{1}{1+b}\left[\frac{2\mu^h}{1-\beta(1-q)}-(1-b)\right]\right)\\ 0 & \text{if } \theta>\frac{1}{1+b}\left[\frac{2\mu^h}{1-\beta(1-q)}-(1-b)\right]\end{cases}$$

$$\tau^c(\mu^h,\theta)=\begin{cases}\text{Coup occurs} & \text{if } \theta<1-\frac{1}{b}\left[1-\frac{\mu^h}{1-\beta(1-q)}\right]\\ 1-b & \text{if } \theta\in\left(1-\frac{1}{b}\left[1-\frac{\mu^h}{1-\beta(1-q)}\right],\frac{1}{1+b}\left[\frac{2\mu^h}{1-\beta(1-q)}-(1-b)\right]\right)\\ max\{\frac{1}{1-\theta}\left[\frac{\mu^h}{1-\beta(1-q)}-\theta\right],0\} & \text{if } \theta>\frac{1}{1+b}\left[\frac{2\mu^h}{1-\beta(1-q)}-(1-b)\right]\end{cases}$$

$$\tau^e(\mu^h,\theta)=\begin{cases}\text{Coup occurs} & \text{if } \theta<1-\frac{1}{b}\left[1-\frac{\mu^h}{1-\beta(1-q)}\right]\\ 0 & \text{if } \theta\in\left(1-\frac{1}{b}\left[1-\frac{\mu^h}{1-\beta(1-q)}\right],\frac{1}{1+b}\left[\frac{2\mu^h}{1-\beta(1-q)}-(1-b)\right]\right)\\ 1-b-\tau^c(\mu^h,\theta) & \text{if } \theta>\frac{1}{1+b}\left[\frac{2\mu^h}{1-\beta(1-q)}-(1-b)\right]\end{cases}$$

8.3 Step 3: Proof of Lemma 5

Consider that $s=E^1$ and $\mu=\mu^h$ so that the political regime is an autocracy under the threat of revolution. Consider that the incumbent elite e^1 proposes a tax schedule $\tau^e(\mu^h,\theta)$. As detailed in Step 1, payoffs in this case depend on what happens at $\mu=\mu^l$ - that is in an autocracy where there is no threat of revolution. Let us focus on what happens if the concurrent elite e^2 does not stage a coup against autocracy at $\mu=\mu^l$.

The incumbent elite e^1 proposes a tax schedule $\tau^e(\mu^h,\theta)=\{\tau^a(\mu^h,\theta),\tau^c(\mu^h,\theta),\tau^r(\mu^h,\theta)\}$. Recall that $\tau^a(\mu^h,\theta)$ redistributes from the incubent elite to the poor, $\tau^c(\mu^h,\theta)$ redistributes from the concurrent elite to the poor, and $\tau^r(\mu^h,\theta)$ redistributes from the concurrent elite to the incubent elite. After observing the tax schedule, the concurrent elite e^2 decide whether or not to stage a coup. If they do not, the poor decide whether or not to stage a revolution. Recall that a coup occurs whenever $\mu=\mu^l$ so that the two elites effectively swap places at such time periods.

If there is neither a coup nor a revolution, I denote payoffs by subscript E. If there is a coup, I denote the subscript by C. If there is a revolution, I denote the subscript by R.

$$
\begin{aligned}
V_E^{e^2}(\mu^h, E^1) &= \frac{1}{1-\beta}\frac{1-\theta}{1-\lambda}\left[[1-\beta(1-q)][1-\tau^r(\mu^h,\theta)-\tau^c(\mu^h,\theta)]+\beta(1-q)(1-\tau^r(\mu^l,\theta))\right] \\
V_E^p(\mu^h, E^1) &= \frac{1}{1-\beta}\left[\frac{\theta}{\lambda}+[1-\beta(1-q)]\frac{1-\theta}{\lambda}\frac{\tau^a(\mu^h,\theta)+\tau^c(\mu^h,\theta)}{2}\right] \\
V_C^{e^2}(\mu^h, E^1) &= \frac{\beta(1-q)(1-\theta)}{(1-\beta)(1-\beta+2\beta q)(1-\lambda)}\left[\beta q[1-\tau^r(\mu^l,\theta)]+[1-\beta(1-q)][1+\tau^r(\mu^l,\theta)]\right] \\
V_C^p(\mu^h, E^1) &= \frac{\beta(1-q)}{1-\beta}\frac{\theta}{\lambda} \\
V_R^{e^2}(\mu^h, E^1) &= 0 \\
V_R^p(\mu^h, E^1) &= \frac{\mu^h}{\lambda(1-\beta)}
\end{aligned}
$$

Suppose that the incumbent elite e^1 proposes the tax rate and the concurrent elite e^2 does not stage a coup. The poor must now decide whether to stage a revolution. The poor will stage a revolution as long as their payoff from doing so is greater than their payoff from accepting the tax schedule. So, a revolution is avoided as long as $V_E^p(\mu^h, E^1) \geq V_E^p(\mu^h, R)$. Rearranging the terms:

$$
\frac{\tau^a(\mu^h,\theta)+\tau^c(\mu^h,\theta)}{2} \geq \frac{1}{1-\beta(1-q)}\frac{\mu^h-\theta}{1-\theta}
$$

This expression has a straightforward interpretation. To prevent a revolution, redistribution towards the poor must be above a certain threshold - given by the RHS of the expression. If redistribution is too low, the poor will rebel.

Now suppose the incumbent elite e^1 proposes a tax rate. The concurrent elite e^2 must now decide whether to stage a coup. The elite e^2 will stage a coup as long as their payoff of doing so is greater than their payoff from accepting the schedule. So a coup is avoided as long as $V_E^{e^2}(\mu^h, E^1) \geq V_C^{e^2}(\mu^h, E^1)$. Rearranging the terms:

$$
1 - \frac{2\beta(1-q)\tau^r(\mu^l,\theta)}{1-\beta+2\beta q} \geq \tau^r(\mu^l,\theta)+\tau^a(\mu^h,\theta)
$$

This expression has a straightforward interpretation. To prevent a coup, the taxation imposed on the concurrent elite must be below a certain threshold - given by the LHS of the expression. Too high a tax rate will incentivise the elite to rebel. Note the LHS of the expression decreases with $\tau^r(\mu^l,\theta)$. Recall $\tau^r(\mu,\theta) \in [b-1, 1-b]$ so that redistribution between the two elites is possible either way. If $\tau^r(\mu^l,\theta) < \frac{b}{2}\frac{1-\beta(1-q)}{\beta(1-q)}$, then the LHS of the expression is larger than$1-b$ so that the maximum taxation imposed on the concurrent elite is simply $1-b$.

In order for autocracy to survive to next period, two conditions must be met:

$$
\begin{aligned}
\tau^e(\mu^h,\theta)+\tau^c(\mu^h,\theta) &\leq 1-\frac{2\beta(1-q)\tau^r(\mu^l,\theta)}{1-\beta+2\beta q} \\
\frac{\tau^a(\mu^h,\theta)+\tau^c(\mu^h,\theta)}{2} &\geq \frac{1}{1-\beta(1-q)}\frac{\mu^h-\theta}{1-\theta}
\end{aligned}
$$

Four cases emerge. First, there is no possible redistribution that can prevent the poor from staging a revolution. This occurs when the minimum redistribution required by the poor is above the tax rate that can be imposed on the two elites, $\frac{1-b}{2}+\frac{1}{2}-\frac{\beta(1-q)\tau^r(\mu^l,\theta)}{1-\beta+2\beta q}$. Rearranging the terms:

$$\frac{\mu^h-[1-\beta(1-q)][1-\frac{b}{2}-\tau^r(\mu^l,\theta)\frac{\beta(1-q)}{1-\beta+2\beta q}]}{1-[1-\beta(1-q)][1-\frac{b}{2}-\tau^r(\mu^l,\theta)\frac{\beta(1-q)}{1-\beta+2\beta q}]}>\theta \tag{5}$$

In this case, a revolution is unavoidable. To prevent a revolution, the concurrent elite stages a coup.

Second, a revolution can be avoided but only if both elites contribute. The incumbent elite e^1 places the maximum possible tax rate on the concurrent elite e^2 without instigating a coup so that $\tau^c(\mu^h,\theta)=1-\tau^r(\mu^l,\theta)\frac{2\beta(1-q)}{1-\beta+2\beta q}$. By doing this, the incumbent elite e^1 foregoes transferring from the concurrent elite to themselves so that $\tau^e(\mu^h,\theta)=0$. The incumbent elite taxes their own income. So that:

$$\begin{aligned}
\tau^c(\mu^h,\theta)&=1-\tau^r(\mu^l,\theta)\frac{2\beta(1-q)}{1-\beta+2\beta q}\\
\tau^a(\mu^h,\theta)&=\frac{2}{1-\beta(1-q)}\frac{\mu^h-\theta}{1-\theta}-1+\tau^r(\mu^l,\theta)\frac{2\beta(1-q)}{1-\beta+2\beta q}\\
\tau^e(\mu^h,\theta)&=0
\end{aligned}$$

This situation occurs when taxation required by the poor lies somewhere on $\left[\frac{1}{2}-\tau^r(\mu^l,\theta)\frac{\beta(1-q)}{1-\beta+2\beta q},\frac{1-b}{2}+\frac{1}{2}-\tau^r(\mu^l,\theta)\frac{\beta(1-q)}{1-\beta+2\beta q}\right]$. Rearranging the terms:

$$\frac{\mu^h-[1-\beta(1-q)][\frac{1}{2}-\tau^r(\mu^l,\theta)\frac{\beta(1-q)}{1-\beta+2\beta q}]}{1-[1-\beta(1-q)][\frac{1}{2}-\tau^r(\mu^l,\theta)\frac{\beta(1-q)}{1-\beta+2\beta q}]}>\theta>\frac{\mu^h-[1-\beta(1-q)][1-\frac{b}{2}-\tau^r(\mu^l,\theta)\frac{\beta(1-q)}{1-\beta+2\beta q}]}{1-[1-\beta(1-q)][1-\frac{b}{2}-\tau^r(\mu^l,\theta)\frac{\beta(1-q)}{1-\beta+2\beta q}]} \tag{6}$$

Third, the incumbent elite e^1 sets no tax burden on themselves so that $\tau^a(\mu^h,\theta)=0$ and instead requires the concurrent elite to bear the full tax burden. The concurrent elite redistributes as much of their income as necessary to prevent a revolution. Any remaining taxation that the concurrent elite can withstand is redistributed towards the incubent. So that:

$$\begin{aligned}
\tau^c(\mu^h,\theta)&=max\{0,\frac{2}{1-\beta(1-q)}\frac{\mu^{h-\theta}}{1-\theta}\}\\
\tau^a(\mu^h,\theta)&=0\\
\tau^e(\mu^h,\theta)&=1-\tau^r(\mu^l,\theta)\frac{2\beta(1-q)}{1-\beta+2\beta q}-\tau^c(\mu^h,\theta)
\end{aligned}$$

This occurs whenever the redistribution required by the poor is below $\frac{1}{2}-\tau^r(\mu^l,\theta)\frac{\beta(1-q)}{1-\beta+2\beta q}$. Rearranging the terms:

$$\theta>\frac{\mu^h-[1-\beta(1-q)][\frac{1}{2}-\tau^r(\mu^l,\theta)\frac{\beta(1-q)}{1-\beta+2\beta q}]}{1-[1-\beta(1-q)][\frac{1}{2}-\tau^r(\mu^l,\theta)\frac{\beta(1-q)}{1-\beta+2\beta q}]} \tag{7}$$

Fourth, the elite e^2 stages a coup. This occurs whenever the maximum taxation born by the concurrent elite is below the minimum possible tax rate, $b-1$. Rearranging the terms:

$$\tau^r(\mu^l,\theta) > \frac{(2-b)(1-\beta+2\beta q)}{2\beta(1-q)} \tag{8}$$

By construction, we require that θ lie in $[0,\lambda]$. Hence, for the thresholds to be relevant, we require they also lie in that interval. To ensure this, the lowest threshold, from Expression (5) must be larger than 0, and the highest threshold, from Expression (7), must be lower than λ. Rearranging the terms, I obtain Assumption D.

ASSUMPTION D: $\lambda > \frac{\mu^h-[1-\beta(1-q)][\frac{1}{2}-\tau^r(\mu^l,\theta)\frac{\beta(1-q)}{1-\beta+2\beta q}]}{1-[1-\beta(1-q)][\frac{1}{2}-\tau^r(\mu^l,\theta)\frac{\beta(1-q)}{1-\beta+2\beta q}]}, \frac{\mu^h-[1-\beta(1-q)][1-\frac{b}{2}-\tau^r(\mu^l,\theta)\frac{\beta(1-q)}{1-\beta+2\beta q}]}{1-[1-\beta(1-q)][1-\frac{b}{2}-\tau^r(\mu^l,\theta)\frac{\beta(1-q)}{1-\beta+2\beta q}]} > 0$

The resulting lemma is:

LEMMA: Suppose Assumption 3 holds. The current state of the world is $s=E^1$ so that there is an autocracy of elite e^1. Consider the elite fights off the threat of revolution at $\mu=\mu^h$ by proposing tax schedule $\tau^e(\mu^h,\theta)=\{\tau^a(\mu^h,\theta),\tau^c(\mu^h,\theta),\tau^r(\mu^h,\theta)\}$. Consider also that agents expect no coup against autocracy when $\mu=\mu^l$, such that there is a tax on the concurrent elite's income $\tau^r(\mu^l,\theta)$. Then, the unique Perfect Markov Equilibrium is:

- If $\tau^r(\mu^l,\theta) > \frac{(2-b)(1-\beta+2\beta q)}{2\beta(1-q)}$, the concurrent elite e^2 stages a coup;
- If $\tau^r(\mu^l,\theta) \leq \frac{(2-b)(1-\beta+2\beta q)}{2\beta(1-q)}$ and $\theta < \frac{\mu^h-[1-\beta(1-q)][1-\frac{b}{2}-\tau^r(\mu^l,\theta)\frac{\beta(1-q)}{1-\beta+2\beta q}]}{1-[1-\beta(1-q)][1-\frac{b}{2}-\tau^r(\mu^l,\theta)\frac{\beta(1-q)}{1-\beta+2\beta q}]}$, the concurrent elite e^2 stages a coup;
- If $\tau^r(\mu^l,\theta) \leq \frac{(2-b)(1-\beta+2\beta q)}{2\beta(1-q)}$ and $\theta \geq \frac{\mu^h-[1-\beta(1-q)][1-\frac{b}{2}-\tau^r(\mu^l,\theta)\frac{\beta(1-q)}{1-\beta+2\beta q}]}{1-[1-\beta(1-q)][1-\frac{b}{2}-\tau^r(\mu^l,\theta)\frac{\beta(1-q)}{1-\beta+2\beta q}]}$, the poor and the concurrent elite e^2 accept the tax schedule and autocracy survives. The tax schedule is given by:

$$\tau^a(\mu^h,\theta) = \begin{cases} \frac{2}{1-\beta(1-q)}\frac{\mu^h-\theta}{1-\theta} - 1 + \tau^r(\mu^l,\theta)\frac{2\beta(1-q)}{1-\beta+2\beta q} & \text{if } \theta < \frac{\mu^h-[1-\beta(1-q)][\frac{1}{2}-\tau^r(\mu^l,\theta)\frac{\beta(1-q)}{1-\beta+2\beta q}]}{1-[1-\beta(1-q)][\frac{1}{2}-\tau^r(\mu^l,\theta)\frac{\beta(1-q)}{1-\beta+2\beta q}]} \\ 0 & \text{otherwise} \end{cases}$$

$$\tau^c(\mu^h,\theta) = \begin{cases} 1-\tau^r(\mu^l,\theta)\frac{2\beta(1-q)}{1-\beta+2\beta q} & \text{if } \theta < \frac{\mu^h-[1-\beta(1-q)][\frac{1}{2}-\tau^r(\mu^l,\theta)\frac{\beta(1-q)}{1-\beta+2\beta q}]}{1-[1-\beta(1-q)][\frac{1}{2}-\tau^r(\mu^l,\theta)\frac{\beta(1-q)}{1-\beta+2\beta q}]} \\ max\{\frac{2}{1-\beta(1-q)}\frac{\mu^h-\theta}{1-\theta},0\} & \text{otherwise} \end{cases}$$

$$\tau^r(\mu^h,\theta) = \begin{cases} 0 & \text{if } \theta < \frac{\mu^h-[1-\beta(1-q)][\frac{1}{2}-\tau^r(\mu^l,\theta)\frac{\beta(1-q)}{1-\beta+2\beta q}]}{1-[1-\beta(1-q)][\frac{1}{2}-\tau^r(\mu^l,\theta)\frac{\beta(1-q)}{1-\beta+2\beta q}]} \\ 1-\tau^r(\mu^l,\theta)\frac{2\beta(1-q)}{1-\beta+2\beta q}-\tau^c(\mu^h,\theta) & \text{otherwise} \end{cases}$$

9 Appendix: Solving the Model

In this section, I use a backward induction type argument to obtain the model solution, using three steps. For simplicity, I exclude the computation of certain payoffs from this section.

9.1 $\mu=\mu^l$ and $s=E^1$

The game begins at state $s=E^1$ with the first elite in power. Consider that, in this first period, $\mu=\mu^l=0$. There is no threat of revolution by the poor, so the incumbent elite are weakly better off not democratizing, and instead proposing tax schedule $\tau^e(\mu^l)=\{\tau^c(\mu^l),\tau^r(\mu^l),\tau^a(\mu^l)\}$. There is no need to redistribute wealth towards the poor, so the incumbent elite set $\tau^a(\mu^l)=\tau^c(\mu^l)=0$. It is on the ruling elite's best interest to set $\tau^r(\mu^l)$ as high as possible without instigating a coup from the concurrent elite. That is, they must ensure that:

$$V^{e^2}(\mu^l, E^1) \geq V^{e^2}(\mu^l, C^2)$$

Where:

$$V^{e^2}(\mu^l, E^1) = (1 - \tau^r(\mu^l))h^e + \beta[qV^{e^2}(\mu^h, E^1) + (1-q)V^{e^2}(\mu^l, E^1)]$$
$$V^{e^2}(\mu^l, C^2) = \frac{2\rho^h}{1-\lambda}h + \beta[qV^{e^2}(\mu^h, E^2) + (1-q)V^{e^2}(\mu^l, E^2)]$$

Focus on term $V^{e^2}(\mu^l, E^2)$ from the above equation. Because the two elites in the model are identical, if the concurrent elite choose to stage a coup against the one in power, the others do the same when their roles are reversed. That is, $V^{e^2}(\mu^l, E^2) = V^{e^2}(\mu^l, C^1)$. I rewrite the above expressions as:

$$V^{e^2}(\mu^l, E^1) = \frac{1}{1-\beta(1-q)}[(1 - \tau^r(\mu^l))h^e + \beta qV^{e^2}(\mu^h, E^1)]$$

$$V^{e^2}(\mu^l, C^2) = \frac{1}{1-\beta^2(1-q)^2)}\left[\frac{2\rho^h h}{1-\lambda} + \beta qV^{e^2}(\mu^h, E^2) + \beta^2 q(1-q)V^{e^2}(\mu^h, E^1)\right]$$

9.2 $\mu = \mu^h$ and $s = E^1$

When $\mu = \mu^h$, the poor pose a credible threat of revolution. The incumbent elite choose between two courses of action: they either extend the franchise, or they propose tax schedule $\tau^e(\mu^h)$. I consider the two cases below.

9.2.1 Franchise extension

Suppose that there is an autocracy, $s = \{E^1, E^2\}$ and that the incumbent elite extends the franchise at $\mu = \mu^h$ and that neither the poor nor the concurrent elite stage a revolution or coup. The poor then set the tax rate to the maximum possible level, $1 - b$, and the state switches to $s = D$. In the next period, a democracy, if $\mu = \mu^h$, the elites pose no threat of a coup, according to Assumption 2. Hence, the poor set $\tau^p(\mu^h) = 1 - b$ as well. This implies $V^i(\mu^h, E^1) = V^i(\mu^h, E^2) = V^i(\mu^h, D)$.

Under democracy, if $\mu = \mu^l$, then the first elite, e^1, pose a credible threat of a coup. If possible, the poor prevent a coup by proposing a lower tax rate $\tau^p(\mu^l) < 1-b$. That is, the poor ease the tax burden on the elites in an attempt to stabilize democracy. However, suppose that there is no feasible $\tau^p(\mu^l)$ that appease the elites, so that the first elite stage a coup. The political state switches to $s = E^1$, as it was in the first period. As detailed in Section 4.1, under autocracy, if $\mu = \mu^l$, the first elite propose a tax schedule to the second elite, $\tau^r(\mu^l)$, and the second elite decide whether to stage a coup. The second elite's payoff from staging a coup against autocracy is:

$$V^{e^2}(\mu^l, C^2) = \frac{2\rho^h h}{1-\theta} + \beta[qV^{e^2}(\mu^h, E^2) + (1-q)V(\mu^l, E^2)]$$

Note that, in this case, $V^{e^2}(\mu^h, E^2) = V^{e^2}(\mu^h, D)$, and $V(\mu^l, E^2) = V^{e^2}(\mu^l, C^1)$. Rearranging the above expression yields

$$V^{e^2}(\mu^l, C^2) = \frac{1 - \beta q - \beta^2(1-q)^2(1-\beta)}{[1 - \beta q - \beta^2(1-q)^2]}\left[\frac{2\rho^h}{1-\lambda}h + bh^e\frac{\beta q[1+\beta(1-q]}{1-\beta q}\right]$$

By a similar logic, the payoff to the second elite from not staging a coup and instead accepting proposed tax rate $\tau^r(\mu^l)$ is:

$$\begin{aligned} V^{e^2}(\mu^l, E^1) &= (1-\tau^r(\mu^l))h^e + \beta[qV^{e^2}(\mu^h, D) + (1-q)V^{e^2}(\mu^l, E^1)] \\ &= \frac{h^e}{1-\beta}\left[(1-\tau^r(\mu^l))\frac{1-\beta+\beta^3 q(1-q)^2}{1-\beta(1-q)} + \beta q b\right] \end{aligned}$$

The second elite foregoes the opportunity to stage a coup if:

$$\begin{aligned} & V^{e^2}(\mu^l, E^1) \geq V^{e^2}(\mu^l, C^2) \\ \iff & \tau^r(\mu^l) \leq 1 - \frac{1-\beta(1-q)}{1-\beta+\beta^3 q(1-q)^2}\left[\frac{2\rho\alpha}{1-\theta} + b\beta q\left(\frac{1+\beta(1-q)}{1-\beta q}\alpha - 1\right)\right] \end{aligned} \tag{1}$$

Where $\alpha = \frac{1-\beta}{1-\beta^2(1-q)^2}\frac{1-\beta q-\beta^2(1-q)^2(1-\beta)}{1-\beta q-\beta^2(1-q)}$. Expression (1) gives the maximum tax rate that can be imposed on the second elite without instigating a coup. Let us refer to the right hand-side of the expression as $\tau^{r*}(\theta)$. A coup can only be avoided if $\tau^{r*}(\theta) \geq 0$. Otherwise, the first elite would have to redistribute from themselves to the second elite to avoid a coup. Such is not possible according to the original set-up of this model, although the the possibility of an autocrat redistributing from themselves to a concurrent elite presents an interesting extension. The topic is discussed further in Section 6. $\tau^{r*}(\theta) \geq 0$ if:

$$\theta \leq 1 - 2\alpha\rho\left[\frac{1-\beta+\beta^3 q(1-q)^2}{1-\beta(1-q)} + b\beta q\left(1 - \frac{1+\beta(1-q)}{1-\beta q}\alpha\right)\right]^{-1} \tag{2}$$

Expression (2) states the incumbent elite can only prevent a coup against autocracy if the poor own a sufficiently low fraction of the assets in the economy. Above this threshold, the poor are relatively well off, while the elites own relatively little assets. Hence, the opportunity cost of a coup is low, causing the concurrent elite to stage one when they can. Here, I make two further assumptions. Assumption 3 assures that the tax rate imposed on the concurrent elite under autocracy is smaller than the one imposed under democracy, so that $\tau^r(\mu^l) = \tau^{r*}(\theta)$ always. Assumption 4 assures that the threshold from Equation (2) is relevant. Given these assumptions, Lemma 1 details the best response of the concurrent elite as a function of θ.

Assumption 3: $\tau^{r*}(\theta) < 1 - b$.

Assumption 4: $\lambda > 1 - 2\alpha\rho\left[\frac{1-\beta+\beta^3 q(1-q)^2}{1-\beta(1-q)} + b\beta q\left(1 - \frac{1+\beta(1-q)}{1-\beta q}\alpha\right)\right]^{-1} > 0$.

Lemma 1: Suppose Assumptions 1 through 4 hold, the franchise is extended and accepted when $\mu = \mu^h$ and $s = \{E^1, E^2\}$, and a coup is staged against $\mu = \mu^l$ and $s = D$. Then, under autocracy $s = \{E^1, E^2\}$ and when $\mu = \mu^l$, the concurrent elite faces a unique best response such that:

- if $\theta \leq 1 - 2\alpha\rho \left[\frac{1-\beta+\beta^3 q(1-q)^2}{1-\beta(1-q)} + b\beta q \left(1 - \frac{1+\beta(1-q)}{1-\beta q}\alpha\right)\right]^{-1}$, the concurrent elite's best response is to accept the proposed tax rate $\tau^r(\mu^l) = \tau^{r*}(\theta)$;
- if $\theta > 1 - 2\alpha\rho \left[\frac{1-\beta+\beta^3 q(1-q)^2}{1-\beta(1-q)} + b\beta q \left(1 - \frac{1+\beta(1-q)}{1-\beta q}\alpha\right)\right]^{-1}$, the concurrent elite's best response is to stage a coup.

Lemma 1 implies the payoff to the fist elite from staging a coup against democracy depends on the concurrent elite's response to autocracy at $\mu = \mu^l$. Let $V_C^{e^1}(\mu^l, C^1)$ be the payoff to staging a coup against democracy, given that a coup is staged against autocracy, and let $V_{NC}^{e^1}(\mu^l, C^1)$ be the payoff to staging a coup against democracy, given that a coup is not staged against autocracy. Some computation yields:

$$V_C^{e^1}(\mu^l, C^1) = \frac{1}{1 - \beta q - \beta^2(1-q)} \left[(1-\beta q)\frac{2\rho^h}{1-\lambda}h + \beta q(1 + \beta(1-q))bh^e\right]$$

$$V_{NC}^{e^1}(\mu^l, C^1) = \frac{[1-\beta q][1-\beta(1-q)]}{1-\beta}\frac{2\rho^h}{1-\lambda} + \frac{\beta h^e}{1-\beta}[qb + (1-q)(1-\beta q)(1 + \tau^{r*}(\theta)]$$

If the first elite choose to accept democracy instead of staging a coup, they obtain payoff:

$$V^{e^1}(\mu^l, D) = \frac{h^e}{1-\beta}[(1-\beta q)(1 - \tau^p(\mu^l)) + \beta q b]$$

First, focus on the case where the concurrent elite stage a coup against autocracy. The first elite refrains from staging a coup against democracy if:

$$\begin{aligned} &V_C^{e^1}(\mu^l, D) \geq V^{e^1}(\mu^l, C^1) \\ \iff\ &\tau^p(\mu^l) \leq 1 - \frac{1-\beta}{1-\beta q - \beta^2(1-q)}\frac{2\rho^h}{1-\theta} \end{aligned} \tag{3}$$

Let $\tau^{p*}(\theta)$ refer to the right hand-side of Expression (3). The expression gives the maximum tax rate the poor can impose on the elites without instigating a coup. Following a similar reasoning to Assumption 3, Assumption 5, further below, states that $\tau^{p*}(\theta) < 1 - b$ for all relevant values of θ, so that $\tau^p(\mu^l) = \tau^{p*}(\theta)$. To prevent a coup, I require $\tau^{p*}(\theta) \geq 0$:

$$\theta \leq 1 - \frac{1-\beta}{1-\beta q - \beta^2(1-q)} \tag{4}$$

The intuition here is similar to Expression (2). Due to the finite amount of assets in the economy, when the poor are relatively well off, the elites are relatively worse off. Hence, taxation under democracy seems even less attractive, making the opportunity cost of a coup low. Hence, at $\mu = \mu^l$, the poor in a democracy face a similar issue to that of the incumbent elite in an autocracy.

The case where the concurrent elite accept autocracy in favor of a coup shares a similar intuition. The elite favour democracy if:

$$\begin{aligned} & V_{NC}^{e^1}(\mu^l, D) \geq V^{e^1}(\mu^l, C^1) \\ \iff & \tau^p(\mu^l) \leq 1 - [1-\beta(1-q)]\frac{2\rho^h}{1-\theta} - \beta(1-q)(1+\tau^{r*}(\theta)) \end{aligned} \tag{5}$$

Let the right-hand side of Expression (5) be equivalent to $\tau^{p**}(\theta)$. Assumption 5 ensures that $\tau^p(\mu^l) = \tau^{p**}(\theta)$ for the relevant values of θ. Note that the tax imposed by the poor on the elites is a decreasing function of both assets owned by the poor, θ and the tax rate imposed under autocracy, $\tau^{r*}(\theta)$. This occurs because, under autocracy, the first elite has the chance to redistribute from the second elite towards themselves. The higher the benefit from returning to autocracy, the more attractive a coup becomes, forcing the poor to relax taxation further to prevent it. As before, the elite refrains from a coup if $\tau^{p**}(\theta) \geq 0$:

$$\theta \leq 1 - \frac{2\rho[1-\beta(1-q)][1+\frac{\beta(1-q)\alpha}{1-\beta+\beta^3 q(1-q)^2}]}{1-2\beta(1-q)+\beta^2 q(1-q)[1-\beta(1-q)]\left(\frac{1+\beta(1-q)}{1-\beta q}\alpha - 1\right)b} \tag{6}$$

Assumption 5: $\tau^{p*}(\theta), \tau^{p**}(\theta) < 1 - b$ for all θ.

Assumption 6: $\lambda > 1 - \frac{1-\beta}{1-\beta q - \beta^2(1-q)} > 1 - 2\alpha\rho\left[\frac{1-\beta+\beta^3 q(1-q)^2}{1-\beta(1-q)} + b\beta q\left(1 - \frac{1+\beta(1-q)}{1-\beta q}\alpha\right)\right]^{-1} >$
$1 - \frac{2\rho[1-\beta(1-q)][1+\frac{\beta(1-q)\alpha}{1-\beta+\beta^3 q(1-q)^2}]}{1-2\beta(1-q)+\beta^2 q(1-q)[1-\beta(1-q)]\left(\frac{1+\beta(1-q)}{1-\beta q}\alpha-1\right)b} > 0$

Assumptions 5 and 6 above are analogous to Assumptions 3 and 4. Assumption 5 guarantees that, in a democracy, taxation under the threat of a coup is always lesser than otherwise. Assumption 6 assures that four different scenarios can occur depending on the value of θ. These scenarios are discussed thoroughly in Lemma 2.

Lemma 2: Suppose Assumptions 1 through 6 hold, and that the franchise is extended and accepted when $\mu = \mu^h$. Then, the unique best responses of the elites at $s = \{E^1, E^2, D\}$ and $\mu = \mu^l$ are such that:

- if $\theta \leq \frac{2\rho[1-\beta(1-q)][1+\frac{\beta(1-q)\alpha}{1-\beta+\beta^3 q(1-q)^2}]}{1-2\beta(1-q)+\beta^2 q(1-q)[1-\beta(1-q)]\left(\frac{1+\beta(1-q)}{1-\beta q}\alpha-1\right)b}$, the best response at $s = E^1$ and $\mu = \mu^l$ is for the first elite to propose tax rate $\tau^r(\mu^l) = \tau^{r*}(\theta)$ and for the second elite to accept it. Once the franchise is extended at the first realization of $\mu = \mu^h$, democracy becomes permanent. The poor tax the elites at rates $\tau^p(\mu^h) = 1 - b$ and $\tau^p(\mu^l) = \tau^{p*}(\theta)$. The economy never reaches state E^2;

- if $\frac{2\rho[1-\beta(1-q)][1+\frac{\beta(1-q)\alpha}{1-\beta+\beta^3 q(1-q)^2}]}{1-2\beta(1-q)+\beta^2 q(1-q)[1-\beta(1-q)]\left(\frac{1+\beta(1-q)}{1-\beta q}\alpha-1\right)b} < \theta < 1 - \frac{2\alpha\rho}{\frac{1-\beta+\beta^3 q(1-q)^2}{1-\beta(1-q)}+b\beta q\left(1-\frac{1+\beta(1-q)}{1-\beta q}\alpha\right)}$, the best response at $s = E^1$ and $\mu = \mu^l$ is for the first elite to propose tax rate $\tau^r(\mu^l) = \tau^{r*}(\theta)$ and for the second elite to accept it. When $s = E^1$ and $\mu = \mu^h$, the franchise is extended and the state switches to $s = D$. When $s = D$ and $\mu = \mu^l$, the first elite e^1 stages a coup and the state switches to $s = E^1$. The economy never reaches E^2;

- if $1 - 2\alpha\rho\left[\frac{1-\beta+\beta^3 q(1-q)^2}{1-\beta(1-q)} + b\beta q\left(1 - \frac{1+\beta(1-q)}{1-\beta q}\alpha\right)\right]^{-1} \leq \theta \leq 1 - \frac{1-\beta}{1-\beta q-\beta^2 q(1-q)}$, the best response at $s = \{E^1, E^2\}$ and $\mu = \mu^l$ is for the concurrent elite to stage a coup. When $s = \{E^1, E^2\}$ and $\mu = \mu^h$, the franchise is extended and the regime switches to $s = D$. Once extended, democracy becomes permanent. The poor tax the elites at rates $\tau^p(\mu^h) = 1 - b$ and $\tau^p(\mu^l) = \tau^{p**}(\theta)$;

- if $\theta > 1 - \frac{1-\beta}{1-\beta q-\beta^2 q(1-q)}$, the best response at $s = \{E^1, E^2\}$ and $\mu = \mu^l$ is for the concurrent elite to stage a coup. When $s = \{E^1, E^2\}$ and $\mu = \mu^h$, the franchise is extended and the regime switches to $s = D$. When $s = D$ and $\mu = \mu^l$, the first elite stages a coup and the regime switches to E^2.

Lemma 2 implies the payoff to democracy depends on the fraction of assets owned by the poor, θ. In turn, the payoff of democracy to the poor, $V^p(\mu^h, D)$, determines whether the poor choose to accept the franchise or stage a revolution when the franchise is extended. Consider the first point of Lemma 2, where θ is such that democracy becomes permanent once the franchise is extended. The value of franchise extension to the poor in this case is:

$$V^p(\mu^h, D) = \frac{1}{1-\beta}\left[h^p + \frac{(1-\theta)}{\lambda}h\left((1-b)[1-\beta(1-q)] + \beta(1-q)\tau^{p*}(\theta)\right)\right]$$

The poor favour democracy over revolution if $V^p(\mu^h, D) \geq V^p(\mu^h, R)$:

$$\theta \geq \frac{\mu^h - (1-b)(1-\beta(1-q)) - \beta(1-q)[1 - 2\rho^h k_\rho - 2\beta(1-q) + k_b b]}{1 - (1-b)(1-\beta(1-q)) - \beta(1-q)[1 - 2\rho^h k_\rho - 2\beta(1-q) + k_b b]} \tag{7}$$

Where $k_\rho = [1-\beta(1-q)]\left[1 + \frac{\beta(1-q)\alpha}{1-\beta+\beta^3 q^2(1-q)^2}\right]$ and $k_b = \beta^2(1-q)[1-\beta(1-q)]\left[\frac{1+\beta(1-q)}{1-\beta q}\alpha - 1\right]$. The intuition behind Equation (7) is that, if the poor own relatively few assets, they face low opportunity cost of revolution. A similar procedure for the last three points of Lemma 2, yields thresholds θ^*, θ^{**} and θ^{***} which carry a similar intuition. Assumption 7 assures that these thresholds are small enough that the poor always favour democracy in these instances. This, along with Assumption 8, guarantees that the threshold from Equation (7) is the only relevant one. These simplifying assumptions guarantee that a revolution only occurs for very low values of θ, thus replicating how, historically, only very unequal societies experience revolutions (Skocpol, 1975).

Assumption 7: $\theta^*, \theta^{**}, \theta^{***} < \frac{2\rho[1-\beta(1-q)][1+\frac{\beta(1-q)\alpha}{1-\beta+\beta^3 q(1-q)^2}]}{1-2\beta(1-q)+\beta^2 q(1-q)[1-\beta(1-q)]\left(\frac{1+\beta(1-q)}{1-\beta q}\alpha-1\right)b}$.

Assumption 8:
$\frac{2\rho[1-\beta(1-q)][1+\frac{\beta(1-q)\alpha}{1-\beta+\beta^3 q(1-q)^2}]}{1-2\beta(1-q)+\beta^2 q(1-q)[1-\beta(1-q)]\left(\frac{1+\beta(1-q)}{1-\beta q}\alpha-1\right)b} > \frac{\mu^h-(1-b)(1-\beta(1-q))-\beta(1-q)[1-2\rho^h k_\rho-2\beta(1-q)+k_b b]}{1-(1-b)(1-\beta(1-q))-\beta(1-q)[1-2\rho^h k_\rho-2\beta(1-q)+k_b b]} > 0$.

Hence, Assumptions 7 and 8 guarantee that the poor stage a revolution if θ is low enough. The concurrent elite, anticipating this behaviour, stage a coup to prevent the revolution. Recall from Assumption 2 that the elite never stage a coup against franchise extension under $\mu = \mu^h$), if they expect democracy. Hence, they only stage a coup against franchise extension if they expect a revolution. The payoff to a concurrent elite from staging a coup in these circumstances is:

$$V^{e^2}(\mu^h, C^2) = \beta[qV^{e^2}(\mu^h, C^1) + (1-q)V^{e^2}(\mu^l, E^2)]$$

There are two-noteworthy points regarding this payoff. First, the motivation behind a coup against franchise extension is different than that behind coups against autocracy and democracy. Here, the concurrent elite does not obtain an immediate benefit from staging a coup, and they expect to be overthrown themselves when next $\mu = \mu^h$. The coup is simply a prevention mechanism: by overthrowing the incumbent elite before the poor have the chance to do so, the concurrent elite prevents the worst possible outcome to themselves. Second, this payoff depends on the value of $V^{e^2}(\mu^l, E^2)$. Note that the conclusions about best responses under autocracy at $\mu = \mu^l$ are conditional on the franchise being extended and accepted at $\mu = \mu^h$. Now that the franchise is prevented, I need to once again compute the best strategy for the elites in these circumstances. The payoffs from staging a coup and accepting autocracy become:

$$V^{e^2}(\mu^l, C^2) = \frac{2\rho^h}{1-\lambda}\frac{1-\beta^2 q}{1-\beta^2}$$

$$\begin{aligned} V^{e^2}(\mu^l, E^1) = & \frac{h^e}{1-\beta(1-q)}\left[(1-\tau^r(\mu^l))\left(1+\frac{\beta^3 q^2(1-q)}{1-2\beta(1-q)+\beta^2(1-2q)}\right)\right. \\ & \left. + \frac{\beta^2 q(1-q)[1-\beta(1-q)](1+\tau^r(\mu^l))}{1-2\beta(1-q)+\beta^2(1-2q)}\right] \end{aligned}$$

The concurrent elite favour the continued autocracy over a coup if:

$$\begin{aligned} & V^{e^2}(\mu^l, E^1) \geq V^{e^2}(\mu^l, C^2) \\ \iff & \tau^r(\mu^l) \leq z - \frac{\phi}{1-\theta} \end{aligned} \tag{8}$$

Where $z = \frac{1-2\beta(1-q)+\beta^2 q(1-q)+\beta^3(1-2q)(1-q)}{1-2\beta(1-q)+\beta^3(1-q-q^2}$ and $\phi = 2\rho^h\frac{[1-\beta(1-q)][1-\beta^2 q]}{1-\beta^2}\frac{1-2\beta(1-q)+\beta^3(1-2q)}{1-2\beta(1-q)+\beta^3(1-q-q^2)}$. Let $\tau^{r**}(\theta) = z - \frac{\phi}{1-\theta}$ and Assumption 9 hold, so that the incumbent autocrat sets $\tau^r(\mu^l) = \tau^{r**}(\theta)$. As before, I require the tax rate to be larger than zero to prevent a coup, which occurs only if:

$$\theta \leq 1 - \frac{\phi}{z} \tag{9}$$

Assumptions 9 and 10 fulfill the similar functions as Assumptions 3 and 5, and 4 and 6 respectively. Lemma 4 presents the results for regime dynamics for the case that the incumbent elite responds to the threat of revolution by extending the franchise when $\mu = \mu^h$.

Assumption 9: $1 - b > \tau^{r**}(\theta)$.

Assumption 10: $\frac{\mu^h - (1-b)(1-\beta(1-q)) - \beta(1-q)[1-2\rho^h k_\rho - 2\beta(1-q) + k_b b]}{1-(1-b)(1-\beta(1-q)) - \beta(1-q)[1-2\rho^h k_\rho - 2\beta(1-q) + k_b b]} > 1 - \frac{\phi}{z} > 0$.

Lemma 3: Suppose the economy begins at regime $s = E^1$, Assumptions 1 through 10 hold, and that the incumbent elite extends the franchise when $\mu = \mu^h$ and $s = \{E^1, E^2\}$. Then, the regime path is characterized as follows:

- if $\theta \leq 1 - \frac{\phi}{z}$, the concurrent elite responds to franchise extension by staging a coup whenever $\mu = \mu^h$. When $\mu = \mu^l$, the incubent elite taxes the concurrent elite's income at rate $\tau^r(\mu^l) = \tau^{r**}(\theta)$. The economy never reaches democracy, and is characterized by a sequence unstable autocracies;
- if $1 - \frac{\phi}{z} < \theta < \frac{\mu^h - (1-b)(1-\beta(1-q)) - \beta(1-q)[1-2\rho^h k_\rho - 2\beta(1-q) + k_b b]}{1-(1-b)(1-\beta(1-q)) - \beta(1-q)[1-2\rho^h k_\rho - 2\beta(1-q) + k_b b]}$, the concurrent elite always responds by overthrowing the incumbent elite, regardless of the value of μ. The economy is characterized by a sequence of unstable autocracies and high political instability;
- if $\frac{\mu^h - (1-b)(1-\beta(1-q)) - \beta(1-q)[1-2\rho^h k_\rho - 2\beta(1-q) + k_b b]}{1-(1-b)(1-\beta(1-q)) - \beta(1-q)[1-2\rho^h k_\rho - 2\beta(1-q) + k_b b]} \leq \theta$ and $\theta \leq \frac{2\rho[1-\beta(1-q)][1+\frac{\beta(1-q)\alpha}{1-\beta+\beta^3 q(1-q)^2}]}{1-2\beta(1-q)+\beta^2 q(1-q)[1-\beta(1-q)]\left(\frac{1+\beta(1-q)}{1-\beta q}\alpha - 1\right)b}$ while $\mu = \mu^l$, $S = E^1$. The first elite stays in power and taxes the second elite's income at rate $\tau^r(\mu^l) = \tau^{r*}(\theta)$. At the first realization of $\mu = \mu^h$, the franchise is extended and nver reversed again. The economy is characterized by a stable democracy, with the poor taxing the rich's income at rates $\tau^p(\mu^h) = 1 - b$ and $\tau^p(\mu^l) = \tau^{p*}(\theta)$;

- if $\frac{2\rho[1-\beta(1-q)][1+\frac{\beta(1-q)\alpha}{1-\beta+\beta^3 q(1-q)^2}]}{1-2\beta(1-q)+\beta^2 q(1-q)[1-\beta(1-q)]\left(\frac{1+\beta(1-q)}{1-\beta q}\alpha-1\right)b} < \theta < 1 - \frac{2\alpha\rho}{\frac{1-\beta+\beta^3 q(1-q)^2}{1-\beta(1-q)}+b\beta q\left(1-\frac{1+\beta(1-q)}{1-\beta q}\alpha\right)}$, the first elite stages a coup against democracy at $\mu = \mu^l$, but no coup is staged against autocracy. That is, democracy is unstable. The incubent elite taxes the concurrent elite at $\tau^r(\mu^l) = \tau^{r*}(\theta)$ under autocracy;
- if $1 - 2\alpha\rho\left[\frac{1-\beta+\beta^3 q(1-q)^2}{1-\beta(1-q)} + b\beta q\left(1 - \frac{1+\beta(1-q)}{1-\beta q}\alpha\right)\right]^{-1} \leq \theta < 1 - \frac{1-\beta}{1-\beta q-\beta^2 q(1-q)}$, the concurrent elite stages a coup against the incubent elite under autocracy at $\mu = \mu^l$. Once the franchise is extended, it is never reversed. The poor tax the elites at rates $\tau^p(\mu^h) = 1 - b$ and $\tau^p(\mu^h) = \tau^{p**}(\theta)$;
- if $\theta > 1 - \frac{1-\beta}{1-\beta q-\beta^2 q(1-q)}$, a coup is staged whenever $\mu = \mu^l$. That is, coups are staged against democracy and autocracy alike.

9.2.2 Proposition of tax schedule $\tau^e(\mu^h)$

Suppose that the incumbent elite fail to extend the franchise when $\mu = \mu^h$. Instead, the incumbent elite respond to the threat of revolution by proposing a tax schedule $\tau^e(\mu^h) = \{\tau^a(\mu^h), \tau^r(\mu^h), \tau^c(\mu^h)\}$. Two discerning cases emerge. First, imagine there is a coup against autocracy whenever $\mu = \mu^l$. Under $\mu = \mu^h$ the payoff to the poor and the rich of accepting autocracy is:

$$V^p(\mu^h, E^1) = \frac{1-\beta(1-q)}{1-\beta}\left[h^p + \frac{(\tau^a(\mu^h)+\tau^c(\mu^h))}{2\lambda}(1-\theta)h\right]$$

$$\begin{aligned}V^{e^2}(\mu^h, E^1) = &\frac{1-\beta q-\beta^2(1-q)^2}{(1-\beta)(1+\beta-2\beta q)}(1-\tau^r(\mu^h)-\tau^c(\mu^h))h^e + \\ &\frac{\beta^2 q(1-q)}{(1-\beta)(1+\beta-2\beta q)}(1-\tau^a(\mu^h)+\tau^r(\mu^h))h^e + \\ &\frac{\beta(1-q)(1-\beta q)}{(1-\beta)(1+\beta-2\beta q)}\frac{2\rho^h h}{1-\lambda}\end{aligned}$$

To prevent a revolution and a coup, I require that both Equations (10) and (11) hold.:

$$\begin{aligned}&V^p(\mu^h, E^1) \geq V^p(\mu^h, R) \\ \iff &\tau^a(\mu^h) + \tau^c(\mu^h) \geq \frac{2}{1-\theta}\left[\frac{\mu^h}{1-\beta(1-q)} - \theta\right]\end{aligned} \tag{10}$$

$$\begin{aligned}&V^{e^2}(\mu^h, E^1) \geq V^{e^2}(\mu^h, C^2) \\ \iff &\tau^r(\mu^h) + \tau^c(\mu^h) \leq 1 + \frac{\beta(1-q)}{1-\beta q-\beta^2(1-q)^2}\times \\ &\left[\frac{1-\beta q-\beta^2(1-q)}{1+\beta}\frac{2\rho^h}{1-\theta} + \beta q(1-\tau^a(\mu^h)+\tau^r(\mu^h))\right]\end{aligned} \tag{11}$$

Equation (10) gives the minimum value of tax rates redistributing from the elites to the poor to prevent a revolution, while Equation (11) gives the maximum tax burden that may be imposed on the concurrent elite without instigating a coup. Note that the right hand-side of Equation (11) is greater than 1 as long as:

Assumption 11: $1 > \beta q + \beta^2(1-q)^2$ and $1 > \beta q + \beta^2(1-q)$.

As the maximum feasible tax rate is $1-b$, such implies that the incumbent elite set $\tau^r(\mu^h) + \tau^c(\mu^h) = 1-b$. Similarly, the maximum taxation that the incumbent elite may impose upon themselves is also $1-b$. The maximum value of taxation that can be directed to the poor is thus $\max\{\tau^a(\mu^h)\} + \max\{\tau^c(\mu^h)\} = 2(1-b)$. In order for the tax schedule to prevent revolution, the minimum taxation required by the poor, given by Equation (10), must be smaller or equal to the maximum possible value of taxation that can directed towards the poor, given by. That is:

$$\begin{aligned} 2(1-b) \geq & \frac{2}{1-\theta}\left[\frac{\mu^h}{1-\beta(1-q)} - \theta\right] \\ \iff \theta \geq & \left[\frac{\mu^h}{1-\beta(1-q)} - (1-b)\right]\frac{1}{b} \end{aligned} \tag{12}$$

Just as in the case of franchise extension, when the poor own relatively little assets, they stage a revolution if given a chance. The concurrent elite, knowing this, stage a coup solely to prevent a revolution. On the other hand, when the poor are relatively well off, the incumbent elite can avoid a revolt by redistributing from the rich to the poor. The question is how they design the tax schedule. Clearly, if they can, the incumbent elite meets the threat of revolution by redistributing solely from the concurrent elite to the poor. Furthermore, if possible, the incumbent elite also redistribute from the concurrent elite to themselves. The ideal tax schedule for the incumbent elite is given by $\tau^{e'}(\mu^h)$ from Equation (13).

$$\tau^{e'}(\theta) = \begin{Bmatrix} \tau^{a'}(\theta) \\ \tau^{c'}(\theta) \\ \tau^{r'}(\theta) \end{Bmatrix} = \begin{Bmatrix} 0 \\ \frac{2}{1-\theta}\left[\frac{\mu^h}{1-\beta(1-q)} - \theta\right] \\ 1-b-\frac{2}{1-\theta}\left[\frac{\mu^h}{1-\beta(1-q)} - \theta\right] \end{Bmatrix} \tag{13}$$

Tax schedlue $\tau^{e'}(\theta)$ requires that redistribution from the concurrent elite alone is enough to satisfy the poor. That is:

$$\begin{aligned} 1-b \geq & \frac{2}{1-\theta}\left[\frac{\mu^h}{1-\beta(1-q)} - \theta\right] \\ \iff \theta \geq & \frac{1}{b}\left[\frac{2\mu^h}{1-\beta(1-q)} - (1-b)\right] \end{aligned} \tag{14}$$

Equation (14) shows that the poor must be well off enough so that the concurrent elite alone can satisfy their needs. Otherwise, the first elite must tax part its own income to prevent a revolution. $\tau^{e''}(\theta)$ illustrates this tax schedule.

$$\tau^{e''}(\theta) = \begin{Bmatrix} \tau^{a''}(\theta) \\ \tau^{c''}(\theta) \\ \tau^{r''}(\theta) \end{Bmatrix} = \begin{Bmatrix} \frac{2}{1-\theta}\left[\frac{\mu^h}{1-\beta(1-q)} - \theta\right] - (1-b) \\ 1-b \\ 0 \end{Bmatrix} \tag{15}$$

Assumption 12 states that the thresholds from Equations (12) and (14) are relevant and that the threshold for revolution is lower than that of the the thresholds associated with the different tax schdules. The assumption allows for three different scenarios to occur under $\mu = \mu^h$, which are outlined in Lemma 4.

Assumption 12: $\lambda > \frac{1}{b}\left[\frac{2\mu^h}{1-\beta(1-q)} - (1-b)\right] > \left[\frac{\mu^h}{1-\beta(1-q)} - (1-b)\right]\frac{1}{b} > 0$

Lemma 4: Suppose Assumptions 11 and 12 hold, the incumbent elite proposes a tax schedule when $\mu = \mu^h$ and $s = \{E^1, E^2\}$, and the concurrent elite stages a coup when $\mu = \mu^l$. The outcomes at $\mu = \mu^h$ are characterized as follows:

- if $\theta < \left[\frac{\mu^h}{1-\beta(1-q)} - (1-b)\right]\frac{1}{b}$, then the concurrent elite stages a coup;
- if $\left[\frac{\mu^h}{1-\beta(1-q)} - (1-b)\right]\frac{1}{b} \leq \theta \leq \frac{1}{b}\left[\frac{2\mu^h}{1-\beta(1-q)} - (1-b)\right]$, then tax schedule $\tau^e(\mu^h) = \tau^{e'}(\theta)$ is enforced;
- if $\theta > \frac{1}{b}\left[\frac{2\mu^h}{1-\beta(1-q)} - (1-b)\right]$, then tax schedule $\tau^e(\mu^h) = \tau^{e''}(\theta)$ is enforced.

Now, consider the case where there is no coup against autocracy when $\mu = \mu^l$. The payoff of continued autocracy to the poor and the concurrent elitebecomes:

$$V^p(\mu^h, E^1) = \frac{1}{1-\beta}\left[h^p + (1-\beta(1-q))\frac{\tau^a(\mu^h) + \tau^c(\mu^h)}{2\lambda}(1-\theta)h\right]$$

$$V^{e^2}(\mu^h, E^1) = \frac{h^e}{1-\beta}[(1-\beta(1-q))(1-\tau^r(\mu^h) - \tau^c(\mu^h)) + \beta(1-q)(1-\tau^r(\mu^l))]$$

To prevent a revolution and a coup respectively, we require:

$$\begin{aligned} &V^p(\mu^h, E^1) \geq V^p(\mu^h, R) \\ \iff &\tau^a(\mu^h) + \tau^c(\mu^h) \geq \frac{2}{1-\theta}\frac{\mu^h - \theta}{1-\beta(1-q)} \end{aligned} \tag{16}$$

$$\begin{aligned} &V^{e^2}(\mu^h, E^1) \geq V^{e^2}(\mu^h, C^2) \\ \iff &\tau^r(\mu^h) + \tau^c(\mu^h) \leq 1 - \tau^r(\mu^l)\frac{2\beta(1-q)}{1-\beta(1-2q)} \end{aligned} \tag{17}$$

To find the tax schedule in this scenario, I require the value of $\tau^r(\mu^l)$. At state $\mu = \mu^l$, under autocracy, the incumbent elite propose tax $\tau^r(\mu^l)$ and the concurrent elite decide whether to accept it, or to reject it and stage a coup. The payoffs from a coup and accepting the tax schedule depend on whether a coup is staged at $\mu = \mu^h$. Four cases arise: everyone accepts tax schedule $\tau^r(\mu^h)$ regardless of the concurrent elite's decision at $\mu = \mu^l$; a coup occurs at $\mu = \mu^h$ regardless of the decision at $\mu = \mu^l$; a coup occurs at $\mu = \mu^h$ if one occurs at $\mu = \mu^l$ but not otherwise; or a coup occurs at $\mu = \mu^h$ if one doesn't occur at $\mu = \mu^l$, but not otherwise. Let us focus on the first case, where autocracy prevails at $\mu = \mu^h$ regardless of what occurs at $\mu = \mu^l$. The concurrent elite gathers the following payoffs from staging a coup or accepting the tax schedule:

$$
\begin{aligned}
V^{e^2}(\mu^l, C^2) =& \frac{(1-\beta q)^2}{[1-\beta][1+\beta(1-2q)]} \frac{2\rho^h h}{1-\lambda} + \\
& \frac{\beta q h^e}{[1-\beta][1+\beta(1-2q)]}[(1-\beta q)(1-\overline{\tau^a}+\overline{\tau^r})+\beta(1-q)(1-\overline{\tau^c}-\overline{\tau^r})] \\
V^{e^2}(\mu^l, E^1) =& \frac{h^e}{1-\beta}[(1-\beta q)(1-\tau^e)+\beta q(1-\tau^r-\tau^c)]
\end{aligned}
$$

Where $\overline{\tau^a}$, $\overline{\tau^r}$ and $\overline{\tau^c}$ are given by Equations (10) and (11) and $\tau^r(\mu^h)$ and $\tau^c(\mu^h)$ are from equation (17). That is, they are such that:

$$
\begin{aligned}
\overline{\tau^a}+\overline{\tau^c} &= \frac{2}{1-\theta}\left[\frac{\mu^h}{1-\beta(1-q)}-\theta\right] \\
\overline{\tau^r}+\overline{\tau^c} &= 1-b \\
\tau^r(\mu^h)+\tau^c(\mu^h) &= 1-\frac{2\beta(1-q)}{1-\beta(1-2q)}\tau^r(\mu^l)
\end{aligned}
\tag{18}
$$

The concurrent elite thus does not stage a coup at $\mu=\mu^l$ if:

$$
\begin{aligned}
& V^{e^2}(\mu^l, E^1) \geq V^{e^2}(\mu^l, C^2) \\
\iff & \tau^r(\mu^l) \leq \frac{1-\beta(1-2q)}{[1-\beta(1+q)+\beta^2 q(3-4q)][1+\beta(1-2q)]} \times \\
& \left[\frac{2(1-\beta q)}{1-\theta}\left(1-\frac{\beta q\mu^h}{1-\beta(1-q)}\right)-b(1-\beta q)\beta q\right]
\end{aligned}
\tag{19}
$$

Let the right inside of Equation (19) be $\tau^r(\theta)$. This amount is positive, and thus a coup is successfully avoided, if:

$$
\theta \geq 1-\frac{2(1-\beta q)}{b\beta q(1-\beta)}\left[1-\frac{\beta q\mu^h}{1-\beta(1-q)}\right]
\tag{20}
$$

Equation (20) shows a coup is prevented when θ is high enough. A similar result emerges for the other three cases above. Let θ', θ'' and θ''' be the thresholds derived from each of these additional cases. Assumptions 13 and 14 guarantee these thresholds are such that the only relevant threshold is that of Equation (20). Assumption (15) in turn guarantees that $\tau^r(\mu^l)=\tau^r(\theta)<1-b$ for all values of θ.

Assumption 13: $\theta', \theta'', \theta''' \leq 0$
Assumption 14: $\lambda > 1-\frac{2(1-\beta q)}{b\beta q(1-\beta)}\left[1-\frac{\beta q\mu^h}{1-\beta(1-q)}\right] > 0$
Assumption 15: $1-b \geq \frac{1-\beta(1-2q)}{[1-\beta(1+q)+\beta^2 q(3-4q)][1+\beta(1-2q)]}\left[\frac{2(1-\beta q)}{1-\theta}\left(1-\frac{\beta q\mu^h}{1-\beta(1-q)}\right)-b(1-\beta q)\beta q\right]$

Now that we know the value of $\tau^r(\mu^l)$, we can return to what happens at $\mu=\mu^h$ when a coup is not staged at $\mu=\mu^l$. The intuition behind the next steps is the same as that behind the steps that led to Lemma 4. From equations (16) and (17), to avoid a revolution and coup when $\mu=\mu^h$, we require:

$$1-b+1-\frac{2\beta(1-q)\tau^r(\mu^l)}{1-\beta(1-2q)} \geq \frac{2}{1-\theta}\frac{\mu^h-\theta}{1-\beta(1-q)}$$
$$\iff \theta \geq \frac{\frac{2\mu^h}{1-\beta(1-q)}+\frac{2\beta(1-q)\alpha_\sigma}{1-\beta(1-2q)}+b-2-\frac{\alpha_b b2\beta(1-q)}{1-\beta(1-2q)}}{\frac{2}{1-\beta(1-q)}+1-\beta(1-2q)+b-2-\frac{\alpha_b b2\beta(1-q)}{1-\beta(1-2q)}} \tag{21}$$

$$\tau^r(\mu^l)+\tau^c(\mu^l) \geq 0$$
$$\iff \theta \leq 1-\frac{2\beta(1-q)\alpha_\theta}{1-\beta(1-q)(1-2\alpha_b b)} \tag{22}$$

As in the previous case, the incumbent elite prefers to redistribute from the concurrent elite towards the poor and themselves. Thus, the ideal tax schedule for the incumbent elite is:

$$\tau^{e'''}(\theta)=\begin{Bmatrix}\tau^{a'''}(\theta)\\ \tau^{c'''}(\theta)\\ \tau^{r'''}(\theta)\end{Bmatrix}=\begin{Bmatrix}0\\ \frac{2}{1-\theta}\frac{\mu^h-\theta}{1-\beta(1-q)}\\ 1-\frac{2\beta(1-q)\tau^e}{1-\beta(1-2q)}-\frac{2}{1-\theta}\frac{\mu^h-\theta}{1-\beta(1-q)}\end{Bmatrix} \tag{23}$$

To ensure the second elite can satisfy the poor on their own we require:

$$1-\frac{2\beta(1-q)\tau^r(\mu^l)}{1-\beta(1-2q)} \geq \frac{2}{1-\theta}\frac{\mu^h-\theta}{1-\beta(1-q)}$$
$$\iff \theta \geq \frac{\frac{2\mu^h}{1-\beta(1-q)}+\frac{2\beta(1-q)(1+\alpha_b b)}{1-\beta(1-2q)}-1}{\frac{2}{1-\beta(1-q)}+\frac{2\beta(1-q)(1+\alpha_b b)}{1-\beta(1-2q)}-1} \tag{24}$$

If θ is not high enough, the incumbent elite must set the following tax schedule instead:

$$\tau^{e''''}(\theta)=\begin{Bmatrix}\tau^{a''''}(\theta)\\ \tau^{c''''}(\theta)\\ \tau^{r''''}(\theta)\end{Bmatrix}=\begin{Bmatrix}\frac{2}{1-\theta}\frac{\mu^h-\theta}{1-\beta(1-q)}+\frac{2\beta(1-q)\tau^e}{1-\beta(1-q)}-1\\ 1-\frac{2\beta(1-q)\tau^e}{1-\beta(1-2q)}\\ 0\end{Bmatrix} \tag{25}$$

Below, Assumption 16 guarantees that the inequality thresholds derived are relevant and that the paths of regime dynamics are characterized as in Lemma 5.

Assumption 16: $\lambda > \frac{\frac{2\mu^h}{1-\beta(1-q)}+\frac{2\beta(1-q)(1+\alpha_b b)}{1-\beta(1-2q)}-1}{\frac{2}{1-\beta(1-q)}+\frac{2\beta(1-q)(1+\alpha_b b)}{1-\beta(1-2q)}-1} > 1-\frac{2\beta(1-q)\alpha_\theta}{1-\beta(1-q)(1-2\alpha_b b)} > \left[\frac{2\mu^h}{1-\beta(1-q)}-(1-b)\right]$

Lemma 5: Suppose Assumptions 11 through 16 hold, and the incumbent elite proposes a tax schedule whenever $\mu=\mu^h$. The regime path outcomes are as follows:

- if $\theta < \left[\frac{\mu^h}{1-\beta(1-q)}-(1-b)\right]\frac{1}{b}$, then the concurrent elite stages a coup regardless of the value of μ;
- if $\left[\frac{\mu^h}{1-\beta(1-q)}-(1-b)\right]\frac{1}{b} \leq \theta \leq \frac{1}{b}\left[\frac{2\mu^h}{1-\beta(1-q)}-(1-b)\right]$, the concurrent elite stages a coup at $\mu=\mu^l$. At $\mu=\mu^h$ tax schedule $\tau^e(\mu^h)=\tau^{e'}(\theta)$ is enforced;

- if $\frac{1}{b}\left[\frac{2\mu^h}{1-\beta(1-q)}-(1-b)\right]<\theta<1-\frac{2\beta(1-q)\alpha_\theta}{1-\beta(1-q)(1-2\alpha_b b)}$, the concurrent elite stages a coup at $\mu=\mu^l$. At $\mu=\mu^h$, tax schedule $\tau^e(\mu^h)=\tau^{e''}(\theta)$ is enforced;
- if $1-\frac{2\beta(1-q)\alpha_\theta}{1-\beta(1-q)(1-2\alpha_b b)}<\theta<\frac{\frac{2\mu^h}{1-\beta(1-q)}+\frac{2\beta(1-q)(1+\alpha_b b)}{1-\beta(1-2q)}-1}{\frac{2}{1-\beta(1-q)}+\frac{2\beta(1-q)(1+\alpha_b b)}{1-\beta(1-2q)}-1}$, tax rate $\tau^r(\mu^l)=\tau^r(\theta)$ and tax schedule $\tau^e(\mu^h)=\tau^{e''}(\theta)$ are enforced. That is, the first elite remains in power forever;
- if $\theta\geq\frac{\frac{2\mu^h}{1-\beta(1-q)}+\frac{2\beta(1-q)(1+\alpha_b b)}{1-\beta(1-2q)}-1}{\frac{2}{1-\beta(1-q)}+\frac{2\beta(1-q)(1+\alpha_b b)}{1-\beta(1-2q)}-1}$, tax rate $\tau^r(\mu^l)=\tau^r(\theta)$ and tax schedule $\tau^e(\mu^h)=\tau^{e''''}(\theta)$ are enforced. That is the first elite remains in power forever.

9.3 Solution

Assumption 17 guarantees that in equilibrium, for high values of θ, the incumbent elite responds to the threat of revolution by proposing tax schedule $\tau^e(\mu^h)$. That is, the assumption assures that only very equal societies can be characterized by stable autocracies. Proposition 1 below characterises the unique pure strategy Markov Equilibrium as a function of θ.

Assumption 17: $\frac{\mu^h}{1-\beta(1-q)}-(1-b)>1-\frac{1-b}{1-\beta q-\beta^2 q(1-q)}$

Proposition 1: Suppose the economy begins at state $s=E^1$ and Assumptions 1 through 17 hold. Then, for all values of $\theta\in[0,\lambda]$, there exists a unique Pure Strategy Markov Equilibrium such that:

1. if $\theta\leq\pi_a$, the economy observes a series of unstable autocracies. Specifically, in periods where $\mu=\mu^h$, the concurrent elite overthrow the incumbent elite through a coup. In periods where $\mu=\mu^l$ the incumbent elite remains in power by proposing tax rate $\tau^r(\mu^l)=\frac{1-2\beta(1-q)+\beta^2 q(1-q)+\beta^3(1-2q)(1-q)}{1-2\beta(1-q)+\beta^3(1-q-q^2}-\frac{2\rho^h\frac{[1-\beta(1-q)][1-\beta^2 q]}{1-\beta^2}\frac{1-2\beta(1-q)+\beta^3(1-2q)}{1-2\beta(1-q)+\beta^3(1-q-q^2)}}{1-\theta}$;
2. if $\pi_a<\theta<\pi_b$, the economy is characterized by a series of very unstable autocracies. A coup occurs in every period, regardless of the value of μ. Hence, the first elite e^1 is in power in odd periods, and the second elite e^2 is in power in even periods;
3. if $\pi_b\leq\theta\leq\pi_c$, the economy eventually reaches a stable democracy. The first elite remains in power as long as $\mu=\mu^l$, taxing the second elite at rate $\tau^r(\mu^l)=\tau^{r*}(\theta)$. At the first realization of $\mu=\mu^h$, the franchise is extended and never reversed. The poor impose tax rates $\tau^p(\mu^h)=1-b$ and $\tau^p(\mu^l)=\tau^{p*}(\theta)$ on the elites;
4. f $\pi_c<\theta<\pi_d$, the economy reaches an unstable democracy. The first elite remains in power as long as $\mu=\mu^l$, taxing the second elite at rate $\tau^r(\mu^l)=\tau^{r*}(\theta)$. At the first realization of $\mu=\mu^h$, the franchise is extended and the regime switches to democracy. However, the first elite stage a coup against democracy whenever $\mu=\mu^l$, so that the regimes changes back to an autocracy ruled by the first elite;
5. if $\pi_d\leq\theta<\pi_e$, the economy eventually reaches a stable democracy. As long as $\mu=\mu^l$, the two elites overthrow each other through coups. At the first realization of $\mu=\mu^h$, the franchise is extended and never reversed. The poor impose tax rates $\tau^p(\mu^h)=1-b$ and $\tau^p(\mu^l)=\tau^{p**}(\theta)$;

6. if $1-\frac{1-\beta}{1-\beta q-\beta^2 q(1-q)}<\theta\leq\frac{1}{b}\left[\frac{\mu^h}{1-\beta(1-q)}-(1-b)\right]$, the economy is characterized by constant switches between democracy and autocracy. Under autocracy, as long as $\mu=\mu^l$, the concurrent elite stages a coup against the incubent. When $\mu=\mu^h$, the incubent elite extends the franchise and the economy reaches a democracy. However, at the next realization of $\mu=\mu^l$, the first elite stages a coup against democracy, so that the regime reverts to an autocracy ruled by the first elite;

7. if $\frac{1}{b}\left[\frac{\mu^h}{1-\beta(1-q)}-(1-b)\right]\leq\theta<1-\frac{2\beta(1-q)\alpha_\theta}{1-\beta(1-q)(1-2\alpha_b b)}$, the economy observes a series of unstable autocracies. Under autocracy, when $\mu=\mu^l$, the concurrent elite stages a coup against the incumbent. When $\mu=\mu^h$, the incumbent elite sets tax schedule $\tau^e(\mu^h)=\tau^{e''}(\theta)$;

8. if $1-\frac{2\beta(1-q)\alpha_\theta}{1-\beta(1-q)(1-2\alpha_b b)}\leq\theta<\frac{\frac{2\mu^h}{1-\beta(1-q)}+\frac{2\beta(1-q)(1+\alpha_b b)}{1-\beta(1-2q)}-1}{\frac{2}{1-\beta(1-q)}+\frac{2\beta(1-q)(1+\alpha_b b)}{1-\beta(1-2q)}-1}$, the economy is characterized by a single stable autocracy, such that the first elite e^1 remains in power forever. The first elite impose tax rate $\tau^r(\mu^l)=\tau^r(\theta)$ and tax schedule $\tau^e(\mu^h)=\tau^{e'''}(\theta)$;

9. if $\theta\geq\frac{\frac{2\mu^h}{1-\beta(1-q)}+\frac{2\beta(1-q)(1+\alpha_b b)}{1-\beta(1-2q)}-1}{\frac{2}{1-\beta(1-q)}+\frac{2\beta(1-q)(1+\alpha_b b)}{1-\beta(1-2q)}-1}$, the economy is characterized by a single stable autocracy, such that the first elite e^1 remains in power forever. The first elite impose tax rate $\tau^r(\mu^l)=^r(\theta)$ and tax schedule $\tau^e(\mu^h)=\tau^{e'''}(\theta)$.