Strategic Concealment in Innovation Races*

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Abstract

We investigate firms' incentives to conceal intermediate research discoveries in innovation races. To study this, we introduce an innovation game where two racing firms dynamically allocate their resources between two distinct research and development (R&D) paths towards a final innovation: (i) developing it with the currently available but slower technology; (ii) conducting research to discover a faster new technology for developing it. We fully characterize the equilibrium behavior of the firms in the cases where their research progress is public and private information. Then, we extend the private information setting by allowing firms to conceal or license their intermediate discoveries. We show that when the reward of winning the race is high enough, firms would conceal their interim discoveries, which inefficiently retards the pace of innovation.

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1 Introduction

In the course of research and development (R&D), firms often discover interim knowledge that brings them closer to successfully producing a final innovation. When multiple firms race towards such innovation, a firm's optimal R&D strategy is likely to be influenced by the information about whether its rivals have made intermediate breakthroughs. Thus, a firm may want to conceal intermediate discoveries in order to hinder its rivals from adjusting their R&D strategies. On the other hand, it may prefer to disclose an intermediate discovery because this can open the opportunity for monetization via licensing the technological breakthrough. In this paper, we introduce and study an innovation race model that captures the tradeoffs between licensing and concealing interim discoveries and characterize firms' equilibrium behavior.

We consider a situation where two firms race towards developing an innovative product, such as a COVID-19 vaccine or a full self-driving (FSD) vehicle. The first firm to develop the product receives a reward (e.g., a transitory flow of monopoly profit) and the other firm does not. At each point in time, the firms allocate their limited resources between two routes for developing the product and incur constant flow costs. One route is to conduct basic *research* to discover a new technology that does not directly deliver the product but makes developing it faster, e.g., messenger RNA (mRNA) or light detection and ranging (LIDAR) technology.[1][2] This route requires two breakthroughs: discovering the new technology and developing the product with it. The other route is to *develop* the product with a currently available but slow technology, namely the incumbent technology. For example, the viral

[1]The mRNA technology was not utilized in practice before the COVID-19 outbreak. Thus, pharmaceutical firms had to first acquire basic knowledge in order to employ this new methodology. The advantage of possessing this intermediate technology is that firms can develop vaccines in a laboratory by using readily available materials. Hence vaccines can be developed faster with mRNA technology than with older methods. Moderna and Pfizer-BioNTech utilize mRNA technology to develop COVID-19 vaccines. For more information, see the web page of the Centers for Disease Control and Prevention (CDC): https://www.cdc.gov/coronavirus/2019-ncov/vaccines/different-vaccines/mrna.html.

[2] LIDAR is a laser radar that can provide extensive and reliable information surrounding a vehicle including an object's distance, size, position, and velocity if it is moving. Most FSD vehicle developers including Waymo—formerly the Google self-driving car project—use LIDAR combined with cameras. The main drawback of LIDAR is its current high cost. Thus, to develop a commercializable FSD vehicle, firms first need to discover a way to make LIDAR less expensive. Once LIDAR becomes affordable, it will be relatively easy to develop a commercializable FSD vehicle. In this sense, successfully developing an FSD vehicle with the LIDAR technology can be understood as a route requiring two breakthroughs.

vector method for developing a COVID vaccine and the camera-based vision technology for developing an FSD vehicle can be considered incumbent technologies.[3][4] This path requires a single breakthrough but the arrival rate is relatively low. We assume that the path with the new technology is more efficient: the total expected completion time of doing research for the new technology and developing the product with it is shorter than that of developing with the incumbent strategy. Thus, the socially efficient policy is to have both firms allocate all their resources to research, and once one of them discovers the new technology, have it share the breakthrough with the other firm to prevent duplication of research costs.

We investigate three different settings in the context of this framework. First, we consider the case where it is public information whether a firm has discovered the new technology or not. In this setting, a firm can condition its strategy not only on its own technological breakthrough but also on its rival's progress. We show that there exists a unique equilibrium and its form is determined by the relative efficiency of the new technology. The efficiency measure is defined to be inversely proportional to the expected total completion time of the path with the new technology, i.e., doing research is more attractive when efficiency is high. It is shown that when efficiency is extreme (high or low), a firm's equilibrium strategy does not depend on its rival's progress. Specifically, when the new technology is highly efficient, both firms allocate all their resources to research (i.e., perform research only); and when the new technology is not much more efficient, both firms allocate all their resources to development (i.e., develop with the incumbent technology only) regardless of their rival's status. On the contrary, when efficiency is intermediate, the equilibrium strategy of each firm does depend on its rival's progress. In this case, both firms begin by conducting research, but once one firm makes the intermediate technological breakthrough, the other switches to developing with the incumbent technology, namely it pursues a *fall-back* strategy.

[3] The viral vector technology was used during recent disease outbreaks including the 2014-2016 Ebola outbreak in West Africa. Many pharmaceutical firms had access to this methodology when the COVID-19 outbreak began. Indeed, this technology was utilized to develop COVID-19 vaccines by Oxford-AstraZeneca and Janssen (Johnson&Johnson). For more information, see the web page of the CDC: https://www.cdc.gov/coronavirus/2019-ncov/vaccines/different-vaccines/viralvector.html.

[4] Unlike other companies, Tesla's approach towards developing an FSD vehicle is to use only cameras without LIDAR (Templeton, 2019). Since camera technology is already very cheap, no cost-saving breakthrough is needed to implement it. However, the quality of information attained from cameras is inferior to that attained from LIDAR, thus it will take more time to develop an FSD vehicle utilizing only cameras.

Next, we analyze the setting where technological discoveries are private information, i.e., a firm cannot observe its rivals' technological progress. As in the public information setting, when efficiency is high, each firm conducts research until it succeeds or its rival produces the final innovation. Similarly, when efficiency is low, both firms endeavor to develop with the incumbent technology. This invariance occurs because, in the extreme cases of very high and very low efficiency, firms do not use the information about their rival's progress even when it is observable. However, in the case of intermediate efficiency, the firms cannot use the fall-back strategy as in the public information setting since they are no longer able to make their resource allocations contingent on their rivals' state of technology. Instead, their resource allocations must depend on their 'beliefs' about their rivals' progress. We characterize the unique symmetric equilibrium that is Markov with respect to these beliefs. The equilibrium strategy has a cutoff structure: firms conduct research exclusively up to a certain date (belief), then they start allocating their resources between developing with the incumbent technology and researching the new one, namely they employ a *stationary fall-back* strategy. The most intriguing feature of this equilibrium is that beliefs remain constant once the allocation of resources to development begins. This stationarity derives from two conflicting effects in the belief evolution. First, as time passes, it becomes more likely that one's rival has found the new technology (the *duration effect*). On the other hand, the lack of one's rival producing the final innovation (which is publically observable) implies that it is less likely that the new technology has been discovered (the *still-in-the-race effect*).

Last, we extend the private information setting by allowing firms to protect their discoveries by using either a *patent* or a *trade secret*. First, when a firm treats the new technology as a trade secret, it conceals the discovery, i.e., its rival still cannot observe its progress. However, this does not prohibit the firm's rival from discovering the new technology independently. Second, when a firm files a patent, it discloses the discovery of the new technology. On the one hand, if its rival has not yet made the technological breakthrough, then the exclusive right to use the new technology is bestowed on the patenting firm. In addition, the patenting firm may *license* the new technology, i.e., it may permit its rival to use the new technology for a fee. Once the licensee pays the fee, both firms race for the final innovation employing the new technology. On the other hand, if the rival firm has already discovered the new technology, i.e., it was protected as a trade secret. Then, the patenting firm cannot claim the exclusive right—rather, the new technology is now considered common property—and both firms can use it without making transfers.[5][6]

We first show that if a firm files a patent and the rival firm does not possess the new technology, the patenting firm always licenses. Thus, both firms develop the final innovation with the new technology, which is socially efficient. Once a firm files a patent, its rival can only try to develop the product with the old slow technology. Given this, the patenting firm can extract rent from its rival by allowing it to use the new technology for a fee. This is an application of the classical result of Coase (1960) in the sense that the socially efficient outcome can be achieved when the property right of the new technology is given to a firm and trade involves no transaction costs. Therefore, disclosing the new technology implies licensing it.

Finally, we explore whether a firm prefers to disclose or conceal the new technology. We show that this decision crucially depends on the size of the reward of winning the race: when the reward is high, firms may prefer to conceal their discoveries, whereas when the reward is low, they disclose and license them. Intuitively, this is because concealment involves a higher chance of winning the race, which is more attractive when the reward is high. Whereas, disclosure delivers an immediate payment from licensing, which is more appealing when the reward is low. More specifically, when a firm conceals a discovery, its rival does not know whether it possesses the new technology. Thus, per the results from the private information setting, the rival firm continues allocating some of its resources to researching the new technology. This is not desirable for the rival, especially when efficiency of the new technology is intermediate, because if it knew that the other firm already possessed the new technology, then its best response would be to allocate all its resources to development with the incumbent technology (i.e., to employ the fall-back strategy). In this sense, concealing the new technology hinders the rival firm from strategically responding to its discovery.

Concealment is detrimental not only to the rival firm but also to social surplus because

[5]When a firm files a patent, the firm with the trade secret can dispute the patent based on 35 U.S. Code §273 - Defense to infringement based on prior commercial use.

[6]For more information about trade secrets and patents, see the web page of the World Intellectual Property Organization: https://www.wipo.int/about-ip/en/. Also, see Lobel (2013) for examples.

it generates duplicate research efforts. This slows down the pace of innovation. On the contrary, the socially efficient outcome could be achieved by disclosing and licensing the new technology. These results on firms' incentives for concealment imply a simple policy intervention. Reducing the reward of winning the race (e.g., weakening the transitory monopoly power in the innovative product market by imposing a tax,) reduces incentives to conceal and promotes licensing, thus speeding up the pace of innovation.

Related Literature

This paper primarily contributes to the literature on patent vs. secrecy by introducing a novel incentive to conceal a firm's discovery: hindering its rival's strategic response. Previous studies mainly focused on the limited protection power of patents. For example, the seminal article by Horstmann et al. (1985) posits that "patent coverage may not exclude profitable imitation." Thus, in their framework, the main reason why a firm may choose secrecy over a patent is not to be imitated.[7] Another limitation of a patent is that it expires in a finite time. For instance, Denicolò and Franzoni (2004) consider a framework where a patent gives the patenting firm monopoly power only for a certain period of time (and no profit after expiration), whereas secrecy can give indefinite monopoly power to a firm but it can be leaked or duplicated by a rival with some probability. On the contrary, in this paper, we abstract from the restrictions of patents and focus analysis on the potential advantages of concealment.

Another hallmark of this paper is its consideration of 'interim' discoveries. Therefore, it is naturally related to the literature on licensing of interim R&D knowledge, e.g., Bhattacharya et al. (1992); d'Aspremont et al. (2000); Bhattacharya and Guriev (2006); Spiegel (2008). In these papers it is assumed that firms already know which of them has superior knowledge, i.e., the firm that will license the technology is exogenously given. Unlike in those studies, we allow firms to choose when to license (and even allow them not to license), i.e., the licensing decision is endogenous.

We also contribute to the innovation literature by introducing a model with two character-

[7] Many subsequent papers study the imitation threat and potential patent infringement, e.g., Gallini (1992); Takalo (1998); Anton and Yao (2004); Kultti et al. (2007); Kwon (2012); Zhang (2012).

istics. First, there are different avenues towards innovation: developing with the incumbent technology and doing research for the new technology. Second, one of the paths involves multiple stages: once a firm discovers the new technology, then the firm develops the innovative product with it.

With respect to the first characteristic, there is a recent branch of the literature that studies races where there are different routes to achieve a final objective. Das and Klein (2020) and Akcigit and Liu (2016) study a patent race where two firms compete for a breakthrough and there are two methods to get the breakthrough: a safe method and a risky method. In Das and Klein (2020) the safe method has a known constant arrival intensity while the risky method has an unknown constant arrival intensity. In Akcigit and Liu (2016), instead, the safe method has a known payoff associated with breakthrough arrival, while there is uncertainty about the payoff if the risky method is used. In this paper, firms face no uncertainty about whether the innovation is feasible. Instead, they are uncertain whether their rival possesses the new and faster technology.

The second characteristic, multi-stage innovation, is also widely studied in the literature, e.g., Scotchmer and Green (1990); Denicolò (2000); Green and Taylor (2016); Song and Zhao (2021). Our paper shares the framework with these in that we use two sequential Poisson discovery processes and ask whether a firm would patent the first discovery or not. A feature setting apart from their works is that there is another path that only requires one but slower breakthrough toward innovation. This feature connects our model to Carnehl and Schneider (2022) and Kim (2022) in the sense that players can choose between a sequential approach—which requires two breakthroughs—and a direct approach, which requires only one breakthrough, but its riskier or slower.[8] Our model mainly differs from theirs in that multiple players compete by choosing between these approaches, whereas Carnehl and Schneider (2022) considers a problem by a single decision maker and Kim (2022) studies a contracting setup between a principal and an agent. In their studies, a key factor for a player to choose the direct approach is a deadline that is either exogenously given or endogenously determined

[8]In Carnehl and Schneider (2022), an agent is uncertain whether the direct approach is feasible or not, i.e., this approach is risky. On the other hand, in Kim (2022), there is no uncertainty on the feasibility of the direct approach, but its completion rate is slower than the ones for the sequential approach. In this sense, our framework is closer to Kim (2022).

to reduce moral hazard. In contrast to these, a deadline is not involved in our model. Rather, the race with the rival firm may induce a firm to develop with the incumbent technology, which can be considered as a direct approach.

Last, this paper is related to the recent literature on information disclosure in priority races, e.g., Hopenhayn and Squintani (2016); Bobtcheff et al. (2017). In those papers, once a firm makes a breakthrough, the innovation value grows as time passes until one of the firms files a patent. Thus, firms face a tradeoff between disclosing to claim the priority and delaying in order to grow the innovation value. On the contrary, in this paper, the value of innovation is fixed and the discovery of the new technology only allows the firm to develop the innovative product faster. Therefore, a firm may delay the disclosure purely to confound the rival's R&D decisions.

Roadmap

We introduce the model in the next section, then characterize equilibria in the private and the public information settings in Section 3 and 4. In Section 5, we extend the private information setting by allowing firms to disclose their discoveries. We conclude in Section 6. All proofs appear in the appendix.

2 Model

We consider a race between two firms, A and B, to develop an *innovative product*. Time is continuous and infinite $t \in [0, \infty)$. The innovative good can be developed using two different technologies: at the start of the race, both firms have access to an incumbent technology, but they can gain access to a faster new technology by conducting research.

Each firm owns a unit of resources per unit of time, which can be used either for research to discover the new technology or for developing the innovative product. We denote by σ_t^i the fraction of time t resources that firm i allocates to 'research' and γ_t^i the resources that firm i allocates to 'develop' the innovative product. Firm i gains access the new technology stochastically at rate $\sigma_t^i \cdot \mu$, where μ is a constant parameter. This access is irreversible. The firm develops the innovative product stochastically at rate $\gamma_t^i \cdot \lambda_t$, where λ_t depends on the firm's technology. Specifically, $\lambda_t = \lambda_L$ if the firm doesn't have access to the new technology and $\lambda_t = \lambda_H > \lambda_L$ if the firm has access to the new technology.

The race ends once one of the firms develops the innovative product. During the race, firms pay a flow cost c. The first firm to develop the innovative product receives a lump-sum reward worth Π.[9] Firms don't discount the future and maximize their expected total payoff.[10] The successful development of the innovative product is publicly observable. Thus, firms know at all times if they are still on the race. However, firms do not observe their opponents' resource allocation.

In the upcoming sections, we will explore different model specifications. These include cases where research breakthroughs are either public or privately observed, situations where firms can voluntarily disclose evidence of a research breakthrough, and the possibility of firms with access to the new technology sharing this access with their opponents.

Change of variables To facilitate the interpretation of the results, we introduce two relevant parameters measuring the efficiency and the relative intensity of the new technology:

$$\eta \equiv \frac{\mathbb{E}[\text{ completion time with the incumbent technology }]}{\mathbb{E}[\text{ total completion time with the new technology }]} = \frac{\frac{1}{\lambda_L}}{\frac{1}{\mu} + \frac{1}{\lambda_H}},$$

$$\delta \equiv \frac{\mathbb{E}[\text{ research completion time with the new technology }]}{\mathbb{E}[\text{ total completion time with the new technology }]} = \frac{\frac{1}{\mu}}{\frac{1}{\mu} + \frac{1}{\lambda_H}}.$$

Note that when η is fixed, a higher δ implies that the new technology is more research-intensive (or less development-intensive). This is because when firms try to achieve innovation via the new technology, they are expected to spend more time in research as δ increases.

For the rest of the paper, we make the following two parametric assumptions:

[9] We model the race as winner-takes-all competition. This payoff structure has been commonly used in the innovation race literature, e.g., Loury (1979); Lee and Wilde (1980); Denicolò and Franzoni (2010). However, as long as firms only care about the identity of the winner and the duration of their participation in the race, we are only assuming that preferences of the firms are separable in these two dimensions and risk-neutral in their participation. Following the literature, we can regard Π as the societal value of having the innovative product, and the first firm that introduces the innovative product becomes the monopolist and captures all the social value, e.g., by using the first-degree price discrimination. In this article, we abstract away from the market after the race and focus on the activities during the race.

[10] With discounting the firms are not risk-neutral over the duration of the race conditional on the outcome. This complicates the closed-form solutions without affecting the qualitative results of the paper.

Assumption A1. The new technology is relatively more efficient: $\eta > 1$.

Assumption A2. The incumbent technology is profitable: $\Pi \geq c/\lambda_L$.

When assumption A1 does not hold, there is no incentive for firms to allocate resources to research. They will instead develop the innovative product using the incumbent technology as long as assumption A2 holds. When assumption A2 fails, there is no incentive for firms to ever use the incumbent technology. We provide a detailed analysis of this case in the appendix for completeness.[11]

First best We want to evaluate the equilibrium resource allocation of the firms in terms of the expected welfare that they produce. we assume that welfare is independent of the identity of the firm that wins the race. Moreover, we assume that the total welfare is linear in the development time. Thus, the social planner is, like the firms, risk-neutral in the development time. The problem of the planner is equivalent to the problem of minimizing the expected completion time.

Observation. *If firms can share their technological breakthroughs, the planner's solution consists of:*

(i) allocating all resources to research, so that the new technology is obtained at rate 2μ*;*

(ii) when any firm discovers the new technology, share it immediately with the opponent so that both firms develop at rate $2\lambda_H$*.*

Hence, the (ex-ante) expected development time is given as follows:

$$L_{FB} = \frac{1}{2\mu} + \frac{1}{2\lambda_H}. \tag{1}$$

3 Public Information Setting

We begin by exploring a setting where firms' research progress is public information, i.e., firms can observe their opponents technological breakthroughs. In this case, the set of firms

[11] TBA.

that have obtained the new technology is common knowledge and we can regard it as a state: $\omega \in \Omega \equiv \{\{A, B\}, \{A\}, \{B\}, \emptyset\}$. We assume the firms employ Markov strategies, i.e., Firm i's strategy is defined by $\sigma^i : \Omega \to [0,1]$. A pair of Markov strategies (σ^A, σ^B) constitutes a Markov perfect equilibrium if, for any state, each firm's strategy is the best response to the opponent's strategy.

Since a firm possessing the new technology derives no value from research, we can restrict attention to the strategies such that $\sigma^i(\omega) = 0$ when $i \in \omega$. Next, we introduce three benchmark Markov strategies that satisfy this condition.

Definition 3.1. (a) The *research strategy* σ_R^i for firm i fully allocates resources to research regardless of the opponent's technology level ($\sigma_R^i(\omega) \equiv \mathbb{1}_{\{i \notin \omega\}}$).

(b) The *fall-back strategy* σ_F fully allocates resources to research unless any firm has the new technology, in which case it fully allocates resources to development ($\sigma_F(\omega) \equiv \mathbb{1}_{\{\omega=\emptyset\}}$)

(c) The *incumbent strategy* fully allocates the resources to development independently of the state ($\sigma_I(\omega) \equiv 0$).

The following proposition shows that it is a Markov perfect equilibrium for both firms to simultaneously use one of the above strategies, depending on parameter values. The proof is in Appendix A.

Proposition 1. *Suppose that technological breakthroughs are public and let* $\overline{\eta}(\delta) \equiv 1 + \delta$ *and* $\underline{\eta}(\delta) \equiv \frac{1}{2}\left(1 + \sqrt{1 + 4\delta(1-\delta)}\right)$. *Then, the Markov perfect equilibrium is uniquely characterized as follows.*

(a) If $\eta \geq \overline{\eta}(\delta)$, *each firm plays its respective research strategy* (σ_R^A, σ_R^B)*;*

(b) If $\eta \in \left(\underline{\eta}(\delta), \overline{\eta}(\delta)\right)$, *both firms play the fall-back strategy* (σ_F, σ_F)*;*

(c) If $\eta \leq \underline{\eta}(\delta)$, *both firms play the incumbent strategy* (σ_I, σ_I)*.*

The above proposition provides a clear picture of how the efficiency of the new technology (η) affects the firms' R&D decisions. When the new technology is sufficiently efficient

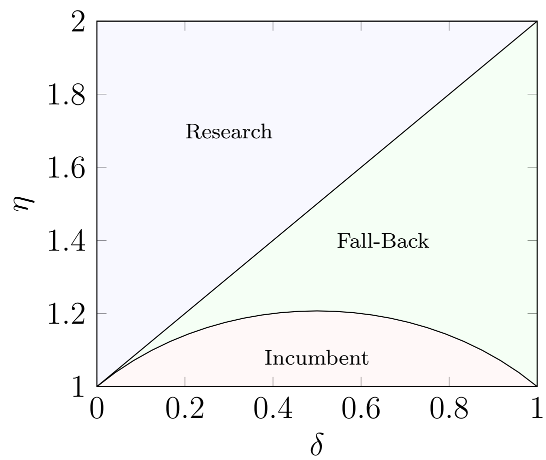


Figure 1: Markov Perfect Equilibrium under the Public Information Setting

and research is relatively easy ($\eta \geq \overline{\eta}(\delta)$), the firms do research regardless of whether their opponent has discovered the new technology. When the new technology is relatively inefficient ($\eta \leq \underline{\eta}(\delta)$), the firms do not engage in research at all. In the intermediate case ($\underline{\eta}(\delta) < \eta < \overline{\eta}(\delta)$), the firms' R&D decisions are affected by the opponent's progress: when neither firm has made the technological breakthrough, both firms do research; but once a firm obtains the new technology, the follower—the firm without the new technology—switches to develop with the incumbent technology.

The proposition also shows that the thresholds depend on δ, the relative intensity of the new technology. Figure 1 illustrates how these thresholds depend on δ. First, to determine the threshold for the equilibrium with the research strategy, we need to consider the case when one firm (the leader) has discovered the new technology and the other (the follower) has not. Say that Firm i is the follower and Firm j is the leader. Firm i needs to determine whether to follow j ($\sigma^i(\{j\}) = 1$) or to switch to the incumbent technology ($\sigma^i(\{j\}) = 0$). When it is difficult to attain the new technology, the follower would choose to follow only if the new technology is efficient enough. Thus, the threshold for the equilibrium with the research strategy is increasing as δ increases.

Second, why is the threshold for the equilibrium with the incumbent strategy hump-shaped? To answer this question, we need to consider the situation where neither firm yet possesses the new technology. The firms allocate resources by taking into account the difficulty and the advantage of becoming a leader. Consider the case with $\delta < 1/2$, i.e., attaining the new technology is relatively easy, but the leader's advantage is relatively weak (due to low λ_H). In this case, the major determinant is the difficulty of becoming a leader. Fix the efficiency (η) and marginally increase δ. Then it becomes more difficult to attain leadership, which makes the incumbent strategy more attractive. Next, consider the case with $\delta > 1/2$, i.e., attaining the new technology is relatively difficult, but it is more advantageous to become a leader (due to high λ_H). In this case, the major determinant is the leader's advantage. If δ decreases, the leader's advantage decreases, which again makes the incumbent strategy more attractive.

The next Corollary characterizes the expected completion time of product development with public technological breakthroughs.

Corollary 3.1. *When technological breakthroughs are public, the expected development time in any MPE is*

$$L_{public} = \begin{cases} \frac{1}{2}\left(\frac{1}{\mu} + \frac{1}{\lambda_H} + \frac{1}{\lambda_H + \mu}\right), & \text{if } \eta \geq \overline{\eta}(\delta), \\ \frac{1}{2\mu} + \frac{1}{\lambda_H + \lambda_L}, & \text{if } \eta \in (\underline{\eta}(\delta), \overline{\eta}(\delta)), \\ \frac{1}{2\lambda_L}, & \text{if } \eta \leq \underline{\eta}(\delta). \end{cases} \tag{2}$$

4 Private Information Setting

We now consider the case in which research breakthroughs are private information, i.e., firms cannot observe whether their opponents have the new technology or not. In this case, the firms can only condition their resource allocation on their own research breakthroughs and calendar time t. As before, a firm that possesses the new technology will fully allocate its resources to development. Thus, we focus on the dynamic resource allocation problem of a firm that has not discovered the new technology yet. Thus, an allocation policy for a player can be therefore described by a function $\sigma : \mathbb{R}_+ \to [0, 1]$ that represents the research allocation at a given time conditional on the new technology not being discovered. Let $\mathcal{S}$ be the set of such policies.

4.1 Evolution of Beliefs and Recursive Formulation

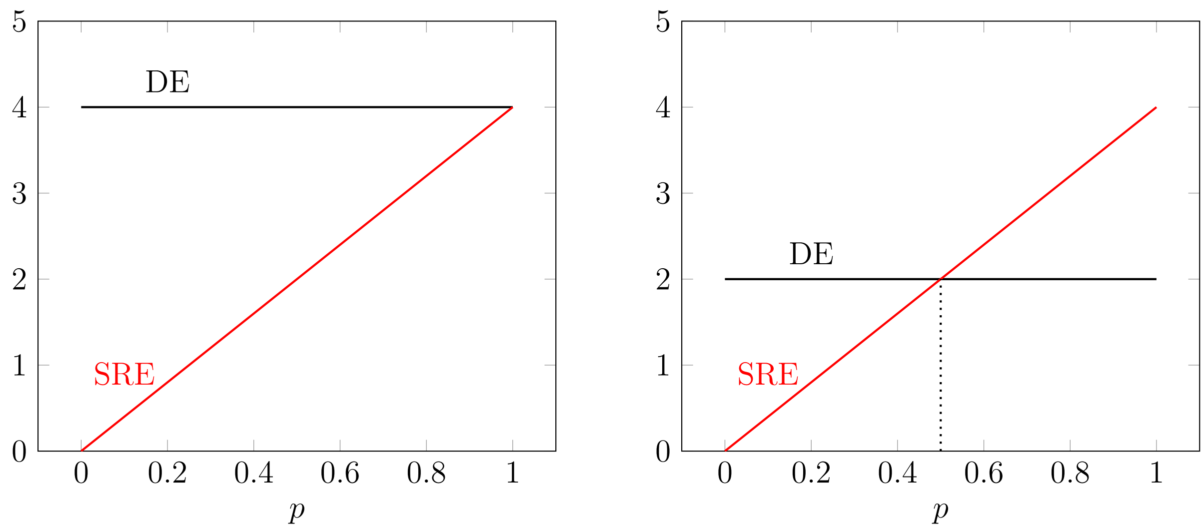
Although firms do not observe their opponents' technology levels, they do form beliefs about whether their opponents have acquired the new technology. In equilibrium, these beliefs should be consistent with the opponents' chosen allocation policy. At any point in time, all that firms know about their opponent's progress stems from the fact that the race is still ongoing, which means that they have not yet successfully developed the innovative product. Given that the race is ongoing by time t, it is possible to calculate the probability p_t that firms have access to the new technology based on their chosen allocation policy. To describe the evolution of these probabilities, we introduce the following lemma.

Lemma 4.1 (Probability of access to the new technology)**.** *Suppose that a firm follows policy $\sigma : \mathbb{R}_+ \to [0,1]$. The probability p_t that the firm has access to the new technology by time t, given that the race is still ongoing at time t, evolves according to the following differential equation:*

$$-\frac{d}{dt}\log(1-p_t) = \frac{\dot{p}_t}{1-p_t} = \mu \cdot \sigma(t) - \{\lambda_H - (1-\sigma(t)) \cdot \lambda_L\} \cdot p_t. \tag{3}$$

The proof is provided in Appendix B. Since none of the firms has access to the new technology at the beginning of the race, we have for any allocation policy σ the initial probability p_0 is zero, which serves as an initial condition for the evolution of p_t. The left-hand side of (3) is the opposite of the time derivative for the log of $(1-p_t)$. The right-hand side of (3) captures two distinct effects in the evolution of the conditional probability. First, given that the firm has not yet attained the new technology by time t, the research succeeds at rate $\mu \cdot \sigma(t)$ and it may raise the belief. The first term of (3) represents this positive effect, which we dub the duration effect (DE). On the other hand, the fact that the race is still ongoing indicates that the firm has not succeeded yet in development and therefore it is less likely to have the new technology in hand. The second term of (3) reflects this effect, which we dub the still-in-the-race effect (SRE).[12] Notice that this term is proportional to

[12] Similar types of the belief updating can be found in the strategic experimentation literature, e.g., Keller et al. (2005); Bonatti and Hörner (2011). The main difference is that the agents form beliefs about whether the project is good or bad in those papers, whereas the firms form beliefs about whereas in our model, firms only form beliefs about the technology access of the rival.



(a) $\sigma = 1$, $\mu = 4$, $\lambda_H = 4$ and $\delta = 1/2$.

(b) $\sigma = 1$, $\mu = 2$, $\lambda_H = 4$ and $\delta = 2/3$.

Figure 2: Duration Effect (Black) and Still-in-the-Race Effect (Red)

$\lambda_H - (1 - \sigma(t))\lambda_L$, which is the rate of successful innovation development given the new technology net of that without the new technology.

A natural benchmark is to consider the allocation policy that only allocates resources to research. We characterize this probability in the following corollary.

Lemma 4.2. *Suppose that a firm follows an allocation policy σ, with $\sigma(s) = 1$ for $s \in [0, t)$. Then, the conditional probability p_t of having access to the technology by time t given that the race is ongoing is:*

$$p_t = q(t) := \frac{\frac{1}{\lambda_H}\left(e^{-\mu t} - e^{-\lambda_H t}\right)}{\frac{1}{\mu}e^{-\mu t} - \frac{1}{\lambda_H}e^{-\lambda_H t}}. \tag{4}$$

In addition, q is increasing, with $\lim_{t\to\infty} q(t) = \min\{1, \mu/\lambda_H\}$.

This result highlights the tradeoff between the duration effect and the still-in-the-race effect. In Figure 2, we illustrate these effects when a firm fully allocates its resources to research ($\sigma_t = 1$ for all $t \geq 0$). Specifically, we provide the graphs of the terms of each effect divided by $(1 - p)$: μ (DE), $\lambda_H p$ (SRE). In Figure 2a, we depict the case where $\mu = \lambda_H$, i.e., $\delta = 1/2$. Observe that, in this case, the duration effect is larger than the still-in-the-race effect for every p. If we fix λ_H and increase μ, we observe that the duration effect continues to dominate the still-in-the-race effect. Hence, when $\delta = \lambda_H/(\lambda_H + \mu) < 1/2$, the probability converges to 1 ($\lim_{T\to\infty} q(T) = 1$). On the other hand, in Figure 2b, we illustrate the case where $\mu < \lambda_H$, i.e., $\delta < 1/2$. In this case, the duration effect is greater than the still-in-the-race effect only when $p < \mu/\lambda_H$. This induces the belief to converge to μ/λ_H.

Firms' best response In this section, we fix the allocation policy σ_j of firm j and derive the set of best responses by firm i. The value for firm i of staying in the race depends on σ_j and the current access to the new technology. We denote $V_t^{1,i}$ the value of firm i at time t that has access to the new technology and $V_t^{0,i}$ the value of firm i at time t when i doesn't have access to the new technology. Next, we explore the dynamics of Firm i's value starting with the value of the firm that has the new technology by time T, what must satisfy the equation

$$\begin{aligned} V_T^{1,i} = & - cdt + \Pi \cdot \lambda_H dt + 0 \cdot \left(\lambda_H p_T^i + \lambda_L(1-p_T^i)(1-\sigma_T^j)\right) dt \\ & + (1 - \lambda_H dt - \lambda_H p_T^i dt - \lambda_L(1-p_T^i)(1-\sigma_T^j)dt)(V_T^{1,i} + \dot{V}_T^{1,i} dt). \end{aligned}$$

Thus, we can derive the Hamilton-Jacobi-Bellman (HJB) equation:

$$0 = \dot{V}_T^{1,i} + \lambda_H(\Pi - V_T^{1,i}) - \left\{\lambda_H p_T^i + \lambda_L(1-p_T^i)(1-\sigma_T^j)\right\} V_T^{1,i} - c. \tag{HJB$_1$}$$

This HJB equation gives a clear interpretation on the evolution of $V_T^{1,i}$: at an instant T (i) Firm i wins the race at rate λ_H, in which case it gets the rent Π but loses the continuation payoff $V_T^{1,i}$; (ii) From the point of view of Firm i, Firm j wins the race at an expected rate $\lambda_H p_T^i + \lambda_L(1-p_T^i)(1-\sigma_T^j)$, in which case Firm i loses the continuation payoff; (iii) the flow cost c is charged.

We continue with the continuation value when Firm i doesn't have access to the new technology by time T, $V_T^{0,i}$. In this case, the firm chooses between doing research and developing with the incumbent technology:

$$\begin{aligned} V_T^{0,i} = \max_{\sigma_T^i \in [0,1]} & - cdt + \Pi \cdot \lambda_L(1-\sigma_T^i)dt + V_T^{1,i} \cdot \mu\sigma_T^i dt + 0 \cdot \left(\lambda_H p_T^i + \lambda_L(1-p_T^i)(1-\sigma_T^j)\right) dt \\ & + \left(1 - \lambda_L(1-\sigma_T^i)dt - \mu\sigma_T^i dt - \lambda_H p_T^i dt - \lambda_L(1-p_T^i)(1-\sigma_T^j)dt\right)(V_T^{0,i} + \dot{V}_T^{0,i} dt). \end{aligned}$$

By using the linearity of σ^i_T, the corresponding HJB equation can be derived as follows:

$$\begin{aligned} 0 = \dot{V}^{0,i}_T - \left\{\lambda_H p^i_T + \lambda_L (1-p^i_T)(1-\sigma^j_T)\right\} V^{0,i}_T - c \\ + \max_{\sigma^i_T \in [0,1]} \left[\sigma^i_T \cdot \mu(V^{1,i}_T - V^{0,i}_T) + (1-\sigma^i_T) \cdot \lambda_L (\Pi - V^{0,i}_T)\right]. \end{aligned} \tag{HJB$_0$}$$

This HJB equation determines whether Firm i allocates the resources to research or development, conditional on not having access to the new technology. If $\mu(V^{1,i}_T - V^{0,i}_T) > \lambda_L(\Pi - V^{0,i}_T)$, Firm i allocates all resources to research: $\sigma^i_T = 1$. If $\lambda_L(\Pi - V^{0,i}_T) > \mu(V^{1,i}_T - V^{0,i}_T)$, Firm i allocates all resources to development: $\sigma^i_T = 0$. If $\mu(V^{1,i}_T - V^{0,i}_T) = \lambda_L(\Pi - V^{0,i}_T)$, Firm i is indifferent between any (potentially interior) level of research and development: any $\sigma^i_T \in [0,1]$ is consistent with a best response.

4.2 Symmetric Markov Equilibrium

To characterize the equilibrium behavior of firms when technological breakthroughs are private, we focus on symmetric equilibria in which policies are Markov with respect to the belief about the opponent's technology level. Formally, we say that (σ^A, σ^B) is an *equilibrium* if for $i \neq j$, σ^i is a best-response to σ^j, i.e. it solves (HJB$_0$) for all $T \geq 0$. Moreover, (σ^A, σ^B) is a *symmetric Markov equilibrium (SME)* if $\sigma^A = \sigma^B$, and σ is measurable with respect to the beliefs that firms have about their opponents' technology, i.e. $p^i_t = p^i_{t'}$ implies $\sigma^i(t) = \sigma^i(t')$ where p_t is derived from $p_0 = 0$ and (3) using σ^j. The following proposition characterizes the symmetric Markov equilibria of the game.

Proposition 2. *When technological breakthroughs are private information, the following statements hold.*

(a) ***(Cutoff Structure)*** *Any SME (σ, σ) is characterized by a cutoff time $T^* \in \mathbb{R}_+ \cup \{\infty\}$ and a stationary allocation $\sigma^* \in [0,1)$ such that both firms fully allocate to research up to time T^* ($\sigma(t) = 1$ for all $t < T^*$), and use σ^* from then on ($\sigma(t) = \sigma^*$ for all $t > T^*$).*

(b) ***(Equilibrium Characterization)*** *Let $\tilde{\eta}(\delta) \equiv \min\{\overline{\eta}(\delta), 2-\delta\}$. The unique symmetric Markov equilibrium is characterized as follows.*

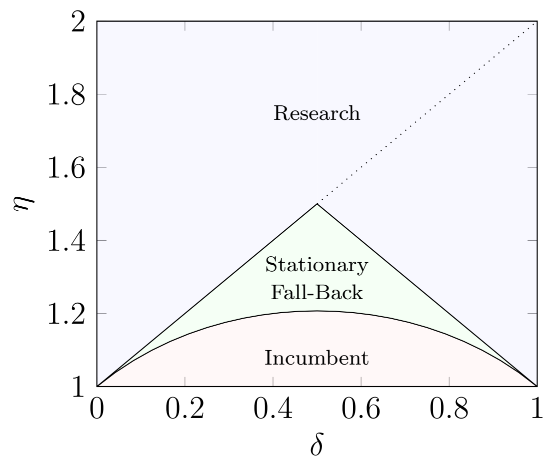


Figure 3: Symmetric Markov Equilibrium under the Private Information Setting

(i) If $\eta \geq \tilde{\eta}(\delta)$, both firms play a ***research policy*** *($T^* = \infty$).*

(ii) If $\eta \leq \underline{\eta}(\delta)$, both firms play an ***incumbent policy*** *($T^* = 0$, $\sigma^* = 0$).*

(iii) If $\eta \in \left(\underline{\eta}(\delta), \tilde{\eta}(\delta)\right)$, both firms play ***stationary fall-back strategy****. Allocate resources fully to research up to time $T^* \in (0, \infty)$ and then choose an interior allocation $\sigma^* \in (0,1)$, where*

$$\sigma^* = \frac{\eta}{1-\delta} - \frac{\eta+\delta}{\eta-\delta}, \qquad q(T^*) = \frac{1}{2}\left\{\frac{\eta}{\delta} - \frac{1-\delta}{\eta-1}\right\}, \tag{5}$$

The equilibrium continuation value and the formal proof of the proposition are relegated to Appendix C. However, a sketch of the proof is provided next. We begin by showing that the belief derived from a symmetric Markov strategy is nondecreasing in time, i.e., $\dot{p}_t \geq 0$ (Lemma C.1). This is because if $\dot{p}_t < 0$ for some $t > 0$, by the Markov property, the belief cannot go above p_t which contradicts $\dot{p}_t < 0$. Note that $\dot{p}_t < 0$ if $p_t > 0$ and $\sigma_t = 0$. This allows us to focus on the following two cases: (i) $\sigma_t = 0$ for all $t \geq 0$; or (ii) $\sigma_t > 0$ for all $t \geq 0$ (Lemma C.2). The first case corresponds to incumbent strategy in the observable-breakthrough benchmark ($T^* = 0$ and $\sigma^* = 0$). Even though the strategy space is different from the benchmark, it is qualitatively equivalent since the firm always develops with the incumbent technology. Similarly, a special case of the second case—$\sigma_t = 1$ for all $t \geq 0$—

corresponds to research strategy ($T^* = \infty$). In the remaining case, on the equilibrium path, $\sigma_S \in (0, 1)$ for some $S \geq 0$, i.e., firms are indifferent between researching to find the new technology and developing the innovation with the old technology at time S. In this case, we show that from then on ($t \geq S$), the firms continue to be indifferent between research and development (Lemma C.3). In addition, we show that to make firms indifferent for all $t \geq S$, the firms' strategies and beliefs should be stationary: $\sigma_t = \sigma^*$ and $p_t = p^*$ for all $t \geq S$ (Lemma C.4). By identifying the earliest time T at which firms are indifferent, we can show that it corresponds to the stationary fall-back strategy: $\sigma_t = 1$ for all $t < T$ and $\sigma_t = \sigma^*$ for all $t > T$. Thus, we have three types of symmetric Markov equilibria: both firms play (i) the research strategy; (ii) the incumbent strategy; or (iii) the stationary fall-back strategy.

Next, we need to identify which regions of the parameter space give rise to each of the three types of equilibrium. First, consider parameter values under which the fall-back policy is not the second-best policy ($\eta \geq \overline{\eta}(\delta)$ or $\eta \leq \underline{\eta}(\delta)$). In this case, firms do not change their resource allocations even if they can observe their opponent's technological breakthroughs. Thus, the same strategy profiles (fully conducting research or fully developing with the incumbent technology) will constitute an equilibrium in the private information setting.

Now consider the remaining case ($\overline{\eta}(\delta) > \eta > \underline{\eta}(\delta)$). In this case, the new technology is efficient enough ($\eta > \underline{\eta}(\delta)$) for both firms to begin by doing only research. However, They need to determine whether to keep fully conducting research indefinitely ($T^* = \infty$) or to hedge their bets by switching to the stationary fall-back strategy at some point ($T^* < \infty$ and $\sigma^* \in (0, 1)$). The answer crucially depends on the relative intensity δ. When the new technology is more development-intensive ($\delta < 1/2$), if firms keep fully conducting research indefinitely, then the beliefs that their opponent has made a breakthrough converge to 1 by Lemma 4.2. Since they are in the parameter region where firms switch to development with the incumbent technology if they know that their opponent possesses the new technology (or equivalently $p = 1$), they will find it better to choose the stationary fall-back strategy when the belief is sufficiently close to 1. Next, consider the case where the new technology is more research-intensive ($\delta > 1/2$). In contrast to the previous case, there exists a region where both firms play the research strategy indefinitely in equilibrium ($2 - \delta < \eta < \overline{\eta}(\delta) = 1 + \delta$).

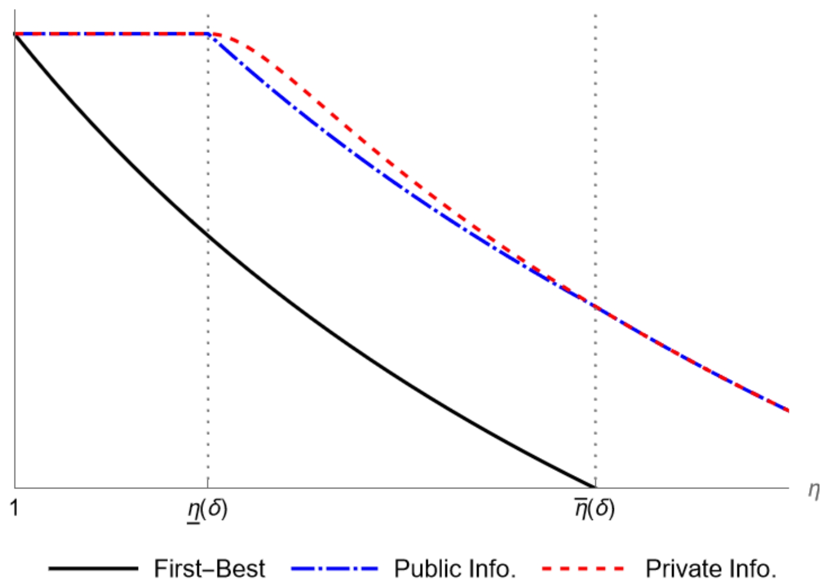


Figure 4: Expected completion times for the first-best case, private and public information settings

This result is also easily understood by considering the belief about technological status. By Lemma 4.2, this belief cannot exceed $(1-\delta)/\delta$ in this region of the parameter space. Since the belief is bounded strictly below 1, firms may find it preferable to keep fully conducting research in the private information setting, even though they would switch to development with the incumbent technology if they were able to observe a breakthrough by their opponent. As δ increases, the upper bound of the belief $(1-\delta)/\delta$ decreases, so the firms will keep fully conducting the research even with the relatively low η. Thus, the threshold between the equilibria with the stationary fall-back strategy and the research strategy decreases in δ.

Remark 1. In Lemma C.6, we characterize the generic forms of the expected payoffs when both firms use $\sigma_t = 1$ for all $t \in [0, T]$. It allows us to derive the expected payoffs at each point in time under the equilibrium with the stationary fall-back policy. By using $V^1_{T^*} = V^*_1$ and $V^0_{T^*} = V^*_0$ in (5) as terminal conditions at time T^*, the constants C_0 and C_1 in (27) and (28) can be determined. Moreover, C_1 is negative (see the proof of Lemma C.8). Thus, the expected payoffs V^1_t and V^0_t for any $t \in [0, T^*]$ can be derived.

4.3 Comparison of Expected Completion Times

Having characterized the firms' equilibrium behavior in the private information setting, we can compare it with the benchmark cases. Recall that the (ex ante) expected completed times for the first-best case and the public information setting are characterized in equations (1) and (2). In Figure 4, we fix δ and display the curves of the expected completion times for the first-best case (black) and the second-best case (blue) with respect to the efficiency measure η. The gap between these two expected completion times is a consequence of the new technology not being shared.

Observe that when $\eta \geq \overline{\eta}(\delta)$ or $\eta \leq \underline{\eta}(\delta)$, the equilibrium policy in the private information setting is consistent with the public information setting, i.e., the expected completion times for these settings are same. When $\eta \in (\underline{\eta}(\delta), \overline{\eta}(\delta))$, in the private information setting, the firms cannot use the fall-back policy because they are not able to observe the rivals' technology levels. Rather, they use stationary fall-back strategies ($\eta \in (\underline{\eta}(\delta), \tilde{\eta}(\delta))$) or research strategies ($\eta \in [\tilde{\eta}(\delta), \overline{\eta}(\delta))$), which are suboptimal compared to the public information case. Therefore, the expected completion time would be longer, i.e., a lack of information transmission about the research status retards the pace of innovation. Figure 4 also illustrates these results: there is a gap between the expected completion times of the public and private information settings only if $\eta \in (\underline{\eta}(\delta), \overline{\eta}(\delta))$.

5 Patents

In this section, we extend the model by allowing the firms to costlessly patent and license the new technology to their opponents.

5.1 Patents with public information

As in Section 3, we consider a setting where research breakthroughs are public. In this framework, there is no reason why a firm would not file a patent and, conditional on filing a patent, there are gains from sharing it with the competitors. Thus, there exists a price such that the firm with the patent (Licensor) is willing to accept to share the technology with the opponent (Licensee) and that the opponent is willing to pay.

Moreover, if the licensor has all the bargaining power (i.e. can extract all the surplus that is created by sharing the new technology) then the incentives to research in the first place are aligned with minimizing the duration of the race and the equilibrium is efficient. We formalize this statement in the following observation.

Observation. *When technological breakthroughs are public, technological breakthroughs are always patented and shared. Moreover, if negotiations take the form of a take-it-or-leave-it offer, the unique equilibrium is first-best efficient.*

5.2 Patents with private information

When technological breakthroughs are private information, a firm that obtains the new technology can choose to conceal its discovery—treat it as a trade secret—or disclose it by filing a patent. In addition, a firm that files a patent can decide whether to license it or not. To simplify the analysis, we assume that the firm that holds the patent can make a take-it-or leave-it offer to the rival firm for the right to use the new technology. As usual, we solve the equilibrium backwards, starting at the subgame in which one of the firms filed a patent.

5.2.1 The Subgame after Patenting

We begin by describing the subgame that takes place after a firm files a patent. There are two possible scenarios, depending on whether the rival had the new technology and was concealing it or didn't had the new technology. Consider first the case where the rival already had the new technology. Then, the trade secret protection allows the rival firm to dispute the patent. Thus, both firms have the right to use the new technology and the expected payoffs of them are $V_C = \frac{\lambda_H \Pi - c}{2\lambda_H}$.

Next, suppose that the rival firm does not possess the new technology. Then, the patenting firm has the exclusive right to use the new technology. If the rival firm rejects the licensing offer, it has to develop with the incumbent technology. Then, the continuation value of the patenting firm and the rival firm are $V_P = \frac{\lambda_H \Pi - c}{\lambda_H + \lambda_L}$ and $V_R = \frac{\lambda_L \Pi - c}{\lambda_H + \lambda_L}$.

Let x be the take-it-or-leave-it (TIOLI) offer that the patent holder makes to the rival firm for the right to use the new technology. After licensing, both firms can use the new technology. Thus, the expected payoffs of the patenting firm and the rival firm after licensing are $V_C + x$ and $V_C - x$. Then, the rival accepts the offer whenever $V_C - x^* = V_R$ and the patent holder chooses x^* such that it s satisfied with equality. With simple algebra, we can derive that

$$x^* = \frac{(\lambda_H - \lambda_L)(\lambda_H \Pi + c)}{2\lambda_H(\lambda_H + \lambda_L)} = \frac{\lambda_H - \lambda_L}{\lambda_H + \lambda_L}\left(V_C + \frac{c}{\lambda_H}\right) > 0. \tag{6}$$

Then, the expected payoff for the patenting firm after licensing is

$$V_L \equiv V_C + x^* = \left(1 + \frac{(\lambda_H - \lambda_L)c}{\lambda_H(\lambda_H \Pi - c)}\right) V_P > V_P. \tag{7}$$

As in the case of public information, a firm that patented the new technology can always be better off by licensing it.

5.2.2 Immediate-Disclosure Equilibrium

We first explore whether the first-best outcome can be achieved by allowing the firms to disclose the new technology and license it. Recall that in the first-best case, both firms do research, and a firm's new technology is immediately spilled over to the rival. Thus, we consider a strategy profile such that a firm with the new technology employs the *immediate-disclosure strategy*—a firm discloses (and licenses) the new technology as soon as it discovers—and a firm without the new technology employs the research strategy ($\sigma_t = 1$ for all $t \geq 0$). Then, we ask whether both firms playing this strategy can be sustained as an equilibrium.

Suppose that a firm (say Firm A) just discovered the new technology and Firm B has not disclosed it yet. Given that Firm B sticks to the immediate disclosure and research strategy, Firm A's belief that Firm B has the new technology is zero. Then, by disclosing the new technology, Firm A expects to license it, i.e., the expected payoff for Firm A after disclosure is V_L. Now consider Firm A's deviation to delay the disclosure by time dt. With the probability $\lambda_H dt$, Firm A wins the race and receives Π. But with the probability μdt, Firm B will discover the new technology and files a patent, but it will be disputed by Firm A's trade secret right. Thus, both firms will race with the new technology from then on and the expected payoff is V_C. With the probability $(1 - \lambda_H dt - \mu dt)$, neither of the events happens and Firm A licenses, then the expected payoff is V_L. Last, the flow cost cdt will be paid. To sum up, Firm A's expected payoff from delaying the disclosure is

$$\begin{aligned}\Pi \cdot \lambda_H dt + V_C \cdot \mu dt + (1 - \lambda_H dt - \mu dt) \cdot V_L - cdt &= V_L + [(\mu + 2\lambda_H)V_C - (\mu + \lambda_H)V_L]\, dt \\ &= V_L + [\lambda_H V_C - (\mu + \lambda_H)x^*]\, dt.\end{aligned}$$

Then, from $\delta = \lambda_H/(\mu+\lambda_H)$, the immediate-disclosure and research strategy can be sustained as an equilibrium if and only if $\delta V_C \leq x^*$. By using (6) and some algebra, this inequality is equivalent to:

$$\frac{(1-\delta)(\overline{\eta}(\delta) - \eta)}{2(\eta - 1 + \delta)} \leq \frac{c}{\lambda_H \Pi - c}. \tag{8}$$

From the assumption that $\lambda_L \Pi \geq c$, observe that (8) always holds if $\eta \geq \overline{\eta}(\delta)$. Recall that firms do research regardless of the rival's progress. It implies that there does not exist any incentive for a firm to conceal its progress. Therefore, the firms would monetize the new technology by licensing it as soon as it discovers, and the first-best outcome would be achieved.

Next, suppose that $\underline{\eta}(\delta) < \eta < \overline{\eta}(\delta)$. Then, (8) is equivalent to:

$$\pi = \frac{\lambda_L \Pi}{c} \leq 1 + \frac{\eta^2 - (1-\delta)^2}{\eta(\overline{\eta}(\delta) - \eta)} \equiv \underline{\pi}. \tag{9}$$

Also note that $\underline{\pi} > 1$ since $\eta > 1 - \delta > 0$. Therefore, when the reward of winning the race is sufficiently low ($1 \leq \pi \leq \underline{\pi}$), the firms would license the new technology as soon as it discovers. The following proposition formally summarizes the above results.

Proposition 3. *Suppose that one of the following conditions holds: (i) $\eta \geq \overline{\eta}(\delta)$; or (ii) $\eta \in (\underline{\eta}(\delta), \overline{\eta}(\delta))$ and $1 \leq \pi \leq \underline{\pi}$. Then, there exists an equilibrium in which firms fully allocate resources to research and license the new technology as soon as they access it.*

5.2.3 No-Disclosure Equilibrium

We now explore whether the worst-case scenario for the planner can be realized, i.e., firms never patent the new technology and the expected completion time corresponds to the private information setting.

First, we consider the case where $\tilde{\eta}(\delta) = \min\{2-\delta, \overline{\eta}(\delta)\} \leq \eta < \overline{\eta}(\delta) = 1+\delta$. By Proposition 2, in the equilibrium under the private information setting, firms do research until it succeeds ($T^* = \infty$ and $\sigma_t = 1$ for all $t \geq 0$). Suppose that both firms stick to this resource allocation strategy and never disclose their discoveries. When Firm A discovers the new technology at time t and never discloses it, the expected payoff of Firm A is $V_1^t = \{1 + \delta(1-q_t)\} \cdot V_C$ by Proposition 2 (b-i). If Firm A discloses the discovery at time t, Firm B has the new technology with the probability q_t. Thus, the expected payoff from the disclosure is $V_C \cdot q_t + V_L \cdot (1-q_t) = V_C + (1-q_t)x^*$. Therefore, the firm will not disclose if $x^* < \delta V_C$. We can also consider the case where Firm A discovers at time t but conceals until t' and decides to disclose or not at time t'. Even in this case, Firm A faces the same problem as before and will not disclose if $x^* < \delta V_C$. Recall that $x^* < \delta V_C$ is equivalent to $\pi > \underline{\pi}$. Therefore, if $\pi > \underline{\pi}(\eta, \delta)$, there exists an equilibrium such that firms never disclose their discoveries and do research until it succeeds.

Next, we consider the case where $\eta \in \big(\underline{\eta}(\delta), \tilde{\eta}(\delta)\big)$. By Proposition 2 (b-iii), in the equilibrium under private information, firms employ the stationary fall-back strategy (for some $T^* \in (0, \infty)$ and $\sigma^* \in (0,1)$, $\sigma_t = 1$ for all $0 \leq t < T^*$ and $\sigma_t = \sigma^*$ for all $t \geq T^*$). Suppose that Firm A discovers the new technology at $t \geq T^*$. If Firm A keeps the discovery secret, the expected payoff of Firm A is $V_1^* = \dfrac{2\delta}{\eta - 1 + \delta} V_C$. In addition, $V_1^* < \overline{V}_1(p^*) = \{1 + \delta(1-p^*)\} V_C$ (see Lemma C.8). On the other hand, if Firm A discloses the discovery, the expected payoff from the disclosure is $V_C + (1-p^*)x^*$. Then, Firm A does not disclose under the condition stronger than $x^* < \delta V_C$. In this case, there exists $\overline{\pi} > \underline{\pi}$ such that Firm A does not disclose when $\pi > \overline{\pi}$. The following proposition formally states this result.

Proposition 4. *Suppose that $\eta \in (\underline{\eta}(\delta), \overline{\eta}(\delta))$. Then, there exists $\overline{\pi} > \underline{\pi}$ such that for all $\Pi/c > \overline{\pi}$, there is an equilibrium in which (i) firms never patent the new technology (ii) firms employ the equilibrium resource allocations from Proposition 2.*

6 Conclusion

In this article, we study the long-lasting question of patent vs. secrecy by highlighting the firm's incentives to conceal breakthroughs to hinder the rival's strategic response. To do so, we introduce an innovation race model with multiple paths and show that firms' disclosing decisions depend on the reward for winning the race.

We show that when interim breakthroughs are public, patent protection is effective in inducing a more efficient allocation of R&D resources. However, when interim breakthroughs are private and stakes are high, patent protection has a limited effect. Based on this result, we can argue that, in some situations, higher stakes may reduce patenting and licensing which would decrease the pace of innovation.

There are many avenues open for further research. For example, we assume that there are exogenously given two paths towards innovation, and one of the paths requires two breakthroughs. However, in practice, there are numerous ways to make an innovation, and it often requires more than two breakthroughs. We also assume that a firm's R&D resources are fixed over time, but we could also allow firms to endogenously choose how much effort to put into each point in time. Finally, we assume the contest structure is given by the winner-takes-all competition, but we might consider a contest designing problem. We leave these intriguing questions and others for future work.

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Appendix

A Proofs for Public Information Setting

A.1 Transformation

It is useful to write conditions in terms of the rate μ. The following lemma summarizes this transformation.

Lemma A.1. *Let* $\overline{\mu} \equiv \frac{2\lambda_L\lambda_H}{\lambda_H-\lambda_L}$ *and* $\underline{\mu} \equiv \frac{\lambda_L(\lambda_H+\lambda_L)}{\lambda_H-\lambda_L}$. *Then,* $\eta \geq \overline{\eta}(\delta)$ *is equivalent to* $\mu \geq \overline{\mu}$ *and* $\eta \leq \underline{\eta}(\delta)$ *is equivalent to* $\mu \leq \underline{\mu}$.[13]

Proof of Lemma A.1. First, we have:

$$\begin{aligned}\eta - \overline{\eta}(\delta) = \eta - (1+\delta) =& \frac{\mu\lambda_H}{\lambda_L(\lambda_H+\mu)} - \frac{\lambda_H}{\lambda_H+\mu} - 1\\ =& \frac{\mu(\lambda_H-\lambda_L) - 2\lambda_L\lambda_H}{\lambda_L(\lambda_H+\mu)} = \frac{\lambda_H-\lambda_L}{\lambda_L(\lambda_H+\mu)}(\mu-\overline{\mu}).\end{aligned}$$

Therefore, $\eta \geq \overline{\eta}(\delta)$ is equivalent to $\mu \geq \overline{\mu}$. Next, we have

$$\begin{aligned}\left(\underline{\eta}(\delta) - \eta\right)\left(\eta - \frac{1-\sqrt{1+4\delta(1-\delta)}}{2}\right) =& -\eta^2 + \eta + (1-\delta)\delta\\ =& -\left(\frac{\mu\lambda_H}{\lambda_L(\lambda_H+\mu)}\right)^2 + \frac{\mu\lambda_H}{\lambda_L(\lambda_H+\mu)} + \frac{\mu\lambda_H}{(\lambda_H+\mu)^2}\\ =& \frac{\mu\lambda_H(\lambda_H-\lambda_L)}{\lambda_L^2(\lambda_H+\mu)^2}\left(\underline{\mu}-\mu\right).\end{aligned}$$

Note that$\eta > 1$ implies $\eta - \left\{1-\sqrt{1+4\delta(1-\delta)}\right\}/2 > 0$. Thus, $\eta \leq \underline{\eta}(\delta)$ is equivalent to $\mu \leq \underline{\mu}$. □

A.2 Proof of Proposition 1

Proposition 1 characterizes the MPE of the setting with observable technologies. A Markov strategy for player i is a mapping $\sigma^i : \Omega \to [0,1]$, where the state Ω represents the set of

[13] Although $\overline{\mu}$ and $\underline{\mu}$ are the functions of λ_L and λ_H, we suppress them to simplify.

firms that possess the new technology. A MPE consists of a profile of Markov strategies such that each of the players is best responding to the strategy of their opponent.[14]

The existence of a best response that is Markov to a Markov opponent strategy is supported by the stationarity of the problem from the firm's perspective. Thus, the best Markov response must also be a best response over all strategies. This means that we need only consider Markov deviations to determine the Markov Perfect Equilibria.

Given a Markov strategy profile, we can compute the expected value for each player at every state. Let U^j_ω denote Firm j's continuation value at the state ω. Notice that for every strategy, a positive transition rate from state ω to ω' implies that $\omega \subseteq \omega'$. This allows us to obtain the equilibrium Markov strategies and continuation value at any MPE via *backward induction*.

We begin with the state $\omega = \{A, B\}$. For any firm that possesses the new technology, it is optimal to develop with it. Thus, in any MPE, $\sigma^A(\{A, B\}) = \sigma^B(\{A, B\}) = 1$ and the continuation value is

$$U^A_{\{A,B\}} = U^B_{\{A,B\}} = \frac{1}{2}\left(\Pi - \frac{c}{\lambda_H}\right) = V_C \tag{10}$$

Next, we consider jointly the states $\omega = \{A\}$ and $\omega = \{B\}$, i.e. the cases in which only one of the firms possesses the new technology. The firm with the new technology will trivially find it optimal to develop with it. Thus, $\sigma^A(\{A\}) = \sigma^B(\{B\}) = 1$ in any MPE. The following lemma characterizes $\sigma^i(\{j\})$, i.e. the opponent firm's optimal action.

Lemma A.2. *If $\eta < \overline{\eta}(\delta)$ then, in any MPE, $\sigma^A(\{B\}) = \sigma^B(\{A\}) = 0$. When $\eta > \overline{\eta}(\delta)$ then, in any MPE, $\sigma^A(\{B\}) = \sigma^B(\{A\}) = 1$.*

Proof of Lemma A.2. Let $\omega = \{i\}$, i.e., only firm i possesses the new technology. Firm i will develop with the new technology. For firm j, the problem is to choose σ to maximize his continuation value:

$$U^j_{\{i\}} = \max_{\sigma\in[0,1]} \frac{\sigma\mu V_C + (1-\sigma)\lambda_L\Pi - c}{\sigma\mu + (1-\sigma)\lambda_L + \lambda_H}$$

Taking the derivative of the previous objective function with respect to the choice variable

[14]Existance of these equilibria for a larger class of continuous-time stochastic games with finite states and actions has been studied in Neyman (2017).

σ, and using (10) we obtain

$$\frac{(\lambda_H \Pi + c) \cdot [\mu(\lambda_H - \lambda_L) - 2\lambda_H \lambda_L]}{2\lambda_H(\lambda_H + \lambda_L + \sigma(\mu - \lambda_L))^2} = \frac{(\lambda_H \Pi + c)(\lambda_H - \lambda_L)(\mu - \overline{\mu})}{2\lambda_H(\lambda_H + \lambda_L + \sigma(\mu - \lambda_L))^2}$$

Note that the above equation is positive if and only if $\mu > \bar{\mu}$, or equivalently by Lemma A.1, $\eta > \overline{\eta}(\delta)$. Therefore, the best response is $\sigma^i(\{j\}) = 1$ if $\eta > \overline{\eta}(\delta)$, and $\sigma^i(\{j\}) = 0$ if $\eta < \overline{\eta}$. In addition, we have

$$U^j_{\{i\}} = \begin{cases} \dfrac{\mu V_C - c}{\mu + \lambda_H}, & \text{if } \eta \geq \overline{\eta}(\delta), \\ \dfrac{\lambda_L \Pi - c}{\lambda_L + \lambda_H}, & \text{if } \eta < \overline{\eta}(\delta). \end{cases}$$

□

Finally, it remains to analyze the state $\omega = \{\emptyset\}$. We divide the analysis into the following three lemmas.

Lemma A.3. *Assume $\eta < \underline{\eta}(\delta)$. Then the unique MPE involves $\sigma^A(\emptyset) = \sigma^B(\emptyset) = 0$.*

Proof of Lemma A.3. Notice that $\eta < \underline{\eta}(\delta) \leq \overline{\eta}(\delta)$. Thus, By Lemma A.2, once a firm finds the new technology, i.e., $\omega = \{A\}$ or $\{B\}$, the other one switches to developing with the incumbent technology since. In these cases, the values of the firms are given as follows:

$$U^A_{\{A\}} = U^B_{\{B\}} = \frac{\lambda_H \Pi - c}{\lambda_L + \lambda_H} \qquad \text{and} \qquad U^B_{\{A\}} = U^A_{\{B\}} = \frac{\lambda_L \Pi - c}{\lambda_L + \lambda_H}. \tag{11}$$

Back to the state $\emptyset$, let $\pi_i(\sigma^i_\emptyset, \sigma^j_\emptyset)$ be the expected continuation payoff of Firm i when firms play the actions $\sigma^i_\emptyset$ and $\sigma^j_\emptyset$ as long as neither firm obtains the new technology, and continue with the optimal actions thereafter.

Given that firm j plays $\hat{\sigma}$ at state $\omega = \emptyset$, the best response of firm i is:

$$\max_{\sigma \in [0,1]} \quad \frac{\sigma \mu U^i_{\{i\}} + (1 - \sigma)\lambda_L \Pi + \hat{\sigma} \mu U^i_{\{j\}} - c}{\sigma \mu + (1 - \sigma)\lambda_L + \hat{\sigma}\mu + (1 - \hat{\sigma})\lambda_L} \tag{12}$$

Taking the derivative of the previous objective function with respect to σ and using eq. 11, we get

$$\frac{[\mu(\lambda_H - \lambda_L) - \lambda_L(\lambda_H + \lambda_L)] \cdot [c + \Pi(\lambda_L + \hat{\sigma}(\mu - \lambda_L)]}{(\lambda_H + \lambda_L)(\lambda_L(2 - \sigma - \hat{\sigma}) + \mu(\sigma + \hat{\sigma}))^2} \tag{13}$$

This is negative since $\eta < \underline{\eta}(\delta)$ implies, by Lemma A.1, $\mu < \underline{\mu} = \frac{\lambda_L(\lambda_L+\lambda_H)}{\lambda_H-\lambda_L}$. Therefore, the best action independently of the action of the opponent at state $\emptyset$ is to use the incumbent technology, given optimal continuation. □

Lemma A.4. *Assume $\eta \in (\underline{\eta}(\delta), \overline{\eta}(\delta))$. Then the unique MPE involves $\sigma^A(\emptyset) = \sigma^B(\emptyset) = 1$.*

Proof of Lemma A.4. The problem of firm i at state $\emptyset$ is, as before,

$$\max_{\sigma\in[0,1]} \quad \frac{\sigma\mu U^i_{\{i\}} + (1-\sigma)\lambda_L\Pi + \hat{\sigma}\mu U^i_{\{j\}} - c}{\sigma\mu + (1-\sigma)\lambda_L + \hat{\sigma}\mu + (1-\hat{\sigma})\lambda_L}$$

By the assumption $\eta < \overline{\eta}(\delta)$ and Lemma A.2, the equations (11) also hold for this case. Thus, the derivative is the same as we obtained in eq. 13. The difference is that now, since $\eta > \underline{\eta}(\delta)$), the derivative changes sign. Thus, independently of the action chosen by the opponent in state $\emptyset$, it is optimal to do research (given optimal continuation).

Thus, the strategy profile that both firms play fall-back strategies constitutes the unique equilibrium. □

Lemma A.5. *Assume $\eta > \overline{\eta}(\delta)$. Then the unique MPE involves $\sigma^A(\emptyset) = \sigma^B(\emptyset) = 1$.*

Proof of Lemma A.5. By Lemma A.2, in any MPE the firms do research once the opponent obtains the new technology. Thus,

$$U^i_{\{i\}} = \frac{\lambda_H\Pi + \mu V_C - c}{\mu + \lambda_H} \qquad \text{and} \qquad U^i_{\{j\}} = \frac{\mu V_C - c}{\mu + \lambda_H}.$$

Using these in the problem of firm i in state $\emptyset$ (eq. 12) and taking derivatives with respect to the choice variable σ we obtain:

$$\frac{\lambda_H\Pi(\mu(\lambda_H-\lambda_L) - \lambda_H\lambda_L)[\lambda_L + \hat{\sigma}(\mu-\lambda_L)] + c(\lambda_H^2(\mu-\lambda_L) + \mu^2(\lambda_H-\lambda_L) - 3\lambda_L\lambda_H\mu)}{\lambda_H(\lambda_H+\mu)(\lambda_L(2-\sigma-\hat{\sigma}) + \mu(\sigma+\hat{\sigma}))^2}$$

Note that the denominator is positive. The numerator can be rewritten as

$$\{\mu(\lambda_H-\lambda_L) - \lambda_H\lambda_L\}\cdot\{\lambda_H\Pi\cdot(\lambda_L + \hat{\sigma}(\mu-\lambda_L)) + c\cdot(\mu+\lambda_H)\}$$

Which is positive since $\mu > \frac{2\lambda_H \lambda_L}{\lambda_H - \lambda_L} > \frac{\lambda_H \lambda_L}{\lambda_H - \lambda_L}$. Thus, the best response at state $\emptyset$, for every $\hat{\sigma}$, is to choose $\sigma = 1$. □

A.3 Proof of Corollary 3.1

When $\underline{\eta}(\delta) \geq \eta$, the expected completion time is $\frac{1}{2\lambda_L}$ since both firms develop with the incumbent technology. Next, when $\eta \in (\underline{\eta}(\delta), \overline{\eta}(\delta))$, the expected time until one of the firms discover new technology is $\frac{1}{2\mu}$. Then, a firm develops with the new technology and the other firm develops with the incumbent technology, thus, the expected completion time from then on is $\frac{1}{\lambda_H + \lambda_L}$. Therefore, the (total) expected completion time is $\frac{1}{2\mu} + \frac{1}{\lambda_H + \lambda_L}$. Last, when $\eta \geq \overline{\eta}(\delta)$, unlike in the previous case, the firm without the new technology keeps doing research. Then, the expected time until either the firm with the new technology develops the product or the firm without the new technology discovers it is $\frac{1}{\lambda_H + \mu}$. With the probability $\frac{\mu}{\lambda_H + \mu}$, the firm without the new technology discovers it earlier than the product development, then it takes an additional expected completion time $\frac{1}{2\lambda_H}$. Therefore, the total expected completion time is

$$\frac{1}{2\mu} + \frac{1}{\lambda_H + \mu} + \frac{\mu}{\lambda_H + \mu} \cdot \frac{1}{2\lambda_H} = \frac{1}{2}\left(\frac{1}{\mu} + \frac{1}{\lambda_H} + \frac{1}{\lambda_H + \mu}\right).$$

B Private Information: Evolution of Beliefs

Proof of Lemma 4.1. Suppose that Firm j allocates σ_t^j attention to research conditional on not having the new technology. Finally, let $\Sigma_t = \int_0^t \sigma_s^j \, ds$.

Let p_t^i be the belief of Firm i that Firm j obtained the new technology by time t.

$$\begin{aligned} \frac{1 - p_t}{p_t} &= \frac{\Pr(\text{no success in research and development})}{\Pr(\text{success in research but no success in development})} \\ &= \frac{e^{-\mu \Sigma_t} \cdot e^{-\lambda_L (t - \Sigma_t)}}{\int_0^t \mu \cdot \sigma_s \cdot e^{-\mu \Sigma_s} \cdot e^{-\lambda_L (s - \Sigma_s)} \cdot e^{-\lambda_H (t-s)} \, ds} \end{aligned} \quad (14)$$

Let U_t and R_t be the numerator and the denominator of the right-hand side of (14). Note that

$$\frac{\partial U_t}{\partial t} = -U_t \cdot \big((\mu - \lambda_L)\sigma_t^j + \lambda_L\big)$$
$$\frac{\partial R_t}{\partial t} = \mu \cdot \sigma_t^j \cdot U_t - \lambda_H \cdot R_t$$

By Differentiating (14) and multiplying R_t/U_t,

$$\begin{aligned} -\frac{\dot{p}_t^i}{(1-p_t^i)p_t^i} &= \frac{-U_t \cdot \big((\mu - \lambda_L)\sigma_t^j + \lambda_L\big) \cdot R_t - \mu \cdot \sigma_t^j \cdot U_t^2 + \lambda_H \cdot R_t \cdot U_t}{R_t^2} \cdot \frac{R_t}{U_t} \\ &= -\big((\mu - \lambda_L)\sigma_t^j + \lambda_L\big) + \lambda_H - \mu \cdot \sigma_t^j (1-p_t^i)/p_t^i \\ &= \big\{\lambda_H - \lambda_L(1-\sigma_t^j)\big\} - \mu \cdot \sigma_t^j / p_t^i. \end{aligned}$$

By multiplying $-(1-p_t^i)p_t^i$, we have (3). □

Proof of Lemma 4.2. By plugging $\sigma_t = 1$ to (3), we have $\dot{p}_t = (\mu - \lambda_H p_t)(1-p_t)$. By rearranging the differential equation, we can derive that

$$\lambda_H - \mu = (\lambda_H - \mu)\frac{\lambda_H \dot{p}_t}{(\mu - \lambda_H p_t)(\lambda_H - \lambda_H p_t)} = \frac{d}{dt}\log\left(\frac{\lambda_H - \lambda_H p_t}{\mu - \lambda_H p_t}\right).$$

Then, from $p_0 = 0$, we can derive that

$$\frac{\lambda_H(1-p_T)}{\mu - \lambda_H p_T} = \frac{\lambda_H}{\mu} e^{(\lambda_H - \mu)T}.$$

By rearranging the above equation, we have (4).

Observe that

$$\dot{q}_T = \frac{\mu(\lambda_H - \mu)^2 e^{(\lambda_H + \mu)T}}{(\lambda_H e^{\lambda_H T} - \mu e^{\mu T})^2} > 0.$$

Thus, q is increasing in T.

When $\mu > \lambda_H$,

$$\lim_{T\to\infty} q_T = \lim_{T\to\infty} \frac{\frac{1}{\lambda_H}\left(e^{(\lambda_H - \mu)T} - 1\right)}{\frac{1}{\mu}e^{(\lambda_H - \mu)T} - \frac{1}{\lambda_H}} = 1.$$

When $\mu < \lambda_H$,

$$\lim_{T\to\infty} q_T = \lim_{T\to\infty} \frac{\frac{1}{\lambda_H}\left(1 - e^{(\mu-\lambda_H)T}\right)}{\frac{1}{\mu} - \frac{1}{\lambda_H} e^{(\mu-\lambda_H)T}} = \frac{\mu}{\lambda_H}.$$

□

C Private Information: Equilibria

C.1 Equilibrium values

In equilibrium, a firm's expected payoffs with and without the new technology are $V_0^t = \overline{V}_0(q_t)$ and $V_1^t = \overline{V}_1(q_t)$ where q is the belief defined in (4) and

$$\overline{V}_1(q) \equiv \frac{1}{2}\left(\Pi - \frac{c}{\lambda_H}\right)(1 + \delta(1-q)), \tag{15}$$

$$\overline{V}_0(q) \equiv \frac{1}{2}\left(\Pi - \frac{c}{\mu} - \frac{c}{\lambda_H}\right)(1 - \delta q) - \frac{c}{2(\lambda_H + \mu)}. \tag{16}$$

In equilibrium, the expected payoff of each firm is $V_t^0 = \frac{\lambda_L \Pi - c}{2\lambda_L}$ for all $t \geq 0$.[15]

Moreover, for all $t \geq T^*$, $\sigma_t = \sigma^*$, $p_t = p^*$, $V_t^1 = V_1^*$ and $V_t^0 = V_0^*$ where

C.2 Cutoff Structure

C.2.1 Lemmas

Lemma C.1. *If the belief process $\{p_t\}$ is derived from a symmetric Markov strategy σ, then $\dot{p}_t \geq 0$ for all $t \geq 0$.*

Proof of Lemma C.1. Suppose that $\dot{p}_t < 0$ for some $t \geq 0$. Note that the belief is nonnegative. Then, $p_{t-\eta} > p_t \geq 0$ for a small $\eta > 0$. Also note that $p_0 = 0$ and $\dot{p}_0 = \mu \cdot \sigma_0 \geq 0$ by (3). By the continuity of p, there exists $t' < t$ such that $p_{t'} = p_t$ and $\dot{p}_{t'} \geq 0$. However, since the strategy is Markov, $\sigma_{t'} = \sigma_t$, which gives $\dot{p}_{t'} = \dot{p}_t$ and contradicts $\dot{p}_{t'} \geq 0 > \dot{p}_t$. Therefore, $\dot{p}_t \geq 0$ for all $t \geq 0$. □

[14] See Remark 1 for the expected payoffs for $t \in [0, T^*]$

[15] When a firm possesses the new technology—though it is off the equilibrium path—the expected payoff is $V_1^t = \frac{\lambda_H \Pi - c}{\lambda_H + \lambda_L}$.

Lemma C.2. *If σ constitutes a symmetric Markov equilibrium, $\sigma_t = 0$ for all $t \geq 0$ or $\sigma_t > 0$ for all $t \geq 0$.*

Proof of Lemma C.2. Consider the case with $p_t > 0$. If $\sigma_t = 0$, by (3), $\dot{p}_t = -(\lambda_H - \lambda_L)p_t(1 - p_t)$. Since p_t cannot be equal to 1, $\dot{p}_t < 0$, which contradicts the previous result. Therefore, $\sigma_t > 0$ whenever $p_t > 0$. If $\sigma_0 = 0$, then $\dot{p}_0 = 0$ and the belief stays at 0. By the Markov property, $\sigma_t = 0$ for all $t \geq 0$. If $\sigma_0 > 0$, then $\dot{p}_0 > 0$ and $p_t > 0$ for a small enough $t > 0$. Then, $\sigma_t = 0$ will never be chosen, i.e., $\sigma_t > 0$ for all $t \geq 0$. □

Lemma C.3. *If σ constitutes a symmetric Markov equilibrium and $\sigma_S \in (0,1)$ for some $S \geq 0$, then $\mu(V_t^1 - V_t^0) = \lambda_L(\Pi - V_t^0)$ for all $t \geq S$.*

Proof of Lemma C.3. By Lemma C.2, if $\sigma_S > 0$ for some $S \geq 0$, $\sigma_t > 0$ for all $t > 0$, thus, $\mu(V_t^1 - V_t^0) \geq \lambda_L(\Pi - V_t^0)$ for all $t \geq 0$.

Assume the contrary. Then, we can properly define $T \equiv \inf\{t > S \mid \mu(V_t^1 - V_t^0) > \lambda_L(\Pi - V_t^0)\}$. Then, $\mu(V_s^1 - V_s^0) = \lambda_L(\Pi - V_s^0)$ holds for $S \leq s \leq T$, and for some $\delta > 0$, $\mu(V_s^1 - V_s^0) > \lambda_L(\Pi - V_s^0)$ for all $T < s < T + \delta$.

When $\mu(V_t^1 - V_t^0) = \lambda_L(\Pi - V_t^0)$, we show that $\mu(\dot{V}_t^1 - \dot{V}_t^0) \leq -\lambda_L \dot{V}_t^0$ if and only if

$$\lambda_H p_t + \lambda_L(1 - p_t)(1 - \sigma_t) \leq \frac{\mu(\lambda_H - \lambda_L)(\Pi - V_t^1) - \lambda_L c}{\lambda_L \Pi}.$$

First, by (HJB$_1$) and (HJB$_0$), we have

$$\mu \dot{V}_t^1 = \mu(\lambda_H + X_t)V_t^1 - \mu(\lambda_H \Pi - c), \tag{17}$$
$$(\mu - \lambda_L)\dot{V}_t^0 = (\mu - \lambda_L)X_t V_t^0 - (\mu - \lambda_L)(\mu(V_t^1 - V_t^0) - c), \tag{18}$$

where $X_t = \lambda_H p_t + \lambda_L(1 - p_t)(1 - \sigma_t)$. Also note that $(\mu - \lambda_L)(V_t^1 - V_t^0) = \lambda_L(\Pi - V_t^1)$. Then, $\mu(\dot{V}_t^1 - \dot{V}_t^0) \leq -\lambda_L \dot{V}_t^0$ is equivalent to:

$$\begin{aligned}
\lambda_L \Pi \cdot X_t =& \left\{\mu V_t^1 - (\mu - \lambda_L)V_t^0\right\} X_t \\
&\leq \mu(\lambda_H \Pi - c) - \mu\lambda_H V_t^1 + (\mu - \lambda_L)c - \mu(\mu - \lambda_L)(V_t^1 - V_t^0) \\
&= \mu(\lambda_H - \lambda_L)(\Pi - V_t^1) - \lambda_L c.
\end{aligned}$$

Let $\sigma_{T^-} := \lim_{t \to T^-} \sigma_t$ and $\sigma_{T^+} := \lim_{t \to T^+} \sigma_t$. Note that $\sigma_{T^+} = 1$. By the continuity of p and V^1, we have $p_{T^-} = p_{T^+} = p_T$ and $V^1_{T^-} = V^1_{T^+} = V^1_T$.

First, consider the case with $\sigma_{T^-} < 1$.[16] In this case, we have

$$\begin{aligned} X_{T^+} =& \lambda_H p_T + \lambda_L (1 - p_T)(1 - \sigma_{T^+}) \\ <& \lambda_H p_T + \lambda_L (1 - p_T)(1 - \sigma_{T^-}) = \frac{\mu(\lambda_H - \lambda_L)(\Pi - V^1_T) - \lambda_L c}{\lambda_L \Pi}. \end{aligned}$$

Then, $\mu(\dot{V}^1_{T^+} - \dot{V}^0_{T^+}) < -\lambda_L \dot{V}^0_{T^+}$. Since $\mu(V^1_T - V^0_T) = \lambda_L(\Pi - V^0_T)$, for small enough $\eta > 0$, $\lambda_L(\Pi - V^0_{T+\eta}) > \mu(V^1_{T+\eta} - V^0_{T+\eta})$ which contradicts $\sigma_{T^+} = 1$.

Next, consider the case with $\sigma_{T^-} = 1$. Note that $\mu(V^1_T - V^0_T) = \lambda_L(\Pi - V^0_T)$ and $\mu(\dot{V}^1_T - \dot{V}^0_T) = -\lambda_L \dot{V}^0_T$. If we show that $\mu(\ddot{V}^1_{T^+} - \ddot{V}^0_{T^+}) < -\lambda_L \ddot{V}^0_{T^+}$, it contradicts $\sigma_{T^+} = 1$. Observe that $\sigma_{T^+} = 1$ and $\dot{\sigma}_{T^+} = 0$, thus, $\dot{X}_{T^+} = \lambda_H \dot{p}_T$. By taking derivatives for (17) and (18), we can derive that

$$\begin{aligned} \mu \ddot{V}^1_{T^+} =& \mu \lambda_H \left[(1 + p_T) \dot{V}^1_T + \dot{p}_T V^1_T \right], \\ (\mu - \lambda_L) \ddot{V}^1_{T^+} =& (\mu - \lambda_L) \left[\lambda_H p_T \dot{V}^0_T + \lambda_H \dot{p}_T V^0_T - \mu(\dot{V}^1_T - \dot{V}^0_T) \right]. \end{aligned}$$

Then, we have

$$\begin{aligned} \mu \ddot{V}^1_{T^+} - (\mu - \lambda_L) \ddot{V}^0_{T^+} =& \lambda_H p_T \left\{ \mu \dot{V}^1_T - (\mu - \lambda_L) \dot{V}^0_T \right\} + \mu \lambda_H \dot{V}^1_T \\ & + \lambda_H \dot{p}_T \left\{ \mu V^1_T - (\mu - \lambda_L) V^0_T \right\} + \mu(\mu - \lambda_L)(\dot{V}^1_T - \dot{V}^0_T) \\ =& \mu(\lambda_H - \lambda_L) \dot{V}^1_T. \end{aligned} \tag{19}$$

Note that $(\lambda_H \Pi - c)/(\lambda_H + \lambda_H p_T)$ is the expected payoff of the firm (with the new technology) at time T if the opponent shuts down when it does not possess the new technology at time T. Then, in equilibrium, the expected payoff cannot exceed this value: $V^1_T < (\lambda_H \Pi - c)/(\lambda_H + \lambda_H p_T)$. From (17), we have $\dot{V}^1_T < 0$. By (19), $\mu(\ddot{V}^1_{T^+} - \ddot{V}^0_{T^+}) < -\lambda_L \ddot{V}^0_{T^+}$, which contradicts $\sigma_{T^+} = 1$. □

[16] It is possible that σ_{T^-} is not properly defined. Even in this case, the following argument still holds by considering a converging subsequence of $\{\sigma_t\}$.

Lemma C.4. *Suppose that there exists* $0 \leq T < \infty$ *such that* σ *is a symmetric Markov equilibrium with* $\sigma_t = 1$ *for all* $t < T$ *and* $\mu(V_t^1 - V_t^0) = \lambda_L(\Pi - V_t^0)$ *for all* $t \geq T$*. Then, for all* $t \geq T$*,* $\sigma_t = \sigma^*$*,* $p_t = p^*$*,* $V_t^1 = V_1^*$*, and* $V_t^0 = V_0^*$*, i.e.,* σ *is a stationary fall-back equilibrium. In addition,* $T = \frac{1}{\lambda_H - \mu} \log\left(\frac{\mu(1-p^*)}{\mu - \lambda_H p^*}\right)$.

Proof of Lemma C.4. To have $0 < \sigma_t < 1$ for all $t \geq T$,

$$\mu(V_t^1 - V_t^0) = \lambda_L(\Pi - V_t^0). \tag{20}$$

By taking a derivative, we also have

$$\mu \dot{V}_t^1 = (\mu - \lambda_L)\dot{V}_t^0. \tag{21}$$

Define $X(p_t, \sigma_t) \equiv \lambda_H p_t + \lambda_L(1 - p_t)(1 - \sigma_t)$. By (HJB$_1$), we have

$$\mu \dot{V}_t^1 = X(p_t, \sigma_t)\mu \cdot V_t^1 - \mu\lambda_H(\Pi - V_t^1) + \mu c. \tag{22}$$

By (HJB$_0$) and (20), we also have

$$\begin{aligned}(\mu - \lambda_L)\dot{V}_t^0 =& X(p_t, \sigma_t)(\mu - \lambda_L)V_t^0 - (\mu - \lambda_L)\mu(V_t^1 - V_t^0) + (\mu - \lambda_L)c \\ =& X(p_t, \sigma_t)(\mu V_t^1 - \lambda_L \Pi) - \mu\lambda_L(\Pi - V_t^1) + (\mu - \lambda_L)c.\end{aligned} \tag{23}$$

By using (21), (22) and (23), we have

$$X(p_t, \sigma_t) = \frac{\mu(\lambda_H - \lambda_L)(\Pi - V_t^1) - \lambda_L c}{\lambda_L \Pi}. \tag{24}$$

By plugging (24) into (HJB$_1$), we can derive that

$$\begin{aligned}0 =& \dot{V}_t^1 - \frac{1}{\lambda_L \Pi}\left\{\mu(\lambda_H - \lambda_L)(\Pi - V_t^1) - \lambda_L c\right\} V_t^1 + \lambda_H(\Pi - V_t^1) - c \\ =& \dot{V}_t^1 - \frac{\mu(\lambda_H - \lambda_L)}{\lambda_L \Pi}\left\{V_t^1 - \frac{\lambda_L(\lambda_H \Pi - c)}{\mu(\lambda_H - \lambda_L)}\right\}(\Pi - V_t^1) \\ =& \dot{V}_t^1 - \frac{\alpha}{\Pi - \beta}(V_t^1 - \beta)(\Pi - V_t^1)\end{aligned} \tag{25}$$

where $\alpha \equiv \frac{\mu(\lambda_H - \lambda_L)(\Pi - \beta)}{\lambda_L \Pi}$ and $\beta \equiv \frac{\lambda_L(\lambda_H \Pi - c)}{\mu(\lambda_H - \lambda_L)}$. Note that

$$\Pi - \beta = \frac{\lambda_L \lambda_H}{\lambda_H - \lambda_L}\left[\left(\frac{1}{\lambda_L} - \frac{1}{\lambda_H} - \frac{1}{\mu}\right)\Pi + \frac{c}{\mu}\right] > 0.$$

Thus, α and β are strictly positive.

Also note that $\Pi - c/\lambda_H > V_t^1$ for all $t \geq 0$ since the firm's expected profit under the competition cannot exceed that without the competition. If $V_T^1 > \beta$, $V_t^1 > \beta$ for all $t \geq T$. If not, there exists $t > T$ such that $V_t > \beta$ and $\dot{V}_t = 0$, which contradict (25). Likewise, if $V_T^1 < \beta$, $V_t^1 < \beta$ for all $t \geq T$. Now suppose that $V_T^1 \neq \beta$. By (25), we have

$$\alpha = \frac{(\Pi - \beta)}{(V_t^1 - \beta)(\Pi - V_t^1)}\dot{V}_t^1 = \frac{d}{dt}\log\left(\frac{|\beta - V_t^1|}{\Pi - V_t^1}\right)$$

By integrating the above equation side-by-side from T to t, we have

$$\begin{aligned} & \alpha(t - T) = \log\left(\frac{|\beta - V_t^1|}{\Pi - V_t^1}\right) - \log\left(\frac{|\beta - V_T^1|}{\Pi - V_T^1}\right) \\ \iff & \frac{|\beta - V_T^1|}{\Pi - V_T^1}e^{\alpha(t-T)} = \frac{|\beta - V_t^1|}{\Pi - V_t^1}. \end{aligned}$$

Notice that the right-hand-side is bounded above and below since $V_t^1 < \Pi - c/\lambda_H$. The left-hand-side diverges to positive or negative infinite. Thus, it must be that $V_T^1 = \beta$.

In addition, by solving (25) with the initial condition $V_T^1 = \beta$, we have $V_t^1 = \beta = V_1^*$ for all $t \geq T$. By plugging $V_t^1 = \beta$ into (20), we also have

$$V_t^0 = \frac{\mu\beta - \lambda_L \Pi}{\mu - \lambda_L} = \frac{\lambda_L}{\mu - \lambda_L}\left(-\Pi + \frac{\lambda_H \Pi - c}{\lambda_H - \lambda_L}\right) = \frac{\lambda_L}{\mu - \lambda_L} \cdot \frac{\lambda_L \Pi - c}{\lambda_H - \lambda_L} = V_0^*$$

Next, by plugging $V_t^1 = \beta$ into (24), we have

$$\lambda_H p_t + \lambda_L(1 - p_t)(1 - \sigma_t) = \frac{\mu(\lambda_H - \lambda_L)(\Pi - \beta) - \lambda_L c}{\lambda_L \Pi} = \frac{\lambda_H - \lambda_L}{\lambda_L}\mu - \lambda_H,$$

or equivalently,

$$1 - \sigma_t = \frac{\left(\frac{\lambda_H - \lambda_L}{\lambda_L}\right)\mu - \lambda_H(1 + p_t)}{\lambda_L(1 - p_t)}. \tag{26}$$

From (3), we have

$$\begin{aligned}
\dot{p}_t =&(1-p_t)\left[\mu-\lambda_H p_t-(1-\sigma_t)(\mu-\lambda_L p_t)\right]\\
=&(1-p_t)(\mu-\lambda_H p_t)-\left\{\left(\frac{\lambda_H-\lambda_L}{\lambda_L}-\lambda_H(1+p_t)\right)\mu\right\}\left(\frac{\mu}{\lambda_L}-p_t\right)\\
=&\frac{\mu(\lambda_H-\lambda_L)}{\lambda_L^2}(\underline{\mu}-\mu)+\left(\frac{\lambda_H-\lambda_L}{\lambda_L}\right)(2\mu-\overline{\mu})p_t\\
=&\frac{(\lambda_H-\lambda_L)(2\mu-\overline{\mu})}{\lambda_L}(p_t-p^*).
\end{aligned}$$

If $p_T \neq p^*$, then the solution of the above differential equation diverges and contradicts $0 \leq p_t \leq 1$ for all $t \geq T$. Therefore, $p_T = p^*$ and it also gives $p_t = p^*$ for all $t \geq T$. Also note that

$$1-p^* = \frac{(\overline{\mu}-\mu)(\mu-\lambda_L)}{\lambda_L(2\mu-\overline{\mu})}.$$

By plugging this into (26), for all $t \geq T$, we have

$$\begin{aligned}
\sigma_t =&1-\frac{\left(\frac{\lambda_H-\lambda_L}{\lambda_L}\right)\mu-2\lambda_H+\lambda_H(1-p^*)}{\lambda_L(1-p^*)}\\
=&1-\frac{\frac{\lambda_H-\lambda_L}{\lambda_L}(\mu-\overline{\mu})+\lambda_H\frac{(\overline{\mu}-\mu)(\mu-\lambda_L)}{\lambda_L(2\mu-\overline{\mu})}}{\frac{(\overline{\mu}-\mu)(\mu-\lambda_L)}{(2\mu-\overline{\mu})}}\\
=&1-\frac{-(\lambda_H-\lambda_L)(2\mu-\overline{\mu})+\lambda_H(\mu-\lambda_L)}{\lambda_L(\mu-\lambda_L)}\\
=&1-\frac{-(\lambda_H-\lambda_L)(\mu-\lambda_L)+\lambda_L(\mu+\lambda_L)}{\lambda_L(\mu-\lambda_L)}=\sigma^*.
\end{aligned}$$

Last, since $\sigma_t = 1$ for all $0 \leq t \leq T$, the belief that the opponent has the new technology at time t is $p^* = p_T = q_T$ where q is defined as in (4). By the definition of q, we can derive that $e^{(\lambda_H-\mu)T} = \frac{\mu(1-p^*)}{\mu-\lambda_H p^*}$, or equivalently, $T = \frac{1}{\lambda_H-\mu}\log\left(\frac{\mu(1-p^*)}{\mu-\lambda_H p^*}\right)$. □

C.2.2 Proof of Proposition 2 (a)

Proof of Proposition 2 (a). By Lemma C.1, σ satisfies $\sigma_t = 0$ for all $t \geq 0$ or $\sigma_t > 0$ for all $t \geq 0$. In the former case, we can set $T^* = 0$ and $\sigma^* = 0$, then we have $\sigma_t = \sigma^*$ for all $t > T^* = 0$. Now consider the latter case. If $\{t \geq 0 | \sigma_t \in (0,1)\}$ is empty, set $T^* = \infty$.

Then, $\sigma_t = 1$ for all $t < T^*$. If $\{t \geq 0 | \sigma_t \in (0,1)\}$ is nonempty, we can properly define $T^* \equiv \inf\{t \geq 0 | \sigma_t \in (0,1)\} < \infty$. Then, $\sigma_t = 1$ for all $t < T^*$. In addition, by Lemma C.3, $\mu(V_t^1 - V_t^0) = \lambda_L(\Pi - V_t^0)$ for all $t \geq T^*$. By Lemma C.4, $\sigma_t = \sigma^*$ for all $t \geq T^*$. □

C.3 Equilibrium Characterization

C.3.1 The Equilibrium with Incumbent Strategies

Lemma C.5. *Suppose that σ is the incumbent strategy, i.e., $\sigma_t = 0$ for all $t \geq 0$. Then, $\sigma^A = \sigma^B = \sigma$ constitutes a symmetric Markov equilibrium if and only if $\mu \leq \underline{\mu}$.*

Proof of Lemma C.5. Suppose that the incumbent strategy constitutes an equilibrium. Since neither firm conducts research, the belief that the other firm possesses the new technology is 0, i.e., $p_t = 0$ for all $t \geq 0$. Observe that $V_t^0 = \frac{\lambda_L \Pi - c}{2\lambda_L}$ since both firms develop with the incumbent technology. If a firm happens to have the new technology and the other firm sticks with the strategy, the expected payoff is $\frac{\lambda_H \Pi - c}{\lambda_H + \lambda_L}$, i.e., $V_t^1 = \frac{\lambda_H \Pi - c}{\lambda_H + \lambda_L}$ for all $t \geq 0$. To support this equilibrium, from (HJB$_0$), $\mu(V_t^1 - V_t^0) \leq \lambda_L(\Pi - V_t^0)$ needs to hold. By plugging V_t^1 and V_t^0 in, we have

$$\begin{aligned}
& \mu\left(\frac{\lambda_H \Pi - c}{\lambda_H + \lambda_L} - \frac{\lambda_L \Pi - c}{2\lambda_L}\right) \leq \lambda_L\left(\Pi - \frac{\lambda_L \Pi - c}{2\lambda_L}\right) \\
\iff \quad & \frac{\mu(\lambda_L \Pi + c)(\lambda_H - \lambda_L)}{2\lambda_L(\lambda_H + \lambda_L)} \leq \frac{\lambda_L \Pi + c}{2} \\
\iff \quad & \mu \leq \frac{\lambda_L(\lambda_H + \lambda_L)}{\mu(\lambda_H - \lambda_L)} = \underline{\mu}.
\end{aligned}$$

Now suppose that $\mu \leq \underline{\mu}$. By the above inequality, the strategy profile with $\sigma_t = 0$ for all $t \geq 0$ constitutes an equilibrium, i.e., the incumbent equilibrium exists. □

C.3.2 The Equilibrium with Research Strategies

Lemma C.6. *Suppose that for some T, both firms play $\sigma_t = 1$ for all $0 \leq t \leq T$. Then, there exist $C_0, C_1 \in \mathbb{R}$ such that the expected payoffs of the firm with and without the new* *technology at time $t \in [0, T]$ is given as follows:*

$$V_t^1 = \overline{V}_1(q_t) + C_1 \cdot (1 - q_t) \cdot \left(\frac{\mu - \lambda_H q_t}{1 - q_t}\right)^{\frac{\mu + \lambda_H}{\mu - \lambda_H}}, \tag{27}$$

$$V_t^0 = \overline{V}_0(q_t) + \left(C_0 \left(\frac{\mu}{\lambda_H} - q_t\right) - C_1 \frac{\mu}{\lambda_H}\right) \cdot \left(\frac{\mu - \lambda_H q_t}{1 - q_t}\right)^{\frac{\mu + \lambda_H}{\mu - \lambda_H}}.[17] \tag{28}$$

Moreover, if both firms play the research strategy ($T = \infty$), $C_1 = C_0 = 0$, i.e., $V_t^1 = \overline{V}_1(q_t)$ and $V_t^0 = \overline{V}_0(q_t)$.

Proof of Lemma C.6. Consider V_t^n as a value function with respect to the belief process q_t defined as in (4): $V_t^n = V_n(q_t)$. Note that $\dot{V}_t^n = V_n'(q_t)\dot{q}_t = V_n'(q_t)(\mu - \lambda_H q_t)(1 - q_t)$. By plugging this into (HJB_1), we have

$$0 = V_1'(q)(\mu - \lambda_H q)(1 - q) - \lambda_H(1 + q)V_1(q) + \lambda_H \Pi - c. \tag{29}$$

By multiplying $(\mu - \lambda_H q)^{-\frac{2\mu}{\mu - \lambda_H}}(1 - q)^{\frac{3\lambda_H - \mu}{\mu - \lambda_H}}$ and rearranging the equation, for all $0 = q_0 \leq q \leq q_T$, we can derive that

$$0 = \frac{d}{dq}\left[\frac{(1 - q)^{\frac{2\lambda_H}{\mu - \lambda_H}}}{(\mu - \lambda_H q)^{\frac{\mu + \lambda_H}{\mu - \lambda_H}}}\left\{V_1(q) - \overline{V}_1(q)\right\}\right]. \tag{30}$$

Therefore, for all $0 = q_0 \leq q \leq q_T$, we have

$$V_1(q) = \overline{V}_1(q) + C_1 \cdot (1 - q) \cdot \left(\frac{\mu - \lambda_H q}{1 - q}\right)^{\frac{\mu + \lambda_H}{\mu - \lambda_H}} \tag{31}$$

for some $C_1 \in \mathbb{R}$. By $V_t^1 = V_1(q_t)$ for all $0 \leq t \leq T$, (27) holds.

[17] If $\mu = \lambda_H$, we need to replace $\left(\frac{\mu - \lambda_H q_t}{1 - q_t}\right)^{\frac{\mu + \lambda_H}{\mu - \lambda_H}}$ to $e^{\frac{2}{1 - q_t}}$.

Next, plug $\dot{V}_t^0 = V_0'(q_t)(\mu - \lambda_H q_t)(1 - q_t)$ into (HJB$_0$):

$$
\begin{aligned}
0 =& V_0'(q)(\mu - \lambda_H q)(1-q) - \lambda_H q V_0(q) - c + \mu(V_1(q) - V_0(q)) \\
=& V_0'(q)(\mu - \lambda_H q)(1-q) - V_0(q)(\lambda_H q + \mu) - c \\
& + \mu\left(\Pi - \frac{c}{\lambda_H}\right)\left(\frac{1}{2} + \frac{\lambda_H(1-q)}{2(\lambda_H + \mu)}\right) + \mu C_1 \cdot (1-q) \cdot \left(\frac{\mu - \lambda_H q}{1-q}\right)^{\frac{\mu+\lambda_H}{\mu-\lambda_H}}.
\end{aligned} \tag{32}
$$

By multiplying $(1-q)^{\frac{2\lambda_H}{\mu-\lambda_H}}(\mu - \lambda_H q)^{-\frac{3\mu-\lambda_H}{\mu-\lambda_H}}$ and rearranging the equation, $0 \le q \le q_T$, we have

$$
0 = \frac{d}{dq}\left[\frac{(1-q)^{\frac{\mu+\lambda_H}{\mu-\lambda_H}}}{(\mu - \lambda_H q)^{\frac{2\mu}{\mu-\lambda_H}}}\left\{V_0(q) - \overline{V}_0(q) + C_1 \cdot \frac{\mu}{\lambda_H} \cdot \left(\frac{\mu - \lambda_H q}{1-q}\right)^{\frac{\mu+\lambda_H}{\mu-\lambda_H}}\right\}\right].
$$

Therefore, we have

$$
V_0(q) = \overline{V}_0(q) + \left(C_0\left(\frac{\mu}{\lambda_H} - q\right) - C_1\frac{\mu}{\lambda_H}\right) \cdot \left(\frac{\mu - \lambda_H q}{1-q}\right)^{\frac{\mu+\lambda_H}{\mu-\lambda_H}}. \tag{33}
$$

for some $C_0 \in \mathbb{R}$. By $V_t^0 = V_0(q_t)$ for all $0 \le t \le T$, (28) holds.

Now suppose that both firms play research-first strategy. Then, (27) and (28) hold for all $t \ge 0$. When $\mu > \lambda_H$, by Lemma 4.2, $\lim_{t\to\infty} q_t = 1$. Since $\lim_{t\to\infty}(1-q_t)\left(\frac{\mu-\lambda_H q_t}{1-q_t}\right)^{\frac{\mu+\lambda_H}{\mu-\lambda_H}} = \infty$ and $\lim_{t\to\infty}\left(\frac{\mu-\lambda_H q_t}{1-q_t}\right)^{\frac{\mu+\lambda_H}{\mu-\lambda_H}} = \infty$, to make the value functions converge, $C_1 = C_0 = 0$. When $\mu < \lambda_H$, by Lemma 4.2, $\lim_{t\to\infty} q_t = \mu/\lambda_H$, which also implies $\lim_{t\to\infty}(1-q_t)\left(\frac{\mu-\lambda_H q_t}{1-q_t}\right)^{\frac{\mu+\lambda_H}{\mu-\lambda_H}} = \infty$ and $\lim_{t\to\infty}\left(\frac{\mu-\lambda_H q_t}{1-q_t}\right)^{\frac{\mu+\lambda_H}{\mu-\lambda_H}} = \infty$. Likewise, we also have $C_1 = C_0 = 0$ in this case to make the value functions converge. □

Lemma C.7. *Suppose that σ is the research strategy, i.e., $\sigma_t = 1$ for all $t \ge 0$. Then, $\sigma^A = \sigma^B = \sigma$ constitutes a symmetric Markov equilibrium if and only if $\mu \ge \min\{\overline{\mu}, \hat{\mu}\}$.*

Proof of Lemma C.7. Suppose that both firms play the research-first strategy. By Lemma C.6, the expected payoffs at time t with and without the new technology are $V_t^1 = \overline{V}_1(q_t)$ and $V_t^0 = \overline{V}_0(q_t)$.

Both firms playing the research strategy constitutes an equilibrium if and only if $\mu(\overline{V}_1(q_t) - \overline{V}_0(q_t)) \geq \lambda_L(\Pi - \overline{V}_0(q_t))$ for all $t \geq 0$. Note that

$$\frac{d}{dq}\left[\mu(\overline{V}_1(q) - \overline{V}_0(q)) - \lambda_L(\Pi - \overline{V}_0(q))\right] = -\frac{\left(\Pi - \frac{c}{\mu} - \frac{c}{\lambda_H}\right) + \frac{c}{\lambda_L}}{2\left(\frac{\lambda_H+\mu}{\lambda_H\lambda_L}\right)} < 0.$$

Therefore, it is enough to check whether the following inequality holds:

$$\lim_{t\to\infty}\left[\mu(\overline{V}_1(q_t) - \overline{V}_0(q_t)) - \lambda_L(\Pi - \overline{V}_0(q_t))\right] \geq 0. \tag{34}$$

When $\mu \geq \lambda_H$, by $\lim_{t\to\infty} q_t = 1$, (34) is equivalent to

$$\mu(\overline{V}_1(1) - \overline{V}_0(1)) - \lambda_L(\Pi - \overline{V}_0(1)) = \frac{(\lambda_H\Pi + c)\mu\lambda_L\left(\mu - \overline{\mu}\right)}{2(\lambda_H + \mu)} \geq 0. \tag{35}$$

When $\lambda_H > \mu$, by $\lim_{t\to\infty} q_t = \mu/\lambda_H$, (34) is equivalent to

$$\begin{aligned}&\mu\left(\overline{V}_1\left(\mu/\lambda_H\right) - \overline{V}_0\left(\mu/\lambda_H\right)\right) - \lambda_L\left(\Pi - \overline{V}_0\left(\mu/\lambda_H\right)\right)\\ &= \frac{(\mu\Pi + c)\lambda_L\lambda_H\left((\lambda_H - 2\lambda_L)\mu - \lambda_L\lambda_H\right)}{2\mu(\lambda_H + \mu)} \geq 0.\end{aligned} \tag{36}$$

Observe that when $\lambda_H < 3\lambda_L$, $\lambda_H < \overline{\mu} < \hat{\mu}$. In this case, by (35), (34) holds iff $\mu \geq \overline{\mu} = \min\{\mu, \hat{\mu}\}$. When $\lambda_H \geq 3\lambda_L$, note that $\lambda_H \geq \overline{\mu} \geq \hat{\mu}$. If $\mu \geq \lambda_H \geq \overline{\mu}$, (34) holds by (35). If $\lambda_H > \mu \geq \hat{\mu}$, (34) holds by (36). Therefore, (34) holds iff $\mu \geq \hat{\mu} = \min\{\mu, \hat{\mu}\}$. □

C.3.3 The Equilibrium with Stationary Fall-back Strategies

Lemma C.8. *Suppose that σ is a stationary fall-back strategy, i.e., for some $T \geq 0$ and $\sigma^* \in [0, 1)$, $\sigma_t = 1$ for all $t < T$ and $\sigma_t = \sigma^*$ for all $t > T$. If $\sigma^A = \sigma^B = \sigma$ constitutes a symmetric Markov equilibrium, then $\underline{\eta}(\delta) < \eta < \min\{1+\delta, 2-\delta\}$. Conversely, if $\underline{\eta}(\delta) < \eta < \min\{1+\delta, 2-\delta\}$, there exists a unique symmetric Markov equilibrium and it is a stationary fall-back strategy.*

Proof of Lemma C.8. Suppose that $\sigma^A = \sigma^B = \sigma$ constitutes an equilibrium. By Lemma

4.2 and C.4, $p^* = p_T = q_T > 0$. Since $2\mu > \overline{\mu}$, to have $p^* > 0$, $\mu > \underline{\mu}$ has to hold.

Next, we show that $\mu < \min\{\hat{\mu}, \overline{\mu}\}$. When $\lambda_H < \mu$,

$$\lim_{\tilde{T}\to\infty} q_{\tilde{T}} = 1 > q_T = p^*.$$

By using the definition of p^*, $\overline{\mu} = \underline{\mu} + \lambda_L$, $1 > p^*$ is equivalent to $(\overline{\mu} - \mu)(\mu - \lambda_L) > 0$, thus, $\overline{\mu} > \mu$. Then, $\overline{\mu} > \mu > \lambda_H$ is equivalent to $3\lambda_L > \lambda_H$, which implies $\hat{\mu} > \overline{\mu}$. Therefore, $\min\{\hat{\mu}, \overline{\mu}\} > \mu$ in this case. Consider the case with $\lambda_H > \mu$ and $3\lambda_L \geq \lambda_H$. In this case, $\hat{\mu} > \overline{\mu} \geq \lambda_H > \mu$, thus, $\min\{\hat{\mu}, \overline{\mu}\} > \mu$. Last, consider the case with $\lambda_H > \mu$ and $3\lambda_L < \lambda_H$. Then, we have $\overline{\mu} > \hat{\mu}$ and

$$\lim_{\tilde{T}\to\infty} q_{\tilde{T}} = \frac{\mu}{\lambda_H} > q_T = p^*.$$

By rearranging the inequality, we have $\lambda_L\lambda_H = \lambda_H\underline{\mu} - \lambda_L\overline{\mu} > (\lambda_H - 2\lambda_L)\mu$, which is equivalent to $\min\{\hat{\mu}, \overline{\mu}\} = \hat{\mu} > \mu$.

Now we assume that $\underline{\mu} < \mu < \min\{\overline{\mu}, \hat{\mu}\}$ and show that the stationary fall-back strategy defined in Lemma C.4 constitutes an equilibrium. By the construction of the strategy, for all $t \geq T$, $\mu(V_t^1 - V_t^0) = \lambda_L(\Pi - V_t^0)$, which supports $\sigma_t \in (0, 1)$. Next, we need to show that $\mu(V_t^1 - V_t^0) \geq \lambda_L(\Pi - V_t^0)$ for all $0 \leq t < T$ to support $\sigma_t = 1$. Assume the contrary: $\mu(V_s^1 - V_s^0) < \lambda_L(\Pi - V_s^0)$ for some $0 \leq s < T$. Since $\mu(V_T^1 - V_T^0) = \lambda_L(\Pi - V_T^0)$, there exists $s < t \leq T$ such that $\mu(V_t^1 - V_t^0) = \lambda_L(\Pi - V_t^0)$ and $\mu(\dot{V}_t^1 - \dot{V}_t^0) > -\lambda_L\dot{V}_t^0$, or equivalently,

$$\lambda_L\dot{V}_t^1 > (\mu - \lambda_L)(\dot{V}_t^0 - \dot{V}_t^1). \tag{37}$$

As a first step, we show that there exists $C_1 < 0$ such that V_t^1 is given as (27) in Lemma C.6 for all $0 \leq t < T$. By $V_T^1 = V_1^*$ and $q_T = p^*$, we have

$$C_1 = \frac{1}{(1-p^*)}\left(\frac{1-p^*}{\mu - \lambda_H p^*}\right)^{\frac{\mu+\lambda_H}{\mu-\lambda_H}}\left(V_1^* - \overline{V}_1(p^*)\right)$$

where $\overline{V}_1$ is defined as in (15). With some algebra and $\min\{\overline{\mu}, \hat{\mu}\} > \mu$, we can derive that

$$\overline{V}_1(p^*) - V_1^* = \left(\Pi - \frac{c}{\lambda_H}\right) \cdot \frac{(\overline{\mu} - \mu)^2\left(\lambda_L\lambda_H - (\lambda_H - 2\lambda_L)\mu\right)}{2\lambda_L\lambda_H(\lambda_H + \lambda_L)(2\mu - \overline{\mu})} > 0. \tag{38}$$

Therefore, $C_1 < 0$. Then, for all $0 \le t < T$, we have

$$\dot{V}_t^1 = \dot{q}_t \left[-\left(\Pi - \frac{c}{\lambda_H} \right) \frac{\lambda_H}{2(\lambda_H + \mu)} + C_1 \cdot \frac{\lambda_H(1+q_t)}{1-q_t} \left(\frac{1-q_t}{\mu - \lambda_H q_t} \right)^{\frac{2\lambda_H}{\lambda_H - \mu}} \right] < 0. \tag{39}$$

By (HJB$_1$) and (HJB$_0$), we have

$$\begin{aligned} \dot{V}_t^1 =& \lambda_H(1+q_t)V_t^1 + c - \lambda_H \Pi \\ \dot{V}_t^0 =& \lambda_H q_t V_t^0 + c - \mu(V_t^1 - V_t^0). \end{aligned}$$

By using $\lambda_L(\Pi - V_t^0) = \mu(V_t^1 - V_t^0)$, we can derive that

$$\begin{aligned} \dot{V}_t^0 - \dot{V}_t^1 =& \lambda_H(1+q_t)(V_t^0 - V_t^1) + \mu(V_t^0 - V_t^1) + \lambda_H(\Pi - V_t^0) \\ =& \left[(\lambda_H - \lambda_L)\mu - \lambda_H \lambda_L (1+q_t) \right] \left(\frac{\Pi - V_t^0}{\mu} \right). \end{aligned}$$

Note that $\Pi > V_t^0$ since the expected payoff cannot exceed the rent Π. By using $\Pi > V_t^0$, $p^* \ge q_t$ and $\min\{\overline{\mu}, \hat{\mu}\} > \mu$, we can derive that

$$\begin{aligned} \dot{V}_t^0 - \dot{V}_t^1 \ge& \left[(\lambda_H - \lambda_L)\mu - \lambda_H \lambda_L (1+p^*) \right] \left(\frac{\Pi - V_t^0}{\mu} \right) \\ =& \frac{(\overline{\mu} - \mu)\left(\lambda_L \lambda_H - (\lambda_H - 2\lambda_L)\mu \right)}{2\mu - \overline{\mu}} \left(\frac{\Pi - V_t^0}{\mu} \right) > 0. \end{aligned} \tag{40}$$

Then, (39) and (40) contradict (37). Therefore, $\mu(V_t^1 - V_t^0) \ge \lambda_L(\Pi - V_t^0)$ for all $0 \le t < T$, and the stationary fall-back strategy constitutes an equilibrium. □