The role of strategic consumers and vertical differentiation on platform compatibility ⋆

Thanos Athanasopoulos[1] and Nick F.D. Huberts[2]

[1] *School of Accounting, Finance and Economics; De Montfort University, The Gateway Leicester, LE1 9BH, United Kingdom*

[2] *School for Business and Society, University of York, Church Lane Building, Heslington, York YO10 5ZF, United Kingdom*

May 2023

Preliminary draft: Please do not cite or circulate

Abstract

This paper considers the pricing decisions and the optimal timing to support platform compatibility in a dynamic duopoly set-up where consumers enjoy network effects. We contribute to the literature by introducing vertical differentiation, market growth, and consumer rationality. We find that in contrast to existing studies, platforms are eager to support compatibility earlier when they are sufficiently differentiated. Then, the threat of the dominant platform's rival attracting consumers is sufficiently reduced. Second, faster-growing markets make platforms more likely to delay their switch to compatibility. Third, if consumers are myopic, both firms support compatibility at the beginning of the game, whereas when consumers are rational, compatibility is delayed until random forces and the installed base differential lead to more asymmetry. In addition, we find that firm value has a U-shaped relationship with the degree of vertical differentiation, and downgrading quality provision from the low-quality seller might lead to higher firm value for both competitors. In such a situation, imposing a minimum quality standard leads both to higher consumer and industry surplus. Our work provides insights into both Sony's decision to delay multihoming for the game Fortnite on

⋆ The authors would like to thank the participants of the ZiF Research Group Workshop in Bielefeld (January 2022), the YARO Workshop at the University of York (December 2022), and the MACCI Conference in Mannheim (March 2023). The paper received the best paper award at the Faculty Research Conference at Leicester Business School/De Montfort University (June 2023).

their platform Playstation, the dominant player in the market for gaming platforms alongside market dynamics in several markets, such as that in the European market for gaming consoles.

Keywords: Vertical differentiation; Compatibility; Multihoming; Network Effects; Market Growth; Duopoly.

JEL Classifications: L15; D43.

1 Introduction

A product is said to be associated with network effects if the value to a customer is increasing in the number of its users–either directly or indirectly. Industries that feature network effects are ubiquitous: gaming consoles, mobile phones, and banking services are just a few.[1] These industries have traditionally experienced positive growth[2] and often comprise asymmetric firms in terms of market shares and other traits, such as the product or service quality. For example, Sony has dominated the market for gaming consoles when looking at market shares within this industry with Playstation 4, offering a console that is considered to be of higher quality as compared to competition both by consumers and independent agencies alike.[3]

Firms' decision to support *compatibility* with competitors within these industries is a strategic choice with important implications as regards the functioning of those markets and welfare. For example, compatibility in the market for banking services means that consumers of a particular bank can use an ATM of another bank without incurring extra fees. Similarly, compatibility in telecommunications would lead mobile service providers to extend "on-net" discounts typically offered to calls within their own network to calls to competitors' networks. In the gaming industry, compatibility enables a Sony PlayStation gamer to play with an Xbox user. This cross-platform play typically occurs via the use of a third platform, such as a mobile phone when a gamer can use the same console account to play a game using a mobile phone and is possible for games whose developer *multihomes* when the same game is available on different platforms.

Although asymmetric firms are not likely to support compatibility in non-growing markets (see, e.g, Chen *et al.* (2009)), the short-run dynamics of the aforementioned growing industries, in the presence of asymmetries, are largely unexplored albeit they provide puzzling illustrations of

[1] See Corts and Lederman (2009) for an article related to the existence of network effects in the markets for gaming consoles.

[2] See here as regards the growth of global users within the Gaming Console market per year.

[3] See here as regards the comparison between Playstation 4 and XBox 1.

firm strategic behavior. A prominent recent example relates to Sony's behavior within the market for gaming consoles that resembles a duopoly with the combined market shares of the two largest firms–Sony and Microsoft–exceeding 95% both in global and many regional markets, most notably in the European market.[4] Whilst Sony was initially reluctant to embrace compatibility for the very successful game "Fortnite" developed by Epic Games in 2017, Sony changed its compatibility stance in 2018, allowing its users to multihome and play with users from other platforms when accessing their mobile phones whilst using their Sony account.[5]

To this end, we analyze the short-run pricing decisions within a duopoly of platform owners that are initially asymmetric–in terms of market shares–and vertically differentiated, alongside their incentives for compatibility in growing industries. Within our framework, firms compete for heterogeneous and forward-looking consumers, while we contrast our results with the scenario consumers are myopic. To understand the strategic value of compatibility in the gaming industry, we look at situations when firms benefit from incompatibility via complementary services. This is a realistic assumption as compatibility in the gaming industry would lead to a loss of income from royalties for Sony and its competitor (i.e., Microsoft) when users use an application with the aid of their mobile phone instead of using the application only within the particular platform with incompatibility. We then relax this assumption and also allow firms to sell standalone products.

Surprisingly, we find that platforms are willing to support compatibility simultaneously at the beginning of the dynamic game when consumers are myopic. In contrast, when consumers are rational, we find that while the smaller as regards its market shares and lower-quality firm is likely to support compatibility early on, the dominant market player is initially reluctant to support compatibility. The installed base differential alongside vertical differentiation is likely to lead to larger asymmetries in installed bases that interestingly trigger the dominant firm's switch to compatibility. In fact, while for lower and intermediate degrees of vertical differentiation, we find that the switching time is increasing in the extent of quality differences between the platforms, this is a non-monotonic relationship: very differentiated firms might find it beneficial to switch faster. In general, however, relative symmetry is gradually restored at the end of the dynamic game. Thus, there is a dynamic within the model preventing the market from getting too asymmetric.

As regards platform pricing, we find a U-shaped relationship between vertical differentiation and the competitors' optimal prices. We also find that the low-quality firm might have the incentive to downgrade the quality it supplies no matter its initial market shares to the benefit of both

[4]See here and here as regards the competitors' market shares within the market for gaming consoles in Europe and globally, respectively.

[5]Seehere as regards Sony's reluctance to support cross-play and the switch in its strategy.

competitors, which is a result that holds when higher quality is not associated with higher costs and also when the marginal cost of production rises linearly with quality. This is because by endogenously creating a larger degree of vertical differentiation within the market, the lesser degree of competition allows both firms to raise their prices to the detriment of consumers. As a result, imposing a minimum standard within the industry will be beneficial for consumers and the industry surplus.

Our results encompass a number of industries with network effects, and the model is tested in the Console Operating Systems industry, explaining the relative prices set by Sony and Microsoft. We also explain Sony's initial reluctance to support compatibility and the switch in its stance alongside the dynamics within particular markets, such as that in Europe.

1.1 Related Literature

The literature has either looked at how strategic consumers affect platform competition and pricing or investigated firms' endogenous multihoming and compatibility decisions in isolation.

Within the broad literature on competition between proprietary networks, there are articles that consider deterministic models, such as Driskill (2007) or models generating dynamics. Within the latter strand of the literature on dynamic competition, the majority of the articles has considered myopic consumers–i.e., Doganoglu (2003), Markovich (2004) and Markovich and Moenius (2009)among others–and a few dynamic models consider rational consumers, such as Cabral (2011) and Zhu and Iansiti (2007). We focus on forward-looking consumers–and contrast our findings with the scenario consumers are myopic–when investigating how *vertical differentiation affects platform pricing and firm value*, albeit our primary goal is to explore the determinants of dominant firms' switch in compatibility strategy in growing markets. More precisely, the important difference of our work with Zhu and Iansiti (2007) is that unlike that paper, where platforms set the same exogenous price, we investigate how consumer expectations, installed bases, and platform quality affect *firms' incentives to differentiate themselves from their competitor by endogenously choosing their pricing strategy.* In contrast to Cabral (2011), where platforms set prices on each date within a model with stationary demand, to match growing markets where competitors' prices might differ but are fixed for some time, such as that for gaming consoles and mobile services, we consider a market with *growing demand* in which platform owners set their prices *at the beginning* of the dynamic game.

We find a U-shaped relationship between the degree of vertical differentiation and both competitors' optimal prices and their value. Interestingly, we also find that for some degrees of vertical differentiation, the lower-quality seller downgrades its quality provision to the benefit of both competitors, which is a result that holds both when higher quality is not associated with higher costs but also when the development costs of production rise linearly with quality. In this case, a MQS (Minimum Quality Standard) would lead to higher consumer and industry surplus, expanding the relevant discussion on the benefits from MQS (Minimum Quality Standards) (see, e.g., Ronnen (1991) to a market with network effects albeit we consider that customers have the same taste for quality.

Early works on compatibility in markets with network effects usually consider a two-stage game, where firms first decide their stance on compatibility and then engage in some form of competition–see, i.e., Katz and Shapiro (1986). This literature usually assumes horizontal differentiation and concludes that compatibility is supported if firms are similar as regards their market shares or other traits (see, i.e., Malueg and Schwartz (2006)). More recent works use this premise as the initial conditions within a dynamic framework and myopic consumers–see, e.g., Chen *et al.* (2009) to check the long-run stability of compatibility.[6] All the aforementioned works consider neither a vertical dimension of differentiation nor a growing market, which are at the core of the industries we investigate, whilst we focus on how strategic consumers affect dynamic competition and firm strategy.

Our results contrast with the existing literature, according to which firms reject compatibility unless they are fairly symmetric. In fact, we find conditions under which very asymmetric firms switch to compatibility either at the beginning of the dynamic game if consumers are myopic, or at later stages of the dynamic game when consumers are rational. In the latter case, we provide insights as regards platforms' relative prices and the evolution of the dynamic system in the market for gaming consoles. The finding that the bilateral switch to compatibility is likely to occur when the market is fairly more asymmetric in terms of firms' market shares as compared to the initial–already asymmetric–conditions, might be alarming as regards the dominance of one of the two platforms at the end of the dynamic game. Unlike the literature on dominance, however, according to which large networks tend to become larger with seminal contributions from Reinganum (1983), Gilbert and Newbery (1982), Budd *et al.* (1993) with more recent contributions in Cabral (2000) and Cabral (2011), we find that under fairly general conditions, there is a dynamic that leads to relative symmetry at the end of our dynamic game.

[6] Also, see Chen (2018) for a dynamic model with myopic consumers, where firms decide prices and compatibility within a market with network effects and switching costs.

2 Model

Although our benchmark model is centered around the motivating example within the Gaming industry, it is generic enough to study many markets with network effects and could easily be generalized further to study other types of industries (see the discussion in section 6).

2.1 State Space and Firms' Decisions

In our discrete time model, we consider two risk-neutral, value maximizing platform owners, which we will denote as P and $X-P$ refers to "Playstation" and X to "Xbox"$-$selling potentially vertically differentiated platforms to a sequence of $N \in \mathbb{N}$ heterogeneous consumers with unit demand. We normalize the cost of production of each console to zero.[7] Our model can be considered to be equivalent to an N-period model, where in each period a new consumer arrives. Denote by $b_i(t) \in \mathbb{N}_0 \equiv \mathbb{N} \cup \{0\}$ the consumer base at time t, $i = X, P$, where we will assume $b_P(0) > b_X(0) \geq 0$ for our main model.

Given their installed bases, the platform sellers initially (t=0) determine the platform quality and their prices in a dynamic setting: these prices are fixed throughout the game. Fixed prices are motivated by the observation that console prices in the gaming industry are typically fixed over time and only start to fall after new generations of consoles are introduced, which is beyond the scope of this paper. Whilst both platforms allow for *seller multihoming* as a game developer offers its game within both consoles, the platforms can also choose whether or not to allow for *consumer multihoming*, in which case consumers (i) can play the game on alternative platforms, such as mobile phones or tablets without losing any information, and (ii) consumers can play games with players from other platforms that also allow for multihoming.[8] Let dummy $d_{i,m}(b_i, b_{-i})$ be equal to 1 when platform owner $i = X, P$ allows for network compatibility and 0 otherwise after $m < N$ consumers have already arrived.

In line with e.g. Cabral and Salant (2014), compatibility is an absorbing state within this model, i.e., as long as firms decide to support compatibility they cannot go back on their decision to reject compatibility in the future. Thereto, define $\mathcal{D}_{i,m} = \{d \in \{0,1\} : d \geq d_{i,m-1}\}$. As in Chen *et al.* (2009), compatibility requires both firms' consent.

On the sellers' side of the market, we assume a multihoming game developer (Epic) that sells its product (Fortnite) on both platforms. For platform owner $i = X, P$, exogenous and fixed

[7] We also relax this assumption to allow for a higher marginal cost of production when producing a platform of higher quality.

[8] We will use the terms consumer multihoming and network compatibility interchangeably within the paper.

instantaneous revenues from royalties under consumer multihoming and singlehoming are given by

$$\tilde{v}_i^{SH}(b_i) = b_i p_E(1-\kappa_0),$$
$$\tilde{v}_i^{MH}(b_i) = b_i p_E(1-\kappa_1)(1-\gamma),$$

respectively, where $p_E \geq 0$ is the average annual expenditure on in-game purchases by consumers of which a fraction $1-\kappa \in [0,1)$ is paid to the platform owner, and γ gives the fraction of in-game purchases made on other platforms, such as tablets and mobile phones. Motivated by the story around Fortnite within the gaming industry, where lost income reflects foregone royalties with network compatibility, we assume that platform rents from services and applications are not lower with incompatibility, i.e., $1-\kappa_0 \geq (1-\kappa_1)(1-\gamma)$. We then discuss the effect of a modified version of this assumption to include industries where either no such royalties are relevant–in which case firms sell their standalone products–or industries, where network compatibility may not necessarily lead to lower income from royalties.

2.2 Demand

Let $\mathcal{N} = \{0, 1, \ldots, N\}$. Assume that by some time $t \geq 0$, $m \in \mathcal{N}$ consumers have arrived and a newly arrived consumer is considering to purchase a console. For brevity, we will omit the variable t in the following descriptions. Then, the instantaneous utility from purchasing platform $i = X, P$ is given by

$$\tilde{u}_{i,m}(b_i, b_{-i}) = u_i + g\left(b_i + b_{-i} d_{i,m}(b_i, b_{-i}) d_{-i,m}(b_i, b_{-i})\right),$$

where u_i represents the fixed intrinsic platform quality that is an average of attributes, such as the quality and number of potentially exclusive games, memory bandwidth, and CPU speed. Although in the benchmark model and to match the story in the introduction around Sony and Fortnite, we assume $u_P \geq u_X$, we also allow for $u_P < u_X$ as we are interested in situations the majority of consumers might have adopted the "wrong" standard. $g : \mathbb{N}_0 \to \mathbb{R}_+$ captures *network effects* with $g(b+1) > g(b)$, for all $b \in \mathbb{N}_0$. For the latter, $b_i + b_{-i} \cdot d_{i,m}(b_i, b_{-i}) d_{-i,m}(b_i, b_{-i})$ is the effective installed base of platform owner i given the support or rejection of compatibility.[9]

Unlike the literature, where consumers are myopic (e.g., Chen *et al.* (2009), Chen (2018)), in

[9] A general shape is assumed for g, so that robustness can be checked. In our numerical explorations we will assume $g(b) = q \cdot (b/N)^L$, for some $q > 0$ and $L \in \mathbb{R}$. The parameter L allows us to study the impact of convex and concave specifications on our results, whereas scalar q allows us to study the impact of network effects relative to the intrinsic platform quality.

our model consumers are forward-looking and have rational expectations. That is, they take into account their expectations about the arrival of future consumers and expectations about when platforms may allow for network compatibility.[10] We can then write down lifetime utility derived from joining platform $i = X, P$. First, for any $m < N$, the deterministic component of consumer utility is given by

$$U_i(b_i, b_{-i}) = \int_0^{\tau(b_i+b_{-i})} \tilde{u}_{i,m}(b_i, b_{-i})e^{-rs}\mathrm{d}s + e^{-r\tau(b_i+b_{-i})} \times \Big(\phi_i U_i(b_i+1, b_{-i}) + \phi_{-i} U_i(b_i, b_{-i}+1) + (1-\phi_i-\phi_{-i})U_i(b_i, b_{-i})\Big),$$

where $\tau(b_X + b_P)$ is the interarrival time depending on market size and where $\phi_i : (b_i, b_{-i}) \mapsto \phi_i(b_i, b_{-i})$ is the probability of subsequent consumer choosing i. Discounting is done under rate r, where we assume that consumers and platforms face the same discount rate. The first term represents the utility that the consumer enjoys until the next consumer arrives. The second term is the present value of the utility received after the next consumer has joined platform i, platform $-i$, or neither platform. We will assume that after the last consumer has arrived, the consoles are supported for a period of normalized length 1 after which no utility can be enjoyed. The interarrival time is assumed to be equal to

$$\tau(b) = \frac{T}{5+b},$$

for some $T > 0$, so that the interarrival time goes down as more consumers are present in the network, but we will relax this assumption later in the paper, considering a fixed time between consumer arrivals at the market. Note that in order to be able to isolate the key forces at play in our set-up, we will assume that the interarrival time does not depend on the compatibility regime. Increasing T has two effects: it delays the arrival of consumers, while at the same time it increases the planning horizon.

Any consumer has utility $U_i + \varepsilon_i$, where $\varepsilon_i \in \mathbb{R}$ is the consumer's idiosyncratic preference shock with zero mean (see, e.g., Chen *et al.* (2009) and Cabral (2011), so that the consumer solves

$$\max\{U_X + \varepsilon_X - p_X, U_P + \varepsilon_P - p_P, \varepsilon_O\},$$

thus, the consumer selects the option that yields the highest utility. The third element captures the

[10]We also look at myopic consumers in an extension.

consumer's outside option with a deterministic instantaneous utility normalized to 0, i.e., not all consumers may decide to purchase a console.[11] Consumers can only observe their own idiosyncratic preferences $\{\varepsilon_X, \varepsilon_P, \varepsilon_O\}$: platform owners cannot observe any. All ε_i are iid distributed and are independent of τ.

Preferences ε_i have a 'Type I extreme value' distribution, so that the probability of choosing platform i is given by

$$\phi_i(b_i, b_{-i}) = \frac{\exp\{(U_i(b_i+1, b_{-i}) - p_i)\}}{\sum_{i=X,P} \exp\{(U_i(b_i+1, b_{-i}) - p_i)\} + \exp\{0\}}, \tag{1}$$

(see, e.g., Maddala (1983)). The probability of the outside option is then $\phi_0 = 1 - \phi_X - \phi_P$.

Figure 1 illustrates the decision process within the model.

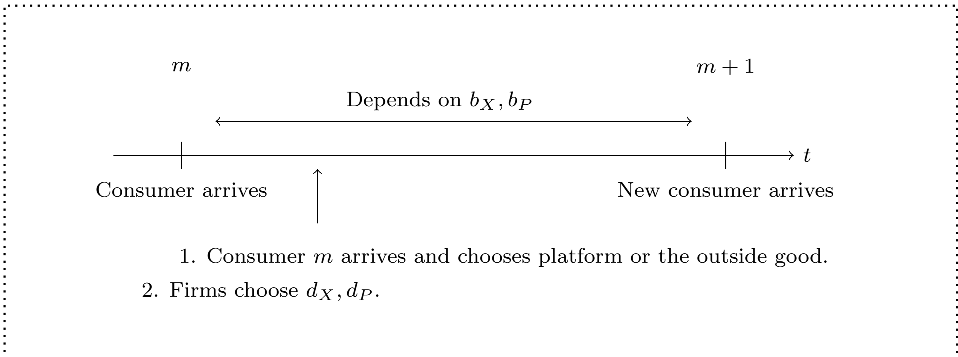


Figure 1: Illustration of the timeline for our model.

2.3 Bellman equations and platform value

In order to find platform value and to characterize the decision process of each firm, first, denote by $\tilde{V}_{i,m}$ the net present value of platform i *after* compatibility decisions have been made and by $V_{i,m}$ the net present value *before* these decisions. Then, for any $m < N$, we have

$$\begin{aligned}\tilde{V}^{MH}_{i,m}(b_i, b_{-i}) = &\int_0^{\tau(b_i+b_{-i})} \tilde{v}^{MH}_i(b_i) e^{-rs} \mathrm{d}s + e^{-r\tau(b_i+b_{-i})} \times \\ &\Big(\phi_i \left[\tilde{V}^{MH}_{i,m+1}(b_i+1, b_{-i}) + p_i\right] + \phi_{-i}\tilde{V}^{MH}_{i,m+1}(b_i, b_{-i}+1) + (1-\phi_i-\phi_{-i})\tilde{V}^{MH}_{i,m+1}(b_i, b_{-i})\Big).\end{aligned}$$

[11] Thus, whether or not the market is fully covered is endogenous: the buyer can also choose to purchase an outside good.

and

$$\tilde{V}^{SH}_{i,m}(b_i, b_{-i}) = \int_0^{\tau(b_i+b_{-i})} \tilde{v}^{SH}_i(b_i) e^{-rs} \mathrm{d}s + e^{-r\tau(b_i+b_{-i})} \times$$
$$\Big(\phi_i \left[V_{i,m+1}(b_i+1, b_{-i}) + p_i\right] + \phi_{-i} V_{i,m+1}(b_i, b_{-i}+1) + (1-\phi_i-\phi_{-i}) V_{i,m+1}(b_i, b_{-i})\Big),$$

for the cases where currently compatibility is and is not supported, respectively. For both cases, the first term represents the cash-inflows in between arrivals and the second term captures the present value of cash-inflows received after the next consumer has arrived. When $m = N$, no more consumers will arrive and the market only exists for one unit of time after the arrival of the N-th consumer. Thus the second term vanishes for both and only the first term remains, where the integral runs between 0 and 1.

Then, platform owner i solves

$$V_{i,m} = \sup_{d' \in \mathcal{D}_{i,m}} \left\{ d' \cdot \tilde{V}^{MH}_{i,m} + (1-d')\tilde{V}^{SH}_{i,m} \right\}.$$

This paper considers a Markov perfect (closed-loop) equilibrium. In particular, both platform owners use Markovian feedback strategies, thereby conditioning their actions on the current value of the state $\mathcal{A} = (m, b_i, b_{-i}, d_{i,m-1}, d_{-i,m-1})$. The strategy sets are then given by $\mathcal{S}_i = \{d_i : \mathcal{A} \to \{0,1\}\}$ for both $i = X, P$. In line with the literature, we refrain from fully characterizing the strategy profiles, however, to determine them, we consider the following. If $d_X + d_P = 1$, only one platform owner has switched to multihoming, which means that the other platform owner can choose its timing independently. However, if $d_X + d_P = 0$, then both platform owners consider switching upon each arrival. In that case we assume that the platform owners play a non-cooperative simultaneous move game where our attention is restricted to pure strategy Nash equilibria in a finite-horizon analogue of Chen *et al.* (2009) and Chen (2018) that use a modified version of the algorithm in Pakes and McGuire (1992)–also see Pakes and McGuire (2001). Figure 2 illustrates all potential transitions that could lead to compatibility within the four possible modes $(d_X, d_P) \in \{0,1\}^2$.

For the scenarios where multiple pure strategy Nash equilibria arise, we make the following assumptions. If (1,1) is included in the set of equilibria, then firms are assumed to select it. In other words, firms are only prone to retain their compatibility stance when (1,1) does not arise as a potential equilibrium. This assumption is made to ensure that a mutual switch to compatibility arises when both firms support it. In the case (1,0) and (0,1) are both equilibria, firms are

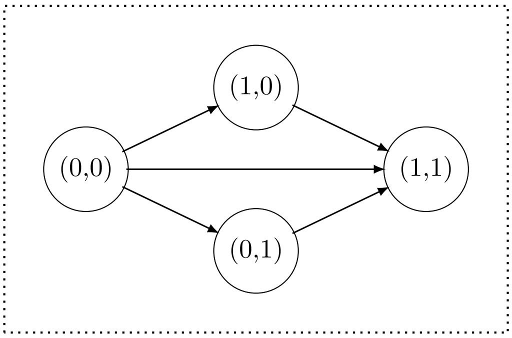


Figure 2: All possible modes and transitions of $(d_X, d_P) \in \{0,1\}^2$.

assumed to flip a coin. Finally, in all other cases, as well as the scenario where no pure strategy Nash equilibria arise, the firms are assumed to stick to their previous stance.

Finally, prices and platform qualities are determined at time $t = 0$, where we assume a simultaneous move game *à la* Bertrand, i.e., prices follow from,

$$\begin{cases} \sup_{p_X} V_X^{SH}(b_X, b_P), \\ \sup_{p_P} V_P^{SH}(b_P, b_X). \end{cases}$$

Following standard procedure, the game is solved by first determining the reaction curves of both platform owners, after which all intersections are considered as potential equilibria.

3 Vertical Differentiation

Section 3.1 investigates the effect of the degree of vertical differentiation on firms' quality and price choices alongside firms' value when (1) compatibility is mandated, for example, due to regulation, or when (2) compatibility is never supported. After having analyzed these two cases, we consider firms' endogenous compatibility stance alongside the optimal timing that triggers firms' "switch" to compatibility in Section 3.2, while Section 3.2.1 and 3.2.2 discuss the generality of the results obtained to encompass situations when higher quality is associated with a higher marginal cost of production alongside one-sided markets and scenarios royalties are not higher with incompatibility. Section 3.3 discusses the impact of mandated quality standards.

3.1 Exogenous Compatibility Regime

This section considers two extreme cases. First, we consider the scenario where compatibility is supported from the outset. Second, an alternative case is studied where firms decide to never support compatibility. The latter case will aid in building the intuition in Section 3.2 as regards why and when firms choose to switch to compatibility.

3.1.1 Markets with compatibility

As outlined in Section 2, in line with industry practice, platforms determine their prices at the start of the game and prices stay fixed throughout. For a given degree of vertical differentiation, uniqueness and existence of equilibrium prices is discussed in Appendix A.[12] For our baseline parameterization, Table 1 summarizes firm prices and values whilst ϕ_X and ϕ_P denote the probabilities of only the first consumer choosing platform X or P, respectively.

p_X	p_P	ϕ_X	ϕ_P	V_X	V_P	U_X	U_P
1.53	2.99	0.1317	0.8683	10.44	35.19	97.53	100.87

Table 1: Optimal prices and resulting probabilities and values for baseline parameterization. $N = 40$, $b_X(t = 0) = 1$, $b_P(t = 0) = 3$, $p_E = 0.40$, $\kappa_1 = 0.80$, $\gamma = 0.30$, $u_X = 4$, $u_P = 4.4$, $r = 0.1$, $L = 1.2$, $q = 4$, $T = 5 + b_X + b_P = 9$.

First, we observe that platform X sets a lower price: to attract new consumers it needs to set a sufficiently lower price than P in order to compensate for the lower quality. It is important to note that within this setting, any price differential between the two platforms is only attributable to vertical differentiation, that is, the initial asymmetry as regards the platforms' installed base plays no role.

For given platform quality, u_P, and a fixed quality $u_X = 4$, Figure 3 illustrates the impact of vertical differentiation, $u_P - u_X$, on optimal platform prices, firm value, and welfare, as well as on the probabilities of the first consumer joining by varying P's platform quality u_P. It is, perhaps, not unexpected that the probability of attracting initial consumers for the high-quality platform P is increasing in the degree of vertical differentiation. This follows directly from consumers caring about the platform intrinsic quality early within the game. As regards firms' optimal prices, when u_P is sufficiently different from $u_X = 4$, relaxed competition due to more differentiation is associated with high market prices both for the high-quality as well as the lower-quality firm, and, as a direct result, firm value is also higher for both competitors. In contrast, when differentiation is low, i.e. when u_P and u_X are close, intensified competition leads to lower platform prices and, correspondingly, firm values. In fact, we find that the relationship between firms' optimal prices

[12] In our default parameterization we assume that $b_X(0) = 1$ and $b_P(0) = 3$ are the platforms' initial installed bases.

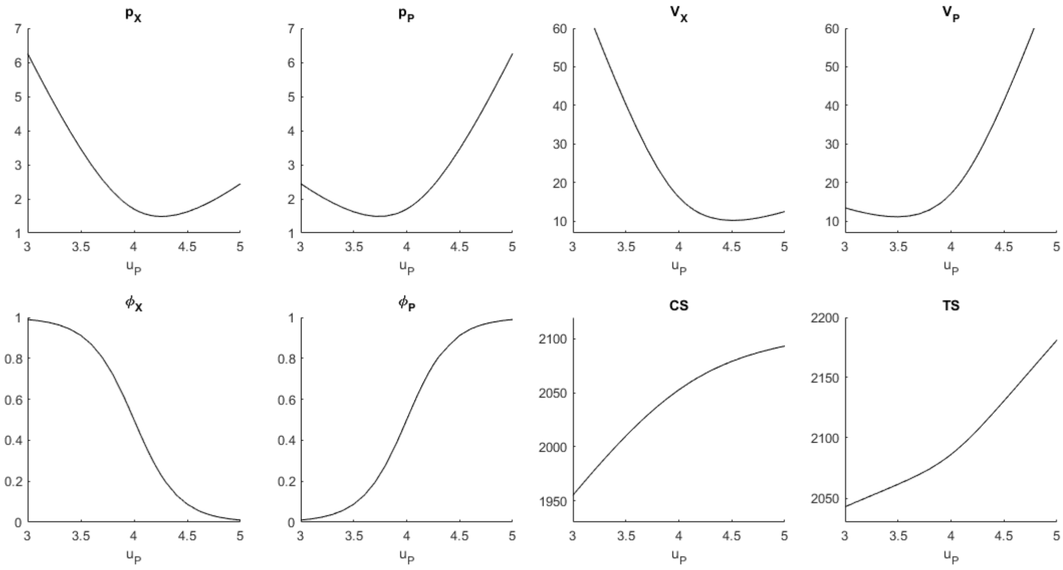


Figure 3: Impact of vertical differentiation.
$N = 40$, $b_X(t = 0) = 1$, $b_P(t = 0) = 3$, $p_E = 0.30$, $\kappa_1 = 0.80$, $\gamma = 0.30$, $u_X = 4$, $r = 0.1$, $\eta = 2$, $T = 5 + b_X + b_P = 8$.

alongside firm values and the degree of vertical differentiation is not monotonic. Instead, Figure 3 highlights a U-shaped relationship between firms' optimal prices–and profits–and the difference in platform quality.

Interestingly, we find that the two competitors may not compete with their highest quality potential. To see this, note that when P is the low-quality seller and $3 \leq u_P \leq 3.5$, an incremental rise in quality u_P would lead P and X to decrease their optimal prices as a result of intensified competition. As a direct result, firm value decreases in u_P for both firms. Thus, the low-quality seller P would be *better* off when it degrades the quality of its offering.[13] Figure 16 in Appendix C illustrates that the same applies when u_P is fixed and platform X decides to lower its quality. Hence, we find that if the reported value of u_X or u_P (depending on which of the two is the low quality platform) gives an upperbound to the technology the platform is able to offer, then incrementally lowering the quality of the platform may lead to an increase in value *for both competitors*.

Result 1. *Both competitors benefit from the low-quality seller degrading its platform quality, to the detriment of consumers, unless the quality difference between the platforms is small.*

Although the probability of platform X attracting the initial consumer, under the default pa-

[13]Figure 17 in Appendix C considers $u_P \in [1, 7]$ and confirms that the same is found when a wider range of u_P is studied.

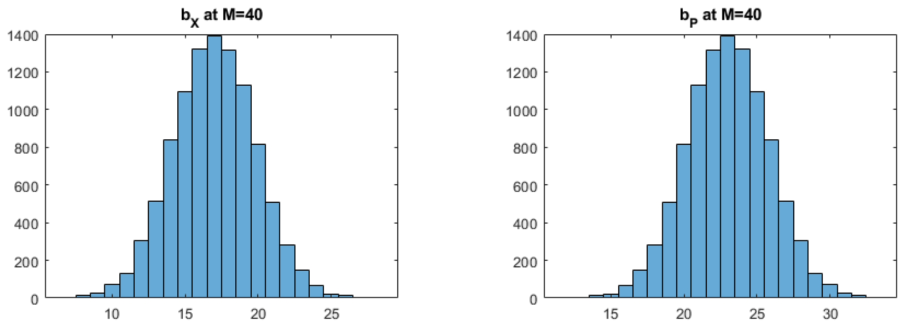


Figure 4: Distribution of consumer base at time N.

rameterization, is only 13.17%, there is relative symmetry as regards the distribution of consumer base at later stages of the game, as portrayed by Figure 4. Figure 5 illustrates this point further and shows how the probabilities of each platform attracting new consumers change over time alongside the expected changes in both platforms' consumer base. Whilst at the earlier stages of the dynamic game, consumers choose the platform with the highest quality with probabilities closer to 1, at later stages of the dynamic game consumers are likely to choose the low-quality platform with the lower price. This happens for two reasons. First, consumers care relatively less about the degree of vertical differentiation at later stages of the game, as there is less time to benefit from the wedge in quality from the durable goods offered. Instead, the difference in price becomes more important, favoring platform X. Second, as the network of consumers grows, network effects start to dominate the consumers' utility-and correct probabilities from any asymmetries.

Result 2. *Market concentration falls towards the end of the game.*

3.1.2 Markets without compatibility

Next, consider a scenario where compatibility is never supported, i.e., until the end of the game both platforms are assumed to not allow consumers to multihome.

Figure 6 highlights the impact of vertical differentiation on prices, the dominant firm's value, and consumer surplus. Under incompatibility, the dominant firm–as regards its market shares–P has a clear advantage: since consumers are forward looking they anticipate a benefit from consumers joining the same network in the future so that consumers become more likely to choose a platform with the largest network. This allows platform P to increase its prices relative to the case with compatibility–the second panel of Figure 6 illustrates that indeed the solid line lies above the dotted line. Platform X has the disadvantage: in order to attract consumers it needs to lower its prices (see first panel). In contrast to the scenario of mandatory compatibility where prices

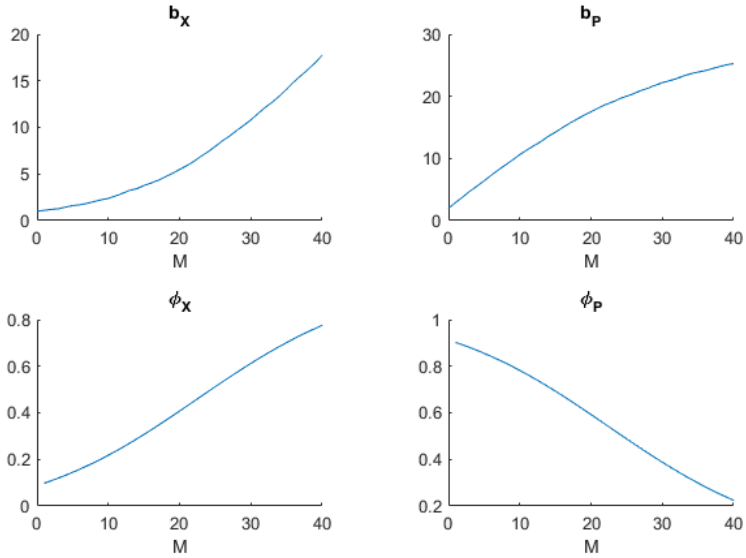


Figure 5: Expected change in probabilities and consumer bases as a function of periods/jumps.

have a U-shaped relationship with vertical differentiation, when compatibility is never supported the relationship between prices and u_P becomes monotonic.

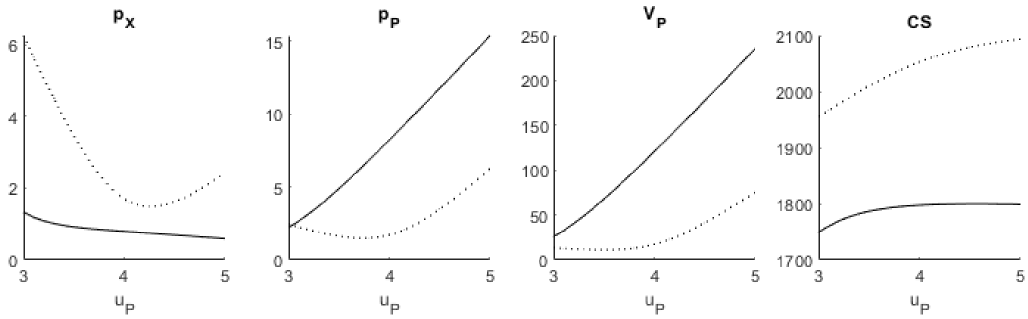


Figure 6: Markets with (dotted) and without (solid) compatibility.
$N = 40$, $b_X(t = 0) = 1$, $b_P(t = 0) = 3$, $p_E = 0.30$, $\kappa_1 = 0.80$, $\gamma = 0.30$, $u_X = 4$, $r = 0.1$, $\eta = 2$, $T = 5 + b_X + b_P = 8$.

The direct effect of vertical differentiation on platform P's value and consumer surplus is visible in the last two panels of Figure 6. Clearly, P benefits from its dominance whereas consumers would be better off with the lower prices from the scenario with compatibility.

Result 3. *When comparing the two cases compatibility is never supported and when it is mandated, the former case yields higher value for platform P whereas consumers are better off with the latter.*

Closer inspection reveals another difference as regards consumer surplus (CS) under multihoming and singlehoming. Figure 7 reveals that CS is monotonically increasing in u_P only when compatibility is mandated. The fact that CS is harmed by a higher intrinsic quality with incompatibility follows directly from the sharp increase in price p_P (also see second panel Figure 6).

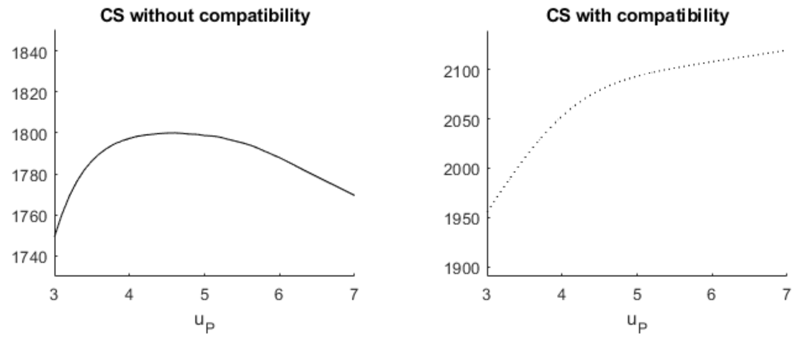


Figure 7: Consumer surplus with (dotted) and without (solid) compatibility.
$N = 40$, $b_X(t = 0) = 1$, $b_P(t = 0) = 3$, $p_E = 0.30$, $\kappa_1 = 0.80$, $\gamma = 0.30$, $u_X = 4$, $r = 0.1$, $\eta = 2$,
$T = 5 + b_X + b_P = 8$.

The analysis in this section raises the question why platform P would consider multihoming in the first place. This question will be addressed in the following section and might shed light on Sony's reluctance to support consumer multihoming as regards Epic's Fortnite that changed later with Sony's switch in compatibility stance.

3.2 Endogenous Compatibility

In this section we analyze firms' compatibility stance and their optimal switching time, that is, we study the question after how many periods it is optimal for both firms to support compatibility. We do this exercise first when firms sell platforms of the same quality, followed by an investigation of how vertical differentiation affects firms' compatibility incentives and the optimal switching time to compatibility.

Platform incentives Perhaps as expected, the low-quality platform X is very likely to support compatibility early in the game. Therefore this section studies the incentives of the high-quality platform P to offer compatibility support under the assumption that X has already switched.

The previous section highlighted that in a scenario where compatibility is never supported, the dominant platform has a clear advantage: due to its larger initial network (and higher quality) consumers are a lot more likely to choose platform P. Thus, at time $t = 0$, in expectation, a scenario where compatibility is never supported is always better for platform P than a scenario where there is support at any future point in time, and if P was to support compatibility, this would have to be triggered by a shift in incentives as the game advances. Appendix B discusses this in more detail and analyzes firm incentives for m close to $m = 40$.

$m=1$	$m=2$	$m=3$	$m=4$	$m=5$	$m=6$	$m=7$	$m=8$	$\cdots$
(1,4)	(2,4)	(3,4)	(4,4)	(5,4)	(6,4)	(7,4)	(8,4)	
	(1,5)	**(2,5)**	**(3,5)**	**(4,5)**	**(5,5)**	**(6,5)**	**(7,5)**	
		(1,6)	(2,6)	(3,6)	(4,6)	(5,6)	(6,6)	
			(1,7)	**(2,7)**	**(3,7)**	**(4,7)**	**(5,7)**	$\cdots$
				(1,8)	(2,8)	(3,8)	(4,8)	
					(1,9)	(2,9)	**(3,9)**	
						(1,10)	(2,10)	
							(1,11)	

Figure 8: Tree with all relevant states of the world and the stopping set (in bold/underlined) for $u_P = u_X = 4$.

Figure 8 highlights the states of the world for which it is optimal for Platform P to start supporting compatibility. Since the probability of consumers choosing the outside option is negligible, the tree considers the part of the state space where in each period a consumer is added to either network. Through numerical simulations, all 5000 sample paths confirmed that a state was reached where P supports multihoming before the end of the planning horizon was reached. These paths can be categorized as follows,

$$
\begin{aligned}
26\% &: (1,4) \to (1,5) \to (1,6) \to \underline{(\mathbf{1},\mathbf{7})} \\
2.5\% &: (1,4) \to (1,5) \to (1,6) \to (2,6) \to \underline{(\mathbf{2},\mathbf{7})} \\
0.2\% &: (1,4) \to (1,5) \to (1,6) \to (2,6) \to (3,6) \to \underline{(\mathbf{3},\mathbf{7})} \\
48\% &: (1,4) \to (1,5) \to \underline{(\mathbf{2},\mathbf{5})} \\
19\% &: (1,4) \to (2,4) \to \underline{(\mathbf{2},\mathbf{5})} \\
2.8\% &: (1,4) \to (2,4) \to (3,4) \to \underline{(\mathbf{3},\mathbf{5})} \\
0.4\% &: (1,4) \to (2,4) \to (3,4) \to (4,4) \to \underline{(\mathbf{4},\mathbf{5})} \\
0.1\% &: (1,4) \to (2,4) \to (3,4) \to (4,4) \to (5,4) \to \underline{(\mathbf{5},\mathbf{5})} \\
0.02\% &: (1,4) \to (2,4) \to (3,4) \to (4,4) \to (5,4) \to (6,4) \to \underline{(\mathbf{6},\mathbf{5})} \\
0.02\% &: (1,4) \to (2,4) \to (3,4) \to (4,4) \to (5,4) \to (6,4) \to (7,4) \to \underline{(\mathbf{7},\mathbf{5})}.
\end{aligned}
$$

In order to gain further understanding, let us consider state (1,5) ($m = 2$). Because consumers act strategically, the next consumer is aware that if it chooses X, then we move from (1,5) to (2,5) and compatibility support is triggered. However, if it chooses P, we move from (1,5) to (1,6) and it needs to wait at least one more period before compatibility is supported. As such, the probability of the consumer choosing P is only 38.46%, whereas it would have been 50% if compatibility was already supported at (1,5). Thus, one would have expected P to support compatibility already at state (1,5). To understand why P chooses to delay compatibility support, we need to look at what happens at the next two consumers. If the first consumer chooses P (with probability 38.46%), then the next consumer chooses P with 90.57%: it would trigger compatibility support as (1,7) is now reached. Thus, the probability of attracting the next two consumers is 34.83% under singlehoming, while this probability would have been 25% under multihoming. Table 2 summarizes these probabilities. It also illustrates that under multihoming, in expectation, P gains 1 consumers, whereas under singlehoming it gains 1.04 consumers in expectation. In other words, singlehoming leads to a higher expected cash inflows from royalties and console sales and, hence, P is better off delaying support.

	Platform P wins			
	2 consumers	1 consumer	0 consumers	Average
Singlehoming	34.83%	34.40%	30.77%	1.04
Multihoming	25.00%	50.00%	25.00%	1.00

Table 2: Assume we are in state $(1,5)$ in period $m = 2$. This table displays the probability of attracting the next two arriving consumers if the firm (does not) support compatibility for $u_P = u_X = 4$.

State (1,5), in $m = 2$, is not part of the stopping set as a result of (1,7) being part of the stopping set. The natural question becomes why (1,7) is a part of the stopping set. In short, as can be observed from Figure 8, states (1,9) ($m = 6$), (2,9) ($m = 7$) et cetera are not part of the stopping set. Consumers are less likely to choose P in $m = 4$ if (1,7) were not part of the stopping set: consumers prefer multihoming to be trigger sooner rather than later. The reason behind (1,9), (2,9), and (1,11) not being part of the stopping set (and similarly other states further down the tree for later periods) is because at that stage P has obtained such a large network that consumers are very likely to choose P anyway, irrespectively of its compatibility stance. In addition, delaying compatibility has the additional advantage of receiving a higher income from royalties. However, although consumers are more likely to choose P in (1,11), they are not in (1,7), which provides Platform P the incentive to change its stance.

An interesting finding is that it is quite likely–almost 30% of the time–that the mutual switch to compatibility occurs when there is considerably larger asymmetry as regards the competitors' market shares as compared to the beginning of the dynamic game.

Result 4. *The switch to compatibility is likely to occur when there is more asymmetry in the market in terms of the competitors' market shares as compared to the beginning of the dynamic game.*

Impact of vertical differentiation For our baseline parameterization with $u_P = u_X = 4$, it takes, in expectation, 3.34 periods before compatibility is supported. When increasing Platform P's quality, we can distinguish two effects. First, the probability of consumers choosing P goes up in any state when u_P goes up: P's quality is higher and thus consumer utility goes up when choosing P over X, directly impacting ϕ_P. When considering, e.g., $u_P = 4.7$, the paths that include (1,6) are more likely:

$$\begin{aligned}
88.6\% &: (1,4) \to (1,5) \to (1,6) \to \underline{(\mathbf{1},\mathbf{7})} \\
0.7\% &: (1,4) \to (1,5) \to (1,6) \to (2,6) \to \underline{(\mathbf{2},\mathbf{7})} \\
8.6\% &: (1,4) \to (1,5) \to \underline{(\mathbf{2},\mathbf{5})} \\
2.1\% &: (1,4) \to (2,4) \to \underline{(\mathbf{2},\mathbf{5})} \\
0.04\% &: (1,4) \to (2,4) \to (3,4) \to \underline{(\mathbf{3},\mathbf{5})}
\end{aligned}$$

One can check that, for $u_P = 4.7$, the average number of periods until support has gone up to 4.01. Thus, an increase in u_P leads to a delay in compatibility.

For the baseline parameterization we observe that the stopping set as illustrated by Figure 8 is unchanged for u_P up to 4.7. However, when $u_P > 4.7$, a second effect can be distinguished. As a direct result of ϕ_P increasing as u_P goes up, for states where P has a considerable network size consumers are very unlikely to choose X: not only is P's network larger, P's platform is perceived to be of (much) higher quality as well. The stopping set then shrinks for states where P has a large network in later periods: it does not need compatibility to attract consumers. This is illustrated by Figure 9. In fact, the probability of consumers choosing P is higher when compatibility is not supported if the difference between b_P and b_X is sufficiently large. For the same reasons as argued above, for earlier states this makes consumers more likely to choose X, as in the other case it might take several periods before compatibility is supported. This forces Platform P to support compatibility early on, potentially as early as in $m = 2$.

$m=1$	$m=2$	$m=3$	$m=4$	$m=5$	$m=6$	$m=7$	$m=8$	$m=9$
(1,4)	(2,4)	(3,4)	(4,4)	(5,4)	(6,4)	(7,4)	(8,4)	(9,4)
	(1,5)	**(2,5)**	**(3,5)**	**(4,5)**	**(5,5)**	**(6,5)**	**(7,5)**	**(8,5)**
		(1,6)	(2,6)	(3,6)	(4,6)	(5,6)	(6,6)	(7,6)
			(1,7)	(2,7)	**(3,7)**	**(4,7)**	**(5,7)**	**(6,7)**
				(1,8)	(2,8)	(3,8)	(4,8)	(5,8)
					(1,9)	**(2,9)**	**(3,9)**	**(4,9)**
						(1,10)	(2,10)	(3,10)
							(1,11)	(2,11)
								(1,12)

Figure 9: $u_P = 4.75$ and $u_X = 4$.

A similar analysis can be done when considering the probabilities of P attracting the next two consumers when the current state of the world is (1,5) ($m = 2$). Table 3 illustrates that, in contrast to Table 2, P is now better off supporting compatibility. Thus, compatibility is supported very early on by Platform P with an expected 2.01 periods,

$$99.1\% : (1,4) \rightarrow \underline{(\mathbf{1},\mathbf{5})},$$
$$0.9\% : (1,4) \rightarrow (2,4) \rightarrow \underline{(\mathbf{2},\mathbf{5})}.$$

	Platform P wins			
	2 consumers	1 consumer	0 consumers	Average
Singlehoming	68.15%	30.02%	1.82%	1.66
Multihoming	89.62%	10.09%	0.28%	1.89

Table 3: Assume we are in state $(1,5)$ in period $m = 2$. This table displays the probability of attracting the next two arriving consumers if the firm (does not) support compatibility for $u_P = 4.75$ and $u_X = 4$.

Hence, for the cases where u_P is sufficiently high, the expected number of periods until both firms support compatibility drops. The jump can be attributed to a change in the stopping sets, while the smooth changes in switching times are due to smooth changes in probabilities.

Figure 10 illustrates the overall impact of vertical differentiation, $u_P - u_X$, on the moment compatibility is supported by both firms and their optimal prices. First we observe that qualitatively, the impact of u_P on prices is the same as in Figure 3. Second, in terms of the impact of u_P on timing, we get a non-monotonic relationship: while for low and medium degrees of vertical differentiation, firms delay switching as the quality difference rises, this is no longer true for a higher degree of differentiation when firms switch earlier.

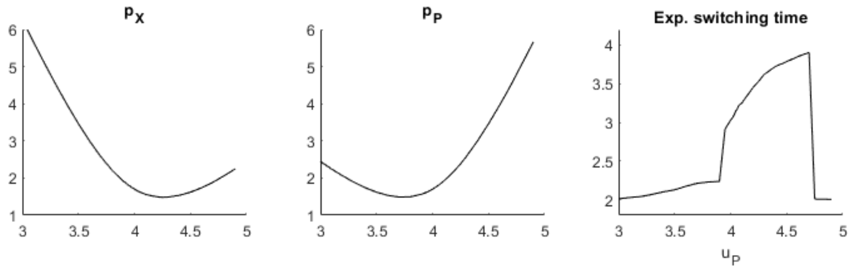


Figure 10: Impact of vertical differentiation.
$N = 40$, $b_X(t = 0) = 1$, $b_P(t = 0) = 3$, $p_E = 0.30$, $\kappa_1 = 0.80$, $\gamma = 0.30$, $u_X = 4$, $u_P = 4.5$, $r = 0.1$, $\eta = 2$, $T = 5 + b_X + b_P = 8$.

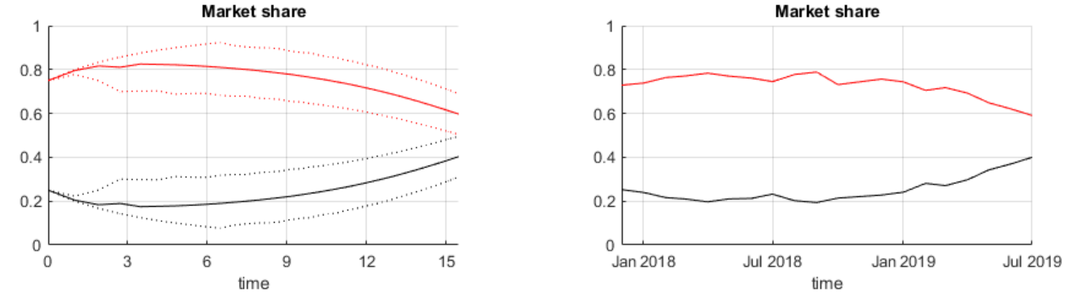
Result 5. *While the switching time to compatibility is increasing in the degree of quality difference for low and medium degrees of vertical differentiation, the relationship breaks down for large quality asymmetries when firms decide to switch early on.*

The initial reluctance from the dominant network to switch to compatibility, followed by the switch to network compatibility later in the dynamic game, is reminiscent of Sony's stance within the market for gaming consoles highlighted in the introduction. In fact, the dominant platform's switch to compatibility when the market is more asymmetric as compared to the beginning of the dynamic game and the gradual convergence to relative market share symmetry were observed in particular markets, such as the European market for gaming consoles as figure 11 highlights. More precisely, in November 2017 (when Fortnite was introduced in the market) Sony had 73% of the market compared to Microsoft's 25%. This asymmetry in market shares expanded further and reached its highest degree in September 2018–81% vs 20% as regards the competitors' market shares–when Sony announced its switch in compatibility strategy. Note that relative symmetry in market shares was restored a few months later in July 2019 (58% vs 42% in Sony and Microsoft market shares, respectively).

Result 6. *The model explains relative prices and market dynamics in the gaming industry in particular markets, such as that in Europe.*

Finally, without further illustration, our numerical explorations have revealed that all other qualitative results from Section 3.1 roll over when the compatibility choice is endogenized.

Result 7. *When firms endogenously choose whether or not to support compatibility alongside the timing of the switch to compatibility, whilst the high-quality firm always competes with the state of the art, the low-quality firm downgrades the quality of its provision for large and medium degrees*



(a) Expected market shares for P (red) and X (black). Dashed line represent 10% and 90% quantiles.

(b) Market shares for gaming consoles PlayStation 4 (red) and Xbox One (black) in Europe.

Figure 11: Market shares over time. Panel (a) is the result of 10,000 simulations. Panel (b) is based on a data set publicly available online.

of differentiation to the benefit of both competitors and to the detriment of consumers. Relative symmetry as regards firms' market shares is restored at the end of the dynamic game.

3.2.1 Cost differential

In the following discussion we relax the assumption of zero marginal costs of production for any platform quality and attribute differences in quality to production costs assuming that unit costs and marginal costs rise linearly with quality.[14] Therefore, we consider that $MC_P = c(u_P - u_X)$ where MC_P is the marginal cost facing seller P, c is a non-negative parameter and u_P, u_X are the competitors' qualities, whilst the marginal cost of seller X is normalized to zero.

Our qualitative results hold when incorporating differences in production costs. As Figure 12 highlights, the U-shaped relationship between the degree of vertical differentiation and firms' prices and value functions are retained. In fact, we find the same cut-off values in platform quality below which the two firms mutually benefit when the low-quality seller downgrades the quality of its product or service. Also, the shift of the system to supporting compatibility follows the exact same pattern. These results hold for various values of the non-negative parameter, $c \leq 0.4$.

[14] See, e.g., Mussa and Rosen (1978) for a seminal work on unit and marginal costs rising with product quality.

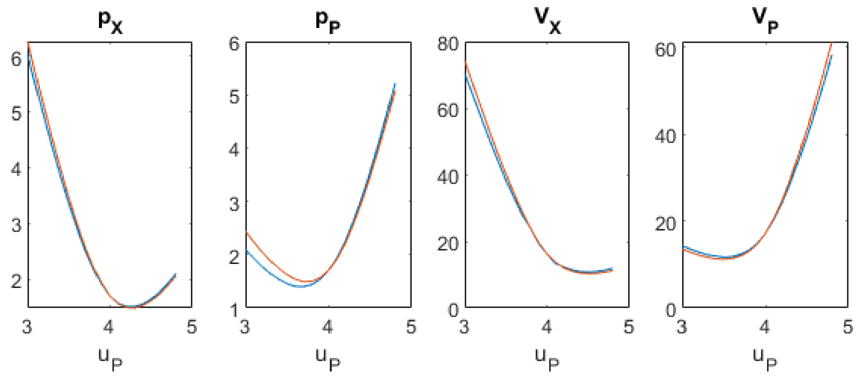


Figure 12: Relative prices and value functions when $c = 0$ (red) and $c = 0.4$ (blue).

3.2.2 Royalties

In this section, we relax the assumptions, according to which firms have income via royalties and incur foregone royalty income with compatibility.

We find that the expected switching time to compatibility alongside the dynamics observed when platforms enjoy royalties from application developers are also present in the absence of royalties when firms sell standalone products or services with network effects. Higher degrees of vertical differentiation usually lead to slower support of a single network from dominant players. This finding applies to industries, such as that related to mobile telecommunications, where asymmetries are often observed in terms of firms' installed bases and potential differences in the quality of service are present. We show that a dominant network has incentives to offer on-net shared discounts to its competitor's network, and this might happen when the asymmetry as regards firms' market shares is considerably high. Similarly, within banking services, the dominant platform might delay the switch to supporting a single network, allowing competitors' consumers to freely use its own ATM machines. In both industries, firms' optimal prices still feature a U-shaped relationship with the degree of vertical differentiation related to the competitors' offered product or service with the immediate undesirable implication that the low-quality seller might downgrade its quality provision to the detriment of consumers and industry surplus in the absence of regulation.

Our simulations showcase that lower royalties–via a lower in-game expenditure, p_E, are associated with higher platform prices. This is expected as platforms are able to recoup losses in one side of the market with a higher price on the other side. Similarly, when the fraction of consumer expenditure on alternative platforms with compatible networks, such as mobile phones, is low, both platforms switch to compatibility early on within the dynamic game, and average platform prices fall. This is a result that holds if compatibility does not lead to foregone royalty income and might be expected given that platforms benefit from compatibility both via royalties and potentially relaxed competition.

3.3 Quality Standards

The fact that for medium and large degrees of vertical differentiation, the low-quality firm downgrades its quality provision to the benefit of the two competitors holds regardless of (a) whether quality differences are attributable to differences in costs or not and (b) the existence of complementary applications that could yield extra revenues via royalty payments for the sellers.

Therefore, this result encompasses one-sided and two-sided markets and makes the discussion aon the potential establishment of quality standards and cutoffs only above which competitors may participate within the market relevant. This would create a twofold benefit for consumers: both the average quality within the market would be higher, and the average price would fall as a result of the standard, provided there is no monopoly arising as a direct result of firms failing to reach the cutoff. Industry surplus would also be higher as a result of the standard, whilst firms would incur losses via a decrease in their profits.

Result 8. *A Minimum Quality Standard that would set a cutoff above which firms can sell their products would lead to a higher average quality supplied in the market, and higher consumer and industry surplus.*

4 Market Growth

In this section we will consider two analyses. In the first analysis, the impact of changes in T is discussed. In the second analysis, the impact of the assumption that changes in the market growth rate over time is studies by assuming a fixed interarrival time. This section is to be written (TBC).

5 Further Analyses

5.1 Consumer Rationality vs Myopia

Section 3.2 illustrated the states where switching to multihoming is optimal under the assumption of consumers being forward-looking. This lead to the result that consumers may act strategically and choose one platform over the other to increase the likelihood that consumers may soon benefit from multihoming and thus larger network effects. If, instead, we assume that consumers act myopically, i.e., they assume that the compatibility stance and the network sizes remain the same in the future, then a typical stopping set looks as in Figure 13.

$m = 1$	$m = 2$	$m = 3$	$m = 4$	$m = 5$	$m = 6$	$m = 7$	$m = 8$	$\cdots$
(1,4)	(2,4)	(3,4)	(4,4)	(5,4)	(6,4)	(7,4)	(8,4)	
	(1,5)	**(2,5)**	**(3,5)**	**(4,5)**	**(5,5)**	**(6,5)**	**(7,5)**	
		(1,6)	(2,6)	(3,6)	(4,6)	(5,6)	(6,6)	
			(1,7)	**(2,7)**	**(3,7)**	**(4,7)**	**(5,7)**	$\cdots$
				(1,8)	(2,8)	(3,8)	(4,8)	
					(1,9)	**(2,9)**	**(3,9)**	
						(1,10)	(2,10)	
							(1,11)	

Figure 13: Tree with all relevant states of the world and the stopping set (in bold/underlined) when consumers are myopic.

The main difference between Figure 13 and the equivalent trees in Section 3.2 is that the stopping sets are extended. Because consumers are no longer forward looking, platform P can no longer induce consumers choosing P through strategically delaying multihoming. Thus, the dynamic considerations and strategic interactions as analyzed before no longer apply and, hence, P loses the incentive to delay multihoming in the states with b_X being small.

This finding highlights that consumer rationality is a novel feature in our setup. In other words, platforms are only willing to delay support for compatibility if consumers are forward-looking. The immediate support of compatibility from asymmetric firms is a new result in the literature investigating the initial conditions under which firms decide to support compatibility and complements existing work, according to which firms support compatibility if they are (close to) symmetric in terms of their installed bases and other traits (Chen *et al.*, 2009).

6 Conclusions

In this paper, we build a model of dynamic competition of vertically differentiated firms in growing markets with network effects and strategic consumers that provides new insights as regards firm optimal pricing and compatibility incentives. Our results encompass a number of industries, where sellers might either compete with standalone products or with platforms that realize revenues on both sides of the market, as is the case in gaming consoles.

We find that dominant market players, who are initially reluctant to support compatibility, change their compatibility stance when there is more asymmetry in the market, whilst relative symmetry–as regards firms' market shares–is restored at the end of the dynamic game. We posit that forward-looking consumers lead dominant market players to switch to compatibility when the market is very asymmetric. In this respect, we explain relative platform prices and Sony's conduct within the market for gaming consoles highlighted in the introduction alongside market dynamics in regions, such as that in the UK, where although the market initially got more asymmetric while Sony was reluctant to change in compatibility stance, relative symmetry was restored within this industry after the tech giant's switch in compatibility stance.

From a policy perspective, since a low-quality firm might find it beneficial to downgrade the quality supplied in the market with negative implications for consumers and total industry surplus, intervention in terms of a quality standard would guarantee that firms compete with the highest quality available to them. This would both benefit consumers and increase total industry surplus–only harming individual firms' profits.

Our model can be easily generalized to include situations where the multihoming decision relates to both sides of the market, for example when a game developer might have a choice to set an exclusive deal with a particular platform. Also, the model can be modified to analyze platform competition in situations when platform owners may not charge a membership fee for the platform on the consumers' side of the market. Instead, platforms might earn revenues from transactions between a consumer and a seller. This is reminiscent of competition between platforms, such as Uber and Bolt (in the U.S. the relevant application is Lyft) in two-sided asymmetric markets in terms of platform network sizes with potential differences in the quality of service supplied, perhaps due to discrepancies in waiting times facing consumers. In such a situation, whilst the platform prices p_X, p_P are zero, the platform owners endogenously choose the price for a ride consumers pay, p_E, that might differ across platforms and the transaction fee (κ_0, κ_1) to drivers, aiming to maximize their expected profits. The intricacy of this market is that a fraction of new and existing consumers *can choose* to multihome by joining both platforms–and picking the one that suits them–and platforms might choose to allow drivers to multihome by introducing flexible contracts.

References

Budd, C., C. Harris, and J. Vickers (1993). A model of the evolution of duopoly: Does the asymmetry between firms tend to increase or decrease? *The Review of Economic Studies*, 60, 543–573.

Cabral, L. (2000). Increasing dominance with no efficiency effect.

Cabral, L. (2011). Dynamic Price Competition with Network Effects. *The Review of Economic Studies*, 78, 83–111.

Cabral, L. and D. Salant (2014). Evolving Technologies and Standards Regulation. *International Journal of Industrial Organization*, 36, 48–56.

Chen, J. (2018). Switching costs and network compatibility. *International Journal of Industrial Organization*, 58, 1–30.

Chen, J., U. Doraszelski, and J. E. Harrington, Jr. (2009). Avoiding market dominance: Product compatibility in markets with network effects. *The RAND Journal of Economics*, 40, 455–485.

Corts, K. S. and M. Lederman (2009). Software exclusivity and the scope of indirect network effects in the u.s. home video game market. *International Journal of Industrial Organization*, 27, 121–136.

Doganoglu, T. (2003). Dynamic price competition with consumption externalities. *Netnomics*, 5, 43–69.

Driskill, R. (2007). Monopoly and oligopoly supply of a good with dynamic network externalities. *Vanderbilt University*, 47.

Gilbert, R. J. and D. M. Newbery (1982). Preemptive patenting and the persistence of monopoly. *The American Economic Review*, 514–526.

Katz, M. L. and C. Shapiro (1986). Product compatibility choice in a market with technological progress. *Oxford Economic Papers*, 38, 146–165.

Maddala, G. (1983). *Limited-Dependent and Qualitative Variables in Econometrics*. Cambridge University Press, Cambridge.

Malueg, D. A. and M. Schwartz (2006). Compatibility incentives of a large network facing multiple rivals. *The Journal of Industrial Economics*, 54, 527–567.

Markovich, S. (2004). Snowball: A dynamic oligopoly model with network externalities", telaviv university. *Technical report*, mimeo.

Markovich, S. and J. Moenius (2009). Winning while losing: Competition dynamics in the presence of indirect network effects. *International Journal of Industrial Organization*, 27, 346–357.

Mussa, M. and S. Rosen (1978). Monopoly and product quality. *Journal of Economic Theory*, 18, 301–317.

Pakes, A. and P. McGuire (1992). Computing markov perfect nash equilibria: Numerical implications of a dynamic differentiated product model.

Pakes, A. and P. McGuire (2001). Stochastic algorithms, symmetric markov perfect equilibrium, and the 'curse'of dimensionality. *Econometrica*, 69, 1261–1281.

Reinganum, J. F. (1983). Uncertain innovation and the persistence of monopoly. *The American Economic Review*, 73, 741–748.

Ronnen, U. (1991). Minimum quality standards, fixed costs, and competition. *The RAND Journal of economics*, 490–504.

Zhu, F. and M. Iansiti (2007). *Dynamics of platform competition: Exploring the role of installed base, platform quality and consumer expectations*. Division of Research, Harvard Business School.

A Optimal response curves

Because the firms engage in pricing in a differentiated Bertrand set-up, optimal prices are found by first setting up the optimal response curves, as illustrated in the second panel of Figure 14. Clearly, there is a unique intersection. The first panel illustrates that the optimal p_P, for a given p_X, is uniquely determined as the interior solution.

B Switching modes

As mentioned in the main text, for this analysis, we will assume that Platform P attracts the first consumer and, thus, X switches immediately.

As strategies are Markovian and as the game is solved backwards, let us start at the end. Starting after the arrival of (potential) consumer $m = 40$, no more consumers will arrive and there

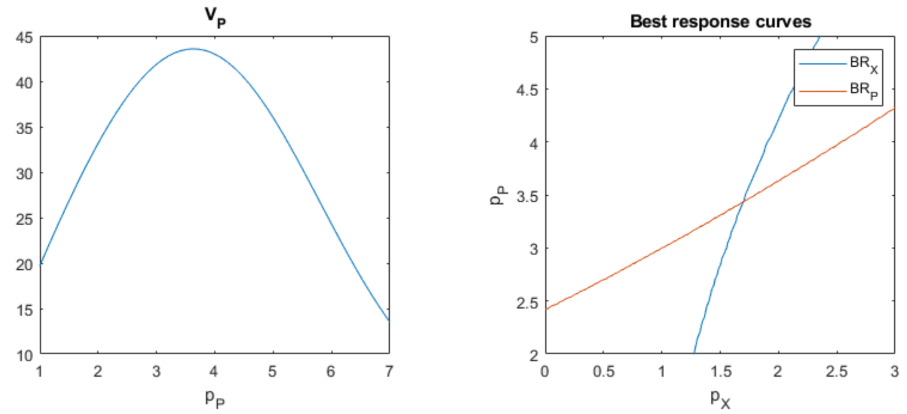


Figure 14: Figures illustrating objective function and best-response curves for optimal pricing.

is only one period left until the end of the horizon. Because $\tilde{v}_i^{MH} < \tilde{v}_i^{SH}$, i.e., because income from royalties is lower under multihoming, clearly, no firm has an incentive to offer support if it is not yet offering support. Next, after the arrival of (potential) consumer $m = 39$, only one more consumer will arrive. Assuming, without loss of generality, that no consumer chose the outside option, then, P has the largest network if $b_P(m) > 21$ (note that $b_X(m) = 43 - b_P(m)$). If $b_P(m) > 21$, P again has no incentive to start offering support. However, if $b_P(m) \leq 21$, then X has a larger network which leads to a high probability of the final consumer choosing X. In that scenario, switching, and thereby increasing the probability of the next consumer choosing P, may outweigh the loss from income in royalties. Our numerical explorations confirm this finding: P switches when $b_P(m = 39) \leq 21$, whereas compatibility will not be supported if $b_P(m = 39) > 21$. Figure 15 illustrates this part of the stopping set by marking the states in a tree.

Next, let us go back one more step to $m = 38$. Consider $b_P = b_X = 21$. Then, if the new consumer chooses P, we move from (21,21) to (21,22) and compatibility will not be supported. If the new consumer, however, chooses X, we move from (21,21) to (22,21) and compatibility will be supported. Thus, the consumer is highly likely to choose X. The only way for P to increase the odds to attract the consumer, is by offering compatibility already in state (21,21) in $m = 38$. This argument also applies for states with $b_P = 21$ for $m < 38$, as illustrated in Figure 15. Shifting our attention to states where $b_P = 20$, an equivalent argument can be made: compatibility will only be supported if the consumer chooses P over X. Thus, P has an incentive not to change its stance (see states $(b_X, 20)$ for periods $M \leq 38$ in Figure 15). For other states in $m = 38$, we find that compatibility is not supported. Since compatibility will be supported from the next period, consumer choice is hardly impacted and the platform owner enjoys one final period with higher income from royalties.[15] Finally, we see similar L-shaped regions for $M \leq 37$. The intuition above applies to these states as well. Nonetheless, for any m, if $b_P > 21$, then compatibility will never be supported.

$m = 34$	$m = 35$	$m = 36$	$m = 37$	$m = 38$	$m = 39$	$m = 40$
(24,14)	**(25,14)**	(26,14)	**(27,14)**	(28,14)	**(29,14)**	(30,14)
(23,15)	**(24,15)**	(25,15)	**(26,15)**	(27,15)	**(28,15)**	(29,15)
(22,16)	**(23,16)**	(24,16)	**(25,16)**	(26,16)	**(27,16)**	(28,16)
(21,17)	**(22,17)**	(23,17)	**(24,17)**	(25,17)	**(26,17)**	(27,17)
(20,18)	(21,18)	(22,18)	**(23,18)**	(24,18)	**(25,18)**	(26,18)
(19,19)	**(20,19)**	**(21,19)**	**(22,19)**	(23,19)	**(24,19)**	(25,19)
(18,20)	(19,20)	(20,20)	(21,20)	(22,20)	**(23,20)**	(24,20)
(17,21)	**(18,21)**	**(19,21)**	**(20,21)**	**(21,21)**	**(22,21)**	(23,21)
(16,22)	(17,22)	(18,22)	(19,22)	(20,22)	(21,22)	(22,22)
(15,23)	(16,23)	(17,23)	(18,23)	(19,23)	(20,23)	(21,23)
(14,24)	(15,24)	(16,24)	(17,24)	(18,24)	(19,24)	(20,24)
(13,25)	(14,25)	(15,25)	(16,25)	(17,25)	(18,25)	(19,25)

Figure 15: States (b_X, b_P)

C Auxiliary Figures

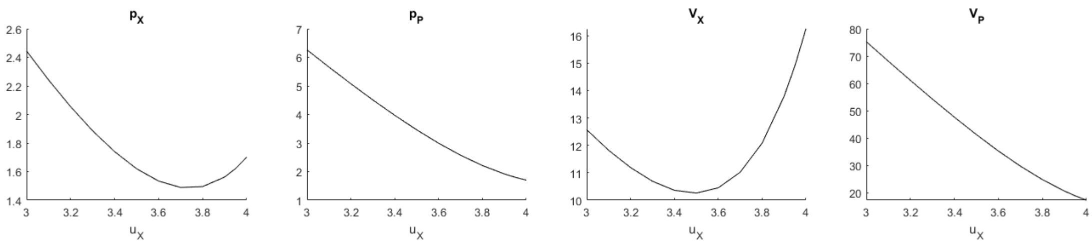


Figure 16: Impact of vertical differentiation.
$N = 40$, $b_X(t = 0) = 1$, $b_P(t = 0) = 3$, $p_E = 0.30$, $\kappa_1 = 0.80$, $\gamma = 0.30$, $u_P = 4$, $r = 0.1$, $\eta = 2$, $T = 5 + b_X + b_P = 8$. This figure illustrates that when u_X is sufficiently small both firms see their values increase when platform X downgrades its quality.

[15] Our analysis in Section 3.2.2 confirms that indeed the platform owner is indifferent when there are no gains or losses from royalties.

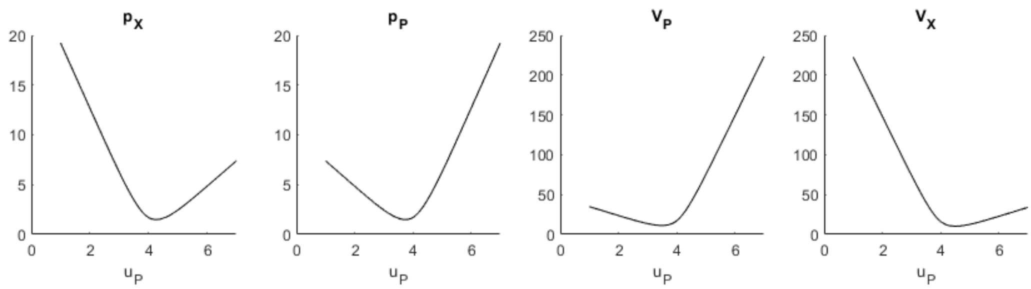


Figure 17: Impact of vertical differentiation.
$N = 40$, $b_X(t=0) = 1$, $b_P(t=0) = 3$, $p_E = 0.30$, $\kappa_1 = 0.80$, $\gamma = 0.30$, $u_X = 4$, $u_P = 4.5$, $r = 0.1$, $\eta = 2$, $T = 5 + b_X + b_P = 8$.