Optimal Regulation of Credit Lines*

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This version: June 20, 2023

Abstract

This paper presents a contract-theoretic model in which banks choose pre-arranged and ex-post funding to finance firms' liquidity needs through credit lines. In states with high liquidity needs, pre-arranged funding is key to sustaining lending and reducing the number of firms liquidated. Yet, in the presence of a pecuniary externality on firms' liquidation values, competitive banks choose insufficient pre-funding compared to a constrained social planner. Constrained efficiency can be restored using regulatory liquidity ratios. The optimal regulatory ratio depends on the frequency of high liquidity need states, the value lost after a firm liquidation, and the premium on pre-arranged funding.

Keywords: credit lines, bank liquidity risk regulation, LCR, NSFR, Basel III.

JEL Codes: G01, G21, G28, G32.

*The author is grateful to Rafael Repullo, Javier Suárez, David Martinez-Miera, Andrés Almazán, Pablo Ruiz-Verdú, Gerard Llobet, Carlos Perez Montes, Jose-Maria Serena, Marcus Opp, and an anonymous referee, as well as participants at the CEMFI Banking and Finance Reading Group, the 2020 European Winter Meeting of the Econometric Society, the 37th meeting of the European Economic Association, the 74th European meeting of the Econometric Society, and the 2022 Annual Meeting of the Latin American and Caribbean Economic Association (LACEA) and the Latin American Meeting of the Econometric Society (LAMES) for insightful feedback. Funding from Spain's Ministerio de Economía, Industria y Competitividad (BES−2017-082176), María de Maeztu Programme for Units of Excellence in R&D (MDM−2016-0684), and from the Santander Research Chair at CEMFI is gratefully acknowledged. This article is the author's exclusive responsibility and does not necessarily reflect the opinion of the Banco de España or the Eurosystem.

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1 Introduction

Firms widely use credit lines for liquidity risk management.[1] In such contracts, firms can draw down funds at will up to an agreed limit at committed pricing terms. However, this lending flexibility exposes banks to liquidity risk, especially after adverse macroeconomic shocks (Jiménez et al., 2009; Mian and Santos, 2011; Greenwald et al., 2020; Kapan and Minoiu, 2021).[2] Consequently, Basel III liquidity requirement ratios consider the liquidity risk arising from credit lines (BCBS, 2013; 2014).[3] Nevertheless, the current framework for liquidity risk regulation does not specify how liquidity requirements on undrawn credit lines should be adapted depending on a country's economic fundamentals. Moreover, despite the large share of bank lending coming from credit lines, the regulation of credit lines has received scant attention in the literature.

This paper provides a rationale for regulating credit lines and studies its main determinants. The existing theoretical literature on credit lines does not consider the need for regulating them, as it assumes that banks always meet credit line drawdowns. On the contrary, this paper justifies a regulatory intervention due to a pecuniary externality on firm liquidation values and the possibility that banks may not meet credit line drawdowns, causing the liquidation of firms needing funds. In the model, firms sign credit line contracts with banks to meet a contingent liquidity need and avert liquidation.[4] Competitive banks finance firms' liquidity demand with pre-arranged and ex-post funding, raised as drawdowns accumulate. However, banks cannot fully meet drawdowns in high liquidity need states as bank revenues may be insufficient to raise enough ex-post funding, thus

[1]According to Sufi (2009) and Demiroglu et al. (2009), 87% of public firms and 64% of large private firms have access to credit lines in the US.

[2]For instance, recent papers document a sudden and sizable increase in drawdowns after the COVID-19 outbreak (Greenwald et al., 2020; Kapan and Minoiu, 2021). Moreover, Greenwald et al. (2020) document that undrawn credit lines are more than 40% of the total used balances on bank credit lines and term loans combined in the US. Thus, exposing banks to liquidity risk arising from a spike in credit line usage.

[3]The Liquidity Coverage Ratio (LCR) requires banks to hold liquid assets between 5% and 30% of undrawn credit lines, which depends on the type of the commitment (credit vs. liquidity facility) and client (non-financial vs. financial firm). The Net Stable Funding Ratio (NSFR) requires banks to fund undrawn credit lines with at least 5% of stable funding (e.g., equity or long-term debt).

[4]For instance, Campello et al. (2010) and Almeida et al. (2012) show that losing access to external funds can negatively affect firms, leading to investment spending cuts or cancellations.

reneging on some credit lines.[5] In this scenario, pre-arranged funding is key to sustaining lending to firms but implies higher ex-ante costs for banks. Firms that do not obtain liquidity from banks have to be liquidated, which implies a negative price pressure on the equilibrium liquidation value. However, banks do not internalize this pecuniary externality and, as a result, choose low levels of pre-arranged funding and tend to renege on credit lines too often. This market failure rationalizes the regulation of credit lines, for instance, in the form of a liquidity requirement.

The paper bears important results. First, I provide a model in which credit lines arising from the optimal contracting between banks and firms might not provide full insurance against liquidity shocks in high liquidity need states. Particularly, if high liquidity need states are rare, securing liquidity to firms in such states will not be efficient as more bank pre-arranged funding will be needed, making the contract too costly. Second, the private arrangement is constrained-inefficient in the presence of a pecuniary externality on firm liquidation values. In particular, competitive atomistic banks do not internalize the value of additional pre-arranged funding in preventing fire-sale losses for everyone by reducing liquidations. Thus, banks choose too little stable funding (pre-arranged funding in the model) and tend to renege on credit lines too often. Third, a minimum requirement on pre-arranged funding, used to partly finance drawdowns, can restore constrained efficiency. This (liquidity) regulation makes credit lines costlier, but welfare improves due to more financing of firms' liquidity needs, especially in high liquidity need states. Finally, the paper shows that the optimal minimum requirement should be higher when the premium on pre-arranged funding is lower, high liquidity need states are more frequent, liquidations are costlier, or firm liquidation values are very sensitive to liquidations.

The contract-theoretic model of contingent lending in this paper has a rich structure. First, the insurance nature of credit lines exposes banks to losses when credit line usage spikes, making it harder for them to meet drawdowns. Note that firms pay fees to the bank even when no drawdown happens in order to have the right to use the credit line when needed. Thus, bank competition implies that a

[5] Credit lines include financial covenants that, if not complied with, allow lenders to restrict access to them. Thus, banks can restrict access to lines by setting stringent covenants and waiving fewer violations of them (Acharya et al., 2020).

credit line drawdown has a negative net return for the lender. Therefore, in aggregate states where liquidity needs are highly correlated, the revenues from credit lines shrink, making it harder for banks to meet the entire demand for drawdowns. Hence, banks renege on some credit lines by invoking financial covenants and leave some cash-strapped firms without access to funds.

Bank pre-arranged funding helps sustain lending to firms via credit lines, especially in high liquidity need states. Yet, it requires an extra return compared to ex-post funding, raised by banks from new investors as drawdowns accumulate. In the model, bank pre-arranged funding is junior to bank ex-post funding. In particular, claims on pre-arranged funding can be diluted to obtain additional funding needed to finance more drawdowns.[6] In exchange for their funds, providers of pre-arranged funding are compensated with higher returns during episodes of low liquidity needs, that is, when the revenues from credit lines are high. Hence, bank pre-arranged funding resembles equity, considered a stable funding source for the computation of the NSFR.

The private arrangement considers the cost and benefit of higher levels of pre-arranged funding. When contracting in an ex-ante stage, the parties acknowledge that a low level of pre-arranged funding cause credit lines to not fully insure firms against liquidity shocks in high liquidity need states. Hence, if high liquidity need episodes are rare, the optimal private arrangement will not always secure funding, saving on costly levels of pre-arranged funding.

Regulation of credit lines is justified due to the pecuniary externality on firms' liquidation values. When choosing pre-arranged funding in the contracting stage, competitive banks do not internalize the value of additional insurance, which by avoiding some additional liquidation, prevents fire-sale losses for everyone. Therefore, banks choose insufficient pre-arranged funding compared to a constrained efficient allocation chosen by a social planner that considers such a positive effect of pre-arranged funding. Thus, the private arrangement provides too little insurance.

The planner's solution can be implemented with a minimum requirement for pre-arranged funding

[6] If only part of the claims could be diluted, the bank's revenue would not be entirely pledgeable to raise ex-post funding from new investors, reducing the bank's ability to meet drawdowns. In such a case, the insurance of the credit line will depend on the funding source that allows a bank to increase its lending capacity in high liquidity need states, and the optimal liquidity regulatory ratio should seek to increase banks' holding of such funding source.

per committed (undrawn) funds in credit lines. Such a requirement resembles the Basel III liquidity ratios. Despite increasing the cost of credit lines, the regulation improves welfare by increasing the insurance of credit lines in high liquidity need states, and in this way, ameliorating the effect on liquidation values of excessive liquidations of firms.

Moreover, the comparative static analysis informs how liquidity requirements on undrawn credit lines could be tuned up for economies with different economic fundamentals. According to the model, the optimal minimum requirement should be increased under the following conditions: a lower premium on pre-arranged funding, more frequent high aggregate liquidity need states, higher costs of liquidating firms, and greater sensitivity of firms' liquidation values to firm liquidations.

As an extension, the demand for credit lines in the baseline model is rationalized within a model of debt overhang with a secondary market for specialized assets. In particular, preexistent senior debt prevents firms from borrowing as they go to meet a contingent liquidity need, forcing them to fire-sell specialized assets in a secondary market populated by entrepreneurs with heterogeneous abilities at managing them. Thus, if many assets are traded in this market, the price decays as less productive entrepreneurs absorb part of the larger supply, causing cash-strapped firms to fire-sell more of their assets and decreasing their continuation payoff. As in Shleifer and Vishny (1992), the immediate need for liquidity does not necessarily transfer assets to the best users, pricing them below their value in best use. By signing a credit line in an ex-ante stage, funding to cover the contingent liquidity need can be obtained without fire-selling specialized assets. In particular, paying a fee when funds are not needed decreases the debt burden when funds are used. However, when liquidity needs are highly correlated, banks cannot accommodate the demand for drawdowns, causing fire sales. Competitive banks do not internalize the effect of their pre-arranged funding choices on preventing welfare losses associated with fire sales, justifying a minimum requirement on bank pre-arranged funding. Specifically, welfare losses arise due to the transfer of specialized assets from more productive (firms) to less productive agents (entrepreneurs). In fact, if a large agent as productive as firms could manage liquidated assets, welfare losses would not exist, and regulation would not be needed.

The paper contributes to the theoretical literature on credit line models based on insurance motives (Campbell, 1978; Boot et al., 1987; Holmström and Tirole, 1998). However, these models do not provide a reason for regulating credit lines, as allocations are constrained-efficient, and assume that banks always meet credit line drawdowns. The model in this paper modifies the seminal model in Holmström and Tirole (1998). In particular, it adds an aggregate state that determines the number of firms needing liquidity. In states with high liquidity needs, banks cannot meet all credit line drawdowns, reneging on some of them and forcing the liquidations of firms left without a loan. Moreover, the model allows each liquidated firm to depress the equilibrium liquidation value. The stochastic aggregate state, which pushes banks to renegue on credit lines in high liquidity need states, and the fire-sale externality on firm liquidation values provide a rationale for regulating credit lines.

The paper also relates to the scarce theoretical literature on bank liquidity regulation, particularly the one that justifies its benefits based on preventing fire-sale externalities (Perotti and Suarez, 2011; Stein, 2012; Dewatripont and Tirole, 2018; Kara and Ozsoy, 2019; Lutz and Pichler, 2021). However, this literature has not analyzed a specific regulation of credit lines. Moreover, as undrawn credit lines are off-balance-sheet items, typical regulation studied in this literature does not necessarily adapt to the specificities of credit lines.[7] Nonetheless, the Basel III liquidity risk regulatory framework considers the treatment of credit lines, though does not consider how it can be adapted to a country's economic fundamentals. To the best of my knowledge, this paper is the first to justify a special liquidity regulation of credit lines and to study its main determinants, which can guide national authorities in the decision to set liquidity requirements for credit lines in their jurisdictions.

The rest of the paper is organized as follows: section 2 describes the basic model; section 3 characterizes the optimal credit line in the laissez-faire equilibrium; section 4 characterizes the social planner's solution and discusses its implementation using a liquidity regulation; an extension of the basic model is studied in section 5; finally, section 6 concludes. The proofs of the results can be found in the Appendix.

[7] For example, a bank may be satisfying on-balance sheet requirements but may not be well covered if a considerable amount of its loan commitments are requested.

2 The basic model

Consider an economy with three dates ($t = 0, 1, 2$) and a large number of three types of risk-neutral agents: *firms*, *investors*, and *banks*. Firms have ongoing investment projects that mature at date 2 and may require cash at date 1 to avert liquidation. The fraction of firms needing cash depends on an aggregate state, which is known at the beginning of date 1. Banks provide contingent lending to firms through credit lines, which are signed at date 0. Banks finance their lending with *pre-arranged* and *ex-post funding* from investors. Pre-arranged funding is raised at date 0 and stored in liquid funds to partially finance credit line drawdowns at date 1. If necessary, additional funds can be raised from investors at date 1. Investors demand an expected return R at $t = 2$ for funds lent at date 1; whereas they demand an extra return $\delta \geq 0$ for funds lent at date 0. As seen next, ex-post funding is riskless, whereas providers of pre-arranged funding are the ones who bear all the risk. Hence, the extra return δ could be interpreted as a return arising from a preference for absolute safety, as in Stein (2012).[8]

2.1 Firms

Each firm has an ongoing investment project that matures at $t = 2$. Moreover, this project faces liquidity risk; a cash injection ℓ could be needed at $t = 1$ to avert liquidation. Let ℓ be the size of the liquidity shock, privately revealed to firms at $t = 1$ after the aggregate state is realized. Conditional on the aggregate state, ℓ is independent and identically distributed as follows

$$\ell = \begin{cases} 1, & \text{with probability } \alpha, \\ 0, & \text{with probability } 1 - \alpha. \end{cases}$$

Hence, firms' aggregate demand for liquidity at date 1 will be equal to α.

An aggregate state determining the fraction α of firms needing liquidity is realized at the beginning

[8] The excess rate δ can also be seen as a foregone return between date 0 and 1 from maintaining funds as liquid assets. Alternatively, a bank moral hazard problem could also justify the assumption. For instance, banks could divert assets like in Gertler and Karadi (2011) once credit lines are signed but before the aggregate state is revealed. To prevent this, banks should receive an additional expected return.

of $t = 1$. Particularly, α is distributed according to a probability density function $g(\cdot)$ over the support $[0, 1]$, which is known when contracting at $t = 0$. Though α is observable, it is not verifiable; hence, contracts cannot be contingent on the aggregate state.

When the cash injection is not met, the firm is liquidated, in which case a liquidation value Q is produced at $t = 2$. The liquidation value depends on the number of firms that do not get liquidity from the bank, denoted by z. Moreover, $Q(\cdot)$ is decreasing in z; the liquidation value decreases when many firms are being liquidated in the economy.

When the cash injection is met, the firm produces a verifiable cash flow X at $t = 2$. Thus, the only source of uncertainty in the model comes from liquidity risk.[9] The cash flow satisfies the following assumption.

Assumption 1. $X - R > Q(0)$.

Assumption 1 states that if a cash need surges at $t = 1$, the net return of meeting ℓ is larger than the liquidation return.

Moreover, firms can pledge at most a portion Y of the cash flow X to outsiders. This *pledgeable income* satisfies the next assumption.

Assumption 2. $Y < R$.

Although financing ℓ is efficient, Assumption 2 implies that firms cannot raise funds from investors to finance ℓ, because pledgeable income is insufficient to pay them for their required return. As in Holmström and Tirole (1998), Assumption 2 justifies the value of credit lines: the bank receives a fee from the firm when the line is not used in exchange for the right to draw down funds when a cash need surges.

Firms have access to a private storage technology, allowing for the possibility to divert funds from their credit lines into consumption. In addition, the parameters satisfy the next assumption.

Assumption 3. $1 < X - Y$ and $1 < Y$.

[9]This simplification does not have major implications in the main results of the analysis.

Assumption 3 implies that, even if the entire pledgeable income Y is promised, a firm in a cash urge prefers meeting ℓ over diverting funds. However, a firm without a cash need could still be tempted to divert funds. As will be seen, the credit line payment scheme will deter this by requesting a payment sufficiently high when the line is used.

This payment structure is derived from a model of debt overhang with a secondary market for specialized assets, which is presented in section 5.

2.2 Banks and credit lines

At $t = 0$ a representative bank from a competitive banking industry offers a credit line contract with sequential service constraint to firms. The contract specifies the amount of committed funds, a payment scheme, and the amount of pre-arranged funding per committed funds held by the bank.

In such a contract, firms can use up to 1 unit of funds. In exchange, the bank and its firms agree at $t = 0$ on a payment scheme (B, f): a gross interest rate B if funds are used and a commitment fee f if not. By making payments contingent on credit line usage, the bank can deter firms from diverting funds by making drawdowns costlier.

The bank finances credit line drawdowns with pre-arranged and ex-post funding. Pre-arranged funding (e.g., equity), denoted by E, is raised from *initial investors* at $t = 0$ and stored as cash to meet drawdowns. If E is not completely used after all drawdowns are served, excess funds can be invested at R. If additional funds are needed, new funding can be raised at $t = 1$ from *new investors*. Investors are compensated from revenues generated at $t = 2$. Moreover, pre-arranged funding is junior to ex-post funding. Thus, the bank can dilute initial investors to obtain more ex-post funding. Hence, the maximum amount of ex-post funding the bank can raise, its *borrowing capacity*, will be determined by date-2 revenues.[10] However, in return for their funds, initial investors are compensated with higher payments in low liquidity need states, that is, when many firms pay fees and do not request funds from their bank.

[10] If dilution were not feasible, the bank's revenue would not be entirely pledgeable to new investors, reducing its borrowing capacity and ability to meet drawdowns.

In high liquidity need states, the bank may not be able to meet the demand for drawdowns. If the bank cannot honor a drawdown, it is forced to terminate the credit line by invoking a financial covenant. However, it is assumed that canceling a contract has a high reputational cost, as in Thakor (2005).[11] As a result, the bank borrows as much as possible to honor drawdowns. Note that the bank suffers a loss whenever a loan is granted because of $B \leq Y < R$. By pooling liquidity risks, firms with a cash need are financed by the fee income generated from those that do not suffer the liquidity shock. However, as α increases, the demand for drawdowns increases, and the fee income shrinks, leaving the bank without enough borrowing capacity to meet the demand for drawdowns. In such a situation, the bank sequentially serves, in random order, drawdowns from credit lines until its borrowing capacity is exhausted. As a consequence, a firm in urge for cash may not get liquidity from its bank. In such a case, the firm is liquidated and is exempted from any payment to the bank. Note that the bank is capable of doing this without technically entering into default due to the presence of financial covenants, which allows the bank to terminate the contract.[12]

Pre-arranged funding helps the bank to better meet the demand for drawdowns, especially in high liquidity need states. Because claims on E can be diluted to obtain new funding, high pre-arranged funding helps to sustain lending to firms, increasing the insurance of the credit line.[13] Even so, excessive holdings of pre-arranged funding are not desirable due to their higher cost ($\delta \geq 0$). Consequently, banks consider this trade-off when pricing credit lines and choosing pre-arranged funding E at date 0.

Figure 1 summarizes the timing of events in the model. At $t = 0$, credit lines are signed. At the beginning of $t = 1$, aggregate and individual uncertainty are revealed. Then, firms decide whether to draw down funds from their credit lines. If demand for drawdowns is higher than pre-arranged funding, banks can raise new funding. Finally, production happens, and payments are made at $t = 2$.

[11] In a consultive document of the BCBS regarding the revisions to the standardized approach for credit risk, it was highlighted that banks appear to be constrained to cancel loan commitments in practice due to consumer protection laws, risk management capabilities, or reputational risk.

[12] To the best of my knowledge, there is no evidence that a bank has defaulted as a consequence of not honoring its credit line drawdowns.

[13] On the contrary, low levels of pre-arranged funding will require the bank to set stricter financial covenants such that the bank retains the option to cancel credit lines in high liquidity need states.

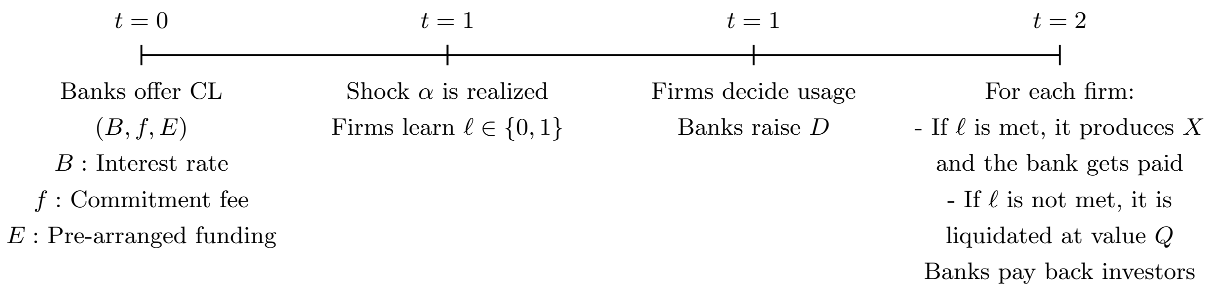

Figure 1: The sequence of events

3 The laissez-faire credit line contract

At date 0 banks compete by offering credit lines with pre-arranged funding E and payment scheme (B, f). When designing their contracts, banks do not internalize the effect of their decisions on aggregate liquidations z and, consequently, the liquidation value $Q(z)$.

To solve for the equilibrium contract, a backward induction approach is followed. First, the payment scheme is set such that firms never divert funds into their private storage technology. Thus, an incentive compatibility constraint is obtained from this step. Next, for a contract satisfying this constraint, liquidations are derived for every α. Finally, a representative bank chooses contractual terms (B, f, E) at date 0 by maximizing the expected payoff of the representative firm subject to the incentive compatibility constraint and the participation constraint of initial investors. In the next analysis, a representative bank takes aggregate liquidations z as given, which is represented by a non-decreasing function $z(\alpha)$.

3.1 Events at date 1

The implementation of a credit line contract requires deterring funds diversion. First, note that diverting funds into the private storage technology is inefficient as $R > 1$. Moreover, risk sharing is only feasible if firms without a cash need do not draw down funds and pay fees to their bank. Thus, the bank uses prices B and f to discourage such behavior entirely.

The next proposition states the condition that the payment scheme (B, f) must satisfy to deter funds diversion.

Proposition 1. *Firms without a need for cash do not draw down funds from credit lines if the payment scheme satisfies*

$$f \leq B - 1. \tag{IC}$$

Recall that by Assumption 3, firms in need of liquidity will prefer meeting the liquidity shock, as the continuation payoff $X - B$ is larger than the payoff from diverting funds. However, drawing down must be sufficiently costly to discourage firms without a cash need from diverting funds. Condition (IC) specifies that the net interest rate of the credit line must be larger than the fee f to discourage an inadequate use of the credit line. Note that the incentive compatibility constraint limits the benefits of risk-pooling, as firms that do not suffer the liquidity shock cannot be charged too much.

If the payment scheme satisfies constraint (IC), the demand for drawdowns is equal to α. In such a situation, the bank first finances drawdowns with cash reserves, created at date 0 using pre-arranged funding E. If insufficient, the bank borrows against date-2 revenues until α is met or its borrowing capacity is exhausted, whichever occurs first.

The next proposition states the region of α's in which the bank can fully accommodate the demand for drawdowns.

Proposition 2. *If the payment scheme satisfies constraint* (IC) *and* $E < \frac{R-B}{R}$, *there exists* $\underline{\alpha} = \frac{f+RE}{f+R-B} \in (0, 1)$ *such that the bank grants a total amount L of loans equal to*

$$L = \begin{cases} \alpha, & \textit{if } \alpha \leq \underline{\alpha}, \\ \dfrac{RE + (1-\alpha)f}{R - B} < \alpha, & \textit{if } \alpha > \underline{\alpha}. \end{cases}$$

When liquidity needs are highly correlated, the risk-sharing mechanism in which firms without a cash need subsidy firms in need of liquidity breaks down. Thus, as α increases, the bank finds it harder to meet the demand for drawdowns as the fee income shrinks and more firms demand liquidity. In particular, the bank can meet at most a demand for drawdowns $\underline{\alpha}$. For a higher realization of α, the bank cannot serve all drawdowns, $L < \alpha$, leaving some firms without a loan and, thus, facing their liquidation. In such a situation, initial investors are fully diluted, as the entire bank's revenue is used to raise new funding and meet as much as possible the demand for drawdowns α.[14]

Pre-arranged funding E can improve the insurance of the credit line. Because claims on E can be diluted to obtain more funding, lending to firms can be sustained for a broader range of $\alpha's$. Note that a higher choice of pre-arranged funding E increases $\underline{\alpha}$. Moreover, if $\alpha > \underline{\alpha}$, increasing E reduces the lending shortfall $\alpha - L$, helping to reduce the number of liquidations. However, for partial insurance of the credit line in high liquidity need states, E cannot be too large ($E < \frac{R-B}{R}$). Otherwise, even when $\alpha = 1$, additional funding to fully meet drawdowns can always be obtained by just diluting initial investors. However, as it is discussed next, choosing high levels of E to insure firms against unlikely high realizations of α is not efficient, as providers of pre-arranged funding demand an extra return δ.

3.2 Events at date 0: the credit line design

At $t = 0$ the competitive representative bank offers a credit line contract (B, f, E) to firms. Such a contract sets the amount of pre-arranged funding E per committed funds and a payment scheme (B, f) that satisfies incentive compatibility constraint (IC).

Pre-arranged funding E is raised from initial investors at $t = 0$, who demand an additional return δ. Because pre-arranged funding is junior to funds raised at $t = 1$, the bank can dilute initial investors to obtain additional funding to meet loan commitments α. As a consequence, initial investors are fully diluted when $\alpha > \underline{\alpha}$. However, initial investors are compensated with payments from low liquidity

[14] Recall that invoking financial covenants to terminate the contract is assumed to be costly due to a high reputational cost.

need states $\alpha \leq \underline{\alpha}$. Formally, raising pre-arranged funding at date 0 equal to E requires

$$(R+\delta)E = \int_0^{\underline{\alpha}} \big(\alpha B + (1-\alpha)f - R(\alpha - E)\big)g(\alpha)\mathrm{d}\alpha. \tag{PC}$$

The right-hand side of (PC) is the expected payment promised to initial investors. In particular, providers of E are entitled to the bank's revenue net of interest payments to new investors.[15] As mentioned, when $\alpha > \underline{\alpha}$, they receive a zero payoff. However, they receive higher payoffs during low liquidity need states coming from large fee revenues.

A credit line contract (B, f, E) delivers an expected payoff V to the representative firm equal to

$$V(B,f,E) = \int_0^{\underline{\alpha}} V_{\mathtt{I}}(B,f,E;\alpha)g(\alpha)\mathrm{d}\alpha + \int_{\underline{\alpha}}^1 V_{\mathtt{II}}(B,f,E;\alpha,z(\alpha))g(\alpha)\mathrm{d}\alpha,$$

where $V_{\mathtt{I}}$ and $V_{\mathtt{II}}$ are the firm's payoffs for a realization of α in regions I and II, respectively, and are defined as

$$V_{\mathtt{I}}(B,E;\alpha) = (1-\alpha)(X-f) + \alpha(X-B),$$
$$V_{\mathtt{II}}(B,E;\alpha,z(\alpha)) = (1-\alpha)(X-f) + \alpha\big(\frac{L}{\alpha}(X-B) + (1-\frac{L}{\alpha})Q(z)\big).$$

In low liquidity need states (Region I, $\alpha \leq \underline{\alpha}$), risk-pooling permits the bank to channel funds from investors to firms in need of cash; $L = \alpha$. Thus, every firm needing liquidity draws down its credit line and receives a loan. However, in Region II ($\alpha > \underline{\alpha}$), such a risk-sharing mechanism breaks down; the bank cannot meet the entire demand for drawdowns. As a consequence, a firm in need of cash requesting a loan from its credit line faces a probability $1 - \frac{L}{\alpha}$ of not receiving the loan and facing liquidation.

Due to competition, the representative bank chooses (B, f, E) to maximize the expected payoff V of the representative firm subject to initial investors' participation constraint (PC) and a payment scheme

[15] Due to competition, the constraint holds with equality. Note that if $\alpha \leq E$, excess of liquidity is invested at rate R. If $\alpha > E$, the bank raises $D = \alpha - E$ from new investors at $t = 1$. To raise such funds, the bank promises a payment P satisfying $RD \leq P$. Due to competition, new investors receive a sure payment of $P = R(\alpha - E)$ from date-2 revenues.

(B, f) satisfying incentive compatibility constraint (IC). However, when choosing the contractual terms, the bank does not consider the effect of its decisions on aggregate liquidations $z(\alpha)$ and, consequently, liquidation value $Q(z(\alpha))$. Hence, from the point of view of the representative bank, aggregate liquidations $z(\alpha)$ are taken as given when designing the credit line contract at date 0.

A full characterization of the equilibrium contract requires a definition of an equilibrium, which is provided as follows.

Definition 1. A *symmetric laissez-faire equilibrium* consists of a choice (B^U, f^U, E^U) for the representative bank and aggregate liquidations $z^U(\alpha)$ such that

1. Given $z^U(\alpha)$, (B^U, f^U, E^U) solves the bank's optimization problem, that is,

$$\max_{B,f,E} V(B, f, E)$$

   subject to the participation constraint (PC) of initial investors and an incentive-compatible payment scheme.

2. Given (B^U, f^U, E^U), aggregate liquidations are computed as $z^U(\alpha) = \alpha - L$ for all α, where L is defined as in Proposition 2.

It is important to remark that no bank finds it optimal to offer a new credit line following the realization of any α. In such a situation, only cash-strapped firms will demand credit. Hence, no credit line arrangement will be feasible as firms without a cash need will not be part of the credit line contract. Thus, contracting exclusively occurs at date 0 before aggregate uncertainty is revealed.

The next proposition characterizes the credit line contract in the laissez-faire equilibrium.

Proposition 3. *The credit line contract in the laissez-faire regime sets $B^U = Y$. Moreover, depending on the parameters, the solution is characterized by one of the two following cases:*

1. ***Interior solution:*** *$f^U < Y - 1$ and E^U are chosen to equalize marginal benefit to marginal*

cost of E, that is,

$$\underbrace{\frac{\partial V}{\partial E}}_{\text{Marginal benefit of } E} = \underbrace{-\frac{\partial V}{\partial f}\frac{df}{dE}\bigg|_{\text{(PC)}}}_{\text{Marginal cost of } E} \tag{UR}$$

2. ***Corner solution:*** *$f^U = Y - 1$ and E^U is pinned down by constraint (PC).*

First, the interest rate of the credit line is set to exhaust pledgeable income; i.e., $B^U = Y$. Recall that fees compensate the bank for the incurred losses when credit lines are drawn down, as financing a drawdown costs R and the bank only receives a payment $B \leq Y < R$. For the same expected payment in Region I, increasing B allows the bank to decrease f; see constraint (PC). However, this increase in B, mainly paid in high liquidity need states, more than offsets the reduction in f for high realizations of α. Such an adjustment in the price of the credit line permits the bank to increase lending in Region II, making the firm better off by improving the insurance of the credit line. Note that despite the higher interest rate, firms benefit from more lending because the payoff of pursuing the project is higher than the liquidation payoff; $X - Y > Q(0)$ due to Assumption 1 and 2. Note as well that in the process of increasing B, the incentive compatibility constraint (IC) becomes less restrictive.

Second, the equilibrium contract considers the trade-off behind the choice of pre-arranged funding E. On the one hand, E helps to sustain lending in high liquidity need states, improving the insurance of the credit line. However, increasing E involves an extra cost, requiring a higher fee f to compensate initial investors; see constraint (PC). Consequently, the representative bank chooses f and E to equalize marginal benefit to the marginal cost of E; see Proposition 3. Note that if high liquidity need states are rare, high levels of pre-arranged funding E are not desirable, as it only increases the credit line's cost without having any major impact on insurance. In such a case, credit lines do not provide full insurance to firms against liquidity shocks when unlikely high realizations of α happen.

Furthermore, the incentive compatibility constraint (IC) can limit the insurance of the credit line. Recall that, in the absence of the liquidity shock, firms cannot be charged very high fees; otherwise, funds are diverted from credit lines into their private storage technology. Although it may be desirable to improve the credit line insurance by increasing E up to the indifference point, constraint (IC) may prevent this from happening, as it limits the maximum fee that can be charged and, thus, the amount of pre-arranged funding. In such a situation, the remaining contractual terms, f^U and E^U, are pinned down by constraints (PC) and (IC).

Note that if loans were granted without incurring any loss ($B \geq R$), neither a fee nor pre-arranged funding would be needed. In such a case, the bank would be able to meet any demand for drawdowns by borrowing at $t = 1$: $B\alpha \geq R\alpha$.[16] Thus, no liquidation would ever occur. However, Assumption 2 rules out this situation and justifies the value of credit lines.

4 Social welfare analysis

This section sets a social planner's problem to determine whether the laissez-faire equilibrium is constrained-efficient. Later, the comparative statics of the social planner's solution is presented. Finally, the implementation of the constrained-efficient allocation by means of regulatory requirements is discussed.

4.1 The social planner's problem

Consider a social planner that maximizes the expected payoff V of the representative firm. This social planner chooses contractual terms (B, f, E) subject to the same constraints as the representative bank in the laissez-faire regime, that is, constraints (PC) and (IC). However, when choosing (B, f, E), the planner internalizes the effect on aggregate liquidations $z(\alpha)$ and, consequently, on liquidation value $Q(z(\alpha))$. Because initial and new investors break even, firms are the only relevant class of agents with a nontrivial stake in social welfare.

[16] Such a situation will be equivalent to borrowing in a spot credit market at $t = 1$; i.e., firms needing cash will request a loan at $t = 1$ without needing to arrange liquidity in advance.

Formally, the social planner's problem consists in maximizing V, that is,

$$\max_{B,f,E} V\big(B, f, E\big),$$

where function V has been defined in the preceding section, subject to the participation constraint (PC) of initial investors, the incentive compatibility constraint (IC), and aggregate liquidations z that is computed as

$$z = \alpha - L \text{ for all } \alpha,$$

where L is defined in Proposition 2.

Let B^*, f^*, and E^* be the payment scheme and the amount of pre-arranged funding that solve the social planner's problem. The next proposition characterizes the *constrained efficient credit line contract* (B^*, f^*, E^*).

Proposition 4. *The constrained efficient credit line contract sets $B^* = Y$. Moreover, depending on the parameters, the solution is characterized by one of the two following cases:*

1. ***Interior solution:*** *$f^* < Y - 1$ and E^* are chosen to equalize marginal social benefit to the marginal social cost of E, that is,*

$$\underbrace{\frac{\partial V}{\partial E}}_{\substack{\textit{Marginal}\\ \textit{private benefit}}} + \underbrace{\frac{\partial V}{\partial z}\frac{\partial z}{\partial E} + \frac{\partial V}{\partial z}\frac{\partial z}{\partial f}\frac{df}{dE}}_{\textit{Pecuniary externality}}\Bigg|_{(\mathrm{PC})} = \underbrace{\frac{\partial V}{\partial f}\frac{df}{dE}\Bigg|_{(\mathrm{PC})}}_{\substack{\textit{Marginal}\\ \textit{private cost}}} \tag{SP}$$

2. ***Corner solution:*** *$f^* = Y - 1$ and E^* is pinned down by constraint (PC).*

As in the laissez-faire regime, payment B exhausts pledgeable income Y. This is because B can be increased and f reduced such that the expected payment in region I is unaltered, but such variation helps to increase lending and reduce costly liquidations in region II.

When choosing f and E, the planner considers the effect of its choice on liquidations. Consequently, if feasible, f and E are chosen so that the marginal social benefit of E equals its marginal social cost; case 1 in Proposition 4. However, the incentive compatibility constraint (IC) may prevent this from occurring, as a very high f cannot be set to finance a higher pre-arranged funding E; see constraint (PC). In such a situation, the solution is determined by constraints (PC) and (IC); case 2 in Proposition 4.

By comparing the conditions for an interior solution in Proposition 3 and Proposition 4, it can be appreciated that the sole difference between them is given by the second term at the left-hand side of condition (SP). This term represents the *pecuniary externality*, and it arises because, as opposed to the representative bank, the planer internalizes the effect of the contractual terms on aggregate liquidations and, consequently, liquidation values. Moreover, the sign of the externality is positive, as pre-arranged funding E helps sustain lending and, thus, reduces aggregate liquidations and ameliorates liquidation values. Additionally, the increase in f used to finance a higher E also helps reduce liquidations as banks can leverage on a higher fee.

Due to the positive pecuniary externality, it is socially desirable to increase pre-arranged funding E relative to the choice of the representative bank in the laissez-faire equilibrium. Yet, increasing E requires to increase commitment fee f. Thus, whether the planner can select $E > E^U$ will depend on the incentive compatibility constraint (IC).

Define the *welfare gain* of implementing the solution of the social planner as

$$\Delta = V(B^*, f^*, E^*) - V(B^U, f^U, E^U),$$

that is the difference between the social welfare achieved in the constrained planner's solution and the social welfare achieved in the laissez-faire equilibrium. The next proposition states under which circumstances such gains are positive.

Proposition 5.

1. *If an interior solution characterizes the laissez-faire contract, welfare gains exist.*

2. *If a corner solution characterizes the laissez-faire contract, no welfare gains exist.*

It is important to recall that the credit line contract in the laissez-faire equilibrium and the one chosen by the planner set payment B equal to pledgeable income Y. However, the contracts may differ in their choice of pre-arranged funding E and, thus, the fee f required to pay for them. As pointed out, due to the presence of the positive pecuniary externality, the firm could benefit from a line with more insurance (higher pre-arranged funding E). However, increasing E requires increasing fee f, which makes the incentive compatibility constraint (IC) less likely to hold with slack. In case 1 of Proposition 5, the planner can increase E compared to the laissez-faire equilibrium because increasing fee f is still feasible; i.e., $f^U < Y - 1$. Hence, the planner chooses pre-arranged funding $E^* > E^U$, that is, the laissez-faire credit line contract is constrained-inefficient as it does not provide enough insurance. However, if fee f^U in the laissez-faire equilibrium is such that $f^U = Y - 1$, the planner will not be able to increase E and make the credit line incentive-compatible at the same time, in spite of the marginal social benefit of E being higher than its marginal social cost. In such a situation, no welfare gain can be achieved by the planner, and the laissez-faire credit line contract is constrained-efficient. Note that in this case, the contractual terms of the credit line chosen by the social planner are pinned down by the same constraints, (PC) and (IC), as in the laissez-faire equilibrium; hence, $f^U = f^*$ and $E^U = E^*$.

4.2 Comparative statics

This subsection discusses the properties of the constrained-efficient credit line. To that purpose, it is assumed that α is beta-distributed with parameters $a = 1$ and $b \geq 1$, that is,

$$\alpha \sim \text{Beta}(1, b), b \geq 1.$$

Such probability density function has support over the range $[0, 1]$ and is decreasing in α. Moreover, as b increases, high realizations of α are less likely to occur. That is to say, extremely high liquidity need states are rare events. Note that the uniform case, in which any realization of α is equally likely, can be obtained as a special case when $b = 1$.

Furthermore, it is assumed that the liquidation value function $Q(z)$ satisfies

$$Q(z) = Q_0(1 - \gamma_0 z^{\gamma_1}),$$

where $Q_0 > 0$, $\gamma_0 > 0$ and $\gamma_1 \geq 1$, that is, $Q(z)$ is decreasing and concave. It is important to remark that γ_0 measures the effect a liquidation has on other firms' liquidation value. For instance, if $\gamma_0 = 0$, no liquidation would depress firms' liquidation values.

Table 1 summarizes the comparative statics of the choice of pre-arranged funding E made by the social planner. The results are derived in Appendix B. The table shows the signs of the derivatives $dE^*/d\theta^k$ with respect to a parameter denoted generically by θ^k.

	θ^k							
	R	δ	γ_0	γ_1	Q_0	Y	X	b
$\frac{dE^*}{d\theta^k}$	$+$	$-$	$+$	$-$	$-$	$-$	$+$	$-$

Table 1: Comparative statics of pre-arranged funding E (in an interior equilibrium)

The cost of facing liquidations is represented by parameters X, Q_0, γ_0, γ_1. For instance, more value is lost after a liquidation if continuation cash flow X is higher, the intercept of the liquidation value function Q_0 is lower, or the slope of the liquidation value function is steeper; i.e., higher γ_0 or lower γ_1. In such situations, it is desirable to increase E to reduce the negative effect of liquidations. On the other hand, the cost of ex-post bank borrowing, denoted by R, positively impacts E. Note that fewer loans can be granted if borrowing at $t = 1$ becomes more expensive, making it harder for the bank to accommodate a particular demand for drawdowns; hence, the need for more pre-arranged funds. Moreover, a higher pledgeable income Y reduces the need for pre-arranged funding. Recall that if $Y = R$, drawdowns do not impose a cost on the bank. In such a situation, demand for drawdowns is always met, even when $E = 0$. In addition, if the additional cost of raising pre-arranged funding, denoted by δ, increases, a lower E is chosen, as it makes the contract more expensive. Finally, an increase in the shape parameter b of the Beta pdf. descreases E. In particular, high realizations of α

become rare; thus, choosing costly high levels of E to secure lending to firms in such states becomes less attractive.

Furthermore, recall that the commitment fee f cannot be set excessively high; otherwise, the incentive compatibility constraint (IC) may not be satisfied. In such a case, the credit line will be characterized by a corner solution (see Proposition 4). Consequently, Table 1 also provides information about when the equilibrium contract will be characterized by a corner solution. For instance, if liquidations are very costly (e.g., high X), a high E is optimally chosen. Simultaneously, the commitment fee f must be increased to finance E, thereby making it less likely that the incentive compatibility constraint (IC) holds with slack. Similarly, a high R, γ_0, or X, or a low Q_0, γ_1, Y, or b make it more likely that the contract will be characterized by a corner solution.

4.3 Implementation

This section focuses on case 1 of Proposition 5, in which welfare gains of implementing the social planer's solution exist.

Consider a bank regulator that sets a minimum requirement $\underline{E}$ on pre-arranged funding used to partly finance credit line drawdowns, that is,

$$\underline{E} \leq E. \tag{LR}$$

Thus, to grant credit lines to firms, banks must comply with this regulatory requirement. As shown, banks in the laissez-faire equilibrium choose insufficient pre-arranged funding. Hence, a regulation that requires banks to maintain a minimum amount of them, such as regulatory requirement (LR), directly addresses the situation.

With the introduction of regulatory requirement (LR), the representative bank chooses (B, f, E), taking aggregate liquidations as given, to maximize the expected payoff of the representative firm subject to the same constraints as in the laissez-faire regime, but with the addition of constraint (LR).

As before, for a full characterization of the contract, equilibrium conditions are required. The next definition enumerates the conditions needed for the construction of a symmetric equilibrium when the new regulation is introduced.

Definition 2. A *symmetric regulated equilibrium* consists of a choice (B^R, f^R, E^R) for the representative bank and aggregate liquidations $z^R(\alpha)$ such that

1. Given $z^R(\alpha)$, (B^R, f^R, E^R) solves the bank's optimization problem, that is,

$$\max_{B,f,E} V(B, f, E)$$

   subject to the participation constraint (PC) of initial investors, an incentive-compatible payment scheme (B, f), and compliance with the regulatory requirement (LR).

2. Given (B^R, f^R, E^R), aggregate liquidations are computed as $z^R(\alpha) = \alpha - L$ for all α, where L is defined as in Proposition 2.

The *optimal regulation* of credit lines is designed by picking the regulatory requirement $\underline{E}$ that attains the social welfare achieved by the social planner. The next proposition defines the optimal regulatory requirement $\underline{E}$ chosen by the bank regulator.

Proposition 6. *If $\underline{E} = E^*$, constrained efficiency is restored.*

Proposition 6 states that a regulation requiring banks to maintain at least E^* units of pre-arranged funding per committed lending implements the planner's solution. As in the laissez-faire regime, the representative bank chooses an interest rate for drawdowns that consumes pledgeable income. Consider the case in which the representative bank chooses E^*; thus, $f = f^*$ in order to satisfy constraint (PC). This choice will constitute an equilibrium if the representative firm's expected payoff V cannot be improved. On the one hand, compliance with the regulation does not permit the bank to choose $E < E^*$. On the other hand, due to the positive pecuniary externality, the marginal private benefit of E is lower than its marginal private cost at $E = E^*$; see condition (SP). Hence, from the bank's perspective, increasing E worsens the representative firm's expected payoff V. As a consequence, when a regulatory requirement $\underline{E} = E^*$ is introduced, the representative bank choosing $E = E^*$ constitutes an equilibrium. Moreover, the regulated credit line coincides with the constrained-efficient contract; i.e., $B = Y$, $E = E^*, f = f^*$.[17]

The pecuniary externality on liquidation values provides a rationale for regulating credit lines. This externality arises because competitive banks do not internalize the effect of their funding decisions on liquidations, leading to insufficient insurance against the liquidity shock in high liquidity need states. Thus, too many firms in need of liquidity are liquidated, depressing other firms' liquidation values.

For a better understanding of the role of the pecuniary externality, assume the following functional form for $Q(z)$,

$$Q(z) = Q_0(1 - \gamma_0 z^{\gamma_1}).$$

Precisely, γ_0 measures the effect a liquidation has on other firms' liquidation value. For instance, if $\gamma_0 = 0$, a firm being liquidated does not affect the liquidation values of other firms. For a certain parametrization of the model, Figure 2 depicts how welfare V and pre-arranged funding E vary with respect γ_0 for the laissez-faire equilibrium and the equilibrium with an optimal regulatory requirement $\underline{E} = E^*$.[18] As it can be appreciated, when $\gamma_0 = 0$, pre-arranged funding E coincides in both circumstances; see panel (A). Thus, welfare under both regimes coincide as well; see panel (B). However, as γ_0 increases, making the pecuniary externality more relevant, the representative bank in the laissez-faire equilibrium chooses sub-optimal levels of pre-arranged funding compared to the planner's choice and, hence, the representative bank in the equilibrium with an optimal regulation; see panel (A). Thus, the welfare attained in the absence of the regulation is lower, as a higher pre-arranged funding E can improve welfare by helping to abate the deleterious effect that liquidations have on liquidation values. Corollary 1 formally states this finding.

[17]If the planner's choice (B, f, E) is characterized by a corner solution, E cannot be increased without violating constraint (IC) even though the marginal social benefit of E is higher than its marginal social cost. In this scenario, only one choice of E will be feasible for the representative bank.

[18]For the sole purpose of illustrating the solution of the model, the following parametrization was chosen $X = 2.1$, $Y = 1.01$, $R = 1.02$, $\delta = 0.02$, $Q_0 = 0.86$, $\gamma_1 = 1.25$, $g \sim \text{Beta}(1, b = 9)$.

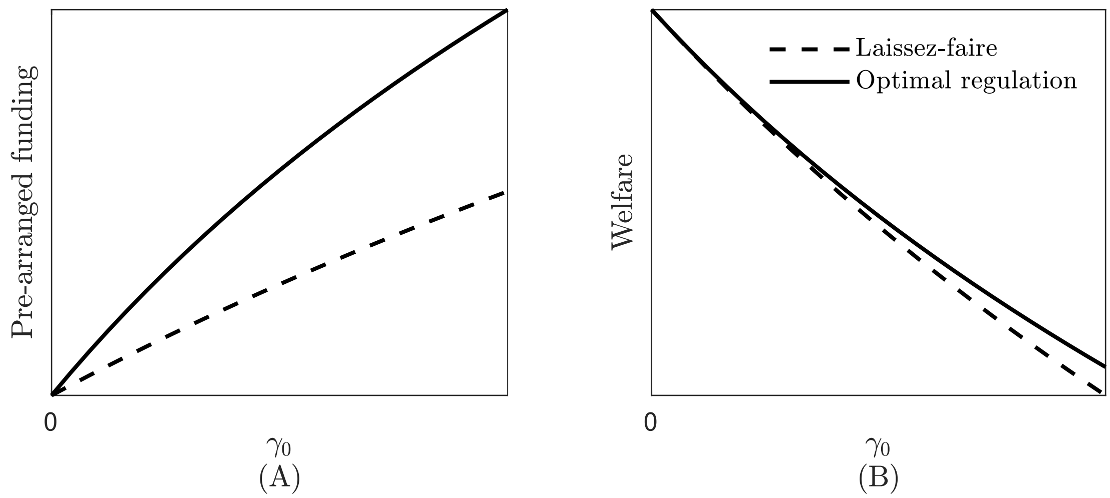

Figure 2: Effect of γ_0 on welfare and pre-arranged funding for each regime

In the elaboration of the figure, the following functional form for the firms' liquidation values was chosen: $Q(z) = Q_0(1 - \gamma_0 z^{\gamma_1})$, where γ_0 measures the effect a liquidation has on other firms' liquidation value. For instance, if $\gamma_0 = 0$, a firm being liquidated does not affect the liquidation values of other firms. Furthermore, for the sole characterization of the solution, the following parametrization was chosen $X = 2.1$, $Y = 1.01$, $R = 1.02$, $\delta = 0.02$, $Q_0 = 0.86$, $\gamma_1 = 1.25$, $g \sim \text{Beta}(1, b = 9)$.

Corollary 1. *If firm liquidations do not to depress firm liquidation values $Q(z)$, $Q_z(z) = 0 \quad \forall z$, credit lines do not need an special regulation.*

4.4 Discussion

The regulatory requirement previously discussed can be matched to Basel III liquidity ratios. For instance, the regulatory requirement in the model can be interpreted as the Basel III Liquidity Coverage Ratio (LCR). Recall that in the model, banks raise pre-arranged funding E at date 0 to store them as cash. Later, these funds are used to partly meet credit line drawdowns at date 1. Thus, regulatory requirement $\underline{E}$ requires banks to hold minimum cash levels to meet loan obligations arising from credit lines.

Similarly, the regulatory requirement can be interpreted as the Basel III Net Stable Funding Ratio (NSFR). Recall that pre-arranged funding E is junior to funding raised at date 1. In particular, banks can dilute initial investors in order to obtain additional funds at $t = 1$ to meet credit line drawdowns.

Thus, in high liquidity need states, losses that arise from an increase in credit line usage are borne by providers of pre-arranged funding. Hence, pre-arranged funding in the model acts as equity, considered a stable funding source in the computation of the NSFR. In particular, the regulatory requirement in the model demands banks to maintain a minimum *required stable funding* (RSF) to finance their loan commitments: a fraction $\underline{E}$ of bank loan commitments has to be financed with stable funding (e.g., equity or long-term debt).

Finally, the optimal regulation can also be implemented by means of a capital requirement for undrawn credit lines. Consider a capital requirement κ for risk-weighted assets (RWA). To compute RWA in the standardized approach for credit risk, off-balance sheet items (OBS) are converted into an on-balance-sheet equivalent by multiplying the OBS by a credit conversion factor (CCF). Thus, assuming a risk weight of 100%, banks are required to maintain a minimum bank capital of $\kappa \times \text{CCF}$ per unit of OBS. Hence, the CCF can be tuned up such that

$$\underline{E} = \kappa \times \text{CCF}^* \Longleftrightarrow \text{CCF}^* = \frac{\underline{E}}{\kappa}.$$

Note that providers of pre-arranged funding E in the model can be equivalently seen as bank equity holders, as they have a residual claim on bank date-2 revenue. Thus, by optimally setting $\text{CCF} = \text{CCF}^*$, banks are required to hold a minimum capital $\underline{E}$ per every unused unit of funds available in credit lines.

The paper rationalizes a product-specific regulation for credit lines. As discussed, such regulation is currently in place as part of the post-crisis reforms initiated with Basel III. However, it does not inform how liquidity requirements on undrawn credit lines can be adopted according to different economic fundamentals. The comparative statics developed in this section can help in this regard. Moreover, one crucial lesson from this simple framework is that excessive requirements of pre-arranged funding could potentially decrease welfare relative to the laissez-faire regime, as shown in Figure 3. Specifically, increasing E can be translated into credit lines with excessive insurance, making them only costlier without having a significant impact on reducing costly liquidations. In the model, such a

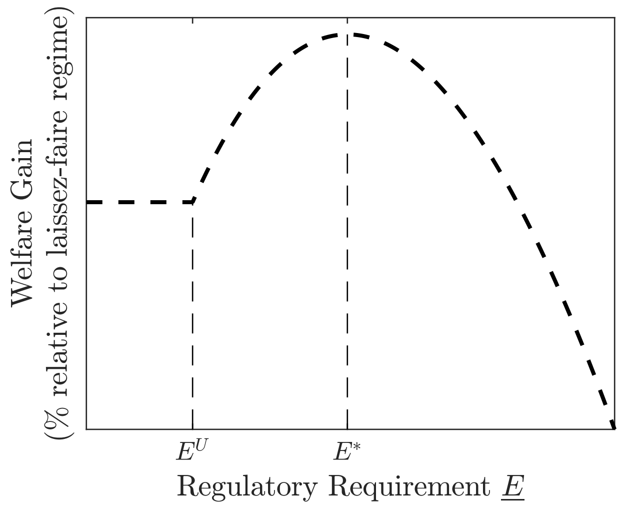

Figure 3: Effect of a regulatory requirement $\underline{E}$ on welfare

In the elaboration of the figure, the following functional form for the firms' liquidation values was chosen: $Q(z) = Q_0(1 - \gamma_0 z^{\gamma_1})$. Moreover, for the sole characterization of the solution, the following parametrization was chosen $X = 2.1$, $Y = 1.01$, $R = 1.02$, $\delta = 0.02$, $Q_0 = 0.86$, $\gamma_0 = 3$, $\gamma_1 = 1.25$, $g \sim \text{Beta}(1, b = 9)$.

situation is described as a very high E such that its marginal social cost more than offsets its marginal social benefit. Contrary, a low requirement of pre-arranged funding may not impact welfare, as such a requirement may not be binding on equilibrium; $\underline{E} < E^U$ in Figure 3.

5 A secondary market for specialized assets

Consider a firm with specialized assets a that return at $t = 2$ a cash flow $X = A \times a$ with probability p and 0, otherwise. Moreover, senior debt holders demand a payment $D < X$. In addition, firms may require a unit of funds at $t = 1$ to prevent bankruptcy, in which case its assets become obsolete.[19] As in the baseline model, the firms' demand for liquidity α distributes according to $g(\cdot)$.

After liquidity risk materializes, the manager chooses whether the firm is run in a diligent or negligent mode. Under the diligent mode, the cash flow in case of success X happens with probability $p = p^H$, whereas under the negligent mode, it happens with probability $p = p^L < p^H$. Moreover,

[19] We could think of a situation in which many uninformed investors have senior claims on the firm's assets, making an orderly liquidation process cumbersome and time-consuming.

the manager enjoys net private benefits $\Gamma > 0$ when running the firm under the negligent mode. It is assumed that meeting the liquidity shock has a positive NPV whenever the firm is run under the diligent mode, but not when negligently run, that is,

$$p^L X + \Gamma - R < 0 < X - R,$$

where it has been assumed, without loss of generality and for the simplicity of notation, that the project run under the diligent mode is safe; i.e., $p^H = 1$. To induce the choice of the diligent mode, the entrepreneur must keep at least

$$X - \frac{\Gamma}{1 - p^L}$$

from the cash flow in case of success. This implies that

$$Y = X - \frac{\Gamma}{1 - p^L} - D$$

is only available for new financiers after considering payment D to senior investors.

If the firm does not get funds to meet the liquidity shock, the firm is forced to sell part of its specialized assets at a price p in a secondary market. The secondary market is populated by a mass S of entrepreneurs with heterogeneous abilities at managing 1 unit of specialized assets: an entrepreneur i gets a payoff B_i per unit of a, where B_i is described by density function $h(\cdot)$ over the support $[\underline{B}, \overline{B}]$, and $\underline{B} \geq \frac{1}{a}$ and $\overline{B} \leq A$. Thus, to obtain a unit of funds, a firm must sell $\frac{1}{p}$ units of its assets; hence, its payoff at $t = 2$ after fire-selling assets at $t = 1$ is equal to

$$Q = A \times \big(a - \frac{1}{p}\big).$$

As appreciated, the payoff Q depends on p; the lower the p, the more the assets needed to be sold. The next proposition states how price p is determined in the secondary market.

Proposition 7. *The equilibrium price in the secondary market for specialized assets satisfies*

$$z\frac{1}{p} = (1 - H(p))S, \tag{MC}$$

where z is the number of firms seeking liquidity, $H(\cdot)$ is the c.d.f of $h(\cdot)$, and S is the total mass of entrepreneurs in the secondary market. Moreover, the price of specialized assets decays with z if the price elasticity of demand ε satisfies $|\varepsilon| > 1$.

Given a price p and a mass z of firms requiring a unit of funds, a certain mass of entrepreneurs is needed to manage specialized assets $z\frac{1}{p}$. In addition, at such a price, only entrepreneurs with abilities $B \geq p$ will demand specialized assets. Thus, the demand for specialized assets in the secondary market is represented by the right-hand side of condition (MC). Consequently, p depends on the number of firms seeking liquidity in the secondary market. Moreover, if the demand for specialized assets is elastic, $|\varepsilon| > 1$, the price of specialized assets is decreasing in z, making payoff Q decreasing in z as well. As in Shleifer and Vishny (1992), the immediate need for liquidity does not necessarily transfer assets to the best users, pricing them below their value in best use.

The previous structure delivers the reduced-form payoffs of firms in the baseline model. Furthermore, the welfare analysis in section 4 remains valid when specialized assets are traded in international markets, as second-best users of specialized assets will have zero weight in the welfare function of a domestic social planner.

However, if specialized assets are traded domestically, the planner's objective function must consider the output delivered by the second-best users of specialized assets. In such a case, the planner's objective becomes

$$V(B, f, E) + \int_{\underline{\alpha}}^{1} \Big(\int_{p}^{\overline{B}} Bh(B)\mathrm{d}B \Big) g(\alpha)\mathrm{d}\alpha,$$

whose second term measures the output delivered by the second-best users.

The next proposition revisits the welfare analysis for the latter situation.

Proposition 8. *If specialized assets are traded domestically, welfare losses due to fire-sale externalities* *become smaller. Furthermore, if $h(\cdot)$ is a degenerate distribution with a point mass at $\overline{B} = A$, welfare losses vanish.*

Proposition 8 shows that welfare losses arise due to a transfer of specialized assets from more productive (firms) to less productive agents (entrepreneurs). In fact, if entrepreneurs in the secondary market were as productive as firms at managing their specialized assets, fire-selling assets would not cause welfare losses. In such a scenario, the minimum regulatory requirement of pre-arranged funding would be unnecessary, as the laissez-faire equilibrium would be constrained-efficient.

6 Concluding remarks

This paper presents a simple model of credit lines in which a rationale for regulating them is provided. In the model, pre-arranged junior funding (stable funding, such as equity) helps banks sustain lending via credit lines, especially when firms' liquidity needs are highly correlated. However, because pre-arranged funding is costly, the bank will not be able to meet all credit line drawdowns in very high liquidity need states, reneging on some credit lines and causing the liquidation of firms left without credit. If such liquidations depress firms' liquidation values (i.e., a pecuniary externality), there exists a scope for regulatory intervention. Specifically, in the laissez-faire equilibrium, competitive banks hold insufficient stable funding compared to a constrained social planner because they do not internalize the effect of their funding decisions on firms' liquidation values. Consequently, laissez-faire credit lines do not provide enough insurance against liquidity risk in high liquidity need states, causing the liquidation of too many firms and depressing liquidation values. This pecuniary externality on firms' liquidation values justifies the regulation of credit lines.

The paper shows that a minimum requirement on pre-arranged funding can restore constrained efficiency. Such a requirement resembles Basel III liquidity ratios. For instance, a liquidity requirement that links pre-funded cash reserves to committed loans, such as the LCR, can implement the constrained efficient allocation. Similarly, requiring banks to finance (future) drawdowns with minimum stable funding, as in the NSFR, can also restore constrained efficiency. However, very conservative requirements can make credit lines excessively costly as stable funding demands an additional return, thus, offsetting the benefit of reducing firms' liquidations in high liquidity need states. The model also indicates that the optimal requirement should be higher when the premium on pre-arranged funding is lower, high liquidity need states are more frequent, liquidations are costlier, or firms' liquidation values are very sensitive to each firm liquidation. Furthermore, the model can serve as a building block for future research regarding credit lines. For instance, how runnable credit lines and runnable deposits interact or how an increase in credit line drawdowns (thus, a reduction in bank liquidity) can reduce other bank investments (e.g., lending to other firms).

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A Proofs

Proof of Proposition 1

Let w be the probability of obtaining a loan at $t = 1$ from the credit line. Firms without a need of cash will not request funding if

$$w(X - B + 1) + (1 - w)(X - f) \leq X - f \longleftrightarrow f \leq Y - 1.$$

Consider a firm needing cash. Such a firm will always request funds to pursue its project, as the payoff of pursuing it is higher than its liquidation value or the payoff of diverting funds, that is,

$$\max\{Q(z), 1\} \leq X - B,$$

due to Assumption 1 and Assumption 3.

Thus, the demand for drawdowns is equal to α.

Proof of Proposition 2

Consider an incentive-compatible payment scheme. Thus, the bank faces a demand for drawdowns α, which is served in a sequential random order until it is fully met or the bank's borrowing capacity is exhausted, whichever happens first.

The bank leverages date-2 revenues to raise additional funding $\alpha - E$. However, revenues are decreasing in α, as $B \leq Y < R$. Let $\alpha = \underline{\alpha}$ be the highest demand for drawdowns that the bank can fully meet, which satisfies

$$\underline{\alpha}B + (1 - \underline{\alpha})f = R(\underline{\alpha} - E) \longleftrightarrow \underline{\alpha} = \frac{f + RE}{R - B + f}.$$

At $\alpha = \underline{\alpha}$, the bank promises its entire revenue (right-hand side term) to new investors to pay for their gross required rate for funds $D = \underline{\alpha} - E$ (left-hand side term). Because the revenue is decreasing in α, the bank will not fully accommodate demand for drawdowns α whenever $\alpha > \underline{\alpha}$. In such a situation, the bank concedes loan requests from credit lines until its borrowing capacity is exhausted, that is,

$$LB + (1 - \alpha)f = R(L - E) \longleftrightarrow L = \frac{RE + (1 - \alpha)f}{R - B} < \alpha \quad \text{for } \alpha > \underline{\alpha}.$$

Proof of Proposition 3

(1) $\boldsymbol{B^U = Y}$. Suppose $B < Y$. Thus, B can be increased and f appropriately reduced such that constraint (PC) is still satisfied. To do that, we apply the implicit function theorem to condition (PC) to get

$$\frac{\mathrm{d}f}{\mathrm{d}B} = -\frac{\mathbb{E}\big[\alpha|\alpha \in [0,\underline{\alpha}]\big]}{1-\mathbb{E}\big[\alpha|\alpha \in [0,\underline{\alpha}]\big]} \equiv -\Delta.$$

To prove $B^U = Y$, we will show that the next expression is positive

$$\frac{\partial V}{\partial B} - \Delta\frac{\partial V}{\partial f} = \int_0^{\underline{\alpha}} \left(\frac{\partial V_{\mathrm{I}}}{\partial B} - \Delta\frac{\partial V_{\mathrm{I}}}{\partial f}\right) g(\alpha)\mathrm{d}\alpha + \big(V_{\mathrm{I}}(\underline{\alpha}) - V_{\mathrm{II}}(\underline{\alpha})\big)g(\underline{\alpha})\left(\frac{\partial \underline{\alpha}}{\partial B} - \Delta\frac{\partial \underline{\alpha}}{\partial f}\right) + \int_{\underline{\alpha}}^1 \left(\frac{\partial V_{\mathrm{II}}}{\partial B} - \Delta\frac{\partial V_{\mathrm{II}}}{\partial f}\right) g(\alpha)\mathrm{d}\alpha, \tag{A.1}$$

that is, the contract can be improved by increasing B and appropriately reducing f.

First, it should be noted that B and f are changed such that the expected payment to the bank in the first region is not altered; hence, the first term in the previous expression is zero. Furthermore, due to the continuity of the payoff function at $\alpha = \underline{\alpha}$, $V_{\mathrm{I}}(\underline{\alpha}) = V_{\mathrm{II}}(\underline{\alpha})$, the second term is also zero.

Because the whole revenue is used to raise ex-post funding, V_{II} can be rewritten as

$$V_{\mathrm{II}} = X - \Big(\alpha\big(X-Q\big) - L\big(X-Q-R\big) - RE\Big).$$

To simplify the exposition, the argument α in functions Q and L has been suppressed. Hence, using the definition of L, the third term in (A.1) is equal to

$$\int_{\underline{\alpha}}^1 \left(\frac{\partial V_{\mathrm{II}}}{\partial B} - \Delta\frac{\partial V_{\mathrm{II}}}{\partial f}\right) g(\alpha)\mathrm{d}\alpha = \int_{\underline{\alpha}}^1 \Big((1-\alpha)\big(f - \Delta(R-B)\big) + RE\Big)\frac{X-Q-R}{(R-B)^2} g(\alpha)\mathrm{d}\alpha.$$

Due to constraint (PC), $f - \Delta(R-B) > 0$. Hence, the entire expression is positive; i.e., it is optimally to increase B and appropriately reduce f until $B^U = Y$.

(2) **Optimal choice of $\boldsymbol{E}$ and $\boldsymbol{f}$**. The optimization problem of the representative bank consists of maximizing

$$\max_{f,E} \int_0^{\underline{\alpha}} \Big(X - (1-\alpha)f - \alpha Y\Big) g(\alpha)\mathrm{d}\alpha + \int_{\underline{\alpha}}^1 \Big(X - (\alpha - L)\big(X-Q\big) - R(L-E)\Big) g(\alpha)\mathrm{d}\alpha$$

subject to the following constraints

$$(R+\delta)E = \int_0^{\underline{\alpha}} \big(\alpha Y + (1-\alpha)f - R(\alpha - E)\big) g(\alpha)\mathrm{d}\alpha, \tag{PC}$$

$$f \leq Y - 1. \tag{IC}$$

Define the lagrangian L for the constrained optimization problem as

$$L(f, E; \theta_1, \theta_2) = V(f, E) + \theta_1 \bigg(\int_0^{\underline{\alpha}} \big(\alpha Y + (1-\alpha)f - R(\alpha - E)\big) g(\alpha)\mathrm{d}\alpha - (R+\delta)E \bigg) + \theta_2 (Y - 1 - f)$$

and whose first-order conditions are

$$\begin{aligned}
\{f\} :& \frac{\partial V}{\partial f} + \theta_1 \int_0^{\underline{\alpha}} (1-\alpha) g(\alpha)\mathrm{d}\alpha - \theta_2 = 0, \\
\{E\} :& \frac{\partial V}{\partial E} - \theta_1 \big(R(1 - G(\underline{\alpha}) + \delta)\big) = 0, \\
\{\theta_1\} :& \int_0^{\underline{\alpha}} \big(\alpha Y + (1-\alpha)f - R(\alpha - E)\big) g(\alpha)\mathrm{d}\alpha - (R+\delta)E = 0, \\
\{\theta_2\} :& (Y - 1 - f)\theta_2 = 0, \text{where } \theta_2 \geq 0 \text{ and } Y - 1 - f \geq 0,
\end{aligned}$$

where θ_1 and θ_2 are the lagrange multipliers of constraints (PC) and (IC), respectively, and $G(\cdot)$ is the cdf of α.

By combining the first order conditions of E and f, we obtain

$$\frac{\partial V}{\partial E} = -\frac{\partial V}{\partial f} \frac{\mathrm{d}f}{\mathrm{d}E}\bigg|_{(\mathrm{PC})} + \lambda^U \tag{A.2}$$

where $\frac{\partial V}{\partial f}$, $\frac{\partial V}{\partial E}$, and $\frac{\mathrm{d}f}{\mathrm{d}E}$ are equal to

$$\begin{aligned}
\frac{\partial V}{\partial f} &= -\int_0^{\underline{\alpha}} (1-\alpha) g(\alpha)\mathrm{d}\alpha + \int_{\underline{\alpha}}^1 \frac{X - Q - R}{R - Y} (1-\alpha) g(\alpha)\mathrm{d}\alpha, \\
\frac{\partial V}{\partial E} &= R \int_{\underline{\alpha}}^1 \frac{X - Q - Y}{R - Y} g(\alpha)\mathrm{d}\alpha, \\
\frac{\mathrm{d}f}{\mathrm{d}E} &= \frac{R(1 - G(\underline{\alpha})) + \delta}{\int_0^{\underline{\alpha}} (1-\alpha) g(\alpha)\mathrm{d}\alpha}
\end{aligned}$$

respectively, and $\lambda^U = \theta_2 \frac{\mathrm{d}f}{\mathrm{d}E}$.

Moreover, due to equilibrium, aggregate liquidations are computed as

$$z(\alpha) = \begin{cases} 0, & \alpha \leq \underline{\alpha} \\ \alpha - L, & \alpha > \underline{\alpha}. \end{cases}$$

Thus, given the equilibrium aggregate liquidation function $z(\alpha)$, equation (A.2) and the first order conditions of θ_1 and θ_2 pin down the equilibrium contractual terms f and E. First, if constraint (IC) is not binding (i.e., $\lambda^U = 0$), the solution is characterized by equations (A.2) and (PC); case 1 in Proposition 3. However, if the commitment fee f required to obtain such pre-arranged funding E violates constraint (IC), a corner solution is obtained, that is, constraints (IC) and (PC) pin down f and E; case 2 in Proposition 3.

Proof of Proposition 4

(1) $\boldsymbol{B^* = Y}$. It follows similar arguments as in the proof of Proposition 3.

(2) **Optimal choice of $\boldsymbol{E}$ and $\boldsymbol{f}$**. The optimization problem of the social planner consists of maximizing V

$$\max_{f,E} \int_0^{\underline{\alpha}} \Big(X - (1-\alpha)B_3 - \alpha Y\Big) g(\alpha) \mathrm{d}\alpha + \int_{\underline{\alpha}}^1 \Big(X - (\alpha - L)\big(X - Q(z)\big) - R(L-E)\Big) g(\alpha) \mathrm{d}\alpha$$

subject to the following constraints

$$(R+\delta)E = \int_0^{\underline{\alpha}} \big(\alpha Y + (1-\alpha) f - R(\alpha - E)\big) g(\alpha) \mathrm{d}\alpha, \tag{PC}$$

$$f \leq Y - 1, \tag{IC}$$

and aggregate liquidations, which are computed as $z = \alpha - L$.

Define the lagrangian L for the constrained optimization problem as

$$L(f, E, \vartheta_1, \vartheta_2) = V(f, E) + \vartheta_1 \Big(\int_0^{\underline{\alpha}} \big(\alpha Y + (1-\alpha) f - R(\alpha - E)\big) g(\alpha) \mathrm{d}\alpha - (R+\delta)E \Big) + \vartheta_2 (Y - 1 - f)$$

and whose first-order conditions are

$$\begin{aligned}
\{f\} :& \frac{\partial V}{\partial f} + \frac{\partial V}{\partial z}\frac{\partial z}{\partial f} + \vartheta_1 \int_0^{\underline{\alpha}} (1-\alpha) g(\alpha) \mathrm{d}\alpha - \vartheta_2 = 0, \\
\{E\} :& \frac{\partial V}{\partial E} + \frac{\partial V}{\partial z}\frac{\partial z}{\partial E} - \vartheta_1 \big(R(1 - G(\underline{\alpha})) + \delta\big) = 0, \\
\{\vartheta_1\} :& \int_0^{\underline{\alpha}} \big(\alpha Y + (1-\alpha) f - R(\alpha - E)\big) g(\alpha) \mathrm{d}\alpha - (R+\delta) E = 0, \\
\{\vartheta_2\} :& (Y - 1 - f)\vartheta_2 = 0, \text{where } \vartheta_2 \geq 0 \text{ and } Y - 1 - f \geq 0.
\end{aligned}$$

where ϑ_1 and ϑ_2 are the lagrange multipliers of constraints (PC) and (IC), respectively.

By combining the first order conditions of f and E, we obtain

$$\frac{\partial V}{\partial E} + \underbrace{\frac{\partial V}{\partial z}\frac{\partial z}{\partial E} + \frac{\partial V}{\partial z}\frac{\partial z}{\partial f}\frac{\mathrm{d}f}{\mathrm{d}E}}_{\text{Pecuniary externality } \chi}\bigg|_{\text{(PC)}} = -\frac{\partial V}{\partial f}\frac{\mathrm{d}f}{\mathrm{d}E}\bigg|_{\text{(PC)}} + \lambda^* \tag{A.3}$$

where the additional term, χ, is equal to

$$\chi = -\int_{\underline{\alpha}}^1 \frac{\alpha - L}{R - Y}\Big(R + (1-\alpha)\frac{\mathrm{d}f}{\mathrm{d}E}\Big) Q_z g(\alpha) \mathrm{d}\alpha$$

and $\lambda^* = \vartheta_2 \frac{\mathrm{d}f}{\mathrm{d}E}$. Thus, if constraint (IC) is not binding (i.e., $\lambda^* = 0$), contractual terms f and E are pinned down from equations (A.3) and (PC); case 1 in Proposition 4. Otherwise, they are pinned down from constraints (IC) and (PC); case 2 in Proposition 4.

Proof of Proposition 5

(1) Assume that $\lambda^U = 0$ in (A.2). In such a case, equations (A.2) and (PC) pin down f^U and E^U. Because the pecuniary externality χ is positive ($Q_z < 0$), it is desirable to increase E in (A.3). Because $f^U < Y - 1$, the social planner can improve welfare by increasing E above E^U.

(2) Suppose that the solution in the laissez-faire equilibrium is given by a corner solution. Hence, $f^U = Y - 1$ and E is pinned down from (PC). Because $\lambda^U > 0$, the marginal private benefit of E is higher than its marginal private cost. Hence, it is desirable to increase E, but incentive compatibility does not permit it as a higher E is financed with a higher f. Moreover, because χ is positive, the marginal social benefit of E is higher than its marginal social cost. Despite this, the planner cannot increase E above E^U, as it will require increasing f above f^U, which is unfeasible due to constraint (IC). Consequently, welfare cannot be improved by the planner in this situation.

Proof of Proposition 6

We will show that, given $\underline{E} = E^*$, the equilibrium credit line contract in the regulated equilibrium will coincide with the constrained efficient credit line contract. Therefore, such regulation implements the social planner's solution.

It can be easily proved that $B^R = Y$. Therefore, taking aggregate liquidations as given, the representative bank chooses f and E such that the representative firm's expected payoff is maximized,

$$\max_{f,E} \int_0^{\underline{\alpha}} \Big(X - (1-\alpha)f - \alpha Y\Big) g(\alpha)\mathrm{d}\alpha + \int_{\underline{\alpha}}^1 \Big(X - (\alpha - L)(X - Q) - R(L - E)\Big) g(\alpha)\mathrm{d}\alpha,$$

subject to the constraints (PC), (IC), and the regulatory requirement $E^* \leq E$.

For $E^R = E^*$ (hence, $f^R = f^*$) to be a solution, it must be the case that the representative bank cannot increase the representative firm's expected payoff by increasing E above E^*. First, assume that the social planner's solution is characterized by an interior solution, that is,

$$\frac{\partial V}{\partial E}(f^*, E^*) + \chi(f^*, E^*) = -\frac{\partial V}{\partial f}(f^*, E^*)\frac{\mathrm{d}f}{\mathrm{d}E}\bigg|_{(\mathrm{PC})}.$$

Therefore, the marginal private benefit at (f^*, E^*) must satisfy

$$\frac{\partial V}{\partial E}(f^*, E^*; z(f^*, E^*)) < -\frac{\partial V}{\partial f}(f^*, E^*; z(f^*, E^*))\frac{\mathrm{d}f}{\mathrm{d}E}\bigg|_{(\mathrm{PC})},$$

that is, increasing E will decrease the representative firm's expected payoff when aggregate liquidations are taken as given. Thus, the representative bank will not find it optimal to increase E above E^*; hence, choosing (f^*, E^*) will constitute an equilibrium. On the other hand, if the social planner's solution is characterized by a corner solution, no choice of $E \geq E^*$ is feasible without violating constraint (IC). In such a situation, the only feasible choice of E is E^*.

Proof of Corollary 1

The definition of the pecuniary externality χ is given by

$$\chi = -\int_{\underline{\alpha}}^1 \frac{\alpha - L}{R - Y}\Big(R + (1-\alpha)\frac{\mathrm{d}f}{\mathrm{d}E}\Big) Q_z g(\alpha)\mathrm{d}\alpha.$$

If $Q_z = 0\ \forall z$, the pecuniary externality is zero and conditions (A.2) and (A.3) coincide.

Proof of Proposition 7

Given a price p in the secondary market, a firm in need of cash will sell x units of its assets to get a unit of funds, that is,

$$x \times p = 1 \Longleftrightarrow x = \frac{1}{p}.$$

Hence, given a price p and a number z of firms searching for liquidity, the total units of specialized assets being supplied in the secondary market are $\frac{z}{p}$.

Given a price p, only entrepreneurs with abilities $B_i \geq p$ will demand specialized assets. Thus, the demand for specialized assets in the secondary market is equal to

$$S \int_{p}^{\overline{B}} h(B)\mathrm{d}B = S(1 - H(p)),$$

where $H(\cdot)$ is the c.d.f. of $h(\cdot)$.

Therefore, market clearing price is determined by

$$\frac{z}{p} = S(1 - H(p)). \tag{MC}$$

Moreover, by total differentiating condition (MC), the derivative of p respect z is equal to

$$\frac{\partial p}{\partial z} = \Big(\frac{1}{1+\varepsilon}\Big)\frac{p}{z},$$

where $\varepsilon = -\frac{Sh(p)p^2}{z}$ is the price elasticity of demand at the equilibrium price in the secondary market. Thus, the equilibrium price in the secondary market is decreasing in z if $|\varepsilon| > 1$.

Proof of Proposition 8

Note that for a realization of $\alpha > \underline{\alpha}$, the planner's payoff is equal to

$$V_{\mathrm{II}}^{SM} = (1-\alpha)(X - f) + \alpha\Big(\frac{L}{\alpha}(X - B) + (1 - \frac{L}{\alpha})Q\Big) + S\int_{p}^{\overline{B}} Bh(B)\mathrm{d}B,$$

where $X = A \times a$ and $Q = A \times (a - \frac{1}{p})$.

If $h(\cdot)$ is a degenerate distribution with a point mass at $\overline{B}$, then $p = \overline{B}$ and the output delivered by second-best users is equal to $\overline{B}\frac{z}{p}$. Thus, V_{II}^{SM} becomes

$$V_{\mathrm{II}}^{SM} = X - (1-\alpha)f - LB - \frac{z}{p}(A - \overline{B}),$$

where $A - \overline{B}$ measures the welfare loss of transferring specialized assets to second-best users. In such a case, if $\overline{B} = A$, then no welfare loss will exist.

If $h(\cdot)$ is not a degenerate distribution, the pecuniary externality considering the output delivered by the second-best users of specialized assets becomes

$$\chi^{SM} = -\int_{\underline{\alpha}}^{1} \frac{z}{R-Y}\Big(R + (1-\alpha)\frac{\mathrm{d}f}{\mathrm{d}E}\Big)Q_z g(\alpha)\mathrm{d}\alpha + S\int_{\underline{\alpha}}^{1} \frac{ph(p)}{R-Y}\Big(R + (1-\alpha)\frac{\mathrm{d}f}{\mathrm{d}E}\Big)\frac{\partial p}{\partial z} g(\alpha)\mathrm{d}\alpha.$$

Note that the first term coincides with the externality in the case where specialized assets are traded in international markets. Note that $Q_z = \frac{A}{p^2}\frac{\partial p}{\partial z}$. Hence, the expression for the externality when specialized assets are traded domestically becomes

$$\chi^{SM} = -\int_{\underline{\alpha}}^{1} \frac{1}{R-Y}\Big(R + (1-\alpha)\frac{\mathrm{d}f}{\mathrm{d}E}\Big)\frac{z}{p}\frac{\partial p}{\partial z}\Big(\frac{A}{p} - |\varepsilon|\Big)g(\alpha)\mathrm{d}\alpha,$$

where $\varepsilon = -Sph(p)/\frac{z}{p}$. Because $\frac{\mathrm{d}f}{\mathrm{d}E} > 0$ and $\frac{\partial p}{\partial z} < 0$, the previous expression is positive. However, such gains of decreasing costly liquidations are partially offset by the production delivered by second-best users. In the extreme case in which an entrepreneur (or many of them) with ability $B = A$ can manage all liquidated assets, the benefit disappears.

B Comparative statics

Proof

Assume that contractual terms are characterized by an interior solution. Hence, according to Proposition 4, f^* and E^* satisfy the following system of equations

$$\Psi_1(f^*,E^*;\theta) \equiv \int_{\underline{\alpha}}^{1} \frac{R(X-Q-Y)+\frac{\mathrm{d}f}{\mathrm{d}E}(X-Q-R)(1-\alpha)}{R-Y} g(\alpha)\mathrm{d}\alpha + \chi + G(\underline{\alpha})R - R_0 = 0, \quad \text{(A.4)}$$
$$\Psi_2(f^*,E^*;\theta) \equiv \int_{0}^{\underline{\alpha}} \big(\alpha Y + (1-\alpha)f^* - R(\alpha - E^*)\big) g(\alpha)\mathrm{d}\alpha - R_0 E^* = 0,$$

where $\theta = \big[X, Q_0, \gamma_0, \gamma_1, Y, R, \delta\big]'$ is a vector of parameters, $R_0 = R+\delta$, and

$$\underline{\alpha} = \frac{f^*+RE^*}{R-Y+f^*}, \frac{\mathrm{d}f}{\mathrm{d}E} = \frac{R(1-G(\underline{\alpha}))+\delta}{\int_0^{\underline{\alpha}}(1-\alpha)g(\alpha)\mathrm{d}\alpha}, \chi = -\int_{\underline{\alpha}}^{1} \frac{\alpha - L}{R-Y}\Big(R+(1-\alpha)\frac{\mathrm{d}f}{\mathrm{d}E}\Big)Q_z g(\alpha)\mathrm{d}\alpha.$$

The effect of a particular element of vector θ on E^* is obtained by total differentiating the system of equations (A.4). In particular, the total effect is computed as

$$\frac{\mathrm{d}E^*}{\mathrm{d}\theta^k} = -\Big(\frac{\partial\Psi_2}{\partial f}\frac{\partial\Psi_1}{\partial\theta^k} - \frac{\partial\Psi_1}{\partial f}\frac{\partial\Psi_2}{\partial\theta^k}\Big)\Big/\Big(\frac{\partial\Psi_1}{\partial E}\frac{\partial\Psi_2}{\partial f} - \frac{\partial\Psi_2}{\partial E}\frac{\partial\Psi_1}{\partial f}\Big),$$

where θ^k is the kth-element of vector θ. Note that the denominator in the latter expression can be rewritten as

$$\frac{\partial\Psi_1}{\partial E} - \frac{\frac{\partial\Psi_2}{\partial E}}{\frac{\partial\Psi_2}{\partial f}}\frac{\partial\Psi_1}{\partial f} < 0,$$

which can be proved to be negative. Moreover, this expression coincides with the second-order condition of the problem when the program is reduced to choose only variable E after incorporating constraint (PC) in the objective. Thus, guaranteeing the presence of a maximum. Therefore, the sign of the derivative of E^* respect to θ_k is determined by

$$\frac{\mathrm{d}E^*}{\mathrm{d}\theta^k} = \mathrm{sign}\Big(\frac{\partial\Psi_2}{\partial f}\frac{\partial\Psi_1}{\partial\theta^k} - \frac{\partial\Psi_1}{\partial f}\frac{\partial\Psi_2}{\partial\theta^k}\Big), \quad \text{(A.5)}$$

It can be shown that $\frac{\partial\Psi_2}{\partial f} > 0$ and $\frac{\partial\Psi_1}{\partial f} < 0$. Moreover, $\frac{\partial\Psi_2}{\partial\theta^k} = 0$ for $\theta^k = X, Q_0, \gamma_0, \gamma_1$. Hence, for such parameters,

$$\mathrm{sign}\Big(\frac{\mathrm{d}E^*}{\mathrm{d}\theta^k}\Big) = \mathrm{sign}\Big(\frac{\partial\Psi_1}{\partial\theta^k}\Big).$$

In particular, $\frac{\partial \Psi_1}{\partial X} > 0$, $\frac{\partial \Psi_1}{\partial Q_0} < 0$, $\frac{\partial \Psi_1}{\partial \gamma_0} > 0$, and $\frac{\partial \Psi_1}{\partial \gamma_1} < 0$.

For the case of δ, it can be shown that terms $\frac{\partial \Psi_1}{\partial \delta}$ and $\frac{\partial \Psi_2}{\partial \delta}$ are negative; hence, by using expression (A.5), $\frac{\mathrm{d}E^*}{\mathrm{d}\delta} < 0$.

For the remaining two parameters, R and Y, it can be demonstrated that

$$\frac{\partial \Psi_2}{\partial f}\frac{\partial \Psi_1}{\partial R} - \frac{\partial \Psi_1}{\partial f}\frac{\partial \Psi_2}{\partial R} > 0,$$
$$\frac{\partial \Psi_2}{\partial f}\frac{\partial \Psi_1}{\partial Y} - \frac{\partial \Psi_1}{\partial f}\frac{\partial \Psi_2}{\partial Y} < 0.$$

Thus, $\frac{\mathrm{d}E^*}{\mathrm{d}R}$ and $\frac{\mathrm{d}E^*}{\mathrm{d}Y}$ are positive and negative, respectively.