# Persuasion of Loss-Averse Buyers Through Early Offers*

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**Abstract**

We study a simple bargaining model in which the seller can make early offers to the buyer. Initially, the seller has private information about the value of the buyer's outside option. The buyer learns this value before she chooses between the seller's early offer and her outside option. Nevertheless, if the buyer is expectation-based loss averse, the seller can persuade her to accept an offer that first-order stochastically dominates her outside option. This result is due to the interaction of two effects: the *attachment effect* that makes it costly for the buyer to reject an offer that she planned to accept, and the *uncertainty effect* which renders the acceptance of the seller's offer as the preferred plan since it creates peace of mind at an early stage. We show that, under mild restrictions, the main result holds for all degrees of loss aversion. Thus, expectation-based loss-averse preferences imply that there is scope for persuasion through signaling even if the buyer has all payoff-relevant information at the decision stage.

**Keywords:** Reference-Dependent Preferences, Signaling, Loss Aversion, Early Offers
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# 1 Introduction

In many settings, a party with superior information tries to influence the choices of a less well-informed decision maker. Economists have extensively studied the scope for *persuasion* of rational decision makers through a variety of mechanisms: signaling (Spence 1973), cheap talk (Crawford and Sobel 1982), and Bayesian persuasion (Kamenica and Gentzkow 2011). A common feature of these mechanisms is that, at the point in time when the decision maker chooses an option, she has incomplete information about the state of world (such as the opponent's type or the realization of a payoff-relevant state variable). She only can infer details about the state of the world based on the actions of or the information provided by the informed party. In an environment with complete information at the decision stage, however, there is no scope for persuasion of rational decision makers with standard preferences.

This changes when the decision maker (she) is rational, but exhibits reference-dependent preferences. The actions of the informed party (he) may endogenously change the decision maker's reference point, which in turn affects her preferences over options at the decision stage. Hence, the informed party may be able to influence the choices of the decision maker with reference-dependent preferences, even if she has complete information about the state of the world at the point in time when she makes a final decision. This type of persuasion does not require any congruence of preferences (as in cheap talk) or commitment on the side of the seller (as in Bayesian persuasion).

In this paper, we study persuasion of buyers with reference-dependent preferences through a signaling-based mechanism. Our analysis will uncover a combination of forces that make persuasion feasible in environments with complete information at the decision stage. The proposed mechanism exhibits properties that are often found in real-world transactions, and it is relevant for many environments where the decision maker acquires full information before her decision becomes final. Our running example is as follows.

> *Shopping.* A buyer is searching for a new laptop at the local electronics store. The store is legally obliged to grant her return rights so that she can reverse her choice even after purchasing a laptop. Over time she learns about all options, in particular when she makes a final decision on whether or not to keep her purchase.

To examine the scope for persuasion in such settings, we consider a simple dynamic model of bargaining between a seller and a buyer. The buyer initially does not know the value of her outside option. She learns this value before she makes a choice. The seller knows the buyer's outside option value right away. Before the buyer learns about her outside option, the seller can make a binding "early offer" to the buyer, i.e., an offer that is valid also in the last stage of the game when the buyer has complete information. The buyer then chooses between this early offer and her outside option. To model reference-dependent preferences, we assume that the buyer is expectation-based loss averse (Kőszegi and Rabin 2006, 2007). After observing the buyer's early offer, she updates beliefs about the value of her outside option. Additionally, she makes a plan under what circumstances she accepts which option. Her beliefs and plan jointly determine her reference point. This reference point defines her preferences at the decision stage.

We show that, in this setting, an equilibrium can exist in which the seller persuades the buyer to accept an early offer that has a lower total value than her outside option (henceforth, an "inferior option"). For this, the seller needs to differentiate his offer from the outside option, i.e., he has to endow his offer with a feature that the outside option does not have. Specifically, we allow the total value of the early offer to consist of a regular value and a transfer. A loss-averse buyer treats the regular value dimension and the transfer dimension separately.[1] The regular value occurs in the same dimension as the outside option value, so that these are directly comparable for the buyer. The transfer occurs in a dimension in which the outside option only offers a zero outcome. For a loss-averse buyer, the seller therefore can differentiate his offer from the outside option through the transfer without changing its total value. For example, suppose the early offer has a regular value below the outside option value and a positive transfer. In this case, if the reference point is defined by the plan "accept the early offer", then choosing the outside option creates a gain in the regular value dimension and a loss in the transfer dimension. Loss aversion (the tendency that losses loom larger than gains of similar size) then reduces the payoff from accepting the outside option. This enables the seller to redistribute surplus from the buyer to himself. Indeed, creating value in multiple dimensions is a common technique in many sales settings.

> *Shopping – Value in Transfer Dimension.* The seller (seller) in the electronics store offers additional services for the laptop, such as an extended warranty, on-site repair services, or the benefits of a loyalty program.

We examine the structure of equilibria in which, for any realization of the outside option value, the seller persuades the buyer to accept an offer that is inferior to her outside option. These equilibria have three noteworthy features. First, such an equilibrium must be a semi-separating signaling equilibrium. The seller's early offer informs the buyer about the range of possible outside option values, but not the precise number. Each such equilibrium has an interval structure, somewhat reminiscent of an equilibrium with information transmission in a cheap talk game (Crawford and Sobel 1982).

[1]This separation is a feature of the expectations-based reference point model of Kőszegi and Rabin (2006, 2007) and is closely linked to mental accounting and the endowment effect. It describes individuals' tendency to assess gains and losses separately across different dimensions (Kahneman et al. 1990, 1991, Thaler 1985, 1999).

The second noteworthy feature is that two different effects – induced by expectation-based loss-averse preferences – interact to enable persuasion: the *attachment effect* and the *uncertainty effect*. The *attachment effect* implies that it is costly for the buyer to choose the outside option when accepting the early offer determines the reference point. As described above, this effect is caused by gain-loss sensations in the regular value and transfer dimension. The *uncertainty effect* makes the acceptance of the early offer relatively more attractive than the acceptance of the outside option. This is due to the fact that the former plan creates peace of mind at an early stage, while the latter plan exposes the buyer to uncertainty which lowers her expected payoff. Both the attachment and the uncertainty effect must be sufficiently strong so that it is optimal for the buyer to plan the acceptance of and eventually accept an offer that with certainty is less valuable for her than the outside option. An equilibrium that is optimal for the seller balances the relative strength of these two effects.

Finally, the third noteworthy feature is that, to enable credible signaling, early offers may exhibit positive outcomes both in the regular value and in the transfer dimension. The outcome in the transfer dimension must be positive in order to create an attachment effect. Note that the maximization of the attachment effect would imply a positive outcome only in the transfer dimension (to distinguish the early offer as much as possible from the outside option). However, in a semi-separating signaling equilibrium, the buyer must be ready to reject disadvantageous off-equilibrium offers. This potentially creates a bound on the strength of the attachment effect. Depending on the prior distribution over outside option values, it therefore may be necessary to offer positive outcomes in both dimensions.

Our main result is that if the loss aversion parameters – the weight of gain-loss sensations $\eta$ and the degree of loss aversion $\lambda$ – are sufficiently large, then an equilibrium exists in which the seller persuades the buyer to accept a value-inferior option even if the buyer has full information at the decision stage. We show that any such equilibrium is a signaling equilibrium with bunching of outside option values, interaction of attachment and uncertainty effect, as well as positive outcomes in the transfer dimension. We show that this also holds for any seller-preferred equilibrium.

In our main result, the required size of the loss aversion parameters is large. We need that $\eta(\lambda - 1) > 3$. As we discuss in the next section, these are empirically relevant degrees of loss aversion as there is substantial evidence for the uncertainty effect. Nevertheless, in most theoretical applications, the assumed levels of loss aversion are typically smaller. In some applications, large levels of loss aversion are ruled out explicitly (e.g., Herweg and Mierendorff 2013). Therefore, we show in an extension that an arbitrary small change in the model that strengthens the uncertainty effect is enough to obtain roughly the same results with any loss aversion parameter values $\eta, \lambda$ that satisfy $\eta(\lambda - 1) > 0$.

The fact that the buyer learns her outside option value before making a final decision of course limits the extent of persuasion in equilibrium. Specifically, the strength of the attachment effect determines the scope for persuasion if the buyer has full information at the decision stage. The seller could potentially benefit if he had the commitment power to force the buyer to accept or reject his early offer before learning her outside option. Indeed, this is done in many sales settings.

> *Shopping – Creating Scarcity.* A popular sales tactic is to artificially create scarcity, e.g., by emphasizing that a product may soon be out of stock or that there is a deadline for sales (Cialdini 2001, Chapter 7). The seller in the electronics store therefore may make an offer that is only valid right now, that is, before the buyer can gather more information.[2]

We show that, if the loss aversion parameters are large enough, then, in the seller-preferred equilibrium, the seller would use this commitment power and request immediate acceptance from the buyer. Only the uncertainty effect matters for the buyer's choice in this case. Nevertheless, for intermediate values of the loss aversion parameters, the seller-preferred equilibrium could again be a signaling equilibrium with a non-trivial interval structure where both attachment and uncertainty effect determine the scope for persuasion.

The rest of the paper is organized as follows. In Section 2, we explain how our paper contributes to the related literature on persuasion, expectation-based loss-averse preferences, and the uncertainty effect. In Section 3, we introduce the formal model and define the equilibrium concept. In Section 4, we derive our main results, characterize the seller-preferred equilibrium, and discuss welfare implications as well as extensions. In Section 5, we consider an extension where the main results obtain for any positive degree of loss aversion. Finally, Section 6 concludes. All proofs are relegated to the appendix.

## 2 Related Literature

*Persuasion with behavioral buyers.* Recently, a number of papers considered persuasion with boundedly rational sellers or buyers, see, for example, Hagenbach and Koessler (2017) as well as Bilancini and Boncinelli (2018) for signaling, and Blume and Board (2013), Glazer and Rubinstein (2012, 2014), Galperti (2019), Giovannoni and Xiong (2019), Hagenbach and Koessler (2020), as well as Eliaz et al. (2021) for cheap talk. In contrast to these papers, we assume that agents have fully rational beliefs, while the buyer exhibits expectation-based loss-averse preferences. The main innovation of our model is that it allows for persuasion

[2] Interestingly, Cialdini (2001) even explains the effectiveness of this sales tactic by referring to loss aversion.

in a setting where the buyer is perfectly informed about the realization of all payoff-relevant variables when she chooses between options.

*Strategic interaction of agents with expectation-based loss-averse preferences.* We also contribute to the literature that analyzes the implications of expectation-based loss aversion for strategic interaction. Heidhues and Kőszegi (2008), Karle and Peitz (2014), Karle and Möller (2020) study imperfect competition with expectation-based loss-averse buyers, and Herweg and Mierendorff (2013), Heidhues and Kőszegi (2014), Rosato (2016), and Karle and Schumacher (2017) study a monopolist's optimal pricing and marketing strategies when buyers are expectations-based loss averse. Rosato (2017) examines a two-period sequential barganing model between risk-neutral seller and a loss-averse buyer who has private information about her type. Further applications of expectation-based loss aversion include Benkert (2022) on bilateral trade; Carbajal and Ely (2016) and Hahn et al. (2018) on monopolistic screening; Herweg et al. (2010) on principal-agent contracts; Lange and Ratan (2010), Dato et al. (2018), Balzer and Rosato (2021) on auctions or tournaments; Dato et al. (2017) on strategic interaction in finite games; and Daido and Murooka (2016) on team incentives. Importantly, the results in these papers only rely on the attachment effect. The uncertainty effect is examined only in a few papers. Dreyfuss et al. (2021) and Meisner and von Wangenheim (2021) refer to it in the context of deferred acceptance mechanisms, and our companion paper Karle et al. (2021) in the context of product switching. The present paper is the first that considers the interaction of attachment and uncertainty effect in a strategic setting.

*Uncertainty Effect.* Several experimental studies find versions of the uncertainty effect. Gneezy et al. (2006) first demonstrated that some individuals value a lottery less than its worst outcome. They applied a between-subject design and obtained the same result for different types of goods, elicitation methods, and implementation. Sonsino (2008) finds in auctions for single gifts and binary lotteries on these gifts that 27 percent of subjects sometimes submit higher bids for the single gift than for the lottery even though the lottery's worst outcome is the gift. In a post-experimental survey, many participants indicate "aversion to lotteries" as their explanation for such behavior. Simonsohn (2009) conducts several within-subject variations of the experiment by Gneezy et al. (2006) and finds that 62 percent of subjects exhibit the uncertainty effect. Yang et al. (2013) show that a pronounced uncertainty effect occurs if the certain outcome is framed as a "gift certificate" while the lottery is framed as "lottery ticket" (or coin flip, gamble, raffle). Mislavsky and Simonsohn (2018) find the uncertainty effect when subjects perceive the certain outcome as more natural transaction than the lottery. They interpret the lottery as a transaction that has an unexplained feature.[3]

# 3 The Model

A loss-neutral seller interacts with a loss-averse buyer in two periods, 1 and 2. In period 1, the seller makes an early offer $(v^s, t^s)$ to the buyer, where $v^s \in \mathbb{R}_+$ is the regular value of the offer to the buyer and $t^s \in \mathbb{R}_+$ a transfer from the seller to the buyer. In period 2, the buyer learns about her outside option $(v^o, 0)$ and then chooses between the seller's offer $(v^s, t^s)$ and her outside option $(v^o, 0)$. The distinction between regular value and transfer allows the seller to offer something that the outside option does not provide (e.g., through differentiation), which will become relevant when we consider gain-loss sensations.

If the buyer accepts the outside option, her consumption utility is $v^o$ and the seller's payoff is zero. If the buyer accepts the seller's offer, her consumption utility is $v^s + t^s$ and the seller's payoff is $1 - v^s - t^s$. The shape of the seller's payoff function ensures that the seller can profitably trade with the buyer even if the buyer's outside option is maximal, and that regular value and transfer are fungible for the seller. For convenience, we will denote by $w^s = v^s + t^s$ the total value of the seller's offer. The outside option value $v^o$ is distributed according to the distribution function $F$ with continuous density $f$ and full support on the interval $[0, 1]$. Moreover, we assume that $F$ is weakly convex and that $f(v^o) < \infty$ for all $v^o \in [0, 1]$. In period 1, the seller observes the realization of $v^o$ and can condition his offer on this value, while the buyer only knows the distribution of $v^o$. Figure 1 shows the timeline of the interaction between seller and buyer.

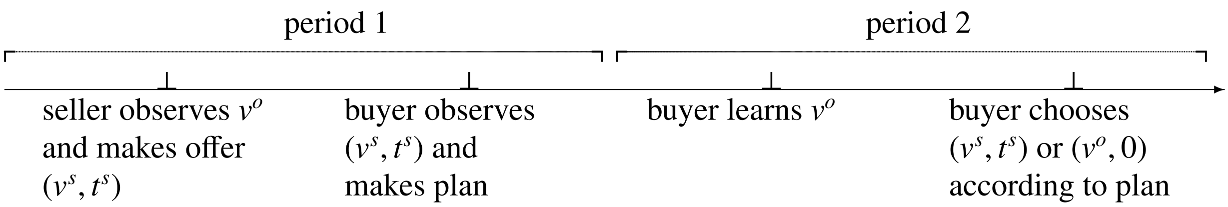


Figure 1: Timeline

*Preferences.* To model the buyer's expectation-based loss aversion we follow Kőszegi and Rabin (2006, 2007). Her payoff from accepting an option in period 2 consists of two compo-

[3] In addition, Andreoni and Sprenger (2011) also find the uncertainty effect in their experimental data. Some studies demonstrate that the uncertainty effect does not show up under certain conditions; see Rydval et al. (2009) and Wang et al. (2013).

nents: consumption utility and gain-loss utility from comparisons of the actual outcome to a reference point. This reference point is defined by the buyer's period-1 expectations.[4] Suppose that in period 1 she expects to accept the option $(\tilde{v}, \tilde{t})$ with certainty in period 2. If she indeed accepts option $(v, t)$, her payoff equals

$$U(v, t \mid \tilde{v}, \tilde{t}) = v + t + \mu(v - \tilde{v}) + \mu(t - \tilde{t}). \tag{1}$$

The function $\mu$ captures gain-loss utility. We assume that $\mu$ is piecewise linear with slope $\eta$ for gains and slope $\eta\lambda$ for losses; $\eta > 0$ is the weight of gain-loss utility relative to consumption utility, and $\lambda > 1$ is the buyer's degree of loss aversion.

The buyer may have stochastic expectations over outcomes. Let the distribution functions $G^v$ and $G^t$ denote her period-1 expectations regarding the outcome in the value and transfer dimension, respectively. The buyer's payoff from accepting option $(v, t)$ is given by

$$U(v, t \mid G^v, G^t) = v + t + \int \mu(v - \tilde{v})\, \mathrm{d}G^v(\tilde{v}) + \int \mu(t - \tilde{t})\, \mathrm{d}G^t(\tilde{t}). \tag{2}$$

Thus, gains and losses are weighted by the probability with which the buyer expects them to occur. This preference model captures the following intuition. If the buyer expects to get either 0 or 1 in the value dimension, each with probability 50 percent, then an allocation of 0.6 feels like a gain of 0.6 weighted with 50 percent probability, and a loss of 0.4 also weighted with 50 percent probability.

*Strategies and Equilibrium.* The seller's strategy defines his offer $(v^s, t^s)$ in period 1 based on the buyer's outside option value $v^o$. It is given by the measurable function[5]

$$\sigma^s : [0, 1] \to [0, 1] \times \mathbb{R}_+. \tag{3}$$

Thus, the seller's offer is potentially informative for the buyer about her outside option value $v^o$. Upon observing the seller's offer $(v^s, t^s)$, the buyer updates her belief about her outside option value to $\hat{F} \equiv F\left(v^o \mid v^s, t^s\right)$. She then makes a plan under what circumstances she accepts either the outside option or the seller's offer. Formally, the buyer's plan is a strategy that defines her choice in period 2 based on the outside option and the seller's offer,

$$\sigma^r : [0, 1] \times [0, 1] \times \mathbb{R}_+ \to [0, 1] \times \mathbb{R}_+. \tag{4}$$

[4]This means that the seller's early offer initiates the expectation formation process of the buyer. In this paper, we consider static reference points. Hence, we use the model of Kőszegi and Rabin (2006, 2007) instead of Kőszegi and Rabin (2009).

[5]We restrict attention to pure strategies in this paper.

Given seller's strategy $\sigma^s$, his offer $(v^s, t^s)$, and the buyer's strategy $\sigma^r$, we can define the buyer's expectations about period-2 outcomes. Let $G^v \equiv G^v(\tilde{v} \mid \sigma^s, \sigma^r, (v^s, t^s))$ denote her expectations about the outcome in the value dimension, and $G^t \equiv G^t(\tilde{t} \mid \sigma^s, \sigma^r, (v^s, t^s))$ her expectations regarding the outcome in the transfer dimension. For a given seller strategy $\sigma^s$, the buyer's strategy $\sigma^r$ is a personal equilibrium (PE) if it is optimal for her in period 2 to always follow this plan. Moreover, strategy $\sigma^r$ is a preferred personal equilibrium (PPE) if it is a PE and there is no alternative PE that yields her a higher expected payoff in period 1. An equilibrium of the game is then given by a perfect Bayesian equilibrium where the buyer's strategy constitutes a preferred personal equilibrium. We state these definitions formally.

**Definition 1.** *For a given seller strategy $\sigma^s$, the buyer's strategy $\sigma^r$ is a personal equilibrium (PE) if for any $v^o$ and seller offer $(v^s, t^s)$ we have*

$$U(\sigma^r(v^o, v^s, t^s) \mid G^v, G^t) \geq U(v, t \mid G^v, G^t)$$

*at all available options $(v, t)$. For a given seller strategy $\sigma^s$, the buyer's strategy $\sigma^r$ is a preferred personal equilibrium (PPE) if it is a personal equilibrium and for any seller offer $(v^s, t^s)$ we have*

$$\mathbb{E}_{\hat{F}}[U(\sigma^r(v^o, v^s, t^s) \mid G^v, G^t)] \geq \mathbb{E}_{\hat{F}}[U(\tilde{\sigma}^r(v^o, v^s, t^s) \mid \tilde{G}^v, \tilde{G}^t)]$$

*at any alternative personal equilibrium $\tilde{\sigma}^r$.*

**Definition 2.** *The triple $\sigma = (\sigma^s, \sigma^r, \hat{F})$ is a perfect Bayesian equilibrium if, for any outside option value $v^o \in [0, 1]$, the seller's offer $\sigma^s(v^o)$ maximizes his expected payoff for given $\sigma^r$, strategy $\sigma^r$ is a PPE for given $\sigma^s$, and $\hat{F}$ is derived from $\sigma^s$ and Bayes' rule whenever possible.*

This model basically describes a signaling game in which the seller (potentially) signals his private information about the buyer's outside option through the early offer to the buyer. In the following, we examine the equilibria of this game.

# 4 Signaling Equilibria

In this section, we study under what circumstances there exists an equilibrium in which the seller benefits from making early offers. In Subsection 4.1, we first discuss the benchmark case when the buyer is loss neutral and then examine some preliminary results for a loss-averse buyer. In Subsection 4.2, we state the main result and explain the structure of equilibria in which the seller persuades the buyer to accept an inferior offer, i.e., an offer that has less total value than her outside option. In Subsection 4.3, we examine the properties of seller-preferred equilibria. In Subsection 4.4, we briefly discuss the welfare consequences of persuasion through early offers in our setting. Finally, in Subsection 4.5, we study how the seller-preferred equilibrium changes when the seller can commit to time-limited offers.

## 4.1 Preliminaries

We consider first the benchmark case when the buyer is loss neutral, $\lambda = 1$. In this case, she is not bothered by gain-loss sensations and accepts a seller's offer $(v^s, t^s)$ only if $v^s + t^s \geq v^o$. In equilibrium, the seller will then, for any $v^o < 1$, make an offer with $v^s + t^s = v^o$ so that his profit equals $1 - v^o$. Making early offers has no particular value for the seller in this setting and there is no scope for persuasion.

In the following, we focus on the case when the buyer is loss averse, $\lambda > 1$. We say that the seller persuades the buyer to accept an inferior option (or offer) at outside option value $v^o$ if the buyer accepts the seller's early offer with total value $v^s + t^s < v^o$. We start our analysis with the following observation. An equilibrium in which the seller benefits from making early offers cannot be a pooling or a separating equilibrium. First, a pooling equilibrium does not exist: The only offer the seller would be willing to make in such an equilibrium is the zero offer $(v^s, t^s) = (0, 0)$. A positive offer with $v^s > 0$ or $t^s > 0$ could not be an equilibrium offer in a pooling equilibrium since at sufficiently low values of the outside option the seller would have an incentive to make a less generous offer. However, the buyer would not accept the zero offer in period 2 as long as her outside option value is positive.

Next, a separating equilibrium does exist.[6] In any such equilibrium, the buyer fully infers her outside option value from the seller's offer and then faces a choice under certainty. A loss-averse buyer would then behave like a loss-neutral one. Hence, in a separating equilibrium, there is no scope for exploiting the buyer's loss aversion through early offers. An equilibrium in which the seller persuades the buyer to accept an inferior offer relative to her outside option therefore must be semi-separating.

In a semi-separating equilibrium, the seller's offer can be informative about the buyer's outside option without revealing its exact value. Suppose that if the buyer gets an early offer $(v^s, t^s)$, this informs her that her outside option is located in the non-empty, open set $V \subset [0, 1]$. Define $\underline{v} = \inf(V)$ and $\bar{v} = \sup(V)$; we will use this notation throughout the paper. The buyer's PE then must be a cut-off equilibrium. The reason for this is that, at any given plan $\sigma^r$, the

[6] Here is an example: At any outside option value $v^o$ the seller makes the offer $(v^s, t^s) = (v^o, 0)$, and the buyer accepts it. If the buyer observes an offer with $t^s > 0$ in period 1, she believes that her outside option value equals $v^o = 1$ and makes a plan that is optimal given this belief. One can show that there is then no profitable deviation for the seller.

buyer's utility from accepting the offer $(v^s, t^s)$ is constant, while her utility from accepting the outside option strictly increases in $v^o$. Hence, for any PE, there exists a value $v^* \in [\underline{v}, \bar{v}]$ so that the buyer chooses the outside option if $v^o > v^*$, and accepts the seller's offer if $v^o \leq v^*$. In general, there can be multiple PEs and it could be cumbersome to determine the PPE. However, since $F$ is weakly convex, we obtain a result that substantially simplifies the analysis.

**Lemma 1.** *Consider any seller strategy $\sigma^s$ where the seller makes the offer $(v^s, t^s)$ if and only if $v_o \in V \subset (0, 1)$, and assume that $v^s + t^s \leq \underline{v}$. Any plan $\sigma^r$ that maximizes the buyer's expected payoff in period 1 at $\sigma^s$ and $(v^s, t^s)$ then specifies either (i) to always accept $(v^s, t^s)$ when $v_o \in V$, or (ii) to always accept the outside option when $v_o \in V$.*

To understand the meaning of this result, note first that in equilibrium the seller will not make an offer $(v^s, t^s)$ with $v^s + t^s > \underline{v}$. Otherwise, he could benefit from offering $(v^o + \varepsilon, 0)$ for some small $\varepsilon > 0$ whenever the outside option value $v^o > \underline{v}$ is close enough to $\underline{v}$. Thus, in equilibrium, we must have $v^s + t^s \leq \underline{v}$. Note further that, in equilibrium, the seller will not make an offer that is going to be rejected by the buyer as long as $v^o < 1$. He (almost) always can make an offer that will be accepted and that yields him a strictly positive payoff. Lemma 1 then implies the following: In order to show that the plan[7] "accept $(v^s, t^s)$ when $v_o \in V$" is a PPE after offer $(v^s, t^s)$ is made, we only have to make sure that it is a PE and that it is weakly better for the buyer than the plan "accept the outside option when $v_o \in V$." This facilitates the subsequent analysis of signaling equilibria.

## 4.2 Main Result

When the seller makes an early offer $(v^s, t^s)$ where the buyer knows that its total value $v^s + t^s$ is lower than that of any possible outside option she may find, why should the buyer plan to accept it? For loss-averse buyers there is an important reason why planning to accept such an offer can be optimal. In period 1, she then enjoys peace of mind as she will not be exposed to gain-loss sensations in period 2. Of course, accepting $(v^s, t^s)$ must also be optimal in period 2, so the total value $v^s + t^s$ of the seller's offer cannot be too small relative to the outside option value $v^o$. In an equilibrium in which the seller persuades the buyer to accept an inferior offer at $v^o$, these forces must be balanced.

We show that there can exists an equilibrium where, at any positive outside option value $v^o$, the seller makes (and the buyer accepts) an inferior offer $(v^s, t^s)$. To state this result and to simplify the subsequent discussion, we refer to a sequence of disjoint intervals $\{V_i\}_{i \in \mathbb{N}}$ and

[7]This is not a fully specified strategy $\sigma^r$. Throughout the paper, we will use this "reduced" description of a strategy whenever it is not necessary to specify all details of the "complete" strategy.

define $\underline{v}_i = \inf(V_i)$ and $\bar{v}_i = \sup(V_i)$ for each interval $i \in \mathbb{N}$. Throughout, we assume that the sequence $\{V_i\}_{i\in\mathbb{N}}$ partitions the unit interval, and that intervals are descending, in the sense that $\underline{v}_i = \bar{v}_{i+1}$. We now can state our main result.

**Proposition 1** (Signaling Equilibria)**.** *If $\eta(\lambda - 1) > 3$, an equilibrium exists in which the seller persuades the buyer to accept an inferior offer at each outside option value $v^o > 0$. Any such equilibrium is characterized by a sequence of disjoint intervals $\{V_i\}_{i\in\mathbb{N}}$ so that the seller makes an offer with total value $w_i < \underline{v}_i$ and positive transfer $t_i > 0$ if $v^o \in V_i$; the buyer always accepts this offer. If $\eta(\lambda - 1) < 3$, there exists no such equilibrium.*

The equilibrium suggested in Proposition 1 is a signaling equilibrium in which the buyer learns from an early offer about the interval in which her outside option value is located. It is shaped by three different forces: the uncertainty effect, the attachment effect, and the seller's incentive to make offers that are as low as possible but are still accepted by the buyer. Below, we explain each force in detail and elaborate what it implies for the structure of the signaling equilibrium. Before we start this discussion, we have to briefly comment on the required degree of loss aversion. An equilibrium in which the seller benefits from making early offers only exists if $\eta(\lambda - 1) > 3$. Thus, the weight of gain-loss sensations $\eta$ and the degree of loss aversion $\lambda$ must be relatively large. Typical values assumed in the loss aversion literature are somewhat smaller (e.g., $\eta = 1$ and $\lambda = 2$). In Section 5, we present a version of the result in Proposition 1 that only requires $\eta(\lambda - 1) > 0$, i.e., that holds for any degree of loss aversion ($\lambda > 1$) as long as there is a positive degree of reference dependence ($\eta > 0$). This considerably increases the generality of our main result.

*The Uncertainty Effect.* Suppose the buyer gets an offer $(v_i^s, t_i^s)$ that informs her that her outside option value is in the interval $V_i$, with $\underline{v}_i = \inf(V_i)$ and $\bar{v}_i = \sup(V_i)$. By Lemma 1, the plan "accept $(v_i^s, t_i^s)$ when $v_o \in V_i$" is a PPE if it is a PE and if its expected payoff in period 1 exceeds the expected payoff from the plan "accept the outside option when $v_o \in V_i$." The uncertainty effect implies that the latter requirement can be met even if accepting the outside option generates strictly more consumption utility than $v_i^s + t_i^s$. The reason is that the plan "accept the outside option when $v_o \in V_i$" has the potential for disappointments, that is, the realized outside option value may be close to the lower bound $\underline{v}_i$ in which case the buyer experiences a loss (relative to higher values of the outside option that were possible ex ante). Formally, the buyer weakly prefers the plan "accept $(v_i^s, t_i^s)$ when $v_o \in V_i$" to "accept the outside option when $v_o \in V_i$" if

$$v_i^s + t_i^s \geq \int_{\underline{v}_i}^{\bar{v}_i} \hat{f}(v)v \,\mathrm{d}v - \eta(\lambda - 1) \int_{\underline{v}_i}^{\bar{v}_i} \hat{f}(v) \int_{v}^{\bar{v}_i} \hat{f}(\tilde{v})(\tilde{v} - v) \,\mathrm{d}\tilde{v} \,\mathrm{d}v. \tag{5}$$

Whether this inequality is satisfied or not depends on the distribution over possible outside option values $\hat{f}(v)$. Given that $F$ is weakly convex, the distribution $\hat{F}(v)$ that minimizes the right-hand side of inequality (5) is the uniform distribution (we show this formally in the proof of Proposition 1). The intuition is that this distribution "maximizes" the uncertainty the buyer is exposed to. If $\hat{F}(v)$ is indeed a uniform distribution, then inequality (5) is satisfied if only if $\eta(\lambda - 1) > 3$ and the value $v_i^s + t_i^s$ is sufficiently close to $\underline{v}_i$.

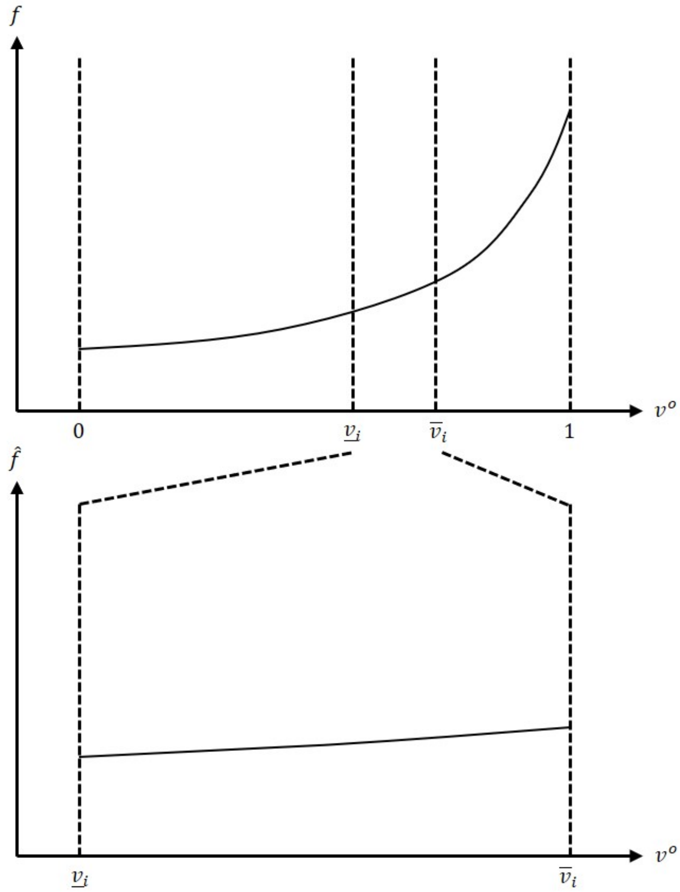


Figure 2: The updated distribution function $\hat{F}$ approaches the uniform distribution when its support $[\underline{v}_i, \bar{v}_i]$ becomes small. We exploit this effect in the proof of Proposition 1 to show that an uncertainty effect always can occur if $\eta(\lambda - 1) > 3$.

Note that the first statement of Proposition 1 holds for all distributions. Hence, we need a further element to ensure that the uncertainty effect unfolds for the full range of positive outside option values. Observe that, by continuity, when we make the interval $(\underline{v}_i, \bar{v}_i)$ small, then the updated density $\hat{f}(v)$ approaches a uniform distribution since $\hat{f}(\underline{v}_i) \to \hat{f}(\bar{v}_i)$; see Figure 2 for an illustration. Hence, the threshold $\eta(\lambda - 1) > 3$ holds for all distributions $F$ since we can always choose the intervals in $\{V_i\}_{i\in\mathbb{N}}$ small enough such that inequality (5) is satisfied for some offer $(v_i^s, t_i^s)$ with $v_i^s + t_i^s < \underline{v}_i$.

*The Attachment Effect.* We consider again the situation where the buyer gets an offer $(v_i^s, t_i^s)$

that informs her that her outside option value is in the interval $V_i$. The buyer follows the plan "accept $(v_i^s, t_i^s)$ when $v_o \in V_i$" only if this plan is a PE. For this, it must be optimal for the buyer to accept the seller's offer in period 2 even if the outside option value equals $\bar{v}_i$. Given the expectations induced by the plan "accept $(v_i^s, t_i^s)$ when $v_o \in V_i$" this is the case if and only if

$$v_i^s + t_i^s \geq \bar{v}_i + \eta(\bar{v}_i - v_i^s) - \eta\lambda t_i^s. \tag{6}$$

If the inequalities in (5) and (6) are satisfied, then accepting $(v_i^s, t_i^s)$ at all outside option values $v^o \in V_i$ is a PPE for the buyer. From inequality (6) we can make two important observations. First, the payoff-maximizing way for the seller to make an offer that satisfies inequality (6) is to create the total value only through the transfer $t_i^s$. Accepting the outside option implies losing the transfer, which through loss aversion is particularly painful for the buyer; we can observe this from the term $\eta\lambda t_i^s$. Intuitively, this means that the seller has to offer something to the buyer that the outside option cannot deliver. As a result, the buyer becomes "attached" to the offer. Second, and relatedly, inequality (6) puts an upper bound on the length of the interval $V_i$. If the total value is smaller than the lowest possible outside option value, $v_i^s + t_i^s < \underline{v}_i$, then both inequalities taken together imply that we must have $\underline{v}_i \geq \frac{1+\eta}{1+\eta\lambda}\bar{v}_i$. As we will see in the next subsection, this restriction may define the shape of the equilibrium that is optimal for the seller.

*Seller Incentives.* The proposed equilibrium is a signaling equilibrium only if the seller has an incentive to make the offer $(v_i^s, t_i^s)$ when $v^o \in V_i$. Specifically, he must not have an incentive to make this offer when $v^o > \bar{v}_i$. Offers and intervals therefore must be chosen such that the buyer rejects an offer $(v_i^s, t_i^s)$ if her true outside option (unexpectedly) exceeds $\bar{v}_i$. Note that upon receiving offer $(v_i^s, t_i^s)$ the buyer expects that $v^o \in V_i$ and that she accepts the offer in period 2. It is then optimal for her to reject $(v_i^s, t_i^s)$ for any $v^o > \bar{v}_i$ if and only if

$$v_i^s + t_i^s \leq \bar{v}_i + \eta(\bar{v}_i - v_i^s) - \eta\lambda t_i^s. \tag{7}$$

The right-hand side of this inequality is the expected payoff from accepting the outside option when $v^o = \bar{v}_i$. Note that this inequality is strictly satisfied if $v^o > \bar{v}_i$. Since the inequality in (6) also needs to be satisfied to ensure sufficient attachment, the offer $(v_i^s, t_i^s)$ for an interval $V_i$ must be chosen such that

$$v_i^s + t_i^s = \bar{v}_i + \eta(\bar{v}_i - v_i^s) - \eta\lambda t_i^s. \tag{8}$$

For given $v_i^s$ and interval $V_i$ the transfer $t_i^s$ must therefore be chosen such that $t_i^s = \frac{1+\eta}{1+\eta\lambda}\bar{v}_i - \frac{1+\eta}{1+\eta\lambda}v_i^s$. Thus, the seller can reduce the total value $v_i^s + t_i^s$ while respecting (8) by substituting transfer $t_i^s$ for value $v_i^s$. However, this may be constrained by the fact that it still must be optimal for the buyer to plan acceptance in period 1, that is, the inequality in (5) must be satisfied as well. In particular, if the interval $V_i$ is very short (which may be necessary for the uncertainty effect to unfold), then $v_i^s + t_i^s$ must be relatively close to $\underline{v}_i$. Consequently, in an equilibrium in which the seller persuades the buyer to accept an inferior offer at all $v^o > 0$, it could be necessary to make offers $(v_i^s, t_i^s)$ with both positive value $v_i^s$ and positive transfer $t_i^s$: The transfer $t_i^s$ is then positive to exploit the attachment effect, and the value $v_i^s$ is positive to enable credible signaling.

The uncertainty effect, the attachment effect, and the seller's incentives constrain the intervals $\{V_i\}_{i\in\mathbb{N}}$ and the offers $\{(v_i^s, t_i^s)\}_{i\in\mathbb{N}}$ of an equilibrium in which the seller persuades the buyer to accept an inferior offer at all outside option values. To construct such an equilibrium, it then only remains to fix off-equilibrium beliefs. It must be optimal for the buyer to reject any off-equilibrium offer $(\tilde{v}_i^s, \tilde{t}_i^s)$ when $\tilde{v}_i^s + \tilde{t}_i^s < v^o$. One option is to assume "optimistic beliefs", that is, the buyer believes in period 1 that her outside option value is maximal, $v^o = 1$, after receiving an off-equilibrium offer $(\tilde{v}_i^s, \tilde{t}_i^s)$.[8] Given this belief, it is then indeed optimal to reject this offer if $\tilde{v}_i^s + \tilde{t}_i^s < v^o$.

The last part of Proposition 1 shows that the seller cannot benefit from early offers if $F$ is weakly convex and $\eta(\lambda - 1) < 3$. The reason for this is that the uncertainty effect is then no longer strong enough so that the buyer would accept any offer $(v_i^s, t_i^s)$ with $v_i^s + t_i^s < \underline{v}_i$, irrespective of how we choose $V_i$. The inequality in (5) would not be satisfied for any such offer. We show in Section 5, that this changes once we consider multi-dimensional outside option values.

## 4.3 Seller-Preferred Equilibria

Like in most signaling games, there are many equilibria and also many persuasion equilibria in our setting. Classic refinements like the Intuitive Criterion (Cho and Kreps 1987) or Undefeated Equilibrium (Mailath et al. 1993) do not reduce the number of equilibria in a meaningful way in our case. The reason is that there can be continuum of offers where each offer is optimal at a certain outside option value given that the buyer plans their acceptance in period 1. In order to select between equilibria, we examine equilibria in which the seller earns the highest possible ex ante expected payoff, that is, the "seller-preferred equilibrium."[9]

For general distributions $F$, the seller-preferred equilibrium is the solution to a complex maximization problem where one has to find the optimal length of the intervals $\{V_i\}_{i\in\mathbb{N}}$. As discussed above, the length of an interval must be chosen so that the uncertainty effect is

[8] This is the analog to "pessimistic beliefs" which are frequently assumed in job-market signaling models in order to have off-equilibrium beliefs that motivate the equilibrium strategies.

[9] Alonso and Câmara (2018) use a similar solution concept.

sufficiently strong. Hence, the optimal configuration of intervals in general depends on the local properties of the distribution $F$. Nevertheless, we can show that the seller-preferred equilibrium must have an interval structure and positive outcomes in the transfer dimension as indicated in Proposition 1.

We can say more about the seller-preferred equilibrium when $F$ is the uniform distribution on the unit interval. This assumption simplifies the problem substantially. If $F$ is the uniform distribution, the updated distribution $\hat{F}$ is also (piecewise) uniform on its support at any equilibrium offer. Given that $\eta(\lambda - 1) > 3$, there is then enough uncertainty in each interval $V_i$ so that the buyer would accept an offer that is worse than the lowest value in the interval. This allows us to characterize the seller-preferred equilibria.

To this end, we define an upper bound on $\underline{v}_i$ for a given value $\bar{v}_i$. Suppose that offer $(v_i^s, t_i^s)$ informs the buyer that her outside option value is in the interval $V_i$. Recall that the smallest total value the seller needs to offer so that the buyer always accepts $(v_i^s, t_i^s)$ in period 2 is $\frac{1+\eta}{1+\eta\lambda}\bar{v}_i$. The corresponding offer creates utility only through the transfer, i.e., $(v_i^s, t_i^s) = (0, \frac{1+\eta}{1+\eta\lambda}\bar{v}_i)$. Provided that the seller makes this offer only if $v^o \in V_i$, it is optimal for the buyer in period 1 to plan accepting $(0, \frac{1+\eta}{1+\eta\lambda}\bar{v}_i)$ if and only if this is weakly better than always accepting the outside option. This is the case if and only if

$$\frac{1+\eta}{1+\eta\lambda}\bar{v}_i \geq \frac{1}{2}(\bar{v}_i + \underline{v}_i) - \frac{1}{6}\eta(\lambda - 1)(\bar{v}_i - \underline{v}_i), \tag{9}$$

where the right-hand side of this inequality equals the right-hand side of the inequality in (5) when $F$ is the uniform distribution. It defines an upper bound on $\underline{v}_i$ for a given value $\bar{v}_i$. This upper bound equals

$$\underline{v}_i \leq \Gamma(\eta, \lambda)\, \bar{v}_i, \tag{10}$$

where

$$\Gamma(\eta, \lambda) = \frac{6\frac{1+\eta}{1+\eta\lambda} - 3 + \eta(\lambda - 1)}{3 + \eta(\lambda - 1)}. \tag{11}$$

The intuition is that $\underline{v}_i$ cannot be closer to $\bar{v}_i$ than indicated in inequality (10), otherwise there is too little uncertainty for the uncertainty effect to unfold. It turns out that this bound also defines the intervals in a seller-preferred equilibrium. We thus obtain the following result.

**Proposition 2** (Seller-Preferred Equilibrium)**.** *If $\eta(\lambda-1) > 3$, the following statements hold.*

*(i) Any seller-preferred equilibrium is characterized by a sequence of disjoint intervals $\{V_i\}_{i\in\mathbb{N}}$ so that the seller makes an offer with total value $w_i \le \underline{v}_i$ and positive transfer $t_i > 0$ if $v^o \in V_i$; the buyer always accepts this offer.*

*(ii) Suppose $F$ is the uniform distribution on the unit interval. Any seller-preferred equilibrium is then characterized by a sequence of disjoint intervals $\{V_i\}_{i\in\mathbb{N}}$, so that the seller offers $(v_i^s, t_i^s) = (0, \frac{1+\eta}{1+\eta\lambda}\bar{v}_i)$ if $v^o \in V_i$; the buyer always accepts this offer. The length of the intervals is minimal subject to the constraint that it is optimal for the buyer to plan acceptance, i.e., $\underline{v}_i = \Gamma(\eta,\lambda)\,\bar{v}_i$ for all $i \in \mathbb{N}$.*

For the case of a uniform distribution the result shows that, in the seller-preferred equilibrium, the seller makes for each interval $V_i$ the least generous offer that is still accepted, and reduces the offered total value as quickly as possible as the outside option value decreases. Figure 3 below displays a seller-preferred equilibrium for the case $\eta = 2$ and $\lambda = 3$.

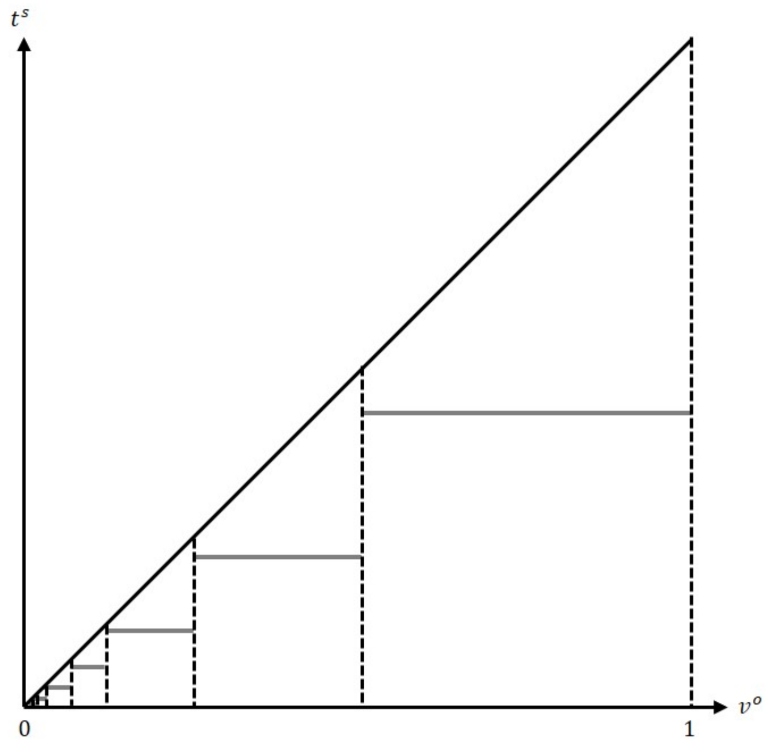


Figure 3: The seller-preferred equilibrium for $\eta = 2$ and $\lambda = 3$ when $F$ is the uniform distribution. The 45-degree line indicates offers with total value $v^o$. The dotted vertical lines indicate the bounds of the intervals $\{V_i\}_{i\in\mathbb{N}}$. The gray lines indicate, for each interval $V_i$, the offered transfer $t_i^s$ (and, hence, also the offered total value).

The proof of Proposition 2 is not as straightforward as it might seem. There is a trade-off between the total value $v_i^s + t_i^s$ that is offered in an interval $V_i$ and the length of this interval. The higher the total value is, the smaller the interval can be (so that a further reduction of the total value is possible in the next interval); see the inequality in (5). It turns out, however, that with a uniform distribution, the seller's expected expenses are minimal if at any given interval he makes the least generous offer and, given this fact, intervals are as short as possible.

The factor $\Gamma(\eta, \lambda)$ defines how informative the seller-preferred signaling equilibrium is. The closer $\Gamma(\eta, \lambda)$ is to 1, the shorter the intervals, and the more informative is the equilibrium. $\Gamma(\eta, \lambda)$ is not monotone in the degree of loss aversion $\lambda$. The reason is that there are two competing forces that influence $\Gamma(\eta, \lambda)$. On the one hand, the larger $\lambda$ is, the smaller is the offered transfer $t_i^s = \frac{1+\eta}{1+\eta\lambda}\bar{v}_i$ in interval $V_i$. This in turn requires more uncertainty. On the other hand, the larger $\lambda$ is, the less uncertainty is needed for the uncertainty effect to unfold at a given offer. Observe that $\Gamma(\eta, \lambda)$ strictly increases in $\lambda$ if $\lambda$ is sufficiently large, and approaches 1 when the buyer becomes very loss averse.

## 4.4 Welfare

We briefly comment on the effect of persuasion through early offers on the individual welfare of the seller and the buyer as well as on aggregate welfare. Welfare statements are so far not common in the applied literature on expectation-based loss-averse preferences. The reason for this is that it is typically not clear to what extent gain-loss utility should be treated as part of normative preferences. We follow Reck and Seibold (2021) as well as Goldin and Reck (2022) and introduce a parameter $\pi \in [0, 1]$ that captures a social planner's judgment about the normative weight of gain-loss utility. The buyer's welfare in period 2 if she expected to accept the option $(\tilde{v}, \tilde{t})$ with certainty and ends up accepting the option $(v, t)$ equals

$$U^*(v, t \mid \tilde{v}, \tilde{t}) = v + t + \pi\mu(v - \tilde{v}) + \pi\mu(t - \tilde{t}). \tag{12}$$

Hence, for $\pi = 0$ gain-loss utility is ignored for welfare judgments, while for $\pi = 1$ they receive the same normative weight as consumption utility. The seller's welfare $G^*$ just equals his payoff. For aggregate welfare we use a simple utilitarian welfare function and add up the seller's and buyer's welfare, $G^* + U^*$.

To evaluate the welfare impact of persuasion through early offers, we first determine the equilibrium outcome in the absence of early offers. Suppose the seller can only make offers in period 2 when the buyer also knows her outside option value. As benchmark equilibrium we use the equilibrium in which the seller just matches the buyer's outside option value: At any outside option value $v^o$, he offers $(v^s, t^s) = (v^o, 0)$ in period 2 and the buyer accepts the seller's offer. The expected welfare of the buyer in this equilibrium equals

$$U_0^* = \int_0^1 f(v)v \,\mathrm{d}v - \pi\eta(\lambda - 1)\int_0^1 f(v)\int_v^1 f(\tilde{v})(\tilde{v} - v)\,\mathrm{d}\tilde{v}\,\mathrm{d}v, \tag{13}$$

while the expected welfare of the seller is given by

$$G_0^* = \int_0^1 f(v)(1 - v)\,\mathrm{d}v. \tag{14}$$

Next, consider a signaling equilibrium in which the seller benefits from making early offers for any outside option value (or a seller-preferred equilibrium) with interval-structure $\{V_i\}_{i\in\mathbb{N}}$ and seller offer $(v_i^s, t_i^s)$ if $v^o \in V_i$. The expected welfare of the buyer in this equilibrium is

$$U^* = \sum_{i=1}^{\infty} \int_{\underline{v}_i}^{\bar{v}_i} f(v)(v_i^s + t_i^s)\,\mathrm{d}v, \tag{15}$$

and the expected welfare of the seller equals

$$G^* = \sum_{i=1}^{\infty} \int_{\underline{v}_i}^{\bar{v}_i} f(v)(1 - v_i^s - t_i^s)\,\mathrm{d}v. \tag{16}$$

From $U_0^*, G_0^*, U^*$, and $G^*$ we obtain the following results. First, if gain-loss sensations do not matter for welfare judgments, $\pi = 0$, then signaling through early offers has no impact on aggregated welfare. Persuading the buyer to accept an inferior offer only redistributes surplus from the buyer to the seller.

Second, this changes as soon as gain-loss sensations are taken into account for welfare judgments, $\pi > 0$. In this case, signaling through early offers increases aggregated welfare. The reason for this is that, in the considered equilibrium, early offers eliminate all gain-loss sensations in period 2. Formally, the increase in welfare is given by the expected gain-loss sensations in the second term on the right-hand side of equation (13).

Third, for any given value $\pi > 0$, signaling through early offers even implies a Pareto-improvement if $\eta(\lambda - 1)$ is large enough. Observe from equation (13) that $U_0^*$ becomes negative if $\eta(\lambda - 1)$ is large enough, while $U^*$ is strictly positive. Intuitively, this means that the buyer also benefits from obtaining early offers as they eliminate the uncertainty about future outcomes. If this benefit is large enough, both parties are strictly better off from signaling through early offers. These welfare statements of course depend on the assumption that offering value $v$ and transfers $t$ is equally costly for the seller.

## 4.5 Requesting Immediate Acceptance

The result in Proposition 1 shows that the seller can persuade the buyer to accept an inferior offer even if the buyer has full information at the decision stage. This begs the question whether the seller can gain even more if the buyer has incomplete information at the point in time when she has to decide between options. For example, he could force the buyer to make a decision already in period 1. In many circumstances, sellers are doing exactly this. A classic sales tactic is to "create time pressure", i.e., urge the buyer to make a quick decision as the offered product may soon be out of stock. In the following, we briefly examine how the seller-preferred equilibrium changes when the seller has the commitment power to request immediate acceptance.[10]

*Updated Setting.* We consider the same model as in Section 3 with the following change. After observing the buyer's outside option value $v^o$, the seller not only chooses an offer $(v^s, t^s)$, but also decides whether or not he requests immediate acceptance. If he requests immediate acceptance, the offer is only valid in period 1. In this case, if the buyer accepts the offer in period 1, payoffs are realized and the game is over. If the buyer rejects the offer in period 1, it is no longer available in period 2 so that she has to accept her outside option. When the seller does not request immediate acceptance, the buyer can accept the offer $(v^s, t^s)$ only in period 2 after learning her outside option value. The buyer observes whether immediate acceptance is requested or not and updates her beliefs about the outside option value accordingly.

*Equilibria with Immediate Acceptance.* Assume first that the seller requests immediate acceptance at any outside option value. Provided that the buyer always accepts the seller's offer, this is possible in equilibrium only if the seller always makes the least generous offer $(v^s, t^s) = (0, 0)$, regardless of the outside option value. In period 2, the buyer would not accept this offer as long as the outside option value is positive. However, in period 1, if the buyer has to decide immediately, it is optimal for her to accept offer $(0, 0)$ if

$$0 \geq \int_0^1 f(v)v\,\mathrm{d}v - \eta(\lambda - 1)\int_0^1 f(v)\int_v^1 f(\tilde{v})(\tilde{v} - v)\,\mathrm{d}\tilde{v}\,\mathrm{d}v. \tag{17}$$

If this inequality is satisfied, the buyer prefers to earn zero with certainty than to being exposed to the gain-loss sensations from accepting the outside option. It is satisfied if $\eta(\lambda - 1)$ is large enough. For a given distribution $F$ over outside option values, denote by $\Lambda_F$ the value of $\eta(\lambda - 1)$ so that (17) is satisfied with equality. Clearly, this value depends on $F$. If $F$ is the uniform distribution, we have $\Lambda_F = 3$. If $\eta(\lambda - 1) \geq 3$, the seller then offers $(0, 0)$ and requests immediate acceptance in the seller-preferred equilibrium. From the second part of Proposition 2 we can derive by how much the seller's commitment power increases his profit: Relative to the seller-preferred equilibrium without commitment, his payoff is raised by $\frac{1+\eta}{1+\eta\lambda}\bar{v}_i$ when

[10] Rosato (2016) also studies the sales tactic of creating scarcity for products that are on sale ("Black Friday deals") in order to induce buyers to purchase options that offer less buyer surplus. The difference to his setting here is that the seller forces the buyer to make a decision when she does not yet know the features of all available options.

$v^o \in V_i$, where $V_i$ is the interval defined in Proposition 2. Next, suppose that $F$ is some strictly convex distribution. We then have $\Lambda_F > 3$ and we obtain a more nuanced description of the seller-preferred equilibrium. The following result is a corollary to Proposition 2.

**Corollary 1** (Immediate Acceptance). *Consider the model in which the seller can request immediate acceptance of an offer in period 1.*

*(i) If $3 < \eta(\lambda - 1) < \Lambda_F$, any seller-preferred equilibrium is characterized by a sequence of disjoint intervals $\{V_i\}_{i \in \mathbb{N}}$ so that the seller makes an offer with total value $w_i \leq \underline{v}_i$ if $v^o \in V_i$; the buyer always accepts this offer. The seller may request immediate acceptance in the highest interval $V_1$, but not in any other interval.*

*(ii) If $\eta(\lambda - 1) \geq \Lambda_F$, then, in any seller-preferred equilibrium, the seller offers $(v^s, t^s) = (0, 0)$ at each outside option value and requests immediate acceptance.*

We already know from Proposition 1 that if $\eta(\lambda - 1) < 3$, the seller cannot persuade the buyer to accept an inferior offer. If the value $\eta(\lambda - 1)$ is in the range between 3 and $\Lambda_F$, the seller can benefit from making early offers, but it is not optimal for him to offer $(0, 0)$ and to request immediate acceptance since the buyer would reject this offer. Instead, the seller-preferred equilibrium again has an interval structure where the equilibrium offer becomes less generous in lower intervals. The seller may request immediate acceptance only in the highest interval $V_1$ where he makes the most generous equilibrium offer. Otherwise, to ensure the credibility of the signal, the buyer first has to learn her true outside option value and then accepts the seller's offer. Finally, if $\eta(\lambda - 1) \geq \Lambda_F$, it is optimal for the buyer to accept a certain payoff of zero instead of accepting an outside option of uncertain value. In a seller-preferred equilibrium, the seller then realizes the maximal possible payoff.

# 5 Extension: Multi-Dimensional Outside Option Values

In our baseline model, the seller only benefits from making early offers if the buyer's loss aversion parameters $\eta, \lambda$ are large enough so that $\eta(\lambda - 1) > 3$. These levels of loss aversion are empirically relevant (e.g., von Gaudecker et al. 2011) and the uncertainty effect has been found in numerous settings (as discussed in Section 2). However, in theoretical applications of expectation-based loss-averse preferences, the assumed levels of loss aversion are typically smaller. In this section, we therefore present an extension of the model in which our main results – persuasion in a signaling equilibrium with bunching of outside option values, interaction of attachment and uncertainty effect, and positive outcomes in the transfer dimension – obtain for all loss aversion parameters $\eta, \lambda$ that satisfy $\eta(\lambda - 1) > 0$.

The idea behind this extension is that the outside option may not only be characterized by the value $v^o$, but also by other attributes that are payoff relevant for the buyer. The buyer may not evaluate the joint value of these different attributes, but narrowly brackets them so that gain-loss sensations can occur in multiple dimensions. We again illustrate this in our running example.

> *Shopping – Uncertainty in Extra Dimensions.* In the discussion with the buyer, the seller at the electronics store can eliminate all uncertainty about his product with respect to delivery times, product specifications, customer support, and so forth. In contrast, a number of issues about other products are unclear when the buyer chooses her plan.

In the following, we consider outside options with multiple value dimensions. Both the expectations-based loss-aversion framework of Kőszegi and Rabin (2006) and the preference model with salience distortions by Bordalo et al. (2013) explicitly allow for multiple value dimensions. We implement such dimensions in a way that leaves the overall consumption utility from the seller's offer and the outside option unchanged. In Subsection 5.1, we describe in detail the changes to our baseline model. In Subsection 5.2, we analyze how they affect our results.

## 5.1 Updated Setting

We consider the same model as in Section 3, with the only difference that any option has values in two extra-dimensions, the $x$-dimension and the $y$-dimension. The seller's offer is now given by $(v^s, t^s, x^s, y^s)$, where $x^s = y^s = 0$. We can interpret these values as non-strategic design choices. As before, the seller chooses the regular value $v^s$ and the transfer $t^s$ of his early offer. The outside option is characterized by the vector $(v^o, 0, x^o, y^o)$, where $v^o$ is again the outside option value and distributed according to $F$ on the unit interval. With probability $\frac{1}{2}$ we have $x^o = \xi$ and $y^o = -\xi$ for some value $\xi > 0$ (state 1), and with probability $\frac{1}{2}$ we have $x^o = -\xi$ and $y^o = \xi$ (state 2). The parameter $\xi$ captures the level of uncertainty in the extra-dimensions the buyer is exposed to if she plans to accept the outside option. For $\xi = 0$ the new version of the model is essentially the same as the baseline model. The consumption utility from any option is $v + t + x + y$. Hence, as in the baseline model, the consumption utility from the seller's offer again equals $v^s + t^s$ and the consumption utility from the outside option equals $v^o$. If in period 1 the buyer expects to accept an offer $(\tilde{v}, \tilde{t}, \tilde{x}, \tilde{y})$ with certainty, and ends up accepting option $(v, t, x, y)$, her utility is

$$U(v, t, x, y \mid \tilde{v}, \tilde{t}, \tilde{x}, \tilde{y}) = v + t + x + y + \mu(v - \tilde{v}) + \mu(t - \tilde{t}) + \mu(x - \tilde{x}) + \mu(y - \tilde{y}). \quad (18)$$

Therefore, the buyer may experience gain-loss sensations in four dimensions (instead of two). The rest of the model proceeds as before. We assume that $\xi$ is not too large so that $\eta(\lambda - 1)\xi < 1 + \eta$. For convenience, we denote in the following a seller's offer by $(v^s, t^s)$ instead of $(v^s, t^s, x^s, y^s)$.

## 5.2 Equilibria with Multi-Dimensional Outside Option Values

We first adapt Lemma 1 to the new environment. Again, the buyer's PE must be a cut-off plan. This plan can be contingent on the state, i.e., the buyer may adopt different cut-off levels at the two states. However, in the proof of the next result, we show that the buyer cannot increase her expected payoff by adopting a plan with state-contingent cut-off levels. Hence, as in Lemma 1, a cut-off plan that maximizes her expected payoff in period 1 is either "always accept the seller's offer when $v_o \in V$" or "always accept the outside option when $v_o \in V$." This result is independent of the level of uncertainty $\xi$ in the extra-dimensions.

**Lemma 2.** *Consider the model with multi-dimensional outside option values. Consider any seller strategy $\sigma^s$ where the seller makes the offer $(v^s, t^s)$ if and only if $v_o \in V \subset (0, 1)$, and assume that $v^s + t^s \leq \underline{v}$. Any plan $\sigma^r$ that maximizes the buyer's expected payoff in period 1 at $\sigma^s$ and $(v^s, t^s)$ then specifies either (i) to always accept $(v^s, t^s)$ when $v_o \in V$, or (ii) to always accept the outside option when $v_o \in V$.*

Next, we characterize under what circumstances an equilibrium exists in which the seller persuades the buyer to accept an inferior offer, regardless of the outside option value. A plan that involves accepting the outside option now implies even more gain-loss sensations due to the uncertainty in the extra-dimensions. This increases the scope for the uncertainty effect. It is therefore conceivable that the critical threshold for the parameter $\eta(\lambda - 1)$ decreases as the uncertainty parameter $\xi$ increases. However, we obtain an even stronger result. As long as $\eta(\lambda - 1) > 0$ and $\xi$ is positive, there exists an equilibrium in which the seller always persuades the buyer through early offers.

**Proposition 3** (Signaling Equilibria with Multi-Dimensional Outside Option Values)**.** *Consider the model with multi-dimensional outside option values. If $\eta(\lambda - 1) > 0$ and $\xi > 0$, an equilibrium exists in which the seller persuades the buyer to accept an inferior offer at each outside option value $v^o > 0$.*

In the proof of Proposition 3, we again construct the desired equilibrium through a sequence of disjoint intervals $\{V_i\}_{i=1,\ldots,n}$ for some finite $n$, so that the seller makes an offer $(v_i^s, t_i^s)$ with total value $v_i^s + t_i^s < \underline{v}_i$ if $v^o \in V_i$. There are two new elements here. First, as we lower the length of an interval $\bar{v}_i - \underline{v}_i$, the magnitude of expected gain-loss sensations converges against a positive value, and not against zero as in the baseline model. This effect is due to the uncertainty in the extra-dimensions. Hence, for any given loss aversion parameters that satisfy $\eta(\lambda - 1) > 0$, if the interval $V_i$ is short enough, we can find an offer $(v_i^s, t_i^s)$ with $v_i^s + t_i^s < \underline{v}_i$ so that the buyer's expected payoff in period 1 is maximal if she plans to accept this offer as long as $v^o \in V_i$. The second new element is that we no longer need an infinite sequence of intervals. If the outside option value is sufficiently small, then it is possible that in equilibrium the buyer is willing to accept the least generous offer $(v^s, t^s) = (0, 0)$. The reason is that (unexpectedly) accepting the outside option always creates a loss sensation in the extra-dimensions.

A crucial difference between Proposition 1 and Proposition 3 is that, with the two extra-dimensions, an equilibrium in which the seller persuades the buyer to accept an inferior option does not necessarily have an interval-structure or positive outcomes in the transfer dimension. We show this by means of a simple example. Suppose that, for any outside option value $v^o \in [0, \frac{\eta(\lambda-1)}{1+\eta}\xi]$, the seller offers $(v^s, t^s) = (0, 0)$, and for any outside option value $v^o \in (\frac{\eta(\lambda-1)}{1+\eta}\xi, 1]$, the seller offers $(v^s, t^s)$ with $t^s = 0$ and

$$v^s = v^o - \frac{\eta(\lambda - 1)}{1 + \eta}\xi. \tag{19}$$

The buyer then can infer the value of her outside option $v^o$ from such an offer. If she plans to accept the offer and – unexpectedly – accepts the outside option in period 2, her payoff is

$$v^o + \eta(v^o - v^s) - \eta(\lambda - 1)\xi, \tag{20}$$

which equals the value in (19) so that the buyer is indifferent between accepting and rejecting the seller's offer. In period 1, it is optimal for the buyer to plan acceptance of the seller's offer if

$$v^s > v^o - \eta(\lambda - 1)\xi, \tag{21}$$

which is equivalent to $\eta > 0$. Hence, we can construct an equilibrium in which the seller persuades the buyer to accept an inferior option and that does neither have an interval-structure nor positive outcomes in the transfer dimension.

This, however, changes when we consider seller-preferred equilibria. We can show that if $\xi$ is small enough, then a seller-preferred equilibrium exhibits both bunching of outside option values and positive outcomes in the transfer dimension, at least for the highest outside option values. The next result states this observation formally.

**Proposition 4** (Seller-Preferred Equilibrium, Multi-Dimensional Outside Option Values)**.** *Consider the model with multi-dimensional outside option values. Suppose that $\eta(\lambda - 1) > 0$ and $\xi > 0$. If for given values $\eta, \lambda$ the parameter $\xi$ is small enough, then in a seller-preferred equilibrium the seller offers the same total value with a positive outcome in the transfer dimension for all $v^o \in (\underline{v}, 1]$ for some $\underline{v} < 1$.*

The intuition for this result will become clear in the discussion of the next proposition. In general, the exact shape of seller-preferred equilibria depends on the loss aversion parameters $\eta, \lambda$, the uncertainty parameter $\xi$, and the distribution over outside option values $F$. Nevertheless, for the special case of a uniform distribution $F$, we can characterize seller-preferred equilibria if the loss aversion parameters $\eta, \lambda$ satisfy

$$\frac{1+\eta}{1+\eta\lambda} < \frac{1}{2} + \frac{1}{6}\eta(\lambda - 1). \tag{22}$$

This inequality holds if $\lambda$ is large enough for given $\eta$ (e.g., $\lambda > 2$ at $\eta = 1$). If additionally $\xi$ is small enough, a seller-preferred equilibrium has the following structure.

**Proposition 5** (Seller-Preferred Equilibrium, Multi-Dimensional Outside Option Values, Uniform Distribution)**.** *Consider the model with multi-dimensional outside option values. Suppose that $\eta(\lambda - 1) > 0$, $\xi > 0$, $\eta\lambda\xi < 1$, the condition in (22) is satisfied, and that $F$ is the uniform distribution on the unit interval. Any seller-preferred equilibrium is then characterized by a sequence of disjoint intervals $\{V_i\}_{i\in\mathbb{N}}$ that partitions the interval $[\eta\lambda\xi, 1]$. In this equilibrium, the seller makes the following offers:*

*(i) If $v^o \leq \frac{\eta(\lambda-1)}{1+\eta}\xi$, the seller offers $(0, 0)$.*

*(ii) If $\frac{\eta(\lambda-1)}{1+\eta}\xi < v^o \leq \eta\lambda\xi$, the seller offers $(0, t^s)$ with*

$$t^s = \frac{(1+\eta)v^o - \eta(\lambda - 1)\xi}{1+\eta\lambda}.$$

*(iii) If $v^o \in V_i \subset [\eta\lambda\xi, 1]$, the seller offers $(0, t_i^s)$ with*

$$t_i^s = \frac{(1+\eta)\bar{v}_i - \eta(\lambda - 1)\xi}{1+\eta\lambda}.$$

*The buyer always accepts the seller's offer. The length of the intervals $\{V_i\}_{i\in\mathbb{N}}$ is minimal subject to the constraint that it is optimal for the buyer to plan acceptance.*

The result groups the interval of outside option values into three domains: small, intermediate, and large values. In the following, we illustrate the intuition behind the offers in each of these domains. This intuition captures the subtle interaction of uncertainty and attachment effect. At small outside option values, the seller makes the least generous offer $(0, 0)$. The buyer accepts this offer in period 2 since rejecting it would generate gain-loss sensations in the two extra-dimensions. At intermediate values $v^o$, the seller makes offers with positive transfers that are fully informative about the buyer's outside option value. In this domain, the uncertainty effect is created only through the extra-dimensions, while the attachment effect is generated trough the transfer as well as the extra-dimensions. Both effects are relatively small (since the transfer is still relatively small) so that it is not necessary to generate additional uncertainty through the bunching of outside option values.

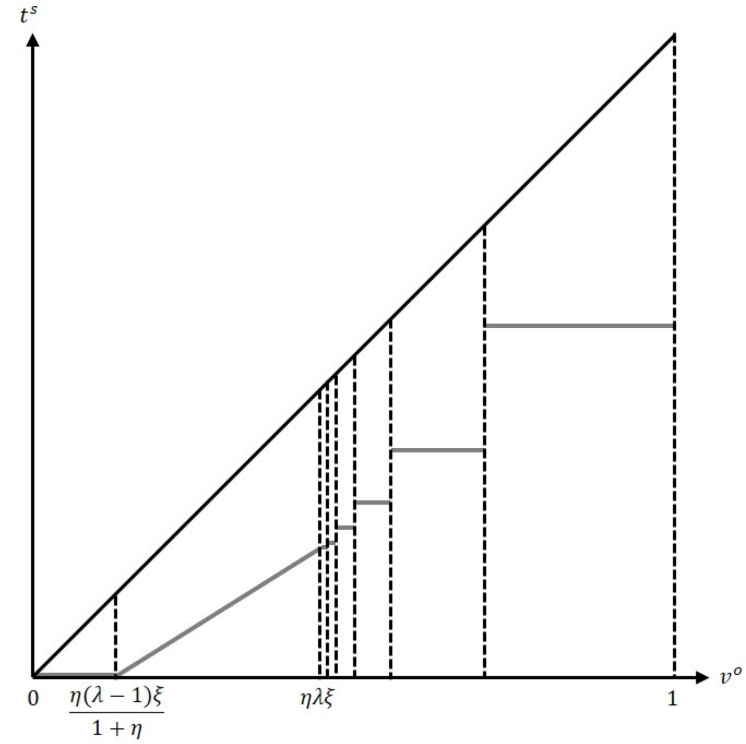


Figure 4: The seller-preferred equilibrium for $\eta = 1$ and $\lambda = 2.25$ when $F$ is the uniform distribution and there is uncertainty $\xi = 0.2$ in two extra-dimensions. The 45-degree line indicates offers with total value $v^o$. The dotted vertical lines indicate the bounds of the intervals $\{V_i\}_{i\in\mathbb{N}}$ and the three cases of Proposition 5. The gray lines indicate, for each interval $V_i$, the offered transfer $t_i^s$ (and, hence, also the offered total value).

Finally, at large outside option values, bunching occurs. Suppose that this were not the case and that the seller would still make offers as in the intermediate domain so that they are fully informative about the outside option. The transfer then would have to be relatively large (following the outside option value). Consequently, the attachment effect – which is generated through the transfer and the extra-dimensions – would be relatively large compared to the uncertainty effect that is still created only through the extra-dimensions. Therefore, in order to reduce the total value necessary to get the buyer's acceptance, the optimal offers employ bunching to strengthen the uncertainty effect.

We again find that the seller can benefit from requesting immediate acceptance, as introduced in Subsection 4.5. Suppose that in period 1 the seller requests immediate acceptance and makes the least generous offer $(0, 0)$ to the buyer. It is then optimal for the buyer to accept this offer if

$$0 \geq \int_0^1 f(v)v\,\mathrm{d}v - \eta(\lambda - 1)\int_0^1 f(v)\int_v^1 f(\tilde{v})(\tilde{v} - v)\,\mathrm{d}\tilde{v}\,\mathrm{d}v - \eta(\lambda - 1). \tag{23}$$

This condition is satisfied if $\eta(\lambda - 1)$ is large enough. Denote by $\Lambda_{FF}$ the value of $\eta(\lambda - 1)$ so that (23) is satisfied with equality. Due to the uncertainty in the extra-dimensions $\Lambda_{FF}$ is strictly smaller than $\Lambda_F$ as defined by inequality (17). We can state the following result.

**Corollary 2** (Multi-Dimensional Outside Option Values, Immediate Acceptance)**.** *Consider the model with multi-dimensional outside option values when the seller can request immediate acceptance of an offer in period 1. If* $\eta(\lambda - 1) \geq \Lambda_{FF}$*, then, in any seller-preferred equilibrium, the seller offers* $(v^s, t^s) = (0, 0)$ *at each outside option value and requests immediate acceptance.*

# 6 Conclusion

In many bargaining situations, parties receive information over time so that initially asymmetric information about possible options becomes symmetric. We showed in this paper that, in this situation, it can be optimal for the better-informed party to make an early offer to an opponent, in particular, if this opponent has reference-dependent loss-averse preferences. The early offer can credibly reveal information and allow the buyer to attain peace of mind at an early stage by planning its acceptance. This allows the seller to persuade the buyer to accept an offer that is inferior to her outside option, even if she has all payoff-relevant information at the decision stage. There would be no such scope for persuasion if the buyer had standard preferences.

The analysis highlighted several factors for when the seller can persuade the buyer to accept an inferior offer. The offer needs to have features that outside options do not have, so that giving up these features creates loss sensations through the attachment effect. Next, early offers must be made in a way so that some uncertainty about the value of the outside option remains. Through the uncertainty effect it then can be optimal for the buyer to plan acceptance of an offer that with certainty is inferior to the outside option. Therefore, an equilibrium with persuasion typically has an interval-structure akin to that in Crawford and Sobel (1982). The seller-preferred equilibrium optimally balances the strength of the attachment and the uncertainty effect. Finally, if the seller has commitment power to request immediate acceptance, this further increases the scope for persuasion.

To show these results, we considered a setup that can be extended in several directions. The literature on persuasion has examined a variety of settings that also could be enriched by taking loss aversion into account, for example, settings with multiple sellers, different incentive structures, or incentives to acquire information. The results of the present paper should be helpful for this analysis.

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# A Appendix

*Proof of Lemma 1.* Consider any offer $(v^s, t^s)$ that the seller makes if and only if $v_o \in V \subset (0, 1)$, with $v^s + t^s \leq \underline{v}$. Let $\hat{F}$ be the updated distribution over outside option values when the buyer observes $(v^s, t^s)$. Consider a cut-off plan $\sigma^r$ where for some $v^* \in [\underline{v}, \bar{v}]$ the buyer accepts $(v^s, t^s)$ if $v^o \in (\underline{v}, v^*]$ and rejects $(v^s, t^s)$ if $[v^*, \bar{v})$. After observing $(v^s, t^s)$, the buyer's expected utility from $\sigma^r$ equals

$$\begin{aligned}
\mathbb{E}_{\hat{F}}[U_R(\sigma^r(v^o, v^s, t^s) \mid G^v, G^t)] \quad = \quad & \hat{F}(v^*)(v^s + t^s) + \int_{v^*}^{\bar{v}} \hat{f}(v) v \, \mathrm{d}v \\
& -\eta(\lambda - 1)\hat{F}(v^*)[1 - \hat{F}(v^*)]t^s \\
& -\eta(\lambda - 1)\hat{F}(v^*) \int_{v^*}^{\bar{v}} \hat{f}(\tilde{v})(\tilde{v} - v^s) \, \mathrm{d}\tilde{v} \\
& -\eta(\lambda - 1) \int_{v^*}^{\bar{v}} \hat{f}(v) \int_{v}^{\bar{v}} \hat{f}(\tilde{v})(\tilde{v} - v) \, \mathrm{d}\tilde{v} \, \mathrm{d}v. \qquad (24)
\end{aligned}$$

We show that this term is maximal either at $v^* = \underline{v}$ or at $v^* = \bar{v}$ or at both points. For this, we differentiate buyer's expected utility with respect to $v^*$:

$$\begin{aligned}
\frac{\partial \mathbb{E}_{\hat{F}}[\cdot]}{\partial v^*} \quad = \quad & \hat{f}(v^*)(v^s + t^s) - \hat{f}(v^*)v^* \\
& -\eta(\lambda - 1)\hat{f}(v^*)[1 - 2\hat{F}(v^*)]t^s \\
& -\eta(\lambda - 1)\left[\hat{f}(v^*) \int_{v^*}^{\bar{v}} \hat{f}(\tilde{v})(\tilde{v} - v^s) \, \mathrm{d}\tilde{v} - \hat{F}(v^*)\hat{f}(v^*)(v^* - v^s)\right] \\
& +\eta(\lambda - 1)\hat{f}(v^*) \int_{v^*}^{\bar{v}} \hat{f}(\tilde{v})(\tilde{v} - v^*) \, \mathrm{d}\tilde{v}. \qquad (25)
\end{aligned}$$

We can simplify this to

$$\frac{\partial \mathbb{E}_{\hat{F}}[\cdot]}{\partial v^*} = -\hat{f}(v^*)\left[(v^* - v^s - t^s) + \eta(\lambda - 1)(1 - 2\hat{F}(v^*))(v^* - v^s + t^s)\right]. \qquad (26)$$

Since $v^s + t^s \leq \underline{v} \leq v^*$, this term is strictly negative for all $v^* > \underline{v}$ with $\hat{F}(v^*) \leq \frac{1}{2}$. Denote by $\Gamma(v^*)$ the term in the squared brackets in (26). The derivative $\frac{\partial \mathbb{E}_{\hat{F}}[\cdot]}{\partial v^*}$ is positive (negative) if and only if $\Gamma(v^*)$ is negative (positive). Consider the derivative

$$\frac{\partial \Gamma(v^*)}{\partial v^*} = 1 + \eta(\lambda - 1)[-2\hat{f}(v^*)(v^* - v^s + t^s) + (1 - 2\hat{F}(v^*))]. \qquad (27)$$

Since $F$ is weakly convex, $\hat{f}$ weakly increases in $v^*$ on its support. Hence, the right-hand side of equation (27) strictly decreases in $v^*$. If $\frac{\partial \Gamma(v^*)}{\partial v^*}$ is negative at $v^* = v^{**}$, it is negative for all $v^* > v^{**}$. By the statement above, if $\frac{\partial \mathbb{E}_{\hat{F}}[\cdot]}{\partial v^*}$ becomes positive at some value $v^* = v^{**}$, it remains positive for all $v^* > v^{**}$, which yields us the result. □

*Proof of Proposition 1.* The proof proceeds by steps. **Step 1.** Consider an interval $V_i = (\underline{v}_i, \bar{v}_i] \subset (0, 1]$ and assume that the seller makes the offer $(v_i^s, t_i^s)$ to the buyer if and only if $v^o \in V_i$. We show that if $\underline{v}_i$ is sufficiently close to $\bar{v}_i$, then we can choose $(v_i^s, t_i^s)$ with $v_i^s + t_i^s < \underline{v}_i$ such that the buyer's PPE specifies to accept this offer whenever $v^o \in V_i$ and to reject it when $v^o > \bar{v}_i$. Lemma 1 implies that the plan "accept $(v_i^s, t_i^s)$ if $v^o \in V_i$" is the payoff-maximizing plan for the buyer if its expected payoff exceeds that from the plan "accept the outside option if $v^o \in V_i$." Her expected utility from the latter plan equals

$$\int_{\underline{v}_i}^{\bar{v}_i} \hat{f}(v)v \, \mathrm{d}v - \eta(\lambda - 1) \int_{\underline{v}_i}^{\bar{v}_i} \hat{f}(v) \int_{v}^{\bar{v}_i} \hat{f}(\tilde{v})(\tilde{v} - v) \, \mathrm{d}\tilde{v} \, \mathrm{d}v. \tag{28}$$

Note that $\hat{f}(v) = \frac{1}{F(\bar{v}_i) - F(\underline{v}_i)} f(v)$, so that we have $\frac{\hat{f}(\bar{v}_i) - \hat{f}(\underline{v}_i)}{\hat{f}(\underline{v}_i)} \to 0$ for $\underline{v}_i \to \bar{v}_i$. Hence, the distribution $\hat{F}(v)$ becomes the uniform distribution with density $\frac{1}{\bar{v}_i - \underline{v}_i}$ as $\underline{v}_i$ get close to $\bar{v}_i$. The expected utility from always choosing the outside option then becomes

$$\frac{1}{\bar{v}_i - \underline{v}_i} \int_{\underline{v}_i}^{\bar{v}_i} 1 \, \mathrm{d}v - \eta(\lambda - 1) \frac{1}{(\bar{v}_i - \underline{v}_i)^2} \int_{\underline{v}_i}^{\bar{v}_i} \int_{v}^{\bar{v}_i} (\tilde{v} - v) \, \mathrm{d}\tilde{v} \, \mathrm{d}v, \tag{29}$$

which we can simplify to

$$\frac{1}{2}(\bar{v}_i + \underline{v}_i) - \eta(\lambda - 1)\frac{1}{6}(\bar{v}_i - \underline{v}_i). \tag{30}$$

The expected utility from accepting the seller's offer equals $v_i^s + t_i^s$. Note that

$$\underline{v}_i > \frac{1}{2}(\bar{v}_i + \underline{v}_i) - \eta(\lambda - 1)\frac{1}{6}(\bar{v}_i - \underline{v}_i) \tag{31}$$

is equivalent to $\eta(\lambda - 1) > 3$. Hence, if $\eta(\lambda - 1) > 3$ and $\underline{v}_i$ is close enough to $\bar{v}_i$, we can find values $(v_i^s, t_i^s)$ with $v_i^s + t_i^s < \underline{v}_i$ such that, in period 1, the plan "accept $(v_i^s, t_i^s)$ if $v^o \in V_i$" is preferred to any other plan. Next, we check when it is optimal for the buyer to also execute this plan in period 2 (so that it is indeed a PPE). Her utility from following the plan is $v_i^s + t_i^s$, while her utility from choosing the outside option equals $v^o + \eta(v^o - v_i^s) - \eta\lambda t_i^s$. If $\underline{v}_i > \frac{1+\eta}{1+\eta\lambda}\bar{v}_i$ we can choose $v_i^s, t_i^s$ with $t_i^s > 0$ and $v_i^s + t_i^s < \underline{v}_i$ such that

$$(1 + \eta)v_i^s + (1 + \eta\lambda)t_i^s = (1 + \eta)\bar{v}_i. \tag{32}$$

It is then optimal for the buyer to accept offer $(v_i^s, t_i^s)$ if $v^o \in V_i$ and to reject it if $v^o > \bar{v}_i$, which completes the proof of the statement. **Step 2**. We construct a signaling equilibrium with the desired property. We can choose a sequence of half-open intervals $\{V_i\}_{i \in \mathbb{N}}$ and a sequence of offers $\{(v_i^s, t_i^s)\}_{i \in \mathbb{N}}$ so that the seller makes the offer $(v_i^s, t_i^s)$ with total value $0 < v_i^s + t_i^s < \underline{v}_i$ to the buyer whenever $v^o \in V_i$ and $v_i^s + t_i^s$ strictly decreases in $i$. By Step 1, this sequence can be chosen so that it is a PPE for the buyer to accept the seller's offer $(v_i^s, t_i^s)$ if $v^o \leq \bar{v}_i$ and to reject it otherwise. For any offer $(v^s, t^s)$ that is not an element of the set $\{(v_i^s, t_i^s)\}_{i\in\mathbb{N}}$ we specify that in period 1 the buyer believes that her outside option value is $v^o = 1$ with certainty. It is then optimal for her to reject an offer $(v^s, t^s)$ in period 2 if $v^s + t^s \leq v^o$. Given this buyer behavior, it is then indeed optimal for the seller to offer $(v_i^s, t_i^s)$ if and only if $v^o \in V_i$. This completes the proof of the first statement of Proposition 1. **Step 3**. We prove the second statement of Proposition 1. Note that in equilibrium it must be the case that the buyer always accepts the seller's offer. Hence, for any two values $v, \hat{v} \in [0, 1]$ with $v > \hat{v}$ the following must hold: Suppose the seller offers the total value $w$ if $v^o = v$ and $\hat{w}$ if $v^o = \hat{v}$. Then we must have $w \geq \hat{w}$. Otherwise, the seller could deviate profitably if $v^o = \hat{v}$ by making the same offer as for $v^o = v$ since the buyer will accept it. Given this result, we can make the following observation: Assume by contradiction that there exists an interval $V = (v_L, v_H) \subset [0, 1]$ so that for any two outside option values $v, \hat{v} \in V$ the seller makes offers with varying total value, $w \neq \hat{w}$. The buyer would then be able to infer her outside option value from these offers so that the seller cannot persuade her accept an inferior offer, a contradiction. Hence, an equilibrium in which the seller always benefits from making early offers must be characterized by a sequence of disjoint intervals $\{V_i\}_{i\in\mathbb{N}}$, so that the seller makes an offer with total value $w_i^s < \underline{v}_i$ and $t_i > 0$ if $v^o \in V_i$. **Step 4**. We show that if $\eta(\lambda - 1) < 3$, there exists no equilibrium in which the seller benefits from making early offers. Note that we can rewrite

$$\int_{\underline{v}_i}^{\bar{v}_i} \hat{f}(v) \int_{v}^{\bar{v}_i} \hat{f}(\tilde{v})(\tilde{v} - v)\,\mathrm{d}\tilde{v}\,\mathrm{d}v = \int_{\underline{v}_i}^{\bar{v}_i} \hat{f}(v) \int_{\underline{v}_i}^{v} \hat{f}(\tilde{v})(v - \tilde{v})\,\mathrm{d}\tilde{v}\,\mathrm{d}v. \tag{33}$$

Moreover, we have

$$\begin{aligned}
&\int_{\underline{v}_i}^{\bar{v}_i} \hat{f}(v) \int_{v}^{\bar{v}_i} \hat{f}(\tilde{v})(\tilde{v} - v)\,\mathrm{d}\tilde{v}\,\mathrm{d}v + \int_{\underline{v}_i}^{\bar{v}_i} \hat{f}(v) \int_{\underline{v}_i}^{v} \hat{f}(\tilde{v})(v - \tilde{v})\,\mathrm{d}\tilde{v}\,\mathrm{d}v \\
= &\int_{\underline{v}_i}^{\bar{v}_i} \hat{f}(v) \left[\int_{v}^{\bar{v}_i} \hat{f}(\tilde{v})(\tilde{v} - v)\,\mathrm{d}\tilde{v} + \int_{\underline{v}_i}^{v} \hat{f}(\tilde{v})(v - \tilde{v})\,\mathrm{d}\tilde{v}\right]\mathrm{d}v \\
= &\int_{\underline{v}_i}^{\bar{v}_i} \int_{\underline{v}_i}^{\bar{v}_i} \hat{f}(v)\hat{f}(\tilde{v}) \mid \tilde{v} - v \mid \mathrm{d}\tilde{v}\,\mathrm{d}v. \qquad (34)
\end{aligned}$$

We therefore can write the expected payoff from the plan "accept the outside option if $v^o \in V_i$" in (28) as

$$\int_{\underline{v}_i}^{\bar{v}_i} \hat{f}(v)\,\mathrm{d}v - \eta(\lambda - 1) \int_{\underline{v}_i}^{\bar{v}_i} \int_{\underline{v}_i}^{\bar{v}_i} \frac{\hat{f}(v)\hat{f}(\tilde{v})}{2} \mid v - \tilde{v} \mid \mathrm{d}\tilde{v}\,\mathrm{d}v. \tag{35}$$

Observe from (35) that the buyer's expected payoff from accepting the outside option is minimal if $\hat{F}$ is a uniform distribution provided that $\hat{f}$ is weakly increasing on its support. To see this, note that the unique maximum of the product $\hat{f}(v)\hat{f}(\tilde{v})$ subject to the constraint that

$\hat{f}(v) + \hat{f}(\tilde{v}) = G$ for some value $G$ is obtained when $\hat{f}(v) = \hat{f}(\tilde{v}) = \frac{G}{2}$. Hence, by Step 1, if the support of $\hat{F}$ is an interval $V$, the buyer accepts an offer with total value $v^s + t^s < \underline{v}_i$ only if $\eta(\lambda - 1) \geq 3$, which completes the proof. □

*Proof of Proposition 2.* The proof proceeds in steps. **Step 1.** We prove the first statement. In an equilibrium, the buyer always accepts the seller's offer if $v^o < 1$. As in Step 3 of the proof of Proposition 1 we can show that if the seller offers the total value $w$ if $v^o = v < 1$ and $\hat{w}$ if $v^o = \hat{v} < v$, then we must have $w \geq \hat{w}$. Assume by contradiction that in a seller-preferred equilibrium $\sigma$ exists an interval $V = (v_L, v_H) \subset [0, 1]$ so that for any two outside option values $v, \hat{v} \in V$ the seller makes offers with varying total value, $w \neq \hat{w}$. The buyer would then be able to infer her outside option value from these offers so that the total value of a seller offer equals the outside option value for each $v^o \in V$. We then can find an alternative equilibrium $\sigma'$ that is identical to $\sigma$ except that there is an interval of outside option values $(v'_L, v'_H) \subset (v_L, v_H)$ at which the seller makes an offer with total value $w < v_L$. This can be shown by following the same steps as in Step 1 of the proof of Proposition 1. The seller's expected payoff in equilibrium $\sigma'$ strictly exceeds that in equilibrium $\sigma$, a contradiction. This implies the first statement. **Step 2.** We prove the second statement. Consider any sequence of disjoint intervals $\{V_i\}_{i \in \mathbb{N}}$ with the property that, for each interval $V_i$, we have $\underline{v}_i = \bar{v}_{i+1}$ and $\frac{1+\eta}{1+\eta\lambda}\bar{v}_i \leq \underline{v}_i \leq \Gamma(\eta, \lambda)\bar{v}_i$. By Step 1, a seller-preferred equilibrium must be characterized by such a sequence. In such an equilibrium, the seller offers $(v_i^s, t_i^s)$ if $v^o \in V_i$, and the buyer accepts this offer. Define $w_i^s = v_i^s + t_i^s$. It is optimal for the buyer to plan acceptance in period 1 if and only if

$$w_i^s \geq \frac{1}{2}(\bar{v}_i + \underline{v}_i) - \frac{1}{6}\eta(\lambda - 1)(\bar{v}_i - \underline{v}_i) \tag{36}$$

for all $i \in \mathbb{N}$. For actual acceptance in period 2, it must be the case that condition (6) is satisfied. Hence, we must have

$$w_i^s \geq \frac{1 + \eta}{1 + \eta\lambda}\bar{v}_i. \tag{37}$$

Note that this inequality is satisfied with equality if and only if $(v_i^s, t_i^s) = (0, \frac{1+\eta}{1+\eta\lambda}\bar{v}_i)$. The inequality in (36) defines an upper bound on $\underline{v}_i$:

$$\underline{v}_i \leq \frac{1}{\frac{1}{2} + \frac{1}{6}\eta(\lambda - 1)}\left[w_i^s + \bar{v}_i\left(\frac{1}{6}\eta(\lambda - 1) - \frac{1}{2}\right)\right]. \tag{38}$$

Consider the average total value offered as a fraction of the average outside option value in interval $V_i$. It is given by

$$av_i = \frac{w_i^s}{\frac{1}{2}(\bar{v}_i + \underline{v}_i)}. \tag{39}$$

We show that for given $\bar{v}_i$ the lowest possible equilibrium value of $av_i$ is obtained when the inequalities in (37) and (38) are satisfied with equality, and that this value only depends on $\eta$ and $\lambda$. Observe that $av_i$ strictly decreases in $\underline{v}_i$. The largest possible equilibrium value of $\underline{v}_i$ is given by the right-hand side of inequality (38). We replace $\underline{v}_i$ by the right-hand side of inequality (38) in equation (39) and obtain

$$av_i = \frac{(1 + \frac{1}{3}\eta(\lambda - 1))w_i^s}{\bar{v}_i \frac{1}{3}\eta(\lambda - 1) + w_i^s}. \tag{40}$$

This expression strictly increases in $w_i^s$. Therefore, the lowest possible equilibrium value of $av_i$ is obtained if $w_i^s = \frac{1+\eta}{1+\eta\lambda}\bar{v}_i$. If we further replace $w_i^s$ by $\frac{1+\eta}{1+\eta\lambda}\bar{v}_i$, the term $\bar{v}_i$ drops out of the resulting expression, which proves the claim. From this, the second statement of Proposition 2 directly follows. □

*Proof of Lemma 2.* Consider an offer $(v^s, t^s)$ that the seller makes if and only if $v_o \in V \subset (0, 1)$, with $v^s + t^s \leq v^o$. Let $\hat{F}$ be the updated distribution over outside option values when the buyer observes $(v^s, t^s)$. Since $F$ is weakly convex, $\hat{f}$ weakly increases on its support. Consider w.l.o.g. a cut-off plan $\sigma^r$ characterized by two values $v_1^*, v_2^* \in [\underline{v}, \bar{v}]$ with $v_1^* \leq v_2^*$ that has the following features: In state 1, the buyer accepts $(v^s, t^s)$ if $v^o \in [\underline{v}, v_1^*]$ and rejects $(v^s, t^s)$ if $(v_1^*, \bar{v}]$. In state 2, the buyer accepts $(v^s, t^s)$ if $v^o \in [\underline{v}, v_2^*]$ and rejects $(v^s, t^s)$ if $(v_2^*, \bar{v}]$. After observing $(v^s, t^s)$, the buyer's expected utility from $\sigma^r$ equals

$$\begin{aligned}
\mathbb{E}_{\hat{F}}[U_R(.)] \;=\;& \left(\frac{1}{2}\hat{F}(v_1^*) + \frac{1}{2}\hat{F}(v_2^*)\right)(v^s + t^s) + \frac{1}{2}\int_{v_1^*}^{\bar{v}} \hat{f}(v)v \,\mathrm{d}v + \frac{1}{2}\int_{v_2^*}^{\bar{v}} \hat{f}(v)v \,\mathrm{d}v \\
& -\eta(\lambda - 1)\left(\frac{1}{2}\hat{F}(v_1^*) + \frac{1}{2}\hat{F}(v_2^*)\right)\left(1 - \frac{1}{2}\hat{F}(v_1^*) - \frac{1}{2}\hat{F}(v_2^*)\right)t^s \\
& -\eta(\lambda - 1)\left(\frac{1}{2}\hat{F}(v_1^*) + \frac{1}{2}\hat{F}(v_2^*)\right)\left(\frac{1}{2}\int_{v_1^*}^{v_2^*} \hat{f}(\tilde{v})(\tilde{v} - v^s)\,\mathrm{d}\tilde{v} + \int_{v_2^*}^{\bar{v}} \hat{f}(\tilde{v})(\tilde{v} - v^s)\,\mathrm{d}\tilde{v}\right) \\
& -\eta(\lambda - 1)\frac{1}{2}\int_{v_1^*}^{v_2^*} \hat{f}(v)\left(\frac{1}{2}\int_{v}^{v_2^*} \hat{f}(\tilde{v})(\tilde{v} - v)\,\mathrm{d}\tilde{v} + \int_{v_2^*}^{\bar{v}} \hat{f}(\tilde{v})(\tilde{v} - v)\,\mathrm{d}\tilde{v}\right)\mathrm{d}v \\
& -\eta(\lambda - 1)\int_{v_2^*}^{\bar{v}} \hat{f}(v)\int_{v}^{\bar{v}} \hat{f}(\tilde{v})(\tilde{v} - v)\,\mathrm{d}\tilde{v}\,\mathrm{d}v \\
& -\eta(\lambda - 1)\left(1 - \frac{1}{2}\hat{F}(v_1^*)^2 - \frac{1}{2}\hat{F}(v_2^*)^2\right)\xi.
\end{aligned} \tag{41}$$

The rest of the proof proceeds in steps. **Step 1.** We show that for given $v_2^* > \underline{v}$ the value in (41) is maximal at $v_1^* = \underline{v}$ or at $v_1^* = v_2^*$ or at both values. The first derivative of (41) with respect to

$v_1^*$ equals

$$\begin{aligned}\frac{\partial \mathbb{E}_{\hat{F}}[U_R(.)]}{\partial v_1^*} &= -\frac{1}{2}\hat{f}(v_1^*)(v_1^* - v^s - t^s) \\ &-\eta(\lambda-1)\frac{1}{2}\hat{f}(v_1^*)(1-\hat{F}(v_1^*)-\hat{F}(v_2^*))t^s \\ &+\eta(\lambda-1)\frac{1}{2}\hat{f}(v_1^*)\left(\frac{1}{2}\hat{F}(v_1^*)+\frac{1}{2}\hat{F}(v_2^*)\right)(v_1^*-v^s) \\ &-\eta(\lambda-1)\frac{1}{2}\hat{f}(v_1^*)\left(\frac{1}{2}\int_{v_1^*}^{v_2^*}\hat{f}(\tilde{v})(\tilde{v}-v^s)\,\mathrm{d}\tilde{v}+\int_{v_2^*}^{\bar{v}}\hat{f}(\tilde{v})(\tilde{v}-v^s)\,\mathrm{d}\tilde{v}\right) \\ &+\eta(\lambda-1)\frac{1}{2}\hat{f}(v_1^*)\left(\frac{1}{2}\int_{v_1^*}^{v_2^*}\hat{f}(\tilde{v})(\tilde{v}-v_1^*)\,\mathrm{d}\tilde{v}+\int_{v_2^*}^{\bar{v}}\hat{f}(\tilde{v})(\tilde{v}-v_1^*)\,\mathrm{d}\tilde{v}\right) \\ &+\eta(\lambda-1)\hat{f}(v_1^*)\hat{F}(v_1^*)\xi, \end{aligned} \tag{42}$$

which can be simplified to

$$\begin{aligned}\frac{\partial \mathbb{E}_{\hat{F}}[U_R(.)]}{\partial v_1^*} &= -\frac{1}{2}\hat{f}(v_1^*)\Bigg[(v_1^*-v^s-t^s)+\eta(\lambda-1) \\ &\times\left((1-\hat{F}(v_1^*)-\hat{F}(v_2^*))t^s+\left(1-\frac{3}{2}\hat{F}(v_1^*)-\frac{1}{2}\hat{F}(v_2^*)\right)(v_1^*-v^s)-2\hat{F}(v_1^*)\xi\right)\Bigg]. \end{aligned} \tag{43}$$

Denote the term in squared brackets by $\Gamma_1(v_1^*, v_2^*)$. The second derivative of (41) with respect to $v_1^*$ equals

$$\begin{aligned}\frac{\partial^2 \mathbb{E}_{\hat{F}}[U_R(.)]}{\partial (v_1^*)^2} &= -\frac{1}{2}\hat{f}'(v_1^*)\Gamma_1(v_1^*,v_2^*)-\frac{1}{2}\hat{f}(v_1^*)\Bigg[1+\eta(\lambda-1) \\ &\times\left(-\hat{f}(v_1^*)\left(\frac{3}{2}(v_1^*-v^s)+t^s+2\xi\right)+\left(1-\frac{3}{2}\hat{F}(v_1^*)-\frac{1}{2}\hat{F}(v_2^*)\right)\right)\Bigg]. \end{aligned} \tag{44}$$

Assume by contradiction that $\mathbb{E}_{\hat{F}}[U_R(.)]$ has a local maximum at $\hat{v}_1^* \in (\underline{v}, v_2^*)$. Note that the first derivative of $\mathbb{E}_{\hat{F}}[U_R(.)]$ with respect to $v_1^*$ is strictly negative at $v_1^* = \underline{v}$. Hence, there must be a local minimum of $\mathbb{E}_{\hat{F}}[U_R(.)]$ at some value $\tilde{v}_1^* \in (\underline{v}, \hat{v}_1^*)$. At a local maximum or minimum, we must have $\Gamma_1(., v_2^*) = 0$. Therefore, the term in squared brackets on the right-hand side of equation (44) must be negative at $v_1^* = \tilde{v}_1^*$ and positive at $v_1^* = \hat{v}_1^*$. This implies that

$$\hat{f}(\hat{v}_1^*)\left(\frac{3}{2}(\hat{v}_1^*-v^s)+t^s+2\xi\right)+\frac{3}{2}\hat{F}(\hat{v}_1^*) < \hat{f}(\tilde{v}_1^*)\left(\frac{3}{2}(\tilde{v}_1^*-v^s)+t^s+2\xi\right)+\frac{3}{2}\hat{F}(\tilde{v}_1^*), \tag{45}$$

which contradicts the fact that $\hat{f}$ weakly increases on its support and $\hat{v}_1^* > \tilde{v}_1^*$. This completes the proof of the statement. **Step 2.** We show that at $v_1^* = \underline{v}$ the expected payoff in (41) is maximal at $v_2^* = \underline{v}$ or at $v_2^* = \bar{v}$ or at both values. The first derivative of (41) with respect to $v_2^*$ equals

$$\begin{aligned}\frac{\partial \mathbb{E}_{\hat{F}}[U_R(.)]}{\partial v_2^*} &= -\frac{1}{2}\hat{f}(v_2^*)\Bigg[(v_2^* - v^s - t^s) + \eta(\lambda-1)\Bigg((1-\hat{F}(v_1^*)-\hat{F}(v_2^*))t^s \\ &+\left(1-\frac{1}{2}\hat{F}(v_1^*)-\frac{3}{2}\hat{F}(v_2^*)\right)(v_2^*-v^s)+\frac{1}{2}\int_{v_1^*}^{v_2^*}\hat{f}(\tilde{v})(2\tilde{v}-v^s-v_2^*)\,\mathrm{d}\tilde{v}-2\hat{F}(v_2^*)\xi\Bigg)\Bigg].\end{aligned} \tag{46}$$

Denote the term in squared brackets by $\Gamma_2(v_1^*, v_2^*)$. The second derivative of (41) with respect to $v_2^*$ equals

$$\begin{aligned}\frac{\partial^2 \mathbb{E}_{\hat{F}}[U_R(.)]}{\partial (v_2^*)^2} &= -\frac{1}{2}\hat{f}'(v_2^*)\Gamma_2(v_1^*, v_2^*) - \frac{1}{2}\hat{f}(v_2^*)\Bigg[1+\eta(\lambda-1) \\ &\quad\times\left(-\hat{f}(v_2^*)\left(v_2^*-v^s+t^s+2\xi\right)+\left(1-2\hat{F}(v_2^*)\right)\right)\Bigg].\end{aligned} \tag{47}$$

Note that, at $v_1^* = \underline{v}$, the first derivative of $\mathbb{E}_{\hat{F}}[U_R(.)]$ with respect to $v_2^*$ is strictly negative. By applying the same arguments as in Step 1, we then can show the result. **Step 3.** We consider the set of cut-off plans with $v_1^* = v_2^* = v^*$ and show that the expected payoff in (41) for these plans is maximal at $v^* = \underline{v}$ or at $v^* = \bar{v}$ or at both values. The first derivative of (41) with respect to $v^*$ is

$$\frac{\partial \mathbb{E}_{\hat{F}}[U_R(.)]}{\partial v^*} = -\hat{f}(v^*)\left[(v_1^* - v^s - t^s) + \eta(\lambda-1)\left((1-\hat{F}(v^*))(v_1^*-v^s+t^s)-2\hat{F}(v^*)\xi\right)\right] \tag{48}$$

and the second derivative of (41) with respect to $v^*$ equals

$$\begin{aligned}\frac{\partial^2 \mathbb{E}_{\hat{F}}[U_R(.)]}{\partial (v^*)^2} &= -\hat{f}'(v^*)\Gamma_3(v^*) - \hat{f}(v^*)\Bigg[1+\eta(\lambda-1) \\ &\quad\times\left(-2\hat{f}(v^*)(v^*-v^s+t^s+2\xi)+(1-2\hat{F}(v^*))\right)\Bigg],\end{aligned} \tag{49}$$

where $\Gamma_3(v^*)$ is the term in squared brackets on the right-hand side of equation (48). We can now apply the same arguments as in Step 1 and Step 2 to prove the statement. **Step 4.** From Steps 1 to 3 it follows that only the following three cut-off plans potentially maximize the expected payoff in equation (41): a plan with $v_1^* = v_2^* = \bar{v}$, a plan with $v_1^* = v_2^* = \underline{v}$, and a plan with $v_1^* = \underline{v}$ and $v_2^* = \bar{v}$. We show that the expected payoff from the last plan is always strictly smaller than the expected payoff of the first or of the second plan. The expected payoff from the first plan is $U_1 = v^s + t^s$. The expected payoff of the second plan is

$$U_2 = \int_{\underline{v}}^{\bar{v}} \hat{f}(v) v \, \mathrm{d}v - \eta(\lambda - 1) \int_{\underline{v}}^{\bar{v}} \hat{f}(v) \int_{v}^{\bar{v}} \hat{f}(\tilde{v})(\tilde{v} - v) \, \mathrm{d}\tilde{v} \, \mathrm{d}v - \eta(\lambda - 1)\xi, \tag{50}$$

and the expected payoff from the third plan is

$$\begin{aligned} U_3 \ = \ & \frac{1}{2}(v^s + t^s) + \frac{1}{2} \int_{\underline{v}}^{\bar{v}} \hat{f}(v) v \, \mathrm{d}v - \eta(\lambda - 1)\frac{1}{2}\frac{1}{2}t^s - \eta(\lambda - 1)\frac{1}{2}\frac{1}{2} \int_{\underline{v}}^{\bar{v}} \hat{f}(\tilde{v})(\tilde{v} - v^s) \, \mathrm{d}\tilde{v} + \\ & -\eta(\lambda - 1)\frac{1}{2}\frac{1}{2} \int_{\underline{v}}^{\bar{v}} \hat{f}(v) \int_{v}^{\bar{v}} \hat{f}(\tilde{v})(\tilde{v} - v) \, \mathrm{d}\tilde{v} \, \mathrm{d}v - \eta(\lambda - 1)\frac{1}{2}\xi. \end{aligned} \tag{51}$$

Note that

$$\int_{\underline{v}}^{\bar{v}} \hat{f}(v) \int_{v}^{\bar{v}} \hat{f}(\tilde{v})(\tilde{v} - v) \, \mathrm{d}\tilde{v} \, \mathrm{d}v < \int_{\underline{v}}^{\bar{v}} \hat{f}(v) \int_{\underline{v}}^{\bar{v}} \hat{f}(\tilde{v})(\tilde{v} - v^s) \, \mathrm{d}\tilde{v} \, \mathrm{d}v = \int_{\underline{v}}^{\bar{v}} \hat{f}(\tilde{v})(\tilde{v} - v^s) \, \mathrm{d}\tilde{v}. \tag{52}$$

We can use this to show that if $U_1 \geq U_2$, then we also have $U_1 > U_3$; and if $U_2 \geq U_1$, then we also have $U_2 > U_3$, which completes the proof. □

*Proof of Proposition 3.* The proof proceeds in two steps. **Step 1.** Consider an interval $V_i = (\underline{v}_i, \bar{v}_i] \subset (\frac{\eta(\lambda-1)}{1+\eta}\xi, 1]$ and assume that the seller makes the offer $(v_i^s, t_i^s)$ to the buyer if and only if $v^o \in V_i$. We show that if $\underline{v}_i$ is sufficiently close to $\bar{v}_i$, then we can choose $(v_i^s, t_i^s)$ with $v_i^s + t_i^s < \underline{v}_i$ such that the buyer's PPE specifies to accept this offer whenever $v^o \in V_i$. Lemma 2 implies that the plan "accept $(v_i^s, t_i^s)$ if $v^o \in V_i$" is the payoff-maximizing plan for the buyer if its expected payoff exceeds that from the plan "accept the outside option if $v^o \in V_i$." Her expected payoff from the latter plan after observing offer $(v_i^s, t_i^s)$ equals

$$\int_{\underline{v}_i}^{\bar{v}_i} \hat{f}(v) v \, \mathrm{d}v - \eta(\lambda - 1) \int_{\underline{v}_i}^{\bar{v}_i} \hat{f}(v) \int_{v}^{\bar{v}_i} \hat{f}(\tilde{v})(\tilde{v} - v) \, \mathrm{d}\tilde{v} \, \mathrm{d}v - \eta(\lambda - 1)\xi. \tag{53}$$

As in the proof of Proposition 1, we can show that for $\underline{v}_i \to \bar{v}_i$ this expression becomes

$$\frac{1}{2}(\bar{v}_i + \underline{v}_i) - \eta(\lambda - 1)\frac{1}{6}(\bar{v}_i - \underline{v}_i) - \eta(\lambda - 1)\xi. \tag{54}$$

By assumption, we have $\eta(\lambda - 1) > 0$. Hence, if $\underline{v}_i$ is sufficiently close to $\bar{v}_i$, then we can find values $(v_i^s, t_i^s)$ with $v_i^s + t_i^s < \underline{v}_i$ such that the payoff-maximizing plan in period 1 is to accept $(v_i^s, t_i^s)$ if $v^o \in V_i$. We examine when this plan is consistent with a PPE. In period 2, the buyer's payoff from accepting the seller's offer is $v^s + t^s$, while the payoff from accepting the outside option value is, in both states, equal to

$$v^o + \eta(v^o - v^s) - \eta\lambda t^s - \eta(\lambda - 1)\xi. \tag{55}$$

The buyer is indifferent between the seller's offer and the outside option at $v^o = \bar{v}_i$ if

$$\bar{v}_i + \eta(\bar{v}_i - v^s) - \eta\lambda t^s - \eta(\lambda - 1)\xi = v_i^s + t_i^s. \tag{56}$$

We can find values $(v_i^s, t_i^s)$ with $v_i^s + t_i^s < \underline{v}_i$ that satisfy this equality if

$$\bar{v}_i + \eta\bar{v}_i - \eta\lambda\underline{v}_i - \eta(\lambda - 1)\xi < \underline{v}_i, \tag{57}$$

which is equivalent to

$$\frac{1+\eta}{1+\eta\lambda}\bar{v}_i - \frac{\eta(\lambda-1)}{1+\eta\lambda}\xi < \underline{v}_i. \tag{58}$$

Hence, if $\underline{v}_i$ is sufficiently close to $\bar{v}_i$, we can find values $(v_i^s, t_i^s)$ with $v_i^s + t_i^s < \underline{v}_i$ such that in the PPE the buyer always accepts $(v_i^s, t_i^s)$ if $v^o \in V_i$. **Step 2.** We can now construct the desired equilibrium. Suppose the seller adopts the following strategy: If $v^o \in [0, \frac{\eta(\lambda-1)}{1+\eta}\xi]$, the seller offers $(v^s, t^s) = (0, 0)$ to the buyer. The interval $(\frac{\eta(\lambda-1)}{1+\eta}\xi, 1]$ is partitioned by a finite sequence of disjoint half-open intervals $\{V_i\}_{i=1,\dots,n}$ so that the seller offers $(v_i^s, t_i^s)$ to the buyer if $v^o \in V_i$. For each $i = 1, \dots, n$, the interval $V_i$ as well as the values $v_i^s, t_i^s$ are chosen such that the buyer is indifferent between the seller's offer and the outside option at $v^o = \bar{v}_i$ in period 2 and accepting on-equilibrium offers characterizes the buyer's PPE. In Step 1, we have shown that this is possible. Assuming optimistic beliefs for off-equilibrium offers then ensures that no party can deviate profitably. □

*Proof of Proposition 4.* Assume by contradiction that there is a seller-preferred equilibrium $\sigma$ in which, for any value $\underline{v}$, there is no bunching of the total value $v^s + t^s$ at the outside option values in $(\underline{v}, 1]$. We modify $\sigma$ so that we obtain an equilibrium with bunching at the highest outside option values, which dominates $\sigma$ in terms of expected payoff for the seller. The assumption on $\sigma$ implies that there is a $v^*$ so that for each $v^o > v^*$ the seller's offers signal the precise outside option value to the buyer. The expected total value that the seller offers to the buyer in equilibrium $\sigma$ given that $v^o > v^*$ is at least

$$\int_{v^*}^{1} v\hat{f}(v)dv - \eta(\lambda - 1)\xi. \tag{59}$$

We find a value $\underline{v}$ so that the following inequalities hold:

$$\underline{v} < \int_{\underline{v}}^{1} v\hat{f}(v)dv - \eta(\lambda - 1)\xi, \tag{60}$$

$$\underline{v} \geq \int_{\underline{v}}^{1} v\hat{f}(v)dv - \eta(\lambda - 1)\int_{\underline{v}}^{1} \hat{f}(v)\int_{v}^{1} \hat{f}(\tilde{v})(\tilde{v} - v)d\tilde{v}dv - \eta(\lambda - 1)\xi, \tag{61}$$

and

$$\underline{v} \geq \frac{1+\eta}{1+\eta\lambda} - \frac{\eta(\lambda-1)\xi}{1+\eta\lambda}. \tag{62}$$

Observe that, if $\xi$ is small enough, then such a value $\underline{v}$ exists. We now modify $\sigma$ as follows. At all outside option values in the interval $(\underline{v}, 1]$ the seller makes an offer $(v^s, t^s)$ with total value $w^s = \underline{v}$ and sufficiently large value $t^s$; otherwise, the seller's strategy remains the same. The three inequalities above ensure that in a PPE the buyer always accepts $(v^s, t^s)$ when $v^o \in (\underline{v}, 1]$, and that the seller's expected payoff under the modified equilibrium is strictly larger than under the original equilibrium. □

*Proof of Proposition 5.* The proof proceeds by steps. In Steps 1 to 3, we show that the assessment stated in Proposition 5 is an equilibrium outcome and characterize some of its properties. In Step 4, we show that any seller-preferred equilibrium is consistent with this assessment.
**Step 1.** We find the interval-structure $\{V_i\}_{i\in\mathbb{N}}$ and offers $(v_i^s, t_i^s)$ for each $i \in \mathbb{N}$ so that the expected total value offered in equilibrium is minimal for outside option values in the interval $(\eta\lambda\xi, 1]$. We denote $w_i^s = v_i^s + t_i^s$. It is optimal for the buyer to plan acceptance in period 1 if and only if

$$w_i^s \geq \frac{1}{2}(\bar{v}_i + \underline{v}_i) - \frac{1}{6}\eta(\lambda-1)(\bar{v}_i - \underline{v}_i) - \eta(\lambda - 1) \tag{63}$$

for all $i \in \mathbb{N}$. The lowest total value that the seller must offer so that the buyer actually accepts $(v_i^s, t_i^s)$ if $v^o = \bar{v}_i$ is defined by

$$t_i^s = (1+\eta)\bar{v}_i - \eta\lambda t_i^s - \eta(\lambda - 1)\xi. \tag{64}$$

We therefore set

$$w_i^s = \frac{(1+\eta)\bar{v}_i - \eta(\lambda-1)\xi}{1+\eta\lambda}. \tag{65}$$

The inequality in (63) defines an upper bound on $\underline{v}_i$:

$$\underline{v}_i \leq \frac{1}{\frac{1}{2} + \frac{1}{6}\eta(\lambda-1)}\left[w_i^s + \bar{v}_i\left(\frac{1}{6}\eta(\lambda-1) - \frac{1}{2}\right) + \eta(\lambda-1)\xi\right]. \tag{66}$$

We consider the average total value offered as a fraction of the average outside option value in interval $V_i$. It is given by

$$av_i = \frac{w_i^s}{\frac{1}{2}(\bar{v}_i + \underline{v}_i)}. \tag{67}$$

As in Step 2 of the proof of Proposition 2, we can now show that for given $\bar{v}_i$ the lowest possible equilibrium value of $av_i$ is obtained when the inequalities in (65) and (66) are satisfied with equality. Importantly, we can show that if $\bar{v}_i > \eta\lambda\xi$, then the value of $\underline{v}_i$ defined by (66) satisfies $\underline{v}_i < \bar{v}_i$ and $\underline{v}_i > \eta\lambda\xi$ so that we indeed obtain an infinite sequence of intervals that partition the interval $(\eta\lambda\xi, 1]$. **Step 2.** For each outside option value $\frac{\eta(\lambda-1)}{1+\eta}\xi < v^o \leq \eta\lambda\xi$ we find the offer $(v^s, t^s)$ that minimizes the total value and that the buyer accepts in equilibrium. At given outside option value $v^o$ the smallest total value that the seller needs to offer so that the buyer accepts the early offer in period 2 is defined by

$$t^s = (1+\eta)v^o - \eta\lambda t^s - \eta(\lambda-1)\xi. \tag{68}$$

The corresponding offer is $(0, t^s)$ with

$$t^s = \frac{(1+\eta)v^o - \eta(\lambda-1)\xi}{1+\eta\lambda}. \tag{69}$$

If the seller makes this offer for any outside option value $v^o$ with $\frac{\eta(\lambda-1)}{1+\eta}\xi < v^o \leq \eta\lambda\xi$, the buyer can infer $v^o$ from it. Planning acceptance in period 1 is then optimal for her if

$$t^s \geq v^o - \eta(\lambda-1)\xi. \tag{70}$$

The restriction $v^o \leq \eta\lambda\xi$ ensures that this inequality is satisfied, which completes the proof. **Step 3.** We show that if $v^o \leq \frac{\eta(\lambda-1)}{1+\eta}\xi$, then the buyer would accept the early offer $(v^s, t^s) = (0, 0)$, provided that the offers for any $v^o > \frac{\eta(\lambda-1)}{1+\eta}\xi$ are those indicated in Step 1 and Step 2. Note that if $v^o \leq \frac{\eta(\lambda-1)}{1+\eta}\xi$, then planning acceptance of $(0, 0)$ is optimal for the buyer. It is then also optimal for her to accept $(0, 0)$ in period 2 if

$$(1+\eta)v^o - \eta(\lambda-1)\xi \leq 0, \tag{71}$$

which is guaranteed by the upper bound on $v^o$. This completes the proof of the desired statement. **Step 4.** The results in Steps 1 to 3 characterize the equilibrium outcome that we stated in Proposition 5. In order to prove that any seller-preferred equilibrium exhibits this outcome, it remains to show that the seller's equilibrium payoff cannot be further increased by making alternative offers at outside option values $v^o \in (\eta\lambda\xi, 1]$. For this, we consider two cases. First, we show that, for any $i \in \mathbb{N}$, it does not pay off for the seller to replace the offer $(v_i^s, t_i^s)$ by offers that signal the precise outside option value to the buyer for all values $v_o \in V_i$. The minimal total value of such an offer at outside option value $v^o$ in equilibrium would have to be

$$v^o - \eta(\lambda - 1)\xi, \tag{72}$$

and the corresponding average total value offered for outside options in the set $V_i$ would be

$$\frac{1}{2}(\bar{v}_i + \underline{v}_i) - \eta(\lambda - 1)\xi \tag{73}$$

The fact that $v^o > \eta\lambda\xi$ ensures that this value is weakly larger than $w_i^s$ as defined in equation (65), which proves the claim. Second, assume by contradiction that there is a seller-preferred equilibrium $\sigma$ in which there is an open set $V \subset (\eta\lambda\xi, 1]$ of outside option values for which the seller's offer signals the precise outside option value. Let $V_i$ be the set of outside option values below those of $V$ so that the seller makes the offer $(v_i^s, t_i^s)$ for all $v^o \in V_i$. By the first case, this set must exist and (since $\sigma$ is a seller-preferred equilibrium) we must have that $w_i^s$ is defined by equation (65). We show that we can find an equilibrium that yields an even higher expected payoff for the seller. To show this, we change the original equilibrium $\sigma$ by slightly increasing $\bar{v}_i$ (all else remains the same). Denote by $w^s$ the infimum of the total values offered to outside option values $v^o \in V$. We must have $w^s \geq \bar{v}_i - \eta(\lambda - 1)\xi$ so that we get

$$w^s - w_i^s \geq \frac{\eta(\lambda - 1)}{1 + \eta\lambda}(\bar{v}_i - \eta\lambda\xi). \tag{74}$$

Increasing $\bar{v}_i$ also increases $w_i^s$ and hence the total value offered for outside option values $v^o \in V_i$. We can calculate

$$\frac{\partial w_i^s}{\partial \bar{v}_i} \times (F(\bar{v}_i) - F(\underline{v}_i)) = \frac{1 + \eta}{1 + \eta\lambda} \frac{1}{\frac{1}{2} + \frac{1}{6}\eta(\lambda - 1)} \frac{\eta(\lambda - 1)}{1 + \eta\lambda}(\bar{v}_i - \eta\lambda\xi). \tag{75}$$

This is the marginal increase in the total value that must be provided to secure the acceptance of the seller's offer by outside option values in $V_i$. The change in the original equilibrium increases the seller's expected payoff if the right-hand side of equation (74) exceeds the right-hand side of equation (75), which is implied by the assumption on the parameters $\eta, \lambda$. This completes the proof of Proposition 5. □