Sources of consumer information *

Frédéric KOESSLER † Regis RENAULT ‡

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Preliminary and incomplete.

Abstract

Consumers can either acquire product information at some cost or base their purchase decision solely on whatever information the seller provides. The optimal information design by a monopoly firm deters information acquisition completely and induces a purchase if and only if the consumer's valuation exceeds a certain threshold. As the information acquisition cost increases, the probability of a purchase increases, consumer welfare deteriorates while profit and social welfare are improved until they reach their first best level. The ability of the firm to inform the consumer may either benefit or hurt the consumer so that, if providing information is costly, the firm informs either too much or too little as compared to a second best social optimum.

KEYWORDS: information disclosure, information acquisition, advertising, search.

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†HEC and CNRS – CNRS
‡CY Cergy Paris Universite- Thema.

1 Introduction

When we contemplate buying an unfamiliar product, we can obtain a wealth of information through online sources such as consumer reviews, forums, specialized blogs or online publications. Acquiring such information however takes time and we might prefer to save the associated cost by only relying on the information the seller of the product willingly provides. The present paper explores how the availability of independent information sources for consumers affects the seller's disclosure strategy and the buyers' well being. Does the buyer benefit from having acess to that information? Does the information provided by the seller improve the consumers' situation by allowing them to save on the cost of acquiring information?

We consider a simple setting where a firm sells a product to a buyer who initially does not know his valuation. The firm may provide some certifiable information to the buyer. After learning that information and the product's price, the buyer decides whether to collect additional information at some cost or to take his purchase decision only relying on the firm's information.

We show that the firm optimally designs its product information so as to completely deter information acquisition by the buyer. The optimal disclosure is characterized by a threshold on the match between the product and the consumer such that the latter learns whether or not the match is above the threshold but learns no farther information. As the cost of information acquisition increases, the information disclosed to a consumer who purchases the product is deteriorated (i.e. the threshold decreases). As a result, the probability that the consumer buys the product increases. This optimal information disclosure allows the firm to increase its price up to a point where it can extract the entire expected consumer surplus. If the information acquisition cost is high enough so that the consumer has zero expected surplus, then a farther increase of that cost induces a decrease in price until information acquisition becomes costly enough so the firm achieves a first best outcome: the consumer buys if and only if he has valuation above marginal cost and the entire surplus is extracted by the seller.

An increased cost of information for the consumer results in a lower consumer welfare. The source of this deteriorated welfare is that, because he buys a product about which the firm provides poorer information, his expected loss from buying with a negative surplus becomes larger. However, profit increases and social surplus always improves if the consumer's information acquisition becomes more costly.

The impact of the seller's optimal information disclosure on the buyer is twofold. On the one hand, he saves on the information cost he would incur if the seller provided no information. On the other hand, optimal information design allows the seller to capture more surplus from the consumer than if the consumer was perfectly informed. We characterize situations where the externality of seller information imparted on the buyer is either positive or negative. Hence, if providing information is costly, the firm will either over provide or under provide information as compared to what is socially optimal.

A similar problem is analyzed in Wang (2017). However, he does not consider optimal information design and restricts the firm's disclosure strategy to be the one introduced by Lewis and Sappington (2014). Our work is also related to the literature on optimal product information disclosure as in Saak (2006) or Anderson and Renault (2006) as well as to the analysis of optimal information design when the receiver can obtain information independently at some cost in Matyskova and Montes (2023).

The model is presented in the next section. Section 3 provides useful preliminary results. The equilibrium is presented in Section 4, which also provides some welfare analysis.

2 Model

The match value is $v \in V = [0, 1]$, drawn from the distribution function G, with strictly positive and twice continuously differentiable density. We assume that $\frac{g(v)}{1-G(v)}$ is strictly increasing, so the profit $p(1 - G(p))$ is strictly quasi-concave and $p_M \in \arg\max_p p(1 - G(p))$ is unique and satisfies

$$p_M = \frac{1 - G(p_M)}{g(p_M)}.$$

Let $\pi_M = \max_p p(1 - G(p)) = p_M(1 - G(p_M))$ be the monopoly profit. The prior expected match is

$$\mu = E(v) = \int_0^1 v dG(v).$$

Timing:

1. nature determines the match v according to the prior G; nobody observes it;

2. the firm chooses a price p and an information disclosure policy $X : V \to \Delta(M)$ for some message space M;

3. the consumer observes the price p, the information disclosure policy X and the realized signal generated by that policy, denoted $\bar{X}$.

4. the consumer chooses $a \in A = \{drop, buy, search\}$;

5. if the consumer drops he gets 0; if he buys he gets $v - p$; if he searches, he pays $s > 0$, observes v, then buys if $v \geq p$ and drops if $v < p$. The firm gets p if the consumer buys, and 0 if the consumer drops.

Remark 1 Wang restricts attention to the following class of information disclosure policies: the firm chooses $\eta \in [0, 1]$: for each v, the signal is $m = v$ with probability η, and m is drawn from the distribution function G with probability $1 - \eta$. Then, the conditional expected match of the consumer when he observes message m is

$$E(v \mid m) = \eta m + (1 - \eta)\mu.$$

Observe that $\eta = 1$ corresponds to full information disclosure and $\eta = 0$ corresponds to no information disclosure. ◇

3 Preliminary results and benchmaks

Proposition 1 *It is without loss of generality to consider a threshold information disclosure policy such that the consumer is recommended to buy when $v \geq \tilde{v}$ and is recommended to drop when $v < \tilde{v}$, for some $\tilde{v} \in [0,1]$. That is, $M = \{drop, buy\}$, $X(buy \mid v) = 1$ if $v \geq \tilde{v}$, $X(drop \mid v) = 1$ if $v < \tilde{v}$, and the consumer follows the recommended action. In addition, at the optimum for the firm we have $p > \tilde{v}$.*

Proof. (Sketch) First, from the revelation principle (**?**), it is without loss of generality to let $M = A$, i.e., the information disclosure policy is a recommendation system $X : [0,1] \to \Delta(A)$ and we let $X(a \mid s)$ denote the probability of receiving signal a if the valuation is v. In addition, it is without loss of generality to require obedience from the consumer, i.e., to require that the consumer chooses action a when getting the signal a. Hence, $X(a \mid s)$ is the probability that the consumer plays a when the match value is v. In particular, we can impose incentive compatibility constraints for those who receive signal *buy*: $E_X(v \mid buy) \geq p$ so they don't drop and $E_X(v \mid buy) - p \geq E_X(v \mid buy; v \geq p) - p - s$ so they don't search (where $E_X(v \mid a)$ is the expectation of the match according to the posterior induced by X when the signal is a). Similarly, incentive compatibility constraints for those who receive signal *drop* are: $E_X(v \mid drop) - p < 0$ so they do not buy and $E_X(v \mid drop; v \geq p) - p - s < 0$ so they do not search.

Now consider some incentive compatible disclosure policy X_0. We show that there exists a disclosure policy X such that $X(search) = 0$ yielding the same probabilities of buying and dropping, as X_0 (where $X(a)$ is the unconditional probability of signal a with policy X). Take X defined as follows: if $v \geq p$ then $X(buy \mid v) = X_0(buy \mid v) + X_0(search \mid v)$ and $X(drop \mid v) = X_0(drop \mid v)$; if $v < p$ then $X(buy \mid v) = X_0(buy \mid v)$ and $X(drop \mid v) = X_0(drop \mid v) + X_0(search \mid v)$. Then we have $E_X(v \mid buy) = \frac{X_0(buy)}{X(buy)} E_{X_0}(v \mid buy) + \frac{X_0(search|v \geq p)[1-G(p)]}{X(buy)} E_{X_0}(v \mid search; v \geq p)$. The first expectation is at least p from IC of X_0 and the second is at least p by construction so a buyer receiving signal *buy* does not deviate to D. He will not search either because his benefit from searching (which only arises when $v < p$) is smaller than with disclosure policy X_0. The arguments for showing that a buyer who receives the *drop* signal will not want to deviate are symmetric.

Next, consider some disclosure policy X_0 such that $X_0(search) = 0$. Let $\tilde{v}$ be the unique solution to $1 - G(\tilde{v}) = X_0(buy)$ and define a disclosure policy X such that $X(buy \mid v) = 1$ if $v \geq \tilde{v}$ and $X(drop \mid v) = 1$ otherwise. First, there is no information acquisition with either X_0 or X and they both yield the same probability of purchase. Furthermore, if $X_0 \neq X$, then there exists a non zero measure subset of $[\tilde{v}, 1]$ such that $X_0(buy \mid v) < 1$ and some non zero measure subset of $[0, \tilde{v}]$ such that $X_0(buy \mid v) > 0$, which implies that $E_{X_0}(v \mid buy) < E_X(v \mid buy)$, $E_{X_0}(v \mid drop) > E_X(v \mid drop)$ and, for $p > \tilde{v}$, $E_X(v \mid buy) - E_X(v \mid buy; v \geq p) = E_X(v \mid buy; v < p) > E_{X_0}(v \mid buy; v < p)$ so that, if X_0 is incentive compatible then so is X.

Given X above, a firm charging $p < \tilde{v}$ could increase p slightly without violating any incentive compatibility constraint and hence make more profit. So we must have $p \geq \tilde{v}$. ■

From this result, the program of the firm is:

$$\max_{p,\tilde{v}} p(1 - G(\tilde{v}))$$

under the obedience constraints

$$E(v \mid v \geq \tilde{v}) - p \geq 0 \tag{1}$$

$$E(v \mid v \geq \tilde{v}) - p \geq \Pr(v \geq p \mid v \geq \tilde{v}) E(v - p \mid v \geq p) - s \tag{2}$$

Observe that the profit is quasi-concave in $\tilde{v}$ and p (iso-profit curves are strictly convex). The first constraint guarantees that the buyer does not prefer to drop when suggested to buy. The second constraint guarantees that the buyer does not prefer to search when suggested to buy. Since $p \geq \tilde{v}$, the obedience contraint when suggested to drop is always satisfied. Notice that the obedience constraints can be rewritten as

$$\int_{\tilde{v}}^{1} (v - p) g(v) dv \geq 0 \tag{Δ}$$

$$s \geq \frac{\int_{\tilde{v}}^{p}(p-v)g(v)dv}{(1-G(\tilde{v}))} := \phi(\tilde{v},p). \tag{Σ}$$

These constraints satisfied at equality are respectively denoted by (Δ_b) and (Σ_b):

$$\int_{\tilde{v}}^{1}(v-p)g(v)dv = 0 \tag{Δ_b}$$

$$s = \frac{\int_{\tilde{v}}^{p}(p-v)g(v)dv}{(1-G(\tilde{v}))} := \phi(\tilde{v},p). \tag{Σ_b}$$

Observe that the LHS of (Δ_b) is the consumer welfare. Social welfare is simply $\int_{\tilde{v}}^{1} vg(v)dv$, which is independent of the price and is increasing in the probability of trade $1-G(\tilde{v})$.

Define the Δ_b (Σ_b, respectively) curve as the set of points $(\tilde{v},p)$ such that (Δ_b) ((Σ_b), respectively) is satisfied.

Full disclosure Full disclosure is equivalent to the case $\tilde{v}=p$. In this case both constraints (Δ) and (Σ) are always satisfied strictly, the firm maximizes $p(1-G(p))$, so $p=p_M$ and $\pi=\pi_M$. Full disclosure is also equivalent to $s=0$.

No disclosure With no disclosure we cannot use the approach above because consumers might search (at least for low s). So for each price $p \leq \mu$, either (Σ) with $\tilde{v}=0$ is satisfied, all buy and the profit is p. Or (Σ) with $\tilde{v}=0$ is not satisfied, they all search, and the profit is $p(1-G(p))$. For $\tilde{v}=0$, (Σ) can be rewritten as

$$\Phi_G(p) := \int_0^p (p-v)g(v)dv \leq s \iff p \leq \Phi_G^{-1}(s),$$

where Φ_G^{-1} is increasing. So under no disclosure we have:

If $s \leq \Phi_G(\pi_M)$, the optimal price is $\min\{p_M,\mu\}$, with profit π_M if $p_M \leq \mu$ or $\mu(1-G(\mu))$ if $p_M \geq \mu$;

If $\Phi_G(\pi_M) \leq s \leq s_2 := \Phi_G(\mu)$, the optimal price is $\Phi_G^{-1}(s) \leq \mu$ with profit $\Phi_G^{-1}(s)$;

If $s \geq s_2$, the optimal price is μ with profit μ

(This is actually almost what is done before Proposition 1 in Wang).

4 Profit maximization

The firm extracts all surplus when both constraints are satisfied for $p = \mu$ and $\tilde{v} = 0$, so the profit is μ. For $p = \mu$ and $\tilde{v} = 0$, (Δ) is binding, so the firm gets the first best if (Σ) is satisfied, i.e.,

$$s \geq \int_0^{\mu} (\mu - v) g(v) dv := s_2$$

Hence, if $s \geq s_2$ the firm gets its first best and if $s < s_2$ then at least one constraint binds.

If $s < s_2$ it cannot be the case that only constraint (Δ) binds because then the program of the firm is to maximize

$$E(v \mid v \geq \tilde{v})(1 - G(\tilde{v})) = \int_{\tilde{v}}^1 v g(v) dv,$$

which is decreasing in $\tilde{v}$, so the optimal threshold would be $\tilde{v} = 0$, with $p = \mu$, which is the first best and is incentive-compatible only for $s \geq s_2$.

Therefore, we have:

Lemma 1 *If $s \geq s_2$ the firm gets its first best: the price and profit is $p = \mu$, no information is disclosed ($\tilde{v} = 0$) and only (Δ) binds. If $s < s_2$, then either both constraints (Δ) and (Σ) bind, or only (Σ) binds.*

Note: If (Δ) and (Σ) bind, then p and $\tilde{v}$ is the solution of the system

$$\int_{\tilde{v}}^1 (v - p) g(v) dv = 0 \tag{3}$$

$$(1 - G(\tilde{v}))s = \int_p^1 (v - p) g(v) dv. \tag{4}$$

Lemma 2 *Along the Σ_b curve, we have*

$$\frac{d\tilde{v}}{dp} = \frac{1 - G(\tilde{v})}{g(\tilde{v})} \frac{G(p) - G(\tilde{v})}{\int_{\tilde{v}}^p 1 - G(v)\, dv} > 1.$$

Proof. First observe that, by integrating by parts, we have

$$\phi(\tilde{v}, p) = \frac{\int_{\tilde{v}}^{p} G(v) - G(\tilde{v})\, dv}{1 - G(\tilde{v})}.$$

Hence,

$$\frac{\partial \phi}{\partial p} = \frac{G(p) - G(\tilde{v})}{1 - G(\tilde{v})} > 0, \tag{5}$$

$$\frac{\partial \phi}{\partial \tilde{v}} = -\frac{g(\tilde{v})}{(1 - G(\tilde{v}))^2} \int_{\tilde{v}}^{p} 1 - G(v)\, dv < 0. \tag{6}$$

It follows that

$$\frac{d\tilde{v}}{dp} = -\frac{\frac{\partial \phi}{\partial p}}{\frac{\partial \phi}{\partial \tilde{v}}} = \frac{1 - G(\tilde{v})}{g(\tilde{v})} \frac{G(p) - G(\tilde{v})}{\int_{\tilde{v}}^{p} 1 - G(v)\, dv}.$$

Because $\frac{g(v)}{1-G(v)}$ is increasing, we have

$$\int_{\tilde{v}}^{p} 1 - G(v)\, dv = \int_{\tilde{v}}^{p} \frac{1 - G(v)}{g(v)} g(v)\, dv < \int_{\tilde{v}}^{p} \frac{1 - G(\tilde{v})}{g(\tilde{v})} g(v)\, dv = \frac{1 - G(\tilde{v})}{g(\tilde{v})} (G(p) - G(\tilde{v})),$$

so

$$\frac{d\tilde{v}}{dp} = \frac{1 - G(\tilde{v})}{g(\tilde{v})} \frac{G(p) - G(\tilde{v})}{\int_{\tilde{v}}^{p} 1 - G(v)\, dv} > \frac{1 - G(\tilde{v})}{g(\tilde{v})} \frac{G(p) - G(\tilde{v})}{\frac{1-G(\tilde{v})}{g(\tilde{v})}(G(p) - G(\tilde{v}))} = 1.$$

■

Lemma 3 *Along the Δ_b curve, we have*

$$\frac{d\tilde{v}}{dp} > 0,$$

and the profit is strictly decreasing in $\tilde{v}$.

Proof. The first part directly follows from the fact that the LHS of (Δ_b) is decreasing in p and increasing in $\tilde{v}$. For the second part, observe that (Δ_b) implies $p = \frac{\int_{\tilde{v}}^{1} vg(v)dv}{1-G(\tilde{v})}$ so the profit is $p(1 - G(\tilde{v})) = \int_{\tilde{v}}^{1} vg(v)dv$ which is decreasing in $\tilde{v}$. ■

From the previous lemmas we know that each constraint (Δ_b) and (Σ_b) defines an increasing relation between $\tilde{v}$ and p. Consider the Δ_b and Σ_b curves in the plane with $\tilde{v}$ in the horizontal axis and p in the vertical axis. The next lemma shows that (for $s < s_2$) the curve Σ_b starts at

$\tilde{v} = 0$ below the curve Δ_b.

Lemma 4 *Under* (Δ_b)*, if* $\tilde{v} = 0$*, then* $p = \mu$*. Under* (Σ_b)*, if* $\tilde{v} = 0$*, then* p *is increasing in* s*, and* $p \in [0, \mu)$ *if* $s < s_2$*.*

Proof. The first part directly follows from (Δ_b). The second part directly follows from the definition of s_2 the fact that, for $\tilde{v} = 0$, (Σ_b) is equivalent to $\phi(0, p) = \int_0^p (p - v)g(v)dv = s$, and $\phi(0, p)$ is increasing in p. ■

Let $\tilde{v}^*$ and p^* be the solution of the firm's program and assume that the first best is not incentive-compatible, i.e., $s < s_2$. From the previous lemmas we know that $\tilde{v}^*$ and p^* is on the Σ_b curve, on the left of the first point of intersection of Δ_b and Σ_b. It is exactly at the point of intersection (when both constraints (Δ_b) and (Σ_b) bind) or strictly before (when only the constraint (Σ_b) binds) .

Lemma 5 *If at the optimum both* (Δ) *and* (Σ) *bind, then* $\tilde{v}^*$ *and* p^* *are strictly decreasing in* s*.*

Proof. Consider the Δ_b and Σ_b curves in the plane with $\tilde{v}$ in the horizontal axis and p in the vertical axis. Notice that the obedience constraints are satisfied for all pairs $(\tilde{v}, p)$ below these curves. From Lemmas 2 and 3 both curves are increasing. From Lemma 3 we also know that the profit on the Δ_b curve increases when moving to the left. Hence, if at the optimum of the firm both (Σ) and (Δ) bind, then it must be at the first point for which the curves cross, i.e., for the lowest value of $\tilde{v}$ for which (Δ_b) and (Σ_b) are satisfied.

At this point $(\tilde{v}^*, p^*)$, we know from Lemma 4 that the Σ_b curve crosses the Δ_b curve from below. If it was not the case, it would be possible for the firm to move along the Δ_b curve to the left (decreasing $\tilde{v}$) by keeping (Σ) satisfied (but not binding), hence increasing its profit.

Next, observe that the Σ curve moves upwards when s increases (i.e., the largest p that satisfies (Σ) is increasing in s), because ϕ is increasing in p and decreasing in $\tilde{v}$. This implies that the first crossing point of the curves, i.e., the optimal solution $(\tilde{v}^*, p^*)$, moves to the left on the Δ_b curve. We conclude that if both (Σ) and (Δ) bind, then $\tilde{v}^*$ and p^* are strictly decreasing in s ■

Lemma 6 *If at the optimum only* (Σ) *binds, then* $p^* > p_M$, $\tilde{v}^*$ *is decreasing in* s, *and* p^* *is increasing in* s.

Proof. For every admissible value of p, let $\tilde{v}(p)$ be the value of $\tilde{v}$ such that (Σ_b) is satisfied. The profit of the firm when (Σ) binds is

$$p(1 - G(\tilde{v}(p)).$$

The FOC is

$$(1 - G(\tilde{v}(p)) - p\tilde{v}'(p)g(\tilde{v}(p)) = 0.$$

Putting $\tilde{v}'(p)$ from Lemma 2 into the FOC yields

$$1 - p\frac{G(p) - G(\tilde{v}(p))}{\int_{\tilde{v}(p)}^{p}(1 - G(v))\,dv} = 0, \tag{7}$$

which simplifies to

$$\int_{\tilde{v}(p)}^{p} 1 - G(v) - pg(v)\,dv = \int_{\tilde{v}(p)}^{p}\left(\frac{1 - G(v)}{g(v)} - p\right)g(v)\,dv = 0. \tag{8}$$

Because $\frac{g(\cdot)}{1-G(\cdot)}$ is increasing and $p > \tilde{v}(p)$ at the optimum $p = p^*$, we have $\frac{1-G(p)}{g(p)} < \frac{1-G(v)}{g(v)}$ for $v > \tilde{v}(p)$, so

$$\int_{\tilde{v}(p)}^{p}\left(\frac{1 - G(p)}{g(p)} - p\right)g(v)\,dv < \int_{\tilde{v}(p)}^{p}\left(\frac{1 - G(v)}{g(v)} - p\right)g(v)\,dv = 0.$$

This inequality implies

$$\frac{1 - G(p^*)}{g(p^*)} < p^*, \tag{9}$$

and therefore $p^* > p_M = \frac{1-G(p_M)}{g(p_M)}$.

Now, taking the derivative of (8) with respect to s at the optimum yields, after simplification:

$$\frac{dp^*}{ds} = \frac{1 - G(\tilde{v}^*) - p^* g(\tilde{v}^*)}{1 - 2G(p^*) + G(\tilde{v}^*) - p^* g(p^*)} \frac{d\tilde{v}^*}{ds}. \tag{10}$$

Note that the denominator is negative,

$$1 - 2G(p^*) + G(\tilde{v}^*) - p^* g(p^*) < 0, \tag{11}$$

because $1-2G(p^*)+G(\tilde{v}^*)-p^*g(p^*) = 1-G(p^*)-p^*g(p^*)+G(\tilde{v}^*)-G(p^*)$, $1-G(p^*)-p^*g(p^*) < 0$ and $G(\tilde{v}^*) - G(p^*) < 0$.

Next, we use the binding constraint (Σ_b), which gives

$$\frac{d\phi}{ds} = \frac{d\tilde{v}^*}{ds} \frac{\partial \phi}{\partial \tilde{v}^*} + \frac{dp^*}{ds} \frac{\partial \phi}{\partial p^*} = 1.$$

Using (5) and (6) we get:

$$\frac{dp^*}{ds} = \frac{1 - G(\tilde{v}^*)}{G(p^*) - G(\tilde{v}^*)} + \frac{g(\tilde{v}^*) \int_{\tilde{v}^*}^{p^*} (1 - G(v))\, dv}{(G(p^*) - G(\tilde{v}^*))(1 - G(\tilde{v}^*))} \frac{d\tilde{v}^*}{ds}. \tag{12}$$

Combining (10) and (12) we get:

$$\frac{d\tilde{v}^*}{ds} \left(\frac{1 - G(\tilde{v}^*) - p^* g(\tilde{v}^*)}{1 - 2G(p^*) + G(\tilde{v}^*) - p^* g(p^*)} - \frac{g(\tilde{v}^*) \int_{\tilde{v}^*}^{p^*} (1 - G(v))\, dv}{(G(p^*) - G(\tilde{v}^*))(1 - G(\tilde{v}^*))} \right) = \frac{1 - G(\tilde{v}^*)}{G(p^*) - G(\tilde{v}^*)}. \tag{13}$$

Observe that the RHS of (13) is positive, the second fraction of the LHS is positive, and the denominator of the first fraction of the LHS is positive as observed in (11). To show that $\frac{d\tilde{v}^*}{ds} < 0$ it remains to show that the numerator of the first fraction of the LHS of of (13) is positive, which amounts to show that $p^* < \frac{1-G(\tilde{v}^*)}{g(\tilde{v}^*)}$. To prove this inequality, we use (7) and the fact that $\frac{1-G(v)}{g(v)}$ is decreasing in v:

$$p^* = \frac{\int_{\tilde{v}^*}^{p^*} \frac{(1-G(v))}{g(v)} g(v)\, dv}{G(p^*) - G(\tilde{v}^*)} < \frac{\int_{\tilde{v}^*}^{p^*} \frac{(1-G(\tilde{v}^*))}{g(\tilde{v}^*)} g(v)\, dv}{G(p^*) - G(\tilde{v}^*)} = \frac{(1 - G(\tilde{v}^*))}{g(\tilde{v}^*)} \frac{\int_{\tilde{v}^*}^{p^*} g(v)\, dv}{G(p^*) - G(\tilde{v}^*)} = \frac{1 - G(\tilde{v}^*))}{g(\tilde{v}^*}. \tag{14}$$

We conclude that $\frac{d\tilde{v}^*}{ds} < 0$, and hence from Equations (10), (11) and (14) that $\frac{dp^*}{ds} > 0$. ■

From these lemmas we get the following proposition:

Proposition 2 *For $s \leq s_2$, if the information acquisition cost s increases, then the probability of a purchase, the profit and the social welfare increase.*

Proof. Caveat: this proposition requires that the optimal solution is continuous, (e.g., this is the case of the problem is convex).

The fact that the probability of a purchase and social welfare increase in s follows from the fact that $\tilde{v}^*$ is decreasing in s at the second best, whether only (Σ) binds (Lemma 6) or both (Δ) and (Σ) bind (Lemma 5).

When only (Σ) binds, the profit is increasing in s because both the price and the probability of purchase increase (Lemma 6).

When both (Δ) and (Σ) bind, the fact that the profit is increasing in s follows from Lemma 3 and the fact that $\tilde{v}^*$ is decreasing in s. ■

Proposition 3 *Consumer welfare is strictly decreasing in s when at the optimum only (Σ) is binding, and is constant (equal to zero) otherwise. In particular, consumer welfare is lower under optimal disclosure than under full disclosure.*

Proof. Trivial when (Δ) is binding. When only (Σ) binds, we know that from Lemma 6 that $\tilde{v}^*$ is decreasing in s, and p^* is increasing in s. Let $s' > s$, $(\tilde{v}, p)$ the optimal solution at s, $(\tilde{v}', p')$ the optimal solution at s', and $CW = \int_{\tilde{v}}^1 (v-p)g(v)\,dv$, $CW' = \int_{\tilde{v}'}^1 (v-p')g(v)\,dv$ the corresponding social welfare. We have $\tilde{v}' < \tilde{v}$ and $p' > p$ and

$$CW' = \int_{\tilde{v}'}^1 (v-p')g(v)\,dv = \int_{\tilde{v}'}^{\tilde{v}} (v-p')g(v)\,dv + \int_{\tilde{v}}^1 (v-p')g(v)\,dv.$$

The first term is negative because $p' > p > \tilde{v}$, and the second term is smaller than CW because $p' > p$. We conclude that $CW' < CW$. ■

Proposition 4 *There exists $s_1 \in (0, s_2)$ such that, at the optimal solution, if $s \in (0, s_1)$ then* *only* (Σ) *is binding, and if* $s \in (s_1, s_2)$ *then both* (Δ) *and* (Σ) *are binding.*

Proof. (sketch) **Regarding the possibility that the drop constraint does not bind for some s although it binds for some lower search cost.** Here is an argument for why, if both constraints bind for some $s < s_2$ then they both bind for all search costs in $[s, s_2]$. It is based on the uniqueness of the solution to the profit maximization problem subject to (Σ) omitting (Δ) (so this would apply for ρ-linear $1 - G$). So take the lowest s at which both constraints bind, call it s_0 and the corresponding solution $(\tilde{v}_0, p_o)$. At that point, (Δ) is only weakly binding in the sense that $(\tilde{v}_0, p_o)$ would still be the solution if that constraint was omitted (in other words, the Lagrange multiplier for (Δ) is zero). Then if we consider the profit maximizing solutions for $s > s_0$ ignoring (Δ), it would involve $\tilde{v} < \tilde{v}_0$ and $p > p_0$. However this would always violate (Δ) because staring at $(\tilde{v}_0, p_0)$ and decreasing $\tilde{v}$, it is necessary to decrease p as well in order for (Δ) to remain satisfied. Hence, for all $s \in [s_0, s_2]$ the two constraints must bind. ■

4.1 Information externality imparted by the firm on the consumer

Will be completed in later versions.

5 Welfare under optimal, full, and no disclosure

We compare profit, consumer and social welfare in different scenarios (full/mandatory disclosure, no disclosure, optimal disclosure, Wang disclosure).

For every v, $q_b(v)$ be the probability that the consumer buys and $q_s(v)$ the probability that the consumer searches. The profit, consumer welfare and social welfare are denoted as follows:

$$\Pi = \int_0^1 q_b(v) p g(v)\, dv$$

$$CW = \int_0^1 \left(q_b(v)(v - p) - q_s(v) s\right) g(v)\, dv$$

$$SW = \Pi + CW = \int_0^1 \left(q_b(v) v - q_s(v) s\right) g(v)\, dv.$$

5.1 Full disclosure

Under full disclosure we have $p = p_M$, $q_b(v) = \mathbb{1}_{v \geq p_M}$, $q_s(v) = 0$ and:

$$\Pi^{FD} = \int_{p_M}^{1} p_M g(v)\, dv$$

$$CW^{FD} = \int_{p_M}^{1} (v - p_M) g(v)\, dv$$

$$SW^{FD} = \int_{p_M}^{1} v g(v)\, dv.$$

5.2 Optimal disclosure

Under optimal disclosure we have $q_s(v) = 0$ and $q_b(v) = \mathbb{1}_{v \geq \tilde{v}}$ so

$$\Pi^{OD} = \int_{\tilde{v}}^{1} p g(v)\, dv$$

$$CW^{OD} = \int_{\tilde{v}}^{1} (v - p) g(v)\, dv$$

$$SW^{OD} = \int_{\tilde{v}}^{1} v g(v)\, dv.$$

5.2.1 First best, $s \geq s_2$

The firm extracts all surplus: $p = \mu$, $\tilde{v} = 0$, $q_b(v) = 1$, $q_s(v) = 0$, $\Pi^{OD} = SW^{OD} = \mu$ and $CW^{OD} = 0$.

5.2.2 Second best, $s_1 \leq s \leq s_2$, both (Δ) and (Σ) bind

Because (Δ) binds we have $CW^{OD} = 0$, so $\Pi^{OD} = SW^{OD} = p(1 - G(\tilde{v}))$ which is increasing in s up to μ when $s \geq s_2$.

5.2.3 Second best, $s < s_1$, only (Σ) binds

CW^{OD} is decreasing in s from $CW^{OD} = CW^{FD} = \int_{p_M}^{1} (v - p_M) g(v)\, dv$ when $s = 0$ to 0 when $s \geq s_1$.

Π^{OD} and SW^{OD} are increasing in s.

Conclusion: For every $s > 0$, $\Pi^{OD} > \Pi^{FD}$, $SW^{OD} > SW^{FD}$ and $CW^{OD} < CW^{FD}$.

5.3 No disclosure

For $s \geq s_2$ it is equivalent to optimal disclosure.

For $\Phi_G(\pi_M) \leq s \leq s_2 := \Phi_G(\mu)$ nobody searches and all buy and we have:

$$\Pi^{ND} = \Phi_G^{-1}(s)$$

$$CW^{ND} = \int_0^1 (v - \Phi_G^{-1}(s))g(v)\,dv$$

$$SW^{ND} = \int_0^1 vg(v)\,dv = \mu.$$

For $s < \Phi_G(\pi_M)$ all search and all v above p buy and we have, with $p = \min\{p_M, \mu\}$:

$$\Pi^{ND} = p(1 - G(p)) = \int_p^1 pg(v)\,dv$$

$$CW^{ND} = \int_p^1 (v - p)g(v)\,dv - s$$

$$SW^{ND} = \int_p^1 vg(v)\,dv - s.$$

OBSERVATION: $SW^{ND} > SW^{OD} > SW^{FD}$ for $\Phi_G(\pi_M) \leq s \leq s_2 := \Phi_G(\mu)$.

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Appendix

Uniform Case

No disclosure

xxx Also do no disclosure

Optimal disclosure

In the uniform case, $\mu = \frac{1}{2}$ and the first best is incentive-compatible iff $s \geq \frac{1}{8}$. Hence, it remains to solve the case $s < \frac{1}{8}$.

When both constraints (Δ) and (Σ) are binding we get $\tilde{v} = 1 - 8s$, $p = 1 - 4s$, and therefore $\pi = p(1 - \tilde{v}) = 8s(1 - 4s)$, which is equal to $\frac{1}{2}$ (the first best) when $s = \frac{1}{8}$ and 0 if $s = 0$. Hence, this is not the optimum when s is small enough. The program when only (Σ) binds is

$$\max_{\tilde{v}} p(1 - G(\tilde{v}))$$

under the constraints

$$E(v \mid v \geq \tilde{v}) - p \geq 0 \tag{D}$$

$$E(v \mid v \geq \tilde{v}) - p = \Pr(v \geq p \mid v \geq \tilde{v}) E(v - p \mid v \geq p) - s \tag{S}$$

In the uniform case it is:

$$\max_{\tilde{v}} p(1 - \tilde{v})$$

under the constraints

$$\frac{1 + \tilde{v}}{2} \geq p \tag{D}$$

$$\frac{1 + \tilde{v}}{2} - p = \frac{(1 - p)^2}{2(1 - \tilde{v})} - s \tag{S}$$

Equation (Σ) has two solutions:

$$\tilde{v}_1 = p - s - \sqrt{s(2 - 2p + s)} \text{ and } \tilde{v}_2 = p - s + \sqrt{s(2 - 2p + s)}.$$

$\tilde{v} = \tilde{v}_2$ is not optimal for the first because then its profit is lower than $\frac{1}{4}$, which is dominated by full disclosure. To see this, observe that when $\tilde{v} = \tilde{v}_2$ the profit of the firm is

$$p(1-\tilde{v}_2) = p(1-p+s-\sqrt{s(2-2p+s)}) = p(1-p)+p(s-\sqrt{s(2-2p+s)}) \leq \frac{1}{4}+p(s-\sqrt{s(2-2p+s)}),$$

and we have

$$p(s-\sqrt{s(2-2p+s)}) \leq 0 \iff s-\sqrt{s(2-2p+s)} \leq 0 \iff s^2 \leq s(2-2p+s)$$

$$\iff s \leq 2-2p+s \iff 0 \leq 2(1-p),$$

which is always satisfied. Hence, let

$$\tilde{v}(p) = p - s - \sqrt{s(2-2p+s)},$$

be the only possible optimal solution when (Σ) is binding. The program of the firm is

$$\max_p p(1-\tilde{v}(p))$$

under the constraint

$$p \leq \frac{1+\tilde{v}(p)}{2}$$

The solution is

$$p = \frac{1}{2} + \frac{3s+\sqrt{s}\sqrt{16+9s}}{16} = \frac{2}{3} - \frac{1}{3}\tilde{v}$$

for $s \leq \frac{1}{10}$. For $s > \frac{1}{10}$ both constraints are binding and we get the previous solution. The solution is continuous: for $s = 0$ we get $p = 1/2$ and for $s = \frac{1}{10}$ we get $p = 3/5$ which is the $1-4s$ found before.

Note that we have at the optimum $1-\tilde{v} > p$ for all $s > 0$ (they coincide at $s = 0$ only).

For $s < \frac{1}{10}$ the price is increasing in s, from $\frac{1}{2}$ to $\frac{3}{5}$, then it is decreasing from $\frac{3}{5}$ to $\frac{1}{2}$ at $s = \frac{1}{8}$.

$\tilde{v}$ is always decreasing in s: it is $\frac{1}{2}$ at $s = \frac{1}{2}$, $\frac{1}{5}$ at $s = \frac{1}{10}$, and it is 0 for $s \geq \frac{1}{8}$.

For all $s \in (0, \frac{1}{8})$ the profit is strictly higher than in Wang.