The Effect of Systemic Rescues on Risk Taking and Systemic Risk

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Abstract

Bailouts increase moral hazard and exacerbate risk taking (strategic effect). However, they also decrease the probability of actual failure, thereby increase firm value, which in turn decreases the individual incentive to take risk (value effect). I study the interplay of these countervailing effects in a stochastic dynamic game. The strategic effect dominates in concentrated markets, but firms take less risk in fragmented markets in the presence of bailouts. The overall effect of bailouts on systemic risk in steady state depends on competitive and entry conditions. Contrary to conventional wisdom, bailouts can reduce systemic risk overall.

Keywords: Systemic Risk, Bailouts, Dynamic Oligopoly

JEL classifications: L13, L16, G01, G21

1 Introduction

Systemic risk is the threat that the market becomes dysfunctional. When policy makers consider a market to be fundamentally important (for macroeconomic and/or political reasons), they often bail out firms to prevent the market from collapsing. Systemic bailouts are not limited to the banking industry: in the past the state has provided financial support to e.g. money market

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mutual funds, insurance companies (AIG) or the Big Three automobile manufacturers (Chrysler, Ford, General Motors).[1] Furthermore, state support does not have to be financial assistance, it can take other forms, such as regulatory forbearance. Consider the audit market: since its indictment brought Arthur Andersen down in 2002, it is often speculated that the remaining four auditors are just "too-few-to-fail" and hence regulators have shied away from firm actions against Big Four auditors on a number of occasions (e.g. Business Week 2003, The Economist 2004 and 2005, WSJ 2004, NYT 2005, Forbes 2013, FT 2015 and 2017).[2] While the forms of systemic rescues can be different, they are a fundamental concern in all markets for the same reason: insulating firms from the risk of failure exacerbates moral hazard and thereby increases risk taking and systemic risk.

Policy makers have long argued that the anticipation of systemic bailouts increases moral hazard.[3] Indeed, strategic complementarities can emerge, because firms anticipating bailouts understand that their ultimate survival also depends on rivals' survival strategies: "if we all fail, we don't fail... so if we all take much risk, we don't actually take much risk".[4] However, there is a countervailing effect at work: systemic bailouts decrease the probability of actual failure and thereby increase the (charter) value of firms, which in turn reduces the individual incentive to take risk.[5] The overall effect of bailouts is therefore ambiguous at both individual firm and systemic level. At firm level, I show that which effect dominates depends on market structure: firms in concentrated markets take less risk in the absence of anticipated systemic bailouts, but they become riskier in fragmented markets compared to the case when they could count on bailouts. At the systemic level, risk taking fuels firm failures, which (along with entry) drives market structure dynamics in my model. I calculate measures of systemic risk in steady state and find that the model confirms conventional wisdom in some parameter regions: bailouts make the market more

[1] There is a push at the Financial Stability Board to extend the existing framework of 'Systemically Important Financial Institutions', currently covering banks and insurers, to all types of financial institutions (FSB 2015).

[2] One cannot help but wonder whether the three leading Credit Rating Agencies, which apply the exact same business model as auditors, emerged unharmed from the recent financial crisis because of similar concerns.

[3] For instance, Brandao-Marques et al (2013) find empirical evidence for higher expected government support increasing banks' risk taking.

[4] There is much anecdotal evidence that firms take risk in a strategic space by taking into account rivals' behaviour. In banking for instance, the famous comment of Chuck Prince (former CEO of Citigroup) in 2007 is a much quoted example: "As long as the music is playing, you've got to get up and dance".

[5] The so-called charter value effect dates back to the seminal work of Keeley (1990), which argued that the introduction of competition reduced charter values and caused banks to take more risk. See also, e.g. Cordella and Yeyati (2003) or Gropp et al (2011), which emphasise the role of the charter value effect in mitigating the effect of moral hazard in banking.

systemically risky. However, the model also cautions in two important ways. First, a market with bailouts can exhibit *less* systemic risk in some parameter regions (low demand and sluggish entry). Second, even when the market with bailouts is more systemically risky, the difference in expected systemic risk is often not large, which highlights the important counteracting effect of firm value.[6]

To fully analyse the implications of systemic bailouts, several modelling features are indispensable. First, because risk taking involves a dynamic trade off between instantaneous profits and survival in a strategic context, the framework of a game with infinite horizon is important. Second, competition, entry, market structure, and risk taking must all be determined endogenously in order to capture the rich set of mechanisms at play and to develop a deeper understanding of industry dynamics.[7] I employ a reduced-form framework whereby firms face a trade-off typical in financial markets: a more conservative (i.e. less risky) business strategy brings lower profits today, but higher probability of survival tomorrow. In my model, the benefit of survival depends on future profits, which are in turn determined by how many other firms will likely compete with the firm in the future. A firm chooses its survival probability taking into account its current and future rivals' surviving strategies, the entry process and the resulting industry dynamics. Note that to focus on the strategic build up of systemic risk, I do not incorporate contagion and/or aggregate risk, firms face identical, but independent idiosyncratic risk and choose their exposure to this risk in a strategic space. I introduce systemic bailouts by assuming that if the system (i.e. all incumbent firms) fails, then firms will be bailed out and can start afresh in the next period.[8] I look at how the presence of bailouts changes strategies and show that bailouts introduce strategic complemen-

[6] The paper analyses the effect of bailout policies on (individual and systemic) risk and leaves aside important issues, such as the direct costs of systemic bailouts, which are typically (but not always) large.

[7] There is long standing research into how the triad of competition, market structure and risk are related in banking. Empirical as well as the theoretical literature have conflicting findings (see surveys e.g. Beck 2008 or VanHoose 2017). Schaeck et al (2009) suggests that both market structure and competition can have independent effects on risk taking. The relationships among these three factors are further complicated by the fact that in banking competition and market structure may be related in an unusual way: Claessens and Laeven (2004) found a positive relationship between competition and market structure. It is therefore important to analyse the incentives of risk taking in a setup, where competition and market structure are two distinct concepts and determined endogenously, as in my model.

[8] Bailouts in systemic crises almost always involve sector wide support schemes, so this blanket bailout policy is arguably close to what we tend to observe in practice. See also e.g. bailout of car manufacturers in the USA 2008-2014, UK bank rescue package 2008, Norwegian, Finnish, Swedish banking crisis in the early 1990s, so-called secondary banking crisis 1973-75 in the UK. In the case of the Troubled Asset Relief Program (TARP), while only 10% of banking organizations received funds through the Capital Purchase Program, recipients accounted for 70% of total assets in the market. Governments have also adopted a mild stance towards all Big Four accounting firms since 2002, which have successfully avoided litigation as a result.

tarities in risk taking. That is, when rivals take more risk, the firm anticipating systemic bailout also has an incentive to increase its risk exposure. I call this reaction the strategic effect. It turns out that the balance between the strategic and charter value effects critically depends on market structure.

Risk creates a dynamic trade off between instantaneous profit and survival. While I offer a model of banking in Section 6, I focus on this dynamic trade off in a stylised model for much of the analysis and do not model the banking business in detail for several reasons. First, a bank can take risk in numerous ways without altering the basic nature of the dynamic trade-off. For instance, reducing the stock of liquid assets increases profits in the short run, but will also expose the bank more in a future liquidity crisis and thereby reduces the probability of survival.[9] Likewise, choosing to lend to riskier borrowers would yield higher interest revenue, but would naturally increase the probability of bankruptcy tomorrow.[10] Similarly, increasing the maturity mismatch between liabilities and assets would increase interest margin, but also the probability of default. And so on. Singling out any of these channels would appear ad hoc; modelling them all would be too complex. Importantly, the channel through which the bank builds up its risk exposure is immaterial in what follows, what matters is that taking more risk today increases current profits at the expense of the probability of survival tomorrow. The second reason for not modelling the banking business in detail is that all firms face the dynamic trade off between current profits and survival probability to some extent. For instance, an automobile manufacturer can save on R&D expenditure and thereby increase profits today, but this will jeopardise the firm's survival in tomorrow's market. Indeed, this was arguably a major factor in the bailout of the car industry between 2008-2014 in the USA: for many years US car manufacturers had not kept pace with the development of small, fuel efficient models, which turned out to be an important problem when fuel prices started to rise.

Systemic rescue is not limited to banking either. For instance, money market mutual funds, insurance companies (AIG) or the Big Three automobile manufacturers (Chrysler, Ford[11], General Motors) were also provided government support in the past. Looking further away, state support

[9] See Section 6 and Appendix B for the details of this model.

[10] See Appendix D.

[11] Ford did not face imminent bankruptcy in 2008, but asked to be included in the bailout program anyway.

does not have to be financial assistance, it can take many forms. Consider the audit (or Credit Rating Agency, CRA) market, where auditors (CRA) face a very similar dynamic trade off: they can increase current fees by yielding to client pressure and issuing biased audit opinions (ratings), while also risking their reputation and ultimately their market survival tomorrow.[12] Since its indictment brought Arthur Andersen down in 2002, it is often speculated that the remaining four auditors are just "too-few-to-fail" and hence regulators have shied away from firm actions against Big Four auditors on a number of occasions (e.g. Business Week 2003, The Economist 2004 and 2005, WSJ 2004, NYT 2005, Forbes 2013, FT 2015 and 2017). Similarly to bailouts, regulatory forbearance is a fundamental concern, because insulating accounting firms (CRA) from the risk of failure exacerbates moral hazard and deteriorates the quality of audit reports (ratings), which play a fundamental role in financial markets.

The current model builds on Tóth (2012) and can be considered a much simplified version of Ericson and Pakes (1995), who developed a framework for the empirical analysis of dynamic oligopoly models with heterogeneous firms. The models in this literature are designed for empirical work and therefore have to be rich enough to be taken to data. For this reason, firm heterogeneity, uncertainty at both firm and market levels, and entry and exit are crucial ingredients in these studies. As a result, the computation of the Markov Perfect Equilibria (MPE) is typically highly complex and thus many of these models allow only for a handful of firms in practice. To ease computational burden, there have been much development in this literature (see for a survey Aguirregabiria et al 2021). I take a fundamentally different approach. Observed and unobserved heterogeneity are of course essential parts of any empirical framework, but I do not take my model to data, so I can sacrifice one of the major hurdles in computation, firm heterogeneity, and assume firms are homogeneous and focus on symmetric equilibrium. Also, in my model the only source of uncertainty external to the firm is rivals' entry costs, the distribution of which is fixed over time. Market conditions (i.e., prices, technology, etc.) are deterministic at the outset, and hence the stochastic evolution of the market is solely governed by firms' survival strategies and the entry that ensuing failures generate. These assumptions vastly simplify computations and I can analyse

[12] Both auditors and credit rating agencies are paid by the companies which they audit and rate. As a result of this conflict of interest, auditors and credit rating agencies have an incentive to provide dishonest audit reports and inflated ratings.

the effect of market structure with arbitrary number of firms. Importantly, these assumptions are also crucial to deliver meaningful analytical results.[13]

Previous articles have analysed the effect of systemic risk and bailouts in banking: see e.g. Suarez (1994), Cordella and Yeyati (2003), Acharya and Yorulmazer (2007, 2008), Diamond and Rajan (2012) and Farhi and Tirole (2012), Allen et al (2018), Dell'Ariccia and Ratnovski (2019). However, none of these articles analyse an oligopolistic setting (imperfect competition), where the effect of market structure can be studied. My contribution to this literature is three-fold: first, I analyse the effect of bailouts on risk taking and systemic risk in the setting of imperfect competition and show that market structure is key to understand the effect of bailouts; second, I develop a general framework which suits a wide range of market settings, including, but not limited to, banking; three, as a microfoundation, I develop a simple model of liquidity risk in Section 6.[14]

A growing number of studies build on the IO literature spearheaded by Ericson and Pakes (1995) and analyses financial intermediation in a stochastic dynamic framework, see e.g. Egan et al (2017), Wang et al (2020), Corbae and D'Erasmo (2021), Clark et al (2021). Corbae and D'Erasmo (2013), which is based on Allen and Gale (2004), study in a stochastic dynamic game, inter alia, the effect of bailout on risk taking. Their study is different from the present paper in two important respects. First, they exogenously impose some features on market structure by defining national and regional banks along with a (non-strategic) competitive fringe, a framework developed in Ifrach and Weintraub (2017) in order to alleviate computational burden. Second, they analyse the effect of "too-big-to-fail" bailout policy: national banks are bailed out if they produce negative profits. They find that while national banks become riskier, smaller banks decrease their risk exposure leading to a net effect of reduction in risk. The concept of systemic risk that I am investigating is very different. In Corbae and D'Erasmo (2013) the strategic aspect of systemic risk build-up due to bailouts is missing (the bailout of a national bank does not depend on the failure of other banks), whereas this strategic effect is the focus in my study.

[13] The dynamic stochastic games which are designed for empirical analysis of industry dynamics are usually too complex for analytical purposes: they deliver "very little in the way of analytical results of applied interest; i.e. just about anything can happen." (Doraszelski and Pakes 2007)

[14] The model is also formally equivalent to, and thus can be thought of as, a dynamic extension of the static banking model of Allen and Gale (2004), as demonstrated in Appendix D. The results of Allen and Gale (2004) are overturned in this dynamic setting, which highlights the crucial role of (truly) dynamic models in understanding the interplay between risk and competition.

The article is organised as follows. In Section 2, I introduce the environment, in Section 3 I analytically derive some of the fundamental properties of the models without entry. Entry is introduced in Section 4, and I discuss computation of the equilibria in detail, derive the steady state distributions and calculate measures of systemic risk numerically for the baseline model in Section 5. In Section 6, I present a microfoundation based on a model of liquidity risk and also demonstrate the robustness of the results discussed in Section 5. Section 7 concludes.

2 Environment

I analyse dynamic strategic interaction among firms. In particular, the firms' current decisions affect their own as well as their rivals' future payoffs and they take into account the implications of their decisions on their own and their rivals' future behaviour, which naturally affect future payoffs. I follow the standard structure of the Ericson and Pakes (1995) framework and assume that competition among firms has two different dimensions. In every period, firms engage in an (for now unmodelled) static "market game", from which they get a symmetric equilibrium payoff $M(n) \geq 0$ (static dimension), where $n \in \{0, 1, 2, ...\}$ is the number of firms present in the market. Crucially, I assume the decisions taken in this market game have no dynamic implications and as a result its equilibrium payoff can be calculated independently and imported into the computation of dynamic policies. Then firms make forward looking investment decisions $x \in [0, \infty)$, and thereby choose their probability of survival $f(x) \in [0, 1)$ (dynamic dimension), where $f(\cdot)$ is continuously differentiable, strictly increasing and concave.

Time is discrete and infinite, firms discount the future with a common factor $\beta \in (0, 1)$. In the full model, each period will consist of two phases, a production and an entry phase, but for now I discuss the production phase below.

Production Phase

At the beginning of each production phase, firms engage in an unmodelled price or quantity competition and realise instantaneous profit $M(n)$ in a symmetric equilibrium of this market game. Similarly to previous literature on industry dynamics (for a survey, see Doraszelski and Pakes 2007 or Aguirregabiria et al 2021), I assume that the distribution of future states, conditional on the current state and investments, is independent of the prices (or quantities) that firms set in the market game. The market game thus can be modelled in a static framework and the reduced form profit function $M(n)$ can simply be fed into the dynamic optimization problem as a primitive of the stochastic dynamic game. The function $M(n)$ essentially captures exogenous factors such as demand conditions, product substitutability, production costs, regulation, etc.

I will assume that $M(n) \leq M(n-1)$ for $n > 2$, $M(2) < M(1)$, $0 \leq M(\cdot) < \infty$, and $\lim_{n\to\infty} M(n) = 0$, which conform with standard models of homogeneous as well as differentiated product (Bertrand or Cournot) competition and hence this specification essentially captures all cases typically considered in the literature. As discussed in the introduction, I will use this reduced form profit function for much of the analysis and leave the market game unmodelled in order to allow the setup to encompass many potential market settings. The focus of my analysis will be on the dynamic trade off between profits and survival. For a possible microfoundation based on a model of banking, see Section 6 below.

In each period, each firm $i = 1, ..., n$ chooses its investment $x_i \in [0, \infty)$ and thereby its probability of survival $f(x_i) \in [0, 1)$ and thus realises its per-period net profit $\pi(x_i; n) = z(M(n), x_i)$. I make the following assumptions on the instantaneous profit function:

Assumption 1.

a) $\pi(x; n) \leq \pi(x; n-1)$ *for* $n > 2$, $\pi(x; 2) < \pi(x; 1)$ *for* $\forall x \in [0, \infty)$, $-\infty < \pi(\cdot, \cdot) < \infty$, $0 \leq \pi(0; n) < \infty$ *for* $\forall n$, *and* $\lim_{n\to\infty} \pi(0; n) = 0$.

b) $\partial\pi(x; n)/\partial x, \partial\pi(x; n)/\partial x\partial x < 0$ *for* $\forall n$ *and* $\forall x \in [0, \infty)$. *Also,* $\partial\pi(0; n)/\partial x = 0$.

c) $f : [0, \infty) \to [0, 1), f(0) = 0, f'(x) > 0, f''(x) \leq 0$ *for* $\forall x \in [0, \infty)$.

The function $\pi(x; n)$ decreases with the number of firms n and it is also decreasing in x: higher probability of survival tomorrow comes at the expense of lower profits today.[15] This simple formulation captures the basic trade off that most firms face in practice: the firm can increase its probability of survival at the cost of sacrificing current profits. The reduced form profit function

[15] The assumptions on the second derivatives of $\pi(\cdot; n)$ and $f(\cdot)$ are sufficient (but not necessary) conditions for the objective functions to be concave.

can be microfounded in several ways. The specification $\pi(x;n) = M(n) - g(x)$, where $g(x)$ is an increasing convex function representing the (fixed) cost of investment, is the workhorse in the IO literature (see e.g. Aguirregabiria et al 2021).[16] In banking, it is also possible to think of $g(x)$ as monitoring cost, where more monitoring ensures a lower probability of default, similarly to e.g. Dell'Ariccia et al (2014), Martinez-Miera and Repullo (2017), or Dell'Ariccia and Ratnovski (2019).[17]

The fixed cost nature, i.e. $g(x)$ enters additively into the profit function, is a common modelling feature both in the literature spearheaded by Ericson and Pakes (1995) and also in studies on unobserved quality. This assumption has technical advantages and also serves an important general purpose. In any static game, firms would never choose to produce negative profits. While this is perfectly reasonable in a static setting, firms in a dynamic environment may find it optimal to operate at a loss today in the hope of profits tomorrow. In order to allow for this possibility, the dynamic leg of the optimisation problem enters additively. However, one may not find the fixed cost nature realistic in general and in models of banking in particular. Therefore, I offer an alternative microfoundation of the market game in Section 6 based on a model of banking, where $\pi(x;n) = M(n)q(1-x)$ for $x \in [0,1]$. As we will see shortly, the firm behaviour generated by this framework will be very different, so Section 6 will also serve as a useful robustness check.

If the production phase started with n firms, then the probability that at the end of the production phase firm i faces a market structure consisting of $n-k$ firms in total is $f(x_i)\Pr(k|f(x_{-i}))$, where k is the number of rivals who have just failed and $\Pr(k|f(x_{-i}))$ is the probability mass function of the convolution of $n-1$ Bernoulli distributions with success probabilities $f(x_{-i})$, where $f(x_{-i}) = [f(x_j)]_{j\neq i}$.

The strategies are assumed to be Markov and I focus on symmetric Markov Perfect Equilibrium

[16] The easiest way to think about this profit function is to consider a market, where survival requires constant innovation. For instance, suppose in each period the old product is displaced by the new product. A firm invests in R&D at the cost of $g(x)$ and successfully innovates tomorrow's new product (and hence survives) with probability $f(x)$. Firms can then play a e.g. homogeneous product Cournot ($M(n) = (\alpha/(n+1))^2$) or a horizontally differentiated Salop game ($M(n) = \alpha/n^2$). Tóth (2012) also provides a microfoundation for this specification based on the unobserved quality literature.

[17] In reality, monitoring costs have both fixed and variable components. The marginal cost of monitoring is implicitly assumed to be constant and captured in $M(n)$, when $\pi(x;n) = M(n) - g(x)$. In other studies, the cost of monitoring is usually related to the number of projects that a bank finances; see Section 6 for such an approach.

(MPE) in pure strategies. That is, strategies depend only on payoff relevant information. The payoff relevant information is condensed into a state variable, which is the number of firms n.

First, I consider the model without entry, so each period only consists of the production phase, and analytically derive some important properties of the dynamic stochastic game. Later on, I introduce entry, so that each period will consist of a production and an entry phase, and analyse the properties of the steady state numerically.

3 The models without entry

In what follows, I introduce the concept of bailout into the model, discuss the dynamic programs with and without bailout in the absence of entry and derive analytical results.

First, note that even a single firm (monopoly) is able to serve the market in this model, the market stops functioning only when all firms fail. Consequently, systemic failure is defined as the event when all incumbent firms fail (i.e. there is no firm at the end of the production phase).[18] Second, the fact that the market is fully functional as a monopoly means that the government need not take action unless systemic failure occurs. Third, the government bails out all firms in a systemic failure. This bailout policy is the most consistent with modelling assumptions (e.g. symmetry) and yields a tractable analytical framework. Arguably, this blanket bailout policy is also the closest to what we observe in practice.[19] Fourth, bailout takes a simple form, the market is restored to its previous state.

In particular, if there are n firms in the market who all invest $y(n) \in [0, \infty)$ and thus survive with probability $f(y(n)) \in [0,1)$ in a symmetric equilibrium, then a systemic failure occurs with

[18] In principle, one could define systemic failure as the state when there is no firm in the market at the end of the entry, rather than the production phase. In other words, a systemic crisis would happen when each incumbent firm has failed and no one has entered the market. This definition, however, would arguably be in conflict with the concept of a bailout: de novo entry is typically slow in practice and bailouts happen, because the market cannot be without firm even for a short time period.

[19] See e.g. bailout of car manufacturers in the USA 2008-2014, UK bank rescue package 2008, Norwegian, Finnish, Swedish banking crisis in the early 1990s, so-called secondary banking crisis 1973-75 in the UK, and governments have also adopted a mild stance towards all Big Four accounting firms since 2002, which have successfully avoided litigation as a result (see references in the Introduction). In the case of the Troubled Asset Relief Program (TARP), while only 10% of banking organizations received funds through the Capital Purchase Program, recipients accounted for 70% of total assets in the market. However, different bailout policies can be introduced in principle as I discuss in the next footnote below.

probability $(1 - f(y(n)))^n$, in which case bailouts ensue and the production phase ends as if no firm has failed, i.e. with n firms again.

The dynamic programs of firm i without and with systemic bailout are as follows

$$\begin{aligned}
v(n; x_{-i}) &= \max_{0 \le x_i \le 1} \left\{ \pi(x_i; n) + \beta f(x_i) \sum_{k=0}^{n-1} V(n-k) \Pr(k | f(x_{-i})) \right\} \\
\widetilde{v}(n; y_{-i}) &= \max_{0 \le y_i \le 1} \left\{ \pi(y_i; n) + \beta f(y_i) \sum_{k=0}^{n-1} \widetilde{V}(n-k) \Pr(k | f(y_{-i})) \right. \\
&\qquad \left. + \beta \widetilde{V}(n) (1 - f(y_i)) \Pr(n-1 | f(y_{-i})) \right\}
\end{aligned}$$

In a symmetric equilibrium, we have that

$$\begin{aligned}
V(n) &= \pi(x(n); n) + \beta \sum_{k=0}^{n-1} V(n-k) \binom{n-1}{k} (1 - f(x(n)))^k (f(x(n)))^{n-k} \\
\widetilde{V}(n) &= \pi(y(n); n) + \beta \sum_{k=0}^{n-1} \widetilde{V}(n-k) \binom{n-1}{k} (1 - f(y(n)))^k (f(y(n)))^{n-k} \\
&\quad + \beta \widetilde{V}(n) (1 - f(y(n)))^n
\end{aligned}$$

where $x(n), y(n)$ are the equilibrium investments in the games without and with bailouts, respectively.[20] First, I establish some basic properties of the value functions. All proofs are relegated to the Appendix.

Proposition 1. *The value functions $V(n), \widetilde{V}(n)$ are strictly positive and decreasing.*

Proof. See Appendix. □

The value functions are decreasing, because the profit from the market game $M(n)$ is decreasing with the number of firms, and also because the prospect of reaching a more concentrated, and thus

[20] In general, if $\omega \in [0, 1]$ portion of the n failing firms get randomly bailed out in a systemic failure, then the last expression in the second program should be modified as $\bar{\omega} \beta \widetilde{V}(n_\omega)(1 - f(y_i)) \Pr(n-1 | f(y_{-i}))$, where $n_\omega = \{u | u - 1 < \omega n \le u\}$, $u \in \mathbb{Z}^+$ and $\bar{\omega} = n_\omega / n$.

lucrative, state (market) is more remote. This, however, does not necessarily mean that firms will invest less in survival in more fragmented markets, as we will see in Section 6.

A crucial difference between the models with and without bailout is the strategic effect that emerges in the presence of bailout. This effect can be easily illustrated in the simplest case, a duopoly without entry:

$$
\begin{aligned}
v(2; x_j) &= \max_{0 \le x_i \le 1} \left\{ \pi(x_i; n) + \beta f(x_i) \left(f(x_j) V(2) + (1 - f(x_j)) V(1) \right) \right\} \\
\widetilde{v}(2; y_j) &= \max_{0 \le y_i \le 1} \Big\{ \pi(y_i; n) + \beta f(y_i) \left(f(y_j) \widetilde{V}(2) + (1 - f(y_j)) \widetilde{V}(1) \right) \\
&\qquad + \beta (1 - f(y_i)) (1 - f(y_j)) \widetilde{V}(2) \Big\}
\end{aligned}
$$

Differentiating the first order condition and using the implicit function theorem yields

$$
\frac{\partial x_i}{\partial x_j} = - \frac{\beta f'(x_i) f'(x_j) (V(2) - V(1))}{\partial \pi(x; n)/\partial x \partial x + \beta f''(x_i) (f(x_j) V(2) + (1 - f(x_j)) V(1))} < 0
$$

This is always negative, because the denominator is negative (Assumption 1) and so is the numerator as the value function is decreasing (Proposition 1). That is, investments in survival are strategic substitutes in the model without bailout, i.e. a firm has an incentive to take less risk if the rival takes more. In the model with bailout, however, we have that

$$
\frac{\partial y_i}{\partial y_j} = - \frac{\beta f'(y_i) f'(y_j) \left(2\widetilde{V}(2) - \widetilde{V}(1) \right)}{\partial \pi(y; n)/\partial y \partial y + \beta f''(y_i) \left(f(y_j) \widetilde{V}(2) + (1 - f(y_j)) \left(\widetilde{V}(1) - \widetilde{V}(2) \right) \right)}
$$

Here the denominator is negative again (Assumption 1 and Proposition 1), but $2\widetilde{V}(2) - \widetilde{V}(1)$ in the numerator may not be negative, and indeed in most cases it is not (see below), even when the value function is decreasing. When $\partial y_i / \partial y_j$ is positive, investments are strategic complements: if a firm takes more risk, then its rival also has an incentive to decrease its probability of survival. This is because the firm counting on bailout recognises that its ultimate survival depends on rival's survival strategy too: "if we both fail, we don't fail... so if we both take much risk, we don't actually take much risk". This has important implications for risk taking behaviour as well as systemic risk.

Next, I prove the presence of a firm value effect, that is, systemic bailouts increase firm values for all market structures.

Proposition 2. *There is a (charter) value effect, i.e.* $\widetilde{V}(n) > V(n)$ *for* $\forall n$.

Proof. See Appendix. □

The result is intuitive: the government effectively reduces the (expected) cost of failure by bailing out firms and thereby increases the continuation value, which naturally increases firm value. Firms with higher value care more about the future and have a natural incentive to invest more in survival. This is the important countervailing force that not only alleviates, but for certain market structures overwhelms the strategic effect discussed above.

In sum, the strategic effect induces firms to invest less, the charter value effect induces them to invest more, and the sum of these effects determines whether firms take more or less risk in the presence of systemic bailouts. The next proposition establishes that whether the strategic or firm value effect is stronger depends on market structure, and thus market structure plays a pivotal role in determining individual as well as systemic risk.

Proposition 3. *For monopoly,* $x(1) > y(1)$. *However, there exists an* n^* *such that firms in the market without bailout take more risk in equilibrium, i.e.* $x(n) < y(n)$ *for all* $n > n^*$.

Proof. See Appendix. □

Proposition 3 suggests that fragmented markets behave differently to concentrated markets. Firms that do not expect systemic bailouts take less risk when the market is concentrated ($x(n) > y(n)$), while they are riskier when there are many firms in the market ($x(n) < y(n)$). In other words, when there are few firms in the market the strategic effect dominates the firm value effect, while in more fragmented markets the reverse is true.

This is not a trivial result. On the one hand, when the market is concentrated the strategic effect is particularly strong, because the probability of a systemic failure ($(1 - f(y(n)))^n$), the effect of a firm's risk taking on the probability of a systemic failure, and the value of the bailed out firm are all relatively large. On the other hand, the value effect is also strong in concentrated markets:

the difference between firm values across the two market settings is large, because bailouts have a significant impact on the *actual* survival probabilities when n is small ($f(x(n))$ vs. $f(y(n)) + (1 - f(y(n)))^n$). Recall that the only difference between the two models is what happens in systemic failure (state zero, i.e. all firms fail), and when the expected value of this state is non-negligible, it will increase both the strategic and value effects. Therefore, both the strategic and value effects are relatively large in concentrated markets, while they are both smaller in fragmented markets. Essentially, Proposition 3 implies that while both effects decrease with n, the strategic effect diminishes faster than the charter value effect and thus the charter value effect becomes dominant for sufficiently fragmented markets. Consequently, firms take less risk when they can count on government bailout compared to the case when they cannot, if n is sufficiently large. This result is surprising, primarily because conventional wisdom would not suggest that firm value can play such a pivotal role when it's so small.

Another way to understand the result in Proposition 3 is to note that a firm can survive in the market with bailout either through the "market process" at the private expense of costly investment, or through the "bailout process" in systemic failure. Firms in all states compare the cost and benefit of surviving through the market process (cost of investment y and the probability of ending up in a more concentrated market, respectively) with the cost and benefit of surviving through the bailout process (zero and the probability of ending up in the same market, respectively), conditional on rivals' strategies. The probability of systemic failure is higher in more concentrated markets by the virtue of fewer firms for any given level of investment. In addition, firm value is already fairly high in concentrated markets, so the status quo, and thus the "bailout process", is an attractive option. Furthermore, the less rivals invest in survival, the larger the probability of systemic failure, which gives a firm an additional incentive to increasingly bet on survival through the bailout process and reduce its investment in more concentrated markets, compared to the market without bailout.

It is easy to see that while the monopoly in the market without bailout has a natural incentive to invest in survival, a monopoly in the market with bailout does not have to invest ($y(1) = 0$), because its failure is a systemic event by definition and thus it will survive with certainty regardless. Consequently, $\widetilde{V}(1) > V(1)$. One may argue that this is the driving force behind much of the results. This is not true, all results go through by assuming and exogenously imposing $\widetilde{V}(1) = V(1)$ (suppose e.g. the monopoly market is always nationalised).[21] The driving force behind the model is emerging strategic complementarities in the presence of bailouts. Furthermore, note that monopoly per se does not play a special role in the model: if the government imposed a floor on the minimum number of firms to prevent the monopoly state altogether and ensure that there are always at least, say, two firms in the market, a duopoly in this case would behave just like a monopoly: they would not invest in survival ($y(2) = 0$), because any failure would trigger the bailout process.[22]

In sum, markets with and without bailouts behave differently, and market structure has a significant impact on individual risk taking across the two market settings. But how does this translate into systemic risk, how does (steady state) market structure affect the risk of systemic failure? In order to answer these questions, I now introduce entry, derive the steady state distributions, and calculate measures of systemic risks numerically.

4 The models with entry

In the previous section a period only consisted of a production phase. I now introduce entry, so each period will consist of a production and an entry phase.

Entry Phase

First, I describe in detail the entry phase for the model without bailout, the case with bailout is analogous. In each period, production is followed by an entry phase, where $N^e \in Z_+$ potential entrants may enter sequentially.[23] Let the fixed cost of entry be F, which are identically and independently distributed across potential entrants with CDF $\rho(\cdot)$ and support $[0, \infty]$. Each

[21] This can be easily illustrated for Proposition 2 when $n = 2$ and $\widetilde{V}(1) = V(1)$. In equilibrium, $V(2) = \pi(x;n) + \beta\left(f(x)^2 V(2) + f(x)(1 - f(x))V(1)\right)$ and $\widetilde{V}(2) \geq \pi(x;n) + \beta\left(f(x)^2 \widetilde{V}(2) + f(x)(1 - f(x))V(1)\right) + \beta(1 - f(x))^2 \widetilde{V}(2)$, because x maximises the programme without bailout. Subtracting the first from the second equation, we get $\left(1 - \beta f(x)^2\right)\left(\widetilde{V}(2) - V(2)\right) \geq \beta(1 - f(x))^2 \widetilde{V}(2) > 0$.

[22] A floor on the minimum number of firms does not change the qualitative results, the duopoly would in effect take over the role of the monopoly state in this case. The duopoly would now be the most lucrative state that firms can reach (the "prize") and firm values would decrease as a result. Duopoly would also attract less entry and this state would therefore linger on longer (i.e. would be less transitory) than monopoly.

[23] Time "stops" within each period; there is discounting only across periods.

entrant knows her own fixed cost of entry before entering. An entrant's fixed cost is private information, but all entrants know the distribution of fixed costs. Thus, entrants are heterogeneous and despite the sequential structure, they are unable to foresee the entry process with certainty within a period. In particular, n firms in the market (incumbents surviving in the production phase and entrants who have already entered in this period) will expect the lth entrant to enter with (endogenous) probability $\rho_{l,n}$, where $l = 0, .., N^e$. Note that heterogeneity before entry induces a non-degenerate distribution of the number of entrants in each state.[24] Let the equilibrium value function at the end of the lth entry round be $W_l(n)$. (That is, n includes the lth entrant if it has entered the market.) Then,

$$W_l(n) = W_{l+1}(n+1)\rho_{l+1,n} + W_{l+1}(n)(1 - \rho_{l+1,n}) \tag{1}$$

The probabilities $\rho_{l,n}$ are determined endogenously: given that the lth entrant enters if $W_l(n+1) - F > 0$, the probability of entry is $\rho_{l,n} = \rho(W_l(n+1))$. The sequential nature of entry facilitates a simple recursive formulation of the entry process. In particular, for the first entrant the value of being in the market is just the expected equilibrium firm value over the distribution of the second entrant's entry decision, which in turn depends on the third entrant's decision and so on until the last entry round. As there is no further entry after the last entry round in a period, $W_{N^e}(n) = V(n)$, where $V(n)$ is the (symmetric) equilibrium value function of the firm in the production phase without bailout.

As before, the strategies are assumed to be Markov and I focus on symmetric Markov Perfect Equilibrium (MPE) in pure strategies. That is, strategies depend only on payoff relevant information and the state variable is the number of firms n. The summary of timing within a period is as follows. Production phase: 1. Firms choose prices/quantities and $M(n)$ is determined in a symmetric equilibrium; 2. firms choose x;[25] 3. failures and exits occur. Entry phase: 4. N^e firms enter sequentially, the lth entrant enters with probability $\rho_{l,n}$ when there are n firms already in the market (surviving incumbents and previous entrants).

[24] An alternative way to obtain a non-degenerate distribution of entrants in each state would be simultaneous entry, where entrants use mixed strategies.

[25] Note that firms can also choose prices (or quantities) and x simultaneously.

The dynamic program of firm i in the market without bailout is as follows

$$v(n;x_{-i}) = \max_{0\leq x_i\leq 1}\left\{\pi(x_i;n)+\beta f\left(x_i\right)\sum_{k=0}^{n-1}W_0(n-k)\Pr(k|f\left(x_{-i}\right))\right\} \tag{2}$$

and the resulting value function in a symmetric equilibrium is

$$V(n) = \pi(x\left(n\right);n)+\beta\sum_{k=0}^{n-1}W_0(n-k)\binom{n-1}{k}(1-f\left(x(n)\right))^k f(x(n))^{n-k} \tag{3}$$

where $W_0(n-k)$ is the value of a (surviving) firm at the end of the production phase, just before the first entrant makes its entry decision, and $f\left(x(n)\right)$ are the (symmetric) equilibrium survival probabilities.

The steady state distribution can be derived observing that at each market structure there are failures ("deaths") and firm entries ("births"), thus the Markov chain produces a birth-death process. The derivation of the steady state distribution can thus follow the standard procedure (see Grimmett and Stirzaker 2001, ch 6.11), see Appendix C for the details.

I make a few comments on existence. In both models, the second order conditions are negative, due to $\partial\pi(\,\cdot\,;n)/\partial\left(\,\cdot\,\right)\partial\left(\,\cdot\,\right)<0$ and $f''\left(\,\cdot\,\right)\leq 0$ (Assumption 1), so the best replies are unique. Moreover, in the model without bailout, the reaction functions are continuous and downward sloping and thus a symmetric equilibrium in pure strategies exists. However, existence in pure strategies in the model with bailout is not ensured, because while the reaction functions are still continuous, in the presence of strategic complementarities nothing guarantees in general that the reaction functions intersect. In the computations I did not come across instances when the equilibrium did not exist in pure strategies.

For the purpose of computations, two sets of numerical results are presented. First, I present results with the specification $\pi(\,\cdot\,;n)=M(n)-g\left(\,\cdot\,\right)$, which is a standard framework in the IO literature. Second, I will also discuss results when $\pi(\,\cdot\,;n)=M(n)q(1-(\,\cdot\,))$ after providing a microfoundation for this specification, a simple banking model.

5 Baseline model: $\pi(\,\cdot\,;n) = M(n) - g\,(\,\cdot\,)$

In what follows, I describe the details of the computation for the baseline model $\pi(\,\cdot\,;n) = M(n) - g\,(\,\cdot\,)$ and also discuss the numerical results. The algorithm for the alternative specification $\pi(\,\cdot\,;n) = M(n)q(1-(\,\cdot\,))$ in Section 6 (banking model) is analogous.

5.1 Computation

The computed Markov Perfect Equilibrium in pure strategies is such that given optimal policies the value functions satisfy the Bellman equations (3) and (7), and given the value functions investment policies satisfy the first order conditions, up to some sufficiently small error.[26] This implies that firms choose optimal policies based on their beliefs on future industry structure, and these beliefs are consistent with rivals' behaviours. For the numerical analysis I use the following cost and survival probability functions:

$$g(h) = \frac{h^2}{1-h}, \quad g'(h) = \frac{1}{(1-h)^2} - 1, \quad f(h) = h, \quad 0 \le h < 1$$

These functions conform to Assumption 1 and lead to simple analytical first order conditions to the programmes (2) and (6), which (after rearranging and imposing symmetry) yield, respectively:

$$x(n) = 1 - \frac{1}{\sqrt{1 + \beta \sum_{k=0}^{n-1} W_0(n-k)\binom{n-1}{k}(1-x(n))^k (x(n))^{n-1-k}}} \tag{4}$$

$$y(n) = 1 - \frac{1}{\sqrt{1 + \beta \sum_{k=0}^{n-1} \widetilde{W}_0(n-k)\binom{n-1}{k}(1-y(n))^k (y(n))^{n-1-k} - \beta \widetilde{W}_0(n)(1-y(n))^{n-1}}} \tag{5}$$

The equilibrium profit function from the market game and the entry probability function that I use for the calculations are as follows, respectively,

[26] This error is typically smaller than 10^{-10} after 30 iterations and after about 100 iterations the error reaches machine precision (10^{-16}).

$$M(n) = \alpha^2 / (n+1)^2, \qquad 0 < \alpha < \infty$$

$$\rho(a) = 1 - e^{-\gamma a}, \qquad 0 < \gamma < \infty$$

For the specification of $M(n)$, I choose the equilibrium profit from a standard Cournot game to illustrate the performance of the dynamic model, where α can be interpreted as a demand parameter or market size. The calculations of course could be executed with the same qualitative results using other static games (e.g. in Salop's circular model $M(n) = \alpha/n^2$, where α is the travel cost and a measure of the level of product differentiation or market power). As mentioned before, $M(n)$ captures exogenous factors in general and hence α can be thought of in numerous ways.[27] In what follows, I refer to α as a demand parameter (high α indicates strong demand). The parameter γ shifts the distribution of the fixed cost of entry. I set the discount factor $\beta = 0.97$. As customary in the literature (see e.g. Aguirregabiria et al 2021), I set the maximum number of firms that can be in the market at any time equal to 20 and allow for 5 potential entrants in each period. The qualitative results are not sensitive to the pre-set maximum number of firms, the only reason for displaying results for (maximum) 20 firms is visual presentation.[28] Furthermore, the main results are remarkably robust to the specification of instantaneous profit and functional forms as we will see in the next section.

Computations are done in Matlab R2018b. The algorithm that solves the model iterates over the value and policy functions until convergence.[29] In the computations, I set all initial values for the value and policy functions equal to zero, results are completely invariant to starting values, so the equilibrium can be regarded as "numerically unique". An iteration consists of the following steps for e.g. the case without bailout (the other case is analogous):

1. Using the values $V(n)$ for all n from the previous iteration, calculate $W_l(n)$ backwards for all l, n, starting at $W_N(n) = V(n)$, and then $W_l(n) = W_{l+1}(n+1)\rho_{l+1,n} + W_{l+1}(n)(1 - \rho_{l+1,n})$ for all $l < N$, where $\rho_{l,n} = \rho(W_l(n+1))$. At the end, one has $W_0(n)$ for all n.

[27] For an alternative specification and a microfoundation for $M(n)$, see Section 6.

[28] The maximum number of firms is immaterial, because in the parameter range investigated states close to 20 are not reached in the steady state (see Figures 1 and 2). If one wishes to investigate other parameter ranges (and/or functional forms) the maximum number of firms can be adjusted so that it is not "binding" (i.e. it is not reached in steady state).

[29] It is possible in principle to solve for the equilibrium by solving the system of non-linear equations, which define the equilibrium. However, this method struggles when the discount factor is high and the number of firms is large ($n > 10$).

2. Using the values of $W_0(n)$ from step 1 above and the values of $x(n)$ from the previous iteration, update the policy function using the first order condition (4) for all n.

3. Using $W_0(n)$ and $x(n)$ from step 1 and 2, update the value function $V(n)$ for all n, using equation (3).

The simple structure of the stochastic dynamic game ensures that the algorithm calculates the equilibrium in a matter of minutes, even with hundreds of firms.

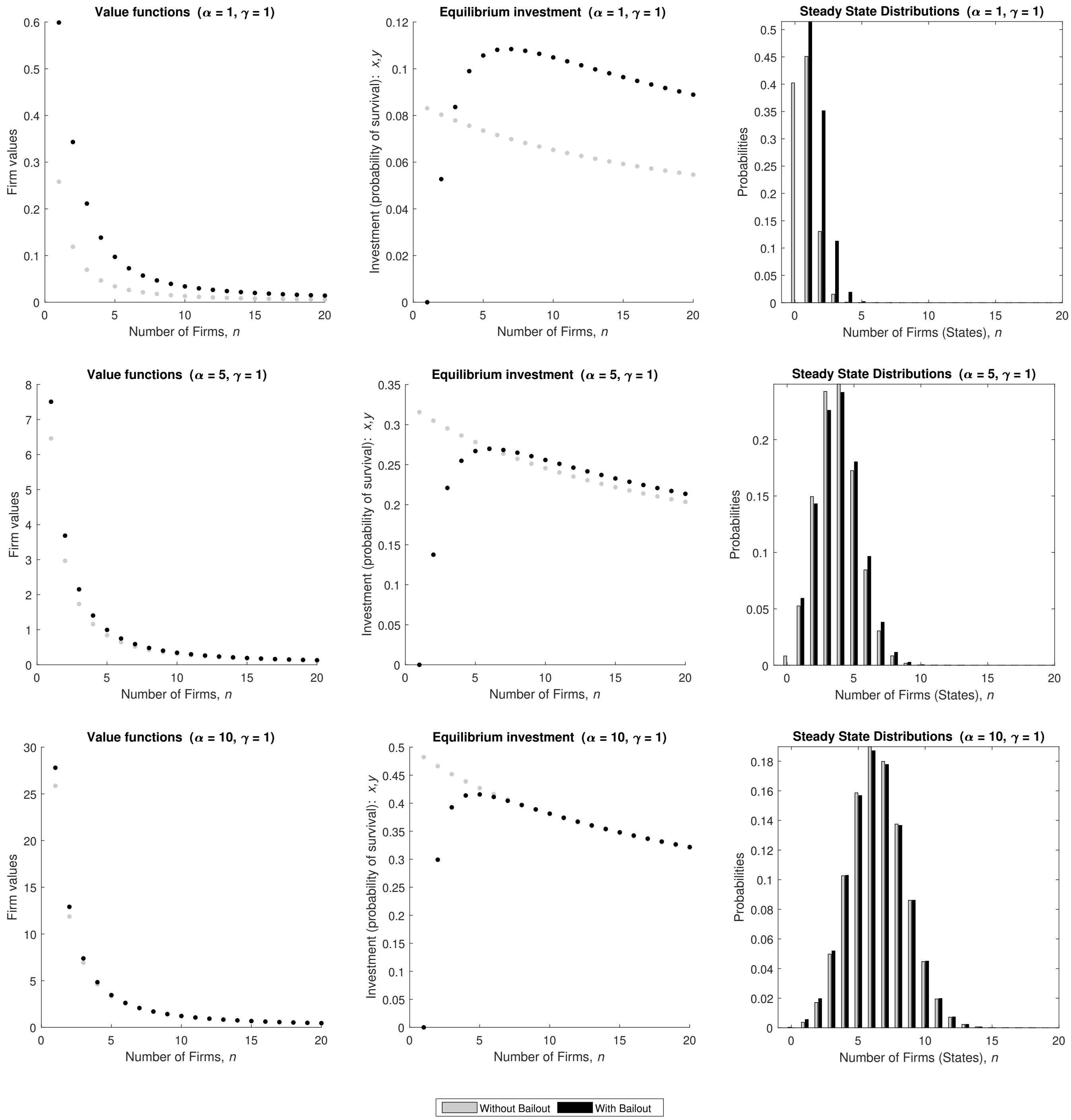
5.2 Numerical results

I focus on two parameters, α and γ, to assess the effect of competition on the effect of individual risk taking and systemic risk. I will interpret α as a demand parameter, higher values indicating stronger demand and leading to higher profits $M(n)$ at all market structures. The other parameter γ measures the intensity of entry: higher γ increases the probability of entry at all market structures and the competitive threat that incumbent firms face ("outside competition").

From the graphs in Figures 1 and 2, several general observations can be made, which are robust across all parameters and parameter ranges. First, there is a clear charter value effect: firms in markets with bailout always have higher values than the firms which operate in a market without bailout. This is very intuitive: by reducing the (expected) cost of failure, the government increases the continuation value of the firm, which naturally increases firm value. Also, observe that firm values decrease with the number of firms, as expected (see the first panels of Figures 1 and 2). Second, the differences between the individual risks (measured by $x(n), y(n)$, the equilibrium investments in survival) that firms take in the two market settings crucially depend on market structure. In particular, in more concentrated markets firms with bailout take more risk; but as the number of firms grow, they tend to take less risk than the firms in the market without bailout. This counterintuitive result is the product of the interplay of the charter value and the strategic effects. *Strategic effect* is the effect due to a firm having an incentive to choose lower probability of survival when rivals do the same. *Charter value effect* stems from the fact that a firm has an incentive to choose higher probability of survival when the (continuation) value of the firm is higher. In Figures 1 and 2, when there are few firms in the market the strategic effect dominates, while in more fragmented markets the reverse is true. A monopoly counting on bailout always invests zero in survival, because its failure is always a systemic event and thus it will survive with certainty, regardless of whether it has failed (and got bailed out) or not.[30] But the probability of survival is less than certain for a duopoly, because now the firm can de facto fail if there is no systemic failure (i.e. its rival survives). Therefore, a duopolist has some incentive to invest in survival, but typically invests less and takes more risk than a duopolist would in the market without bailout. This is the strategic effect. However, the event that everyone fails and gets bailed out becomes ever less likely in more fragmented markets. As a result, the strategic effect diminishes with the number of firms. Because the strategic effect diminishes faster than the charter value effect (firm values decrease with n), the charter value effect becomes dominant for sufficiently fragmented markets and consequently firms take less risk when they can count on government bailout compared to the case when they cannot. This result is surprising, primarily because conventional wisdom would not suggest that firm value can play such a pivotal role when it's so small. Lastly, notice that the market with bailout is always less concentrated in steady state (see the third panels of Figures 1 and 2). This is the result of entry (higher firm values attract more entry) and of course the fact that systemic failures and the resulting bailouts work against market concentration by setting the market back to its original state.

Figure 1 analyses the effect of demand conditions. As α and thus the potential profit from the market game $M(n)$ increases, firm values naturally increase (first column of graphs). This has the unsurprising effect of firms taking less risk in general (i.e. they increase investments in survival, second columns of graphs). Consequently, the strategic effect plays a much more muted role, because the event of a systemic failure and the resulting government bailout becomes more remote for any given $n > 1$. This in turn means that the difference in firm values across the two market settings narrows as α increases (first column of graphs). As a result, in the region when the charter value starts to dominate the strategic effect (i.e. in fragmented markets), the difference in charter values are so tiny that the difference between the investment profiles across the two market

[30] It is possible to take the monopoly out of the game (by saying e.g. if there is only one firm left, then the government nationalises the firm), set $\widetilde{V}(1) = V(1)$ and feed this exogenous value into the optimisation problems. The results are qualitatively the same, as the problem of strategic complementarities does not disappear in the market with bailout.



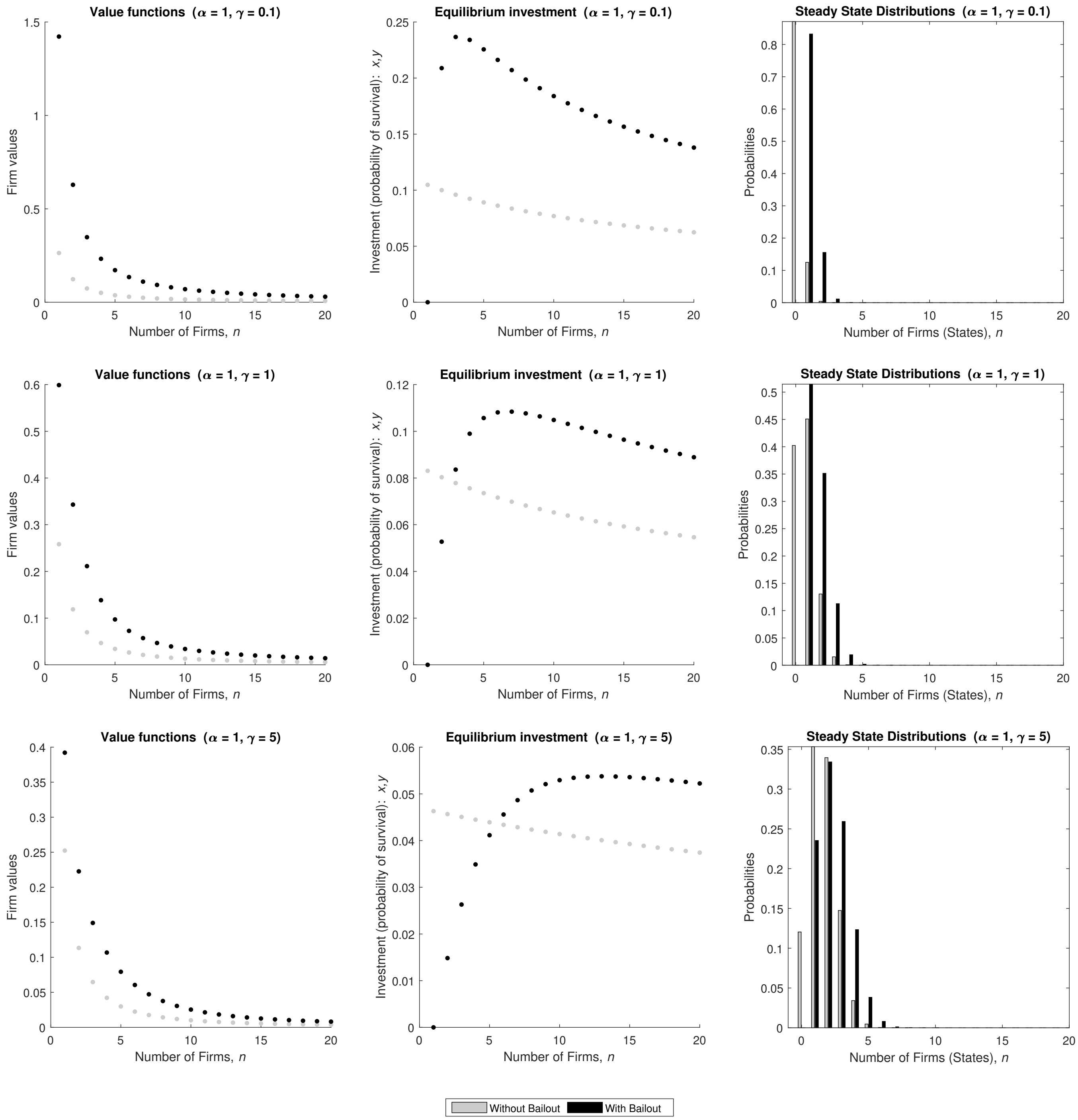
Notes: per-period profit function: $\pi(x;n) = (\alpha/(n+1))^2 - x^2/(1-x)$, $\alpha = 1, 5, 10$ (for the first, second and third row of graphs, respectively); survival probability: $f(x) = x$; probability function of entry: $\rho(a) = 1 - \exp(-a)$; $\beta = 0.97$; maximum number of firms: 20 (potential entrants: 5).

Figure 1: The effect of demand ($\alpha = 1, 5, 10$)

settings vanishes (although it is still true that $x(n) < y(n)$ for large n). In sum, the two types of markets still exhibit important differences in concentrated markets, but for more fragmented market structures, the two market types look very much alike for large α. Especially so, when we consider the steady state distributions in more detail (third column of graphs). For low values of α, strategic effects play an important role, hence the apparent differences in firm values, investment profiles, and the resulting steady state distributions. However, as α increases, firm values and investments increase, which in turn result in both higher entry and lower failure rates, leading to relatively fragmented markets in both settings. As discussed, when the markets are fragmented, the differences across the two market types are very small, hence the almost identical steady state distributions. In other words, for high α the two types of markets look strikingly alike and bailout policy does not seem to make (much) difference.

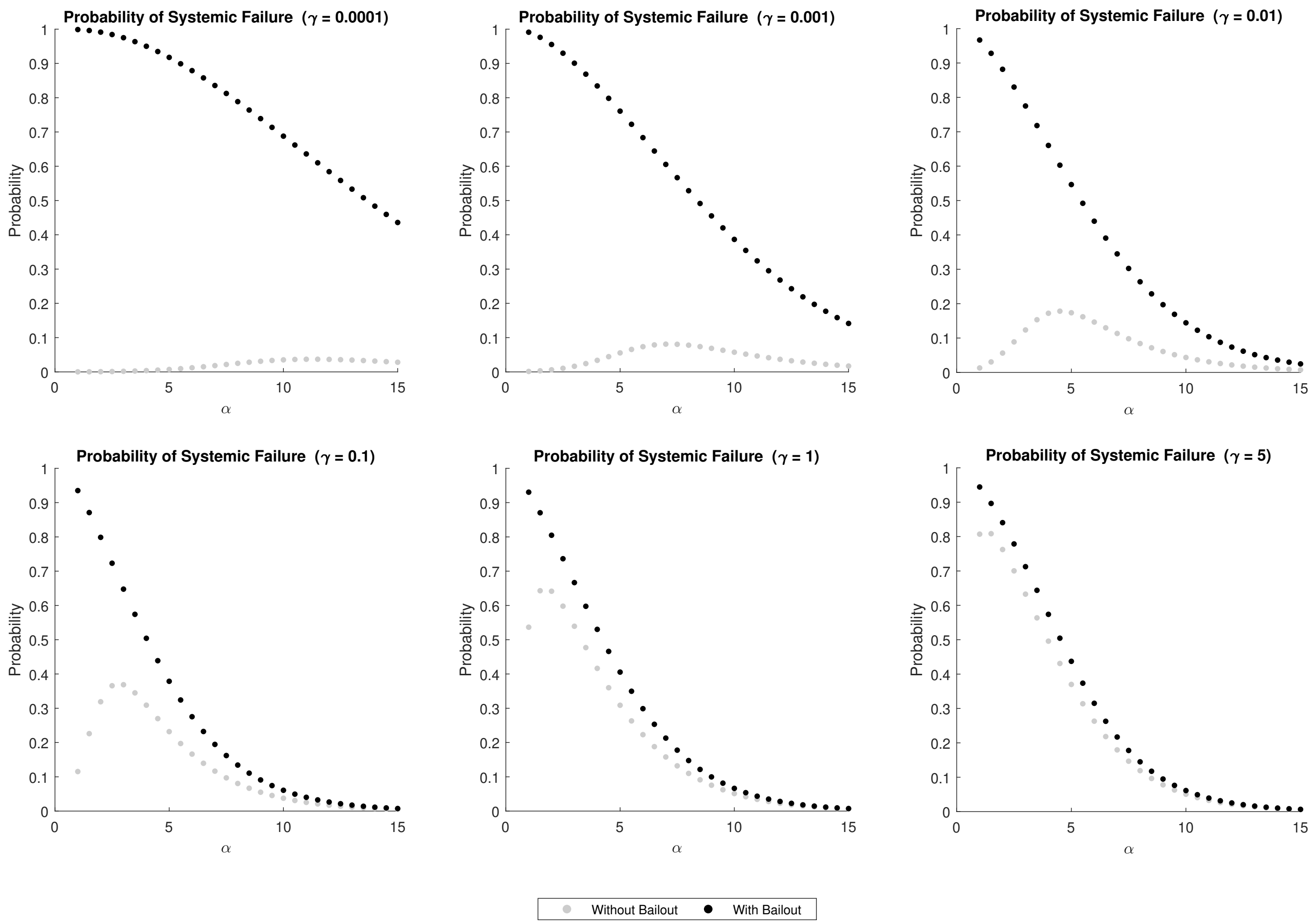
Figure 2 examines the effect of outside competition. For low γ, entry is costly in general and thus happens infrequently. This increases firm values in general, because firms can hold on to their positions longer in expectation (e.g. it's more probable that a surviving monopolist can start the next period as a monopolist again). Because firm values are high, investments are relatively high, which means that systemic failure is less likely, and hence the strategic effect is muted and the charter value effect dominates even when markets are more concentrated. However, as entry intensifies (i.e. γ increases), the charter value effect starts to dominate only in more fragmented markets while the difference in the investment profiles between the two market types narrows too due to the smaller difference in firm values at these states.

I analyse systemic risk in Figures 3 and 4. In particular, for different parameter values I calculate the weighted average of systemic risk, where the weights are the steady state probabilities. From the analyses above, the effect on the average systemic risk (i.e. the probability that all firms fail) is unclear. On the one hand, firms in the market with bailout invest less and fail with higher probability when the market is concentrated. On the other hand, they invest more and fail less often in more fragmented markets, although the difference is typically fairly small. Moreover, the market with bailout is less concentrated in steady state, suggesting the investment profiles in fragmented markets would weigh more in the average of systemic risks across states. In Figure 3,



Notes: per-period profit function: $\pi(x;n) = (1/(n+1))^2 - x^2/(1-x)$; survival probability: $f(x) = x$; probability function of entry: $\rho(a) = 1 - \exp(-\gamma a)$ for $\gamma = 0.1, 1, 5$ (for the first, second and third row of graphs, respectively); $\beta = 0.97$; maximum number of firms: 20 (potential entrants: 5).

Figure 2: The effect of entry ($\gamma = 0.1, 1, 5$)



Notes: Systemic failure is defined as the probability of *entering* into the state with zero firms at the end of the production phase.

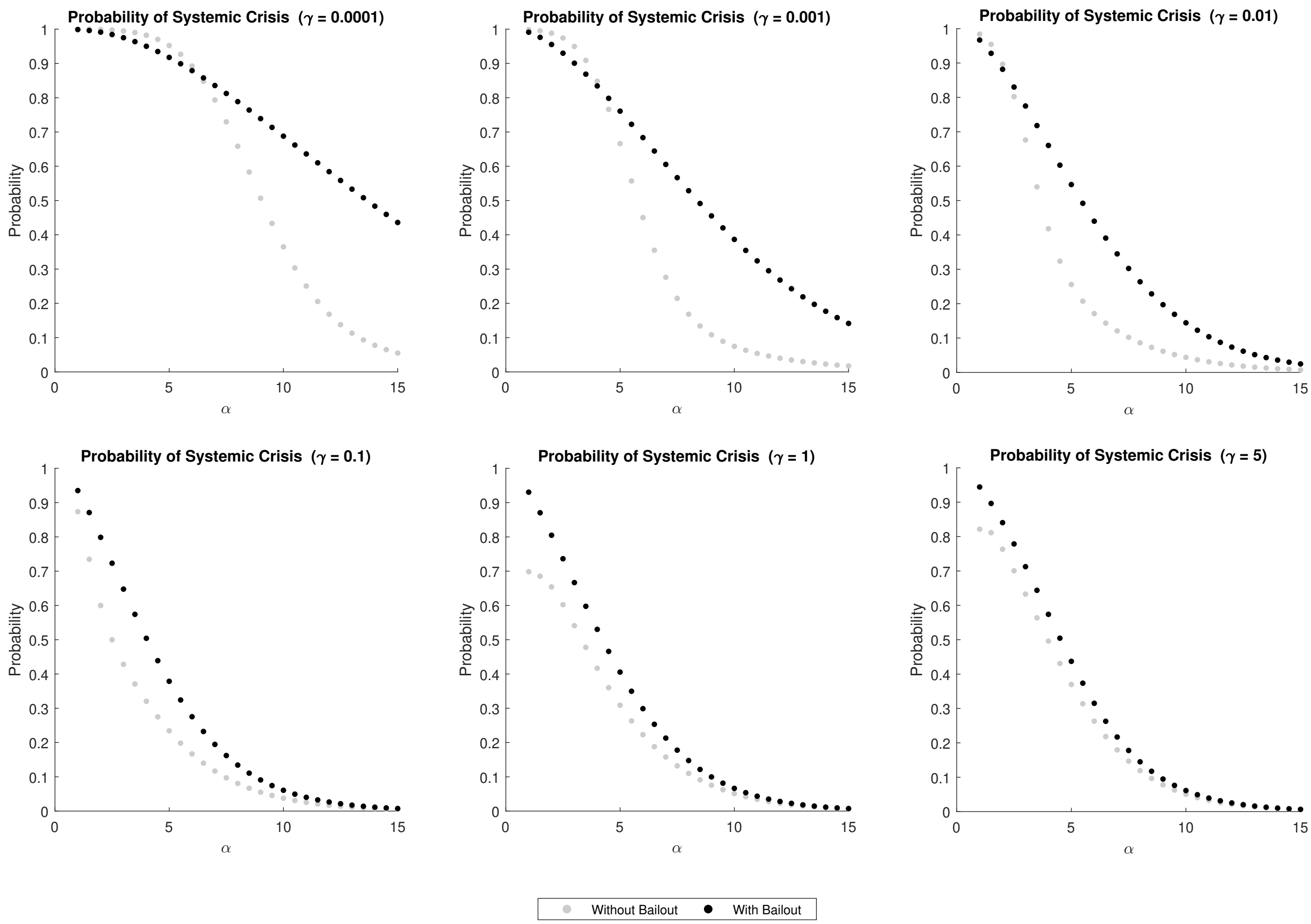
Figure 3: Probability of systemic failure

I define systemic risk for the case without bailout as the probability of *systemic failure*, i.e. the probability that the market *enters* (in the case with bailout: *would enter*) into state zero at the end of a production phase and it is calculated as $P_1(1-x(1)) + P_2(1-x(2))^2 + ...$, where P_j is the steady state probability of state j and $x(j)$ is the equilibrium investment in state $j \geq 1$. The difference between the two types of markets in Figure 3 is large when entry is very sluggish (small γ) and it disappears as I allow for a more intense entry process (increasing γ). However, this is deceptive. Consider the first panel in Figure 3, where the probability of systemic failure in the market without bailout is so low (near zero). Here the market (almost) never enters into state zero, simply because the market is (almost) always in state zero (see e.g. the third panel in the top row in Figure 2).

For this reason, I also calculated the probability of *systemic crisis* in Figure 4, i.e. for the case without bailout the probability that the market *is* (in the case with bailout: *would be*) in state zero at the end of a production phase and it is calculated as $P_0 \cdot 1 + P_1(1-x(1)) + P_2(1-x(2))^2 + ...$[31] This shows a very different picture (first graph in Figure 4): surprisingly, systemic crisis is more probable without bailout in adverse demand conditions (low α) and when the market faces little threat of entry (low γ). This is primarily because, under these market conditions, firms with bailout invest substantially more in survival even in concentrated markets as discussed above. However, in good demand conditions (high α) conventional wisdom is restored and the market without bailout is substantially less (systematically) risky. This is because high α ensures that firms in both market settings invest a lot more in survival and the difference between survival probabilities are fairly small - except in state 1, where the monopoly in the market with bailout still invests zero. Because entry is suppressed, state 1 occurs with high probability in steady state and thus has a large impact on the average probability of systemic crisis. As we allow for more entry, however, the difference between the two types of markets disappear for all market conditions α (last graph in Figure 4).

What is clear from both Figure 3 and 4 is that stronger demand or more intense entry result in

[31] For the case with bailout, both for systemic failure and systemic crisis the probability is $\widetilde{P}_1(1-y(1)) + \widetilde{P}_2(1-y(2))^2 + ...$ (where $\widetilde{P}_j$ is the steady state probability of state j and $y(j)$ is the equilibrium investment in state j), because state zero is of course never reached.



Notes: Systemic crisis is defined as the probability of *being* in the state with zero firms at the end of the production phase.

Figure 4: Probability of systemic crisis

lower systemic risk in general and also that the differences between the two market settings vanish with α and γ. This is the result of the fact that higher α and γ, together or alone, lead to an (expected) market structure that is fragmented in steady state, and this in turn keeps the strategic effect at bay. The key, therefore, appears to be (endogenous) market structure. As long as there is government bailout, more concentrated markets will typically exhibit more systemic risk, which is primarily due to the strategic effect at work. However, if circumstances in the market, such as benign demand conditions, lax competition (e.g. high product differentiation, high switching costs, etc), or intense entry nudge the market towards a fragmented structure, then the fact that firms can count on bailouts is not overly concerning from a systemic risk perspective.[32]

6 Banking models: $\pi(\,\cdot\,; n) = M(n)q(1 - (\,\cdot\,))$

In the baseline model, the cost function of survival $g(\,\cdot\,)$ enters additively into the profit function. The fixed cost nature of $g(\,\cdot\,)$ is a common modelling feature both in the literature spearheaded by Ericson-Pakes (1995) and also in studies on unobserved quality, but it can be found in the banking literature too (see e.g. Dell'Ariccia et al, 2014 or Martinez-Miera and Repullo, 2017). This assumption has technical advantages and also serves an important general purpose. In any static game, firms would never choose to produce negative profits. While this is perfectly reasonable in a static setting, firms in a dynamic environment may find it optimal to operate at a loss today in the hope of profits tomorrow. In order to allow for this possibility, the dynamic leg of the optimisation problem entered additively in the previous section. However, one may find the fixed cost nature unrealistic in general and in models of banking in particular. Therefore, in what follows I discuss an alternative specification based on a model of liquidity risk.

The primary goal of this section, therefore, is to provide a simple microfoundation of the market game, which is closely aligned with traditional models of banking. The secondary objective is robustness check: we will see that while the model presented here produces fundamentally different firm behaviour and market structure dynamics, the main conclusions are unchanged in terms of

[32] It is perhaps important to emphasise that high α does not necessarily mean that firms make (much) profit in a period, as profit (i.e. $M(n) - g(x)$) also depends on investment and (endogenous) market structure.

systemic risk.

6.1 The environment

In the following model of liquidity risk, banks compete for deposits in a Cournot setting and trade off higher profits (lower cash reserves) with smaller probability of survival.[33] The setup formalises in a simple way that long term investments make bank liquidity difficult to adjust in the short run, and while a liquidity buffer is costly (in terms of foregone profits), it is essential to weather uncertain liquidity shocks. The model is also formally equivalent to a dynamic extension of the (static) credit risk model of Allen and Gale (2004), where banks also compete in the deposit market in a Cournot setting and trade off higher returns with lower probability of survival, as demonstrated in Appendix D. In what follows, I present a brief description of the game, further details can be found in Appendix B.

In each period, bank i collects deposits d_i and promises to pay back $r(D)$ on each (per unit) withdrawal, where $D = \sum_{i=1}^{n} d_i$ and $r'(\cdot) > 0$. Withdrawals can occur intra-period or at the end of the period. At the beginning of the period, bank i leaves $(1 - z_i)\, d_i$ in cash and invests z_i portion of its deposits $z_i d_i$ in assets, which earn repayment rate $R > 1$ at the end of the period and zero if the investment is liquidated before. I assume that cash pays no interest, but it is the only way to meet random intra-period withdrawals, because investments cannot be liquidated within a period. The portion of deposits withdrawn intra-period from bank i is a random variable $w_i \in [0, 1]$ with $\Pr(w_i \leq 1 - z_i) = \psi(z_i)$ and thus the bank is able to meet intra-period liquidity demand with probability $\psi(z_i)$, $\psi'(\cdot) < 0$.

Depositors don't observe z_i, they do not know a bank's exposure to liquidity risk. Depositors play an equilibrium strategy where they never deposit in a bank which has ever failed to meet intra-period withdrawals, banks do not use deposit bases to signal risk and, given this, consumers rightly ignore deposits when they form their beliefs about unobserved risk. See Appendix B for a detailed description of strategies and e.g. Rob and Fishman (2005) for a similar example in the

[33] See e.g. Egan et al (2017) for a recent study where rivalry among banks is modelled as (imperfect) competition for deposits in a dynamic (albeit stationary) setting. However, in Egan et al (2017) a bank cannot choose its risk profile.

unobserved quality literature.

If the bank is not able to meet all intra-period withdrawals, it will not operate next period, because rational depositors will favour rivals with a history of successful repayments in a symmetric equilibrium (alternatively, one can assume the regulator closes the bank). The bank thus essentially chooses its level of exposure to an uncertain intra-period liquidity shock by choosing the size of its liquidity buffer: choosing less cash increases profits today at the expense of the probability of survival tomorrow. Bank i's per-period profit is equal to $(R-r(D))z_id_i - r(D)(1-z_i)d_i$, which simplifies to $(Rz_i - r(D))d_i$. All depositors who did not want to or could not withdraw intra-period will be fully paid at the end of the period (when R is paid).[34]

Assuming $r(\cdot) = (\cdot)^\theta$ and $\theta \geq 1$, each period bank i maximises the following profit function when choosing d_i with n banks in the market:

$$\left(Rz - \left(\sum_{i=1}^{n} d_i\right)^\theta\right) d_i$$

The first order condition after imposing symmetry and the resulting solution are thus:

$$Rz - \theta (nd)^{\theta-1} d - (nd)^\theta = 0$$

$$d_* = \left(\frac{Rz}{(\theta+n)\, n^{\theta-1}}\right)^{\frac{1}{\theta}}$$

Profit in symmetric equilibrium is as follows:

$$\begin{aligned}\left(Rz - (nd_*)^\theta\right) d_* &= \left(Rz - \frac{Rzn^\theta}{(\theta+n)\, n^{\theta-1}}\right)\left(\frac{Rz}{(\theta+n)\, n^{\theta-1}}\right)^{\frac{1}{\theta}} \\ &= \theta n^{-\frac{\theta-1}{\theta}} \left(\frac{Rz}{\theta+n}\right)^{\frac{1+\theta}{\theta}}\end{aligned}$$

Suppose the probability of survival is defined as $f(x) = x = \psi(z) = 1 - z^\eta$, where $\eta \geq 2$. It is slightly more convenient in the current framework to set up the dynamic optimisation problem

[34] Note again that there is no point liquidating the bank intra-period, as liquidation is value destroying.

using the inverse function $z = (1-x)^{\frac{1}{\eta}}$ and so the loan rate can be written as $R(1-x)^{\frac{1}{\eta}}$ and the per period profit thus takes the form of

$$\pi(x;n) = M(n)\, q(1-x)\,, \text{ where } \quad M(n) = \theta n^{-\frac{\theta-1}{\theta}} \left(\frac{R}{\theta+n}\right)^{\frac{1+\theta}{\theta}}, \quad q(1-x) = (1-x)^{\frac{1+\theta}{\theta\eta}}$$

It can be easily verified that most properties of $\pi(x;n)$ in Assumption 1 hold, i.e. $\pi(x;n)$ is decreasing in n and $\partial\pi(x;n)/\partial x < 0$, $\partial\pi(x;n)/\partial x\partial x < 0$, provided $\theta \geq 1$ and $\eta \geq 2$. The technical assumption $\partial\pi(0;n)/\partial x = 0$, which ensures interior solution, does not hold, but I will choose parameter values such that interior solutions result.

The dynamic optimisation problem without bailout is as follows (the case with bailout is analogous):

$$v(n;x_{-i}) = \max_{0\leq x_i\leq 1}\left\{\pi(x_i;n) + \beta x_i \sum_{k=0}^{n-1} W_0(n-k)\Pr(k|x_{-i})\right\}$$

Note that this formulation of the bank's dynamic optimisation problem closely resembles the dynamic banking models in e.g. Suarez (1994), Faia et al (2021) or Freixas and Rochet (2008, Chapter 3.5.1). The first order condition to the programme above is then:

$$-\frac{(1+\theta)\,M(n)}{\theta\eta}(1-x_i)^{\frac{1-\theta(\eta-1)}{\theta\eta}} + \beta\sum_{k=0}^{n-1} W_0(n-k)\Pr(k|x_{-i}) = 0$$

to which the solution is

$$x_* = 1 - \left(\frac{1+\theta}{\theta\eta}\cdot\frac{M(n)}{\beta\sum_{k=0}^{n-1} W_0(n-k)\Pr(k|x_{-i})}\right)^{\frac{\theta\eta}{\theta(\eta-1)-1}}$$

The corresponding second order condition is as follows:

$$-\frac{(1+\theta)\,(\theta(\eta-1)-1)\,M(n)}{\theta^2\eta^2}(1-x)^{\frac{1-\theta(2\eta-1)}{\theta\eta}} < 0$$

given $\theta(\eta-1) > 1$. In the numerical calculations, I set $\theta = 1$ and $\eta = 4$, so that the resulting

$M(n)$ in this model is similar to that of the baseline model and also to ensure that interior solutions result. The parameter γ shifts the distribution of the fixed cost of entry and I choose its value so that the support of the steady state distribution stays within the range of 20 firms to facilitate visual presentation of the results. I solve the game using the same algorithm as discussed in the Computation Section 5.1.

6.2 Banking model, numerical results

As is clear from the Figures 5-8, the main difference compared to the baseline model is that banks behave differently, investment in survival increases with n, in contrast to the findings in Section 5. This is simply because the cost of survival $q(1-x)$ enters now multiplicatively, rather than additively. As a result, lower $M(n)$ naturally reduces the cost of survival, which in turn induces banks to invest more in fragmented markets.[35]

In the present banking model, the equilibrium investment profiles $x(n), y(n)$ increase with n, because the marginal cost of investment decreases faster with n than the marginal benefit of survival. The marginal cost of investment is a function of $M(n)$ and $M(n) \to 0$ as $n \to \infty$. The marginal benefit of survival, however, is not only a function of $M(n)$, but also firm values at all possible future states, including the lucrative (concentrated) states (i.e. $M(1), M(2), ...$). Hence, the investment profile increasing with n. In contrast, the marginal cost of investment in survival is independent of $M(n)$ in the baseline model with per period profits $M(n) - g(\cdot)$.

Interestingly, while firm behaviours are very different in the baseline and banking models, the comparison of the markets with and without bailouts yield strikingly similar results. I present the graphs without detailed analyses, I only highlight the most important features. Similarly to the baseline model discussed in the previous section, there is always a charter value effect ($\widetilde{V}(n) > V(n)$), banks counting on bailout invest less in concentrated and invest more in fragmented

[35] This is in contrast to the findings of Allen and Gale (2004), who find that higher n implies more risk taking. It is, once again, a reminder that static models have important limitations when decisions are made in a dynamic context. Moreover, simple extensions of static models to dynamic settings with stationary equilibrium are not quite satisfactory either, because in these dynamic models the future is much like the present, the continuation value is simply the present discounted value of today's static profits, and thus these models tend to produce very similar results to static models. In my model, however, decisions about survival affect not only whether future profits could be collected, but they also have dynamic implications for tomorrow's strategic space and thus for the level of future profits.

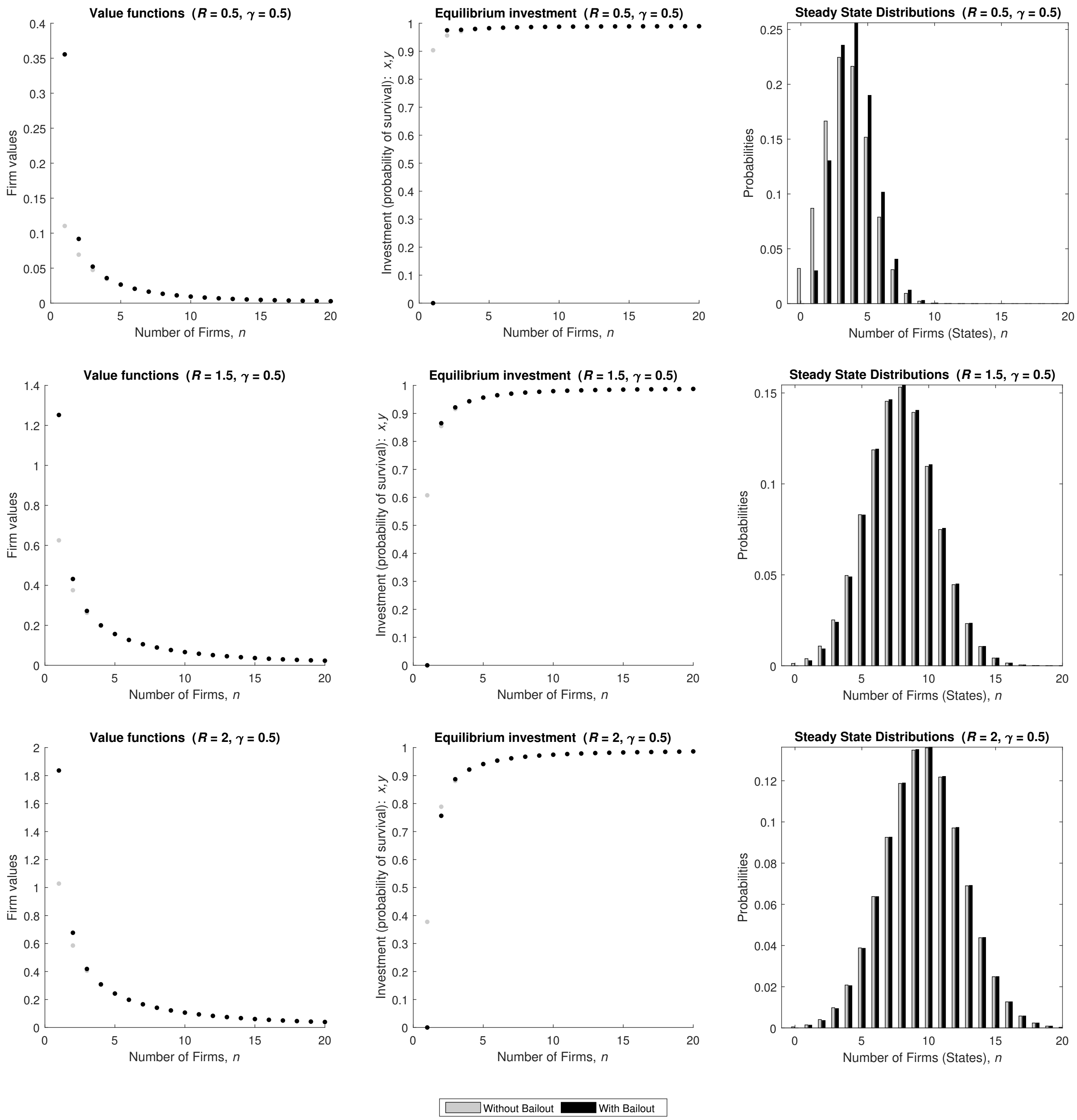
markets (albeit the difference is vanishingly small), and the probability of systemic crisis with bailout is lower for low demand (low R) than without bailout. That is, there is a parameter region where the market with bailout exhibits less systemic risk, in contrast to conventional wisdom.

7 Conclusions

The stochastic dynamic games presented highlight a complex relationship between market structure, competition and risk taking, which has important implications for systemic risk. I analyse two channels through which bailout policies affect risk taking behaviour: the strategic effect increases the incentive to take risk, while the charter value effect reduces it. The interplay of these two effects determine market structure, while market structure in turn drives the interplay between them. Regarding systemic risk, conventional wisdom is challenged in some parameter regions where bailout policy *reduces* systemic risk; in others it is confirmed that the presence of bailout increases systemic risk, as policy makers and academics often argue in public debates. However, the effect of bailout policies on systemic risk seem to be small overall, which points to the important countervailing role of charter value.

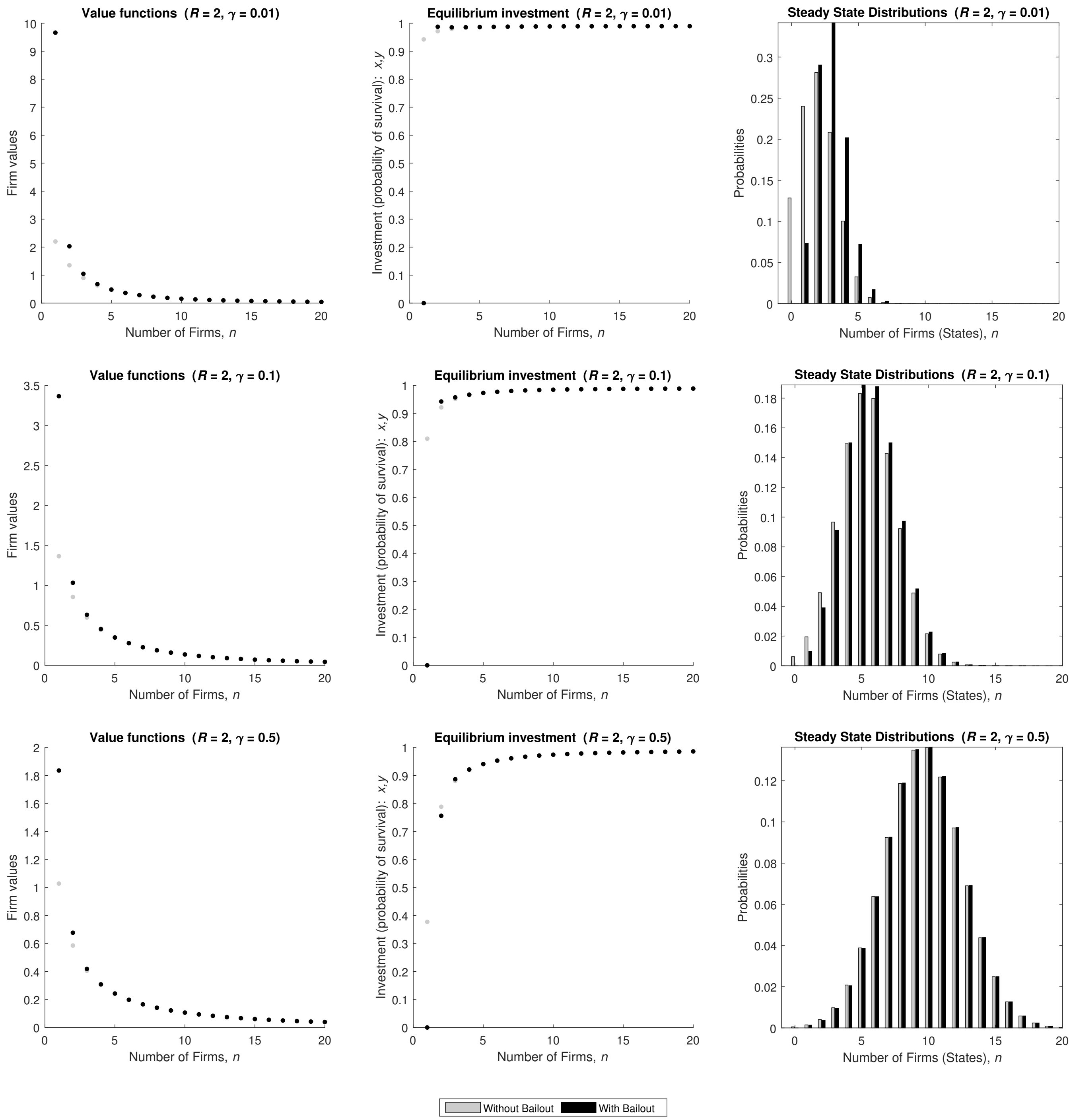
The model is not calibrated for several reasons. First, it is designed to be a general framework to encompass many market settings and thus it is not rich enough in its current form to bring it to data. Second, model parameters are not suitable for calibration, because they cannot be directly mapped into observational data, they are best estimated in a richer structural model with firm heterogeneity and uncertainty at both firm and market levels. Third, complex models are often calibrated, inter alia, because many parameters need to be pinned down first in a meaningful way to allow the researcher to analyse the variables of interest. However, one advantage of a general setup and the limited number of parameters is that results can be illustrated for the full set of parameters and model performance can therefore be comprehensively assessed across all parameters and their relevant values, as done in this study.

The model and the accompanying Matlab code present a useful framework for future research and policy analyses. This framework, for instance, can be used to assess and compare different bailout policies. In particular, this paper analyses the most conservative policy option, where



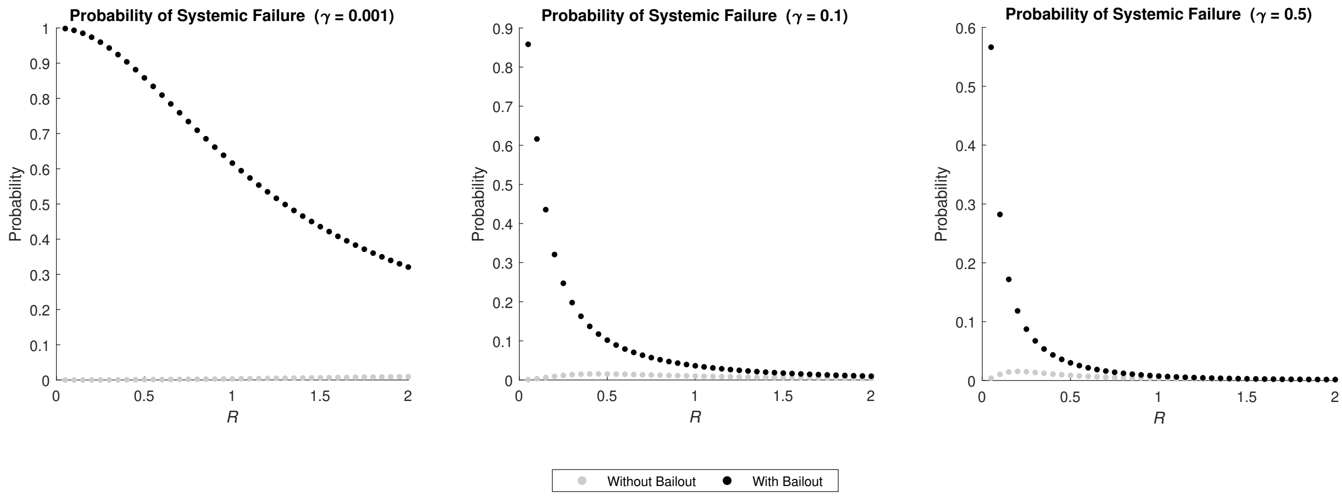
Notes: per-period profit function: $\pi(x;n) = (R/(n+1))^2 \cdot \sqrt{1-x}$, $R = 0.5; 1.5; 2$ (for the first, second and third row of graphs, respectively); survival probability: $f(x) = x$; probability of entry: $\rho(a) = 1 - \exp(-0.5a)$; $\beta = 0.97$; maximum number of firms: 20 (potential entrants: 5).

Figure 5: Banking model: The effect of demand ($R = 0.5; 1.5; 2$)



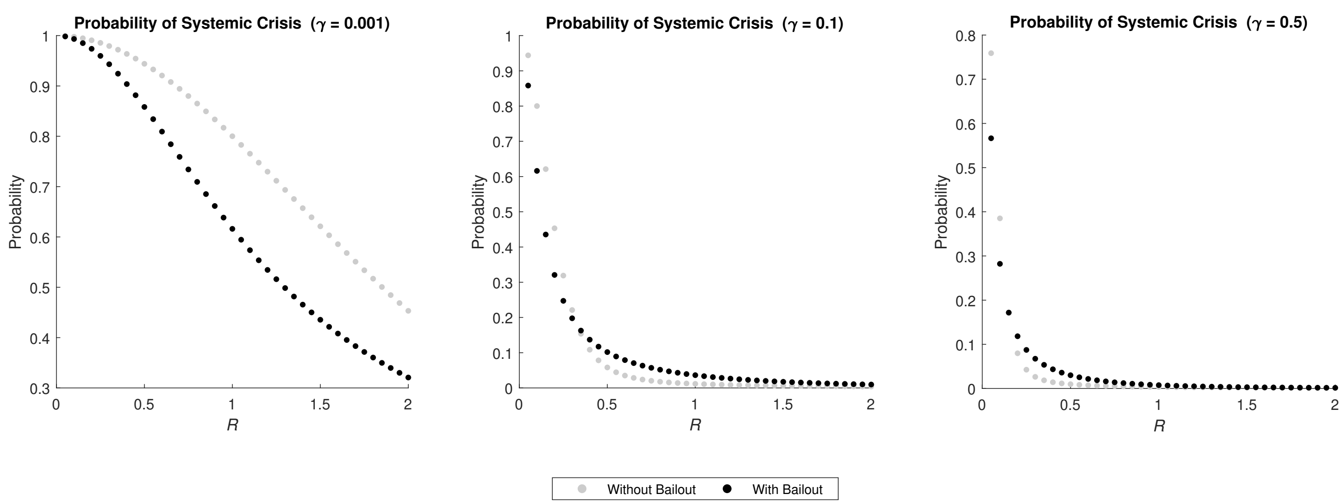
Notes: per-period profit function: $\pi(x; n) = (2/(n+1))^2 \cdot \sqrt{1-x}$; survival probability: $f(x) = x$; probability of entry: $\rho(a) = 1 - \exp(-\gamma a)$ for $\gamma = 0.01; 0.1; 0.5$ (for the first, second and third set of three graphs, respectively); $\beta = 0.97$; maximum number of firms: 20 (potential entrants: 5).

Figure 6: Banking model: The effect of entry ($\gamma = 0.01; 0.1; 0.5$)



Notes: Expected systemic failure is defined as the probability of *entering* into the state with zero firms at the end of the production phase.

Figure 7: Banking model: probability of systemic failure



Notes: Expected systemic crisis is defined as the probability of *being* in the state with zero firms at the end of the production phase.

Figure 8: Banking model: probability of systemic crisis

bailout occurs only when all incumbent firms fail. It would be interesting to consider the effect of alternative policies, when the government bails out firms under less dramatic scenarios, e.g. when 50% of incumbents fail. The effect of a more generous bailout policy is uncertain, because it amplifies both the charter value and the strategic effects in an unpredictable way, while it also pushes the market towards a more fragmented structure. I leave these questions for future research.

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Appendix

A PROOFS

Proof of Proposition 1

I show by induction that the value function $\widetilde{V}(n)$ is strictly decreasing, the proof for the case $V(n)$ is similar. First, I show that $\widetilde{V}(2) < \widetilde{V}(1)$. Then assuming that $\widetilde{V}(i) < \widetilde{V}(i-1)$ for all $i = 2, ..., n$, I will show that $\widetilde{V}(n+1) < \widetilde{V}(n)$.

First, note that $\widetilde{V}(n)$ are non-negative for all n, because choosing $y(n) = 0$ always ensures at least $\pi(0;n) \geq 0$ (Assumption 1). For the monopoly,

$$\begin{aligned}
\widetilde{V}(1) &= \max_{0\leq y\leq 1}\left\{\pi(y;1)+\beta f(y)\widetilde{V}(1)+\beta\widetilde{V}(1)(1-f(y))\right\}\\
&= \max_{0\leq y\leq 1}\left\{\pi(y;1)+\beta\widetilde{V}(1)\right\}
\end{aligned}$$

and the solution to this program is simply $y(1)=0$, given that $\pi(y;1)$ is decreasing in y (Assumption 1).

For the duopoly, we have in equilibrium,

$$\begin{aligned}
\widetilde{V}(2)-\widetilde{V}(1) &= \pi(y;2)+\beta\widetilde{V}(1)f(y)(1-f(y))+\beta\widetilde{V}(2)f(y)^2+\beta\widetilde{V}(2)(1-f(y))^2-\widetilde{V}(1)\\
&= \frac{\pi(y;2)+\beta\widetilde{V}(1)f(y)(1-f(y))+\beta\widetilde{V}(1)f(y)^2+\beta\widetilde{V}(1)(1-f(y))^2-\widetilde{V}(1)}{1-\beta\left(f(y)^2+(1-f(y))^2\right)}\\
&= \frac{\pi(y;2)-\pi(y;1)-\beta\widetilde{V}(1)f(y)(1-f(y))}{1-\beta\left(f(y)^2+(1-f(y))^2\right)}\\
&< 0
\end{aligned}$$

where in the penultimate line in the numerator $\pi(y;2)-\pi(y;1)<0$ by Assumption 1 and the last expression is non-positive, because $0\leq f(y)\leq 1$ and $\widetilde{V}(1)\geq 0$, while the denominator is clearly positive.

Now, assume that $\widetilde{V}(i)<\widetilde{V}(i-1)$ for all $i=2,...,n$. The following inequality holds for the value function in a symmetric equilibrium when the RHS is evaluated at $y=y(n+1)$, rather than the maximiser $y(n)$:

$$\widetilde{V}(n)\geq\pi(y;n)+\beta\sum_{k=0}^{n-1}\widetilde{V}(n-k)\binom{n-1}{k}f(y)^{n-k}(1-f(y))^k+\beta\widetilde{V}(n)(1-f(y))^n$$

Then, adding and subtracting the continuation value for the $n+1$ case and also observing that $\pi(y;n+1)\leq\pi(y;n)$ by Assumption 1a,

$$\begin{aligned}\widetilde{V}(n) \geq & \pi(y;n+1) + \beta \sum_{k=0}^{n} \widetilde{V}(n+1-k) \binom{n}{k} f(y)^{n+1-k} (1-f(y))^k + \beta \widetilde{V}(n+1) (1-f(y))^{n+1} \\ & + \beta \sum_{k=0}^{n-1} \widetilde{V}(n-k) \binom{n-1}{k} f(y)^{n-k} (1-f(y))^k + \beta \widetilde{V}(n) (1-f(y))^n \\ & - \beta \sum_{k=0}^{n} \widetilde{V}(n+1-k) \binom{n}{k} f(y)^{n+1-k} (1-f(y))^k - \beta \widetilde{V}(n+1) (1-f(y))^{n+1}\end{aligned}$$

On the RHS, the first line is just $\widetilde{V}(n+1)$ and so

$$\begin{aligned}\widetilde{V}(n) - \widetilde{V}(n+1) \geq & \beta \sum_{k=0}^{n-1} \widetilde{V}(n-k) \binom{n-1}{k} f(y)^{n-k} (1-f(y))^k + \beta \widetilde{V}(n) (1-f(y))^n \\ & - \beta \sum_{k=0}^{n} \widetilde{V}(n+1-k) \binom{n}{k} f(y)^{n+1-k} (1-f(y))^k - \beta \widetilde{V}(n+1) (1-f(y))^{n+1}\end{aligned}$$

Then switching indexes, $k = i$ in the first and $k = i+1$ in the second sums, we can rewrite the above as

$$\begin{aligned}\left(1-\beta(1-f(y))^n\right)\left(\widetilde{V}(n) - \widetilde{V}(n+1)\right) \geq & \beta \sum_{i=0}^{n-1} \widetilde{V}(n-i) \binom{n-1}{i} f(y)^{n-i} (1-f(y))^i \\ & - \beta \sum_{i=-1}^{n-1} \widetilde{V}(n-i) \binom{n}{i+1} f(y)^{n-i} (1-f(y))^{i+1} \\ & + \beta \widetilde{V}(n+1) f(y) (1-f(y))^n\end{aligned}$$

Then, using Pascal's identity and the fact that $\binom{n}{l} = 0$ for $l < 0$ and $l > n$, we have that

$$
\begin{aligned}
\left(1-\beta\left(1-f\left(y\right)\right)^{n}\right)\left(\widetilde{V}(n)-\widetilde{V}(n+1)\right) \geq & \beta\sum_{i=0}^{n-1}\widetilde{V}(n-i)\binom{n-1}{i}f\left(y\right)^{n-i+1}\left(1-f\left(y\right)\right)^{i} \\
& -\beta\sum_{i=-1}^{n-1}\widetilde{V}(n-i)\binom{n-1}{i+1}f\left(y\right)^{n-i}\left(1-f\left(y\right)\right)^{i+1} \\
& +\beta\widetilde{V}(n+1)f\left(y\right)\left(1-f\left(y\right)\right)^{n}
\end{aligned}
$$

Now, switching back the indexes,

$$
\begin{aligned}
& \left(1-\beta\left(\left(1-f\left(y\right)\right)^{n}+f\left(y\right)^{n+1}\right)\right)\left(\widetilde{V}(n)-\widetilde{V}(n+1)\right) \geq \\
& \beta\sum_{k=1}^{n-1}\left[\widetilde{V}(n-k)-\widetilde{V}(n-k+1)\right]\binom{n-1}{k}f\left(y\right)^{n-k+1}\left(1-f\left(y\right)\right)^{k} \\
& +\beta\widetilde{V}(n+1)f\left(y\right)\left(1-f\left(y\right)\right)^{n}>0
\end{aligned}
$$

Note that the RHS is strictly positive, because in the summation sign we have the convex linear combination of strictly positive values by the inductional hypothesis and note also that $0<\beta\left(\left(1-f\left(y\right)\right)^{n}+f\left(y\right)^{n+1}\right)<1$ on the LHS.

Lastly, observe that because $\widetilde{V}(n)$ is strictly decreasing and non-negative, $\widetilde{V}(n)$ must be strictly positive. *QED*

Proof of Proposition 2

The proof is by induction. First, I show that $\widetilde{V}(1)-V(1)>0$, then assuming $\widetilde{V}(i)-V(i)>0$ for all $i=1,...,n-1$, I will prove that $\widetilde{V}(n)-V(n)>0$.

Observe that for the case of a monopoly, the two maximisation problems are as follows

$$
\begin{aligned}
V(1) &= \max_{0\leq x\leq 1}\left\{\pi(x;1)+\beta f\left(x\right)V(1)\right\} \\
\widetilde{V}(1) &= \max_{0\leq y\leq 1}\left\{\pi(y;1)+\beta f\left(y\right)\widetilde{V}(1)+\beta\widetilde{V}(1)\left(1-f\left(y\right)\right)\right\} \\
&= \max_{0\leq y\leq 1}\left\{\pi(y;1)+\beta\widetilde{V}(1)\right\}
\end{aligned}
$$

As $\pi(y;n)$ is strictly decreasing in y, by strict concavity of the objective function (Assumption 1) the unique solution to the second program is simply $y(1)=0$ and thus $\widetilde{V}(1)=\pi(0;1)/(1-\beta)$. If the optimal solution to the maximisation problem without bailout is $x(1)=0$, then $V(1)=\pi(0;1)$ and so $\widetilde{V}(1)-V(1)>0$. When $x(1)>0$, then $\widetilde{V}(1)=\pi(0;1)/(1-\beta)>\pi(x(1);1)/(1-\beta f(x(1)))=V(1)$, because the numerator is decreasing in x (Assumption 1b) and the denominator is increasing since $0<f(x(1))\leq 1$.

Now, I assume that $\widetilde{V}(i)-V(i)>0$ for all $i=1,...,n-1$ and show that this must imply that $\widetilde{V}(n)-V(n)>0$. Since $x=x(n)$ maximises $V(n)$, rather than $\widetilde{V}(n)$,

$$\widetilde{V}(n) \geq \pi(x;n)+\beta\sum_{k=0}^{n-1}\widetilde{V}(n-k)\binom{n-1}{k}f(x)^{n-k}(1-f(x))^{k}+\beta\widetilde{V}(n)(1-f(x))^{n}$$

Subtract $V(n)$ from both sides to get

$$\widetilde{V}(n)-V(n)\geq\beta\sum_{k=0}^{n-1}\left[\widetilde{V}(n-k)-V(n-k)\right]\binom{n-1}{k}f(x)^{n-k}(1-f(x))^{k}+\beta\widetilde{V}(n)(1-f(x))^{n}$$

Take the first element from the sum on the RHS and rearrange it to the LHS to get

$$\begin{aligned}(1-\beta f(x)^{n})\left[\widetilde{V}(n)-V(n)\right] \geq&\beta\sum_{k=1}^{n-1}\left[\widetilde{V}(n-k)-V(n-k)\right]\binom{n-1}{k}f(x)^{n-k}(1-f(x))^{k}\\ &+\beta\widetilde{V}(n)(1-f(x))^{n}\\ >&0\end{aligned}$$

On the RHS, the last term in the second line is positive by Proposition 1 and the sum in the first line is also positive by the inductional hypothesis. *QED*

Proof of Proposition 3

I already established in the proof of the previous proposition that $y(1)=0$. The assumption $\partial\pi(0;n)/\partial x=0$ (Assumption 1b) ensures interior solution for the market without bailout, therefore

$x(1) > y(1)$. Now, I show that $x(n) < y(n)$, when n is large enough. The proof is by contradiction. Suppose $x(n) > y(n)$ for all n. The FOCs for the case n are as follows

$$\beta f'(x) \sum_{k=0}^{n-1} V(n-k) \binom{n-1}{k} f(x)^{n-k-1} (1-f(x))^k = -\partial\pi(x;n)/\partial x$$

$$\beta f'(y) \left[\sum_{k=0}^{n-1} \widetilde{V}(n-k) \binom{n-1}{k} f(y)^{n-k-1} (1-f(y))^k - \widetilde{V}(n) (1-f(y))^{n-1} \right] = -\partial\pi(y;n)/\partial y$$

where $x(n)$ and $y(n)$ solve the first and the second equation above, respectively. If $x(n) > y(n)$, it must be that the first FOC evaluated at $y = y(n)$ is as follows

$$\beta f'(y) \sum_{k=0}^{n-1} V(n-k) \binom{n-1}{k} f(y)^{n-k-1} (1-f(y))^k > -\partial\pi(y;n)/\partial y$$

because the objective functions are strictly concave (Assumption 1). Subtracting the second FOC from this inequality and rearranging yields

$$\sum_{k=0}^{n-1} \left[\widetilde{V}(n-k) - V(n-k) \right] \binom{n-1}{k} f(y)^{n-k-1} (1-f(y))^k - \widetilde{V}(n) (1-f(y))^{n-1} < 0$$

This inequality always holds for $n = 1$, but it cannot hold for a large enough n. To see this, take out the last element from the summation sign and rewrite

$$\sum_{k=0}^{n-2} \left[\widetilde{V}(n-k) - V(n-k) \right] \binom{n-1}{k} f(y)^{n-k-1} (1-f(y))^k + \left(\widetilde{V}(1) - V(1) - \widetilde{V}(n) \right) (1-f(y))^{n-1} < 0$$

The first expression on the LHS is clearly positive, because the summation is a linear combination of strictly positive values (Proposition 2). The second expression is positive too for large enough n, because $\widetilde{V}(1) - V(1) > 0$ (Proposition 2) and $\widetilde{V}(n) \to 0$ as $n \to \infty$, which follows from Assumption 1 and Proposition 1. Also, once the term $\widetilde{V}(1) - V(1) - \widetilde{V}(n)$ becomes positive, it will remain so, because $\widetilde{V}(n)$ is strictly decreasing. *QED*

B Details of the Liquidity Risk model in Section 6

As mentioned before, competition among banks is modelled as a Cournot game in the deposit market, similarly to much of the previous literature. There are three dates $T = 0, 1, 2$ within the production phase of each period t, there is no discounting within a period. The set of banks is denoted N^t, with cardinality $|N^t| = n^t$.

Depositors. After each depositor deposits one unit of cash at $T = 0$, the depositor faces an idiosyncratic liquidity shock and withdraws her money "early" ($T = 1$) with probability w or "late" ($T = 2$) with probability $1 - w$. Assuming very large number of depositors, bank i thus faces a withdrawal rate w_i at $T = 1$. The withdrawal rates w_i are random variables, which are identically and independently distributed across banks and have non-degenerate support on $[0, 1]$ with $\Pr(w_i \leq 1 - z_i) = \psi(z_i)$.[36] Liquidity risk is unobserved, i.e. depositors do not observe the liquidity buffers of banks, they only observe if a bank has successfully met its intra-period (i.e. $T = 1$) withdrawals previously. Assuming early consumers do not benefit from late consumption and vice versa, the expected payoff of a depositor is $(wq_i + 1 - w)\, r(D)$, where $q_i \in [0, 1]$ is the depositor's belief about bank i being able to meet all early withdrawals and $r(D)$ is the deposit rate, where $D = \sum_{i=1}^{n} d_i$.[37]

Banks. Bank i faces a large pool of depositors in period t. Each period, bank i chooses its deposit base d_i, the amount to invest in loans $z_i d_i$ and consequently its cash reserves $(1 - z_i)\, d_i$. Loans pay nothing at $T = 1$, but pay a return R at $T = 2$. Assets cannot be liquidated at $T = 1$ (liquidation is value destroying). Cash earns no interest. For simplicity, I assume that the bank makes its decision about the liquidity buffer at the beginning of the period, observes the number of early withdrawals subsequently, and the bank does not open intra-period ($T = 1$) if early withdrawals exceed its cash reserve (i.e. $w_i > 1 - z_i$), there are no payments at all at $T = 1$ in this case.[38]

[36] As discussed in the Introduction, I do not incorporate contagion or aggregate risk in the model, hence withdrawal rates (w_i) are uncorrelated.

[37] Note that late consumers do not gain from withdrawing early in this model, regardless of other depositors' withdrawal strategy. Late consumers always get paid fully, because the bank's assets cannot be liquidated before maturity and R is deterministic (no credit risk).

[38] This is a simplifying assumption, the results are not dependent on it. Alternatively, one could assume that the available cash is distributed equally among early withdrawals at the expense of a slightly more complicated ex ante utility of depositors. What matters is that ex ante utility of depositors increases in q_i.

Timing within a period. At $T=0$: 1) bank i decides to quit or stay, 2) bank i receives deposits d_i and chooses to reserve $(1-z_i)\,d_i$ as cash, then lend a total sum of $z_i d_i$ to entrepreneurs, who will pay return R at $T=2$. At $T=1$: 3) depositor type is (privately) revealed, early withdrawals are attempted, 4) bank i observes the proportion of early withdrawals w_i and if $w_i \leq 1-z_i$, then it opens and pays out early withdrawals, otherwise it remains closed. At $T=2$: the loan pays R and all consumers who did not want to, or manage to, withdraw at $T=1$ get now paid $r\,(D)$, 5) all depositors and banks observe which bank failed to meet early withdrawals.

Markov strategies. I will look at Markov strategies that depend on last period payout history. The last period withdrawal history is defined as $H^t = \times_{i \in N^{t-1}} \{1, 0\}$, where 1 corresponds to the bank successfully meeting all early withdrawal at $t-1$ and zero otherwise. Then a depositor's strategy can be described by the mapping $H^t \rightarrow N^t$. The strategy of banks comprises of three mappings. Having observed last period withdrawal history, they decide to quit or stay, $\varsigma_i : H^t \rightarrow \{Quit, Stay\}$, then incumbents choose $d_i : H^t \times N^t \rightarrow \mathbb{R}_+$ and the proportion of deposits to be lent $z_i : H^t \times N^t \rightarrow [0,1]$.

Symmetric Nash Equilibrium in Markov strategies

Depositors. Do not deposit in a bank that has not met its intra-period withdrawal demand before. The equilibrium beliefs of depositors are as follows: (i) if a bank has ever failed to meet its intra-period withdrawals, then it will always fail to do so with probability one (i.e., $q_i^t, q_i^{t+1}, ... = 0$), (ii) banks in period t meet their intra-period withdrawal demand with probability $q_i^t = q^t = \Pr\left(w < 1 - z_*^t\right) = \psi\left(z_*^t\right) \equiv x_*^t$ if they have successfully met intra-day period withdrawals previously.

Banks. Quit if you failed to meet intra-period withdrawals last period. Otherwise stay and choose your deposit base and probability of survival d, x such that

$$\left(d_*^t, x_*^t\right) = \arg\max_{\left(d_i^t, x_i^t\right)} \left(R\varphi^{-1}\left(x_i^t\right) - r\left(d_i^t + \sum_{j \neq i} d_j^t\right)\right) d_i^t + \beta x_i^t \sum_{k=0}^{n^t-1} W_0(n^t-k)\Pr(k^t | x_{-i*}^t)$$

These Markov strategies constitute a symmetric subgame perfect Nash equilibrium, because they maximise the utility of depositors, given their beliefs and the optimal policies of banks, and they also maximise the value of the bank, given depositor strategies, beliefs, and the optimal policies of rival banks, while the beliefs of depositors are correct in equilibrium, i.e. beliefs represent the probability of the actual behaviour of other players conditional on the information available.[39]

C Derivation of the Steady State distributions

Each period of the game is taken to be infinitely small, and thus the infinitesimal transition probabilities at time t are given by

$$\Pr\left(z\left(t+dt\right)=n+m|z\left(t\right)=n\right)=\begin{cases}\lambda_n dt+o\left(dt\right) & \text{if} \quad m=1\\ \mu_n dt+o\left(dt\right) & \text{if} \quad m=-1\\ o\left(dt\right) & \text{if} \quad |m|>1\end{cases}$$

where $\lim_{dt\to 0} o\left(dt\right)/dt=0$, and the intensity parameters of the Markov process are "average birth" $\lambda_n=\sum_{l=1}^{N}\rho_{l,n}$ and "average death" $\mu_n=n\left(1-f\left(x\left(n\right)\right)\right)$.[40] For a stationary steady state distribution $P=[P_0,P_1,...]$, the flow into a state must be equal to the flow out of the state, i.e.

$$(1-\lambda_0)P_0+\mu_1 P_1=P_0$$

$$\lambda_{n-1}P_{n-1}+(1-\lambda_n-\mu_n)P_n+\mu_{n+1}P_{n+1}=P_n \text{ for } n\geq 1$$

Solving this system of equations yields the steady state probabilities as follows

$$P_n=\frac{\lambda_{n-1}}{\mu_n}P_{n-1}=\frac{\lambda_0\lambda_1\cdots\lambda_{n-1}}{\mu_1\mu_2\cdots\mu_n}P_0 \quad \text{for } n\geq 1, \text{ and } P_0=\left(1+\sum_{n=1}^{\infty}\prod_{i=0}^{n-1}\frac{\lambda_i}{\mu_{i+1}}\right)^{-1}$$

In the model without bailout, there is no government intervention, the market evolves undisturbed. As a result, the state when there is no firm in the market (i.e. state zero) happens with positive probability in the steady state. State zero, however, cannot be reached by definition when there are systemic bailouts. The modified dynamic program of a firm expecting systemic bailouts

[39]Note that a Nash equilibrium in Markov strategies is necessarily subgame perfect.
[40]For a detailed derivation of the transition probabilities, see Tóth (2012).

is as follows:

$$\widetilde{v}(n;y_{-i}) = \max_{0\leq y_i\leq 1}\left\{\pi(y_i;n) + \beta f\left(y_i\right)\sum_{k=0}^{n-1}\widetilde{W}_0(n-k)\Pr(k|f\left(y_{-i}\right))\right.$$
$$\left. + \beta\widetilde{W}_0(n)\left(1-f\left(y_i\right)\right)\Pr(n-1|f\left(y_{-i}\right))\right\} \tag{6}$$

where the entrants' values and entry probabilities are defined analogously to the case without bailout.[41] The resulting value function in a symmetric equilibrium is then

$$\widetilde{V}(n) = \pi(y\left(n\right);n) + \beta\sum_{k=0}^{n-1}\widetilde{W}_0(n-k)\binom{n-1}{k}(1-f\left(y(n)\right))^k f(y(n))^{n-k}$$
$$+ \beta\widetilde{W}_0(n)\left(1-f\left(y(n)\right)\right)^n \tag{7}$$

The steady state probabilities are computed similarly to the market without bailouts, except that one needs to account for the fact that state zero (i.e. no firm in the market) is never reached by design (i.e. there is no entry into and exit from state zero), so the modified steady state probabilities are $\widetilde{P}_n = \left(\widetilde{\lambda}_{n-1}/\widetilde{\mu}_n\right)\widetilde{P}_{n-1}$ for $n \geq 2$ with $\widetilde{P}_1 = \left(1+\sum_{n=1}^{\infty}\prod_{i=1}^{n-1}\left(\widetilde{\lambda}_i/\widetilde{\mu}_{i+1}\right)\right)^{-1}$, where the parameters of average "births" (entry) $\widetilde{\lambda}_i$ and average "deaths" (failure) $\widetilde{\mu}_i$ are defined analogously to λ_i and μ_i.

D A dynamic extension of Allen and Gale (2004)

As in Allen and Gale (2004), each bank chooses a portfolio of perfectly correlated loans in each period (i.e. idiosyncratic risk is assumed to be diversified away). The bank charges loan interest rate Rz, which comprises of two components: risk $0 \leq z \leq 1$ is chosen by the bank and $0 < R \leq 1$ is a constant, which can be thought of as a measure of market power in the loan market, because lower R reduces loan rates for all risk classes (i.e. z). Thus higher loan rate Rz is associated

[41] That is, $\widetilde{W}_l(n) = \widetilde{W}_{l+1}(n+1)\widetilde{\rho}_{l+1,n} + \widetilde{W}_{l+1}(n)(1-\widetilde{\rho}_{l+1,n})$, where $\widetilde{\rho}_{l,n} = \rho(\widetilde{W}_l(n+1))$ for $l = 0,..,N$ and $n \in \{0,1,2,...\}$.

with higher risk z and R is the loan rate that the riskiest portfolio commands (i.e. when $z = 1$). Furthermore, I assume that the repayment of loans is structured in a way that first interest Rz and last the principal is paid back (i.e. 1). In particular, the production phase is divided into two subphases: in the first subphase loan is made and interest is paid with certainty; in the second subphase, the principal (the larger sum) is paid in full with probability $x(z)$ and 0 otherwise, where $x'(z) < 0, x''(z) \leq 0, x(0) = 1, x(1) = 0$, as in Allen and Gale (2004). If the principal too is paid back, then the bank is able to repay depositors who demand $(1 + r(D))d_i$ in total at the end of the period, where $D = \sum_{i=1}^{n} d_i$ and $r(D)$ is the deposit rate with $r(\cdot), r'(\cdot) > 0, r''(\cdot) \geq 0$, and d_i is the amount of deposit that bank i collects. If the principal is not paid back, then the bank fails, because the bank profit $(Rz - r(D))d_i$ is never enough to pay depositors in full in case the project fails: $(Rz - 1 - r(D))d_i < 0$, because $Rz \leq 1$ by construction.

The assumption that interest is paid first with certainty and the uncertain principal last is meant to capture the fact that debt almost never defaults at the beginning of its lifetime, it typically generates a steady stream of profits for the bank before default: for instance, if a bank makes a single loan, which then defaults halfway through its life, then the bank still realises interest margin and produces profit on this loan for a number of years prior to bankruptcy.[42] In particular, over the lifetime of a loan, the first installments typically have low default probabilities in practice, while the default probabilities of later payments are significantly higher. Given that the lifetime of a loan is a single period in the current model (otherwise z could not be chosen independently across periods), one simple way to capture this feature is that there are two payments in a period, the first (interest) payment is certain and the second (principal) is stochastic. Note that most debts have the "interest-first-principal-last" feature to some degree.[43] This assumption also ensures that the general model structure of the dynamic game in this section, which is similar to the dynamic banking models of e.g. Perotti and Suarez (2002) or Freixas and Rochet (2008, Chapter 3.5.1),

[42] The average maturity of corporate debt at origination is around 12 years and the average maturity at default is around 5 years, suggesting defaulting firms tend to service a debt for 7 years on average before default (see e.g. Davydenko et al 2012).

[43] For instance, bonds typically pay interest until maturity and principal is paid back only at maturity. Similarly, debt services on loans are most often structured with loan amortization: initially, interest payments constitute the dominant part of the annual debt service (in fact, there are many loans with substantial so-called "principal repayment holiday") and over the years the share of principal increases gradually and it only becomes the dominant part of the payments towards the end of maturity. "Interest only" loans also exist (about 30% of UK mortgage market), where debt service before maturity consists solely of interest payments.

also keeps in line with the basic framework presented in the paper.

In each period, the bank thus makes profit equal to $(Rz_i - r(D))d_i + x(z_i)(1-1)d_i$. The second term is the principal payment occurring with probability $x(z_i)$, which is returned to depositors at the end of the period when paid. Banks compete for deposits in a static and will choose z in a dynamic framework, higher z implying higher risk, i.e. lower probability of survival. This yields a formally equivalent program to the liquidity risk model above.