Ad-valorem Taxes, Prices and Content Diversification in the News Market

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February 8, 2023

Abstract

In this paper, we look at two questions. First, can lower ad-valorem taxes (on the selling of news and on the selling of advertising) conduce to lower prices in the media sector? Second, can lower ad-valorem taxes stimulate firms to increase the diversity of content that they offer? We show that when firms do not diversify content, ad-valorem taxes on selling of news and on selling of advertising have different effects on prices of news: ad-valorem taxes on selling of news reduce prices; ad-valorem taxes on selling of advertising increases prices. The difference arises because ad-valorem taxes on selling of news increases price competition, while ad-valorem taxes on selling of advertising have no effects on price competition, but since they reduce advertisement revenues, media platforms try to recoup these losses by increasing prices to consumers. Second, we show that when we allow for firms to diversify content, ad-valorem taxes on advertising also increase prices. However, ad-valorem taxes on selling of news do not anymore always increase prices relatively to the no ad-valorem taxes scenario. This will depend on consumer's preference for their ideal variety. Accordingly, if consumers have a strong preference for their ideal variety, ad-valorem taxes on selling of news increases prices relatively to the case with no ad-valorem taxes. Third, we show that both ad-valorem taxes on selling of news and on selling of advertising reduce content provision by firms, reducing therefore media diversity.

Keywords: Ad-Valorem Taxes; Content Provision; Advertising; Two-Sided Markets.

JEL Classification: D11; D21; H25, L13; L82.

1 Introduction

Newspapers sell news to readers and advertising space do advertisers, i.e. news media operate in a two-sided market. On one hand, the more readers a newspaper attracts, the more valuable a newspaper is for advertisers. On the other

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hand, the more advertisers a newspaper attracts, the more revenues a newspaper gets to finance the production of news. In turn, many argue that media diversity is essential to democracy and the society (see for instance, Strömberg, 2001, 2004a,b, 2007, 2008; Sunstein, 2007, 2016; Gentzkow et al., 2014)[1].

Given their importance to democracy, some countries offer special tax treatment to newspapers, like reduced VAT rates (see Foros et al., 2019; European Commission, 2021). The idea of the special tax treatment is twofold. First, lower taxes can help to increase newspaper revenues and, in this way, allow them to reduce the price of news so that readers consume more news (and in this way, with better informed consumers to improve the functioning of democracy). Second, in the same way, higher revenues due to lower taxes, can promote newspapers to invest more in media diversity (for instance, to cover a broader set of news stories and political leanings).

In this paper, we analyze the role of ad-valorem taxes when applied to the two sides of the news market: ad-valorem taxes on the selling of news; and ad-valorem taxes on the selling of advertising. We are interested on the effects of these two taxes on the prices of news and on media diversity. In particular, we try to answer the following two questions:

(1) Can lower ad-valorem taxes (on the selling of news and on the selling of advertising) conduce to lower prices in the media sector?

(2) Can lower ad-valorem taxes (on the selling of news and on the selling of advertising) promote firms to increase the diversity of content that they offer?

In order to answer these two questions, we adopt the Hotelling model (1929) of horizontal product differentiation (see also d'Aspremont et al., 1979). This model is usually used to analyze two-sided markets (see for instance, Rochet and Tirole, 2003; Anderson and Coate, 2005; Armstrong, 2006). We consider both single-homing consumers (that consume from only one media firm) and multi-homing consumers (that consume from two media firms). For models with multi-homing consumers see for instance Doganoglu and Wright (2006, 2010); Kim and Serfes (2006); Choi (2010); Anderson et al. (2017).

In turn, content diversification is introduced in the following way. In the standard Hotelling model, firms only supply the market with one variety, i.e. one point in the line. Instead, like in Dewan et al. (2003) and Alexandrov (2008), we allow firms to supply the market with different varieties, i.e. a line segment.

In this set-up, we obtain three main results. First, we show that when we do not allow firms to diversify content, ad-valorem taxes on the selling of news and ad-valorem taxes on the selling of advertisement have different effects on prices. While ad-valorem taxes on selling of news decreases prices (as in Foros et al., 2019), ad-valorem taxes on selling of advertising increase prices of newspapers. This difference arises because ad-valorem taxes on selling of news increases price

[1] The advent of Internet has made media diversity even more central since news content is sometimes offered for free. This has lead media firms to loose revenues from both subscriptions and advertising, which in turn has led to the exit of many newspapers (especially local ones) and the reduction of journalists in news rooms and as a result of investigative journalism (see Pew Research Center, 2016a,b; 2017, 2018a,b,c).

competition. In turn, ad-valorem taxes on selling of advertising have no effects on price competition, but since they reduce advertisement revenues, media platforms try to recoup these losses by increasing prices to consumers.

Second, when we allow firms to diversify content, ad-valorem taxes on selling of news do not anymore always increase the prices of news relatively to the no taxes scenario. This will depend on consumer's preference for their ideal variety. Accordingly, if consumers have a strong preference for their ideal variety, ad-valorem taxes on selling of news increases prices relatively to the case with no taxes.

Third, we show that both ad-valorem taxes on selling of news and ad-valorem taxes on selling of advertising reduce content diversification by firms, reducing therefore media diversity.

Our paper is then closely related with the work of Foros et al. (2019). We differ from Foros et al. (2019) in two ways. First, Foros et al. (2019) only look to the effects of ad-valorem taxes on the selling of news. In other words, they do not analyze ad-valorem taxes on selling of advertising. Foros et al. (2019) find that lower ad-valorem taxes on the selling of news can led to higher prices, because this reduces price competition. We then show that this result does not always hold when: (1) ad-valorem taxes fall on advertising; (2) when firms invest in content diversification.

Second, Foros et al. (2019), contrary to us, do not analyze the effects of ad-valorem taxes on the diversity of content offered by media firms. They focus only on the effects of ad-valorem taxes (on the selling of news) on multi-homing. Accordingly, they consider that if consumers single-home, consumers have access to less content than when they multi-home, since with single-homing readers only consume news from one media source, while with multi-homing they have access to different media sources. In this sense, multi-homing by readers, according to Foros et al. (2019), leads to more media diversity relatively to single-homing. In this set-up, they show that ad-valorem taxes increase multi-homing, because relatively to the no tax scenario, prices are lower. Accordingly, with lower prices, a reader is more likely to buy two newspapers, instead of just one.

We extend this view of media diversity by considering the effects of taxes on media firms' incentives to diversify content. The more content a media firm supplies the market, more content diversity, and vice-versa. The idea is that in what concerns media diversity, it is not only important how many sources consumers access, but also how much content each media firms supply the market.

Besides Foros et al. (2013), our paper is also related with other papers that look at taxes in two-sided media markets. Kind et al. (2013) and Kind and Koethenbuerger (2018), like Foros et al. (2018), also show that lowering taxes in a two-sided market can increase prices. In addition, Kind et al. (2013) demonstrate that a low tax regime increases product differentiation relatively to the social optimum. Kind et al. (2008), in turn, show that a monopolist may have a higher output relatively to the social optimum. They argue that this can be corrected by a subsidy or by a specific tax. Belleflamme and Toulemonde (2018) look to a broader set of taxes besides ad-valorem taxes. They show that specific taxes are passed to the agents on the side on which they are imposed,

transaction taxes hurt agents on both sides and benefit media firms, ad-valorem taxes allow tax authorities to capture part of the media firms' profits, and asymmetric taxes benefit agents on the untaxed side[2].

The rest of the paper is organized as follows. In the next section, we present the theoretical model. We then look to the case with no content diversification for the benchmark case with no taxation, then with ad-valorem taxes on the selling of news, and with ad-valorem taxes on the selling of advertising. After, we consider the case with content diversification for the benchmark case with no taxation, then with ad-valorem taxes on the selling of news, and the with ad-valorem taxes on the selling of advertising. We conclude by discussing our results.

2 The Model

In order to allow a more direct comparison with Foros et al. (2019), we follow their model closely. In this sense, like in Foros et al. (2019), we adopt the Hotelling (1929) model to compare a market with ad-valorem taxes with a market with no ad-valorem taxes. When necessary, the case with taxation is labeled with the superscript T and the case with no taxation is labeled with the superscript N. Also, similarly to Foros et al. (2019) we consider single-homing and multi-homing. When needed, the case with single-homing is labeled with the superscript S and the case with multi-homing with the superscript M. In addition, differently from Foros et al. (2019), we also consider how taxes affect media firms' incentives to diversify content[3].

In this sense, we consider different scenarios: no taxation versus taxation, single-homing versus multi-homing, and no content diversification versus content diversification. We have a case with no content diversification in order to replicate the results in Foros et al. (2019) and to show that their results do not hold when ad-valorem taxes fall on advertising (instead of on selling of news). We have a case with content diversification to show that taxes affect the diversity of content provided in the market, and that media diversity is more than just single-homing versus multi-homing (i.e. readers having access to one source or two sources of news), but should also include how much content each media outlet supplies the market. Given the different cases considered, it can be helpful to make a list of the different cases right away. The following first

[2] Other papers that analyze taxation in media markets look to the effects of taxation on the collection of personalized data (Bourreau et al., 2018); on tax competition/coordination (Bacache Beauvallet, 2018, Gauthier, 2018); on privacy protection (Bloch and Demange, 2018), and on transfer pricing and taxation of royalty payments (Juranek, et al. 2018).

[3] When we talk about media diversity, we usually think primarily about newspapers, because many news items focus on political issues. However, besides newspapers other type of media firms (magazines, books, films, television, or media platforms) can also be important for media diversity even when they are more entertainment based. For example, movies about minorities can contribute as much to media diversity as an op-ed talking about immigrants. In this paper, then, despite the focus is newspaper, our results could be relevant for other type of media markets besides newspapers that also operate in a two-sided market.

three cases do not consider content diversification, the last three cases consider content diversification.

Case 1. No Content Diversification and No Ad-Valorem Taxes. This case is used as a benchmark to compare with the taxation cases 2 and 3, in what respects the effects of taxes on prices.

Case 2. No Content Diversification and Ad-Valorem Taxes on Selling of News. This is the case considered by Foros et al. (2019). The aim of looking at this case is to show that our model without content diversification replicates the results of Foros et al. (2019).

Case 3. No Content Diversification and Ad-Valorem Taxes on Selling of Advertisement. The aim of looking at this case is to show that the results of Foros et al. (2019) do not hold with ad-valorem taxes on advertising.

Case 4. Content Diversification and No Ad-Valorem Taxes. This case is used as a benchmark to compare with the taxation cases 5 and 6, in what respects the effects of taxes now both in terms of prices and content diversification.

Case 5. Content Diversification and Ad-Valorem Taxes on Selling of News. The aim of looking at this case is to show that when firms invest in content diversification, the results of Foros et al. (2019) do not always hold and that taxes have effects on media plurality apart from just single-homing versus multi-homing.

Case 6. Content Diversification and Ad-Valorem Taxes on Selling of Advertisement. The objective of this case is to look to the effects of taxes on selling of advertising on content diversification and prices.

We consider two media firms, L and R, that compete in the Hotelling (1929) fashion for consumers. Like in Foros et al. (2019), media firm L is located at the left extreme of the Hotelling line, and media firm R is located at the right extreme of the Hotelling line. The two competing media firms derive revenues from advertising and from selling of news (for instance, selling of newspapers and subscriptions).

Differently from Foros et al. (2019), besides providing content that mirrors their location in the line (a point in the line), media firms can also provide content along the line (a line segment). Accordingly, instead of just providing a point in the line, a media firm can provide a line segment of content (see Dewan et al., 2003). When a media firm supplies a line segment of content, we say that firms diversify content. We represent content diversification by $0 \leq k_L \leq 1$ for firm L and $0 \leq k_R \leq 1$ for firm R. In this way, a media firm can choose to be single-content, i.e. a point in the line, $k_L = 0$ and $k_R = 1$, or to be multi-content, i.e. a line segment, $[0, k_L]$ for media firm L and $[1 - k_R, 1]$ for media firm R. See figure 1 for the single-homing case and figure 2 for the multi-homing case (note that these figures do not necessarily represent the equilibrium of the model).

As in Hotelling (1929), we assume that consumers are uniformly distributed in a line of length one: $[0, 1]$. The line represents consumers' preferences in terms of content. As we have said, we are going to consider both the case where consumers are single-home (i.e. they patronize just one outlet) and the case where consumers can choose to multi-home (they buy content from the two

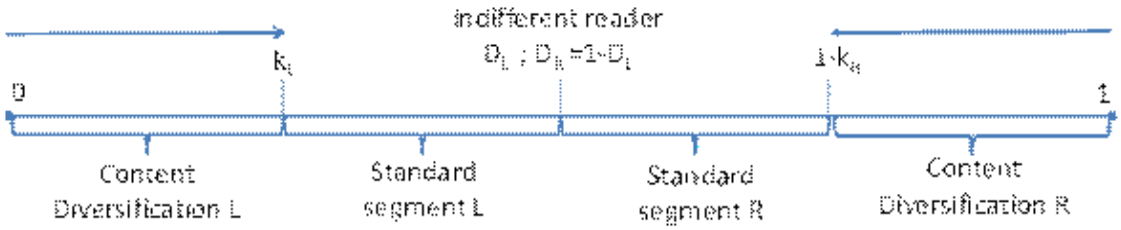

Figure 1: Content Diversification: Single-Homing

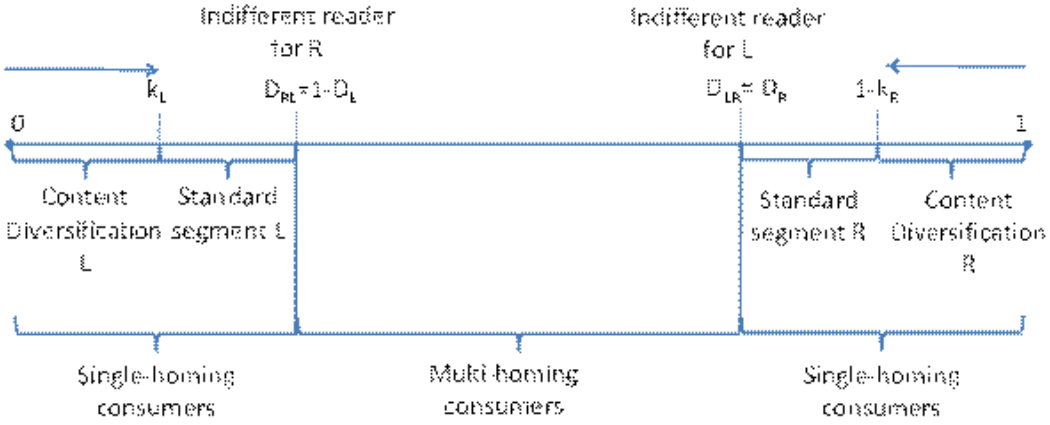

Figure 2: Content Diversification: Multi-Homing

outlets). In both cases, consumers incur in a disutility from buying content that differs from their ideal one. We capture this disutility as in Hotelling with the parameter t, which represents the intensity of consumers' content preferences, i.e.: transport costs. To illustrate, consider consumer x located in the left side of the line. If consumer x is outside the line segment of content provided by media firm L, his disutility is $t(x - k_L)$. However, if this consumer is inside the line segment of content provided by firm L, he does not suffer any disutility since his ideal variety is supplied by the newspaper[4].

In the following, we present our model with no ad-valorem taxes and at the end we show how the model changes with the introduction of ad-valorem taxes (on selling of news and on selling of advertisement). We present then the model in an encompassing way that can include all the cases mentioned above. When we later solve for each case, we mention how the case in question differs from the encompassing model.

Start with single-homing (no taxation). In this case, the utility of consumers in the left and right segments are as follows[5]:

$$\begin{aligned} U_L^S &= v - p_L - t(x - k_L) \\ U_R^S &= v - p_R - t(1 - x - k_R) \end{aligned} \tag{1}$$

Where v is the reservation price of consumers, t transport costs, k_i $(i = L, R)$ diversity of content offered by firm i. Remember that the superscript S stands for single-homing. Note that in the no content diversification cases (cases 1 to 3 above), $k_i = 0$ $(i = L, R)$.

In the multi-homing case, following Foros et al. (2019), if a consumer buys content from both media firms, he has the following utility:

$$U_{L+R}^M = U_L + U_R - d \tag{2}$$

Where d represents the loss of utility due to overlap of content by consuming from the two media firms. Remember that the superscript M stands for multi-homing.

In order to provide a line segment of content, as in Alexandrov (2008), a media firm has to incur in a cost (see also Garcia Pires, 2014):

$$C_i = \frac{\gamma k_i^2}{2},\ i = L, R \tag{3}$$

[4] We are implicitly assuming that when a consumer buys a newspaper (or an online subscription of a newspaper) he can potentially consume all news pieces offered by the newspaper. Obviously, some of these news pieces do not conform totally with the preferred variety of this consumer, but some do. In terms of the model, our results do not change if we account for the news pieces that do not conform with this reader preferences. It can be easily seen that even if we account for the news that differ from this reader ideal variety, he has a lower disutility if his ideal variety is supplied than if it is not supplied.

[5] As in Foros et al. (2019), we do not consider disutility from advertising. As argued by Foros et al. (2019) this assumption is consistent with empirical evidence. See for instance Gentzkow (2007), Fan (2013) and Gentzkow et al. (2014).

Where γ is a parameter that captures the informational and flexibility costs to adapt to the consumers' preferences.

In what concerns the advertising market, we follow Gabszewicz et al. (2001), Anderson and Coate (2005) and Peitz and Valletti (2008). The demand for ads for media firm i equals:

$$r_i = \alpha - \beta a_i,\ i = L, R \tag{4}$$

Where r_i is the price of advertising per reader, a_i is the advertising volume. The parameters α and β reflect the size of the advertising market. Accordingly, a high α and a low β represent a large advertising market. Then as in Anderson and Coate (2005), media firms extract all surplus from advertisers.

To calculate gross advertising income, we have to take into consideration demand for each media firm. Demand, though, depends on if consumers single-home or they multi-home. Start with single-homing (and no taxation). In this case, gross advertising revenues are:

$$A_i^{SN} = (\alpha - \beta a_i)\, a_i D_i^S,\ i = L, R \tag{5}$$

Where D_i is the demand for media firm i under single-homing. In this sense, the indifferent consumer between buying from L and R equals, D_L. In turn the indifferent consumer between buying from R and L equals $D_R = (1 - D_L)$. Figure 1 depicts the indifferent consumer under single-homing. It can be shown that the indifferent consumer under single- homing is the one that makes:

$$v - p_L - t\,(x - k_L) = v - p_R - t\,(1 - x - k_R) \tag{6}$$

Again note that in the no content diversification cases (cases 1 to 3), $k_i = 0$ $(i = L, R)$. Solving for x in the previous equation, we obtain the indifferent consumer, D_L for firm L and $D_R = (1 - D_L)$ for firm R.

Turn now to the multi-homing case. Differently from the single-homing case, in the multi-homing case there are two indifferent consumers. One for media firm L and another for media firm R. To see this, start with the indifferent consumer for media firm L. It can be seen that the demand for L consists of its exclusive consumers (single-homing consumers of L, that we represent by D_L) plus the shared consumers with media firm R (multi-homing consumers). The shared consumers equal: $D_R - D_L$, where $1 - D_R$ represents the exclusive consumers of R. Then demand for media firm L in the multi-homing case equals:

$$D_L^M = D_L + (D_R - D_L) = D_R. \tag{7}$$

Therefore, $D_{LR} = D_R$ is the indifferent consumer between buying only from L or from both L and R.

In turn, demand for media firm R equals the exclusive consumers of R (single-homing consumers of R, that as we have just said are represented by $1 - D_R$) and the shared consumers (multi homing consumers). The shared consumers as we have seen above equal $D_R - D_L$. Then demand for media firm R in the multi-homing case is:

$$D_R^M = 1 - D_R + (D_R - D_L) = 1 - D_L. \tag{8}$$

Then $D_{RL} = 1 - D_L$ is the indifferent consumer between buying only from R or from both R and L. Figure 2 depicts the indifferent consumers under multi-homing.

In this way, in the multi-homing case (with no taxation), we have that advertising revenues for media firm L and R equal:

$$\begin{aligned} A_L^M &= (\alpha - \beta a_L)\, a_L D_{LR} \\ A_R^M &= (\alpha - \beta a_R)\, a_R D_{RL} \end{aligned} \tag{9}$$

With $D_{LR} = D_R$ and $D_{RL} = 1 - D_L$.

From the above we have that profits for media firm i ($i = L, R$) in the single-homing case with no taxation equals:

$$\begin{aligned} \Pi_L^{SN} &= (p_L + (\alpha - \beta a_L)\, a_L)\, D_L - C_L \\ \Pi_R^{SN} &= (p_R + (\alpha - \beta a_R)\, a_R)\, D_R - C_R \end{aligned} \tag{10}$$

In turn, in the multi-homing case, profits with no taxation are:

$$\begin{aligned} \Pi_L^{MN} &= (p_L + (\alpha - \beta a_L)\, a_L)\, D_{LR} - C_L \\ \Pi_R^{MN} &= (p_R + (\alpha - \beta a_R)\, a_R)\, D_{RL} - C_R \end{aligned} \tag{11}$$

Taxation of advertising As we have mentioned above, to consider the effects of taxation on content diversification in a two-sided market, we consider two cases: with no taxation (see above) and with ad-valorem taxes on selling of news and on selling of advertising. The only difference between taxation and no taxation case is on what concerns revenues from selling news and from selling advertising.

In the case of ad-valorem taxes on selling of news (which is the one studied by Foros et al., 2019), we have that revenues from selling news are now:

$$s_i^T = \tfrac{p_i}{1+T},\ i = L, R \tag{12}$$

In turn, in the case of ad-valorem taxes on selling of advertising, we have that advertising revenues equal now:

$$r_i^T = \tfrac{\alpha - \beta a_i}{1+T},\ i = L, R \tag{13}$$

Next, we derive the equilibrium of the different cases.

3 No Content Diversification and No Ad-Valorem Taxes

In this section, we consider the case with no taxes. This means that we have the model above without content diversification (i.e. $k_i = 0$, $i = L, R$). This can be considered a benchmark case that we will use later to compare with the taxation cases. We start with the single-homing case and then turn to the multi-homing case.

3.1 Single-Homing

The first thing to note, is that if there is no content diversification, the indifferent consumer equals:

$$D_L = \frac{1}{2t}\left(t - p_L + p_R\right) \tag{14}$$

We can now solve for advertising levels. The First Order Conditions (FOCs) for advertising equals (all Second Order Conditions, SOCs, are in appendix):

$$\begin{array}{rcl} \frac{d\Pi}{da_L} & = & \left(\alpha - 2\beta a_L\right)\frac{t - p_L + p_R}{2t} \\ \frac{d\Pi}{da_R} & = & \left(\alpha - 2\beta a_L\right)\frac{t - p_R + p_L}{2t} \end{array} \tag{15}$$

Solving the FOCs for advertising for a_L and a_R, we get:

$$a_L = a_R = \frac{\alpha}{2\beta} \tag{16}$$

In turn, the FOCs for prices equal:

$$\begin{array}{rcl} \frac{d\Pi}{dp_L} & = & \frac{4\beta(t - 2p_L + p_R) - \alpha^2}{8t\beta} \\ \frac{d\Pi}{dp_R} & = & \frac{4\beta(t - 2p_R + p_L) - \alpha^2}{8t\beta} \end{array} \tag{17}$$

Solving for prices, we get:

$$p_L = p_R = t - \frac{\alpha^2}{4\beta} \tag{18}$$

3.2 Multi-Homing

We now turn to multi-homing case. We start again with the indifferent consumer. As noted above, in the multi-homing case, we have two indifferent consumers.

Start by noticing that the utility of consuming from both L and R is:

$$U_{L+R} = 2v - p_L - p_R - t - d \tag{19}$$

The consumer that is indifferent from buying from L or both L and R has a utility $U_{L+R} - U_L = 0$, with $U_L = (v - p_L - t(x))$. Solving for x, we get the indifferent consumer from buying from L or both L and R:

$$D_{LR} = 1 - \frac{v - p_R - d}{t} \tag{20}$$

In turn, the consumer that is indifferent from buying from R or both R and L has a utility $U_{L+R} - U_R = 0$, with $U_R = v - p_R - t(1 - x)$. Solving for x, we get the indifferent consumer from buying from R or both R and L:

$$D_{RL} = \frac{v - p_L - d}{t} \tag{21}$$

The FOCs for advertising then equal:

$$\begin{aligned} \frac{d\Pi}{da_L} &= (\alpha - 2\beta a_L)\frac{v - d - p_L}{t} \\ \frac{d\Pi}{da_R} &= (\alpha - 2\beta a_L)\frac{v - d - p_R}{t} \end{aligned} \tag{22}$$

As we can see, an important difference relatively to single-homing case is that prices for the rival firm do not enter the FOCs for advertising. The reason is that with multi-homing, consumers buy from both media firms, and therefore competition is reduced. This is a known result from multi-homing literature, that multi-homing can soften competition, since consumers by consuming from all the firms, reduces firms' competition for consumers. See for instance, Doganoglu and Wright (2006, 2010); Kim and Serfes (2006); Choi (2010); Anderson et al. (2017).

Solving the FOCs for advertising levels, we get the same levels of advertising as under single-homing, $a_L = a_R = \frac{\alpha}{2\beta}$.

In turn the FOCs for prices equal:

$$\begin{aligned} \frac{d\Pi}{dp_L} &= \frac{4\beta(v - d - 2p_L) - \alpha^2}{4t\beta} \\ \frac{d\Pi}{dp_R} &= \frac{4\beta(v - d - 2p_R) - \alpha^2}{4t\beta} \end{aligned} \tag{23}$$

As for the FOCs for advertising, prices of the rival do not enter the FOCs for prices. This is as we have just explained because multi-homing reduces price competition.

Solving the FOCs for prices we get:

$$p_L = p_R = \frac{v - d}{2} - \frac{\alpha^2}{8\beta} \tag{24}$$

4 No Content Diversification and Ad-Valorem Taxes on Selling of News

In this section, we look to the case with ad-valorem taxes on the selling of news (and no content diversification). We continue then to have the model above

without content diversification (i.e. $k_i = 0$, $i = L, R$). We present this case to replicate the results of Foros et al. (2019) with ad-valorem taxes on the selling of news. With this exercise, we want to show that what drives our results are not the (small) differences between our model and that of Foros et al. (2019). We start with the single-homing case and then turn to the multi-homing case. In the end of this section, we compare this taxation case with the no taxation case from the previous section.

4.1 Single-Homing

The first thing to note is that the indifferent consumer under ad-valorem taxes on selling of news is the same as with no taxation above. Note also that FOCs for advertising are the same under taxation and no taxation. Then, advertising levels are also the same.

In turn, the FOCs for prices equal now:

$$\begin{aligned} \frac{d\Pi}{dp_L} &= \frac{4\beta(t-2p_L+p_R)-\alpha^2(T+1)}{8t\beta(T+1)} \\ \frac{d\Pi}{dp_R} &= \frac{4\beta(t-2p_R+p_L)-\alpha^2(T+1)}{8t\beta(T+1)} \end{aligned} \tag{25}$$

Solving for prices, we get:

$$p_L = p_R = -\tfrac{1}{4\beta}\left(\alpha^2\left(T+1\right) - 4t\beta\right) \tag{26}$$

4.2 Multi-Homing

We turn now to the multi-homing case. Again, the indifferent consumers under multi-homing with ad-valorem taxes on selling of news are the same as for the no taxation case above. We can also see that FOCs for advertising under ad-valorem taxes on selling of news are the same as above with no taxation. Then once again we get the same levels of advertising levels.

In what concerns the FOCs for prices, we get:

$$\begin{aligned} \frac{d\Pi}{dp_L} &= \frac{4\beta(v-d-2p_L)-\alpha^2(T+1)}{4t\beta(T+1)} \\ \frac{d\Pi}{dp_R} &= \frac{4\beta(v-d-2p_R)-\alpha^2(T+1)}{4t\beta(T+1)} \end{aligned} \tag{27}$$

Again, relatively to single-homing case, prices for the rival firm do not enter the FOCs for prices. The reasons for this are the same as pointed out above for the no taxation case: multi-homing reduces competition, since consumers buy from all firms.

Solving for p_L and p_R, we get:

$$p_L = p_R = -\frac{\alpha^2(T+1)-4\beta(v-d)}{8\beta} \tag{28}$$

4.3 Taxation versus No Taxation

We can now compare the taxation and the no taxation case in what concerns prices. Start with single-homing. We can see that the difference in prices under no taxation and taxation equals:

$$p_i^{SN} - p_i^{ST} = T\frac{\alpha^2}{4\beta} > 0 \tag{29}$$

Then, as in Foros et al (2019), prices are higher under no taxation than under taxation. This occurs for the reasons pointed out in Foros et al. (2019): in a two-sided market, taxation increases price competition. Accordingly, reducing the tax rate on the selling of news increases the profitability of the consumer market, but does not change the profitability of the advertising market. This reduces the pressure on price competition to attract more demand, and therefore also to increase advertising revenues.

In what relates to the multi-homing case, we have.

$$p_i^{MN} - p_i^{MT} = T\frac{\alpha^2}{8\beta} > 0 \tag{30}$$

Then again as in Foros et al. (2019), prices are higher under no taxation than under taxation. This occurs for the reasons just pointed out above for the single-homing case: ad-valorem taxes on selling of news reduces price competition.

We can then see that our model with just ad-valorem taxes on selling of news replicates the results of Foros et al. (2019). Next, we will see if the same occurs when we consider ad-valorem taxes on selling of advertising.

5 No Content Diversification and Ad-Valorem Taxes on Selling of Advertisement

In this section, we introduce ad-valorem taxes on advertising (but continue to not consider content diversification). Again, we first look to the case with single-homing consumers and then look to the case with multi-homing consumers. After, we compare this case with taxation with the case above with no taxation.

5.1 Single-Homing

The first thing to note is that the indifferent consumer is again the same as for the no taxation case.

In turn, the FOCs for advertising equal now:

$$\begin{aligned} \frac{d\Pi}{da_L} &= (\alpha - 2\beta a_L)\,\frac{t - p_L + p_R}{2t(T+1)} \\ \frac{d\Pi}{da_R} &= (\alpha - 2\beta a_R)\,\frac{t - p_R + p_L}{2t(T+1)} \end{aligned} \tag{31}$$

Solving for a_L and a_R, we get the same advertising levels as with no taxation, i.e. $a_L = a_R = \frac{\alpha}{2\beta}$.

In turn, the FOCs for prices equal:

$$\begin{aligned} \frac{d\Pi}{dp_L} &= \frac{4\beta(t-2p_L+p_R)(T+1)-\alpha^2}{8t\beta(T+1)} \\ \frac{d\Pi}{dp_R} &= \frac{4\beta(t-2p_R+p_L)(T+1)-\alpha^2}{8t\beta} \end{aligned} \tag{32}$$

Solving p_L and p_R, we get:

$$p_L = p_R = t - \frac{\alpha^2}{4\beta(T+1)} \tag{33}$$

5.2 Multi-Homing

Once more, the indifferent consumers under multi-homing are the same as under the no taxation case above.

In turn, the FOCs for advertising equal now:

$$\begin{aligned} \frac{d\Pi}{da_L} &= (\alpha - 2\beta a_L)\,\frac{v-d-p_L}{t(T+1)} \\ \frac{d\Pi}{da_R} &= (\alpha - 2\beta a_L)\,\frac{v-d-p_R}{t(T+1)} \end{aligned} \tag{34}$$

Note again that under multi-homing prices of the rival do not enter the FOCs for advertising. This results for the same reason pointed out previously: under multi-homing, consumers buy from all firms and therefore price competition is reduced.

In turn the FOCs for prices equal:

$$\begin{aligned} \frac{d\Pi}{dp_L} &= \frac{4\beta(v-d-2p_L)(T+1)-\alpha^2}{4t\beta(T+1)} \\ \frac{d\Pi}{dp_R} &= \frac{4\beta(v-d-2p_R)(T+1)-\alpha^2}{4t\beta(T+1)} \end{aligned} \tag{35}$$

Once more under multi-homing prices of the rival do not enter the FOCs for prices. As we have already said, this is because multi-homing reduces price competition.

Solving the FOCs for prices we get:

$$p_L = p_R = \frac{v-d}{2} - \frac{\alpha^2}{8\beta(T+1)} \tag{36}$$

5.3 Taxation versus No Taxation

We can now compare the effects of taxation on prices. Start with single-homing. Under single-homing, we have that the difference in prices with no taxation and taxation equal:

$$p_i^{SN} - p_i^{ST} = -T\frac{\alpha^2}{4\beta(T+1)} < 0,\ i = L, R. \tag{37}$$

We then see that with ad-valorem taxes on selling of advertising, prices are now lower under no taxation. This is the opposite result of Foros et al (2019), where, as we have seen above, prices are higher under no taxation when we consider ad-valorem taxes on selling of news. The reason for this difference is that ad-valorem taxes on selling of news softens price competition. However, ad-valorem taxes on selling of advertising have no effect on price competition. As a result, ad-valorem taxes on selling of advertising, by reducing advertising revenues make firms to compensate this loss by increasing prices of newspapers relatively to the no taxation case.

In turn, under multi-homing, we have that the difference in prices with taxation and no taxation equal:

$$p_i^{MN} - p_i^{MT} = -T\frac{\alpha^2}{8\beta(T+1)} < 0,\ i = L, R. \tag{38}$$

We then see that when taxes are on advertising, prices are again lower under no taxation. Once more then, this is the opposite result of Foros et al (2019). As we mentioned above the reason for this is that while ad-valorem taxes on selling of news softens price competition, ad-valorem taxes on selling of advertising have no effect on price competition but reduce revenues, which firms compensate by increasing prices of news.

6 Content Diversification and No Ad-Valorem Taxes

We now in addition to price competition consider also content competition, and allow therefore firms to diversify content, i.e., $k_i \geq 0$, $i = L, R$. We start with the no taxation case to have a benchmark to compare afterwards with the cases of taxation (ad-valorem taxes on selling of news and ad-valorem taxes on selling of advertising). Again, we consider first the case with single-homing and then multi-homing.

6.1 Single-Homing

With single-homing, we can show that the indifferent consumer is the one that makes:

$$v - p_L - t\left(D_L - k_L\right) = v - p_R - t\left(1 - D_L - k_R\right) \tag{39}$$

From the previous expression we can derive the demand for media firm i ($i = L, R$):

$$\begin{aligned} D_L &= \frac{t(k_L - k_R + 1) - p_L + p_R}{2t} \\ D_R &= 1 - D_L = 1 - \frac{t(k_L - k_R + 1) - p_L + p_R}{2t} \end{aligned} \tag{40}$$

To solve the game, we start with advertising volumes, a_i $(i = L, R)$. The First Order Conditions (FOC) in relation to a_i $(i = L, R)$ equal (SOCs are in appendix):

$$\begin{aligned} \frac{d\Pi_L}{da_L} &= (\alpha - 2\beta a_L)\,\frac{t(k_L - k_R + 1) - p_L + p_R}{2t} \\ \frac{d\Pi_R}{da_R} &= (\alpha - 2\beta a_R)\,\frac{t(k_R - k_L + 1) - p_R + p_L}{2t} \end{aligned} \tag{41}$$

Solving for a_L and a_R, we get the same levels of advertising as with no content diversification, $a_L = a_R = \frac{\alpha}{2\beta}$.

The FOCs for prices are:

$$\begin{aligned} \frac{d\Pi_L}{dp_L} &= \frac{4\beta(t(k_L - k_R + 1) - 2p_L + p_R) - \alpha^2}{8t\beta} \\ \frac{d\Pi_R}{dp_R} &= \frac{4\beta(t(k_R - k_L + 1) - 2p_R + p_L) - \alpha^2}{8t\beta} \end{aligned} \tag{42}$$

Solving for p_L and p_R, we obtain:

$$\begin{aligned} p_L &= \frac{4t\beta(k_L - k_R + 3) - 3\alpha^2}{12\beta} \\ p_R &= \frac{4t\beta(k_R - k_L + 3) - 3\alpha^2}{12\beta} \end{aligned} \tag{43}$$

We can now compute the FOC for content diversification, k_i $(i = L, R)$. Start by noticing that FOC for content diversification can be decomposed into the direct effect of k_i on the demand of media firm i ($\frac{\delta D_i}{\delta k_i}$, $i = L, R$) and an indirect effect of k_i on the demand of firm i via the effect on the price of the rival j ($\frac{\delta D_i}{\delta p_j}\frac{dp_j}{dk_i}$, $i = L, R$ and $i \neq j$):

$$\begin{aligned} \frac{d\Pi_L}{dk_L} &= (p_L + (\alpha - \beta a_L)\,a_L)\left(\frac{\delta D_L}{\delta k_L} + \frac{\delta D_L}{\delta p_R}\frac{dp_R}{dk_L}\right) - \gamma k_L \\ \frac{d\Pi_R}{dk_R} &= (p_R + (\alpha - \beta a_R)\,a_R)\left(\frac{\delta D_R}{\delta k_R} + \frac{\delta D_R}{\delta p_L}\frac{dp_L}{dk_R}\right) - \gamma k_R \end{aligned} \tag{44}$$

We can show that the direct effect and the indirect effect equal:

$$\begin{aligned} \frac{\delta D_L}{\delta k_L} &= \frac{\delta D_R}{\delta k_R} = \frac{1}{2} > 0 \\ \frac{\delta D_L}{\delta p_R} &= \frac{\delta D_R}{\delta p_L} = \frac{1}{2t} > 0 \\ \frac{dp_R}{dk_L} &= \frac{dp_L}{dk_R} = -\frac{1}{3}t < 0 \\ \frac{\delta D_L}{\delta k_L} + \frac{\delta D_L}{\delta p_R}\frac{dp_R}{dk_L} &= \frac{\delta D_R}{\delta k_R} + \frac{\delta D_R}{\delta p_L}\frac{dp_L}{dk_R} = \frac{1}{3} > 0 \end{aligned} \tag{45}$$

We can then see that the direct effect of content diversification on demand of the media firm is positive. Accordingly, more content increases demand for the media firm. In turn, the indirect effect is negative. Accordingly, more content

increases price competition, i.e. it reduces the price of the rival, which has in turn a negative effect on the demand of the media firm. Even so, the direct effect dominates the indirect effect, and more content diversification has therefore a total positive impact on the demand for the media firm.

We can simplify the FOCs for content diversification and show that they equal:

$$\begin{array}{rcl} \frac{d\Pi_L}{dk_L} & = & \frac{t(k_L-k_R+3)-9\gamma k_L}{9} \\ \frac{d\Pi_R}{dk_R} & = & \frac{t(k_R-k_L+3)-9\gamma k_R}{9} \end{array} \tag{46}$$

Solving for k_L and k_R, we obtain:

$$k_L = k_R = \frac{t}{3\gamma} \tag{47}$$

Then, content diversification increases with the intensity of consumers preferences for their ideal variety (t) and decreases with the costs to provide content (γ).

We can now also solve for prices p_L and p_R.

$$p_L = p_R = t - \frac{\alpha^2}{4\beta} \tag{48}$$

6.2 Multi-Homing

With multi-homing, the first thing to note, as shown above, is that there are two indifferent consumers, one for media firm L (D_{LR}) and another for media firm R (D_{RL}). Where D_{LR} is the indifferent consumer between buying only from L or from both L and R, and D_{RL} is the indifferent consumer between buying only from R or from both R and L.

The indifferent consumer for media firm L is the one that makes:

$$U_{L+R} - U_L = 0 \tag{49}$$

Where:

$$\begin{array}{c} U_{L+R} = (v - p_L - t\,(x - k_L)) + (v - p_R - t\,(1 - x - k_R)) - d \\ U_L = v - p_L - t\,(x - k_L) \end{array} \tag{50}$$

Then:

$$U_{L+R} - U_L = v - d - p_R - t\,(1 - x - k_R) \tag{51}$$

Solving for x, we get the indifferent consumer for L (i.e. the one that is indifferent from buying only from L or from both L and R):

$$D_{LR} = (1 - k_R) - \frac{v - p_R - d}{t} \tag{52}$$

In turn for media firm R, we have that the indifferent consumer is the one that makes:

$$U_{L+R} - U_R = 0 \tag{53}$$

Where U_{L+R} is as above and U_R equals:

$$U_R = (v - p_R - t\,(1 - x - k_R)) \tag{54}$$

Then:

$$U_{L+R} - U_R = v - d - p_L - t\,(x - k_L) \tag{55}$$

Solving for x, we get the indifferent consumer for R (i.e. the one that is indifferent from buying only from R or from both R and L):

$$D_{RL} = k_L + \tfrac{v - p_L - d}{t} \tag{56}$$

We again start by solving the model for advertising volumes, a_i $(i = L, R)$. The FOCs in relation to a_i $(i = L, R)$ equal (SOCs are in appendix):

$$\begin{array}{rcl} \frac{d\Pi_L}{da_L} & = & (\alpha - 2\beta a_L)\,\frac{v - d - p_L + tk_L}{t} \\ \frac{d\Pi_R}{da_R} & = & (\alpha - 2\beta a_R)\,\frac{v - d - p_R + tk_R}{t} \end{array} \tag{57}$$

We can then see that now under multi-homing, it is not only prices of the rival that do not enter the FOCs for advertising but also content diversification of the rival. This shows that multi-homing reduces competition not only price, but also on content diversification. Again, this results from the fact that multi-homing reduces competition, since readers that multi-home buy from both newspapers, and therefore firms do not need to compete for these consumers.

Solving for a_L and a_R, we get the same advertising levels as under single-homing, $a_L = a_R = \frac{\alpha}{2\beta}$.

The FOCs for prices are:

$$\begin{array}{rcl} \frac{d\Pi_L}{dp_L} & = & \frac{4\beta(v - d - 2p_L + tk_L) - \alpha^2}{4t\beta} \\ \frac{d\Pi_R}{dp_R} & = & \frac{4\beta(v - d - 2p_R + tk_R) - \alpha^2}{4t\beta} \end{array} \tag{58}$$

We can see again that the FOCs for prices under multi-homing differ from the single-homing case, since now prices and content diversification of the rival do not enter the FOCs. This shows once more that multi-homing softens competition, not only on prices but also for content diversification. This is so, as we have already said, since when consumers multi-home (i.e. they consume from all the firms), competition for consumers is softened.

Solving for p_L and p_R, we obtain:

$$\begin{aligned} p_L &= \tfrac{4\beta(v-d+tk_L)-\alpha^2}{8\beta} \\ p_R &= \tfrac{4\beta(v-d+tk_R)-\alpha^2}{8\beta} \end{aligned} \tag{59}$$

We can now solve for the FOC of content diversification, k_i ($i = L, R$). Start again by noticing that, as for the single-homing case, the FOCs for content diversification can be decomposed into a direct effect of k_i on the demand of media firm i ($\frac{\delta D_i}{\delta k_i}$, $i = L, R$) and an indirect effect of k_i on the demand of firm i via the effect on the price of the rival ($\frac{\delta D_i}{\delta p_j}\frac{dp_j}{dk_i}$, $i = L, R$ and $i \neq j$):

$$\begin{aligned} \tfrac{d\Pi_L}{dk_L} &= (p_L + (\alpha - \beta a_L)\, a_L)\left(\tfrac{\delta D_{RL}}{\delta k_L} + \tfrac{\delta D_{RL}}{\delta p_R}\tfrac{dp_R}{dk_L}\right) - \gamma k_L \\ \tfrac{d\Pi_R}{dk_R} &= (p_R + (\alpha - \beta a_R)\, a_R)\left(\tfrac{\delta D_{LR}}{\delta k_R} + \tfrac{\delta D_{LR}}{\delta p_L}\tfrac{dp_L}{dk_R}\right) - \gamma k_R \end{aligned} \tag{60}$$

We can show that the direct effect and the indirect effect equal:

$$\begin{aligned} \tfrac{\delta D_L}{\delta k_L} &= \tfrac{\delta D_R}{\delta k_R} = 1 > 0 \\ \tfrac{\delta D_L}{\delta p_R} &= \tfrac{\delta D_R}{\delta p_L} = 0 \\ \tfrac{dp_R}{dk_L} &= \tfrac{dp_L}{dk_R} = 0 \\ \tfrac{\delta D_L}{\delta k_L} + \tfrac{\delta D_L}{\delta p_R}\tfrac{dp_R}{dk_L} &= \tfrac{\delta D_R}{\delta k_R} + \tfrac{\delta D_R}{\delta p_L}\tfrac{dp_L}{dk_R} = 1 > 0 \end{aligned} \tag{61}$$

We can then see that, as in the single homing case, the direct effect of content diversification on demand of the media firm is positive. Accordingly, more content increases demand for the media firm. However now under multi-homing this occurs since the indirect effect is canceled. This is so because as we have mentioned above, multi-homing reduces competition for readers that multi-home, since these consume from the two media firms, and therefore firms do not need to compete for them.

We can simplify the FOCs for content diversification and show that they equal:

$$\begin{aligned} \tfrac{d\Pi_L}{dk_L} &= \tfrac{4\beta(v-d+k_L(t-2\gamma))+\alpha^2}{8\beta} \\ \tfrac{d\Pi_R}{dk_R} &= \tfrac{4\beta(v-d+k_R(t-2\gamma))+\alpha^2}{8\beta} \end{aligned} \tag{62}$$

Then, we have that the content diversification of the rival does not enter the FOCs for content diversification, showing once again that multi-homing reduces competition not just on prices but also on content provision.

Solving for k_L and k_R, we obtain:

$$k_L = k_R = \tfrac{\alpha^2+4\beta(v-d)}{4\beta(2\gamma-t)} \tag{63}$$

Since $v > d$ (the market is covered), and since the SOC for content diversification demands that $t < 2\gamma$ (see appendix), then $k_L = k_R > 0$.

We can now also solve for prices p_L and p_R:

$$p_L = p_R = \frac{\gamma(v-d)}{(2\gamma-t)} - \frac{\alpha^2(\gamma-t)}{4\beta(2\gamma-t)} > 0 \tag{64}$$

7 Content Diversification and Ad-Valorem Taxes on Selling of News

We now look to the case with ad-valorem taxes on selling of news. We first look to the single-homing case, then to the multi-homing case. We close this section by comparing the no taxation case with the taxation case. We are interested to check if the result in Foros et al. (2019), i.e. that ad-valorem taxes on selling of news reduces prices of news, also holds when firms invest in content diversification.

7.1 Single-Homing

We now derive the equilibrium condition of the single-homing case with ad-valorem taxes on selling of news when firms invest in content. The first thing to note is that the indifferent consumer is the same as under no taxation.

It can also be seen that the First Order Conditions (FOCs) in relation to a_i $(i = L, R)$ under taxation are the same as in the no taxation case. Then advertising levels under taxation and no taxation are also equal.

In what concerns prices, we can see that the FOCs in relation to p_i $(i = L, R)$ are:

$$\begin{aligned} \frac{d\Pi_L}{dp_L} &= \frac{4\beta(t(k_L-k_R+1)-2p_L+p_R)-\alpha^2(T+1)}{8t\beta(T+1)} \\ \frac{d\Pi_R}{dp_R} &= \frac{4\beta(t(k_R-k_L+1)-2p_R+p_L)-\alpha^2(T+1)}{8t\beta(T+1)} \end{aligned} \tag{65}$$

Solving for p_L and p_R, we obtain:

$$\begin{aligned} p_L &= \frac{4t\beta(k_L-k_R+3)-3\alpha^2(T+1)}{12\beta} \\ p_R &= \frac{4t\beta(k_R-k_L+3)-3\alpha^2(T+1)}{12\beta} \end{aligned} \tag{66}$$

In what concerns the FOCs for content diversification, as above, these can be divided into a direct and an indirect effect:

$$\begin{aligned} \frac{d\Pi_L}{dk_L} &= \left(\frac{p_L}{1+T} + (\alpha - \beta a_L)\, a_L\right)\left(\frac{\delta D_L}{\delta k_L} + \frac{\delta D_L}{\delta p_R}\frac{dp_R}{dk_L}\right) - \gamma k_L \\ \frac{d\Pi_R}{dk_R} &= \left(\frac{p_R}{1+T} + (\alpha - \beta a_R)\, a_R\right)\left(\frac{\delta D_R}{\delta k_R} + \frac{\delta D_R}{\delta p_L}\frac{dp_L}{dk_R}\right) - \gamma k_R \end{aligned} \tag{67}$$

It can be seen that the direct and the indirect effect under the taxation case are equal to the no taxation case. This means that also with taxation, content diversification has a positive impact on demand of the media firm. Furthermore, the FOCs for content diversification can be simplified to:

$$\begin{aligned} \frac{d\Pi_L}{dk_L} &= \frac{t(k_L-k_R+3)-9\gamma k_L(T+1)}{9(T+1)} \\ \frac{d\Pi_R}{dk_R} &= \frac{t(k_R-k_L+3)-9\gamma k_R(T+1)}{9(T+1)} \end{aligned} \tag{68}$$

Solving for k_L and k_R, we obtain:

$$k_L = k_R = \frac{t}{3\gamma(T+1)} \tag{69}$$

We can now solve for prices to obtain:

$$p_L = p_R = t - \frac{\alpha^2(1+T)}{4\beta}$$

7.2 Multi-Homing

In this sub-section, we look to the case of multi-homing with ad-valorem taxes on selling of news. The first thing to note is that the indifferent consumers for firm L and firm R in the taxation scenario are the same as in the no taxation case.

The second thing to note is that the FOCs for advertising volumes, a_i ($i = L, R$) under taxation are also equal to the FOCs under no taxation. As a result, advertising levels under multi-homing with taxation are also the same, $a_L = a_R = \frac{\alpha}{2\beta}$.

In turn the FOCs for prices under the taxation case equal:

$$\begin{aligned} \frac{d\Pi_L}{dp_L} &= \frac{4\beta(v-d-2p_L+tk_L)-\alpha^2(T+1)}{4t\beta(T+1)} \\ \frac{d\Pi_R}{dp_R} &= \frac{4\beta(v-d-2p_R+tk_R)-\alpha^2(T+1)}{4t\beta(T+1)} \end{aligned} \tag{70}$$

As for the no taxation case under multi-homing, with taxation, the FOCs for prices under multi-homing differ from ones under the single-homing case, since now prices and content diversification of the rival do not enter the FOCs. This results from the fact that, as we have already said previously, multi-homing reduces competition on both prices and content, since multi-home consumers consume from the two media firms.

Solving for p_L and p_R, we obtain:

$$\begin{aligned} p_L &= \frac{4\beta(v-d+tk_L)-\alpha^2}{8\beta} \\ p_R &= \frac{4\beta(v-d+tk_R)-\alpha^2}{8\beta} \end{aligned} \tag{71}$$

We can now solve for the FOC of content diversification, k_i $(i = L, R)$. Start again by noticing that the FOC for content diversification can be again decomposed once more into a direct effect and an indirect effect:

$$\begin{aligned} \frac{d\Pi_L}{dk_L} &= \left(\frac{p_L}{T+1} + (\alpha - \beta a_L)\, a_L\right)\left(\frac{\delta D_{RL}}{\delta k_L} + \frac{\delta D_{RL}}{\delta p_R}\frac{dp_R}{dk_L}\right) - \gamma k_L \\ \frac{d\Pi_R}{dk_R} &= \left(\frac{p_R}{T+1} + (\alpha - \beta a_R)\, a_R\right)\left(\frac{\delta D_{LR}}{\delta k_R} + \frac{\delta D_{LR}}{\delta p_L}\frac{dp_L}{dk_R}\right) - \gamma k_R \end{aligned} \tag{72}$$

We can show that the direct effect and the indirect effect under the taxation case are equal to the no taxation case. Then as for the no taxation case, multi-homing softens price competition.

We can simplify the FOCs for content diversification and show that they equal:

$$\begin{aligned} \frac{d\Pi_L}{dk_L} &= \frac{4\beta(v-d+k_L(t-2\gamma(T+1)))+\alpha^2(T+1)}{8\beta(T+1)} \\ \frac{d\Pi_R}{dk_R} &= \frac{4\beta(v-d+k_R(t-2\gamma(T+1)))+\alpha^2(T+1)}{8\beta(T+1)} \end{aligned} \tag{73}$$

Note again that content diversification of the rival does not enter the FOCs of the firm in relation to content diversification. As we have mentioned, this is due to the fact that multi-homing reduces competition, not only on prices, but also on content provision.

Solving for k_L and k_R, we obtain:

$$k_L = k_R = \frac{\alpha^2(T+1)+4\beta(v-d)}{4\beta(2\gamma(T+1)-t)} \tag{74}$$

Since $v > d$ (the market is covered), and since the SOC for content diversification demands that $t < 2\gamma\,(T+1)$ (see appendix), then $k_L = k_R > 0$.

We can now also solve for prices p_L and p_R.

$$p_L = p_R = (T+1)\,\frac{4\beta\gamma(v-d)-\alpha^2(\gamma(T+1)-t)}{4\beta(2\gamma(T+1)-t)} \tag{75}$$

7.3 Taxation versus No Taxation

We can now compare the effects of taxation on prices and content diversification. Start with the single-homing-case.

In what concerns prices, we have:

$$p_i^{SN} - p_i^{ST} = T\frac{\alpha^2}{4\beta} > 0,\ i = L, R. \tag{76}$$

Then, similar to Foros et al. (2019), under single-homing, prices are higher in the no taxation case. As in Foros et al. (2019), and as we have mentioned above this is the consequence of the two-sidedness of the market.

Foros et al. (2019) then argue that reducing taxation could have a double negative impact in the media market since it would not only reduce demand (because of higher prices with no taxation) but also media plurality, because

some consumers would stop to multi-home due to higher prices. We will look at multi-homing next, however, as we mentioned in the Introduction, in Foros et al. (2019) media plurality is only about two media outlets providing one variety of content, i.e. media firms do not diversify content. When this is the case, media plurality increases if consumers buy two varieties (i.e. they multi-home), and media plurality decreases if consumers just buy one variety (i.e. they single-home). In our model, though, media plurality is more than just multi-homing or single-homing, i.e. buying two varieties or just one. Media plurality is also about how much content media firms provide. This part of media plurality, we can already analyze for the case of single-homing.

In this regard, we can show that the difference in content diversification with taxation and no taxation equals:

$$k_i^{SN} - k_i^{ST} = T\frac{t}{3\gamma(T+1)} > 0,\ i = L, R. \tag{77}$$

Taxation then unambiguously reduces content diversification of media firms. This then puts into light that content diversification is more than just a question of single-homing and multi-homing (demand side) but also about how much media firms provide of content (supply side). In this sense, under single-homing, ad-valorem taxes on selling of news can be positive for prices but are negative for media plurality.

We turn now to the multi-homing case. We start again with prices. We can show that difference in prices between taxation and no taxation equal:

$$p_i^{MN} - p_i^{MT} = T\frac{4t\beta\gamma(v-d)+\alpha^2\left(2\gamma^2+(2\gamma-t)(T\gamma-t)\right)}{4\beta(2\gamma-t)(2\gamma(T+1)-t)},\ i = L, R. \tag{78}$$

We can see that prices are higher under no taxation than with taxation when $t < T\gamma$ (i.e. consumers do not have a very strong preference for their own ideal variety relatively to ad-valorem taxes and the costs to provide content). This is a similar result to the single-homing case above, and to Foros et al. (2019). However, for $t > T\gamma$ (i.e. consumers have a strong preference for their own ideal variety relatively to ad-valorem taxes and the costs to provide content), this is no longer necessarily the case. In particular, prices with no taxation can be smaller than under taxation. This differs from both the single-homing case above, and from Foros et al. (2019). The reason for this result is that when consumers have a strong preference for their own ideal variety, increasing prices might not be anymore an option under no taxation, because by doing so the firm can lose consumers to the rival.

In what concerns content diversification, we can show that the difference in content diversification with taxation and no taxation equals:

$$k_i^{MN} - k_i^{MT} = T\frac{t\alpha^2+8\beta\gamma(v-d)}{4\beta(2\gamma-t)(2\gamma(T+1)-t)} > 0,\ i = L, R. \tag{79}$$

We then have the same result under single homing and multi-homing: taxation reduces content diversification. Again, what we can take from this result is that we cannot see media plurality just in terms of single-homing (buying from one media firm, just having access to one news source) and multi-home (buying

from two media firms, having access to two news source), but also how much diversity of content media firms provide to the market. Furthermore, while the effect of ad-valorem taxes on selling of news can be ambiguous, the effect on content diversification is always negative, since taxes always decrease content provision.

8 Content Diversification and Ad-Valorem Taxes on Selling of Advertisement

In this section, we look to the case with ad-valorem taxes on selling of advertising when firms can diversify content. We again start with the single-homing case, then look to the multi-homing case. We close this section by comparing the no taxation and the taxation case in what respects prices and content diversification.

8.1 Single-Homing

The first thing to note is that the indifferent consumer with taxes on advertisement under single-homing is again the same as with no taxation.

With taxation, however, the FOCs for advertising equal:

$$\begin{aligned} \frac{d\Pi_L}{da_L} &= (\alpha - 2\beta a_L)\,\frac{t(k_L - k_R + 1) - p_L + p_R}{2t(T+1)} \\ \frac{d\Pi_R}{da_R} &= (\alpha - 2\beta a_R)\,\frac{t(k_R - k_L + 1) - p_R + p_L}{2t(T+1)} \end{aligned} \tag{80}$$

Solving for a_L and a_R, we get the same advertising values as in the no taxation case, $a_L = a_R = \frac{\alpha}{2\beta}$.

In turn, the FOCs for prices equal:

$$\begin{aligned} \frac{d\Pi_L}{dp_L} &= \frac{4\beta(t(k_L - k_R + 1) - 2p_L + p_R)(T+1) - \alpha^2}{8t\beta(T+1)} \\ \frac{d\Pi_R}{dp_R} &= \frac{4\beta(t(k_R - k_L + 1) - 2p_R + p_L)(T+1) - \alpha^2}{8t\beta(T+1)} \end{aligned} \tag{81}$$

Solving for p_L and p_R, we obtain:

$$\begin{aligned} p_L &= \frac{4t\beta(k_L - k_R + 3)(T+1) - 3\alpha^2}{12\beta(T+1)} \\ p_R &= \frac{4t\beta(k_R - k_L + 3)(T+1) - 3\alpha^2}{12\beta(T+1)} \end{aligned} \tag{82}$$

We can now solve for the FOC of content diversification, k_i $(i = L, R)$. As above, the FOCs for content diversification can be decomposed into the direct effect of k_i on the demand of media firm i ($\frac{\delta D_i}{\delta k_i}$, $i = L, R$) and an indirect effect of k_i on the demand of firm i via the effect on the price of the rival j ($\frac{\delta D_i}{\delta p_j}\frac{dp_j}{dk_i}$, $i = L, R$, $i \neq j$):

$$\begin{aligned}\frac{d\Pi_L}{dk_L} &= \left(p_L + \frac{(\alpha-\beta a_L)a_L}{1+T}\right)\left(\frac{\delta D_L}{\delta k_L} + \frac{\delta D_L}{\delta p_R}\frac{dp_R}{dk_L}\right) - \gamma k_L \\ \frac{d\Pi_R}{dk_R} &= \left(p_R + \frac{(\alpha-\beta a_R)a_R}{1+T}\right)\left(\frac{\delta D_R}{\delta k_R} + \frac{\delta D_R}{\delta p_L}\frac{dp_L}{dk_R}\right) - \gamma k_R \end{aligned} \tag{83}$$

It can be shown that the direct effect and the indirect effect are the same as above with no taxation. Therefore the (positive) direct effect dominates the (negative) indirect effect. Then, the FOCs for advertising under taxation and under no taxation are also the same. This means that under single-homing, content diversification under taxation and no taxation are equal, $k_L = k_R = \frac{t}{3\gamma}$. In other words, ad-valorem taxes on advertising do not affect content provision in the market.

Solving for prices, we have:

$$p_L = p_R = t - \frac{\alpha^2}{4\beta(T+1)} \tag{84}$$

8.2 Multi-Homing

The first thing to note with the multi-homing case is that once more the indifferent consumers under multi-homing are the same as for the no taxation case above. In turn, the FOCs for advertising under multi-homing, equal:

$$\begin{aligned}\frac{d\Pi_L}{da_L} &= (\alpha - 2\beta a_L)\,\frac{v-d-p_L+tk_L}{t(T+1)} \\ \frac{d\Pi_R}{da_R} &= (\alpha - 2\beta a_R)\,\frac{v-d-p_R+tk_L}{t(T+1)} \end{aligned} \tag{85}$$

Again, with multi-homing, prices and content diversification of the rival do not show up in the FOCs for advertising. This results from multi-homing reducing competition. Solving for a_L and a_R we get the same advertising values as in the no taxation case, $a_L = a_R = \frac{\alpha}{2\beta}$.

In turn, the FOCs for prices equal:

$$\begin{aligned}\frac{d\Pi_L}{dp_L} &= \frac{4\beta(v-d-2p_L+tk_L)(T+1)-\alpha^2}{4t\beta(T+1)} \\ \frac{d\Pi_R}{dp_R} &= \frac{4\beta(v-d-2p_R+tk_R)(T+1)-\alpha^2}{4t\beta(T+1)} \end{aligned} \tag{86}$$

Note once more that multi-homing reduces competition on prices and content, since prices and content of the rival do not come up in the FOCs for prices.

Solving for p_L and p_R, we obtain:

$$\begin{aligned} p_L &= \frac{4\beta(v-d+tk_L)(T+1)-\alpha^2}{8\beta(T+1)} \\ p_R &= \frac{4\beta(v-d+tk_R)(T+1)-\alpha^2}{8\beta(T+1)} \end{aligned} \tag{87}$$

In what relates the FOCs for advertising, again these can be decomposed into a direct effect and an indirect effect:

$$\begin{aligned} \frac{d\Pi_L}{dk_L} &= \left(p_L + \frac{(\alpha-\beta a_L)a_L}{1+T}\right)\left(\frac{\delta D_{RL}}{\delta k_L} + \frac{\delta D_{RL}}{\delta p_R}\frac{dp_R}{dk_L}\right) - \gamma k_L \\ \frac{d\Pi_R}{dk_R} &= \left(p_R + \frac{(\alpha-\beta a_R)a_R}{1+T}\right)\left(\frac{\delta D_{LR}}{\delta k_R} + \frac{\delta D_{LR}}{\delta p_L}\frac{dp_L}{dk_R}\right) - \gamma k_R \end{aligned} \tag{88}$$

It can be easily shown that the direct and indirect effect are equal to the no taxation case. Then again content diversification increases demand of a media firm.

The FOCs for content diversification can then be simplified to:

$$\begin{aligned} \frac{d\Pi_L}{dk_L} &= \frac{4\beta(v-d+k_L(t-2\gamma))(T+1)+\alpha^2}{8\beta(T+1)} \\ \frac{d\Pi_R}{dk_R} &= \frac{4\beta(v-d+k_R(t-2\gamma))(T+1)+\alpha^2}{8\beta(T+1)} \end{aligned} \tag{89}$$

Once more with multi-homing, content diversification of the rival does not show up in the FOCs for content diversification. As mentioned several times now, this results from the fact that multi-homing reduces competition.

Solving for k_L and k_R, we obtain:

$$k_L = k_R = \frac{\alpha^2+4\beta(v-d)(T+1)}{4\beta(2\gamma-t)(T+1)} \tag{90}$$

Since $v > d$ (the market is covered), and since the SOC for content diversification demands that $t < 2\gamma$ (see appendix), then $k_L = k_R > 0$.

We can now also solve for prices p_L and p_R:

$$p_L = p_R = \frac{\gamma(v-d)}{(2\gamma-t)} - \frac{\alpha^2(\gamma-t)}{4\beta(T+1)(2\gamma-t)} \tag{91}$$

8.3 Taxation versus No Taxation

We can now compare the effects of taxation on prices. Under single-homing, we have that the difference in prices with taxation and no taxation equal:

$$p_i^{SN} - p_i^{ST} = -T\frac{\alpha^2}{4\beta(T+1)} < 0,\ i = L, R. \tag{92}$$

We can then see that with ad-valorem taxes on selling of advertising, prices are lower under no taxation. Once more then, this is the opposite result of Foros et al. (2019) with ad-valorem taxes on selling of news, where prices are higher under no taxation. So, the result we obtained before with no content diversification is robust to the introduction of content diversification.

In what concerns content diversification, as we have already noted, we have that:

$$k_i^{SN} - k_i^{ST} = 0,\ i = L, R. \tag{93}$$

Then, taxation of advertising under single-homing has no effects on content diversification.

In turn, under multi-homing, we have that the difference in prices with taxation and no taxation equal:

$$p_i^{SN} - p_i^{ST} = -T\alpha^2 \frac{\gamma - t}{4\beta(2\gamma - t)(T+1)},\ i = L, R. \tag{94}$$

With multi-homing then we have that for $0 < t < \gamma$, prices are higher with no taxation. In turn, for $\gamma < t < 2\gamma$, prices are lower with no taxation. In other words, when consumers do not have a strong preference for their ideal variety in relation to the cost of diversifying content (low t relatively to γ), we can obtain the same result as in Foros et al. (2019) that prices are lower under taxation. However, when consumers have a strong preference for their own ideal variety relatively to the costs to diversify content (high t relatively to γ), again prices are lower under no taxation. This result is intuitive, when consumers have a low preference to their own ideal variety, firms can increase prices without fearing losing consumers to the rival. The opposite occurs, when consumers have a strong preference for their own ideal variety.

We can also show that the difference in content diversification with taxation and no taxation equals:

$$k_i^{MN} - k_i^{MT} = T\frac{\alpha^2}{4\beta(2\gamma - t)(T+1)} > 0,\ i = L, R. \tag{95}$$

Since the SOC for content diversification demands that $t < 2\gamma$, then there is more content diversification (and therefore more media plurality) with no taxation. Then in spite of the fact that with multi-homing prices are no longer always lower with no taxation, we still have that content diversification is always higher with no taxation.

9 Conclusion

In this paper, we have analyzed the effects of ad-valorem taxes (on selling of news and on selling of advertising) on prices and content diversification. The motivation for this exercise comes from the fact that it has been shown that in a two-sided market, ad-valorem taxes on selling of news can contribute to fiercer price competition (and therefore lower prices) and as a result promote multi-homing by consumers, which is positive to media diversity (Foros et al., 2019). We have shown that this is not the case when we consider ad-valorem taxes on selling of advertisement. In this case, ad-valorem taxes on selling of advertising increase prices relatively to the no taxation scenario, which then promote consumers to single-home. The reason for this different result is that ad-valorem taxes on selling of advertising does not affect price competition but reduces advertising revenues which media firms try to recover with higher prices of newspapers.

Another contribution of this paper is to show that when we look to media diversity, we should consider not only how many news sources a consumer has

access to (single-homing versus multi-homing), but also the diversity of content that each media firm supplies the market. In order to investigate this, we have also look to the incentives of firms to diversify content when they face ad-valorem taxes on selling of news and ad-valorem taxes on selling of advertising. In this respect, we have found the following. First, if firms can diversify content, ad-valorem taxes on selling of news do not anymore unambiguously increase prices. This will depend on consumers' intensity of preferences for their ideal variety. Accordingly, if consumers have a strong preference for their ideal variety, prices can be lower in the no taxation scenario relatively to the scenario with ad-valorem taxes on selling of news. This is so because when consumers have a strong preference for their ideal variety, price competition increases.

Second, we show that both ad-valorem taxes on selling of news and ad-valorem taxes on selling of advertising reduces content diversification in the media market relatively to the no taxation case. The reason for this is that to diversify content is costly, and taxation of advertising, by reducing advertising revenues, also reduces firms' capacity to finance content diversification. This shows that even in the cases where ad-valorem taxes reduce prices (i.e. with ad-valorem taxes on selling of news) promoting consumers to multi-home, media diversity can even so be reduced, because media firms decrease content diversification in the media market.

In this sense, the policy implications of this paper are the following. First, if governments are afraid of the negative effects that ad-valorem taxes on selling of news can have on prices (Foros et al., 2019), governments should use instead ad-valorem taxes on selling of advertising, since taxes on advertising have no effects on price competition. Second, governments must however keep in mind that taxation in media markets can have negative effects on content diversification and therefore media plurality.

A Appendix: No Content Diversification and No Ad-Valorem Taxes

A.1 Single-Homing

Second-Order Conditions (SOCs). The SOCs for advertising equal:

$$\begin{aligned} \frac{d^2\Pi}{da_L^2} &= \beta\frac{p_L-p_R-t}{t} < 0 \\ \frac{d^2\Pi}{da_R^2} &= \beta\frac{p_R-p_R-t}{t} < 0 \end{aligned} \tag{96}$$

At the symmetric equilibrium ($p_L = p_R$), the SOCs for advertising are always satisfied.

The SOCs for prices equal:

$$\frac{d^2\Pi}{dp_L^2} = \frac{d^2\Pi}{dp_R^2} = -\frac{1}{t} < 0 \tag{97}$$

The SOCs for prices are always satisfied.

A.2 Multi-Homing

A.2.1 Second-Order Conditions (SOCs).

The SOCs for advertising equal:

$$\begin{aligned} \frac{d^2\Pi}{da_L^2} &= -2\beta\frac{v-d-p_L}{t} \\ \frac{d^2\Pi}{da_R^2} &= -2\beta\frac{v-d-p_R}{t} \end{aligned} \tag{98}$$

We can see that substituting for p_L and p_R in the SOCs for advertising, these SOCs are always satisfied.

The SOCs for prices are the same as for the single-homing case. Then they are always satisfied.

B Appendix: No Content Diversification and Ad-Valorem Taxes on Selling of News

B.1 Single-Homing

Second-Order Conditions (SOCs). The SOCs for advertising are the same as for the no taxation case.

The SOCs for prices equal:

$$\frac{d^2\Pi}{dp_L^2} = \frac{d^2\Pi}{dp_R^2} = -\frac{1}{t(T+1)} < 0 \tag{99}$$

Then, the SOCs for prices are always satisfied.

B.2 Multi-Homing

Second-Order Conditions (SOCs) The SOCs for advertising are the same as for the no taxation case.

The SOCs for prices equal:

$$\frac{d^2\Pi}{dp_L^2} = \frac{d^2\Pi}{dp_R^2} = -\frac{2}{t(T+1)} < 0 \tag{100}$$

Then, the SOCs for prices are always satisfied.

C Appendix: No Content Diversification and Ad-Valorem Taxes on Selling of Advertisement

C.1 Single-Homing

Second-Order Conditions (SOCs). The SOCs for advertising equal:

$$\begin{aligned}\frac{d^2\Pi}{da_L^2} &= \beta\frac{p_L-p_R-t}{t(T+1)} < 0\\ \frac{d^2\Pi}{da_R^2} &= \beta\frac{p_R-p_L-t}{t(T+1)} < 0\end{aligned} \tag{101}$$

We can see that at the symmetric equilibrium ($p_L = p_R$), the SOCs for advertising are always satisfied.

The SOCs for prices are the same as in the no taxation case. Then, the SOCs for prices are always satisfied.

C.2 Multi-Homing

Second-Order Conditions (SOCs) The SOCs for advertising equal:

$$\begin{aligned}\frac{d^2\Pi}{da_L^2} &= 2\beta\frac{d-v+p_L}{t(T+1)}\\ \frac{d^2\Pi}{da_R^2} &= 2\beta\frac{d-v+p_R}{t(T+1)}\end{aligned} \tag{102}$$

We can see that substituting for p_L and p_R in the SOCs for advertising, these SOCs are always satisfied.

The SOCs for prices are the same as for the non taxation case and are therefore always satisfied.

D Appendix: Content Diversification and No Ad-Valorem Taxes

D.1 Single-Homing

Second-Order Conditions (SOCs). SOCs for advertising:

$$\begin{aligned}\frac{d^2\Pi}{da_L^2} &= \beta\frac{p_L-p_R-t(k_L-k_R+1)}{t} < 0\\ \frac{d^2\Pi}{da_R^2} &= \beta\frac{p_R-p_R-t(k_L-k_R+1)}{t} < 0\end{aligned} \tag{103}$$

At the symmetric equilibrium $p_L = p_R$, the SOCs are always satisfied since content diversification cannot be bigger than the size of the line (the Hotelling line has size one).

SOCs for prices:

$$\frac{d^2\Pi}{dp_L^2} = \frac{d^2\Pi}{dp_R^2} = -\frac{1}{t} < 0 \tag{104}$$

The SOCs for prices are always satisfied.

SOCs for content diversification:

$$\frac{d^2\Pi}{dk_L^2} = \frac{d^2\Pi}{dk_R^2} = \frac{t-9\gamma}{9} \tag{105}$$

The SOCs for content diversification are satisfied, i.e. $\frac{d^2\Pi}{dk_L^2} = \frac{d^2\Pi}{dk_R^2} < 0$, for $t < 9\gamma$.

D.2 Multi-Homing

Second-Order Conditions (SOCs). SOCs for advertising:

$$\begin{aligned} \frac{d^2\Pi}{da_L^2} &= -2\beta\frac{v-d-p_L+tk_L}{t} \\ \frac{d^2\Pi}{da_R^2} &= -2\beta\frac{v-d-p_R+tk_R}{t} \end{aligned} \tag{106}$$

Since $v > d$, then the SOCs for advertising are satisfied if $t > \frac{p_L}{k_L}$. In other words, consumers need to have a strong preference for their ideal variety. When this does not happen, consumers will choose to single-home.

SOCs for prices:

$$\frac{d^2\Pi}{dp_L^2} = \frac{d^2\Pi}{dp_R^2} = -\frac{2}{t} < 0 \tag{107}$$

The SOCs for prices are always satisfied.

SOCs for content diversification:

$$\frac{d^2\Pi}{dk_L^2} = \frac{d^2\Pi}{dk_R^2} = \frac{1}{2}(t-2\gamma) \tag{108}$$

The SOCs for content diversification are satisfied for $t < 2\gamma$.

E Appendix: Content Diversification and Ad-Valorem Taxes on Selling of News

E.1 Single-Homing

Second-Order Conditions (SOCs). The SOCs for advertising under taxation are the same as above with no taxation.

SOCs for prices:

$$\frac{d^2\Pi}{dp_L^2} = \frac{d^2\Pi}{dp_R^2} = -\frac{1}{t(T+1)} < 0$$

The SOCs for prices are always satisfied.

SOCs for content diversification:

$$\frac{d^2\Pi}{dk_L^2} = \frac{d^2\Pi}{dk_R^2} = \frac{t-9\gamma(T+1)}{9(T+1)}$$

The SOCs for content diversification are satisfied for $t < 9\gamma(T+1)$.

E.2 Multi-Homing

Second-Order Conditions (SOCs). The SOCs for advertising under the taxation case equal the SOCs for advertising under no taxation.

SOCs for prices:

$$\frac{d^2\Pi}{dp_L^2} = \frac{d^2\Pi}{dp_R^2} = -\frac{2}{t(T+1)} < 0$$

The SOCs for prices are always satisfied.

SOCs for content diversification:

$$\frac{d^2\Pi}{dk_L^2} = \frac{d^2\Pi}{dk_R^2} = \frac{t-2\gamma(T+1)}{2(T+1)}$$

Then the SOCs for content diversification are satisfied if $t < 2\gamma\left(T+1\right)$.

F Appendix: Content Diversification and Ad-Valorem Taxes on Selling of Advertisement

F.1 Single-Homing

Second-Order Conditions (SOCs). SOCs for advertising:

$$\begin{aligned} \frac{d^2\Pi}{da_L^2} &= \beta\frac{p_L-p_R-t(k_L-k_R+1)}{t(T+1)} \\ \frac{d^2\Pi}{da_R^2} &= \beta\frac{p_R-p_L-t(k_R-k_L+1)}{t(T+1)} \end{aligned} \tag{109}$$

We can see that at the symmetric equilibrium $p_L = p_R$, these SOCs are always satisfied.

SOCs for prices:

$$\frac{d^2\Pi}{dp_L^2} = \frac{d^2\Pi}{dp_R^2} = -\frac{2}{t} < 0 \tag{110}$$

The SOCs for prices are always satisfied.

F.2 Multi-Homing

Second-Order Conditions (SOCs). SOCs for advertising:

$$\begin{aligned} \frac{d^2\Pi}{da_L^2} &= -2\beta\frac{v-d-p_L+tk_L}{t(T+1)} \\ \frac{d^2\Pi}{da_R^2} &= -2\beta\frac{v-d-p_R+tk_R}{t(T+1)} \end{aligned}$$

We can see that substituting for p_L and p_R in the SOCs for advertising, these SOCs are always satisfied.

SOCs for prices:

$$\frac{d^2\Pi}{dp_L^2} = \frac{d^2\Pi}{dp_R^2} = -\frac{2}{t} < 0$$

The SOCs for prices are then always satisfied.

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