Informative Advertising and Consumer Search in Markets for New Products

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March 15, 2023

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Abstract

We consider consumer search in markets for new products, where consumers are initially unaware of the product's existence. Two firms sell differentiated products and may advertise to inform some fraction of consumers. Yet, consumers that are informed about the existence of the product in that manner, may also decide to seek out other firms selling a similar product. In the context of a search model with differentiated products, we show that if search costs are low, firms are likely to underadvertise. Informing an additional consumer then implies that it is likely that she will also check out the competitor's product, giving firms an incentive to try to free ride on the advertising effort of their competitor. Yet, if search costs are high, firms will overadvertise. Advertising then becomes worthwhile even if it reaches consumers already informed by the other firm, as that makes it more likely that this consumer will visit you first. This leads to socially wasteful duplication of advertising efforts.

JEL-codes: C72, L13

Keywords: Consumer search, advertising, new products

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1 Introduction

Models of informative advertising typically assume that a consumer only knows that a firm sells a particular product when she sees it advertised by that firm. The classic models in this literature typically assume that a consumer compares the firms from which she has received an offer, and buys from the one offering the best deal (see in particular Butters, 1977, and Grossman and Shapiro, 1984.).

Yet, especially in the case of novel products, an ad may not just inform a consumer that a firm sells a particular product: it may also inform her of the very existence of that product. Suppose for example that a consumer sees brand A advertising a smartwatch. Before seeing that ad, she may not have been familiar with the existence of smartwatches altogether. After having seen the ad of brand A, she may consider its product, only to realize that there may well be other firms that also sell smartwatches as well. She may then decide to check out these firms as well, and ultimately end up buying brand B's product. In that case, brand B has effectively free ridden on the advertising effort of brand A.

It is exactly this scenario that we consider in this paper. We study a duopoly model of consumer search with differentiated products along the lines of Anderson and Renault (1999) or Wolinsky (1986). A consumer is initially unaware of the existence of the product. She becomes aware if she sees it advertised. The costs of advertising to a firm are convex in the fraction of consumers that is informed.

A classic question in the literature on advertising is whether the market provides too much or too little advertising from a welfare perspective, see e.g. the excellent survey in Bagwell (2007). We also ask that question in our model. The answer is not a priori clear: on the one hand, the possibility of free riding may result in an advertising effort that is too low, but on the other hand, the market may also provide a wasteful duplication of advertising effort.

If the consumer only sees an ad of firm A, she will visit that firm first. After having done so, she may decide to figure out whether there is a second firm offering the same product. The costs of doing so and visiting that firm are her search costs. If these are sufficiently low, she may indeed visit firm B to see whether its product is more to her liking. On the other hand, if she only receives an ad from firm B, she will visit that firm first, but may also decide to visit firm A afterwards. If she receives ads from both firms, the first visit is determined at random, Hence, in our model, firm B may free ride on the advertising efforts of firm A, But, if that occurs, firm A at least has the advantage that a consumer that sees its ad will visit firm A first.

We find that, in a competitive market, the fraction of consumers that is informed in equilibrium is increasing in search costs. As search costs increase, a consumer that visits a firm is less likely to walk away and check the other firm. Hence, from the point of view of an individual firm, the returns to investing in advertising increase in consumer search costs. Next, consider a social planner that could dictate advertising levels. She would choose to *decrease* the fraction of informed consumers as search costs increase. Higher search costs mean that the net surplus of any consumer will be lower. Hence, from the point of view of a social planner, the returns to investing in advertising decrease in search costs.

The above implies that, under some weak conditions on the advertising technology and the distribution of match values, we have that there is underadvertising if search costs are sufficiently low, but overadvertising if they are sufficiently high. With low search costs, firms have little market power and consumers are likely to also visit the other firm. Hence, firms are tempted to free ride on each other's advertising. For high enough search costs, this effect vanishes and instead there is duplication of wasteful advertising efforts.

We are surely not the first to study the interplay of advertising and consumer search. Most closely related to our paper is probably Haan and Moraga-González (2011), where the consumers' order of search is determined by the number of advertising messages they see from a particular firm. In that paper, advertising is purely combative; it does not affect the market outcome and hence there is always overadvertising. In our paper, advertising has a social function, as it informs consumers of the mere existence of a product.

Other work that looks at the relation between advertising and match values include Meurer and Stahl (1994), where firms can advertise match values, and Johnson and Myatt (2006), where advertising affects the distribution of match values. In Johnson (2009) firms can use ads to send signals about match values, but consumers can choose to block those ads. In Loginova (2009) consumers have an imperfect memory for the ads that they have seen. Papers that combine advertising and consumer search but in the context of homogeneous products include Robert and Stahl (1993), Janssen and Non (2008), McCarthy (2016), and Shelegia and Wilson (2021).

The remainder of this paper is structured as follows. In Section 2, we present the model. We solve for the market equilibrium in Section 3. The problem of the social planner is considered in Section 4, where we also compare the market outcome with the social optimum. Section 5 illustrates our model for the case of a uniform distribution of match values. Section 6 concludes.

2 The Model

The basis of our analysis is the canonical model of search with differentiated products introduced by Wolinsky (1986), and further popularized by Anderson and Renault (1999). Consider a market where 2 firms sell differentiated products to a unit mass of consumers. Marginal costs are constant and normalized to zero. Consumer i derives utility

$$u_{ij} = v + \varepsilon_{ij} - p_j \tag{1}$$

if she buys from firm j at a price p_j, $j \in \{1, 2\}$. The parameter v is the stand-alone utility of consuming the product: it is sufficiently high such that the market is always covered (in the sense that all informed consumers buy) in equilibrium. A necessary condition to have that is that $v \geq 1$. The match value ε_{ij} reflects how much this consumer likes the product of firm j, and is the realisation of a random variable with distribution F and continuously differentiable density f with support normalized to $[0, 1]$. We follow the convention in the literature and assume that $1 - F$ is log-concave. Firms cannot observe ε_{ij} so price discrimination is not feasible. Consumers must incur a search cost s to visit firm j and learn price p_j and match value ε_{ij}. The consumer searches sequentially with perfect recall.

In this framework, we assume that each firm can engage in advertising to inform consumers about the existence of the product sold on this market, and the fact that this particular firm sells that product. Crucially, we assume that firms do not advertise their price, and can also not provide information that would allow consumers to learn their match value.[1]

A firm can inform a fraction φ of consumers at a cost $c(\varphi)$, with $c'(\varphi) > 0$ if $\varphi > 0$, $c'(0) = 0$, and $c''(\varphi) > 0$ for all φ. A consumer that only receives an ad from firm k will visit firm k first, and has to incur search costs s to do so. After having visited the first firm, she decides whether to also search for an alternative firm that offers the same product. For simplicity, we assume that the search costs of finding that firm and checking out its offer are also s. A consumer that

[1] To allow for price advertising, we could use a directed search model as the basis for our analysis, see e.g. Haan et al. (2018). But that would greatly complicate the analysis.

is informed by both firms randomly determines which firm to visit first. Also in that case, each firm that she checks out will cost her s. After having visited the second firm, a consumer may decide to go back to the first firm as, with hindsight, she prefers the offer of that firm. Doing so does not entail any additional costs.

The timing is thus as follows. First, firms simultaneously decide on advertising and prices. Second, consumers search sequentially.

3 Solving the model

We look for an equilibrium price p^* and an equilibrium fraction of consumers φ^* that each firm informs. In equilibrium, suppose that an informed buyer visits firm j first, and finds a match value ε_{ij}. Her expected benefit from visiting the other firm is given by $g(\varepsilon_{ij}) = \int_{\varepsilon_{ij}}^1 (\varepsilon - \varepsilon_{ij}) f(\varepsilon) d\varepsilon$. An additional search is worthwhile only if this benefit exceeds search costs s. We denote the ε_{ij} for which this holds as $\hat{\varepsilon}$. Hence $\hat{\varepsilon}$ is implicitly defined by

$$s = \int_{\hat{\varepsilon}}^1 (\varepsilon - \hat{\varepsilon}) f(\varepsilon) d\varepsilon. \tag{2}$$

With the usual arguments, such a $\hat{\varepsilon}$ always exists, provided that $s < E(\varepsilon_{ij})$.

To find the equilibrium, suppose that firm k uses (p^*, φ^*), but j sets some (p_j, φ_j). For ease of exposition, denote $\Delta \equiv p_j - p^*$. Demand for firm j then equals

$$\begin{aligned} D_j(p_j, \varphi_j; p^*, \varphi^*) = &\left[\varphi_j \left(1 - \frac{1}{2}\varphi^*\right) + \varphi^* \left(1 - \frac{1}{2}\varphi_j\right) F(\hat{\varepsilon})\right] (1 - F(\hat{\varepsilon} + \Delta)) \\ &+ [\varphi_j + (1 - \varphi_j)\varphi^*] \int_0^{\hat{\varepsilon}+\Delta} F(\varepsilon - \Delta) dF(\varepsilon). \end{aligned} \tag{3}$$

This can be seen as follows. A consumer that visits firm j will buy there if $\varepsilon_{ij} > \hat{\varepsilon} + \Delta$. If j is visited first, it thus makes a sale with probability $1 - F(\hat{\varepsilon} + \Delta)$. If the other firm, k, is visited first, j will make a sale with probability $F(\hat{\varepsilon})(1 - F(\hat{\varepsilon} + \Delta))$, as this consumer first has to decline k's offer. A consumer visits j first if she receives an ad from j and not from k, and with probability $1/2$ if she receives an ad from both. The probability that j is visited first thus equals $\varphi_j(1 - \varphi^*) + \varphi_j \varphi^*/2 = \varphi_j(1 - \varphi^*/2)$. Similarly, k is visited first with probability $\varphi^*(1 - \varphi_j/2)$. Taken together, this implies the first term. The second term reflects consumers that find a match value that is too low at both firms, but return to j as they have the best match there. This applies to all informed consumers, which is a fraction $\varphi_j + (1-\varphi_j)\varphi^*$.

Profits of firm j are given by

$$\pi_j(p_j, \varphi_j; p^*, \varphi^*) = p_j \cdot D_j(p_j, \varphi_j; p^*, \varphi^*) - c(\varphi_j). \tag{4}$$

Maximizing with respect to φ_i and p_j yields first-order conditions

$$\begin{aligned} \frac{\partial \pi_j}{\partial \varphi_j} &= p_j \frac{\partial D_j}{\partial \varphi_j} - c'(\varphi_j) = 0 & (5)\\ \frac{\partial \pi_j}{\partial p_j} &= D_j + p_j \frac{\partial D_j}{\partial p_j} = 0, & (6) \end{aligned}$$

Imposing symmetry and assuming an interior solution for φ^*, we have:

$$\begin{aligned} D_j\left(p^*, \varphi^*\right) &= \varphi^*\left(1 - \frac{1}{2}\varphi^*\right) \\ \left.\frac{\partial D_j}{\partial \varphi_j}\right|_{p^*,\varphi^*} &= \left[1 - \frac{1}{2}\varphi^*\left(1 + F\left(\hat{\varepsilon}\right)\right)\right]\left(1 - F\left(\hat{\varepsilon}\right)\right) + \frac{1}{2}\left(1 - \varphi^*\right) F\left(\hat{\varepsilon}\right)^2 \\ &= \frac{1}{2}\left(1 - F\left(\hat{\varepsilon}\right)\right)^2 + \frac{1}{2}\left(1 - \varphi^*\right) \\ \left.\frac{\partial D_j}{\partial p_j}\right|_{p^*,\varphi^*} &= -\varphi^*\left(1 - \frac{1}{2}\varphi^*\right)\left[(1 - F(\hat{\varepsilon}))f(\hat{\varepsilon}) + 2\int_0^{\hat{\varepsilon}} f^2(\varepsilon)d\varepsilon\right] \end{aligned}$$

Plugging these back into (5) and (6) and solving for the equilibrium then yields the usual expression for the equilibrium price:

$$p^* = \frac{1}{(1 - F(\hat{\varepsilon}))f(\hat{\varepsilon}) + 2\int_0^{\hat{\varepsilon}} f^2(\varepsilon)d\varepsilon}, \tag{7}$$

Note that in equilibrium, both firms inform the same fraction of consumers, and hence receive the same share of first visitors. Hence, in equilibrium the only difference between the standard model of search with differentiated products and our model, is that the standard model has a mass of consumers 1, and ours has an equilibrium mass of consumers that equals $\varphi^*(2 - \varphi^*)$ – the fraction of consumers that is informed in equilibrium. But of course, the mere fact that we have a different mass of consumers does not affect equilibrium prices.

From the analysis above, the equilibrium fraction of informed consumers is implicitly defined by:

$$p^* \left[\frac{1}{2}(1 - F(\hat{\varepsilon}))^2 + \frac{1}{2}(1 - \varphi^*) \right] = c'(\varphi^*) \tag{8}$$

We can now show the following:

Proposition 1 (Comparative statics market equilibrium)**.** *The equilibrium price p^* and the equilibrium fraction of informed consumers per firm φ^* are both increasing in search costs s.*

Proof. Note that an increase in search cost implies a decrease in the search threshold $\hat{\varepsilon}$. We thus have to show that p^* and φ^* are decreasing in $\hat{\varepsilon}$. Since (7) is the same equilibrium price as in Anderson and Renault (1999), it follows from their Proposition 1 that p^* is decreasing in $\hat{\varepsilon}$ due to log-concavity of $1 - F$. Using (8) and making the dependency of p^* and φ^* on $\hat{\varepsilon}$ explicit, we have:

$$p(\hat{\varepsilon}) \left[\frac{1}{2}(1 - F(\hat{\varepsilon}))^2 + \frac{1}{2}(1 - \varphi(\hat{\varepsilon})) \right] = c'(\varphi(\hat{\varepsilon})).$$

Differentiate both sides with respect to $\hat{\varepsilon}$ and solve for $\varphi'(\hat{\varepsilon})$:

$$\varphi'(\hat{\varepsilon}) = \frac{p'(\hat{\varepsilon}) \left[\frac{1}{2}(1 - F(\hat{\varepsilon}))^2 + \frac{1}{2}(1 - \varphi(\hat{\varepsilon})) \right] - p(\hat{\varepsilon}) f(\hat{\varepsilon})(1 - F(\hat{\varepsilon}))}{c''(\varphi(\hat{\varepsilon})) + \frac{1}{2} p(\hat{\varepsilon})}.$$

Since $p'(\hat{\varepsilon}) < 0$ and $c''(\cdot) > 0$, we see that $\varphi'(\hat{\varepsilon}) < 0$ and the fraction of informed consumers is decreasing in $\hat{\varepsilon}$. ∎

First note that our result on the equilibrium price is standard in this literature: an increase in search costs gives a firm more market power vis-à-vis the consumers that visit its firm; having higher search costs means they are less likely to walk away. The effect on equilibrium advertising is novel. As higher search costs mean that each visiting consumer is more valuable (first, because they are less likely to walk away, and second, because equilibrium prices will also be higher), firms are willing to invest more in trying to attract those consumers.

Note that the comparative statics for an upward shift of the marginal cost of informing consumers is straightforward. Equilibrium price (cf. eq. 7) is unchanged. The left-hand side of (8) is the marginal profit of informing consumers and it is decreasing in φ^*. The right-hand side of (8) is the marginal increase of informing consumers and it is increasing in φ^*. Hence an upward shift of the marginal cost leads to a lower fraction of consumers being informed. It is a standard result that equilibrium profit decreases.

4 Welfare

In this section, we derive the advertising levels that would be preferred by a social planner. We thus assume that the two firms still use the same advertising technology and use the same levels of advertising, and prices are set competitively.

For simplicity, we will henceforth assume that $c(\varphi) = \frac{1}{2}a\varphi^2$, where $a > 0$ is a parameter. This allows us to find an explicit expression for φ^*. From (8) we have:

$$\varphi^* = \frac{p^*(1 + (1 - F(\hat{\varepsilon}))^2)}{2a + p^*}$$

Note that $\varphi^* \leq 1$ if and only if $a \geq \frac{1}{2}p^*(1 - F(\hat{\varepsilon}))^2$. In the remainder of this paper, we assume that to be the case. For lower values of a, we would have a corner solution with $\varphi^* = 1$.

Consider a consumer that is informed. Denote the (ex ante expected) gross utility she will obtain as U: this includes stand-alone value v, plus the expected match value she will end up with, minus her expected search costs. The total number of consumers that ends up informed equals $1 - (1 - \varphi)^2 = \varphi\,(2 - \varphi)$. Total welfare then equals

$$W = \varphi\,(2 - \varphi)\,U - a\varphi^2.$$

Conditional on being informed, all consumers buy in equilibrium, which implies that the equilibrium price p^* does not enter the expression for welfare. Hence, total welfare only depends on the number of informed consumers and the gross utility they end up with on the one hand, and the total costs of advertising on the other.

Maximizing W with respect to φ yields

$$\tilde{\varphi} = \frac{U}{a + U},$$

which is always an interior solution.

To further evaluate $\tilde{\varphi}$, we have to pin down U. Note that with probability $F(\hat{\varepsilon})$, a consumer will do a second search, hence we can write

$$U = v + E\left(\varepsilon_s\right) - \left(1 + F\left(\hat{\varepsilon}\right)\right) s,$$

with ε_s the match value she will end up with. With probability $F(\hat{\varepsilon})^2$, she finds a match value at both firms that is lower than $\hat{\varepsilon}$; in that case, she gets the highest of the two. With probably $1 - F(\hat{\varepsilon})^2$, she ends up with a draw above $\hat{\varepsilon}$. Hence, her expected match value is

$$E(\varepsilon_s) = F(\hat{\varepsilon})^2 E(\max\{\varepsilon_1, \varepsilon_2\} \,|\, \max\{\varepsilon_1, \varepsilon_2\} < \hat{\varepsilon}) + \left(1 - F(\hat{\varepsilon})^2\right) E(\varepsilon | \varepsilon > \hat{\varepsilon})$$

From (2) we have that $s = (1 - F(\hat{\varepsilon}))E(\varepsilon - \hat{\varepsilon} | \varepsilon > \hat{\varepsilon})$. This implies

$$E(\varepsilon_s) - (1 + F(\hat{\varepsilon}))\, s = F(\hat{\varepsilon})^2 E(\max\{\varepsilon_1, \varepsilon_2\} \,|\, \max\{\varepsilon_1, \varepsilon_2\} < \hat{\varepsilon}) + \hat{\varepsilon}(1 - F(\hat{\varepsilon}))$$

which implies

$$U = v + 2\int_0^{\hat{\varepsilon}} \varepsilon f(\varepsilon) F(\varepsilon) d\varepsilon + \hat{\varepsilon}(1 - F(\hat{\varepsilon})). \tag{9}$$

We can now show:

Proposition 2 (Comparative statics social optimum)**.** *The optimal fraction of informed consumers per firm $\tilde{\varphi}$ is decreasing in search costs s and advertising parameter a.*

Proof. We first derive the comparative statics with respect to s. Differentiating the expression for $\tilde{\varphi}$ with respect to U yields:

$$\frac{\partial \tilde{\varphi}}{\partial U} = \frac{a}{(U + a)^2} > 0.$$

Hence it suffices to show that U is decreasing in s. For any value of s, there is an optimal strategy associated with it, i.e. continue searching if the match value is below $\hat{\varepsilon}(s)$ (to make the dependence on s explicit). The (ex ante expected) gross utility $U(s, \hat{\varepsilon})$ depends on both the threshold $\hat{\varepsilon}$ and search cost s. Note that $\hat{\varepsilon}(s) \in \arg\max_{\hat{\varepsilon}} U(s, \hat{\varepsilon})$. Suppose $s' > s$. Then:

$$U(s', \hat{\varepsilon}(s')) < U(s, \hat{\varepsilon}(s')) \leq U(s, \hat{\varepsilon}(s)),$$

where the first inequality follows from the fact that the consumer follows the same search strategy and hence has the same distribution over the number of searches (but the search cost is less). This establishes that U is decreasing in s.

For the comparative statics with respect to a, differentiating the expression for $\tilde{\varphi}$ with respect to a yields:

$$\frac{\partial \tilde{\varphi}}{\partial a} = -\frac{U}{(U + a)^2} < 0$$

$\blacksquare$

Hence, contrary to the market outcome, we have that the socially optimal level of advertising is *decreasing* in search costs. As search costs increase, the gross utility U that each consumer can expect to earn from participating in the market, decreases. Hence, the returns from investing in advertising also decrease, so a social planner would do less of it.

We now have:

Proposition 3. *Comparing the market outcome with the social optimum, we have the following:*

1. *For sufficiently low search costs, there is underadvertising.*

2. *For sufficiently high search costs, there is overadvertising if and only if*

$$f(0) \leq \frac{2a+v}{2av}. \tag{10}$$

3. *If condition* (10) *is satisfied, there is a cut-off value of search costs $\tilde{s}$ such that there is underadvertising if $s < \tilde{s}$, and overadvertising if $s > \tilde{s}$. If it is not, there is always underadvertising.*

Proof. First note that we have overadvertising, hence $\varphi^* > \tilde{\varphi}$ whenever

$$\frac{1+(1-F(\hat{\varepsilon}))^2}{2a+p^*} \cdot p^* > \frac{U}{a+U}$$

Rewriting yields

$$U < \frac{ap^*\left(1+(1-F(\hat{\varepsilon}))^2\right)}{2a-(1-F(\hat{\varepsilon}))^2 p^*} \tag{11}$$

(Note that the denominator is strictly positive under the assumption that $\varphi^* < 1$).

Suppose $s \to 0$, hence $\hat{\varepsilon} \to 1$. We then have

$$E(\varepsilon_s) = E\left(\varepsilon_{(2)}\right),$$

with $\varepsilon_{(2)}$ the highest-order statistic of two draws from F. Condition (11) then collapses to

$$v + E\left(\varepsilon_{(2)}\right) < \frac{p^*}{2}. \tag{12}$$

With $\hat{\varepsilon} \to 1$, we have $p^* = 1/2 \int_0^1 f^2(\varepsilon) d\varepsilon$, Necessarily $\int_0^1 (f(\varepsilon) - 1)^2 d\varepsilon \geq 0$, which implies $\int_0^1 \left(f(\varepsilon)^2 - 2f(\varepsilon) + 1 \right) d\varepsilon \geq 0$. But $\int_0^1 f(\varepsilon) d\varepsilon = 1$, hence $\int_0^1 f(\varepsilon)^2 d\varepsilon \geq 1$ so $p^* \leq 1/2$. With $v = 1$, this implies that (12) is never satisfied. Hence, there is underadvertising in that case.

Now suppose $\hat{\varepsilon} \to 0$, which implies $s \to E(\varepsilon)$. In that case $E(\varepsilon_s) = E(\varepsilon | \varepsilon > 0) = E(\varepsilon)$, so $U = v + E(\varepsilon) - s = v$. Condition (11) then collapses to

$$v < \frac{2ap^*}{2a - p^*}.$$

With $v = 1$, this implies that we need $p^* > \frac{2a}{2a+1}$. This is always satisfied if $p^* = 1/f(0) \geq 1$ or $f(0) \leq 1$.

Monotonicity of the difference between φ^* and $\tilde{\varphi}$ follows directly from Propositions 1 and 2. Since φ^* is increasing in s and $\tilde{\varphi}$ is decreasing in s, there is at most one point, $s = \tilde{s}$ where the two coincide.

■

Note that condition (10) is relatively mild. For example, with $v = 1$, it is always satisfied if $f(0) < 1$. Sufficient for that to hold is that $f' \geq 0$ – a condition that is often imposed in similar models.

Proposition 3 shows that, under mild conditions, the market provides overadvertising if search costs are sufficiently high, but underadvertising if search costs are sufficiently low. Intuitively, with very low search costs, any additional consumer that a firm informs is likely to also check out the product that is offered by its competitor. Hence, firms can do a lot of free riding on the advertising effort of their competitor, which implies that equilibrium advertising will be too low. With high search costs, however, free riding will not occur. This gives firms an incentive to inform consumers, even if these are likely to already be informed by their competitor. After all, this increases the probability that this firm will be visited first to 1/2 rather than virtually zero. But that implies that there will be a lot of wasteful duplication of advertising effort. Hence, from a welfare perspective, the amount of advertising provided by the market will then be too high.

5 Numerical illustration

In this section, we consider the case of a uniform distribution of match values. Moreover, we assume that the advertising cost function is quadratic, as in the previous section: $c(\varphi) = a\varphi^2/2$. Again consider that firm k uses the tentative equilibrium strategy (p^*, φ^*), and allow firm j to defect some (p_j, φ_j) where we define $\Delta \equiv p_i - p^*$. We then have, following

$$D_j = \left(\varphi_j \left(1 - \frac{(1+\hat{\varepsilon})\,\varphi^*}{2}\right) + \hat{\varepsilon}\varphi^*\right)(1 - \hat{\varepsilon} - \Delta) + \frac{1}{2}(\varphi_j\,(1-\varphi^*) + \varphi^*)(\hat{\varepsilon}^2 - \Delta^2) \tag{13}$$

so

$$\begin{aligned}
\frac{\partial D_j}{\partial \varphi_j} &= \left(1 - \frac{(1+\hat{\varepsilon})\,\varphi^*}{2}\right)(1 - \hat{\varepsilon} - \Delta) + \frac{1}{2}(1-\varphi^*)(\hat{\varepsilon}^2 - \Delta^2) \\
\frac{\partial D_j}{\partial p_j} &= -\left(\varphi_j \left(1 - \frac{(1+\hat{\varepsilon})\,\varphi^*}{2}\right) + \hat{\varepsilon}\varphi^*\right) - \Delta(\varphi_j\,(1-\varphi^*) + \varphi^*)
\end{aligned}$$

Imposing symmetry:

$$\begin{aligned}
D_j\,(p^*, \varphi^*) &= \varphi^* \left(1 - \frac{\varphi^*}{2}\right) \\
\left.\frac{\partial D_j}{\partial \varphi_j}\right|_{p^*,\varphi^*} &= 1 - \frac{1}{2}\varphi^* - \hat{\varepsilon}\left(1 - \frac{\hat{\varepsilon}}{2}\right) \\
\left.\frac{\partial D_j}{\partial p_j}\right|_{p^*,\varphi^*} &= -\varphi^*\,(1+\hat{\varepsilon})\left(1 - \frac{\varphi^*}{2}\right)
\end{aligned}$$

Plugging these back into (5) and (6) yields

$$\begin{aligned}
\left.\frac{\partial \pi_j}{\partial \varphi_j}\right|_{p^*,\varphi^*} &= p^* \left(1 - \frac{1}{2}\varphi^* - \hat{\varepsilon}\left(1 - \frac{\hat{\varepsilon}}{2}\right)\right) - a\varphi^* = 0 \\
\left.\frac{\partial \pi_j}{\partial p_j}\right|_{p^*,\varphi^*} &= \varphi^* \left(1 - \frac{\varphi^*}{2}\right)(1 - p^*\,(1+\hat{\varepsilon})) = 0.
\end{aligned}$$

Solving for the equilibrium then yields

$$\begin{aligned}
p^* &= \frac{1}{1+\hat{\varepsilon}} \\
\varphi^* &= \frac{1 + (1-\hat{\varepsilon})^2}{1 + 2a\,(1+\hat{\varepsilon})},
\end{aligned}$$

where, with a uniform distribution, we have using $g(\hat{\varepsilon}) = \hat{\varepsilon}$ and (2) that $\hat{\varepsilon} = 1 - \sqrt{2s}$.

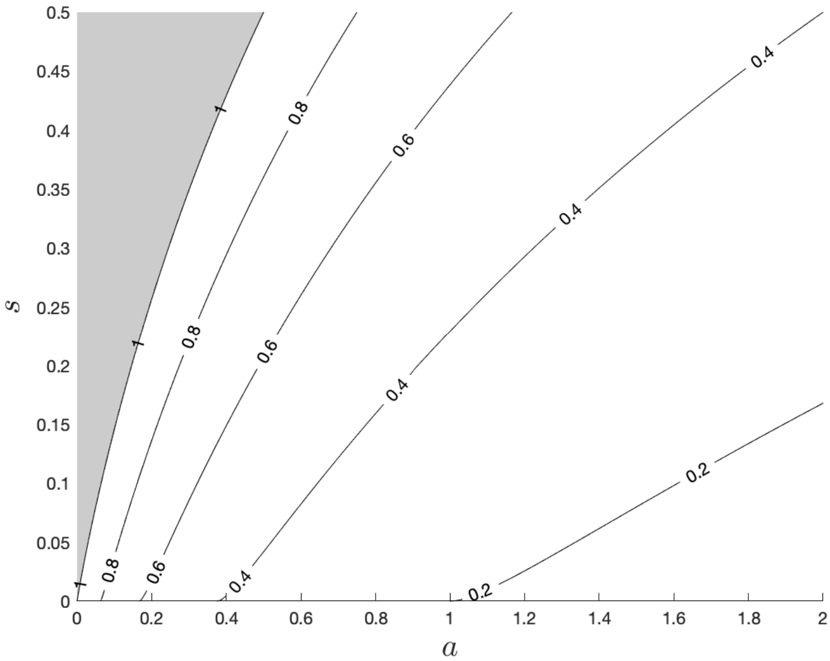

Figure 1: *Advertising intensity in the market equilibrium as function of a and s. In the grey area, we have the boundary solution $\varphi^* = 1$.*

Figure 1 shows the equilibrium fraction of informed consumers as function of a and s. This confirms the comparative statics derived in Section 3: higher advertising cost leads to fewer informed consumers, and higher search cost lead to more informed consumers.

Figure 2 depicts equilibrium profits as function of a and s. The general picture is that more market power (higher s) leads to more profit and an increase in advertising cost lowers profit. Observe that we were unable to derive monotone comparative statics for profit as function of search cost. It turns out that there is a small area where an increase in market power leads to lower profit (this happens near $s = 0.5$ and $a = 0.5$). The reason that the comparative statics are ambiguous is the following. Revenue is increasing in s, at it leads to a higher price and a higher fraction of informed consumers (see Proposition 1). But total advertising costs are also increasing in s, again via an equilibrium increase of informed consumers. When almost all consumers receive an ad from either you or your competitor (around $s = 0.5$ and $a = 0.5$, φ^* is close to one), then the marginal effect of advertising on revenue is relatively small whilst the marginal cost of advertising is close to its maximum. When all these effects are combined, it can yield the counter-intuitive result that an increase in market power lowers profit.[2]

[2]But note that Haan and Moraga-González (2011) also find that results in their paper.

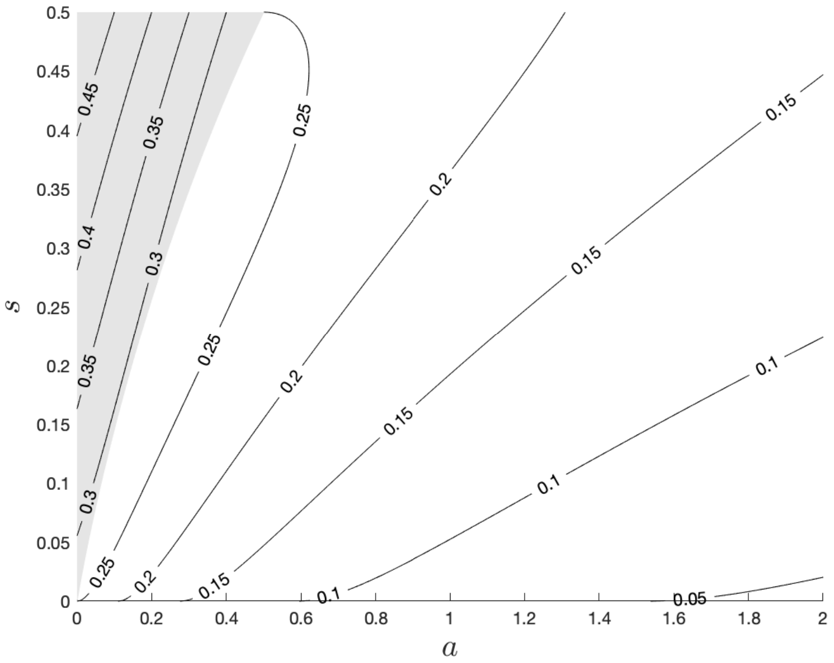

Figure 2: *Profit in the market equilibrium as function of a and s. In the grey area, we have the boundary solution $\varphi^* = 1$.*

To derive the social optimum, we first have to compute the expected gross utility of an informed consumer. Using (9), this works out at

$$U = v + 2\int_0^{\hat{\varepsilon}} \varepsilon^2 d\varepsilon + \hat{\varepsilon}(1-\hat{\varepsilon}) = v + \frac{2}{3}\hat{\varepsilon}^3 + \hat{\varepsilon}(1-\hat{\varepsilon}). \tag{14}$$

This implies that the socially optimal amount of advertising is

$$\tilde{\varphi} = \frac{v + \frac{2}{3}\hat{\varepsilon}^3 + \hat{\varepsilon}(1-\hat{\varepsilon})}{a + v + \frac{2}{3}\hat{\varepsilon}^3 + \hat{\varepsilon}(1-\hat{\varepsilon})}.$$

Figure 3 shows the socially optimal fraction of informed consumers as a function of a and s. This confirms the comparative statics as derived in Proposition 2 – although the effect of s on $\tilde{\varphi}$ turns out to be weak.

Figure 4 compares the market equilibrium to the social optimum. Confirming Proposition 3, we see that for the uniform distribution and a given value of a, there is always a threshold level of $\tilde{s}$ such that there is overadvertising if and if $s > s$.

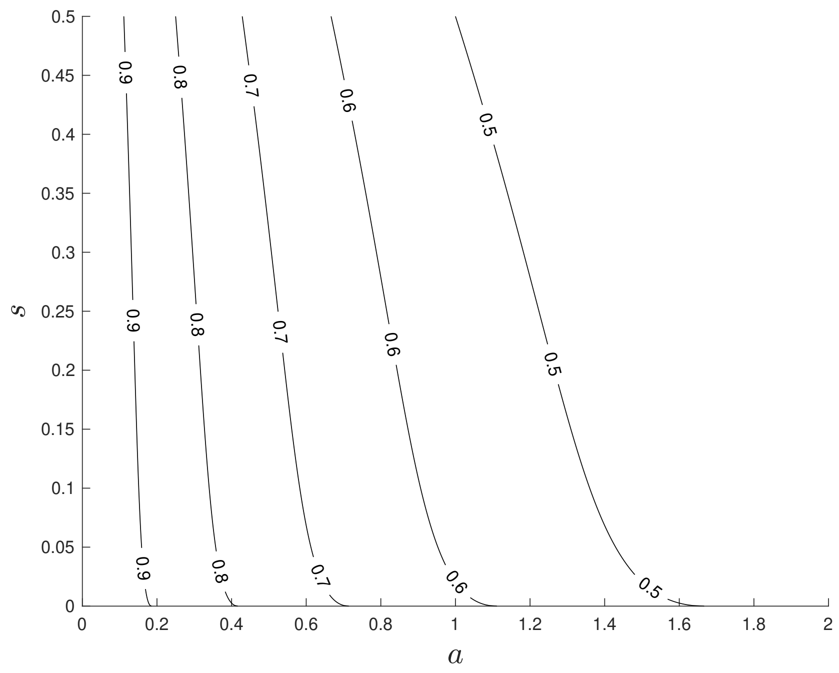

Figure 3: *Advertising intensity in the social optimum as function of a and s, with $v = 1$*

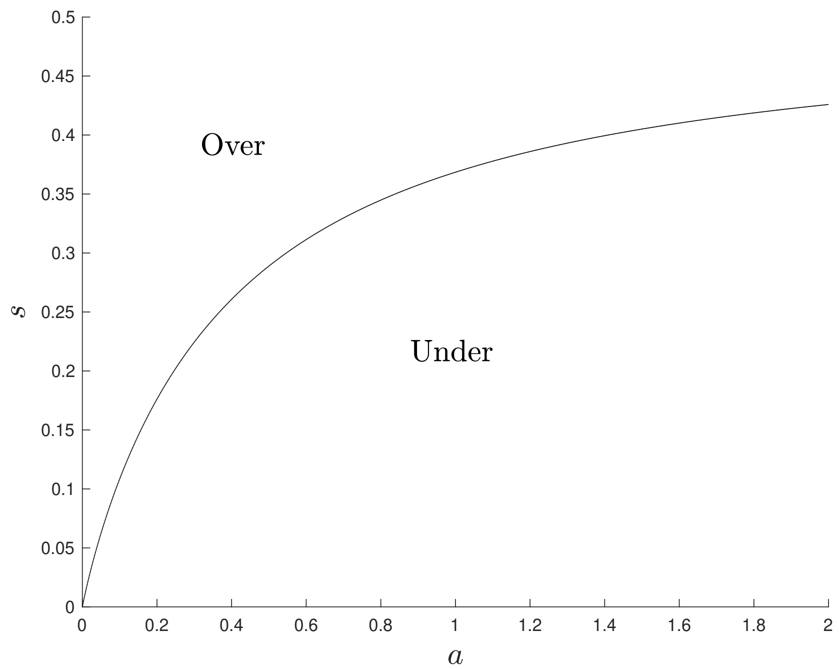

Figure 4: *Comparing advertising intensity in the market equilibrium and the social optimum as function of a and s, with $v = 1$. In the area labeled "over" there is too much advertising in the market equilibrium – in the are labeled "under" there is too little.*

6 Conclusion

In this paper, we studied a search market where two firms inform consumers of the existence of the new product by means of advertising. A consumer that observes the ad of a firm learns about the existence of the product — which may also trigger her to search the other firm. Firms may thus have an incentive to free ride on each other's advertising effort.

We find equilibrium advertising levels are increasing in search costs. Higher search costs imply higher prices, and make it less likely that a consumer also checks out the other firm. Both factors imply that consumers become more valuable, hence firms are willing to invest more to inform them. But the socially optimal levels of advertising are decreasing in search costs. Higher search costs imply that the social value of informing a consumer is lower, hence a social planner would be less willing to invest to inform them. Under some weak conditions we show that there is underadvertising for low search costs, but overadvertising for high search costs.

This paper is definitely work in progress. We plan to expand it further in the near future, along at least three lines. First, we will study the possibility of having more than 2 firms. Second, we will study the case in which the market is not fully covered - so an increase in the number of informed consumers does not necessarily increase social value. Third, we will consider the desirability of an cartel in advertising. Fourth, we will relax the assumption that being informed about one firm *always* makes consumers realize that there is also a second firm selling a similar product.

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