Shakeouts and Staggered Exits from an R&D Race between Externally-Funded Startups *

Samuel Häfner†

March 13, 2023

Abstract

This article analyzes a continuous-time R&D race with moral hazard. Cash-constrained, loss-making startups compete to access a profitable market requiring a stochastic breakthrough. Each startup is funded by a different early-stage investor. R&D efforts are non-verifiable, and the investors make mutually optimal incentive-compatible contract offers. A contract specifies the funding period, when the firm should undertake R&D, and the equity stake in case of market entry. The profitable market accommodates more than one firm. A strong early-mover advantage induces asymmetric funding deadlines, resulting in staggered exits from the race. If the early-mover advantage is less distinct, some deadlines are symmetric, leading to shakeouts. The comparative statics follow from an interplay of contractual externalities among investors and the particular cost of moral hazard. The model provides an explanation for asymmetries in VC contracts and for industry shakeouts.

Keywords: Continuous-Time Race, Multiple Prizes, Market Entry, External Funding, Moral Hazard, Multiple Principals.

JEL Classification Numbers: C73, D86, O31.

*I benefited from discussions with Stefan Bühler, Giacomo Calzolari, Florian Dendorfer, and Curtis Taylor. I greatfully acknowledge financial support from the Swiss National Science Foundation (Grant P300P1_167648) and thank the Economics Department at Duke for its hospitality.

†University of St. Gallen, School of Economics and Political Science, samuel.haefner@unisg.ch.

1 Introduction

Startup firms often rely on early-stage equity funding before turning profitable. The market for such funding is informal yet sizable. For example, the audit and advisory firm KPMG estimates global venture capital investments in 2021 to be $671 billion across roughly 40′000 deals. A typical contract between a firm and its early investor determines the duration, amount of funding, and the specifics of the investor's equity share if the firm is eventually sold or holds an IPO (Sohl, 1999).

A major issue for early investor-firm relations is moral hazard. Even though investors typically seek an active role in the firm they invest in, firm owners can always covertly divert some funding to their benefit (e.g., Kelly and Hay, 2003). Or, as David Cohen, co-founder of Techstars, bluntly put it:

> "Founders can live for a year or two on seed capital, have some fun and punch their lottery ticket. If things don't take off immediately, they can simply move on."[1]

The challenges that early-stage investors face are not restricted to the relation with their startup. Startups developing new business ideas typically compete with other startups over the entry into a common market. So, the contract an investor offers to a firm will also depend on how they expect the other firms to progress over time. But the other firms are likely funded by external parties, too, and their progress depends on the funding that they receive. Consequently, the investors' contract offers are part of an interdependent choice problem.

This article develops a model that captures this two-fold choice problem of investors. There are a finite number of exogenously given, symmetric firm-investor dyads. Investors make a take-it-or-leave-it contract offer to their cash-constrained firms, which then compete in a complete information, continuous-time R&D race for breakthrough opportunities. Once a firm takes up a breakthrough opportunity, it enters a profitable market, garners a payoff, and leaves the game. If doing so, the firm and its investor share the profit according to the contractual terms. Crucially, the dyads not only compete against each other for a breakthrough opportunity, but they also need to solve a hidden action problem concerning the costly R&D efforts on the part of the firm, which are necessary to produce a breakthrough opportunity.

The model gives rise to some intriguing equilibrium predictions. Most notably, depending on the market the firms aim for, some investors may keep their firms longer in the race than others. Put differently, both staggered exits and shakeouts are possible equilibrium outcomes.

[1] See the Financial Times article "The moral hazard created by buckets of start-up funding" from September 1, 2015.

Asymmetric contract offers square with empirical evidence on early-stage equity contracts, suggesting that there is quite some variation in the duration and the volume of funding. For example, Sohl (1999) and Mason and Harrison (2002) report funding periods that range from 2 to 8 years. They rationalize the variation by differences in the investors' ability to screen startup quality (cf. also Wiltbank et al., 2009). However, recent research suggests that early-stage investors might not be as good at picking successful projects as previously thought (Nanda et al., 2020; Davenport, 2022). So, my model provides a complementary explanation for asymmetric contracts which focuses on the interplay between competition and moral hazard rather than on asymmetric information.

My model might also provide a novel perspective on the phenomenon of industry shakeouts; i.e., the sudden consolidation of an industry or sector through simultaneous exit (cf., among others, Gort and Klepper, 1982; Klepper, 1996; Klepper and Simons, 2000; Lenox et al., 2007). The literature typically focuses on more mature markets than I have in mind for my model. Also, theoretical explanations for shakeouts tend to focus on the uncertainty about the evolution of the firms' production technologies (cf. also Hopenhayn, 1993; Cabral, 2011). Agency frictions are absent in these models. Nevertheless, the role of venture capital in industry shakeouts has been discussed prominently among practitioners in the past.[2]

The details of the model are as follows. A finite number of symmetric firms compete for market entry by obtaining a stochastically arriving breakthrough opportunity. The R&D technology is the same for all firms and produces breakthrough opportunities at a fixed and commonly known rate. Doing R&D is costly, and the firms can freely switch between periods of R&D and periods without R&D. Besides the variable R&D costs, the firms also incur a fixed flow cost from running daily business. Once they stop day-to-day business operations, the firms have to leave the race for good. Firms are cash-constrained, each relying on the flow of funding from a different investor. Firm-investor dyads are exogenously given and fixed throughout the race.

Daily business is contractible, yet doing R&D is not verifiable and subject to moral hazard. I model moral hazard by following Green and Taylor (2016) and assuming that the firm owners can covertly decide not to engage in R&D and instead divert part of the intended funds to their benefit.[3] Both investors and firm owners are risk-neutral and discount the future at the same rate. Firms are protected by limited liability. Investors have all the bargaining power and make take-it-or-leave-it contract offers. A contract specifies a (possibly infinite) deadline on the funding provided for daily business, the periods of R&D, as well as

[2] See, for example, the 1997 HBR article "Strategies for Surviving a Shakeout" by George Day (https://hbr.org/1997/03/strategies-for-surviving-a-shakeout.)

[3] Green and Taylor (2016) study a principal contracting with an agent to complete a two-stage project (Wolf, 2017; Moroni, 2019, add experimentation to this setup). Their one-stage benchmark corresponds to the single-dyad case in my setting with no discounting and no costs for daily business.

a (time-dependent) share of the profits from market entry left to the investor.

The market that the dyads compete for may ultimately accommodate more than one firm. Specifically, I assume the market consists of two sequentially available spots. Each spot comes with an exogenous, time-independent, and publicly known payoff, which we might interpret as the net present value of future firm profits. If a firm takes up a breakthrough opportunity, it collects this payoff, leaves the game, and the next spot becomes available. Races with multiple prizes are also analyzed in Denicolo (1996, 1999), who does not consider moral hazard.

There is limited commitment and sparse information about competitors. I assume the only public event during the race is a firm taking up a breakthrough opportunity. This assumption is motivated by the fact that the environment of young firms is often chaotic and dynamically evolving. Further, I assume that investors can commit to the terms of the contract only until the next firm takes up a breakthrough opportunity. That is, whenever new information arrives, then the contracts have to be renegotiated. In reality, contract renegotiation between early investors and their startups happens quite frequently (e.g., Bengtsson and Sensoy, 2015).

An equilibrium in this setting is a profile of mutually optimal contracts that satisfy limited liability and incentive compatibility of the firms' R&D decisions. Throughout, I restrict attention to equilibria in which investors ask their firms to do R&D whenever they are in the race. As I show, such contracts are mutually optimal if daily business operations are sufficiently costly.

Whether or not the equilibrium contracts are asymmetric depends on the market structure. The equilibrium contracts specify two relevant deadlines. The first deadline specifies the maximum duration of funding provided to race for the first spot. The second deadline specifies the maximum duration of funding to race for the second spot, provided some other firm has obtained the first spot. If the ratio of the first spot's value to that of the second spot exceeds a certain threshold, then the mutually optimal deadlines to obtain the first spot turn out asymmetric. This means that there are staggered exits from the race, provided no breakthrough opportunity arrives for a sufficiently long time. On the other hand, if the first spot is not too attractive relative to the second spot, then some deadlines are symmetric and simultaneous exits or a shakeout may occur.

The main reason for these comparative statics is two opposing effects that arise from moral hazard and multiple prizes. Moral hazard leads to a particular cost function for keeping the firm in the race. Multiple prizes imply that seizing on a breakthrough opportunity not only removes the current spot from the set of prizes but also enables competition for the second spot, thus generating an externality for the remaining dyads in the race.

If the relative value of the current spot is high, then it is always optimal for some investors to marginally increase the deadline over those of her opponents and to ask for a little more R&D from her firm. A more extended deadline implies a higher agency rent for the firm, yet it also increases the chances of obtaining the current spot. On the other hand, if the value of the second spot is relatively high, then an investor with a lower deadline is always incentivized to increase her deadline. Doing so increases the chances of obtaining the first spot and, more importantly, getting the second spot. Crucially, these gains are more significant than the additional agency cost only until the deadline matches that of her rivals, beyond which the second prize does not play a role at the margin.

The equilibrium predictions I obtain differ fundamentally from the equilibria in existing R&D races without moral hazard. In races where the R&D technology's hazard rate is commonly known, it is without loss to assume that firms continue until at least one of the firms obtains a breakthrough (Loury, 1979; Denicolo, 1996, 1999; Erkal and Scotchmer, 2009). Exit without breakthroughs can occur in non-stationary settings, e.g., when the hazard rate of the R&D technology is uncertain and agents experiment (Reinganum, 1981, 1982; Choi, 1991; Malueg and Tsutsui, 1997; Moscarini and Squintani, 2010; Awaya and Krishna, 2020). However, in the standard symmetric experimenting agent setting without private information, which is closest to the setting in this article, the agents' beliefs all decrease at the same rate and, hence, exit is symmetric (Choi, 1991).

My analysis relates to other areas of research. Naturally, moral hazard features prominently in existing models of early startup funding. For example, Elitzur and Gavious (2003) analyze the incentives of a single angel investor when a venture capitalist provides follow-up funding. Casamatta (2003) examines the double role of venture capitalists both as a source of funding and a source of advice. De Bettignies and Brander (2007) consider the problem of a firm deciding between bank finance and venture capital. Neither of these models analyzes the optimal funding duration nor considers competition between VC-backed firms. Drover et al. (2017) provide an exhaustive overview of the empirical and theoretical literature on different types of startup funding.

The literature on delegation in static contests is also related. In a series of articles, Kräkel (2002, 2004, 2005) and Kräkel and Sliwka (2006) analyze the incentives of owner-manager dyads that compete for market shares. In particular, Kräkel (2005) finds asymmetric equilibria among symmetric players for the rank-order tournament. The motive for choosing higher efforts is preemption, which is quite different from the motive that I find in my dynamic model. Another important analysis of delegation in contests is by Wärneryd (2000), who considers a litigation setting. In a similar model, Konrad et al. (2004) shows that the delegation equilibrium is symmetric for the special case of an all-pay auction, even though players may be asymmetric.

Another model often used to analyze dynamic competition is the war of attrition. Under complete information, the war of attrition typically has a multitude of asymmetric pure strategy equilibria. Due to the discontinuity in the payoffs, there cannot be a deterministic exit at intermediate times, though; i.e., firms always either concede right at the beginning or very late (Hendricks et al., 1988). Exit at intermediate times only happens in (symmetric) mixed strategy equilibria or in settings that are non-stationary (Georgiadis et al., 2020) or feature incomplete information (Bulow and Klemperer, 1999).

2 The Model

Time is continuous and denoted by $t \geq 0$. There are $n \geq 2$ firm-investor dyads labeled $i = 1, , ..., n$. Firm-investor dyads are exogenously given and remain fixed throughout the game. Firms have the technological expertise for R&D, but they lack funding; investors can provide funding, but they lack the required expertise to run the technology themselves.

2.1 R&D and Prizes

The firms are not yet profitable, and daily business comes at a flow cost of $\theta > 0$. The firms can engage in R&D at an additional flow cost of $c > 0$, which produces random breakthrough opportunities that allow them to enter a profitable market. While a firm can freely switch between periods with R&D and periods without R&D, it has to leave the game for good once it stops running daily business. If a firm engages in R&D, it obtains breakthrough opportunities at a rate $\lambda > 0$. Breakthrough opportunities arrive independently among firms. If a firm does not engage in R&D, it obtains no breakthrough opportunities.

If a firm obtains a breakthrough opportunity, it can take it up and enter the profitable market or discard it. Discarded breakthrough opportunities depreciate immediately. The profitable market can ultimately accommodate two firms. The time-independent monetary value of entering first is $\Pi_1 \geq 0$, and the value of entering second is $\Pi_2 \geq 0$. It seems reasonable to assume $\Pi_1 > \Pi_2$; yet the model also allows for strong second-mover advantages involving $\Pi_1 \leq \Pi_2$.[4] Throughout, I use $\sigma \in \{1, 2\}$ to index the values of market entry, Π_σ. If a firm is the σ-th firm to enter the profitable market, it receives Π_σ and leaves the race.

[4]One way to rationalize decreasing prizes is to assume that, after the first breakthrough, the winner enjoys monopoly profits until the second breakthrough arrives. One way to rationalize increasing prizes is to posit that every new entrant in the market attracts a growing number of additional customers. In any case, determining the prizes in a full-fledged model of the profitable market would require taking the expected arrival times of breakthroughs into account. As these arrival times are endogenous, this would add another layer of complexity to the analysis, from which I abstract in the model.

2.2 Moral Hazard

The firm owners lack the required capital both to run daily business and to engage in R&D. Instead, they rely on the funding provided by their respective investors. The investors have all the bargaining power and make simultaneous take-it-or-leave-it contract offers, which I specify below. Firm owners are protected by limited liability. The firm owners and the investors are risk-neutral and discount the future at a rate $\rho > 0$.

Within the dyads, the investor can verify whether the firm runs daily business operations and whether a breakthrough opportunity arrives, yet there is moral hazard concerning the R&D decision.[5] Specifically, if provided with the required means for R&D, the firm owner can covertly decide not to undertake R&D but rather divert a positive flow $\phi \leq c$ for personal benefits. That is ϕ measures the severity of the moral hazard problem. If $\phi = 0$, then the owner does not gain anything from shirking, whereas $\phi = c$ means that the owner can fully divert the funding for R&D. Throughout, I assume

$$\lambda \Pi_\sigma - (c + \theta + \phi) > 0, \ \forall \sigma. \tag{A}$$

As we will see, Assumption (A) ensures that funding the firm at least for some time is optimal for every investor and, hence, that all investors offer a contract in equilibrium.

2.3 Contracts

Whether or not firm owners accept their contract offer is publicly observed. The only other publicly-observed event during the race is a firm taking up a breakthrough opportunity and thus leaving the game. The actual terms of the contract between investor and firm, any decision to stop daily business without a breakthrough opportunity, and the actual arrivals of breakthrough opportunities are unobserved outside a dyad. Further, investors possess limited commitment power and can commit to the contract terms only up to the opponent firm taking up a breakthrough opportunity.

As we will see, it is thus without loss to restrict attention to contracts that only condition on a public state $s = (\sigma, N)$, where σ corresponds to the index of the current spot in the market and $N \subseteq \{1, ..., n\}$ corresponds to the subset of dyads starting in that state.[6]

The contract prescribes for any time in any relevant state whether to invest in daily business, engage in R&D, take up a breakthrough opportunity if it arrives, and any transfers

[5] Truthful reporting of breakthrough arrivals is not an issue. In Section 5.1, I briefly discuss an extension in which firms can hide arrivals from the investor. It turns out that incentive compatibility always induces the right incentives for truthful reporting.

[6] The reason is that the mutually optimal contract when $\sigma = 2$ is unique and, hence, by limited commitment, independent of calendar time. With more than two prizes, the state space would be more involved (cf. the discussion in Section 5.3).

between the firm and the investor. A couple of observations about the optimal contracts follow directly from the environment described so far.

1. As the firms have to leave the game for good when funding for daily business dries up, it is without loss to assume that any contract entails a deadline up to which funding for daily business operations is provided, where the deadline may be infinite.

2. Funding for daily business equals θ because a higher payment is unnecessary. For the same reason, the additional flow payment to the firm when asked to engage in R&D equals c.

3. Because the firm lacks cash, the only way to compensate the investor is to use an equity stake she retains if the firm eventually takes off.

4. Because the investor can always suspend the funding for R&D when she deems entering the profitable market currently not worthwhile and because breakthrough opportunities depreciate immediately if not taken up, it is without loss to assume that breakthrough opportunities must always be taken up when they arrive.

Let $S_i = \{s = (\sigma, N) \in S : i \in N\}$ be the set of states where dyad i is active. The relevant parts of the contract offered by investor i can thus be described by a triple

$$C_i = \{T_i, R_i, a_i\},$$

which consists of a deadline profile $T_i = \{T_{is}\}_{s \in S_i}$, a transfer profile $R_i = \{R_{is}\}_{s \in S_i}$, and an action profile $a_i = \{a_{is}\}_{s \in S_i}$. The deadlines $T_{is} \geq 0$ specify the time to which funding in state s is provided. The transfer functions $R_{is} : \mathbb{R}_+ \to \mathbb{R}$ return the transfer $R_{is}(t)$ from the firm to the investor (her equity stake) in case the firm obtains a breakthrough opportunity at $t \geq 0$. And the action functions $a_{is} : \mathbb{R}_+ \to \{0, 1\}$ indicate whether the firm should do R&D ($a_{is}(t) = 1$) at time t or not ($a_{is}(t) = 0$).[7] Throughout, I take the transfer functions $R_{is}(t)$ and the action functions $a_{is}(t)$ to be measurable. Both the transfer functions and the action functions satisfy $R_{is}(t) = a_{is}(t) = 0$ for $t > T_{is}$. Further, I adopt the convention that at the beginning of each state, the clock is set back to zero; i.e., each state s starts anew at time $t = 0$.

To summarize, the timing and the information are as follows:

$\tau = 1$: Each investor i offers a contract $C_i = \{T_i, R_i, a_i\}$ to firm owner i, who then decides whether to accept. Contract offers are only observed within the respective firm-investor dyad. Acceptance decisions are publicly observed.

[7]Mixing — i.e., $a_{is}(t) \in [0, 1]$ — could, in principle, be integrated into the analysis. Yet, it is unclear how a contract that asks the firm to mix should be implemented in practice. For the sake of exposition, I thus refrain from modeling mixing.

$\tau = 2$: The firms who accepted the contract in $\tau = 1$ enter the race, which starts at date $t = 0$ in state $s = (1, N)$ with $N \subseteq \{1, ..., n\}$. The actual arrivals of breakthrough opportunities and any exit before a breakthrough opportunity arrives are only observed within the respective firm-investor dyads. R&D decisions are only observed by the respective firms. If a breakthrough opportunity is taken up at some $t \geq 0$ by firm $j \in N$ and $|N| \geq 2$, then the clock is set back to $t = 0$, and the game continues in (public) state $s = (2, N \setminus j)$. If just one firm accepted its contract, $|N| = 1$, then the game ends after it takes up the first breakthrough opportunity.

As the game is one of complete information, all dyads of course correctly anticipate the contracts and decisions by the other dyads on the equilibrium path.

2.4 Payoffs and Equilibrium

For a state $s = (\sigma, N) \in S_i$, write $U_{is}(t; \hat{a}_{is}, C_i, C_{-i})$ for the utility of firm owner i in state $s \in S_i$ at time t who is given a contract $C_i = \{T_i, R_i, a_i\}$ and chooses a (given the funding of the investor, feasible) action $\hat{a}_i$ when the other firms adhere to their contracts in the profile $C_{-i} = (C_1, ..., C_{i-1}, C_{i+1}, ..., C_n)$. The utility $U_{is}(t; \hat{a}_i, C_i, C_j)$ is obtained by integrating the product of the flow payoff at a point in time τ when choosing $\hat{a}_{is}(\tau)$ and the (discounted) probability that no breakthrough opportunities have arrived between t and τ. Letting $N_s(t; C)$ be the set of dyads still in the race at t, we have

$$
\begin{aligned}
U_{is}(t; \hat{a}_i, C_i, C_{-i}) = \int_t^{T_{is}} \Bigg[& a_{is}(\tau) \Big[\hat{a}_{is}(\tau) \lambda (\Pi_\sigma - R_{is}(\tau)) + (1 - \hat{a}_{is}(\tau)) \phi \Big] \\
& + \lambda \sum_{j \in N_s(\tau; C) \setminus i} a_{js}(\tau) U_{i(\sigma+1, N_s(\tau; C) \setminus j)}(0; \hat{a}_i, C_i, C_{-i}) \Bigg] \\
& \times e^{-\int_t^{\tau} \left[(a_{is}(\hat{\tau}) \hat{a}_{is}(\hat{\tau}) + \sum_{j \in N_s(\hat{\tau}; C) \setminus i} a_{js}(\hat{\tau})) \lambda + \rho \right] \mathrm{d}\hat{\tau}} \, \mathrm{d}\tau, \quad \text{(U)}
\end{aligned}
$$

with $U_{i(3,.)}(.) = 0$. The flow payoff from being in the race at some point τ consists of the current flow from racing plus the expected continuation (flow) payoff from the contract if an opponent takes up a breakthrough opportunity.

Incentive compatibility of a contract C_i prescribing action a_i requires that for all states $s \in S_i$, for all times $t \in [0, T_{is}]$, and for all (given the funding of the investor, feasible) action profiles $\hat{a}_i \neq a_i$,

$$U_{is}(t; a_i, C) \geq U_{is}(t; \hat{a}_i, C), \quad \text{(IC)}$$

where $C = (C_i, C_{-i})$. Further, limited liability requires that the transfer from the firm to the investor cannot exceed the value of the breakthrough opportunity; i.e., for all dates t and states $s \in S_i$ it must hold

$$R_{is}(t) \leq \Pi_\sigma. \tag{LLC}$$

Analogous to the firm owner utility, let $V_{is}(t; C_i, C_{-i})$ be the value of the incentive compatible contract C_i to investor i when she is still in the race at time t in state $s \in S_i$ and the other investors offer incentive-compatible contracts C_{-i}. We have

$$\begin{aligned} V_{is}(t; C_i, C_{-i}) = \int_t^{T_{is}} \Bigg[& a_{is}(\tau)[\lambda R_{is}(\tau) - c] - \theta \\ & + \lambda \sum_{j \in N_s(\tau;C)\setminus i} a_{js}(\tau) V_{i(\sigma+1, N_s(\tau;C)\setminus j)}(0; C_i, C_{-i}) \Bigg] e^{-\int_t^{\tau} \left[\sum_{j \in N_s(\hat{\tau};C)} a_{js}(\hat{\tau})\lambda + \rho\right] \mathrm{d}\hat{\tau}} \mathrm{d}\tau, \end{aligned} \tag{V}$$

with $V_{i(3,.)}(.)) = 0$. Then, an equilibrium is a profile of feasible contracts that are mutually optimal at the beginning of every state s. I only consider pure strategy Nash equilibria.

Definition 1 (Equilibrium)**.** *An equilibrium is a profile of contracts $C^* = (C_1^*, ..., C_n^*)$ satisfying* (LLC) *and* (IC) *such that, for all investors $i \in \{1, ..., n\}$ and all states $s \in S_i$, it holds*

$$V_{is}(0; C_i^*, C_{-i}^*) \geq V_{is}(0; C_i, C_{-i}^*),$$

for all $C_i \neq C_i^*$ that also satisfy (LLC) *and* (IC)*.*

Throughout, I restrict attention to what I call *full-R&D* contracts. Under such contracts, the firms always engage in R&D when in the race. Formally,

Definition 2 (Full-R&D Contract)**.** *A full-R&D contract is a contract C_i such that $a_{is}(t) = 1$ holds for all $t \in [0, T_{is}]$ and all states $s \in S_i$.*

Full-R&D contracts may not be mutually optimal when daily business is too cheap (i.e., θ is too low), the future is hardly discounted ($\rho \approx 0$), or entering second provides a considerably higher payoff than entering first ($\Pi_2 \gg \Pi_1$). In the following analysis, I first determine the mutually optimal contract profile when investors are restricted to offering full-R&D contracts. In the second step, I present a sufficient condition for the mutual optimality of offering full-R&D contracts.

3 Mutually Optimal Deadlines

Because investors can commit to the terms of their contract only up to the first breakthrough, we can proceed backward. In the following, we will thus first determine the optimal continuation contract for the states where firms compete for the second prize, $\sigma = 2$. Using the continuation payoffs from these states, we then solve for the mutually optimal contract in the states s with $\sigma = 1$.

The necessary tools to find and characterize the mutually optimal contract offers are developed in Appendix A. As is standard for models with dynamic moral hazard, incentive compatibility allows us to substitute the payments R_{is} from the utility functions of the firm. This then gives us investor utility $V_{is}(0; C)$ under the cost-minimizing transfers as a function $V_{is}^*(T)$ of the deadline profile $T = (T_{is}, \{T_{js}\}_{j \in N \setminus i})$ in state s alone. The main contribution in Appendix A is to delineate a link between the structure of $V_{is}^*(T)$ and the set of mutually optimal deadlines. Section 3.2 below provides more details.

3.1 The Race for the Second Prize

The following result characterizes the mutually optimal deadlines and the resulting continuation payoffs in states where the dyads race for the second spot. The statement restricts investors to full-R&D contracts as defined in Definition 2 above. At the end of this section, Proposition 3 then gives a condition under which an equilibrium in this class of contracts exists. As with all the results in this section, the proof is in Appendix B.

Proposition 1. *In any state $s = (\sigma, N)$ involving $\sigma = 2$, the optimal full-R&D contract offers are symmetric between dyads. All investors offer a deadline T_2^*, which is independent of the number of opponent dyads and given by*

$$T_2^* = \frac{1}{\lambda} \ln \left(\frac{\lambda \Pi_2 - (c + \theta)}{\phi} \right). \tag{1}$$

Under the optimal contract, the firms' utilities, $U_{2,|N|}^*$, and entrepreneurs' utilities, $V_{2,|N|}^*$, from entering state s depend on the number of dyads, $|N|$, and are given by

$$U_{2,|N|}^* = \frac{\phi}{\lambda(|N| - 1) + \rho} \left[1 - e^{-(\lambda(|N|-1)+\rho)T_2^*} \right] \tag{2}$$

$$V_{2,|N|}^* = \frac{\lambda \Pi_2 - (c + \theta)}{\lambda |N| + \rho} \left[1 - e^{-(\lambda|N|+\rho)T_2^*} \right] - U_{2,|N|}^*. \tag{3}$$

In the optimal contract, the respective firms' incentive compatibility constraints bind and, hence, firm utility (2) is equal to the expected net present value of shirking under the deadline T_2^*. The expression for investor utility in (3) corresponds to the difference between total dyad welfare and firm utility.

The optimal deadline (1) is independent of the number of dyads, $|N|$, and the discount factor, ρ. This independence may appear counterintuitive, yet the reason for it is pretty simple. Without a further spot to race for, the number of opponents only affects the effective discount rate but not the expected flow payoff from being in the race for the investors. Yet, the effective discount rate does not play a role at the margin when comparing the marginal costs and the marginal benefits of changing the deadline.

Further, note that the statement is silent on the equilibrium payments from firms to investors in case of market entry, R_s^*. After solving the full contract, I provide a discussion of the payments in Section 5.1. For the case of $\sigma = 2$, we will find that R_s^* is increasing in t; i.e., the investor's stake in the firm increases over time. This is a standard result (Green and Taylor, 2016); the closer the firm is to the deadline, the less continuation payoff the investor has to leave to the firm to prevent them from shirking.

Finally, some comparative statics of the optimal deadline are apparent. If the cost of racing, $c + \theta$, increases, the investor has less to gain, and the deadline becomes shorter. On the other hand, if the value of the spot, Π_2, increases or if the agency problem becomes less severe (i.e., ϕ decreases), then there is more to gain for the investor and the deadline T_2^* increases. Indeed, if ϕ vanishes, then T_2^* diverges to infinity, which shows that finite deadlines are solely due to moral hazard.

As regards λ, Assumption (A) gives us that the deadline T_2^* increases for low λ, is unimodal, and approaches zero as λ diverges to infinity. The reason is that an increase in the arrival rate of the breakthroughs has two opposing effects. On the one hand, it increases the per-period value of racing, which would call for a more extended deadline. On the other hand, observing no breakthrough for a given period becomes more indicative of shirking, which would call for a shorter deadline. For low values of λ, the first effect dominates, whereas the second effect dominates for high values.

3.2 The Race for the First Prize

We now turn to the states where the firms race for the first prize, $\sigma = 1$. The characterization of the mutually optimal deadlines is more involved than in the last section, because the continuation payoffs depend on the number of dyads that move on to the next state.

I proceed in several steps. The first step, discussed in detail in Appendix A.1, characterizes the marginal utility of investor i under the cost-minimizing transfers for a deadline profile $T = (T_{is}, \{T_\kappa\}_{\kappa=1}^{|N|-1})$. The opponent deadlines are collected such that $\{T_\kappa\}_{\kappa=1}^{|N|-1}$ decreases in κ. The particular dyads' identities corresponding to these deadlines do not matter for investor i, because all players are symmetric, and the mutual optimal play in the continuation games (i.e., the race for prize $\sigma = 2$) is independent of the dyad's identities taking part.

Lemma 1. *Fix a dyad i and a state $s = (\sigma, N) \in S_i$ with $\sigma = 1$. Let $V_{is}^*(T)$ be investor utility* *under the cost-minimizing transfers for deadlines* $T = (T_{is}, \{T_\kappa\}_{\kappa=1}^{|N|-1})$*, where* $\{T_\kappa\}_{\kappa=1}^{|N|-1} = \{\emptyset\}$ *if* $|N| = 1$ *and the elements of* $\{T_\kappa\}_{\kappa=1}^{|N|-1}$ *decrease in* κ *otherwise. Further, let* $T_0 = \infty$ *and* $T_{|N|} = 0$*. Then, for all* $\kappa \in \{1, .., |N|\}$ *such that* $T_\kappa < T_{\kappa-1}$*,*

$$sgn\left(\frac{\partial V_{is}^*(T)}{\partial T_{is}}\bigg|_{T_{is}\in[T_\kappa, T_{\kappa-1})} \right) = sgn\Bigg(\left[\lambda\Pi_1 - (c+\theta) + \lambda(\kappa-1)\left[V_{2,\kappa-1}^* + U_{2,\kappa-1}^*\right]\right] e^{-\lambda T_{is}} - \left[\phi + \lambda(\kappa-1)U_{2,\kappa-1}^*\right] \Bigg). \quad (4)$$

The expression in the sign function on the right side in (4) is proportional to the difference between the marginal gain of extending the deadline and the marginal cost of doing so. Other than in the race for the second prize, the number of dyads still in the race matters for the incentives. The marginal benefit of staying in the race increases in the flow benefit from racing for the first spot, $\lambda\Pi_1 - (c + \theta)$, and the dyad welfare that accrues in expectation when racing for the second spot together with $\kappa - 2$ other dyads, $V_{2,\kappa-1}^* + U_{2,\kappa-1}^*$. The marginal cost of doing so is determined by the shirking payoff, ϕ, plus the continuation utility that the firm obtains, $U_{2,\kappa-1}^*$, in case a competitor takes the first spot. The firm must be compensated for both to follow the contract.

Observe that the argument in the sign function on the right side in (4) strictly decreases in T_{is}. This implies that the utility of the investor is unimodal on any (proper) interval $[T_\kappa, T_{\kappa-1})$. As I discuss in Appendix A.2, we can characterize the set of mutually optimal deadline profiles from the relation of the functions' roots that we obtain by extending the argument on the right side in (4) to the positive real line.[8] For the following, observe that these roots are given by $\ln(G_\kappa)/\lambda$, where

$$G_\kappa \equiv \frac{\Pi_1 - \dfrac{c+\theta}{\lambda} + (\kappa-1)\left[V_{2,\kappa-1}^* + U_{2,\kappa-1}^*\right]}{\dfrac{\phi}{\lambda} + (\kappa-1)U_{2,\kappa-1}^*}. \quad (5)$$

To characterize the mutually optimal deadlines, I let $\mathcal{T}_s$ be set of all mutually optimal deadline profiles $\{T_\kappa^*\}_{\kappa=1}^{|N|}$ in state s that are decreasing in the index κ. Formally,

Definition 3 (Set of Weakly Decreasing Mutually Optimal Deadline Profiles, $\mathcal{T}_s$)**.** *Let* $\mathcal{T}_s$ *be the set of all deadline profiles* $\{T_\kappa^*\}_{\kappa=1}^{|N|}$ *in state* s *for which the following holds:*

(a) *If* $|N| \geq 2$*, then they are decreasing:* $T_{\kappa-1}^* \geq T_\kappa^*$ *for all* $\kappa \in \{2, ..., |N|\}$*.*

[8] In Section 4 below, I give some intuition for the approach by considering a two-dyad case (Remark 2).

(b) *They are mutually optimal: For every $\kappa \in \{1, ..., |N|\}$, it holds*

$$T_\kappa^* \in \arg\max_{T_i} V_{is}(T_i, \{T_{\hat{\kappa}}^*\}_{\hat{\kappa}\in\{1,...,|N|\}\setminus i}).$$

Except for the case with two dyads (cf. Section 4 below), explicit solutions for the mutually optimal deadlines are not feasible for general levels of moral hazard, $\phi > 0$. So, for the main result in this section, I focus on the case when the agency problem vanishes, $\phi \to 0$. For the following, let

$$\Phi \equiv \frac{\lambda\Pi_1 - (c+\theta)}{\lambda\Pi_2 - (c+\theta)}. \tag{6}$$

When ϕ vanishes, the optimal deadlines diverge to infinity. However, the degree of deadline asymmetry as we approach the limit profile varies in Φ. We have the following characterization.

Proposition 2. *Fix a state $s = (\sigma, N)$ with $\sigma = 1$ and let $\{T_\kappa^*\}_{\kappa=1}^{|N|} \in \mathcal{T}_s$. The mutually optimal deadlines satisfy*

$$\lim_{\phi\to 0} T_\kappa^* = \infty$$

for all κ. Further, suppose $|N| \geq 2$ and $\rho > \lambda$. Then, for every $\phi > 0$ sufficiently close to zero:

(a) *If $\Phi < \rho/(\rho+\lambda)$, then any profile $\{T_\kappa^*\}_{\kappa=1}^{|N|} \in \mathcal{T}_s$ is completely symmetric,*

$$T_1^* = T_2^* = ... = T_{|N|}^* \in \left[\frac{1}{\lambda}\ln(G_1), \frac{1}{\lambda}\ln\left(G_{|N|}\right)\right],$$

where $\frac{1}{\lambda}\ln\left(G_{|N|}\right) \geq \frac{1}{\lambda}\ln(G_1)$.

(b) *If $\Phi > (\rho+\lambda)/(\rho-\lambda)$, then there is a unique profile $\{T_\kappa^*\}_{\kappa=1}^{|N|} \in \mathcal{T}_s$, which is completely asymmetric,*

$$T_1^* > T_2^* > ... > T_{|N|}^*,$$

where $T_\kappa^* = \frac{1}{\lambda}\ln(G_\kappa)$ for all $\kappa \in \{1, ..., |N|\}$.

(c) *For a.e. $\Phi \in (\rho/(\rho+\lambda), (\rho+\lambda)/(\rho-\lambda))$, there are natural numbers $\underline{k}, \overline{k} \leq |N|$ satisfying $\overline{k} \geq 2$ and $\underline{k} \leq \overline{k}$ such that, for any number $\ell \in \{\underline{k}, \underline{k}+1, ..., \overline{k}\}$, there is a profile $\{T_\kappa^*\}_{\kappa=1}^{|N|} \in \mathcal{T}_s$ satisfying*

$$T_1^* > T_2^* > ... > T_\ell^* = T_{\ell+1}^* = ... = T_{|N|}^* \in \left[\frac{1}{\lambda}\ln(G_\ell), \frac{1}{\lambda}\ln\left(G_{|N|}\right)\right], \tag{7}$$

where $T_\kappa^* = \frac{1}{\lambda}\ln(G_\kappa)$ for all $\kappa \in \{1, ..., \ell-1\}$ and $\frac{1}{\lambda}\ln\left(G_{|N|}\right) \geq \frac{1}{\lambda}\ln(G_\ell)$. Moreover, to any profile $\{T_\kappa^*\}_{\kappa=1}^{|N|} \in \mathcal{T}_s$ there is a number $\ell \in \{\underline{k}, \underline{k}+1, ..., \overline{k}\}$ such that (7) holds.

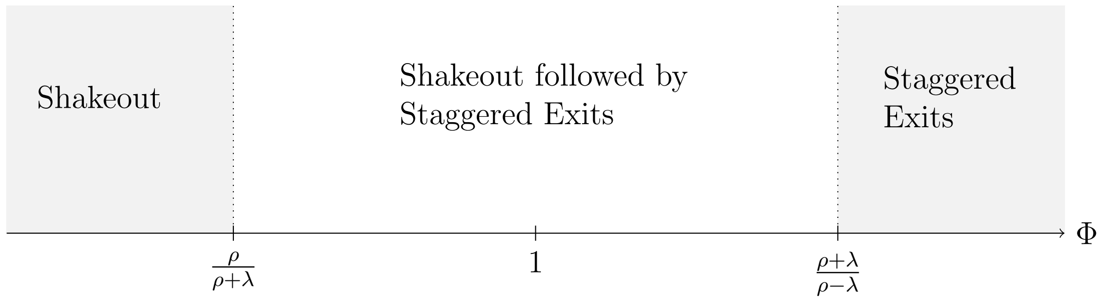

Figure 1: *Mutually optimal deadlines in states with $\sigma = 1$. For low Φ, the deadlines are symmetric in any mutually optimal deadline profile, implying a shakeout. For high values of Φ, the deadlines are always asymmetric, implying staggered exits. For intermediate values, staggered exits follow a one-time shakeout involving a subset of the dyads.*

Both $\underline{k}$ and $\overline{k}$ are non-decreasing in Θ. For any relevant Φ, $\overline{k} < |N|$ holds whenever $|N|$ is sufficiently large; if $\overline{k} < |N|$ holds, then $\overline{k}$ does not change when increasing $|N|$.

If the first spot is of relatively low value, then the mutually optimal deadlines are symmetric. In this case, which is stated in (a) of Proposition 2, there are multiple mutually optimal deadlines that the dyads can coordinate on. Conversely, the optimal deadlines are asymmetric if the first spot is of high value. In this case, as stated in (b), the deadline profile is unique up to permutations of the dyads.

For generic intermediate values of Φ, the statement in (c) says we may have a multiplicity of asymmetric mutually optimal deadline profiles.[9] These deadline profiles all have a similar structure: provided that no breakthrough occurs, we first see a shakeout, when some of the firms leave at once, and then staggered exits by the remaining ones. The "size" of the shakeout, i.e., the number of firms leaving at the same time, as well as the time until a shakeout occurs can vary between the different mutually optimal deadline profiles. Figure 1 shows the different ranges for Φ.

The lower and the upper bound on the number of possible distinct deadlines, $\underline{k}$ and $\overline{k}$, increase in the value of the first spot and decrease in the value of the second spot. The higher the first-mover advantage in the market, the higher the asymmetry in the funding horizons among investors. The last part of statement (c) says that when the number of dyads is large enough, there will be a shakeout for a.e. $\Phi \in (\rho/(\rho+\lambda), (\rho+\lambda)/(\rho-\lambda))$. Moreover, if we hypothetically add a new dyad to the game, then the lowest number of firms in a shakeout across all possible mutually optimal contract profiles increases by one.

Remark 1. *Proposition 2 only covers the case of a high discount factor, $\rho > \lambda$. As shown*

[9] The strategy of the proof requires that the elements in $\{G_\kappa\}_{\kappa=1}^{|N|}$ be distinct, which might fail for non-generic values of Π_1 and Π_2 — hence the qualification w.r.t Φ in the statement of Part (c).

in the Appendix C, asymmetry is limited to at most two different deadlines when $\rho \leq \lambda$. Specifically, I find that the equilibrium deadline profile is entirely symmetric for low values of Π_1. For high values of Π_1, there are $|N| - 1$ investors choosing the same deadline and one investor choosing a strictly longer deadline. For intermediate values, both profiles are mutually optimal.

3.3 Equilibrium Existence

To complete the analysis, it remains to show that an equilibrium in full-R&D contracts exists. Technically, existence of an equilibrium in the class of full-R&D contracts follows directly from the characterization results that I obtain in Appendix A. Condition (8) in the following statement then ensures that doing R&D is indeed the optimal choice for any investor when funding the firm.

Proposition 3. *Suppose it holds for both $\sigma \in \{1, 2\}$ that*

$$\Pi_\sigma - \frac{c+\phi}{\lambda} \geq \frac{\lambda}{\lambda+\rho}\left[\max_{\sigma' \geq \sigma} \Pi_{\sigma'} - \frac{c+\theta}{\lambda}\right]. \tag{8}$$

Then, an equilibrium in full-R&D contracts exists. In particular, any equilibrium is in full-R&D contracts, and all dyads $i = 1, ..., n$ are active at the beginning of the race.

For $\sigma = 1$, condition (8) holds whenever $\rho > 0$ and Π_1 is sufficiently high or when ρ is sufficiently high. A high ρ and a high Π_1 make the first prize attractive (relative to the second prize), such that doing R&D is worthwhile. Moreover, the condition holds for $\sigma = 2$ if $\phi \leq \theta$ or if ρ is high enough. If staying in the race is sufficiently expensive, delaying R&D for the second spot is never attractive.

For sufficiently low $\phi \geq 0$, as assumed for Proposition 2, then the condition in (8) always holds for $\sigma = 2$, because $\theta > 0$. For $\sigma = 1$, the condition holds whenever $\Pi_1 \geq \Pi_2$. And for $\Pi_1 < \Pi_2$, it can be rewritten as

$$\Phi \geq \frac{\lambda}{\lambda+\rho}\frac{\lambda\Pi_1 - (c+\theta)}{\lambda\Pi_1 - c}.$$

So, because the second fraction on the right is strictly below one and $\rho > \lambda$ in Proposition 2, there are values of Φ in any of the three ranges depicted in Figure 1 for which an equilibrium in full-R&D contracts exists. For $\rho < \lambda$, a sufficiently high θ ensures that full-R&D equilibria exist for all the ranges described in Proposition C.5.

4 The Incentives in the Two-Dyad Case

In this section, we look at the two-dyad case, $n = 2$. This case allows us to discuss the intuition for the varying deadline profiles in some more depth. With two dyads, we have the following set of states

$$S \equiv \{(1, \{1, 2\}), (1, \{1\}), (1, \{2\}), (2, \{1\}), (2, \{2\})\}.$$

We will focus on the state $s = (1, \{1, 2\})$, where both firms are present and race for the first spot.

4.1 Asymmetric vs. Symmetric Deadlines

Let us consider the incentives of investor i. If her deadline is below that of the opponent, $T_{is} \leq T_{js}$, then investor i has to consider both the presence of the other firm and the agency problem with her firm. When $T_{is} > T_{js}$, then only the agency problem matters. Formally, combining the result from Lemma 1 with the continuation payoffs obtained in Proposition 1, we get that under the cost-minimizing transfers, the marginal gain from increasing the deadline satisfies,

$$\operatorname{sgn}\left(\frac{\partial V_{is}^*(T)}{\partial T_{is}}\right) = \begin{cases} \operatorname{sgn}\Bigg(\left[\Pi_1 - \frac{c+\theta}{\lambda} + \frac{\lambda}{\lambda+\rho}\left[\Pi_2 - \frac{c+\theta}{\lambda}\right] \times \left[1 - e^{-(\lambda+\rho)T_2^*}\right]\right] e^{-\lambda T_{is}} - \left[\frac{\phi}{\lambda} + \frac{\phi}{\rho}\left[1 - e^{-\rho T_2^*}\right]\right]\Bigg) & \text{if } T_{is} < T_{js} \\ \operatorname{sgn}\left(\left[\Pi_1 - \frac{c+\theta}{\lambda}\right] e^{-\lambda T_{is}} - \frac{\phi}{\lambda}\right) & \text{if } T_{is} \geq T_{js}. \end{cases} \tag{9}$$

As already observed after Lemma 1, the arguments in the sign function on the right in (9) are monotone in T_{is} and, thus, the marginal gain from increasing the deadline has at most one sign change in T_{is} on either of the intervals $[0, T_{js})$ and $[T_{js}, \infty)$. Writing (T_{1s}^*, T_{2s}^*) for a profile of mutually optimal deadlines, it is immediate that the necessary and sufficient conditions for $T_{1s}^* = T_{2s}^*$ are

$$\lim_{\hat{T}_{i1} \uparrow T_{js}^*} \left.\frac{\partial V_{is}^*(T)}{\partial T_{is}}\right|_{\substack{T_{is}=\hat{T}_{i1} \\ T_{js}=T_{js}^*}} \geq 0 \text{ and } \left.\frac{\partial V_{js}^*(T)}{\partial T_{js}}\right|_{\substack{T_{is}=T_{js}^* \\ T_{js}=T_{js}^*}} \leq 0, \ \forall i \neq j \in \{1, 2\}. \tag{10}$$

There are multiple symmetric, mutually optimal deadlines that the investors can coordinate on. Specifically, from (9), we obtain that the symmetric mutually optimal deadlines $T^*_{1s} = T^*_{2s}$ must lie in the interval $[\underline{T}, \overline{T}]$, where

$$\underline{T} = \frac{1}{\lambda} \ln \left(\frac{\lambda \Pi_1 - (c+\theta)}{\phi} \right) \tag{11}$$

and

$$\overline{T} = \frac{1}{\lambda} \ln \left(\frac{\left[\lambda \Pi_1 - (c+\theta) + \dfrac{\lambda}{\lambda+\rho} [\lambda \Pi_2 - (c+\theta)] \left[1 - e^{-(\lambda+\rho)T^*_2} \right] \right.}{\phi + \lambda \dfrac{\phi}{\rho} \left[1 - e^{-\rho T^*_2} \right]} \right). \tag{12}$$

By Assumption (A) it holds $\underline{T}, \overline{T} > 0$. Moreover, we have $\overline{T} \geq \underline{T}$, and thus that an interval of mutually optimal symmetric deadlines exists if and only if

$$\frac{\lambda \Pi_1 - (c+\theta)}{\lambda \Pi_2 - (c+\theta)} \leq \frac{\rho}{\lambda + \rho} \frac{1 - e^{-(\lambda+\rho)T^*_2}}{1 - e^{-\rho T^*_2}}. \tag{13}$$

Clearly, keeping Π_2 fixed, there is $\bar{\Pi}_1$ such that inequality (13) holds for all $\Pi_1 \leq \bar{\Pi}_1$ but is violated for all $\Pi_1 > \bar{\Pi}_1$. That is, symmetric, mutually optimal deadlines require that the first spot in the profitable market is not too attractive. To sum up:

Observation 1. *There are symmetric mutually optimal deadlines in state* $s = (1, \{1, 2\})$ *if and only if* (13) *holds. In particular, any profile* (T^*_{1s}, T^*_{2s}) *satisfying* $T^*_{1s} = T^*_{2s} \in [\underline{T}, \overline{T}]$*, where* $\underline{T}$ *and* $\overline{T}$ *are given in* (11) *and* (12)*, is a profile of mutually optimal deadlines.*

Let's turn to the case of asymmetric deadline profiles. A profile of asymmetric deadlines, $T^*_{is} \neq T^*_{js}$ is mutually optimal if and only if

$$\left. \frac{\partial V^*_{is}(T)}{\partial T_{is}} \right|_{\substack{T_{is} = T^*_{is} \\ T_{js} = T^*_{js}}} = 0 \quad \text{and} \quad \left. \frac{V^*_{js}(T)}{\partial T_{js}} \right|_{\substack{T_{is} = T^*_{is} \\ T_{js} = T^*_{js}}} = 0, \quad \text{for } i \neq j \in \{1, 2\}. \tag{14}$$

The above equations have two asymmetric solutions. For $T^*_{is} > T^*_{js}$, this yields $T^*_{is} = \underline{T}$ and $T^*_{js} = \overline{T}$ from (11) – (12) above. Consequently, $T^*_{is} > T^*_{js}$ holds whenever (13) fails to hold. That is, symmetric and asymmetric mutually optimal deadline profiles never co-exist.

Observation 2. *There are asymmetric mutually optimal deadlines in state* $s = (1, \{1, 2\})$ *if and only if* (13) *fails. In particular, any profile* (T^*_{1s}, T^*_{2s}) *satisfying* $T^*_{is} = \underline{T}$ *and* $T^*_{js} = \overline{T}$ *for* $i \neq j$*, where* $\underline{T}$ *and* $\overline{T}$ *are given in* (11) *and* (12)*, is a profile of mutually optimal deadlines.*

As in the symmetric case, multiple profiles of mutually optimal deadlines exist. The reason for multiplicity is a different one, though. Here, multiplicity follows from permuting the dyads rather than coordinating on an identical deadline from an interval of possible such deadlines.

4.2 Discussion

To get an intuition for when symmetric deadlines and when asymmetric deadlines are mutually optimal in $s = (1, \{1, 2\})$, it is instructive to look at the sign of the marginal utility in (9). In either of the two cases in (9), the sign of the marginal utility is equal to the sign of a difference proportional to the difference between the marginal benefit from increasing the deadline and the marginal agency cost thereof.

If the deadline is longer than that of the opponent ($T_{is} \geq T_{js}$), then the marginal benefit consists of the expected flow from racing for the first spot. In the case of having a shorter deadline than the opponent ($T_{is} < T_{js}$), the marginal benefit additionally consists of the value from keeping the option to work towards the second spot once the first spot becomes unavailable. That is, the presence of the opponent dyad has a positive externality. In a sense, it keeps the race open for the second prize.

The opponent's presence also has a negative externality, though. This negative externality manifests itself in the form of an additional marginal cost whenever $T_{is} < T_{js}$. The extra cost stems from the fact that the firm owner, anticipating the possibility of racing for the second spot alone, has a higher continuation utility than when no opponent is present. Incentive compatibility then requires that the investor promises to leave a higher share of Π_1 to the firm owner.

Now, in the case of a sufficiently valuable second spot, the overall effect of having the opponent in the race is positive. This gives an investor with a shorter deadline an incentive to match the deadline of the opponent and thus induces symmetric, mutually optimal deadlines. In contrast, when the first spot is sufficiently valuable, the incentive to free-ride on the efforts of the other firm is not so strong. Instead, a competition effect dominates, giving one investor (but not the other) an incentive to choose a strictly more extended deadline.

Remark 2. *On a technical note, observe from* (11)–(12) *together with Observations 1–2 that asymmetric deadlines occur if and only if the root of the argument in the first case of* (9)*, extended to the positive reals, lies to the left of the root of the argument in the second case of* (9)*, extended to the positive reals. This observation generalizes to the case of more than two dyads, giving us a useful link between the utility function of the investors and the structure of the mutually optimal deadline profiles (cf. Proposition A.4 in Appendix A).*

5 Discussion

5.1 Payments

As formally established in Lemma A.1 in Appendix A.1, the transfer from the firm to the investor if taking up a breakthrough opportunity at t in equilibrium C^* is given by

$$R^*_{is}(t) = \Pi_\sigma - \frac{\phi}{\lambda} - U_{is}(t; a_{is}, C^*). \tag{15}$$

From the expression for $U_{is}(t; a_{is}, C)$ in Lemma A.1, it is immediate that whenever the profile of deadlines is symmetric ($T_{is} = T_{js}$ for all $i, j \in N$), then $U_{is}(t; a_{is}, C^*)$ is strictly decreasing in t. Given (15), this implies that the transfer $R^*_{is}(t)$ — i.e., the equity share of the investor in case of a breakthrough — is increasing over time.

In the case of asymmetric deadlines, increasing $R^*_{is}(t)$ is also straightforward to verify for any firm i and any $t \in [\hat{T}, T^*_{is}]$, where $\hat{T}$ is the time of the latest opponent's exit from the race before the exit of dyad i.[10] Indeed, as $t \to T^*_{is}$ the continuation payoff, $U_{is}(t; a_{is}, C^*)$, vanishes, and the expected share of the firm value that the firm retains after a breakthrough decreases to ϕ. The closer to the deadline, the lower the agency rent of the firm, and a breakthrough in the last instance is only worth as much as shirking.

5.2 Truthful Reporting

A natural extension of the model is to assume that the firm privately observes the arrival of breakthroughs and that only the move into the profitable market is contractible. In such a case, the investor might be worried about truthful reporting on the part of the firm.

In particular, there might be breakthrough opportunities that the investor wants the firm owner to take up but the owner prefers to discard, and wait for a better spot instead. To avoid this, the net return from the current breakthrough for the firm owner has to be higher than the continuation value from the contract,

$$\Pi_\sigma - R_{is}(t) \geq U_{is}(t; a_i, C^*).$$

From the binding incentive compatibility constraint (15), we see that this condition is automatically satisfied in any incentive-compatible contract.

[10] As can be seen from the second case in (A.1), whether or not $R^*_{is}(t)$ is increasing for $t < T_\ell$, where T_ℓ is the longest time up to which ℓ opponents are in the race, depends on the expected utility in the continuation game, $\mathbf{U}_\ell$.

5.3 More than Two Prizes

It is possible to extend the model to more than two prizes. In this case, an additional complication arises due to the multiplicity of mutually optimal contract offers. In the model analyzed in this paper, solving for the mutually optimal contract offers relied on the fact that the respective continuation values are independent of the dyads' identities staying in the race. For states involving the first spot, this independence resulted from the symmetric contract when dyads race for the second spot (Proposition 1). For more than two spots, this independence of identities breaks down whenever the continuation contract is asymmetric, which may occur for states involving spots before the second-to-last one.

One way to circumvent the problem of multiple mutually optimal continuation contracts is to introduce a randomization device that publicly draws a random variable anytime a firm takes up a spot. With such a public randomization device, sun-spot like equilibria come into focus in which the remaining dyads mix between the different mutually optimal continuation profiles to ensure the continuation games provide continuation utilities that depend on the number of dyads but not on their identities. I show existence of such equilibria in an earlier version of this paper.[11]

5.4 Concluding Remarks

This article analyzed a race between firm-investor dyads that seek to obtain breakthrough opportunities to enter a profitable market. Combining moral hazard and multiple prizes in an otherwise standard, symmetric continuous-time race, I obtained equilibrium behavior that may exhibit staggered exits, shakeouts, or intermediate cases depending on the prize structure.

We have focused on the case where multiple firm-investor dyads race for two sequentially available spots in a profitable market. If the first spot in the market is sufficiently valuable, then the equilibrium contracts generally exhibit asymmetric funding deadlines. On the other hand, if the second spot is sufficiently valuable, then the equilibrium contracts are symmetric. Partially symmetric contract profiles might occur in intermediate cases.

The presence of opponent dyads creates both a positive and a negative externality. When the second spot is sufficiently valuable, then a free-riding effect dominates. Choosing a shorter deadline than the opponents is always worse than matching their deadlines, as the presence of the opponents keeps the race open for the second spot. When the second spot is not that valuable, that positive externality is outweighed by a negative externality, which comes in the form of additional agency rent from competition in the second stage. Here, competition induces asymmetric deadlines.

[11] I am happy to provide the earlier version upon request.

A Auxiliary Results

A.1 Payoff Characterization

In this section, I derive the utility functions of the firms and the investors in a state $s = (\sigma, N)$ under the cost-minimizing transfers. As we will see shortly, it is without loss to do so under the assumption that the continuation values (i.e., the values of remaining in the game once an opponent dyad has taken up a spot and left the game) only depend on the number of remaining players but not on their identities.[12] So, for states with $|N| \geq 2$, I collect the investors' continuation values of states with $\kappa \in \{1, ..., |N|\}$ players remaining in the race by the vector $\mathbf{V} = \{\mathbf{V}_\kappa\}_{\kappa \in \{1,...,|N|-1\}}$. Analogously, the firms' continuation values are given by $\mathbf{U} = \{\mathbf{U}_\kappa\}_{\kappa \in \{1,...,|N|-1\}}$.

Lemma A.1 (Firm Owner Utility). *Fix a dyad i and a state $s = (\sigma, N) \in S_i$. Let C be a profile of full-R&D contracts. Then:*

(a) *Let $\mathbf{U}_0 = 0$, $T_{|N|} = 0$, and $T_0 = \infty$. Further, if $|N| \geq 2$, let $\{T_\kappa\}_{\kappa=1}^{|N|-1}$ be a sequence whose elements T_κ decrease in κ and assume it corresponds to the profile of opponent deadlines in that state, $\{T_{js}\}_{j \in N \setminus i} = \{T_\kappa\}_{\kappa=1}^{|N|-1}$. Suppose investor i sets a deadline $T_{is} \in [T_\kappa, T_{\kappa-1})$ for some $\kappa \in \{1, ..., |N|\}$. Then, under cost-minimizing transfers*

$$U_{is}(t; a_i, C) = \begin{cases} \dfrac{\phi + \lambda(\kappa - 1)\mathbf{U}_{\kappa-1}}{\lambda(\kappa-1) + \rho} \times \left[1 - e^{-(\lambda(\kappa-1)+\rho)(T_{is}-t)}\right] & \text{if } t \in [T_\kappa, T_{is}] \\ U_{is}(T_{\ell-1}; a_i, C) e^{-(\lambda(\ell-1)+\rho)(T_{\ell-1}-t)} + \dfrac{\phi + \lambda(\ell-1)\mathbf{U}_{\ell-1}}{\lambda(\ell-1)+\rho} \times \left[1 - e^{-(\lambda(\ell-1)+\rho)(T_{\ell-1}-t)}\right] & \text{if } t \in [T_\ell, T_{\ell-1}) \text{ for } \ell \in \{\kappa+1, ..., |N|\}. \end{cases} \tag{A.1}$$

(b) *The cost-minimizing transfers R_{is} satisfy*

$$R_{is}(t) = \Pi_\sigma - \frac{\phi}{\lambda} - U_{is}(t; a_i, C), \quad \text{for a.e. } t \geq 0. \tag{A.2}$$

[12] This holds in all equilibria for the race with two prizes. The reason is that in the states with $\sigma = 2$, the continuation values are zero for everyone. If so, the mutually optimal contract offers are symmetric (cf. Corollary A.1 below), implying symmetry of the continuation values in the states with $\sigma = 1$, too.

Proof of Lemma A.1. Suppose $T_{is} \in [T_\kappa, T_{\kappa-1})$ for some $\kappa \in \{1, ..., |N|\}$ and consider $t \in [T_\ell, \min\{T_{is}, T_{\ell-1}\})$ for some $\ell \in \{\kappa, ..., |N|\}$ and suppose $a_{is}(t) = 1$. Expanding (U), we get that firm owner i's utility for small $\mathrm{d}t > 0$ approximately satisfies

$$U_{is}(t; \hat{a}_i, C) = [(1 - \hat{a}_{is}(t))\phi + \hat{a}_{is}(t)\lambda[\Pi_\sigma - R_{is}(t)] + (\ell - 1)\lambda \mathbf{U}_{\ell-1}]\,\mathrm{d}t \\ + (1 - (\lambda(\ell - 1 + \hat{a}_{is}(t)) + \rho)\mathrm{d}t)U_{is}(t + \mathrm{d}t; \hat{a}_i, C)$$

for small $\mathrm{d}t > 0$. Presuming optimality of $\hat{a}_{is}$ implies

$$U_{is}(t; \hat{a}_i, C) = \max_{a \in \{0,1\}} \Bigg\{ [(1 - a)\phi + a\lambda[\Pi_\sigma - R_{is}(t)] + (\ell - 1)\lambda \mathbf{U}_{\ell-1}]\,\mathrm{d}t \\ + (1 - (\lambda(\ell - 1 + a) + \rho)\mathrm{d}t)U_{is}(t + \mathrm{d}t; \hat{a}_i, C)\Bigg\}.$$

Rearranging and taking the limit $\mathrm{d}t \to 0$ gives

$$0 = \max_{a \in \{0,1\}} \Big\{ a\left[\lambda[\Pi_\sigma - R_{is}(t) - U_{is}(t; \hat{a}_i, C)] - \phi\right] \Big\} \\ + \phi + (\ell - 1)\lambda \mathbf{U}_{\ell-1} - (\lambda(\ell - 1) + \rho))U_{is}(t; \hat{a}_i, C) + U'_{is}(t; \hat{a}_i, C), \quad \text{(A.3)}$$

from which we see that in order for $\hat{a}_{is}(t) = 1$ to be chosen by the firm at all $t \in [0, T_{is}]$, incentive compatibility requires

$$\Pi_\sigma - R_{is}(t) - U_{is}(t; a_i, C) \geq \phi/\lambda \quad \text{(A.4)}$$

for all $t \in [0, T_{is}]$. Now, observe that the constraint (A.4) must bind at (almost) every t in any equilibrium, for, if not, then there would be an alternative, feasible contract with strictly higher payments, $\hat{R}_{is}(t)$, whenever the constraint does not bind on a strictly positive measure of points t. Because the contract is otherwise unchanged this would yield a strictly higher profit to the investor. This gives (A.2).

Since a contract under which the constraint (A.4) binds everywhere also satisfies (LLC) and is payoff equivalent to a contract under which it binds almost everywhere, we can rewrite (A.3) as

$$0 = \lambda \left[\frac{\phi}{\lambda} + (\ell - 1)\mathbf{U}_{\ell-1}\right] - (\lambda(\ell - 1) + \rho)U_{is}(t; a_i, C) + U'_{is}(t; a_i, C).$$

Solving gives

$$U_{is}(t; a_i, C) = e^{(\lambda(\ell-1)+\rho)t} \left[-\int_0^t \lambda \left[\frac{\phi}{\lambda} + (\ell-1)\mathbf{U}_{\ell-1} \right] e^{-(\lambda(\ell-1)+\rho)\tau} \mathrm{d}\tau + \mathcal{C} \right],$$

where $\mathcal{C}$ is a constant. For $\ell = \kappa$ it must hold $U_{is}(T_{is}; a_{is}, C) = 0$, yielding the first case in (A.1). For $\ell \in \{\kappa+1, ..., |N|\}$, value matching requires $\lim_{t \uparrow T_{\ell-1}} U_{is}(t; a_{is}, C) = U_{is}(T_{\ell-1}; a_{is}, C)$, yielding the second case in (A.1). □

The incentive compatibility constraint (A.2) allows to substitute the payments in the investor utility function. As the proof to the following lemma shows, this gives the following characterization result for investor utility under the cost-minimizing transfers.

Lemma A.2 (Investor Utility)**.** *Fix a dyad $i \in \{1, ..., n\}$ and a state $s = (\sigma, N) \in S_i$. Let C be a profile of full-R&D contracts. Let $\mathbf{V}_0 = 0$, $T_{|N|} = 0$, and $T_0 = \infty$. Further, if $|N| \geq 2$, let $\{T_\kappa\}_{\kappa=1}^{|N|-1}$ be a sequence whose elements T_κ decrease in κ and assume it corresponds to the profile of opponent deadlines in that state, $\{T_{js}\}_{j \in N \setminus i} = \{T_\kappa\}_{\kappa=1}^{|N|-1}$.*

(a) *Suppose investor i sets a deadline $T_{is} \in [T_{|N|}, T_{|N|-1})$, then under cost-minimizing transfers*

$$V_{is}(0; C) = \int_0^{T_{is}} \lambda \left[\Pi_\sigma - U_{is}(\tau; a_i, C) - \frac{c+\theta+\phi}{\lambda} + (|N|-1)\mathbf{V}_{|N|-1} \right] e^{-(\lambda|N|+\rho)\tau} \mathrm{d}\tau. \quad \text{(A.5)}$$

(b) *Suppose investor i sets a deadline $T_{is} \in [T_\kappa, T_{\kappa-1})$ for some $\kappa \in \{1, ..., |N|-1\}$, then under cost-minimizing transfers*

$$V_{is}(0; C) = \mathcal{V}_{|N|-1} e^{-(\lambda|N|+\rho)T_{|N|-1}} + \int_0^{T_{|N|-1}} \lambda \left[\Pi_\sigma - U_{is}(\tau; a_i, C) - \frac{c+\theta+\phi}{\lambda} + (|N|-1)\mathbf{V}_{|N|-1} \right] e^{-(\lambda|N|+\rho)\tau} \mathrm{d}\tau, \quad \text{(A.6)}$$

where $\mathcal{V}_\ell$ is recursively given by

$$\mathcal{V}_\ell = \mathcal{V}_{\ell-1} e^{-(\lambda\ell+\rho)(T_{\ell-1}-T_\ell)} + e^{(\ell\lambda+\rho)T_\ell} \times \int_{T_\ell}^{T_{\ell-1}} \lambda \left[\Pi_\sigma - U_{is}(\tau; a_i, C) - \frac{c+\theta+\phi}{\lambda} + (\ell-1)\mathbf{V}_{\ell-1} \right] e^{-(\lambda\ell+\rho)\tau} \mathrm{d}\tau \quad \text{(A.7)}$$

for $\ell \in \{\kappa+1, ..., |N|-1\}$*, and for* $\ell = \kappa$ *by*

$$\mathcal{V}_\ell = e^{(\lambda\ell+\rho)T_\ell} \times \int_{T_\ell}^{T_{is}} \lambda \left[\Pi_\sigma - U_{is}(\tau; a_i, C) - \frac{c+\theta+\phi}{\lambda} + (\ell-1)\mathbf{V}_{\ell-1}\right] e^{-(\lambda\ell+\rho)\tau} \mathrm{d}\tau. \quad \text{(A.8)}$$

Proof of Lemma A.2. Suppose $T_{is} \in [T_\kappa, T_{\kappa-1})$ for some $\kappa \in \{1, ..., |N|\}$ and consider $t \in [T_\ell, \min\{T_{is}, T_{\ell-1}\})$ for some $\ell \in \{\kappa, ..., |N|\}$. Then, expanding (V), the utility of the investor, $V_{is}(t; C)$, under a profile C of full-R&D contracts approximately satisfies for small $\mathrm{d}t > 0$,

$$V_{is}(t; C) = a_{is}(t)[\lambda R_{is}(t) - c] + (\ell-1)\lambda\mathbf{V}_{\ell-1} - \theta + (1 - ((\ell - 1 + a_{is}(t))\lambda + \rho)\mathrm{d}t)V_{is}(t + \mathrm{d}t).$$

Using $a_{is}(t) = 1$ and (A.2) and taking the limit $\mathrm{d}t \to 0$ yields

$$0 = \lambda \left[\Pi_\sigma - U_{is}(t; a_i, C) - \frac{c+\theta+\phi}{\lambda} + (\ell-1)\mathbf{V}_{\ell-1}\right] - (\lambda\ell+\rho)V_{is}(t; C) + V'_{is}(t; C).$$

Solving gives

$$V_{is}(t; C) = e^{(\lambda\ell+\rho)t} \times \left[-\int_0^t \lambda \left[\Pi_\sigma - U_{is}(\tau; a_i, C) - \frac{c+\theta+\phi}{\lambda} + (\ell-1)\mathbf{V}_{\ell-1}\right] e^{-(\lambda\ell+\rho)\tau} \mathrm{d}\tau + \mathcal{C}\right], \quad \text{(A.9)}$$

where $\mathcal{C}$ is a constant. If $T_{is} < T_{\ell-1}$, then using the boundary condition $V_{is}(T_{is}; C) = 0$ we obtain

$$V_{is}(t; C) = e^{(\lambda\ell+\rho)t} \times \left[\int_t^{T_{is}} \lambda \left[\Pi_\sigma - U_{is}(\tau; a_i, C) - \frac{c+\theta+\phi}{\lambda} + (\ell-1)\mathbf{V}_{\kappa-1}\right] e^{-(\lambda\ell+\rho)\tau} \mathrm{d}\tau\right]. \quad \text{(A.10)}$$

If, on the other hand, $T_{is} \geq T_{\ell-1}$, then value matching requires $\lim_{t\uparrow T_{\ell-1}} V_{is}(t; C) = V_{is}(T_{\ell-1}; C)$, so that defining $\mathcal{V}_\ell \equiv V_{is}(T_\ell; C)$ gives us

$$V_{is}(t; C) = \mathcal{V}_{\ell-1}e^{-(\lambda\ell+\rho)(T_{\ell-1}-t)} + e^{(\lambda\ell+\rho)t}$$

$$\times \left[\int_t^{T_{\ell-1}} \lambda \left[\Pi_\sigma - U_{is}(\tau; a_i, C) - \frac{c+\theta+\phi}{\lambda} + (\ell-1)\mathbf{V}_{\ell-1} \right] e^{-(\lambda\ell+\rho)\tau} \mathrm{d}\tau \right]. \quad \text{(A.11)}$$

Now, to obtain (A.5) we set $\ell = |N|$ and evaluate (A.10) at $t = 0$. This gives (a). To obtain (A.6) we set $\ell = |N|$ and evaluate (A.11) at $t = 0$. Then (A.7) follows from recursively using (A.11) for $\ell \in \{\kappa+1, ..., |N|\}$ and (A.8) follows from (A.10) for $\ell = \kappa$. This gives (b) and completes the proof. □

A.2 Equilibrium Characterization Results

This section uses the expressions for the firms' and the investors' utilities obtained in the last sectoin to characterize the mutually optimal deadlines. The next lemma is instrumental in doing so.

Lemma A.3. *Fix a dyad i and a state $s = (\sigma, N) \in S_i$. Let $V^*_{is}(T)$ be investor utility under the cost-minimizing transfers for deadlines $T = (T_{is}, \{T_\kappa\}_{\kappa=1}^{|N|-1})$, where $\{T_\kappa\}_{\kappa=1}^{|N|-1} = \{\emptyset\}$ if $|N| = 1$ and the elements of $\{T_\kappa\}_{\kappa=1}^{|N|-1}$ decrease in κ otherwise. Further, let $T_0 = \infty$, $T_{|N|} = 0$, and $\mathbf{U}_0 = \mathbf{V}_0 = 0$. Then, for all $\kappa \in \{1, .., |N|\}$ such that $T_\kappa < T_{\kappa-1}$,*

$$sgn\left(\left. \frac{\partial V^*_{is}(T)}{\partial T_{is}} \right|_{T_{is}\in[T_\kappa, T_{\kappa-1})} \right) = sgn\Bigg(\left[\lambda\Pi_\sigma - (c+\theta) + \lambda(\kappa-1)\left[\mathbf{V}_{\kappa-1} + \mathbf{U}_{\kappa-1}\right]\right] e^{-\lambda T_{is}} - \left[\phi + \lambda(\kappa-1)\mathbf{U}_{\kappa-1}\right] \Bigg). \quad \text{(A.12)}$$

Proof of Lemma A.3. Let $V^*_{is}(T) = V_{is}(0; C)$ be investor i's utility in state s under cost-minimizing transfers and deadlines $T = (T_{is}, \{T_{js}\}_{j\in N\setminus i})$ as characterized in Lemma A.2. That is, if $|N| = 1$, then the opponent deadline profile is $\{T_{js}\}_{j\in N\setminus i} = \{\emptyset\}$. And if $|N| \geq 2$, then the opponent deadline profile corresponds to $\{T_\kappa\}_{\kappa=1}^{|N|-1}$, where the elements of $\{T_\kappa\}_{\kappa=1}^{|N|-1}$ weakly decrease in κ. Last, recall that $\mathbf{V}_0 = \mathbf{U}_0 = 0$ and that $T_{|N|} = 0$ and $T_0 = \infty$.

First, consider $T_{is} \in [T_\kappa, T_{\kappa-1})$ for some $\kappa \in \{1, ..., |N|-1\}$. From (A.6) in part (b) of Lemma A.2, we obtain that

$$\frac{\partial V^*_{is}(T)}{\partial T_{is}} = \frac{\partial \mathcal{V}_{|N|-1}}{\partial T_{is}} e^{-(\lambda|N|+\rho)T_{|N|-1}} - \lambda \int_0^{T_{|N|-1}} \frac{\partial U_{is}(\tau; a_i, C)}{\partial T_{is}} e^{-(\lambda|N|+\rho)\tau} \mathrm{d}\tau.$$

Consequently, plugging in for $\partial \mathcal{V}_{|N|-1}/\partial T_{is}$ and recalling that $T_{|N|} = 0$, (A.7) gives

$$\frac{\partial V_{is}^*(T)}{\partial T_{is}} = \frac{\partial \mathcal{V}_{|N|-2}}{\partial T_{is}} e^{-\sum_{m=|N|-1}^{|N|}(\lambda m+\rho)(T_{m-1}-T_m)} - \lambda \sum_{m=|N|-1}^{|N|} e^{-\lambda \sum_{j=m}^{|N|} T_j} \int_{T_m}^{T_{m-1}} \frac{\partial U_{is}(\tau; a_i, C)}{\partial T_{is}} e^{-(\lambda m+\rho)\tau} \mathrm{d}\tau.$$

Repeating, we thus obtain more generally,

$$\frac{\partial V_{is}^*(T)}{\partial T_{is}} = \frac{\partial \mathcal{V}_{\kappa}}{\partial T_{is}} e^{-\sum_{m=\kappa+1}^{|N|}(\lambda m+\rho)(T_{m-1}-T_m)} - \lambda \sum_{m=\kappa+1}^{|N|} e^{-\lambda \sum_{j=m}^{|N|} T_j} \int_{T_m}^{T_{m-1}} \frac{\partial U_{is}(\tau; a_i, C)}{\partial T_{is}} e^{-(\lambda m+\rho)\tau} \mathrm{d}\tau,$$

where (A.8) yields

$$\frac{\partial \mathcal{V}_{\kappa}}{\partial T_{is}} = e^{-(\lambda\kappa+\rho)(T_{is}-T_\kappa)} \lambda \left[\Pi_\sigma - \frac{c+\theta+\phi}{\lambda} + (\kappa-1)\mathbf{V}_{\kappa-1}\right] - \lambda e^{(\lambda\kappa+\rho)T_\kappa} \int_{T_\kappa}^{T_{is}} \frac{\partial U_{is}(\tau; a_i, C)}{\partial T_{is}} e^{-(\kappa\lambda+\rho)\tau} \mathrm{d}\tau.$$

This can be expressed more succinctly as

$$\frac{\partial V_{is}^*(T)}{\partial T_{is}} = \lambda \left[\Pi_\sigma - \frac{c+\theta+\phi}{\lambda} + (\kappa-1)\mathbf{V}_{\kappa-1}\right] e^{-(\lambda\kappa+\rho)T_{is} - \sum_{m=\kappa+1}^{|N|} \lambda T_{m-1}} - \lambda \sum_{m=\kappa}^{|N|} e^{-\lambda \sum_{j=m}^{|N|} T_j} \int_{T_m}^{\min\{T_{is}, T_{m-1}\}} \frac{\partial U_{is}(\tau; a_i, C)}{\partial T_{is}} e^{-(\lambda m+\rho)\tau} \mathrm{d}\tau. \quad \text{(A.13)}$$

Next, from Lemma A.1 we obtain for $\tau \in (T_m, T_{m-1})$, $m \in \{\kappa+1, ..., |N|\}$, that

$$\frac{\partial U_{is}(\tau; a_i, C)}{\partial T_{is}} = \frac{\partial U_{is}(T_{m-1}; a_i, C)}{\partial T_{is}} e^{-(\lambda(m-1)+\rho)(T_{m-1}-\tau)}.$$

Plugging in for $\partial U_{is}(T_{m-1}; a_i, C)/\partial T_{is}$, we obtain

$$\begin{aligned}\frac{\partial U_{is}(\tau; a_i, C)}{\partial T_{is}} &= \frac{\partial U_{is}(T_{m-2}; a_i, C)}{\partial T_{is}} e^{-(\lambda(m-2)+\rho)(T_{m-2}-T_{m-1})} e^{-(\lambda(m-1)+\rho)(T_{m-1}-\tau)} \\ &= \frac{\partial U_{is}(T_{m-2}; a_i, C)}{\partial T_{is}} e^{-(\lambda(m-2)+\rho)T_{m-2} - \lambda T_{m-1} + (\lambda(m-1)+\rho)\tau}.\end{aligned}$$

Repeating, and using

$$\frac{\partial U_{is}(T_\kappa;a_i,C)}{\partial T_{is}} = \left[\phi + \lambda(\kappa-1)\mathbf{U}_{\kappa-1}\right] e^{-(\lambda(\kappa-1)+\rho)(T_{is}-T_\kappa)},$$

we finally obtain

$$\frac{\partial U_{is}(\tau;a_i,C)}{\partial T_{is}} = \left[\phi + \lambda(\kappa-1)\mathbf{U}_{\kappa-1}\right] e^{-(\lambda(\kappa-1)+\rho)T_{is}-\lambda\sum_{j=\kappa+1}^{m}T_{j-1}+(\lambda(m-1)+\rho)\tau}.$$

Plugging above expression into (A.13) then yields

$$\begin{aligned}\frac{\partial V_{is}^*(T)}{\partial T_{is}} &= \lambda\left[\Pi_\sigma - \frac{c+\theta+\phi}{\lambda} + (\kappa-1)\mathbf{V}_{\kappa-1}\right] e^{-(\lambda\kappa+\rho)T_{is}-\sum_{m=\kappa+1}^{|N|}\lambda T_{m-1}} \\ &\quad - \lambda\sum_{m=\kappa}^{|N|} e^{-\lambda\sum_{j=m}^{|N|}T_j} \int_{T_m}^{\min\{T_{is},T_{m-1}\}} \left[\phi + \lambda(\kappa-1)\mathbf{U}_{\kappa-1}\right] e^{-(\lambda\kappa+\rho)T_{is}-\lambda\sum_{j=\kappa+1}^{m}T_{j-1}-\lambda\tau}\mathrm{d}\tau.\end{aligned}$$

Integrating gives

$$\begin{aligned}\frac{\partial V_{is}^*(T)}{\partial T_{is}} &= \lambda\left[\Pi_\sigma - \frac{c+\theta+\phi}{\lambda} + (\kappa-1)\mathbf{V}_{\kappa-1}\right] e^{-(\lambda\kappa+\rho)T_{is}-\sum_{m=\kappa+1}^{|N|}\lambda T_{m-1}} \\ &\quad + \sum_{m=\kappa}^{|N|} e^{-\lambda\sum_{j=m}^{|N|}T_j}\left[\phi + \lambda(\kappa-1)\mathbf{U}_{\kappa-1}\right] \\ &\quad \times \left[e^{-(\lambda\kappa+\rho)T_{is}-\lambda\sum_{j=\kappa+1}^{m}T_{j-1}-\lambda\min\{T_{is},T_{m-1}\}} - e^{-(\lambda\kappa+\rho)T_{is}-\lambda\sum_{j=\kappa+1}^{m}T_{j-1}-\lambda T_m}\right],\end{aligned}$$

which, recalling $T_{|N|}=0$, can be rewritten as

$$\begin{aligned}\frac{\partial V_{is}^*(T)}{\partial T_{is}} &= \lambda\left[\Pi_\sigma - \frac{c+\theta+\phi}{\lambda} + (\kappa-1)\mathbf{V}_{\kappa-1}\right] e^{-(\lambda\kappa+\rho)T_{is}-\sum_{m=\kappa+1}^{|N|}\lambda T_{m-1}} \\ &\quad + \sum_{m=\kappa}^{|N|}\left[\phi + \lambda(\kappa-1)\mathbf{U}_{\kappa-1}\right] \\ &\quad \times \left[e^{-(\lambda\kappa+\rho)T_{is}-\lambda\sum_{j=\kappa+1}^{|N|}T_{j-1}-\lambda\min\{T_{is},T_{m-1}\}} - e^{-(\lambda\kappa+\rho)T_{is}-\lambda\sum_{j=\kappa+1}^{|N|}T_{j-1}-\lambda T_m}\right],\end{aligned}$$

or

$$\frac{\partial V_{is}^*(T)}{\partial T_{is}} =$$

$$e^{-(\lambda\kappa+\rho)T_{is}-\sum_{j=\kappa+1}^{|N|}\lambda T_{j-1}}\Bigg[\left[\lambda\Pi_\sigma - (c+\theta+\phi) + \lambda(\kappa-1)\mathbf{V}_{\kappa-1}\right] e^{-\lambda T_{is}}$$

$$+ \left[\phi + \lambda(\kappa-1)\mathbf{U}_{\kappa-1}\right] \sum_{m=\kappa}^{|N|} \left[e^{-\lambda \min\{T_{is}, T_{m-1}\}} - e^{-\lambda T_m}\right] \Bigg],$$

which is equivalent to

$$\frac{\partial V_{is}^*(T)}{\partial T_{is}} = e^{-(\lambda\kappa+\rho)T_{is} - \sum_{j=\kappa+1}^{|N|} \lambda T_{j-1}} \Bigg[\left[\lambda \Pi_\sigma - (c+\theta+\phi) + \lambda(\kappa-1)\mathbf{V}_{\kappa-1}\right] e^{-\lambda T_{is}} - \left[\phi + \lambda(\kappa-1)\mathbf{U}_{\kappa-1}\right]\left[1 - e^{-\lambda T_{is}}\right] \Bigg],$$

finally yielding

$$\frac{\partial V_{is}^*(T)}{\partial T_{is}} = e^{-(\lambda\kappa+\rho)T_{is} - \sum_{j=\kappa+1}^{|N|} \lambda T_{j-1}} \times \Bigg[\left[\lambda \Pi_\sigma - (c+\theta) + \lambda(\kappa-1)\left[\mathbf{V}_{\kappa-1} + \mathbf{U}_{\kappa-1}\right]\right] e^{-\lambda T_{is}} - \left[\phi + \lambda(\kappa-1)\mathbf{U}_{\kappa-1}\right] \Bigg],$$

thus giving us the claim when $T_{is} \in [T_\kappa, T_{\kappa-1})$ for $\kappa \in \{1, ..., |N|-1\}$.

Second, we repeat above steps for $\kappa = |N|$, now starting from part (a) rather than part (b) of Lemma A.2. This gives

$$\frac{\partial V_{is}^*(T)}{\partial T_{is}} = e^{-(\lambda\kappa+\rho)T_{is}} \times \Bigg[\left[\lambda \Pi_\sigma - (c+\theta) + \lambda(\kappa-1)\left[\mathbf{V}_{\kappa-1} + \mathbf{U}_{\kappa-1}\right]\right] e^{-\lambda T_{is}} - \left[\phi + \lambda(\kappa-1)\mathbf{U}_{\kappa-1}\right] \Bigg],$$

and thus completes the proof. □

Now, consider the definition of G_κ in (5), momentarily replace the continuation values $V_{2,\kappa-1}^*$ and $U_{2,\kappa-1}^*$ with our more general continuation values $\mathbf{V}_{\kappa-1}$ and $\mathbf{U}_{\kappa-1}$, and construct a sequence $\{G_\kappa\}_{\kappa=1}^{|N|}$ containing these G_κ. I am interested in subsequences of the sequence $\{G_\kappa\}_{\kappa=1}^{|N|}$ that have some specific properties. For the following, I define a set $\mathcal{G}_s$ collecting all index sets of the subsequences that have these properties for some state s. Specifically,

Definition 4 (Set of Subsequences, $\mathcal{G}_s$)**.** *Let $\mathcal{G}_s$ be the set of all index sets $\{\kappa_1, ..., \kappa_\ell\}$ with $\kappa_\ell = |N|$ to which there is a subsequence of $\{G_\kappa\}_{\kappa=1}^{|N|}$ with the following properties:*

(a) *For all $q \in \{1, ..., \ell\}$, it holds $G_{\kappa_{q-1}+1} \leq G_{\kappa_q}$, where $\kappa_0 = 0$.*

(b) *For all $q \in \{0, ..., \ell-1\}$, it holds $G_{\kappa_q} > G_{\kappa_q+1}$, where $G_{\kappa_0} = \infty$.*

For example, if the sequence $\{G_\kappa\}_{\kappa=1}^{|N|}$ is strictly decreasing, then the unique set in $\mathcal{G}_s$ is $\{1, ..., |N|\}$. On the other hand, if the sequence $\{G_\kappa\}_{\kappa=1}^{|N|}$ is increasing, then the unique set in $\mathcal{G}_s$ is $\{|N|\}$. Either of these cases can occur in equilibrium, but also mixed cases play an important role in which $\{G_\kappa\}_{\kappa=1}^{|N|}$ is U-shaped and the set $\mathcal{G}_s$ consists of multiple elements. For example, if $|N| = 4$, all elements in $\{G_\kappa\}_{\kappa=1}^{4}$ are distinct, and they satisfy $G_1 > G_4 > G_2 > G_3$, then $\mathcal{G}_s = \{\{1, 4\}, \{1, 2, 4\}\}$. The next lemma gives a more systematic characterization that will become important in the following.

Lemma A.4. *Fix a state $s = (\sigma, N)$. Suppose the elements of the corresponding sequence $\{G_\kappa\}_{\kappa=1}^{|N|}$ are distinct and that the sequence is U-shaped with lowest element $G_{\overline{k}}$. Further, let $\underline{k}$ be the lowest index $\kappa \leq \overline{k}$ of the elements satisfying $G_\kappa < G_{|N|}$ (setting $\underline{k} = |N|$ if there is no such element). Then, an index set $\{\kappa_1, \kappa_2, ..., \kappa_\ell\}$ with $\kappa_\ell = |N|$ is an element of $\mathcal{G}_s$ if and only if: (a) $\kappa_q = q$ for all $q < \ell$ and (b) $\ell \in \{\underline{k}, \underline{k} + 1, ..., \overline{k}\}$.*

Proof of Lemma A.4. The claim follows from the following four observations:

1. Whenever some index κ satisfying $2 < \kappa < \overline{k}$ is part of an index set in $\mathcal{G}_s$ then so must be $\kappa - 1$.

   (Proof: By assumption, the sequence $\{G_\kappa\}_{\kappa=1}^{|N|}$ is strictly decreasing until $\kappa = \overline{k}$. The observation then follows from (a) in Definition 4.)

2. For any index set $\{\kappa_1, \kappa_2, ..., \kappa_\ell\} \in \mathcal{G}_s$ with $\ell \geq 2$ it must hold that $\kappa_{\ell-1} < \overline{k}$.

   (Proof: Suppose to the contrary that $\kappa_{\ell-1} \geq \overline{k}$. Then, $\overline{k} < |N|$ because $\kappa_{\ell-1} < \kappa_\ell = |N|$. Point (b) in Definition 4 implies $G_{\kappa_{\ell-1}} > G_{\kappa_{\ell-1}+1}$, which contradicts the fact that G_κ is increasing for $\kappa \in \{\overline{k}, \overline{k} + 1, ..., |N|\}$.)

3. For any index set $\{\kappa_1, \kappa_2, ..., \kappa_\ell\} \in \mathcal{G}_s$ with $\ell \geq 2$ it must hold that $\kappa_{\ell-1} \geq \underline{k} - 1$.

   (Proof: If $\underline{k} = 1$, then the claim is evidentially true. As regards $\underline{k} \geq 2$, suppose to the contrary that $\kappa_{\ell-1} < \underline{k} - 1$. Then (a) in Definition 4 implies $G_{\kappa_{\ell-1}+1} \leq G_{|N|}$. But the definition of $\underline{k}$ gives $G_{\kappa_{\ell-1}+1} > G_{|N|}$, a contradiction.)

4. For every $\kappa \in \{\underline{k}, ..., \overline{k}\}$ there is an index set $\{1, 2, ..., \kappa - 1, |N|\} \in \mathcal{G}_s$ (which is taken to be $\{|N|\}$ if $\kappa = 1$).

   (Proof: This is a straightforward consequence of the facts that $\{G_\kappa\}_{\kappa=1}^{|N|}$ is strictly decreasing until $\kappa = \overline{k}$ and that for all $\kappa \in \{\underline{k}, ..., \overline{k}\}$ we have $G_\kappa < G_{|N|}$. Consequently, any index set $\{1, 2, ..., \kappa - 1, |N|\}$ satisfies conditions (a) and (b) in Definition 4.)

Observation 1 gives that any element in $\{\kappa_1, \kappa_2, ..., \kappa_\ell\} \in \mathcal{G}_s$ has $\kappa_1 = 1$ and – up to the $(\ell - 1)$-th element — consists of consecutive elements only. Observation 2 provides an upper bound on the length of any element in $\mathcal{G}_s$ and Observation 3 provides a lower bound. Finally, Observation 4 establishes that these bounds are tight. □

For any sequence $\{G_\kappa\}_{\kappa=1}^{|N|}$, the following proposition gives a full characterization of the corresponding set of mutually optimal deadline profiles, as defined in Definition 3 in the text.

Proposition A.4. *Fix a state $s=(\sigma,N)\in S$ and take any weakly decreasing deadline profile $\{T_\kappa^*\}_{\kappa=1}^{|N|}$. Then, $\{T_\kappa^*\}_{\kappa=1}^{|N|}\in\mathcal{T}_s$ if and only if there is $\{\kappa_1,...,\kappa_\ell\}\in\mathcal{G}_s$ such that $\{T_\kappa^*\}_{\kappa=1}^{|N|}$ satisfies*

$$T_{\kappa_q}^* > T_{\kappa_q+1}^* = T_{\kappa_{q+1}}^*, \forall q\in\{0,...,\ell-1\},\ \textit{where } T_{\kappa_0}^*=\infty, \tag{A.14}$$

and

$$\frac{1}{\lambda}\ln\left(G_{\kappa_{q-1}+1}\right)\le T_{\kappa_q}^*\le\frac{1}{\lambda}\ln\left(G_{\kappa_q}\right), \forall q\in\{1,...,\ell\},\ \textit{where } \kappa_0=0. \tag{A.15}$$

Moreover, for every $\{\kappa_1,...,\kappa_\ell\}\in\mathcal{G}_s$ there is at least one $\{T_\kappa^*\}_{\kappa=1}^{|N|}\in\mathcal{T}_s$.

Proof of Proposition A.4. First, I show the only-if part, then the if part.
Only-If Part: If $\{T_\kappa^*\}_{\kappa=1}^{|N|}\in\mathcal{T}_s$ then there is $\{\kappa_1,...,\kappa_\ell\}\in\mathcal{G}_s$ such that $\{T_\kappa^*\}_{\kappa=1}^{|N|}$ satisfies the conditions (A.14) and (A.15). To show this, consider an index set $\{\kappa_1,\kappa_2,...,\kappa_\ell\}$ with $\kappa_\ell=|N|$ and a corresponding deadline profile $\{T_\kappa^*\}_{\kappa=1}^{|N|}$ satisfying

$$T_1^*=...=T_{\kappa_1}^*>T_{\kappa_1+1}^*=...=T_{\kappa_2}^*>...>T_{\kappa_{\ell-2}+1}^*=...=T_{\kappa_{\ell-1}}^*>T_{\kappa_{\ell-1}+1}^*=...=T_{|N|}^*.$$

I will derive necessary conditions on the sequence $\{G_\kappa\}_{\kappa=1}^{|N|}$ for this profile to be one of mutually optimal deadlines and then verify that they imply $\{\kappa_1,\kappa_2,...,\kappa_\ell\}\in\mathcal{G}_s$ and condition (A.15) (condition (A.14) is satisfied by construction).

From Lemma A.3, optimality of $T_{\kappa_j}^*$ implies

$$\left[\lambda\Pi_\sigma-(c+\theta)+\lambda(\kappa_j-1)\left[\mathbf{V}_{\kappa_j-1}+\mathbf{U}_{\kappa_j-1}\right]\right]e^{-\lambda T_{\kappa_j}^*}\ge\phi+\lambda(\kappa_j-1)\mathbf{U}_{\kappa_j-1}, \tag{A.16}$$

for every $j\in\{1,...,\ell\}$. That is, all investors choosing $T_{\kappa_j}^*$ must find it optimal to choose a deadline of at least $T_{\kappa_j}^*$ when κ_j-1 opponents are in the race. Further, for every $j\in\{1,...,\ell\}$ it must hold (setting $\kappa_0=0$) that

$$\left[\lambda\Pi_\sigma-(c+\theta)+\lambda\kappa_{j-1}\left[\mathbf{V}_{\kappa_{j-1}}+\mathbf{U}_{\kappa_{j-1}}\right]\right]e^{-\lambda T_{\kappa_j}^*}\le\phi+\lambda\kappa_{j-1}\mathbf{U}_{\kappa_{j-1}}, \tag{A.17}$$

ensuring that neither of the investors choosing a deadline of lenght $T_{\kappa_j}^*$ find it profitable to extend the deadline beyond $T_{\kappa_j}^*$, where they would face κ_{j-1} opponents. Last, all dyads choosing $T_{\kappa_{j-1}}^*$, $j\in\{2,...,\ell\}$, must find it strictly optimal to extend the deadline beyond

$T^*_{\kappa_j}$, where they face $\kappa_{j-1}-1$ opponents,

$$\left[\lambda\Pi_\sigma-(c+\theta)+\lambda(\kappa_{j-1}-1)\left[\mathbf{V}_{\kappa_{j-1}-1}+\mathbf{U}_{\kappa_{j-1}-1}\right]\right]e^{-\lambda T^*_{\kappa_j}} \\ >\phi+\lambda(\kappa_{j-1}-1)\mathbf{U}_{\kappa_{j-1}-1}. \quad \text{(A.18)}$$

Combining (A.16) and (A.17) gives (a) in Definition 4. Combining (A.17) and (A.18) gives (b) in Definition 4. Hence, we have that $\{\kappa_1,\kappa_2,...,\kappa_\ell\}\in\mathcal{G}_s$. Moreover, the first inequality in condition (A.15) follows from rearranging (A.17) while the second inequality in condition (A.15) follows from rearranging (A.16).

If Part: If there is $\{\kappa_1,...,\kappa_\ell\}\in\mathcal{G}_s$ such that $\{T^*_\kappa\}_{\kappa=1}^{|N|}$ satisfies the conditions (A.14) and (A.15), it holds $\{T^*_\kappa\}_{\kappa=1}^{|N|}\in\mathcal{T}_s$. To show this consider a decreasing profile $\{T^*_\kappa\}_{\kappa=1}^{|N|}$ and suppose there is $\{\kappa_1,...,\kappa_\ell\}\in\mathcal{G}_s$ such that $\{T^*_\kappa\}_{\kappa=1}^{|N|}$ satisfies the conditions (A.14) and (A.15). First, note that $\{\kappa_1,...,\kappa_\ell\}\in\mathcal{G}_s$ implies that the outer inequalities in (A.15) are consistent; i.e., that $\frac{1}{\lambda}\ln\left(G_{\kappa_{q-1}+1}\right)\leq\frac{1}{\lambda}\ln\left(G_{\kappa_q}\right)$ holds for all $q\in\{1,...,\ell\}$, where $\kappa_0=0$.

Next, consider an investor i that faces an opponent deadline profile

$$\{T_{js}\}_{j\in N\setminus i}=\{T^*_\kappa\}_{\kappa\in\{1,...,|N|\}\setminus\hat{\kappa}}$$

for some $\hat{\kappa}=\kappa_\iota$, where $\iota\in\{1,...,\ell\}$. I will show that setting $T_{is}=T^*_{\hat{\kappa}}$ is optimal for that investor. Writing $T=(T_{is},\{T_{js}\}_{j\in N\setminus i})$, we have for all $q\in\{\iota,...,\ell\}$ that

$$\operatorname{sgn}\left(\lim_{\hat{T}\uparrow T^*_{\kappa_q}}\left.\frac{\partial V^*_{is}(T)}{\partial T_{is}}\right|_{T_{is}=\hat{T}}\right)= \\ \operatorname{sgn}\Bigg(\left[\lambda\Pi_\sigma-(c+\theta)+\lambda(\kappa_q-1)\left[\mathbf{V}_{\kappa_q-1}+\mathbf{U}_{\kappa_q-1}\right]\right]e^{-\lambda T^*_{\kappa_q}} \\ -\left[\phi+\lambda(\kappa_q-1)\mathbf{U}_{\kappa_q-1}\right]\Bigg)\geq 0,$$

where the equality follows from Lemma 1 (appreciating the fact that the number of opponents on $[T^*_{\kappa_{q+1}},T^*_{\kappa_q})$ is equal to κ_q-1 for all $q\in\{\iota,...,\ell\}$ where $T^*_{\kappa_{\ell+1}}=0$) and the inequality follows from condition (A.15), ensuring that $\lambda^{-1}\ln(G_{\kappa_q})\geq T^*_{\kappa_q}$ holds for all $q\in\{\iota,...,\ell\}$. From Lemma 1 we also know that $\partial V^*_{is}(T)/\partial T_{is}$ is unimodal in the intervals between the opponent deadlines. Consequently, it holds $\partial V^*_{is}(T)/\partial T_{is}\geq 0$ for all $T_{is}\in[0,T^*_{\kappa_\iota})$.

Further, observe that, for all $q\in\{1,...,\iota\}$, we have

$$\operatorname{sgn}\left(\lim_{\hat{T}\downarrow T^*_{\kappa_q}} \left.\frac{\partial V^*_{is}(T)}{\partial T_{is}}\right|_{T_{is}=\hat{T}}\right) =$$
$$\operatorname{sgn}\Bigg(\left[\lambda\Pi_\sigma - (c+\theta) + \lambda\kappa_{q-1}\left[\mathbf{V}_{\kappa_{q-1}} + \mathbf{U}_{\kappa_{q-1}}\right]\right] e^{-\lambda T^*_{\kappa_q}} - \left[\phi + \lambda\kappa_{q-1}\mathbf{U}_{\kappa_{q-1}}\right] \Bigg) \le 0,$$

where the equality follows from Lemma 1 (appreciating the fact that the number of opponents on $[T^*_{\kappa_q}, T^*_{\kappa_{q-1}})$ is equal to κ_{q-1} for all $q \in \{1, ..., \iota\}$ where $T^*_{\kappa_0} = \infty$) and the inequality follows from condition (A.15), ensuring that $G_{\kappa_{q-1}+1} \le e^{-\lambda T^*_{\kappa_q}}$ holds. Thus, again by the fact that $\partial V^*_{is}(T)/\partial T_{is}$ is unimodal in the intervals between the opponent deadlines, it holds $\partial V^*_{is}(T)/\partial T_{is} \le 0$ for $T_{is} \in [T^*_{\kappa_\iota}, \infty)$. Together we obtain that choosing a deadline $T_{is} = T^*_{\kappa_\iota}$ maximizes $V^*_{is}(T)$, as desired.

It remains to show that for every $\{\kappa_1, ..., \kappa_\ell\} \in \mathcal{G}_s$ there is at least one $\{T^*_\kappa\}^{|N|}_{\kappa=1} \in \mathcal{T}_s$. In view of the only-if part it suffices to show that for a given $\{\kappa_1, ..., \kappa_\ell\} \in \mathcal{G}_s$ there exists a decreasing deadline profile such that (A.14) and (A.15) hold. Consider the profile $\{T^*_\kappa\}^{|N|}_{\kappa=1}$ satisfying

$$T^*_1 = T^*_2 = ... = T^*_{\kappa_1} = \hat{T}^*_{\kappa_1} > T^*_{\kappa_1+1} = ... = T^*_{\kappa_2} = \hat{T}^*_{\kappa_2} >$$
$$...$$
$$> T^*_{\kappa_{\ell-2}+1} = ... = T^*_{\kappa_{\ell-1}} = \hat{T}^*_{\kappa_{\ell-1}} > T^*_{\kappa_{\ell-1}+1} = ... = T^*_{|N|} = \hat{T}^*_{\kappa_\ell},$$

where
$$\hat{T}^*_{\kappa_j} = \frac{1}{\lambda}\left(\frac{\lambda\Pi_\sigma - (c+\theta) + \lambda(\kappa_j - 1)\left[\mathbf{V}_{\kappa_j-1} + \mathbf{U}_{\kappa_j-1}\right]}{\phi + \lambda(\kappa_j-1)\mathbf{U}_{\kappa_j-1}}\right), \; j \in \{1, ..., \ell\}.$$

This trivially satisfies condition (A.14). Appreciating that $\{\kappa_1, ..., \kappa_\ell\} \in \mathcal{G}_s$ implies $G_{\kappa_{q-1}+1} \le G_{\kappa_q}$ by Point (a) in Definition 4 then finally gives condition (A.15) and, hence, the claim. □

The last result in this section is a straightforward corollary of Proposition A.4. Part (a) follows directly from Part (c), appreciating when $\mathbf{U}_\kappa = \mathbf{V}_\kappa = 0$ for all κ, then $G_\kappa = (\lambda\Pi_\sigma - (c+\theta))/\phi$ for all κ. The proofs for the other parts make use of Lemma A.4.

Corollary A.1. *Fix a state $s = (\sigma, N) \in S$.*

(a) *If all continuation payoffs are zero ($\mathbf{U}_\kappa = \mathbf{V}_\kappa = 0$ for all κ), then there is a unique profile $\{T^*_\kappa\}^{|N|}_{\kappa=1} \in \mathcal{T}_s$. This profile is completely symmetric with $T^*_\kappa = \frac{1}{\lambda}\ln\left((\lambda\Pi_\sigma - (c+\theta))/\phi\right)$ for all $\kappa \in \{1, ..., |N|\}$.*

(b) *Suppose $\{G_\kappa\}^{|N|}_{\kappa=1}$ is strictly decreasing. Then, there is a unique profile $\{T^*_\kappa\}^{|N|}_{\kappa=1} \in \mathcal{T}_s$. This profile is completely asymmetric with $T^*_\kappa = \frac{1}{\lambda}\ln(G_\kappa)$ for all $\kappa \in \{1, ..., |N|\}$.*

(c) *Suppose* $\{G_\kappa\}_{\kappa=1}^{|N|}$ *is weakly increasing. Then, any profile* $\{T_\kappa^*\}_{\kappa=1}^{|N|} \in \mathcal{T}_s$ *is completely symmetric; i.e.,* $T_\kappa^* = T^*$ *for all* $\kappa \in \{1, ..., |N|\}$*. Specifically,* $T^* \in [\underline{T}, \overline{T}]$*, where* $\overline{T} \equiv \frac{1}{\lambda} \ln\left(G_{|N|}\right) \geq \frac{1}{\lambda} \ln\left(G_1\right) \equiv \underline{T}$.

Proof of Corollary A.1. Part (b) — Complete asymmetry and uniqueness is a direct consequence of Lemma A.4: When $\{G_\kappa\}_{\kappa=1}^{|N|}$ is strictly decreasing, then the unique element in $\mathcal{G}_s$ is $\{1, 2, ..., |N|\}$. The characterization then follows directly from Proposition A.4. Part (c) — Complete symmetry of the deadlines is also a direct consequence of Lemma A.4: When $\{G_\kappa\}_{\kappa=1}^{|N|}$ is strictly increasing, then the unique element in $\mathcal{G}_s$ is $\{|N|\}$. Again, the characterization follows directly from Proposition A.4. □

B Proofs of Propositions in Text

Proof of Proposition 1. The optimal deadline T_2^* follows directly from (a) in Corollary A.1, appreciating that for $\sigma = 2$ all continuation payoffs are zero ($\mathbf{U}_\kappa = \mathbf{V}_\kappa = 0$ for all relevant κ). The utility of the firm follows from Lemma A.1. As noted in the text, the utility of the investor is total dyad welfare minus the rent left to the firm. □

Proof of Proposition 2. To begin, observe that under the mutuall optimal continuation contract offers (cf. Proposition 1) we have from (5),

$$\lim_{\phi \to 0} \phi \cdot G_\kappa = \frac{\lambda \Pi_1 - (c + \theta) + \dfrac{\lambda(\kappa - 1)}{\lambda(\kappa - 1) + \rho} [\lambda \Pi_2 - (c + \theta)]}{1 + \dfrac{\lambda(\kappa - 1)}{\lambda(\kappa - 2) + \rho}} \equiv \bar{G}_\kappa. \tag{B.19}$$

This follows from expanding the fraction in (5) by λ and appreciating

$$\lim_{\phi \to 0} \left[V_{2,\kappa-1}^* + U_{2,\kappa-1}^*\right] = \frac{[\lambda \Pi_2 - (c + \theta)]}{\lambda(\kappa - 1) + \rho}$$

and

$$\lim_{\phi \to 0} \frac{U_{2,\kappa-1}^*}{\phi} = \frac{1}{\lambda(\kappa - 2) + \rho},$$

both of which follow from plugging in the expressions of the firm's and the investor's utilities (2)–(3) and appreciating that T_2^* diverges to infinity when $\phi \to 0$.

By construction, we have $\text{sgn}(G_\kappa - G_{\kappa'}) = \text{sgn}(\bar{G}_\kappa - \bar{G}_{\kappa'})$ for all sufficiently low ϕ whenever $\bar{G}_\kappa \neq \bar{G}_{\kappa'}$. This allows me to use $\{\bar{G}_\kappa\}_{\kappa=1}^{|N|}$ rather than $\{G_\kappa\}_{\kappa=1}^{|N|}$ to construct $\mathcal{G}_s$ in order to draw conclusions on the properties of the profile of mutually optimal deadlines when ϕ becomes small.

Fix κ, κ' satisfying $\kappa' > \kappa \geq 2$. Then we have from (B.19) that $\bar{G}_\kappa > \bar{G}_{\kappa'}$ is equivalent to

$$\Phi\left[\frac{\lambda(\kappa'-1)}{\lambda(\kappa'-2)+\rho} - \frac{\lambda(\kappa-1)}{\lambda(\kappa-2)+\rho}\right] > \frac{\lambda(\kappa'-1)}{\lambda(\kappa'-1)+\rho}\left[1+\frac{\lambda(\kappa-1)}{\lambda(\kappa-2)+\rho}\right] - \frac{\lambda(\kappa-1)}{\lambda(\kappa-1)+\rho}\left[1+\frac{\lambda(\kappa'-1)}{\lambda(\kappa'-2)+\rho}\right],$$

where Φ is defined in (6). Straightforward yet tedious calculations reveal that this is equivalent to

$$\Phi\lambda(\rho-\lambda)[\lambda(\kappa-1)+\rho][\lambda(\kappa'-1)+\rho] > \lambda(\rho-\lambda)\left[[\lambda(\kappa-1)+\rho][\lambda(\kappa'-1)+\rho]-\rho\lambda\right] + 2\lambda^4(\kappa-1)(\kappa'-1).$$

From this we get that $\bar{G}_\kappa > \bar{G}_{\kappa'}$ for $\kappa' > \kappa \geq 2$ is equivalent to $f(\kappa, \kappa') > 0$, where $f(\kappa, \kappa')$ is given by

$$f(\kappa,\kappa') = \lambda^2(\kappa-1)(\kappa'-1)\left[(\Phi-1)(\rho-\lambda)-2\lambda\right] + \rho(\rho-\lambda)\left[(\Phi-1)\left[\lambda(\kappa+\kappa'-2)+\rho\right]+\lambda\right]. \quad \text{(B.20)}$$

To continue, treat κ as a real number where necessary, define the binomial $g(\kappa) \equiv f(\kappa, \kappa+1)$ and observe that

$$g'(\kappa) = \lambda^2(2\kappa-1)\left[(\Phi-1)(\rho-\lambda)-2\lambda\right] + 2\rho(\rho-\lambda)\lambda[\Phi-1]. \quad \text{(B.21)}$$

Consequently, it holds $g''(k) \geq 0$ with $\lim_{k\to\infty} g(k) = \infty$ when $\Phi \geq (\rho+\lambda)/(\rho-\lambda)$ and $g''(\kappa) < 0$ with $\lim_{k\to\infty} g(k) = -\infty$ when $\Phi < (\rho+\lambda)/(\rho-\lambda)$. Moreover, for $\kappa = 2$ we have

$$g(2) = 2\lambda^2\left[(\Phi-1)(\rho-\lambda)-2\lambda\right] + \rho(\rho-\lambda)\left[(\Phi-1)(3\lambda+\rho)+\lambda\right],$$

which is increasing in Φ, $\partial g(2)/\partial\Phi > 0$, because we consider the case $\rho > \lambda$.

Claim (a). Plugging in $\Phi = \rho/(\lambda+\rho)$ in the expression for $g(2)$ above, we obtain $g(2) < 0$, which together with the fact that $g(2)$ increases in Φ implies that $g(2) < 0$ for all $\Phi \leq \rho/(\lambda+\rho)$. Further, because $\rho/(\lambda+\rho) < 1 < (\rho+\lambda)/(\rho-\lambda)$, we obtain from (B.21) that $g(\kappa)$ is strictly decreasing for all $\kappa \geq 2$. Together, we have $g(\kappa) < 0$ for all $\kappa \geq 2$, implying that $\bar{G}_\kappa$ is strictly increasing for all $\kappa \geq 2$. Finally, observe that $\Phi < \rho/(\lambda+\rho)$ implies $\bar{G}_1 < \bar{G}_2$. Indeed,

$$\bar{G}_1 < \bar{G}_2 \iff \Phi < \frac{\Phi + \frac{\lambda}{\lambda+\rho}}{1+\frac{\lambda}{\rho}} \iff \Phi < \frac{\rho}{\lambda+\rho}.$$

So, we obtain that $\bar{G}_\kappa$ (and, hence, G_κ for all sufficiently small $\phi > 0$) is also strictly increasing for all $\kappa \geq 1$. Together with Corollary A.1 we then have the claim.

Claim (b). Observe that for $\Phi = (\rho+\lambda)/(\rho-\lambda)$ it holds $g(2) > 0$. Because $g(2)$ increases in Φ, this must hold for all $\Phi \geq (\rho+\lambda)/(\rho-\lambda)$. Further, because $(\rho+\lambda)/(\rho-\lambda) > 1$, we obtain from (B.21) that $g(\kappa)$ is strictly increasing for all $\kappa \geq 2$. This implies that $\bar{G}_\kappa > \bar{G}_{\kappa+1}$ for all $\kappa \geq 2$. Moreover, recall from the proof of claim (a) above that $\bar{G}_1 > \bar{G}_2$ iff $\Phi > \rho/(\rho+\lambda)$. Appreciating that $(\rho+\lambda)/(\rho-\lambda) > \rho/(\rho+\lambda)$ then yields that $\bar{G}_\kappa$ (and, hence, G_κ for all sufficiently small $\phi > 0$) is strictly decreasing for all $\kappa \geq 1$. Together with Corollary A.1 we then have the claim.

Claim (c). It remains to analyze the case $\Phi \in (\rho/(\lambda+\rho), (\rho+\lambda)/(\rho-\lambda))$. Doing so, I suppose Π_1 and Π_2 are such that no two elements in the sequence $\{\bar{G}_\kappa\}_{\kappa=1}^{|N|}$ are equal, which holds for generic values of Π_1 and Π_2, as is readily verified from (B.19).

I first argue for the existence of $\overline{k}$. We know from above that $\Phi > \rho/(\lambda+\rho)$ implies $\bar{G}_1 > \bar{G}_2$. Also, we know that, as a consequence of $\Phi < (\rho+\lambda)/(\rho-\lambda))$, the binomial $g(\kappa)$ is strictly concave and that $g(\kappa)$ diverges to minus infinity when κ grows large. Together with the assumption that all elements in $\{\bar{G}_\kappa\}_{\kappa=1}^{|N|}$ are distinct, the following cutoff is well defined:

$$\overline{k} = \min\{\kappa \in \{2, 3, ..., |N|\} : g(\kappa) < 0\}, \tag{B.22}$$

with the usual convention that $\min\{\emptyset\} = |N|$. Because $g(\kappa) > 0$ for all natural $\kappa < \overline{k}$ and $g(\kappa) < 0$ for all natural $\kappa \geq \overline{k}$, together with $\bar{G}_1 > \bar{G}_2$, it thus holds $\bar{G}_\kappa > \bar{G}_{\kappa+1}$ for all $\kappa \in \{1, ..., \overline{k}-1\}$ and $\bar{G}_\kappa \leq \bar{G}_{\kappa+1}$ for all $\kappa \in \{\overline{k}, ..., |N|\}$. That is, the cutoff index $\overline{k}$ refers to the lowest element in the sequence $\{\bar{G}_\kappa\}_{\kappa=1}^{|N|}$. Also, because the value of $g(\kappa)$ for $\kappa < |N|$ does not depend on $|N|$ and $g(\kappa)$ diverges to minus infinity when κ grows large, it is immediate that if $\overline{k} < |N|$, then $\overline{k}$ does not change when we increase $|N|$.

Next I show existence of $\underline{k}$. Because the sequence $\{\bar{G}_\kappa\}_{\kappa=1}^{|N|}$ is first decreasing, up to $\overline{k}$, and then increasing, the following cutoff is also well defined:

$$\underline{k} = \begin{cases} 1 & \text{if } \bar{G}_1 < \bar{G}_{|N|} \\ \min\{\kappa \in \{2, 3, ..., \overline{k}\} : f(\kappa, |N|) < 0\} & \text{if } \bar{G}_1 > \bar{G}_{|N|}, \end{cases} \tag{B.23}$$

again with the convention that $\min\{\emptyset\} = \overline{k}$. By construction, the index $\underline{k}$ refers to the earliest element in $\{\bar{G}_\kappa\}_{\kappa=1}^{|N|}$ that is below $\bar{G}_{|N|}$. Recalling that we have $\text{sgn}(G_\kappa - G_{\kappa'}) = \text{sgn}(\bar{G}_\kappa - \bar{G}_{\kappa'})$ for all sufficiently low ϕ whenever $\bar{G}_\kappa \neq \bar{G}_{\kappa'}$, the characterization in (c) then follows from Lemma A.4 together with Proposition A.4.

Last, I argue that both $\underline{k}$ and $\overline{k}$ are non-decreasing ceteris paribus in Φ. Observe that both $f(\kappa, \kappa')$ and $g(\kappa)$ are increasing in Φ. The latter, i.e., $g(\kappa)$ increasing in Φ, implies that if $g(\hat{\kappa}) > 0$ for some $\hat{\kappa}$ and Φ, then $g(\hat{\kappa}) > 0$ for $\Phi' > \Phi$. In view of (B.22), the fact that $g(\kappa)$ is strictly concave, and that $g(\kappa)$ diverges to minus infinity when κ grows large, this observation implies that $\overline{k}$ is non-decreasing in Φ. From the former we obtain that $f(\kappa, |N|)$ is increasing in Φ. We also know that $\bar{G}_\kappa$ is U-shaped with lowest element $\overline{k}$. This implies that, keeping $|N|$ fixed, $f(\kappa, |N|)$ changes signs at most once on $\{2, 3, ..., \overline{k}\}$, from positive to negative. But then, it follows from (B.23) that $\underline{k}$ is non-decreasing in Φ, too. □

Proof of Proposition 3. The proof proceeds in three steps. I begin by showing that, for every state $s = (\sigma, N)$, the set $\mathcal{G}_s$ is non-empty. In a second step, I restrict attention to contract profiles C in full-R&D contracts and show that there is an equilibrium in such restricted contracts. Finally, I show that under condition (8), asking $a_{is}(t) = 1$ for all $t \in [0, T_{is}]$ from the firm is indeed mutually optimal for the investors in any state s.

Step I: I want to show that, for every state $s = (\sigma, N)$, the set $\mathcal{G}_s$ is non-empty. I show this by constructing a particular index set $\{\kappa_1, \kappa_2, ..., \kappa_\ell\} \in \mathcal{G}_s$. Consider a state s and the corresponding sequence $\{G_\kappa\}_{\kappa=1}^{|N|}$, with elements defined in (5). Then, apply the following algorithm:

1. Pick the largest element in $\{G_\kappa\}_{\kappa=1}^{|N|}$ (if there are multiple, take the element with the highest κ), and set κ_1 to its index, κ.

2. From the remaining elements in $\{G_\kappa\}_{\kappa=1}^{|N|}$ having a higher κ than the one just picked, take again the largest element (and again, if there are multiple, take the element with the highest κ), and set κ_2 equal to its index, κ.

3. Repeat Step 2 until the set of remaining elements in $\{G_\kappa\}_{\kappa=1}^{|N|}$ having a higher index κ than the one just picked is empty.

First, observe that the last index, κ_ℓ, thus chosen always corresponds to $|N|$. For $q \in \{1, ..., \ell\}$, condition (a) in Definition 4 must hold for otherwise κ_q would not have been chosen in step q. For the same reason, condition (b) in Definition 4 must hold for $q \in \{1, ..., \ell - 1\}$. Last, observe that condition (b) holds for $q = 0$ by construction.

Step II: I now argue that from Step I we obtain that, within the class of full-R&D contracts, a profile of mutually optimal contracts exists. We know that, in all states $s \in S$ with $\sigma = 2$, the optimal deadline T^*_{is} exists and is the same for all firms and, consequently, so is the payment function R^*_{is}. Hence, the continuation value for any dyad in any state s with $\sigma = 1$ when an opponent takes up a breakthrough opportunity is the same and independent of the remaining dyads' identities. Step I together with Proposition (A.4) then gives existence of a set of mutually optimal deadlines for that state. The set of mutually optimal deadlines pins down the firm owners' utilities and, hence, their payment functions, R^*_{is}. Existence of a full-R&D equilibrium thus follows.

Step III: I now establish that under condition (8) asking $a_{is}(t) = 1$ from the firm for all $t \in [0, T_{is}]$ is indeed always optimal for all investors in any state s. Recall from the proof of Lemma A.2 that under a contract profile C the value of the investor, $V_{is}(t; C)$ on some interval $t \in [T_\kappa, T_{\kappa-1})$ when κ dyads are present, approximately satisfies for small $\mathrm{d}t > 0$,

$$V_{is}(t; C) = a_{is}(t)[\lambda R_{is}(t) - c] + \lambda(\kappa - 1)\mathbf{V}_{\kappa-1} - \theta \\ + (1 - (\lambda(\kappa - 1 + a_{is}(t)) + \rho)\mathrm{d}t)V_{is}(t + \mathrm{d}t; C).$$

From the limit $\mathrm{d}t \to 0$, we see that $a_{is}(t) = 1$ for all $t \in [T_\kappa, T_{\kappa-1})$ maximizes $V_{is}(T_\kappa; C)$ for any initial condition on $V_{is}(T_{\kappa-1}; C)$ if

$$R_{is}(t) - \frac{c}{\lambda} - V_{is}(t; C) \geq 0.$$

Any contract must be incentive compatible. Hence, substituting from (A.2) for $R_{is}(t)$, the above condition is equivalent to

$$\Pi_\sigma - \frac{c + \phi}{\lambda} \geq U_{is}(t; a_i, C) + V_{is}(t; C). \tag{B.24}$$

The right side of (B.24) corresponds to total dyad welfare at $t \geq 0$. To finish the proof it suffices to appreciate that

$$\frac{\lambda}{\lambda + \rho}\left[\max_{\sigma' \geq \sigma} \Pi_{\sigma'} - \frac{c + \theta}{\lambda}\right]$$

is an upper bound on total dyad welfare under any feasible profile C, because it corresponds to the maximum welfare the dyad could secure if it were allowed to freely choose the spot for which to run in isolation. □

C More on Vanishing Moral Hazard

This appendix treats the case $\rho \leq \lambda$. We have the following result:

Proposition C.5. *Fix a state $s = (\sigma, N)$ with $\sigma = 1$ and let $\{T^*_\kappa\}_{\kappa=1}^{|N|} \in \mathcal{T}_s$. The mutually optimal deadlines satisfy*

$$\lim_{\phi \to 0} T^*_\kappa = \infty$$

for all κ. Further, suppose $|N| \geq 2$ and $\rho \leq \lambda$. Then, for every $\phi > 0$ sufficiently close to zero:

(a) *If $\Phi < \rho/(\lambda+\rho)$, then $\{G_\kappa\}_{\kappa=1}^{|N|}$ is strictly increasing and, hence, any profile $\{T_\kappa^*\}_{\kappa=1}^{|N|} \in \mathcal{T}_s$ is completely symmetric.*

(b) *If $\Phi > (\lambda(|N|-2)+\rho)/(\lambda(|N|-1)+\rho)$, then all $\{T_\kappa^*\}_{\kappa=1}^{|N|} \in \mathcal{T}_s$ have $|N|-1$ investors choosing the same deadline and one investor choosing a strictly longer deadline.*

(c) *For a.e. $\Phi \in (\rho/(\lambda+\rho), (\lambda(|N|-2)+\rho)/(\lambda(|N|-1)+\rho))$, both kinds of profiles described in (a) and (b) above are mutually optimal.*

Proof of Proposition C.5. Consider $\bar{G}_\kappa$ defined in (B.19) from the proof of Proposition 2. While the numerator of $\bar{G}_\kappa$ strictly increases in κ, the denominator weakly decreases in $\kappa \geq 2$ if an only if $\rho \leq \lambda$. As a consequence, $\bar{G}_\kappa$ strictly increases in $\kappa \geq 2$ if $\rho \leq \lambda$. Further, note that $\bar{G}_1 < \bar{G}_\kappa$ holds if and only if $\frac{\lambda(\kappa-2)+\rho}{\lambda(\kappa-1)+\rho} < \Phi$, where Φ is defined in (6).

Consequently, in case (a) we have $\bar{G}_1 < \bar{G}_2$, giving that $\{\bar{G}_\kappa\}_{\kappa=1}^{|N|}$ is strictly increasing. But then, recalling from the proof of Proposition 2 that we have $\text{sgn}(G_\kappa - G_{\kappa'}) = \text{sgn}(\bar{G}_\kappa - \bar{G}_{\kappa'})$ for all sufficiently low ϕ whenever $\text{sgn}(\bar{G}_\kappa - \bar{G}_{\kappa'}) \in \{-1, 1\}$, the sequence $\{G_\kappa\}_{\kappa=1}^{|N|}$ is strictly increasing for all $\phi > 0$ sufficiently small. The claim thus follows from Corollary A.1.

In case (b) we have $\bar{G}_1 > \bar{G}_k$. Consequently, the elements of $\{G_\kappa\}_{\kappa=1}^{|N|}$ are distinct for all $\phi > 0$ sufficiently small. Letting $\underline{k} = \overline{k} = 2$, the claim then follows from Lemma A.4 together with Proposition A.4.

Last, in case (c), we have $\bar{G}_{|N|} > \bar{G}_1 > \bar{G}_2$. Since $\{\bar{G}_\kappa\}_{\kappa=1}^{|N|}$ strictly increases in $\kappa \geq 2$, it might be that $\bar{G}_1$ is equal to some $\bar{G}_\kappa$, $\kappa \geq 3$. As can easily be verified from (B.19), this only happens for non-generic values of Π_1 and Π_2. Consequently, for generic values of Φ, all elements of $\{G_\kappa\}_{\kappa=1}^{|N|}$ are distinct for all $\phi > 0$ sufficiently small. Letting $\underline{k} = 1$ and $\overline{k} = 2$, the claim then follows again from Lemma A.4 together with Proposition A.4. □

References

Awaya, Y. and V. Krishna (2020): "Startups and Upstarts: Disadvantageous Information in R&D," *Journal of Political Economy*.

Bengtsson, O. and B. A. Sensoy (2015): "Changing the Nexus: The Evolution and Renegotiation of Venture Capital Contracts," *Journal of Financial and Quantitative Analysis*, 50, 349–375.

Bulow, J. and P. Klemperer (1999): "The Generalized War of Attrition," *American Economic Review*, 89, 175–189.

Cabral, L. (2011): "Technology Uncertainty, Sunk Costs, and Industry Shakeout," *Industrial and Corporate Change*, 21, 539–552.

Casamatta, C. (2003): "Financing and Advising: Optimal Financial Contracts with Venture Capitalists," *The Journal of Finance*, 58, 2059–2085.

Choi, J. P. (1991): "Dynamic R&D Competition under "Hazard Rate" Uncertainty," *The RAND Journal of Economics*, 596–610.

Davenport, D. (2022): "Predictably Bad Investments: Evidence from Venture Capitalists," Working Paper.

De Bettignies, J.-E. and J. A. Brander (2007): "Financing entrepreneurship: Bank finance versus venture capital," *Journal of Business Venturing*, 22, 808–832.

Denicolo, V. (1996): "Patent Races and Optimal Patent Breadth and Length," *The Journal of Industrial Economics*, 249–265.

——— (1999): "The Optimal Life of a Patent when the Timing of Innovation is Stochastic," *International Journal of Industrial Organization*, 17, 827–846.

Drover, W., L. Busenitz, S. Matusik, D. Townsend, A. Anglin, and G. Dushnitsky (2017): "A Review and Road Map of Entrepreneurial Equity Financing Research: Venture Capital, Corporate Venture Capital, Angel Investment, Crowdfunding, and Accelerators," *Journal of Management*, 43, 1820–1853.

Elitzur, R. and A. Gavious (2003): "Contracting, Signaling, and Moral Hazard: A Model of Entrepreneurs, 'Angels', and Venture Capitalists," *Journal of Business Venturing*, 18, 709–725.

Erkal, N. and S. Scotchmer (2009): "Scarcity of Ideas and R&D Options: Use it, Lose it or Bank it," National Bureau of Economic Research Working Paper No. 14940.

Georgiadis, G., Y. Kim, and H. D. Kwon (2020): "The Absence of Attrition in the War of Attrition under Complete Information," Working Paper.

Gort, M. and S. Klepper (1982): "Time Paths in the Diffusion of Product Innovations," *The Economic Journal*, 92, 630–653.

Green, B. and C. R. Taylor (2016): "Breakthroughs, Deadlines, and Self-Reported Progress: Contracting for Multistage Projects," *American Economic Review*, 106, 3660–99.

Hendricks, K., A. Weiss, and C. Wilson (1988): "The War of Attrition in Continuous Time with Complete Information," *International Economic Review*, 663–680.

Hopenhayn, H. A. (1993): “The Shakeout,” Unpublished Manuscript.

Häfner, S. (2021): “On Shakeouts and Staggered Exit: An R&D Race with Moral Hazard and Multiple Prizes,” Working Paper.

Kelly, P. and M. Hay (2003): “Business Angel Contracts: The Influence of Context,” *Venture Capital*, 5, 287–312.

Klepper, S. (1996): “Entry, Exit, Growth, and Innovation over the Product Life Cycle,” *The American economic review*, 562–583.

Klepper, S. and K. L. Simons (2000): “The Making of an Oligopoly: Firm Survival and Technological Change in the Evolution of the US Tire Industry,” *Journal of Political economy*, 108, 728–760.

Konrad, K. A., W. Peters, and K. Wärneryd (2004): “Delegation in First-Price All-Pay Auctions,” *Managerial and Decision Economics*, 25, 283–290.

Kräkel, M. (2002): “Delegation and Strategic Incentives for Managers in Contests,” *Managerial and Decision Economics*, 23, 461–470.

——— (2004): “R&D Spillovers and Strategic Delegation in Oligopolistic Contests,” *Managerial and Decision Economics*, 25, 147–156.

——— (2005): “Strategic Delegation in Oligopolistic Tournaments,” *Review of Economic Design*, 9, 377–396.

Kräkel, M. and D. Sliwka (2006): “Strategic Delegation and Mergers in Oligopolistic Contests,” *Journal of Economics and Business*, 58, 119–136.

Lenox, M. J., S. F. Rockart, and A. Y. Lewin (2007): “Interdependency, Competition, and Industry Dynamics,” *Management Science*, 53, 599–615.

Loury, G. C. (1979): “Market Structure and Innovation,” *The Quarterly Journal of Economics*, 395–410.

Malueg, D. A. and S. O. Tsutsui (1997): “Dynamic R&D Competition with Learning,” *The RAND Journal of Economics*, 751–772.

Mason, C. M. and R. T. Harrison (2002): “Is it Worth it? The Rates of Return from Informal Venture Capital Investments,” *Journal of Business Venturing*, 17, 211–236.

Moroni, S. (2019): “Experimentation in Organizations,” Working Paper.

Moscarini, G. and F. Squintani (2010): "Competitive Experimentation with Private Information: The Survivor's Curse," *Journal of Economic Theory*, 145, 639–660.

Nanda, R., S. Samila, and O. Sorenson (2020): "The Persistent Effect of Initial Success: Evidence from Venture Capital," *Journal of Financial Economics*, 137, 231–248.

Reinganum, J. F. (1981): "Dynamic Games of Innovation," *Journal of Economic Theory*, 25, 21–41.

——— (1982): "A Dynamic Game of R and D: Patent Protection and Competitive Behavior," *Econometrica*, 671–688.

Sohl, J. (1999): "The Early-Stage Equity Market in the USA," *Venture Capital*, 1, 101–120.

Wärneryd, K. (2000): "In Defense of Lawyers: Moral Hazard as an Aid to Cooperation," *Games and Economic Behavior*, 33, 145–158.

Wiltbank, R., S. Read, N. Dew, and S. D. Sarasvathy (2009): "Prediction and Control under Uncertainty: Outcomes in Angel Investing," *Journal of Business Venturing*, 24, 116–133.

Wolf, C. (2017): "Informative Milestones in Experimentation," Working Paper.