Pushing towards shared mobility

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Abstract

This paper provides a theoretical argument for preferential treatment of shared vehicles (SV) over private ones by municipal parking authorities. When all parked vehicles are treated equally, multiple equilibria may exist: (i) a "private" one, where travelers are reluctant to share their vehicle, due to lack of alternatives for their next trip, and (ii) a "shared" equilibrium, where travelers release their vehicles for use by others, due to abundance of other SV for their next trip. The latter equilibrium, if exists, is shown to deliver a higher welfare. Municipal parking discounts for vacant SV are shown to make the private equilibrium unstable, so even a small initial fleet of SV pushes the economy towards the shared equilibrium.

Keywords: Shared mobility, Parking policy, Multiple equilibria, Frictions of space, Repeated matching

JEL codes: C78, L91, R48

1. Introduction

Commercial vehicle sharing (i.e. per-minute or per-hour automated vehicle rental service, SV henceforth) offers a great promise for the future of ground transportation. According to back-of-envelope calculation by Zakharenko (2022), in a large city, sharing cars enables the economy to meet the same transportation demand with six times fewer vehicles and eight times less parking space, dramatically reducing the capital cost of the industry. Jochem et al. (2020), by analyzing survey data from multiple European cities, offer an even more optimistic conclusion that each free-floating SV substitutes from 7 to 18 private vehicles, depending on the city. Jochem et al. (2020) also provide a substantial number of references to other studies measuring this ratio in various cities of the world.

While the SV technology has gained some momentum in many places, primarily in large cities of Europe, it still remains a fringe transportation option for most people in the world. For example in the U.S., the largest provider of round-trip SV (i.e. vehicles that have to be returned to the same location) had only 12000 vehicles in 2019,[1] and the largest provider of free-floating SV (i.e. those that can be dropped off anywhere within a certain area) has only 1000 vehicles and serves only a handful of cities.[2]

Although shared vehicles serve many more people per day than private ones, they still spend a considerable amount of time being parked. Zakharenko (2022) estimates that in Moscow (Russia), shared cars are parked and available for booking 70% of all time. The success of shared mobility is therefore highly sensitive to municipal parking policy. For example Car2Go, a prominent SV provider of its time, chose to discontinue serving its 80000 customers in Toronto (Canada) after the city hall introduced parking fees for shared vehicles.[3]

This paper offers a theoretical analysis of how people with travel demand (*travelers* henceforth) make their choice between private and shared vehicles. Vehicle sharing allows to meet the same transportation demand with fewer vehicles and less parking space, but requires travelers to search for a vehicle before use. The cost of search depends on the density of vacant SV across space, which in turn depends on how many of other travelers choose shared mobility. Thus, vehicle sharing is a coordination problem with potential multiple equilibria. In the "private" equilibrium, the number of vehicles equals the number of travelers, which results in high costs of transportation (vehicles capital costs and parking), but allows to always have an available vehicle nearby. The "shared" equilibrium reduces the capital costs

[1] https://en.wikipedia.org/wiki/Zipcar

[2] "GIG Car Share Thanking Members for Big Win with Big Expansion News," prnewswire.com, December 3, 2020.

[3] "Car2Go to shut down in Toronto, blaming new city rules", CBC News, May 24, 2018.

of transportation industry but introduces frictions of vehicle search. The model developed in this paper shows that, when the shared equilibrium exists, it results in a higher social welfare.

The above results imply that the economy can be stuck in a bad "private" equilibrium, where travelers are essentially hoarding vehicles due to expected difficulty of search for alternatives. This paper shows that a local government parking policy that favors vacant SV can push the economy towards the better equilibrium: such preferential treatment makes the private equilibrium unstable. That is, an introduction of arbitrarily small fleet of SV leads to a snowball of additional travelers using shared mobility, as well as an additional supply of SV. Eventually, the economy converges to the better shared equilibrium.

There exists a vast research on various aspects of shared mobility in various fields of science; Nansubuga and Kowalkowski (2021) offer a review of nearly 200 papers, published mainly in last 10 years. While a substantial part of this review covers the hurdles limiting the success of shared mobility, the municipal parking policy is omitted from the discussion. Therefore, the current paper is probably the first theoretical paper to formalize the link between parking policy for shared vehicles and social welfare.

A substantial body of economics literature on optimal parking regulations is also silent about shared vehicles; for example Inci (2015), the most recent literature review in the field, implicitly assumes throughout the paper that each parked vehicle is used exclusively by a specific individual. The same assumption is made in existing theoretical studies of preferential parking policy, e.g. Zakharenko (2020) or Jakob and Menendez (2020). While some studies have discussed parking management for futuristic autonomous shared vehicles (e.g. Winter et al. (2021)), no study could be found that theoretically analyzes the optimal parking policy for already existing non-autonomous SV services.

The general policy advice in the parking economics literature, e.g. in Zakharenko (2016), is that every parked vehicle should be charged the congestion externality it causes for other vehicles searching for parking. van Ommeren et al. (2021) apply this methodology to calculate optimal parking rates in Melbourne. Because the instantaneous externality of a parked vehicle does not depend on whether it is private or shared, this school of thought would recommend that the per-minute parking rates for all vehicles should be equal. The current paper offers a theoretical counterargument, that endogenous choices between shared and private vehicles may lead to multiple equilibria, and that parking discounts for shared vehicles may push the economy to the better equilibrium.

This paper also contributes to the Economics literature on repeated matching. In transportation economics, numerous studies offered models of one-sided repeated matching, i.e. where the supplier, usually a taxi or ridehailing driver, is long-lived and matched repeatedly, while passengers are short-lived and matched once. Examples include Lagos (2000), Buchholz (2021), Zakharenko (2022). The current paper is probably the first paper in this field to analyze two-sided repeated matching, i.e. where both demand and supply side seek to be matched repeatedly. Two-sided repeated matching models have been proposed in other contexts (e.g. marriage and re-marriage, as in Kadam and Kotowski (2018)). These models typically assume that both sides are willing to be matched continuously and dissolve matches only to find a better match. The current paper, in contrast, makes such assumption only for the supply side (operators of shared vehicles); the demand side need to be matched to vehicles occasionally, rather than continuously.

2. Existing regulation practices

Because car sharing is an emerging market, local regulations of SV parking vary considerably. For example the city of Los Angeles apparently adheres to the "all cars created equal" philosophy, as its parking fees for shared automobiles are roughly equal to those of private automobiles.[4] In sharp contrast, San Francisco requires developers to provide a certain amount of free parking spaces to qualified carsharing organizations.[5] Not surprisingly, the success of carsharing in San Francisco far exceeds that in Los Angeles despite much smaller population. For example, Zipcar has about 200 carsharing stations in the former, versus about 60 in the latter.[6] Some cities impose non-monetary constraints, for example Toronto allows a maximum of 2 parked free-floating SV per residential block, in addition to yearly per-vehicle permit fee.[7] Diversity of regulations may be explained by lack of theory of optimal regulation, a gap filled by the current paper.

3. The model

The model is based on that in Zakharenko (2022), but with some modifications. This is a dynamic model with infinite time horizon, where all parties do not discount the future. The reason for non-discounting is high frequency of transactions, e.g. each vehicle being used multiple times per day. It is implausible to assume, for example, that profit earned by vehicle operator in the evening has any lower value than profit in the morning. The objective of SV operators is therefore the average profit per unit of time, while that of travelers is minimization of the average travel cost per trip.

The model also has spatial heterogeneity in the form of an arbitrary number of geographical *zones* with symmetric travel demand between any two of them. All results of the model are also applicable to a single zone, with round-trip travel demand.

Each zone is a single-dimensional circular space, with travelers having a specific origin location in the origin zone and destination location in the destination zone, for each trip. Figure 1 illustrates travel demand. We assume the destinations of inbound trips are dis-

[4]LA municipal code, SEC. 80.58.1 (e).
[5]San Francisco planning code, Sec. 166.
[6]Based on count of Google Maps results for “Zipcar San Francisco” and “Zipcar Los Angeles”, respectively.
[7]Free-Floating Car-Share Parking Permit Program by the city of Toronto

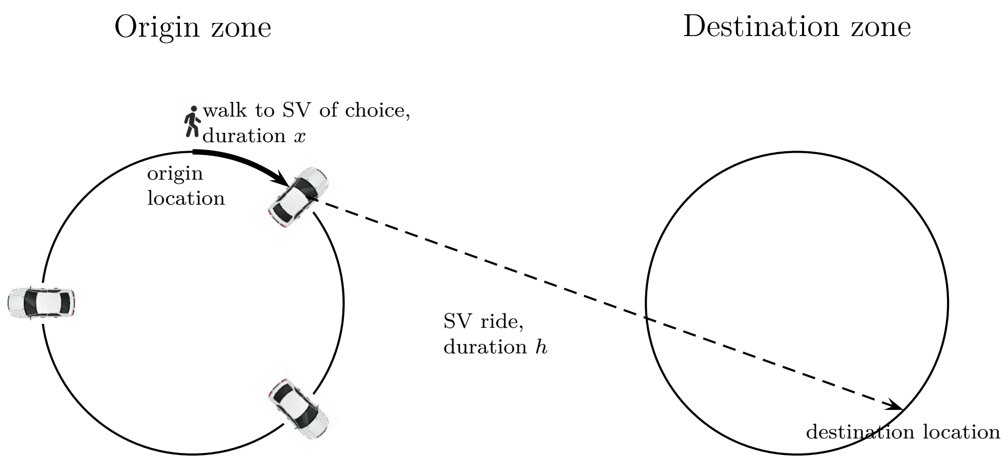

Figure 1: Illustration of a typical trip, reproduced from Zakharenko (2022). Vehicle image courtesy of Macrovector/Freepik.

tributed uniformly around the zone, such that an exogenous mass L of travelers arrive per unit of zone space, per unit of time. Unlike Zakharenko (2022) who considers one-time trip demand, here we assume that all travelers have recurrent trip demand, i.e. will demand another trip, originating from the location of previous arrival, after some period of stay. For mathematical tractability of the results that follow, we will assume that the duration of stay t of a traveler in each zone is distributed exponentially with mean τ. We also assume that the next departure is a Poisson process: travelers do not know in advance when their stay ends and the next trip begins, hence make identical choices at any time during their stay. Given these assumptions, the origins of outbound trips are also distributed uniformly with density L, per unit of zone space per unit of time.

All travel requires the use of a personal vehicle, owned or rented by the traveler. We assume the cost of vehicle use does not depend on ownership, so we can also assume without loss of generality that all vehicles are rented and can be transferred from one traveler to another at any moment when not in use. A "private" vehicle is then equivalent to a rented vehicle that always remains under control of a single traveler.

Every trip takes h units of time. Vehicle costs include, per unit of time: social cost of parking g, when the vehicle is vacant or reserved; cost of use c when in transit; capital cost ϕ at all times.

Upon arrival and beginning of their stay, travelers decide whether they want to *hold* the vehicle until the next trip (which is equivalent to private ownership), or *release* (i.e. drop off) to make it available for booking by other travelers. Denote by λ the endogenous share of travelers who choose to release.

Denote by μ the endogenous density of vacant vehicles. For travelers who released their vehicle upon arrival, two random outcomes are possible for the next departure: (i) the same vehicle is still available for booking, and (ii) another vehicle has to be found. In the latter case, almost surely, another vehicle is located some distance away, and the traveler has to walk to the vehicle. Denote by x the idiosyncratic walking time to such vehicle; assuming unitary walking speed, it is also equal to walking distance. Denote by w the disutility of walking, per unit of time/distance. The vehicle has to be *reserved* while a traveler is walking towards it.

Among travelers who previously released a vehicle, those who return to the same vehicle are referred to as *returnees*, while those who search for another one are *walkers*. For a representative vacant vehicle, denote by q the endogenous rate of its booking by the walkers.[8]

4. Social optimum

What are the socially optimal release decision λ and vacant vehicle density μ? Because the number of passenger-kilometers traveled (and therefore the number of vehicles in transit) is assumed exogenous, maximization of social welfare amounts to minimization of costs related to parked vehicles. These vehicles fall into one of three categories:

[8] More complex strategies of travelers are possible, when they screen for vacant vehicles continuously during their stay, and make release/reserve decisions upon appearance of more convenient vacant vehicles. This paper assumes all decisions are made only in the beginning or at the end of stay, for mathematical tractability.

- Vehicles held by arriving travelers. $(1-\lambda)L$ of such vehicles are expected to emerge, per unit of time, per unit of space; each remains parked for τ units of time in expectation.

- Vacant vehicles: density μ per unit of space.

- Vehicles reserved by the walkers. The mass of travelers who release their previous vehicle is λL. It is socially optimal that travelers always walk to the most proximate vehicle. For a traveler who stayed t units of time before the next trip, the probability that the previous vehicle is still vacant is $\exp(-qt)$. In this case, the traveler is a returnee and zero walking time/cost is incurred. With the remaining probability $1 - \exp(-qt)$, the traveler becomes a walker. Given exponential distribution of t with p.d.f. $\frac{1}{\tau}\exp\left(\frac{t}{\tau}\right)$, the mass of walkers is

$$\lambda L \int_{t=0}^{\infty} (1 - \exp(-qt)) \frac{1}{\tau} \exp\left(-\frac{t}{\tau}\right) \mathrm{d}t = \lambda L \frac{q\tau}{1+q\tau}.$$

  Given vacant vehicle density μ, the expected walking distance (and time) is $\frac{1}{2\mu}$; the coefficient 2 here is because the traveler can walk in two directions from her initial position.

The social cost of any parked vehicle per unit of time is $g + \phi$; when a traveler is walking toward a reserved vehicle, an additional cost of w is incurred.

Given this analysis, the total social cost of all parked vehicles, per unit of time per unit of space, is given by

$$C \equiv (g+\phi)(L(1-\lambda)\tau + \mu) + (g+\phi+w)\lambda L \frac{q\tau}{1+q\tau}\frac{1}{2\mu}. \tag{1}$$

The equilibrium value of q is found as follows: it is equal to the flow of newly emerging walkers, $\lambda L \frac{q\tau}{1+q\tau}$, divided by the density of vacant vehicles μ. Thus, q is found from equation

$$q \equiv \lambda L \frac{q\tau}{1+q\tau} \frac{1}{\mu}. \tag{2}$$

One solution to this equation is $q = 0$; it corresponds to the case when all travelers who release their vehicles return to the same vehicles. We will refer to such state as *quasi-shared*, because it essentially makes all vehicles private: even if they are in the vacant status, they will not be demanded by anyone except their previous user. This means that, in the quasi-shared economy, the number of vehicles should be no less than the number of travelers, hence the mass of vacant vehicles μ must be greater or equal to the flow of travelers releasing their vehicles λL times their expected duration of stay τ.

Replacing $q = 0$ into (1) trivially yields

$$C = (g + \phi)(L(1 - \lambda)\tau + \mu).$$

Minimization of social cost then amounts to minimization of μ (i.e. removal of excess vacant vehicles) and/or maximization of λ (i.e. removal of vehicles previously held by travelers, and making these travelers switch to excess vacant vehicles). Thus, when $q = 0$, the lowest social cost is achieved when $\mu = \lambda L\tau$ (i.e. there is exactly one vehicle per traveler) and thus

$$C = (g + \phi)L\tau, \tag{3}$$

i.e. the social cost of parked vehicles is equal to that in a private vehicle economy. Can the society do better than that?

When $q > 0$, the solution to (2) is

$$q = \frac{\lambda L}{\mu} - \frac{1}{\tau}, \tag{4}$$

which is only possible when

$$\mu < \lambda L\tau. \tag{5}$$

Given (4), the problem of minimization of (1) is

$$\min_{\lambda,\mu}(g+\phi)(L(1-\lambda)\tau+\mu)+\frac{g+\phi+w}{2}\left(\frac{\lambda L}{\mu}-\frac{1}{\tau}\right), \tag{6}$$

subject to (5). The first-order conditions of optimal μ and λ are

$$\frac{\mathrm{d}C}{\mathrm{d}\mu} = g+\phi-\frac{g+\phi+w}{2}\frac{\lambda L}{\mu^2}=0, \tag{7}$$

$$\frac{\mathrm{d}C}{\mathrm{d}\lambda} = -(g+\phi)L\tau+\frac{g+\phi+w}{2}\frac{L}{\mu}\begin{cases} =0, & \lambda\in\left(\frac{\mu}{L\tau},1\right) \\ \leq 0, & \lambda=1\end{cases}. \tag{8}$$

Denote by $\bar{\mu}$ the value of μ that makes (8) an equality, $\bar{\mu}\equiv\frac{1}{2\tau}\frac{g+\phi+w}{g+\phi}$. Note that at point $\mu=\bar{\mu}$ and $\bar{\lambda}=\frac{\bar{\mu}}{L\tau}$, (7) also holds. Also note that equality (7) and inequality (5) can hold simultaneously only when $\mu>\bar{\mu}$.

The comparison of $\bar{\mu}$ to $L\tau$, i.e. maximum possible vehicle density (cf.(5)), yields two types of optimal solutions. When $\bar{\mu}\geq L\tau$, the economy cannot do better than the all-private-vehicle scenario. This may be due to low travel demand L, or short durations of stay τ, or high walking costs w, all of which make vehicle sharing less attractive compared to personal vehicles. Technically, any point along the quasi-shared line $\mu=\lambda L\tau$ (bold line on figure 2, right panel) delivers the same (lowest possible) social cost (3).

When $\bar{\mu}<L\tau$, the social cost can be reduced to a lower level by vehicle sharing. The global minimum is characterized by $\lambda=1$ (i.e. all travelers release vehicles) and $\mu^*=\sqrt{\frac{(g+\phi+w)L}{2(g+\phi)}}$, as shown by black dot on figure 2, left panel. The social cost at this point is lower than (3), i.e. vehicle sharing makes the economy better off, despite the fact that positive walking costs are incurred. At the same time, any combination (λ,μ) that satisfies

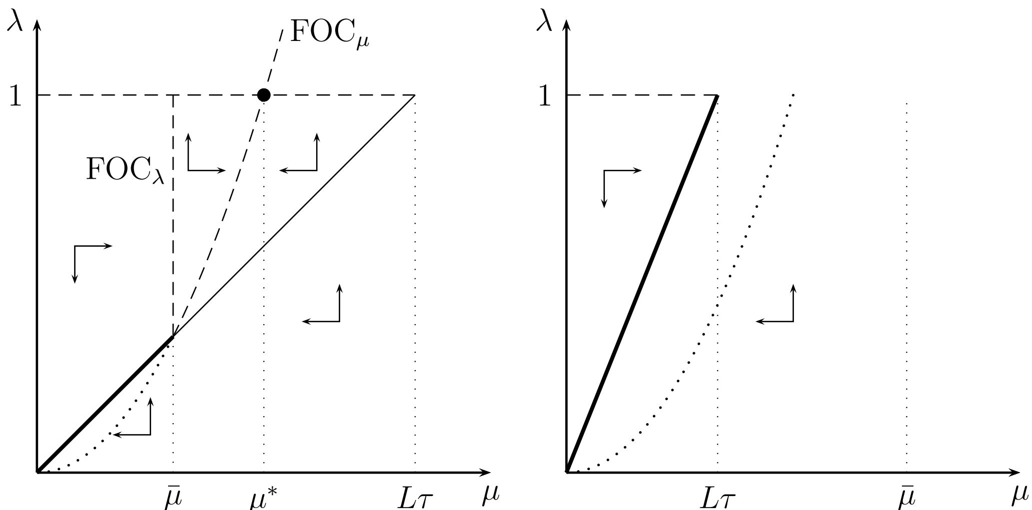

Figure 2: Socially optimal vehicle density and traveler decisions, high L (left panel) vs. low L (right panel).

$\mu = \lambda L\tau, \mu \leq \bar{\mu}$ (bold line on figure 2, left panel) is also weakly locally optimal, in the sense that a small deviation from any such point cannot increase social welfare. Furthermore, if there are marginal welfare gains of holding, rather than releasing, a vehicle (e.g. due to cost of equipment required for vehicle sharing), the point $\lambda = 0, \mu = 0$ (i.e. pure private-vehicle economy) becomes strictly preferred to any point in the vicinity.

To summarize, the social cost function (6) may have multiple local minima. One local optimum is the pure private-vehicle economy, where all travelers hold their vehicles at all times. If demand L and parking duration τ are sufficiently high while walking cost w sufficiently low, there is also a global optimum with vehicle sharing, such that all travelers release their vehicles after use, and $\mu = \mu^\star$. The existence of multiple local optima is due to problems of coordination: when few travelers release their vehicles, there are few vacant vehicles, which in turn makes it optimal to hold them after each trip.

5. Market equilibrium and optimal policies

This section studies decentralized equilibrium with provision of SV by competing operators, and government policies that help to achieve the global optimum. I assume free entry of SV providers, which ensures their zero profits in equilibrium. I also assume a small market share of each provider, meaning that its particular vacant vehicle may compete with vehicles by other providers, but not with those by the same provider. The government can regulate the market by introducing fees for parked vehicles; denote by g_r the parking rate for reserved vehicles, and by g_v the rate for vacant vehicles.

The travelers at the end of each trip decide whether to release their vehicle. Denote by λ_i the probability of release by individual i, and by λ the share of travelers who release. At the beginning of the next trip, travelers who previously released should choose one of vacant vehicles; they choose the one that minimizes the sum of long-term monetary and walking costs.

Operator tariffs include a trip fare p^a, charged during the h units of transit time, and also the reservation rate p' per unit of reservation time. We will focus on symmetric equilibria where all competing operators have the same tariffs $\{p^a, p'\}$. To show that such tariffs maximize individual profit, we will consider small deviations $\{p^a_j, p'_j\}$ of a particular operator j from the equilibrium values.

Define by *vehicle cycle* the time from the end of the previous vehicle trip until the end of the next trip. If the vehicle was released at the end of the previous trip, its cycle consists of three phases: vacancy, (possible) reservation, and use. If held, the vehicle cycle consists of the time it is held, and the time of use.

We analyze separately two candidate equilibria: with and without vehicle sharing.

5.1. Equilibrium without sharing

First, observe that when traveler i holds their vehicle (i.e. $\lambda_i = 0$), competition between operators implies that the traveler expected cost per cycle equals that of the operator,

$$(c + \phi)h + (g_r + \phi)\tau. \tag{9}$$

Next, consider the quasi-shared state, where $\lambda_i = \frac{\mu}{L\tau} \in (0, 1]$. As elaborated in section

4, such state is characterized by $q = 0$, that is, all vehicles are demanded only by their respective last users, and travelers normally return to their last vehicle for the next trip.

In the quasi-shared state, travelers always pick up the vehicle where they previously dropped them off, there are no walking or reservation costs, and the only traveler expense is the trip fare p^a. Appendix Appendix A shows that for any $\lambda > 0$, any operator j has an incentive to set p^a_j marginally below the prevailing rate p^a, given any positive value of the latter. Such fare undercutting will lead to negative profits and operator exit (i.e. reduction of vacant vehicle density μ), meaning that a quasi-shared state cannot be sustained in equilibrium.

5.2. Equilibrium with sharing

This section looks for possible shared equilibria, i.e. those that satisfy (5) and $q > 0$.

5.2.1. Traveler problem

In this section, we solve the problem of a traveler i without a previously held SV, who is searching for a vehicle for her next trip.

Suppose the walking distance to vehicle j is $x \geq 0$, so the reservation cost for i, in case of booking such vehicle, is $x(p'_j + w)$. In case of booking vehicle j, traveler i may release and re-book it repeatedly in the future. The probability that i can use the same vehicle j for the second trip is approximately (cf. section 4) $\frac{1}{1+q\tau}$, i.e. the probability that no other traveler books vehicle j between i's trips.[9] By induction, the expected number of times that traveler

[9]Note that when p^a_j marginally differs from p^a, the probability of repeated vehicle use also marginally differs from $\frac{1}{1+q\tau}$, for the following reasons: (i) the rate of booking of vehicle j by other walkers marginally differs from q; (ii) if $p^a_j > p_a$, there is a marginal probability that traveler i chooses another vehicle for her second trip, even if vehicle j is still available; (iii) if $p^a_j < p_a$, there is marginal probability that other travelers book vehicle j even if their previous vehicle is still available to them. Such deviations also marginally change the expected number of trips by traveler i with vehicle j. However, because the change in the cost per trip $p^a_j - p^a$ is marginal too, the change in traveler costs due to changing number of trips is of smaller order of magnitude and can be ignored.

i will use vehicle j is approximately

$$1+\frac{1}{1+q\tau}+\frac{1}{(1+q\tau)^2}+\cdots=1+\frac{1}{q\tau}.$$

and the total expected cost of all trips using the same vehicle j is

$$\left(1+\frac{1}{q\tau}\right)p_j^a+(p_j'+w)x. \tag{10}$$

Vehicle j will be chosen if no vehicle by other operators (i.e. those with equilibrium tariff $\{p^a,p'\}$) offers a lower cost of the same expected number of trips. This means that all competing vehicles should be further than distance $z(p_j^a,p_j',x)$ from the traveler i's position, where z is defined by (10) being equal to $\left(1+\frac{1}{q\tau}\right)p^a+(p'+w)z$:

$$z(p_j^a,p_j',x)=\frac{1}{p'+w}\left(\left(1+\frac{1}{q\tau}\right)(p_j^a-p^a)+(p_j'+w)x\right). \tag{11}$$

Given density μ of alternative vacant vehicles, the probability that all alternative vacant vehicles are indeed further than z, and that traveler i indeed books vehicle j, is given by $D(p_j^a,p_j',x)=\exp(-2\mu z(p_j^a,p_j',x))$. The coefficient 2 here is because the traveler can walk in two directions from her initial position.

We can also find the expected walking distance by i, conditional on having to walk, given by

$$\bar{x}(p_j')=\frac{\int_0^\infty xD(p_j^a,p_j',x)\mathrm{d}x}{\int_0^\infty D(p_j^a,p_j',x)\mathrm{d}x}=\frac{1}{2\mu}\frac{p'+w}{p_j'+w} \tag{12}$$

5.2.2. Operator problem

The rate of booking of vehicle j by walkers, q_j, is equal to the density of travelers who previously released their vehicles and now demand travel, λL, times the probability of having to walk, $\frac{q\tau}{1+q\tau}$, times the probability that vehicle j is more convenient than other vehicles,

$2\int_0^\infty D(p_j^a, p_j', x)\mathrm{d}x$, where coefficient 2 is due to two directions of search. Thus, q_j is defined by

$$q_j(p_j^a, p_j') = 2\lambda L \frac{q\tau}{1+q\tau}\int_0^\infty D(p_j^a, p_j', x)\mathrm{d}x$$
$$= \frac{\lambda L}{\mu}\frac{q\tau}{1+q\tau}\exp\left(-2\mu\left(1+\frac{1}{q\tau}\right)\frac{p_j^a - p^a}{p'+w}\right)\frac{p'+w}{p_j'+w}. \quad (13)$$

The duration of vacancy of vehicle j is the inverse of the rate of vehicle booking; the latter is the sum of q_j (booking by walkers) and $\frac{1}{\tau}$ (the rate of booking by the last user of vehicle j). The cost of vacancy for the operator is $g_v + \phi$ per unit of time.

The probability that vehicle j is booked by a walker and remains reserved for some time is $\frac{q_j\tau}{1+q_j\tau}$. The expected duration of such reservation is given by (12); the operator's net profit during reservation is $p_j' - g_r - \phi$ per unit of time.

Finally, the net profit during the phase of vehicle use is $p_j^a - (c+\phi)h$.

The operator's net profit from the entire vehicle cycle (vacancy, reservation, use) is thus defined by

$$\pi(p_j^a, p_j') = p_j^a - (c+\phi)h + \frac{q_j(p_j^a, p_j')\tau}{1+q_j(p_j^a, p_j')\tau}\frac{1}{2\mu}\frac{p'+w}{p_j'+w}(p_j' - g_r - \phi) - \frac{\tau}{1+q_j(p_j^a, p_j')\tau}(g_v + \phi). \quad (14)$$

Operator j chooses their tariff $\{p_j^a, p_j'\}$ to maximize (14). The first-order conditions of optimal tariff, at symmetric equilibrium point $p_j^a = p^a, p_j' = p'$, can be shown to be as follows:

$$\frac{\partial \pi}{\partial p_j^a}(p^a, p') = 1 - \frac{1}{1+q\tau}\frac{p' - g_r - \phi}{p'+w} - \frac{\tau}{1+q\tau}\frac{2\mu}{p'+w}(g_v + \phi) = 0, \quad (15)$$

$$\frac{\partial \pi}{\partial p_j'}(p^a, p') = \frac{q\tau}{1+q\tau}\left[-\frac{p' - g_r - \phi}{2\mu(p'+w)}\left(\frac{1}{1+q\tau} + 1\right) + \frac{1}{2\mu} - \frac{\tau}{1+q\tau}\frac{g_v+\phi}{p'+w}\right] = 0. \quad (16)$$

In the symmetric equilibrium, (13) implies the same solution for q as (2). The system (15,16) then defines the equilibrium reservation fee p' and vehicle density μ. Specifically, we have that $p' = g_r + \phi$, i.e. operators should optimally charge the marginal cost during the reservation phase and not attempt to earn profit, the same result as in Zakharenko (2022). Given this result, we can modify (15) to

$$2\mu^2(g_v + \phi) = (g_r + \phi + w)\lambda L. \tag{17}$$

The latter condition coincides with (7) when parking rates are equal to parking social cost, $g_r = g_v = g$. Note that (17), jointly with shared equilibrium condition (5), implies that such shared equilibrium is possible only if

$$\mu > \bar{\mu}(g_r, g_v) \equiv \frac{g_r + \phi + w}{2\tau(g_v + \phi)}. \tag{18}$$

Also note that $\bar{\mu}(g, g)$ equals $\bar{\mu}$ defined in section 4.

The equilibrium trip fare p^a is pinned down by operator free entry, which makes profit $\pi(p^a, p')$ equal to zero (cf.(4)):

$$p^a = (c + \phi)h + \frac{\tau}{1 + q\tau}(g_v + \phi) = (c + \phi)h + \frac{\mu}{\lambda L}(g_v + \phi). \tag{19}$$

Such trip fare includes the cost of vehicle use $(c + \phi)h$ plus a markup that compensates the costs incurred during vehicle vacancy.

5.2.3. Release decision

The decision whether to release or hold the vehicle by traveler i at the end of previous trip is made by comparing the expected cost of the two options. If released, i's expected cost of the next trip includes the trip fare p^a; with probability $\frac{q\tau}{1+q\tau}$, there is also a need to reserve a vehicle for $\frac{1}{2\mu}$ units of time in expectation, which incurs costs $p' + w = g_r + \phi + w$

per unit of time. Thus, the total expected cost is (cf.(19))

$$(c+\phi)h + \frac{\tau}{1+q\tau}(g_v+\phi) + \frac{q\tau}{1+q\tau}\frac{g_r+\phi+w}{2\mu}. \tag{20}$$

The per-trip cost of travel when vehicles are held is given by (9). Comparing the latter with (20) implies that the release decision is determined by comparison

$$q\left[\frac{g_r+\phi+w}{2\mu} - \tau(g_r+\phi)\right] \lesseqgtr g_r - g_v, \tag{21}$$

with $\lambda_i = 1$ when the left-hand side is smaller. Note that q is a function of λ and μ that solves (2). We now seek, for every μ, for an equilibrium release decision that satisfies $\lambda_i = \lambda$.

When $g_r = g_v$, i.e. government parking rates are non-discriminatory, the shared equilibrium (i.e. with $q > 0$ and $\lambda > 0$) is possible only when the term in square brackets in (21) is non-positive, that is, $\mu \geq \bar{\mu}(g_r, g_r)$. Note that when (18) is true, the latter inequality is strict, which implies $\lambda_i = \lambda = 1, \forall i$.

Few things are worth noting. First, the shared equilibrium can exist (i.e. equilibrium μ exceeds $\bar{\mu}(g_r, g_r)$, but is still below $\lambda L\tau = L\tau$) only when L is sufficiently high. Second, setting $g_r = g$ makes the shared equilibrium socially optimal.

When $g_v < g_r$, i.e. the government offers a discount for vacant shared vehicles, release is strictly preferred ($\lambda_i = 1$) when $\mu \geq \bar{\mu}(g_r, g_r)$, i.e. when the left-hand side of (21) is negative. In case $\mu < \bar{\mu}(g_r, g_r)$, from (21), λ_i equals unity (zero) when q is sufficiently low (high), which further implies that (cf.(4)) λ is sufficiently low (high). But then, a unique equilibrium traveler strategy $\lambda_i = \lambda$ for given μ is given by equality in (21), which implies (cf.(4))

$$\lambda = \frac{\mu}{L\tau} + \frac{2\mu^2}{L}\frac{g_r - g_v}{g_r + \phi + w - 2\mu\tau(g_r+\phi)}. \tag{22}$$

Such release probability satisfies (5) for any $\mu > 0$, meaning that if any vacant vehicles exist,

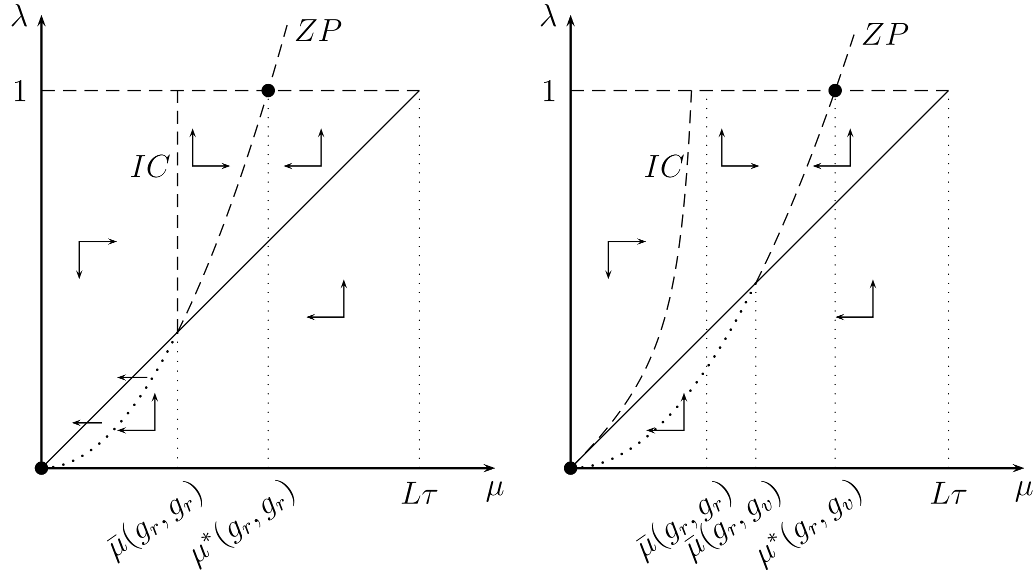

Figure 3: Equilibria when vacant SV are treated equally ($g_v = g_r$, left panel) and subsidized ($g_v < g_r$, right panel). Equilibria are shown as black dots.

they will indeed be shared by several travelers.

The final case $g_v > g_r$ (vehicle vacancy is penalized) clearly cannot contribute to vehicle sharing and is omitted.

5.3. Summary of equilibria

We now discuss the market equilibrium where λ is driven by traveler choices while μ by operator choices. The equilibria are illustrated as black dots on figure 3. The ZP (zero-profit) curves represent equilibrium supply of vacant SV, given by (17). The IC (indifference) curves represent the optimal mixed strategy for the release decision λ.

The equilibria when all parked vehicles are treated equally ($g_r = g_v$) are on the left panel. The private equilibrium ($\lambda = 0, \mu = 0$) always exists, but when L is large enough, a shared equilibrium with $\lambda = 1$, $\mu = \mu^*(g_r, g_v) \equiv \sqrt{\frac{(g_r+\phi+w)L}{2(g_v+\phi)}}$ also exists. When parking rates g_r are equal to the social cost of parking g, the shared equilibrium, if exists, maximizes social welfare. Both equilibria are stable, meaning that the economy may remain in the inferior private equilibrium unless some kind of "big push" increases the density of vacant shared vehicles beyond $\bar{\mu}(g_r, g_r)$.

The right panel of figure 3 illustrates the phase diagram for discounted parking of vacant SV, $g_v < g_r$. The same two equilibria are still present. However, the equilibrium release strategy λ, given by (22), now satisfies (5), so that vehicles are actually shared for any positive value of μ. This makes the private equilibrium unstable: introduction of arbitrarily small number of shared vehicles leads to emergence of sufficiently high demand for these vehicles, so their operations are profitable, their number increases until the shared equilibrium is reached. In other words, instead of a "big push" necessary in the equal-parking-policy scenario, only a small push in SV supply is sufficient to move the economy away from inefficient private equilibrium. This paper therefore advises governments to offer free or cheap parking to shared vehicles, when they are in the vacant state, while the SV industry is emerging.

Although traveler decision to always release vehicles in the shared equilibrium is socially optimal, the vehicle density $\mu^*(g_r, g_v)$ is generally not. For example when reserved vehicles pay the social cost of parking while vacant vehicles pay less, $g_v < g_r = g$, the equilibrium vehicle density $\mu^*(g_r, g_v)$ will exceed the social optimum $\mu^*(g, g)$. Therefore, discounted or free parking for vacant vehicles should be used as a temporary solution to push travelers into releasing and sharing their vehicles; when the density of vacant vehicles becomes sufficiently high, parking discounts for such vehicles can be eliminated.

6. Extensions

6.1. Convex social cost of parking

The model above has assumed that the marginal social cost of a parked vehicle does not depend on aggregate parking demand. It is likely though that such marginal cost is increasing: as the number of parked vehicles rises, the economy has to transition from cheaper surface parking lots to more expensive multi-story garages. But then, transition of the economy towards the shared equilibrium, by reducing overall demand for parking, will allow to make parking cheaper for all. This further increases the social value of shared mobility and the government incentives to push toward such mobility.

6.2. Traveler risk aversion

One simplifying assumption made in the above model is that travelers are risk-neutral with respect to their walking distance to the next vehicle. If any risk aversion exists, it may have a negative impact on the willingness to release and share vehicles by travelers, thus making the shared equilibrium more difficult to achieve. However, SV operators may counter this problem by offering some kind of insurance for the next vehicle reservation. For example it could offer free or even subsidized reservation time for the walk, in excess of some distance, from the location of previous vehicle release to the nearest available vehicle for the next ride. Further research is needed to formulate the optimal insurance policy to counter uncertainty in the location of the next vehicle.

6.3. Public transit

While public transit (PT) is generally viewed as a substitute to personal car, it may in some circumstances become a complement to shared vehicles. For example CoMoUK, a British non-profit organization that promotes shared mobility, argues in its website that "Car sharing schemes generally work best where there are good public transport links".[10]

In the context of the model developed in this paper, PT could be modeled as a fixed-cost transportation option that is inferior (more costly) than a private car. Then, PT would have no effect on the private equilibrium, when everyone uses the same vehicle perpetually and there are no vacant SV available.

But in the presence of actual vehicle sharing (i.e. when $q > 0$ in the notation of this paper), PT would be used by travelers who found themselves without a vehicle within certain walking distance, effectively imposing a cap on the walking time x. This would have a

[10]"Would car sharing work in your area?"at https://knowledge.como.org.uk, accessed on October 17, 2022.

twofold effect on optimal decisions. First, the fact that a fraction of travelers use another method of transportation would lead to a reduction of equilibrium density μ of vacant SV, for every given release decision λ. At the same time, existence of alternative transportation method would hedge travelers from worst-case outcomes (very long walking times) in case they previously release their vehicles. That would increase the release probability λ for every given SV supply μ.

To sum up, public transit reduces the long-term scope of SV popularity; at the same time, it makes it easier for the industry to overcome coordination failures and take off from the private equilibrium. Empirically, cities of Europe with better public transit have seen far more success in shared mobility (especially its free-floating form) than car-friendly American cities: there are several European free-floating SV operators with 5000+ vehicles each, compared to a single U.S. operator (GIG carshare) with estimated fleet of 1000 vehicles. In the future however, car cities like Los Angeles can become global leaders in shared mobility, provided that they overcome the coordination problems discussed in this paper.

7. Conclusion

This paper analyzes whether preferential treatment of shared vehicles by local governments is socially optimal. The answer is positive: in the early stages of industry growth, such preferential treatment helps to overcome coordination problems in transition from private to shared use of vehicles. In later stages, when the density of shared vehicles becomes sufficiently high, such preferential treatment can be removed to avoid over-supply of shared vehicles in equilibrium.

Appendix A. Price competition is quasi-shared state

This section investigates optimal pricing of an arbitrary operator j in the quasi-shared state, i.e. the one characterized by $\mu = \lambda L\tau$, all other operators setting some trip fare p^a, and $q = 0$ (i.e. a vacant vehicle always used by previous user) for all operators except j.

Suppose an arbitrary operator j deviates from prevailing prices and sets a trip fare p_j^a marginally different from p^a. The case $p_j^a > p^a$ would imply that j's existing customer would have an incentive to switch to another vehicle (because otherwise the extra cost $p_j^a - p^a$ would have to be paid infinitely many times, with no discounting of future losses, while the cost of search for another vehicle is finite). In a quasi-shared state, no other traveler would ever use vehicle j, hence the loss of the existing customer would lead to a permanent loss of revenue for j, which is clearly suboptimal.

In case $p_j^a = p^a - \epsilon < p^a$ for some $\epsilon > 0$, the last user i of vehicle j always wants to return to the same vehicle. The Poisson rate of last user return to j is $\frac{1}{\tau}$. In addition, other travelers, sufficiently proximate to vehicle j, are incentivized to switch to j from their previous vehicle. Denote by $q_j > 0$ the endogenous Poisson rate of vehicle j booking by travelers other than i.

What is the expected cost saving for traveler i from using vehicle j rather than another vehicle? The probability that vehicle j remains available for i's another trip is $\frac{\frac{1}{\tau}}{\frac{1}{\tau}+q_j} = \frac{1}{1+q_j\tau}$. Then, the expected number of trips by traveler i with vehicle j is $1 + \frac{1}{1+q_j\tau} + \frac{1}{(1+q_j\tau)^2} + \cdots = 1 + \frac{1}{q_j\tau}$; i's cost saving for every such trip is ϵ.

Because other travelers have the same expected cost saving from using vehicle j, they will prefer to abandon their previous vehicle and walk to j if the walking time does not exceed $\hat{x}$ given by $(p_j' + w)\hat{x} = \epsilon\left(1 + \frac{1}{q_j\tau}\right)$, where p_j' is the per-minute reservation rate by operator j. But then, the rate q_j of vehicle j reservation by walkers is the product of the Poisson rate of traveler departure λL and length of the two-sided walking range $2\hat{x}$:

$$q_j = 2\lambda L\hat{x} = 2\lambda L\left(\frac{\epsilon}{p_j' + w}\right)\left(1 + \frac{1}{q_j\tau}\right) \tag{A.1}$$

If ϵ is a small quantity, so is q_j; but then $1 + \frac{1}{q_j\tau}$ in (A.1) is approximately equal to $\frac{1}{q_j\tau}$.

Then, q_j can be approximated by $\sqrt{\frac{2\lambda L}{\tau}\frac{\epsilon}{p'_j+w}}$, which is a higher order of magnitude than ϵ.

What is the effect of price undercutting ϵ on operator j's profit per cycle? The profit gain during the vehicle use phase is $-\epsilon$ (i.e. the loss in collected revenue). The extra profit during vehicle reservation by walkers is the product of the (i) probability that the vehicle is reserved by a walker, $\frac{q_j\tau}{1+q_j\tau}$, (ii) the expected walking time $\frac{\hat{x}}{2} = \frac{1}{2}\frac{\epsilon}{p'_j+w}\left(1+\frac{1}{q_j\tau}\right)$, and (iii) the profit per minute of reservation $p'_j - g_r - \phi$. This product is equal to $\frac{1}{2}\frac{\epsilon}{p'_j+w}(p'_j - g_r - \phi)$, the same order of magnitude as ϵ. Finally, the profit gain during the vehicle vacancy phase stems from the reduction of expected vacancy duration from τ to $\frac{\tau}{1+q_j\tau} = \frac{\tau}{1+\sqrt{2\lambda L\tau\frac{\epsilon}{p'_j+w}}}$, with a saving of $g_v + \phi$ per minute of reduced vacancy.

Note that profit gains from reduced duration of vacancy are of higher order of magnitude than losses in revenue per cycle: this is because travelers expect to use cheaper service repeatedly for many cycles, hence have higher-order-of magnitude incentives to switch to vehicle j from their previous vehicle. But than means that the net profit gain from undercutting the price p^a_j below p^a is always positive, and no equilibrium p^a exists. In a quasi-shared state, competition will always lead to negative profit, operator exit, and eventually to fully private equilibrium with $\mu = 0$ (no vacant SV) and $\lambda = 0$ (nobody releasing their vehicle).

References

Buchholz, N., 2021. Spatial Equilibrium, Search Frictions, and Dynamic Efficiency in the Taxi Industry. The Review of Economic Studies .

Inci, E., 2015. A review of the economics of parking. Economics of Transportation 4, 50–63.

Jakob, M., Menendez, M., 2020. Optimal parking occupancy with and without differentiated parking: A macroscopic analysis. Preprints , 2020060227.

Jochem, P., Frankenhauser, D., Ewald, L., Ensslen, A., Fromm, H., 2020. Does free-floating carsharing reduce private vehicle ownership? the case of share now in european cities. Transportation Research Part A: Policy and Practice 141, 373–395.

Kadam, S.V., Kotowski, M.H., 2018. Multiperiod matching. International Economic Review 59, 1927–1947.

Lagos, R., 2000. An alternative approach to search frictions. Journal of Political Economy 108, 851–873.

Nansubuga, B., Kowalkowski, C., 2021. Carsharing: A systematic literature review and research agenda. Journal of Service Management .

van Ommeren, J., McIvor, M., Mulalic, I., Inci, E., 2021. A novel methodology to estimate cruising for parking and related external costs. Transportation Research Part B: Methodological 145, 247–269.

Winter, K., Cats, O., Martens, K., van Arem, B., 2021. Parking space for shared automated vehicles: How less can be more. Transportation Research Part A: Policy and Practice 143, 61–77.

Zakharenko, R., 2016. The time dimension of parking economics. Transportation Research Part B: Methodological 91, 211 – 228.

Zakharenko, R., 2020. The merits of privileged parking. Transportation Research Part B: Methodological 140, 193–209.

Zakharenko, R., 2022. Pricing shared vehicles. mimeo .