The Effects of Monetary Policy Shocks on Risk

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Abstract

I use a combination of a structural dynamic factor model and quantile regressions to study how monetary policy affects the predicted distributions of GDP growth and inflation in the US. The non-linear methodology leads to the following new findings: first, the conditional distributions of both GDP growth and inflation display multimodality as the norm rather than the exception. Second, contractionary monetary policy shocks shift the expected distribution of GDP growth to the left and deepen its two humps. The expected distribution of inflation is marginally shifted left as well, but importantly, the second mode related to high inflation is exacerbated. Expansionary policy almost resolves the bimodality of GDP growth in favor of the "good" equilibrium and decreases the spread of the inflation distribution.

Keywords: Monetary Policy, Quantile Regression, Dynamic Factor Models, Growth-at-Risk, Inflation-at-Risk
JEL classification: C3, E31, E52

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For a very helpful discussion I thank Fabio Cesare Bagliano. I am grateful to Luca Gambetti and participants of the Bellaterra Macro Club, the Vilfredo Pareto PhD Workshop in Turin, and the BSE PhD Jamboree for very helpful comments and suggestions. I acknowledge funding through grant FPI PRE2020-093941.

1 Introduction

Central bankers need to assess the risks of extreme events, such as very poor (or very good) expected economic performances and price rises or falls to conduct policy.[2] For example, after the Great Recession, special focus has been dedicated to monitoring financial market conditions to expose potential threats to economic stability coming from the banking and financial services sector. As a consequence, the academic literature on growth-at-risk (Adrian et al., 2019) and inflation-at-risk (Lopez-Salido & Loria, 2024) has expanded significantly. This literature studies how the tails of the expected distributions of GDP growth and inflation changes over time. In this paper, I look at the question whether adjustments in the monetary policy stance of the Federal Reserve can combat the buildup of risks of unusually bad economic outcomes. To do this, I study the response of the entire conditional distributions of GDP growth and inflation to monetary policy shocks in the US, using a combination of a structural dynamic factor model (DFM) and quantile regressions.

A significant hurdle in the x-at-risk literature is the selection of the set of variables that constitute the information set which produces the expected distribution of macroeconomic variables of interest. Instead of relying on hand-picking or statistical variable selection I resolve this issue by simply using a set of factors extracted from the FRED-QD data base. These capture information on industrial production, employment, financial markets and other segments of macroeconomic importance and arguable constitute a comprehensive information set to be used to compute conditional distributions. Equipped with the factors, I rely on the quantile regression methodology of Adrian et al. (2019) and fit individual quantiles of the annual percentage change of GDP and of the PCE price index. Obtaining the impulse responses (IRFs) of the individual quantiles and of the full distribution is achieved using a version of the methodology of Forni et al. (2021) and Adrian et al. (2021).

I document the following new findings: contrary to Adrian et al. (2019), the left tail of expected GDP growth is not significantly more volatile than the right tail, suggesting that variable selection matters strongly for their result. Moreover, multimodality is the norm rather than the exception for GDP growth and strongly present for inflation, although in the latter case it is more a phenomenon around recessions. Contractionary monetary policy shocks shift the expected GDP distribution to the left, in line with standard findings and emphasizes the two humps in the steady state distribution. Expected inflation is reduced, but a significant probability mass associated with unchanged or even increased inflation after the monetary policy shock persists. Therefore, the inflation distribution spreads out. The Federal Reserve induces changes in the economy which reduce inflation on average but is unable to control tail risks. Expansionary monetary policy briefly shifts the probability mass into the "good" mode of GDP growth and reduces the spread of the inflation distribution.

This paper adds to the literature on growth-at-risk and inflation-at-risk. Much of the existing research in this area has focused on determining the best set of conditioning variables to accurately describe the expected distributions of inflation and growth, for the purpose of in-sample analysis (see Giglio et al. (2016), Adrian et al. (2019), Adrian

[2] Minutes of FOMC meeting on 28/29 January 2020 p.9: link.

et al. (2022), and Lopez-Salido & Loria (2024)) or out-of-sample now-/forecasting (see Ghysels et al. (2018), Plagborg-Møller et al. (2020), Adams et al. (2021), Carriero et al. (2022)). Some important contributions for causal analysis of macro shocks include Duprey & Ueberfeldt (2020), Boire et al. (2021), Forni et al. (2021), and Loria et al. (2023). Given the focus on identifying causal effects in this paper, I opt for a factor approach as this has been shown to resolve important empirical puzzles in causal analysis of monetary policy (see Forni & Gambetti (2010) and Kerssenfischer (2019)) and captures a large set of information that can be used consistently for the prediction of the conditional quantiles of a large number of time series, without having to manually adjust the information set each time. Factors in quantile regressions have been studies by Ando & Tsay (2011) and can produce good forecasting results for the mean (J. Stock & Watson, 2002) and reasonable out-of-sample results for conditional quantiles (Carriero et al., 2022).

Multimodality of the conditional distribution of GDP growth has been shown in Adrian et al. (2021) and Forni et al. (2021). It appears to arise during recessionary periods more than in normal times, an observation echoed in this paper as well. Adrian et al. (2021) discuss how poor financial conditions induce the multimodality and conjecture that bad policy decisions can lead to persistent selection of the worse equilibrium. This does not seem to hold for monetary policy which can actually select the "good" equilibrium if it is expansionary, but does not select the bad one if it is contractionary. The authors' research differs from this paper in the sense that I study the effect of monetary policy shocks on the predicted distributions of GDP growth and inflation, while they study the joint behavior of economic and financial conditions with no particular focus on monetary policy. Forni et al. (2021) on the other hand establish the methodological basis for this paper but focus on how changes in the tails of the predicted distributions affect real and financial outcomes. The multimodality in their paper obtains again in recessionary periods, although the set of predictors and their estimation procedure differ.

In a paper closely related to this one, Boire et al. (2021) use a QR-SVAR to study the response of the GDP growth distribution in six developed countries to monetary and fiscal policy shocks. They find mostly location effects for monetary policy and shape effects for fiscal shocks. In this paper, on the other hand, my approach suggests shape effects to be present in the short run. In another study close to this one, Loria et al. (2023) analyze how different macroeconomic shocks – monetary policy included – affect the conditional quantiles of GDP growth using the quantile regression model of Adrian et al. (2019) as a case study. They find that monetary policy drives the conditional distribution of forecasted GDP growth to the left and that the effect on the 5^{th} quantile is more negative than the effect on the 95^{th} quantile. Their study differs from this paper in the following ways: first, they use the conditional quantiles of Adrian et al. (2019), whereas I construct new forecasted quantiles using a factor approach. Second, they use local projections to gauge the effect of the monetary policy shock, whereas I use a combination of quantile regression and a dynamic factor model. Lopez-Salido & Loria (2024) study inflation-at-risk for a panel of OECD countries using a Phillip's Curve predictive setup. They find that inflation expectations have been the key predictor of inflation-at-risk over the past 20 years and that asymmetry in the expected distribution of inflation is induced by financial variables. I take their observations as an important motivation for including factors in my setup that summarize information in forward looking variables. Third, I construct quantile forecasts also for inflation and study their reaction to the monetary policy shock. Such forecasts are constructed also in Adams et al. (2021) using financial conditions and forecast errors from the Survey of Professional Forecasters (SPF) as predictors instead of the factor approach used in this paper. The findings in Loria et al. (2023) are broadly in line with those of this paper, in the sense that the forecasted distribution of GDP growth moves to the left in response to a monetary contraction. This research is also related to the literature on the effects of monetary policy on different measures of uncertainty. For instance, Bekaert et al. (2013) study the effect of monetary policy shocks on a "risk aversion" and an "uncertainty" component of the Chicago Board Options Exchange (CBOE) volatility index (VIX). They find that expansionary monetary policy shocks lead to a decreases in both risk aversion and uncertainty, pointing to a risk-taking channel for monetary policy.

The rest of this paper is organsied as follows: section 2 presents how the DFM is combined with quantile regressions to obtain quantile IRFs. Section 3 describes the data set, transformations, and model specification. Section 4 presents the results. Section 5 offers a discussion of the results and section 6 concludes.

2 Econometric Approach

The main relationship that I use in this paper is an extension of the QR-SVAR of Forni et al. (2021) and is given by

$$Q_{t+h,\tau} = \beta_\tau(L)Wf_t = \beta_\tau(L)WH(L)u_t \tag{1}$$

$Q_{t+h,\tau}$ stands for the h period ahead prediction of the conditional quantile of a variable of interest. In this paper, the variables of interest are GDP growth and inflation. $\beta_\tau(L)$ is a polynomial in the lag-operator L of the coefficients $\beta\tau$ obtained from a quantile regression of the variable of interest on a set of predictors, selected by the matrix W. The set of predictors is made up of the static factors f_t that are extracted from a large macroeconomic data set. The factors are assumed to follow an invertible VAR relationship and therefore, can be cast into the corresponding moving average (MA) representation. The structural MA representation is then given by $H(L)u_t$, where $H(L)$ are the structural impulse responses and u_t are the structural shocks. The monetary policy shock is identified using the dynamic factor model approach of Forni & Gambetti (2010). This relationship exploits the high information content in the static factors as potentially useful predictors for the conditional quantiles of the variables of interest and simultaneously makes use of large information approach to the identification of monetary policy shocks that has been shown to perform similarly to other accepted methods such as high-frequency identification (Kerssenfischer, 2019).

In what follows I explain in more detail the two main relationships – quantile regression and factor model – their specification, treatment and identification of the structural shocks.

2.1 The Dynamic Factor Model

The methodology used to estimate factors and identify the structural shocks is based on the dynamic factor model of Forni & Gambetti (2010). Given a large data set of macroeconomic variables with cross sectional dimension N and time span T the following relationship is assumed to be present in the set:

$$x_{it} = \chi_{it} + \xi_{it} \tag{2}$$

Here, x_{it} is one of the N variables in the data set at time t. χ_{it} is the common component and ξ_{it} is the idiosyncratic component. Common and idiosyncratic components are assumed to be orthogonal. The idiosyncratic components of different variables are allowed to be mildly correlated. The latter implies that the factor model is approximate and note exact (Bai & Ng, 2002). In addition, I assume that the common component χ_{it} is driven by r common factors f_t. The importance each of these r factors has for the common component is captured by a vector of loadings a_i. Hence,

$$\chi_{it} = a_i f_t \tag{3}$$

Next, I assume that the common factors follow a VAR which is assumed to be invertible and given by

$$D(L)f_t = \epsilon_t \tag{4}$$

with $D(L)$ a matrix polynomial of coefficients in the lag operator L and ϵ_t a vector of zero-mean reduced form shocks. As in Forni & Gambetti (2010), the number of structural shocks u_t driving the common factors f_t need not be equal to the number of factors r. Instead, it is possible that the number of deep shocks q is smaller than the number of factors, i.e. $q \leq r$. This is captured by a rotation R such that $\epsilon_t = Ru_t$.

$$D(L)f_t = Ru_t \tag{5}$$

The vector u_t is assumed to be vector white noise and holds the deep shocks in the economy. One of these shocks is assumed to be a monetary policy shock which can be identified in the following way.

2.2 Identification of the Monetary Policy Shock

Beginning from equation (5), invertibility provides that the factors can be written as

$$f_t = D(L)^{-1}Ru_t \tag{6}$$

For any orthogonal matrix H equation (6) can be rewritten as

$$f_t = D(L)^{-1}RH'Hu_t = D(L)^{-1}SHu_t = S(L)Hu_t = H(L)u_t \tag{7}$$

which is the relation used in model (1) with $S = RH'$, $S(L) = D(L)^{-1}S$ and $H(L) = S(L)H$. Similar to the SVAR literature, the model is not identified without further assumptions on the matrix H. The objective is to identify a monetary policy shock in the same way that traditional recursive SVARs, as for example, Christiano et al. (1999) do – by imposing that the monetary policy shock does not contemporaneously affect slow moving variables such as GDP or prices, but can contemporaneously affect financial variables and interest rates. However, it is not possible to impose such restrictions on the factors f_t directly as they lack a concrete interpretation. Hence, I impose the restriction with a view to the common component χ_{it} of the macro panel instead.

Using equations (7) and (3) we obtain the structural IRFs of the common components $b_i(L)$ from

$$\chi_{it} = a_i D(L)^{-1} R H' H u_t = a_i S(L) v_t = c_i(L) v_t = c_i(L) H u_t = b_i(L) u_t \tag{8}$$

where $c_i(L) = a_i S(L)$ and $b_i(L) = c_i(L)H$. Therefore, the IRFs in $b_i(L)$ have to fulfill the identifying assumptions. In the Cholesky case preferred in this paper, this is achieved by selecting exactly $m = q$ variables from x_{it} and ordering them according to the assumed recursive structure. Since,

$$B_m(L) = C_m(L) H \tag{9}$$

the restrictions are imposed contemporaneously by setting

$$H = C_m(0)^{-1} G_m \tag{10}$$

where $C_m(0)^{-1}$ is the inverse of the non-structural IRF on impact and G_m is the lower triangular Cholesky factor of $C_m(0)C_m(0)'$.

2.3 Quantile Regressions

The second step consists in running a predictive regression of the variables of interest – GDP growth and inflation – on the respective sets of predictors, the factors in model (1). While OLS regressions are concerned with the mean response of the dependent variable to changes in different explanatory variables, quantile regression (Koenker & Bassett Jr, 1978) is concerned with the median and other quantiles of the distribution of the dependent variable. In that sense, it is more robust to potential outliers that may distort the mean and, in addition, allows the study of the drivers of more extreme outcomes of GDP growth and inflation, such as $\tau = 0.05, 0.1, 0.9, 0.95$, which are the common quantiles studied in the literature on "growth-at-risk".

Koenker (2005) shows that $\hat{\beta}\tau$ solves the minimization problem of the conditional sample quantile function $Q_{t,\tau}$ as in

$$\min_{\beta \in \mathbb{R}^p} \sum_{i=1}^{n} \rho_\tau (y_i - x_i' \beta) \tag{11}$$

where p is the number of regressors, ρ is the loss function $\rho_\tau(u) = u(\tau - \mathbb{1}(u < 0))$ with $u_i = y_i - x_i'\beta$ and y_i being the dependent variable. This is a linear-programming problem that can be solved using simplex methods. Hence, the quantile regression that I run is given by

$$Q_{t+h,\tau} = \beta_\tau(L) W f_t \tag{12}$$

Throughout the paper, the generic issue of “quantile-crossing” is dealt with using the methodology of Chernozhukov et al. (2010).

2.4 Combining Quantile Regression and Dynamic Factor Model

From the estimated coefficients $\widehat{\beta_\tau(L)}$ and given the selection of regressors Wf_t I can pin down the impulse response functions of the conditional quantiles of GDP growth and inflation for a monetary policy shock. This allows an assessment of whether the uncertainty around future economic conditions is reduced, increased or not affected by policy action. Moreover, since I study the three quantiles individually it may be that downside risk (as measured by Q05) is affected differently than upside risk (as measured by Q95) by the shock.

The IRFs are given by

$$IRF_\tau^{GDP} = \widehat{\beta_\tau^{GDP}(L)} W_{GDP} H(L) \tag{13}$$

$$IRF_\tau^{\pi} = \widehat{\beta_\tau^{\pi}(L)} W_{\pi} H(L) \tag{14}$$

Since I include all factors as regressors, the matrices W are identity matrices in both cases. To repeat, the matrix polynomial $H(L)$ is given by $D(L)^{-1}SH$ as described in equation (7). The above expressions can also be used to carry out an analysis of the forecast error variance of the quantiles and decompose the conditional time series for each quantile into the contributions coming from the monetary policy shock and other, undetermined factors.

3 Data and Model Specification

The data set for the factor model is the collection of US quarterly macroeconomic variables of McCracken & Ng (2020), which is gathered by FRED of the Federal Reserve Bank of St. Louis. The data set contains 248 variables from Q1:1959 to Q4:2023 and is built on top of the data set used in J. Stock & Watson (2012). Variables are transformed to induce stationarity. Price series are kept in single differences since the quantile regression outcome for inflation-at-risk is also single differenced. The appendix reports all variables that are included, their transformations and the variation explained by the static factors.

The data set contains missing values and outliers. Outliers are problematic for principal components analysis and are dealt with as suggested in McCracken & Ng (2020) by omitting values whose difference from the median is larger than ten times the interquartile range. These outliers are replaced by missing values. To deal with the missing values I use the expectation maximization algorithm of McCracken & Ng (2020). I tweak their procedure by using the selection criterion of Alessi et al. (2010) which is a refined version of the criterion of Bai & Ng (2002) to determine the optimal number of factors at each iteration. The optimal number of static factors to include in f_t is determined to be between $r = 8$ and $r = 15$. For parsimony, I prefer the lower of the two as my benchmark and check robustness of the results against the upper bound. Factors are estimated using the principal component estimator.

In a second step, I determine the lag length to be used in the VAR for the factors using the Hannan-Quinn, Schwarz and Akaike information criteria. These suggest $p = 1$ (HQ, SC) and $p = 3$ (AIC). Again, for parsimony, I choose $p = 1$ and check robustness of the results for $p = 3$. Finally, the number of dynamic factors or primitive shocks is determined using the criterion of Bai & Ng (2007), which suggests to set $q = 4$ when $r = 8$. For the purpose of identification $m = q = 4$ identifying variables are required. I use the variables for real GDP (GDPC1), the PCE price index (PCECTPI), the Federal Funds Rate (FEDFUNDS), replaced by the shadow rate (Wu & Xia, 2016) between Q4:2008 and Q1:2022, and the USD-CHF exchange rate (EXSZUS), in that order. This setup is similar to Forni & Gambetti (2010) and implies that the monetary policy shock is the third one in u_t. It does not drive GDP or prices on impact, but can contemporaneously influence the Federal Funds Rate and the exchange rate.

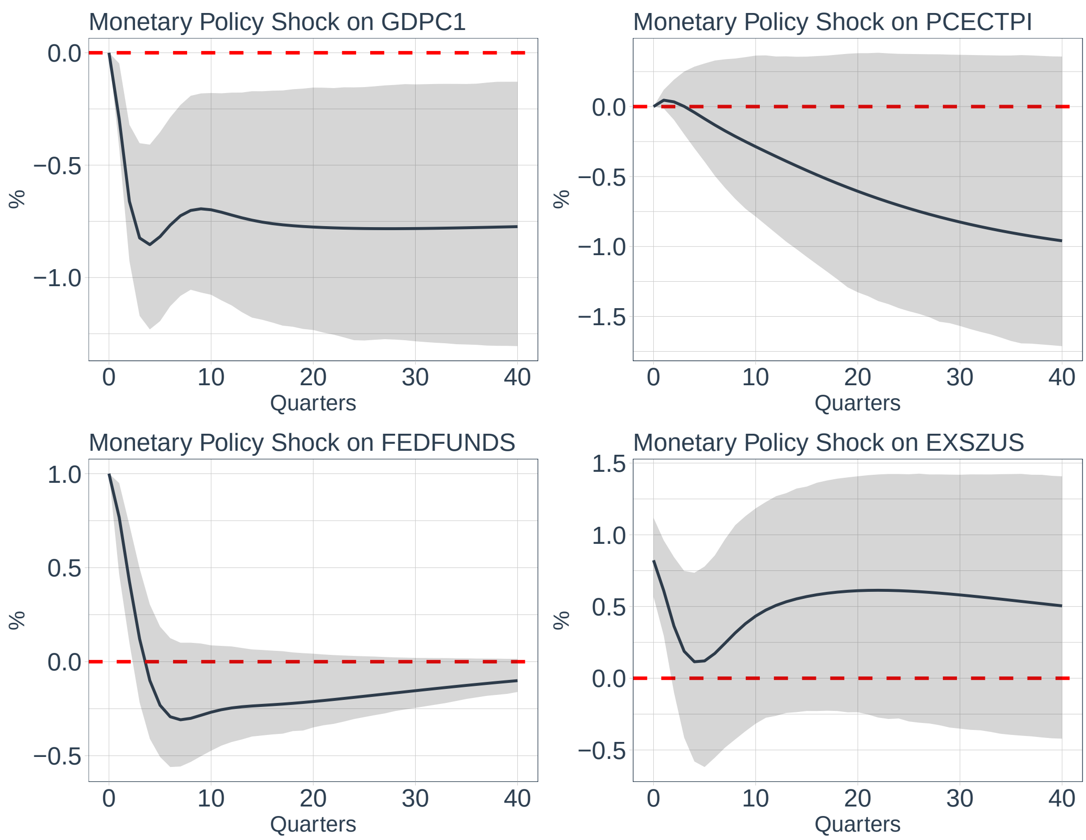
In the quantile regression exercise I follow Adrian et al. (2019) and set $\tau = 0.05, 0.5, 0.95$ to capture the tail behavior of the conditional distribution of the dependent variables real GDP growth (GDPC1) and PCE inflation (PCECTPI). Four quarter ahead growth rates ($h = 4$) are computed as the difference of the logs between t and $t - 4$ and multiplied by 100. The median is useful for comparison to the mean and to compute descriptive statistics such as the Kelley skewness of the conditional distributions implied by the estimates. The forecast horizon is $h = 4$ quarters. For the estimation of the quantile regression part in model (1), there are $r = 8$ static factors available as potential predictors, as well as their lags. To keep the model simple I do not include additional predictors and do not add lags to the quantile regressions. Hence, the matrix W in equation (1) is an identity matrix and $\beta_\tau(L)$ is just β_τ.

4 Results

4.1 Monetary Policy Shock IRFs

Figure 1 shows the impulse responses of the four identifying variables to the Cholesky monetary policy shock, scaled to a 100 basis points increase in the Federal Funds Rate. The shock decreases the levels of GDP and prices and affects the exchange rate between Swiss Francs and US Dollars in line with the overshooting principle of Dornbusch (1976). The classical "price puzzle" and "delayed-overshooting puzzle" do not obtain and the results are in line with Cholesky-DFM results presented in Forni & Gambetti (2010) as well as Kerssenfischer (2019). Notice that this can be shown to obtain regardless of using the real exchange rate or nominal one as I do (Kim et al., 2017). We conclude that the shock is well identified.

Figure 1: Point estimates of the impulse responses of the identifying variables to a contractionary 100bp monetary policy shock with bootstrapped 90% confidence intervals.



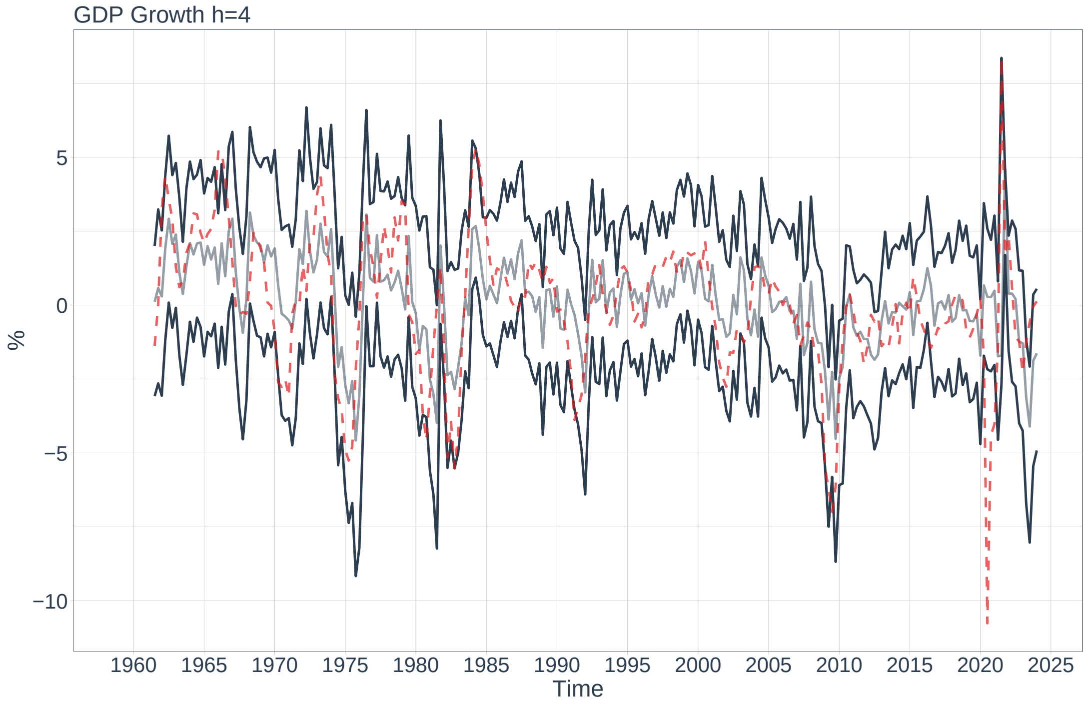
4.2 Quantile Regression Predictions

Figure 2 shows the time-varying quantiles as predicted using the factor approach from equation (12) for annual real GDP growth and PCE inflation. As distinct from other forecasts in the literature, the median and Q95 predictions of GDP growth appear more volatile, while the higher volatility of Q05 that is observed in Adrian et al. (2019) and Forni et al. (2021) is also featured as shown in Table 1. These differences may be due to conditioning on a larger set of information. In fact, for every quantile considered here at least three of the factors are highly significant predictors. Moreover, the factors do not suffer from collinearity by construction which may influence results as well. Especially the downside growth risk increases around the recessionary periods in the mid 1970s, early 1980s, early 1990s and 2000s as well as the Great Recession are clearly visible. However, the model predicts a very rapid recovery of downside growth risk and median risk after the 2007-2008 recessionary episode. The COVID period is not included in the quantile range at all. Moreover, the interquantile range decreases markedly with the Great Moderation as shown in Figure 3. During most periods the expected distribution of GDP growth was negatively (left) skewed which suggests a longer left tail relative to the right tail.

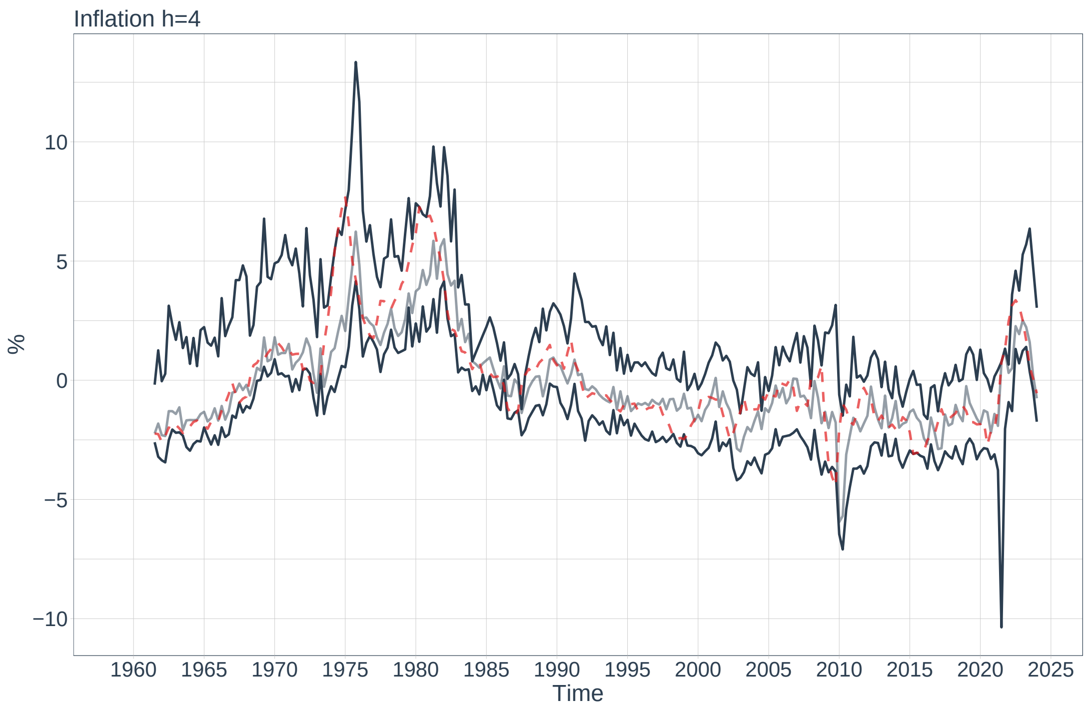
As for the predicted quantiles of PCE inflation, a downward trend is observable with sharp increases in the right tail in the recessions of the 1970s and 1980s but a pronounced drop in the lower tail during the Great Recession and COVID-19. The fall in the interquantile range is not as clear cut as with GDP growth, but during the period of the

Great Moderation, the spread measure is less volatile up to the Great Recession. According to the non-normalized Kelley skewness, PCE inflation predictions were almost always positively (right) skewed, which suggests a longer right tail with more extremely high predictions. By a similar token, Table 1 shows that the upper tail was much more volatile than the lower tail over the considered horizon.

Figure 2: h=4 predicted quantiles (Top line: Q95, Light grey line: Q50, Bottom line: Q05) of GDP growth and PCE inflation. Red dashed line is the realized series.

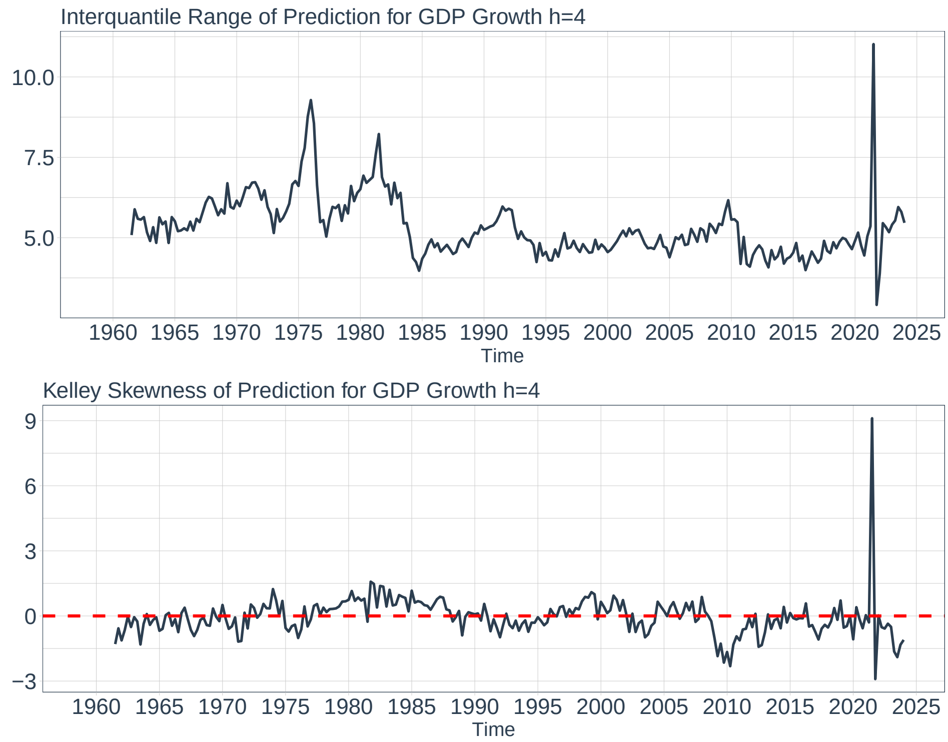


(a) ΔGDP h=4

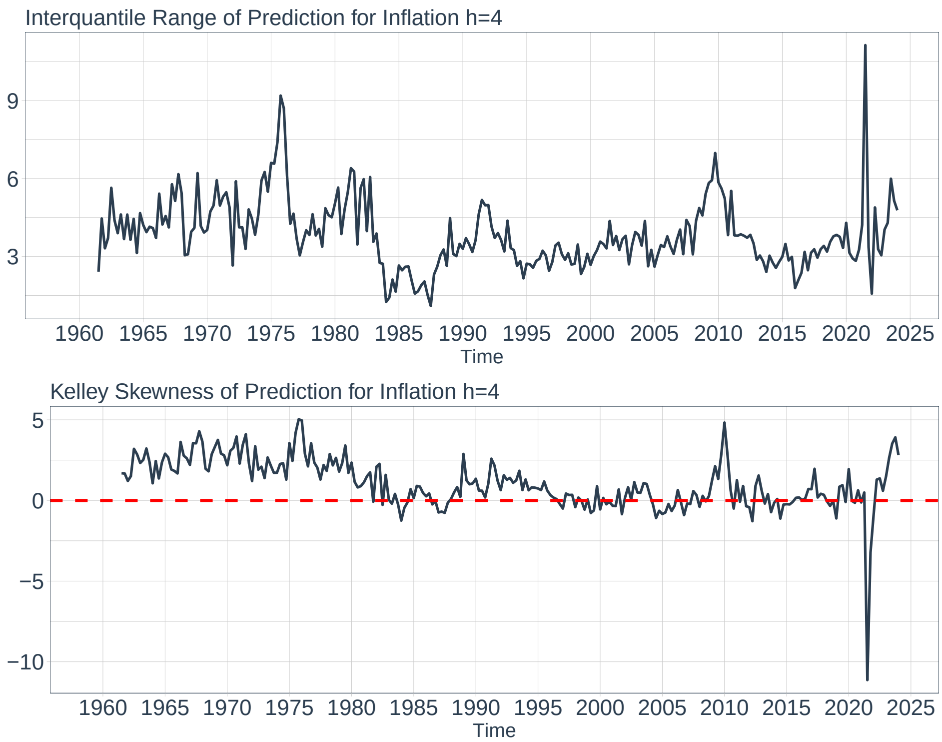


(b) PCE Inflation h=4

Figure 3: Top graph is interquantile range over time (Q95-Q05), bottom graph is non-normalized Kelley skewness (Q95+Q05-2×Q50).



(a) Moments ΔGDP h=4



(b) Moments ΔPCE h=4

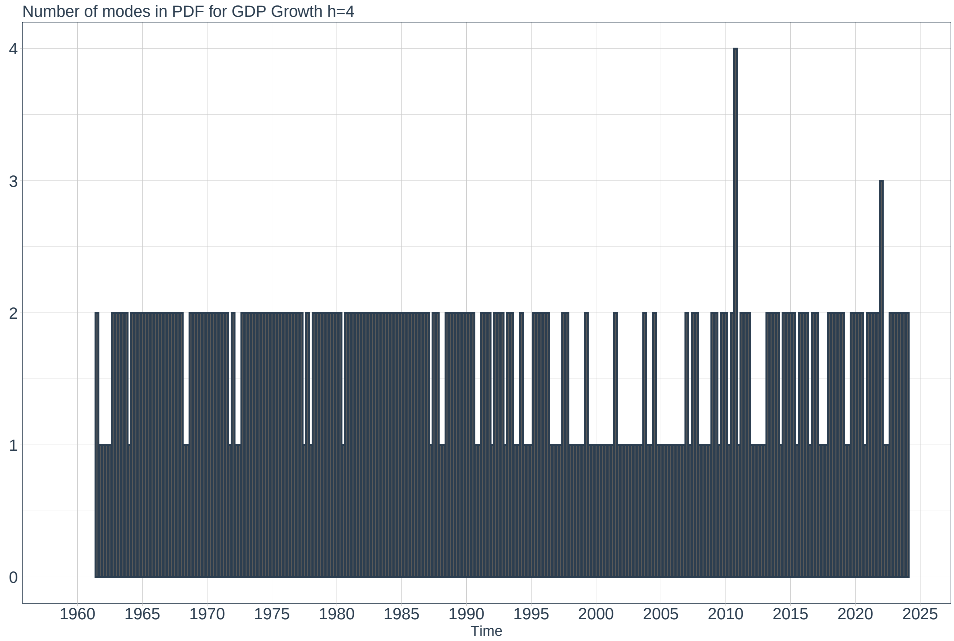
Table 1: Standard deviations of predicted quantiles and correlation coefficient for the extreme quantiles.

Metric	ΔGDP h=4	ΔPCE h=4
σ_{05}	1.80	2.00
σ_{50}	1.48	1.91
σ_{95}	1.61	2.62
$\rho_{5,95}$	0.85	0.87

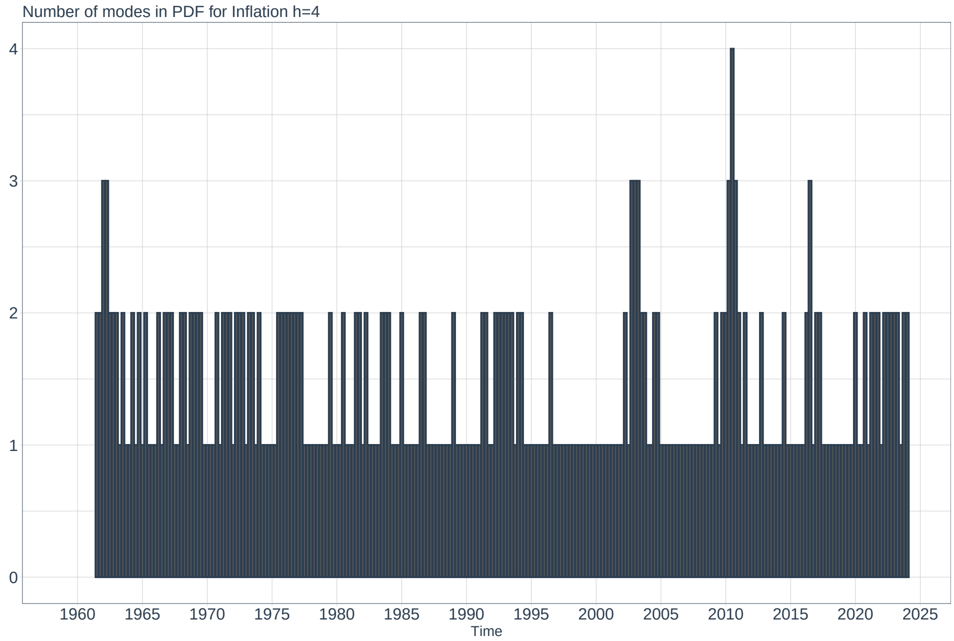
Multimodality: I use the adaptive kernel method of Portnoy & Koenker (1989) to fit a Gaussian kernel to the implied conditional quantiles from the factor approach for both GDP growth and inflation. The initial bandwidth is computed by the rule of thumb in Silverman (1986) (p.48). This initial bandwidth is used to get a sense of regions of low density in the distribution which should receive larger bandwidths. In a second step, the adaptive procedure allows the "bumps" of the fitted densities to have different bandwidths (Silverman, 1986) (p. 101). The sensitivity of the adaptive bandwidths is set to $\alpha = 0.5$ as suggested in Silverman (1986) (p. 102).[3] This approach is more flexible than the t-skew distribution of Azzalini & Capitanio (2003) which Adrian et al. (2019) use as it does not force the distribution to be unimodal. In fact, using this approach, Figure 4 shows that the GDP growth distribution is bimodal most of the time and the same goes for PCE inflation, although this is more pronounced in recessionary periods. The finding echoes Adrian et al. (2021) and Forni et al. (2021) and suggests the presence of good and bad states the economy can be expected to converge to given current economic conditions.

[3]The procedure is adapted from the R package quantreg, function *akj*.

Figure 4: Count of modes in the kernel fitted distributions implied by the conditional quantiles from factor regressions for GDP growth and PCE inflation.



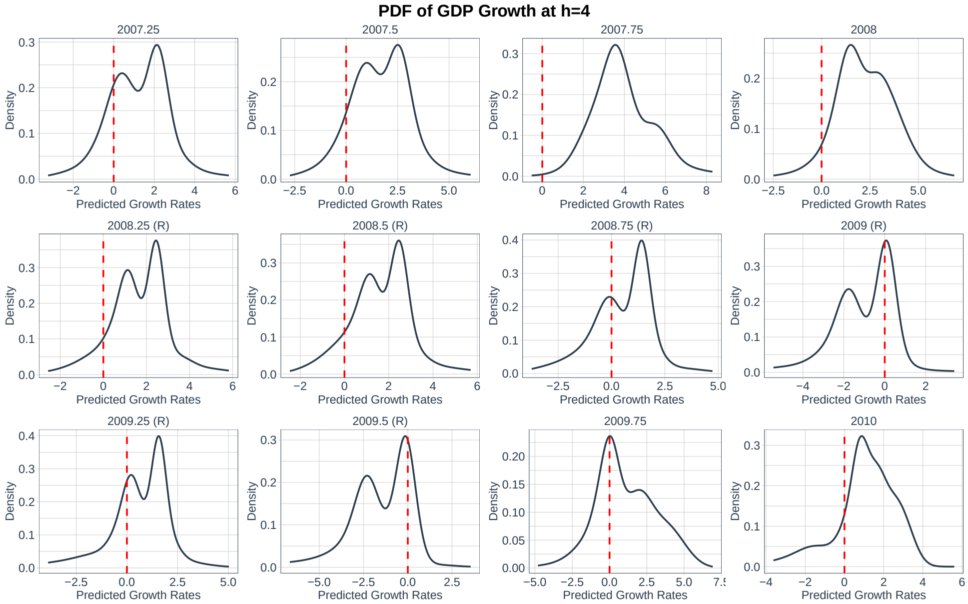
(a) ΔGDP h=4



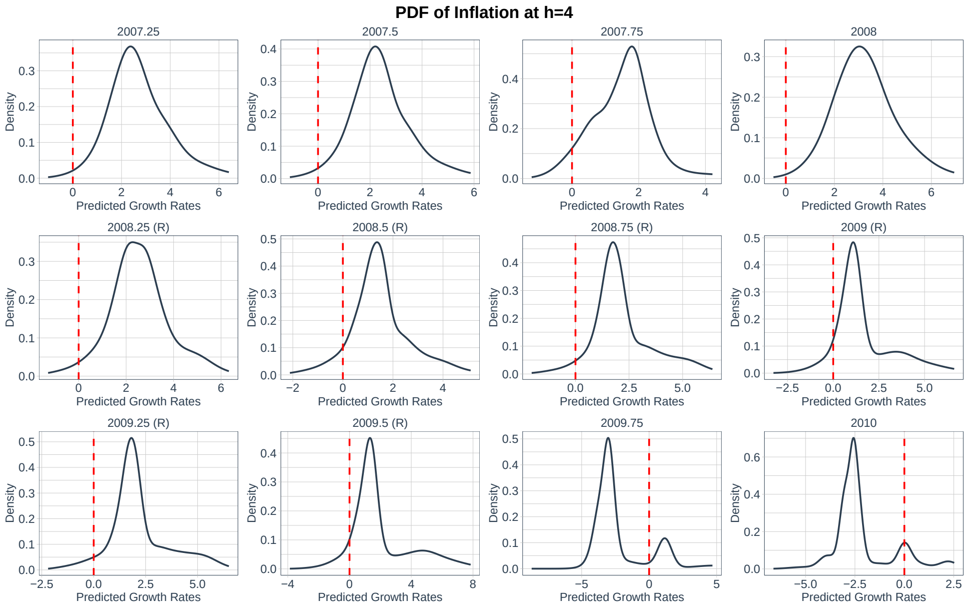
(b) PCE inflation h=4

Conditional distributions during the financial crisis: Figure 5 shows the Gaussian kernel fitted distributions of expected GDP growth and inflation for the years of the Great Recession. The evolution of the distribution shows that even before the crisis, the GDP growth distribution had been bimodal, with a mode near zero growth and a positive mode. As the recession unfolds, the information set captured by the factors leads the expected distribution to shift leftwards while maintaining the bimodality feature. This only disappears in the recovery. Contrary to this, the expected distribution of inflation is unimodel almost throughout the crisis as prices only dropped signficantly towards its end. The model anticipated the deflationary episode of the third quarter of 2009 reasonably well, but overpredicts its persistence.

Figure 5: Four quarters ahead predicted PDFs of GDP growth and PCE Inflation between Q2:2007 and Q1:2010. The year in the title of each plot is the year for which the expected distribution is computed, not the year when the expectation is formed, which would be $h = 4$ quarters earlier.



(a) ΔGDP h=4



(b) PCE inflation h=4

Model evaluation: Evaluating the fit of the model is not straightforward in the case of quantile regressions, as no direct equivalent to the usual R^2 measure and its adjusted versions exists. Koenker & Machado (1999) propose a metric called R^1. This statistic evaluates the loss implied by the residuals of the quantile regression (V1) at each quantile against the alternative of running the regressions only on a constant (V0).

$$V1_\tau = \sum_{i=1}^{T} \rho_\tau(u_i^1)$$
$$V0_\tau = \sum_{i=1}^{T} \rho_\tau(u_i^0)$$
$$R^1 = 1 - \frac{V1}{V0}$$

The values will be close to one if the loss of the V0 model is large relative to the V1 model. Table 2 reports the R^1 measures for the quantile regressions of GDP growth and the inflation measure at the prediction horizon of four. The performance of the model over the simple model is very good for the extreme quantiles and for the median. Especially for PCE inflation the inclusion of additional information in the predictive regressions is beneficial. In addition, Table 3 compares the fit of the model of Adrian et al. (2019) and a QAR(1) for both variables of interest using the maximimum available sample including the NFCI from 1971. Both score consistently worse than the factor approach. The results suggest that the factor approach is suitably information rich.

Table 2: Measures of fit from factor quantile regressions computed for the quantiles of interest.

Quantile R^1	ΔGDP h=4	ΔPCE h=4
Q_5	0.9853	0.9940
Q_{50}	0.9413	0.9634
Q_{95}	0.9859	0.9838

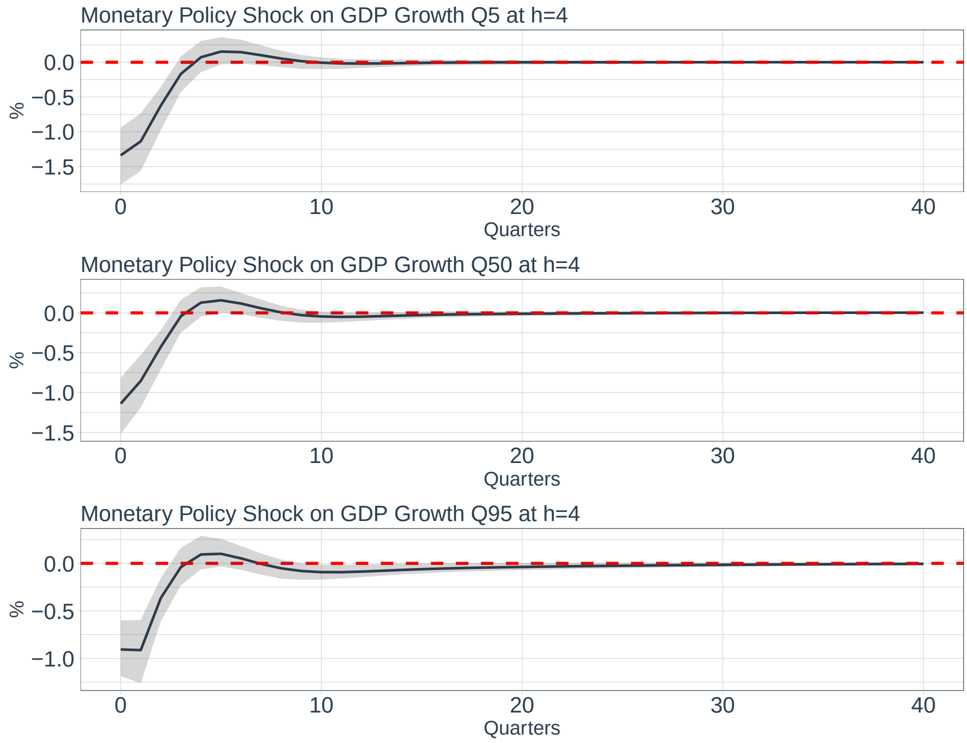
Table 3: Measures of fit computed for the quantiles of interest using the specification of Adrian et al. (2019) which includes the lagged dependent variable and the NFCI or a QAR(1). Forecast horizon is $h = 4$, the column names are the regressors used.

Quantile R^1	ΔGDP + NFCI	ΔPCE + NFCI	ΔGDP	ΔPCE
Q_5	0.9430	0.9780	0.9367	0.9780
Q_{50}	0.8364	0.8940	0.8302	0.8940
Q_{95}	0.9566	0.9548	0.9556	0.9507

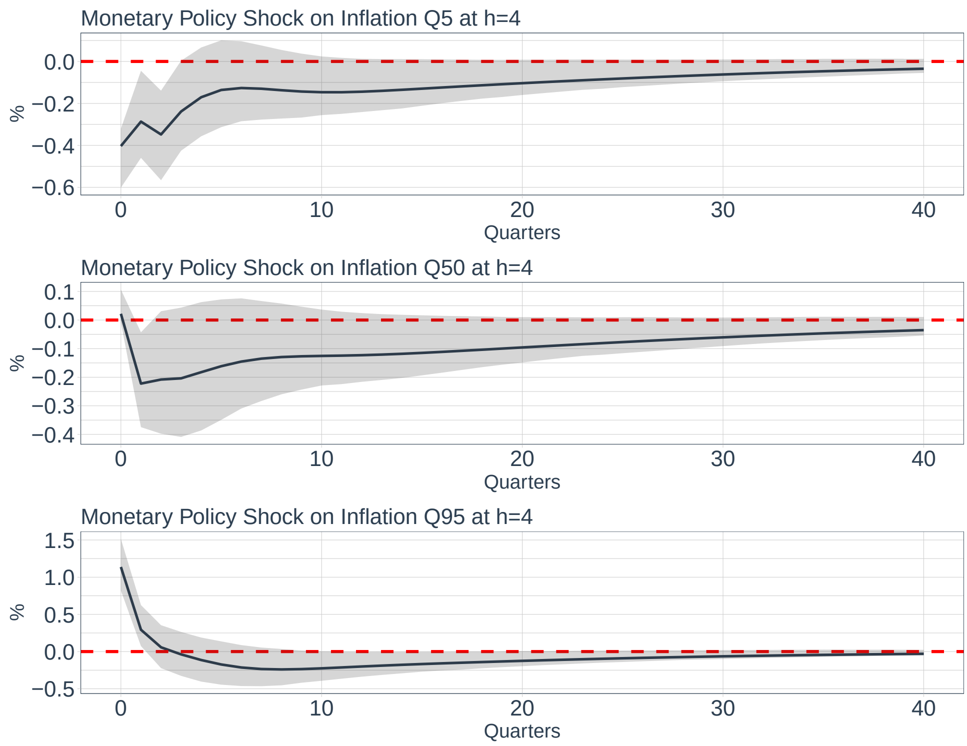
4.3 Quantile Impulse Responses

Figure 6 shows the responses of the predicted conditional quantiles to the 100bp contractionary monetary policy shock at horizon $h = 4$. The IRFs can be interpreted as follows: a 100 bp monetary polic shock alters current macroeconomic conditions such that the expected year-on-year distribution of GDP growth and inflation is affected as portrayed. For example, all forecasted quantiles for GDP growth decrease on impact and return to zero after around five quarters. The initial reaction of downside risk is significantly weaker than for upside risk. The median result is in line with the mean effect depicted in Figure 1. This suggests that the predictions for GDP growth become more pessimistic and the distribution shifts to the left. As distinct from this, we observe a clear spreading of the predicted distribution of PCE inflation, which is a result of the left tail moving further left and the right tail moving further right. Since the median also moves to the left (again in line with the mean results), this implies a skewing of the distribution as well as a spreading – the right tail becomes longer.

Figure 6: Point estimates of the impulse responses of the quantiles of h=4 GDP growth and PCE inflation to a 100bp monetary policy shock. Bootstrapped 90% confidence intervals. Top graph is Q5 response, middle graph is Q50 response, bottom graph is Q95 response.



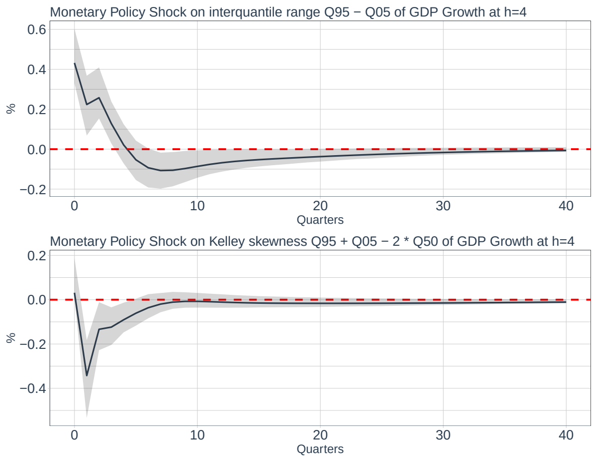
(a) ΔGDP h=4



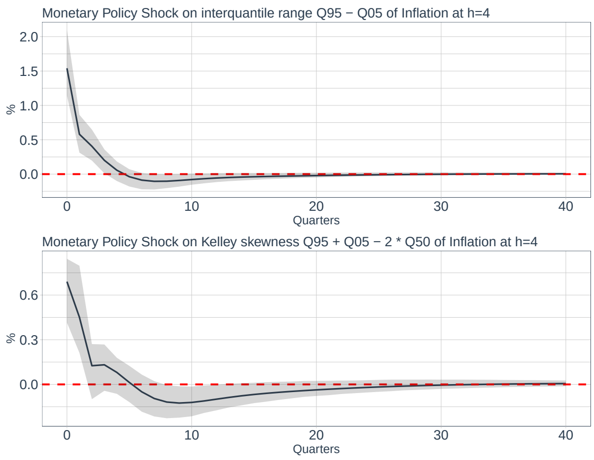
(b) PCE inflation h=4

From the responses of the conditional quantiles we can compute how the contractionary monetary policy shock affects the interquantile range and the (Kelley) skewness in each case. Both the predicted GDP growth and inflation distributions get spread out. The skewness of GDP growth decreases relative to the steady state, for PCE inflation it becomes more positive, as shown in Figure 7.

Figure 7: Response of interquantile range (top) and non-normalized Kelley skewness (bottom) to the 100 bp contractionary monetary policy shock. All responses are at $h = 4$. 90% bootstrap intervals shaded in grey.



(a) ΔGDP h=4



(b) PCE inflation h=4

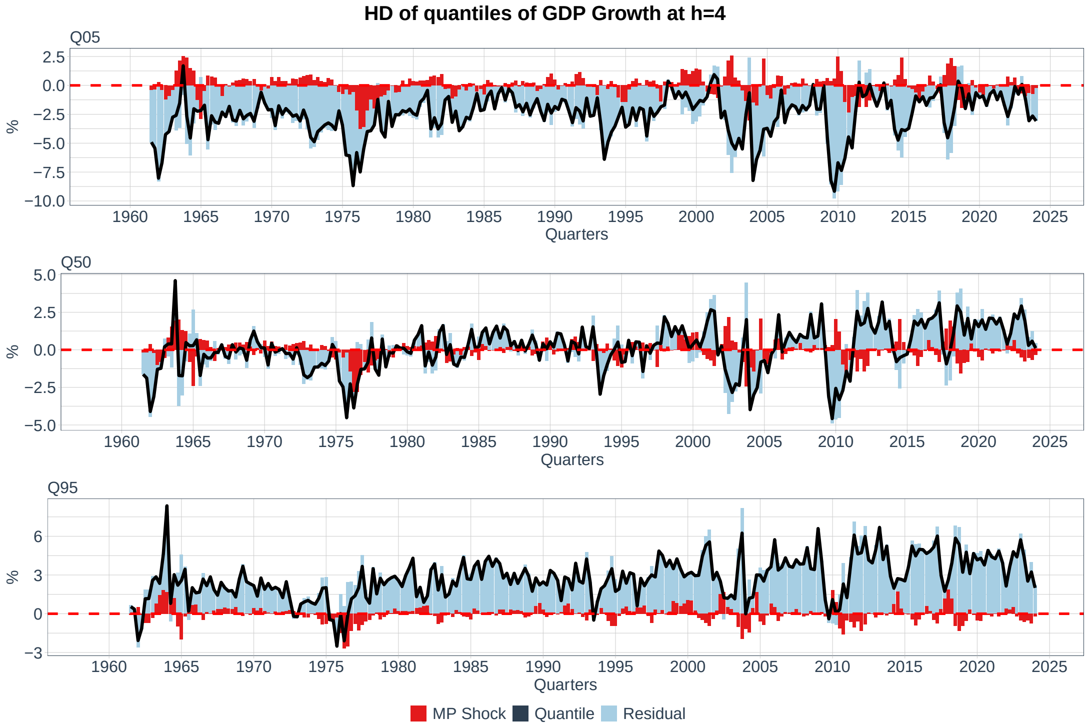
To explore the effects of monetary policy shocks on the conditional quantiles in more detail, I compute the forecast error variances as well as a historical decomposition for the fitted time series' of each quantile for the $q = 4$ primitive shocks. This provides some information about whether the movement in the quantiles is largely attributable to monetary policy or rather to the other three unidentified shocks. The results are reported in Table 4. The monetary policy shock explains a sizable share of the variation in the conditional quantiles of GDP growth, especially of the lower tail. This suggests that policymakers should indeed take the ramifications of their decisions for growth risks into account. Furthermore, the short term importance of monetary policy for the upper tail of the predicted distribution of inflation is around 45% which decreases to 16% over the horizon of 40 periods.

Table 4: Forecast error variance decomposition of GDP growth quantiles and inflation quantiles to a monetary policy shock at different forecast horizons, j periods after the event of the shock.

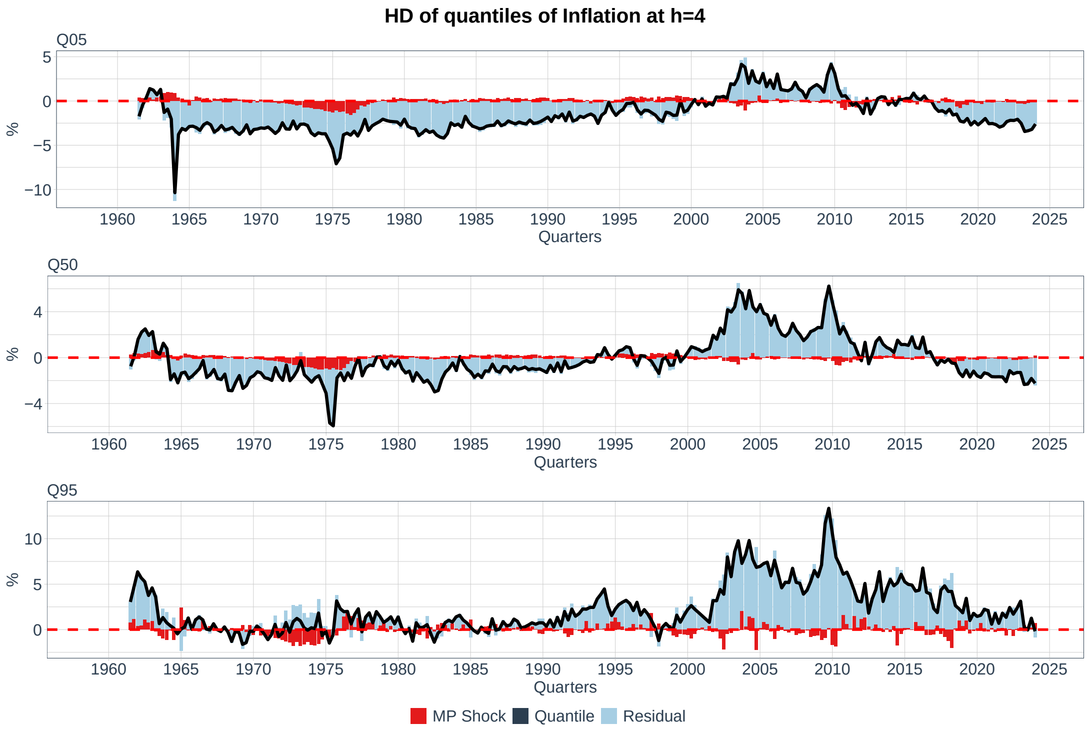
	Horizon	Q05	Q50	Q95		Q05	Q50	Q95
ΔGDP h=4	j=0	0.14	0.15	0.36	ΔPCE h=4	0.10	0.00	0.45
	j=1	0.37	0.43	0.20		0.10	0.03	0.32
	j=8	0.41	0.44	0.30		0.11	0.05	0.16
	j=40	0.36	0.38	0.28		0.11	0.07	0.16

For the historical decomposition in Figure 13 I compute the contribution of the monetary policy shocks according to equations (14) together with the time series of the structural shocks u_t, the residual is the difference between the fitted quantile and the monetary policy shock's contribution.

Figure 8: Historical decomposition of the quantiles of GDP growth and inflation into the contribution of the unit variance monetary policy shock and the residual component.



(a) ΔGDP h=4



(b) PCE inflation h=4

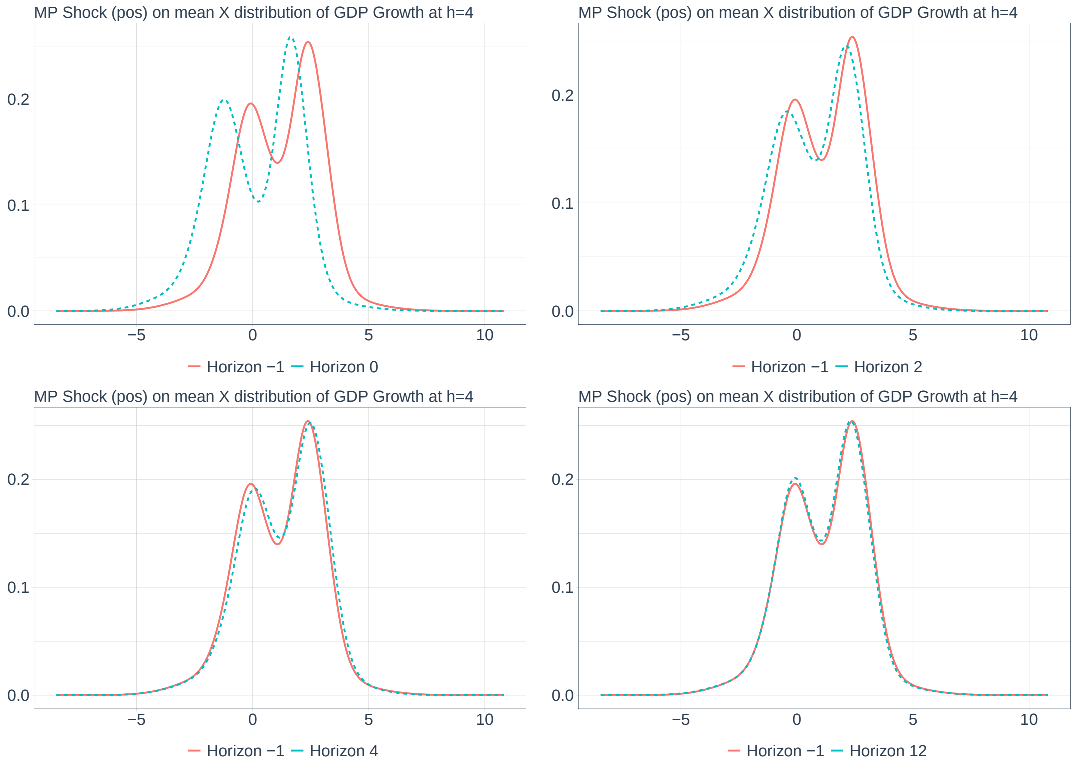
The results of this exercise are shown in Figure 8. The plots reveal that the monetary policy shock played an important role in the downside risk during the 1970s recession, but has been keeping the 5th quantile higher during the Great Moderation. Its overall impact on the quantiles for inflation is limited to the early sample between the 1960s and the 1980s. If anything, it seems to have somewhat contained excessive upside risk after the year 2000. In summary, while there are some signs that monetary policy can combat downside growth risk and upside inflation risk, it seems far fetched to call for monetary policy as a means of containing macroeconomic risk, given its historically negligible role.

4.4 Distributional IRFs

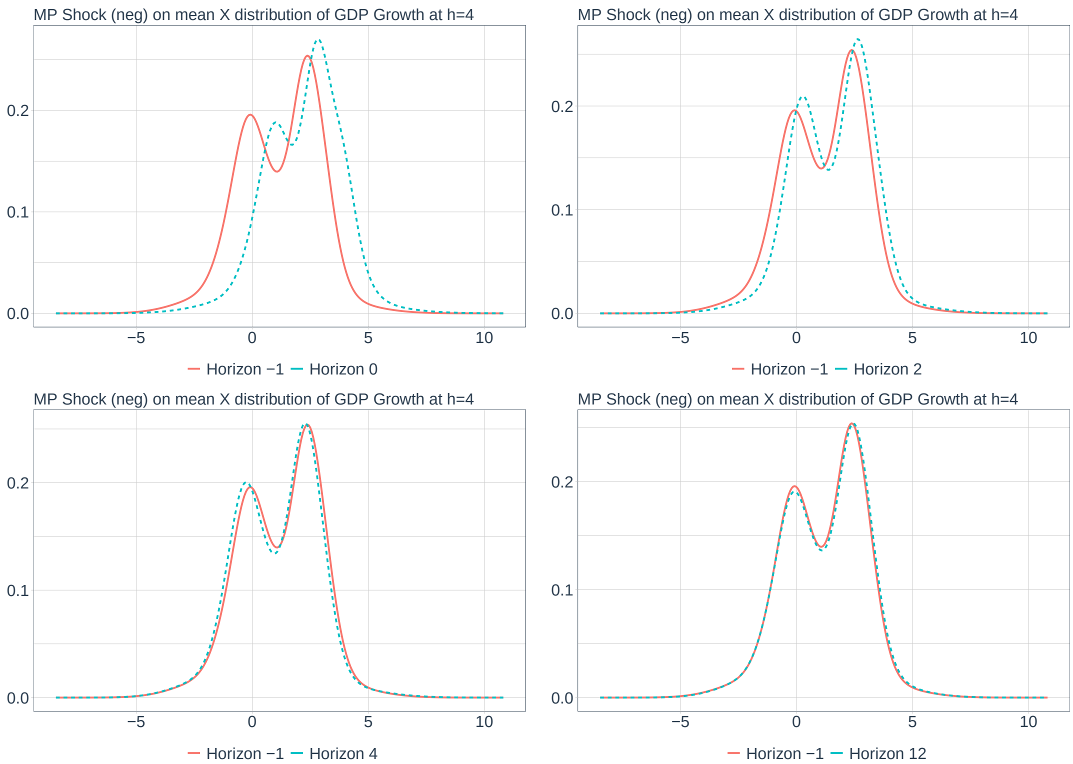
The quantile IRFs above are marginal moments that cannot be used to characterize the movement of the full expected distributions of GDP growth and inflation as the central bank alters the economic conditions through the release of the monetary policy shock. To construct such distributional IRFs, I opt for an alteration of the method of Adrian et al. (2021) which consists in perturbing the quantile regression inputs and then computing the implied fitted value for all quantiles of interest. This is achieved by deviating the factors away from their mean for $h = 40$ periods according to the impulse response coefficients estimated for equation (6). Next, the original quantile coefficients are used to obtain the fitted values. To these newly fitted quantiles I can then use the Portnoy & Koenker (1989) adaptive kernel procedure to obtain the full distribution for each of the $h = 40$ periods. Given that the kernel fitting step is non-linear, this procedure allows for size and sign effects of the monetary policy shock.

The distributional changes are depicted in Figure 9. In response to the monetary policy shock, the predicted distribution of GDP growth shifts left and the left mode becomes more distinguished. The effect recedes after around two quarters and the distribution is nearly back to its initial shape after eight quarters. Interestingly, this result does not fully mirror for an expansionary shock. The "bad" mode is essntially dissolved as the distribution shifts right. Again, the expected distribution returns to its initial position relatively quickly. In the case of inflation, the initial distribution is slightly right-skewed. In line with the description above, this skewness is further increased by the monetary policy contraction, but reversed by an expansion. Moreover, we can see the second mode being pushed furhter to the right for the contraction which suggests that the centrla bank is unable to contain upside inflation risk while seemingly reducing prices on average. The contraction rather induces additional uncertainty into the economy which makes a wider range of future inflation values likely. This puts into question the usefulness of monetary policy as a risk management tool.

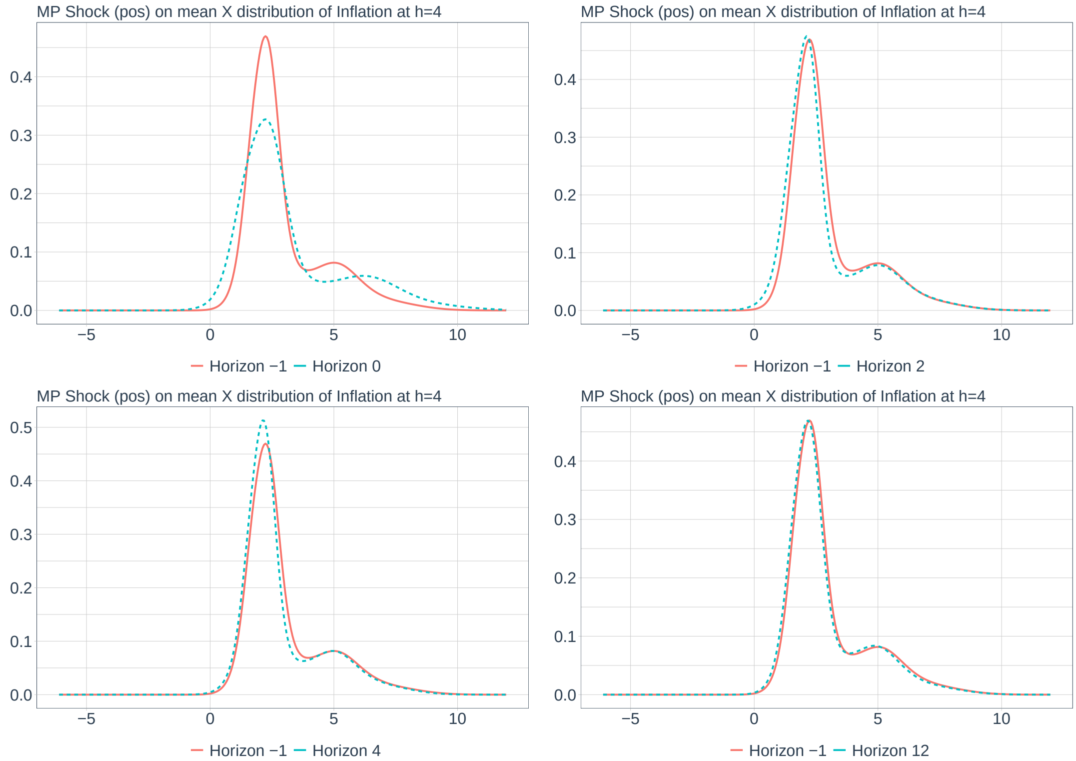
Figure 9: Response of fitted distribution to 100 bp monetary policy shocks. Steady state distribution is fitted to the predicted distribution using the median values of the regressors. All forecast distributions are at h=4.



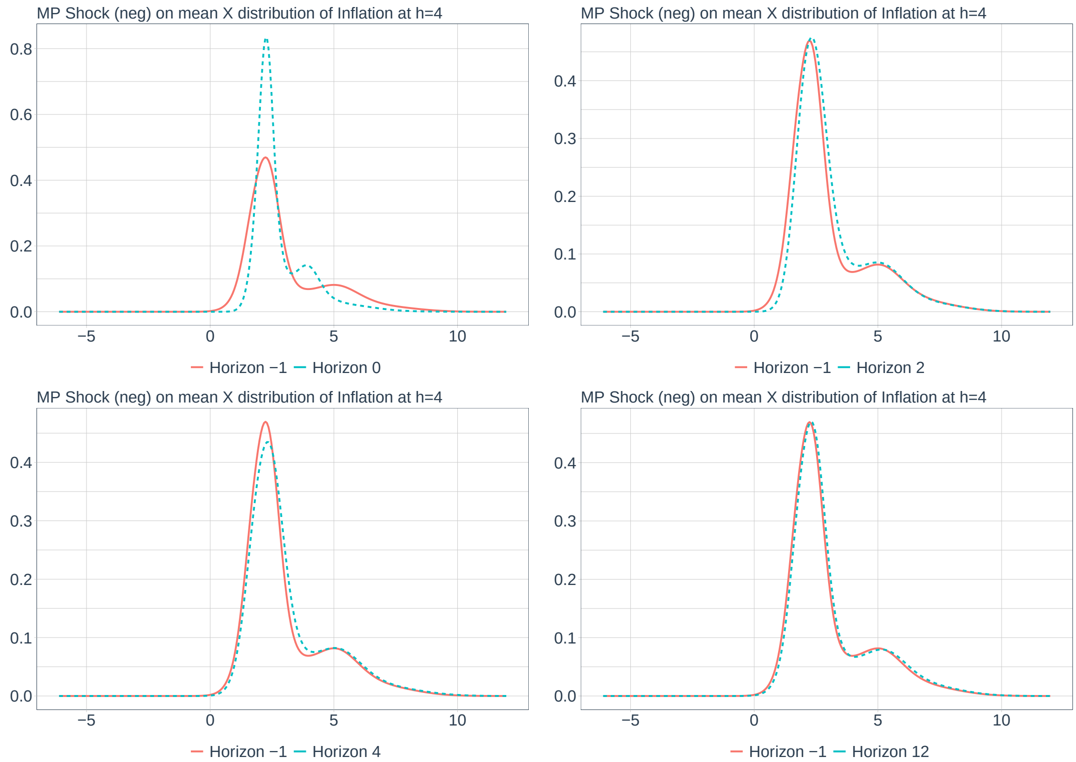
(a) ΔGDP (positive)



(b) ΔGDP (negative)



(c) PCE inflation (positive)



(d) PCE inflation (negative)

5 Discussion

The results presented above taken together suggest that contractionary monetary policy is an inadequate tool for controlling macroeconomic risk. Rather, it appears to exacerbate the "bad" equilibria existing in the conditional distributions of GDP growth and inflation. However, expansionary shocks can indeed reduce the interquantile range of the distribution and may in fact select the better of the two dominant probability accumulation points. Therefore, our results can be interpreted as evidence for a supporting role of looser monetary policy for extended growth and less price volatility, whereas its role in returning inflation to lower levels is associated with significant macroeconomic and uncertainty costs. Possible explanations for these mechanisms can be traced back as far as Bernanke (1983).Given that contractionary monetary policy increases the range of possible outcomes, this may activate an "options" channel which induces agents to save instead of invest which contributes to lower growth. Explanations for non-linear amplification of monetary policy are usually drawn from financial frictions (see Brunnermeier & Sannikov (2014) or Adrian et al. (2021)). For example, if financial conditions are tightened after the monetary policy intervention, this can cause constraints on borrowing to start to bind pushing the economy away from the flexible equilibrium. Another explanation for asymmetric effects of monetary policy shocks is given in Debortoli et al. (2023). The authors show that downward wage rigidity in the labor market can keep the economy away from full employment for longer after contractionary shocks than after expansionary ones. Claus & Nguyen (2020) offer a treatment of the asymmetric responses of consumer expectation to monetary policy shocks. They show that not only inflation expectations, but also economic expectations of activity and readiness to spend react quickly and non-linearly to monetary tightening and easing. The plethora of possible amplification mechanisms – spanning financial markets, labor markets and consumer confidence channels – as well as the possibly involved non-linearities that can arise in response to monetary policy shocks suggests that it is necessary to allow for a rich set of conditioning variables to characterize the conditional distributions of GDP growth and inflation as I have done here.

A striking feature of the results is the continued presence of multiple modes in the distributions of GDP growth and inflation. The emergence of additional modes in the conditional distributions of GDP growth and inflation can be seen as evidence that monetary policy can lead to multiplicity of economic equilibria. Such multiplicity can arise for example if the central bank's reaction to inflation is insufficiently strong in a New-Keynesian framework (Lubik & Schorfheide, 2004). This leads to an indeterminate propagation of monetary policy with potentially many outcomes receiving positive probability. Lubik & Schorfheide (2004) show that depending on the selected equilibrium contractionary shocks can decrease growth by different magnitudes, which is in line with the two modes in the predicted GDP growth distribution representing different equilibria where the contraction has different intensities. Similarly, they show that depending on the selected equilibrium prices can in fact increase shortly after the introduction of the contractionary policy, which is observable in the right mode of Figure 9 in panel c).

Multiple equilibria are a salient feature of economic models which feature some form of coordination failure (Cooper & John, 1988). A discussion in the context of global games is provided in Morris & Shin (2002). Essentially, multiplicity arisis if the optimal strategies of the involved players are affected the others' strategies, for example in supplier networks, demand across different economic sectors or trade. Special cases of such failures are the famed bank runs of Diamond & Dybvig (1983) as extended to the financial panics of Gertler et al. (2020) or the uncertainty traps of Fajgelbaum et al. (2017). All these models have in common a deviation from a deterioration of fundamentals which keeps agents from jointly coordinating on the first-best actions which leads them to get stuck in a less desirable equilibrium. The research in the present paper shows that there exists the possibility to select among different equilibria almost throughout the entire sample period since the 1960s. While multiplicity is stronger during the first half of the sample as suggested in Lubik & Schorfheide (2004), strategic complementarities are omnipresent throughout and expansionary monetary policy can help selecting the better state.

6 Conclusion

In this paper I propose a new method to assess how monetary policy shocks influence the conditional distributions of GDP growth and inflation in the US by combining a dynamic factor model with predictive quantile regressions. The methodology is general enough to study the response of any variable included in the data set. I show that the predictive distributions of both variables shift to the left in response to the shock and that downside growth risks are exacerbated by the shock even more than upside risks are reduced. Moreover, the short horizon predicted inflation distribution spreads out. This suggests that monetary policy may open up uncertainty channels which can have real consequences. While the policy achieves a reduction in median expected inflation, it creates potential for more extreme future inflation outcomes as well. Moreover, I show that contractionary shocks can create additional modes in the expected distributions of interest, which lead to significant increase in the probability that growth will decline by much more than on average and even carries the potential for the policy being ineffective in reducing inflation at all. The finding suggests that the multiplicity of equilibria, which theory links to weak policy responses, may only obtain when policy is not expansionary.

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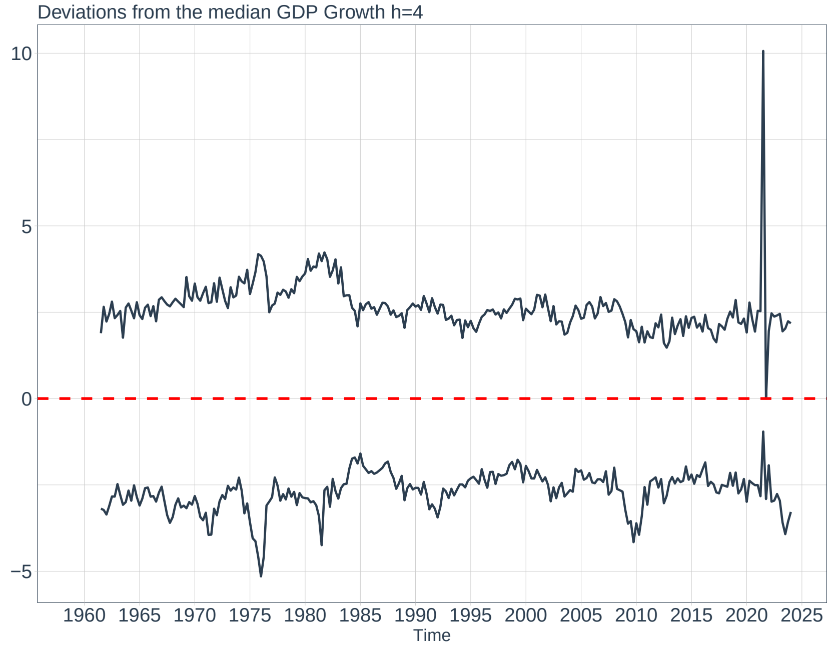
Appendix A: Additional evidence on conditional quantiles

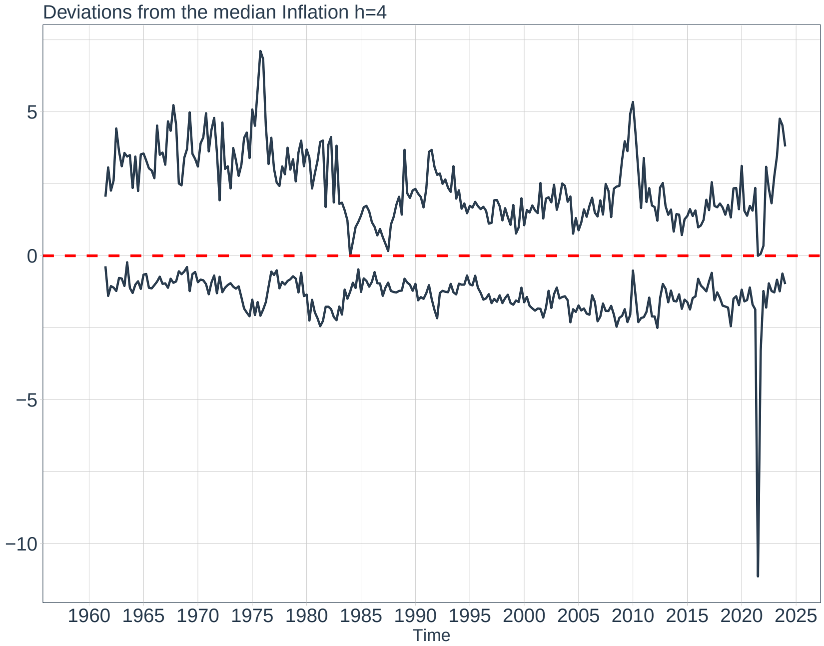
A.1: Deviations of tails from the median

To highlight the tail movement implied by the factor quantile regressions, I compute the deviations of the 5th and the 95th quantiles from the median. This is important as a prevailing feature of the literature is that the lower tail is much more volatile than the upper tail for GDP growth. I repeat this also for inflation.

Figure 10: Quantile IRFs with differing numbers of static factors.



(a) ΔGDP h=4



(b) PCE inflation h=4

Figure 10 clearly shows that there is more variation in the lower tail than in the upper tail of GDP growth, but not as starkly as in Adrian et al. (2019) or Forni et al. (2021) where the upper tail is essentially flat. This suggests a key role for the information set used in the quantile regressions. I find several factors to be highly significant for the upper quantile. For inflation, on the other hand, the upper tail is in fact much more volatile than the lower tail. This points to significant upward price pressures being captured by the factors from the large macroeconomic data base. Again, many of these are significant. Table

Table 5: Number of significant factors by quantile. 90% significance is assessed using the rank confidence intervals of Koenker (1994).

Quantile	GDP Growth	Inflation
0.05	3	7
0.5	4	7
0.95	3	4

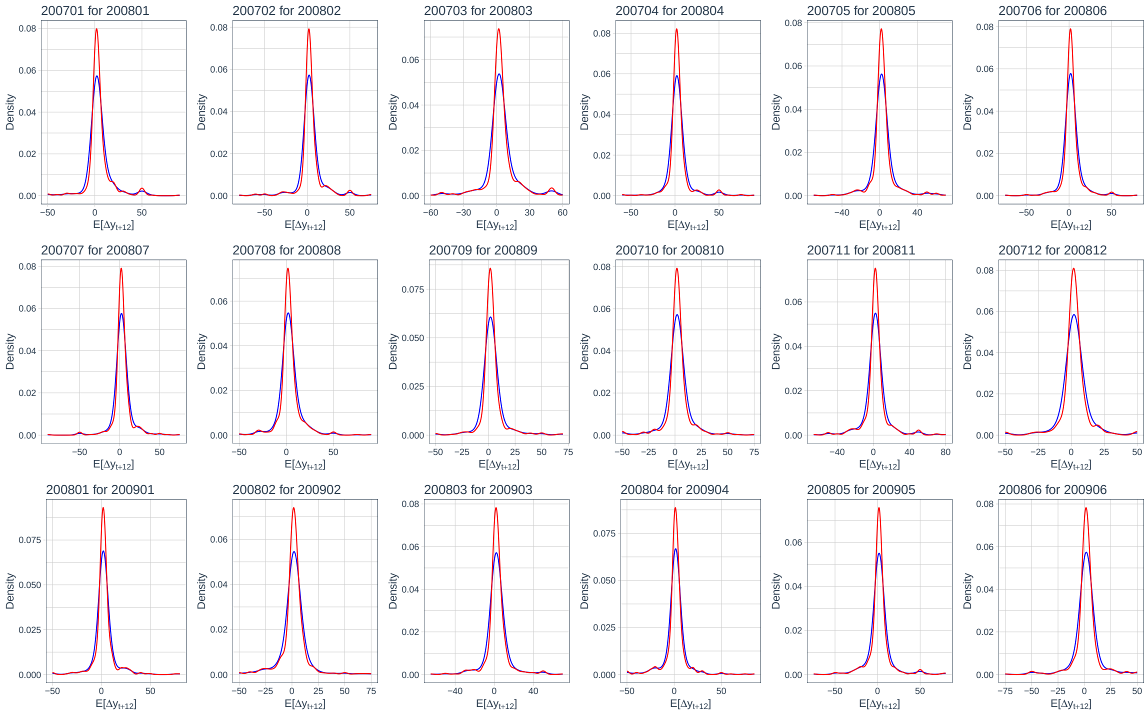
A.2: Multimodality in the Michigan Survey

Bimodality also obtains in the Michigan Survey of Consumers. I consider the cross sectional responses over time for the following two questions:

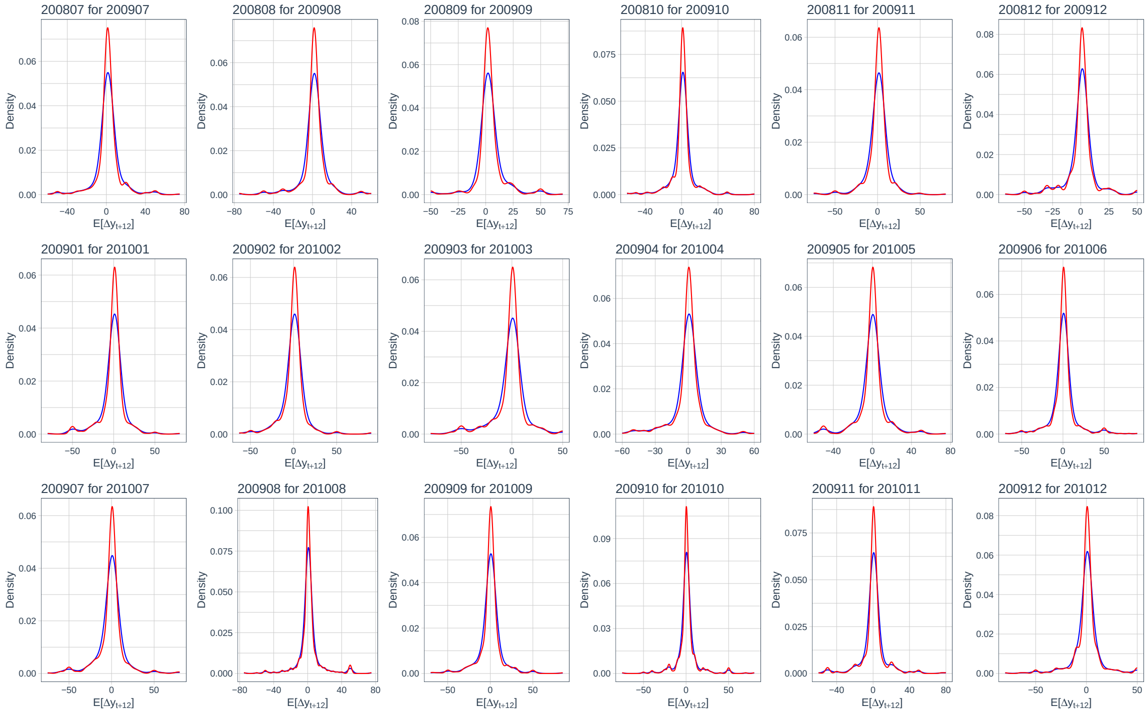
1. By about what percent do you expect your income to (increase/decrease) during the next 12 months?
2. By about what percent do you expect prices to go (up/down) on the average, during the next 12 months?

The monthly kernel fitted distributions (using two rule of thumb bandwidths and Gaussian kernels) for the years of the Great Recession are depicted in Figure **??**. Clearly there is bimodality in inflation expectations in the consumer cross section over time. Given that many respondents do not expect any changes in income, there is a strong mode at zero for expected income changes, but there are signs of additional humps also in these cross section distributions. I take this observations as indicative of multimodality in expected distributions beyond the ones computed through the lens of the quantile DFM used in this paper and as a sign that the expectational channel put forward in Claus & Nguyen (2020) may be an important amplification mechanism for monetary policy shocks.

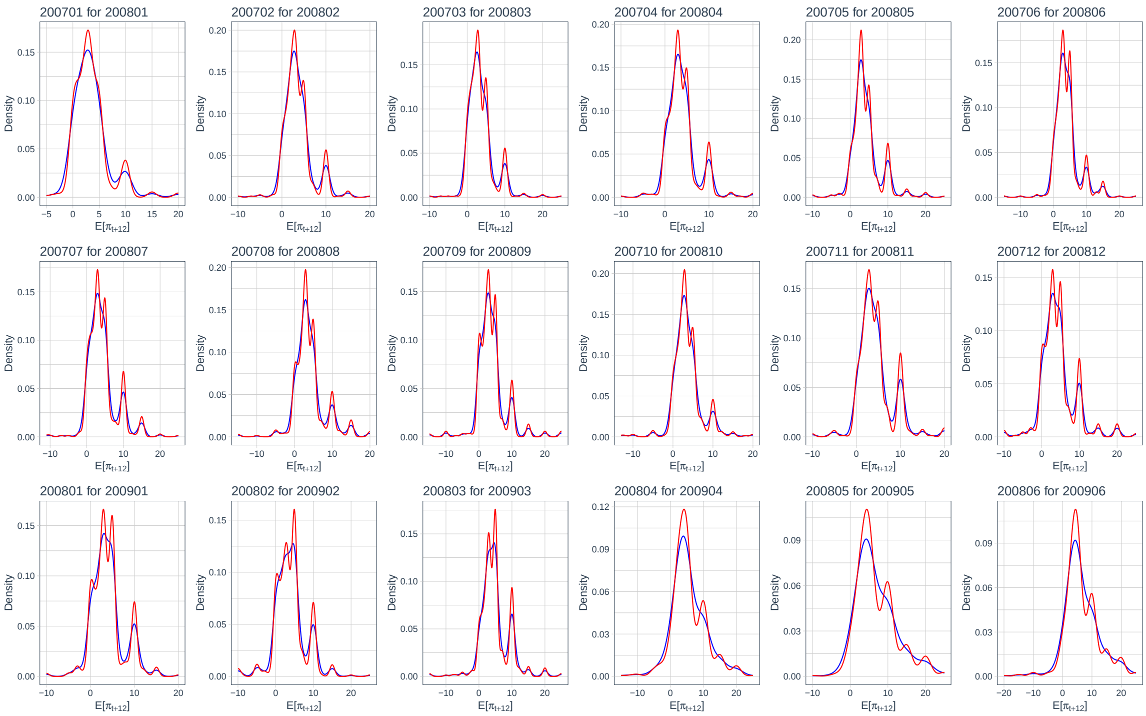
Figure 11: Cross section distributions from Michigan Survey of Consumers. Kernel fits with standard bandwidths (red line) and bandwidths scaled up for stronger smoothing (blue line).



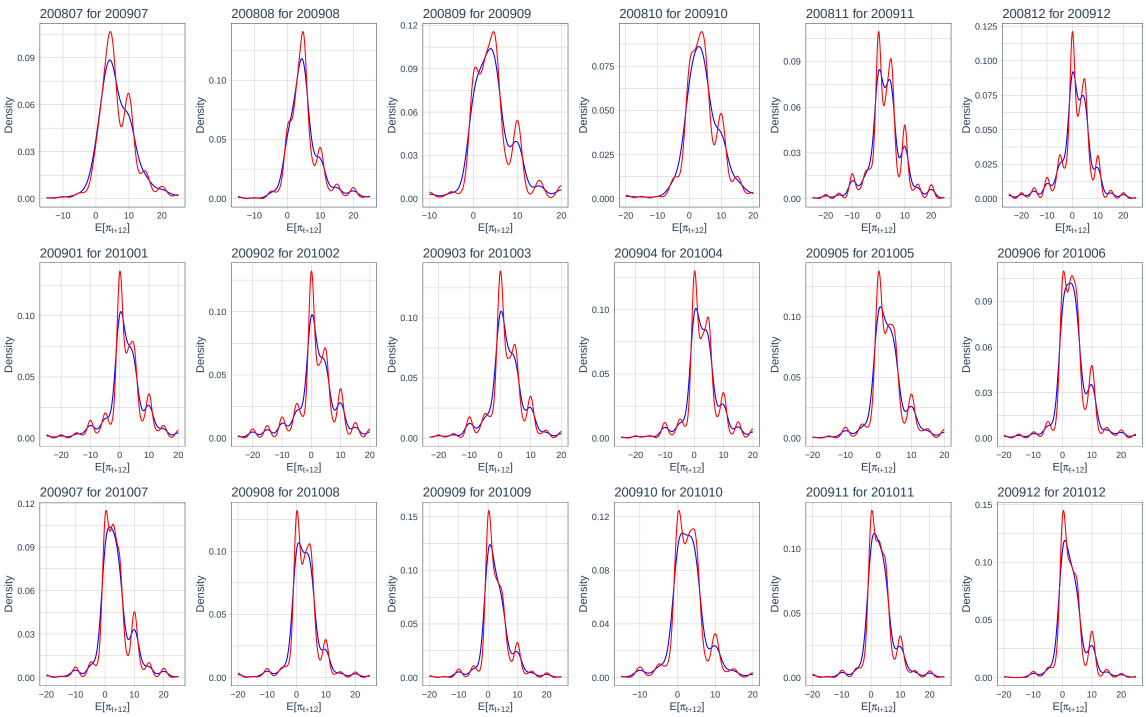
(a) Income expectations Jan 2007 - Jun 2008



(b) Income expectations Jul 2008 - Dec 2009



(c) Inflation expectations Jan 2007 - Jun 2008



(d) Inflation expectations Jul 2008 - Dec 2009

Appendix B: Robustness

I conduct some robustness checks to verify the validity of the approach used in the main specification. First, I identify the monetary policy shock by using between seven and fifteen static factors as suggested by the criterion of Alessi et al. (2010). The same approach is used for the computation of the quantile IRFs. Including more factors in the model increases the information content, but may also lead to overfitting the quantile regression and loss of degrees of freedom. Second, I estimate the monetary policy shock with $p = 3$ lags for the VAR in equation (5) as suggested by the AIC.

B.0:.1 Changing the number of static factors

Figure (12) shows the responses of the identifying variables to the monetary policy shock for different numbers of static factors $r \in [7, 15]$. There hardly any qualitative differences.

Figure 12: Responses of identifying variables to 100 basis points monetary policy shock with different numbers of static factors.

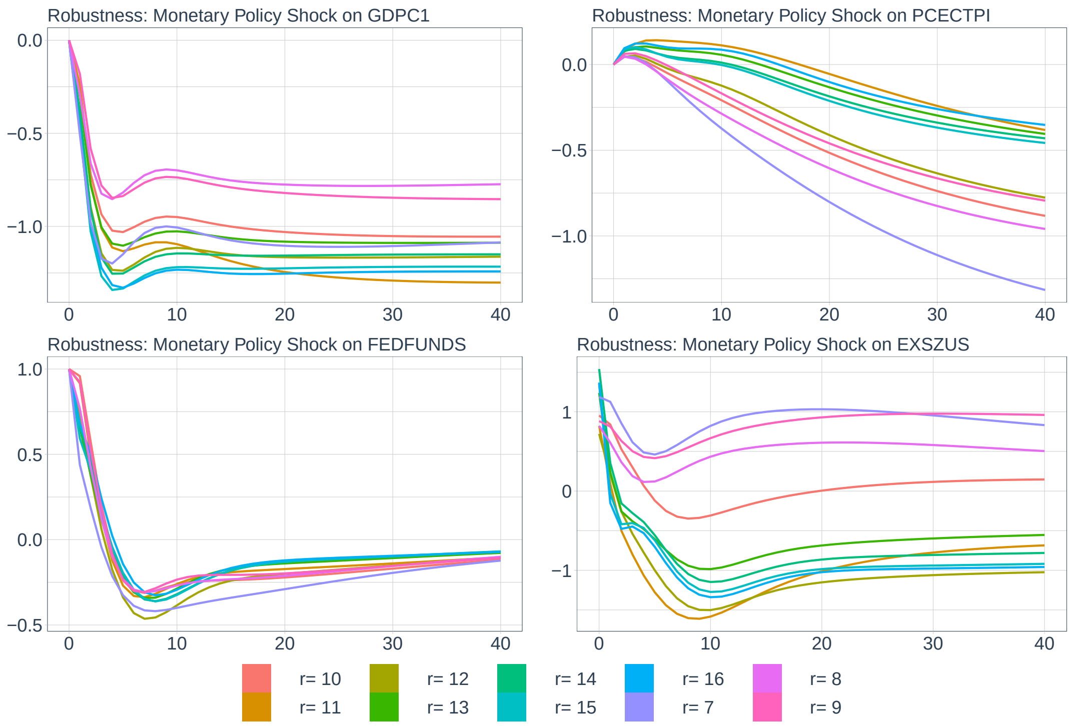
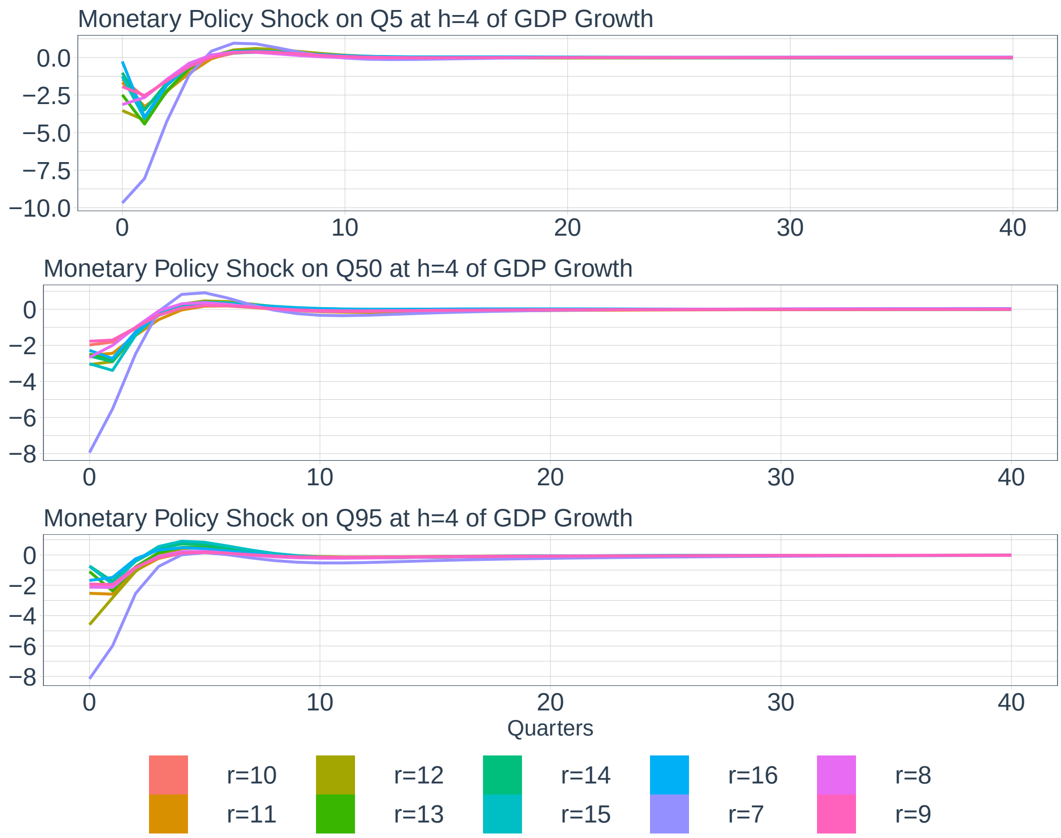
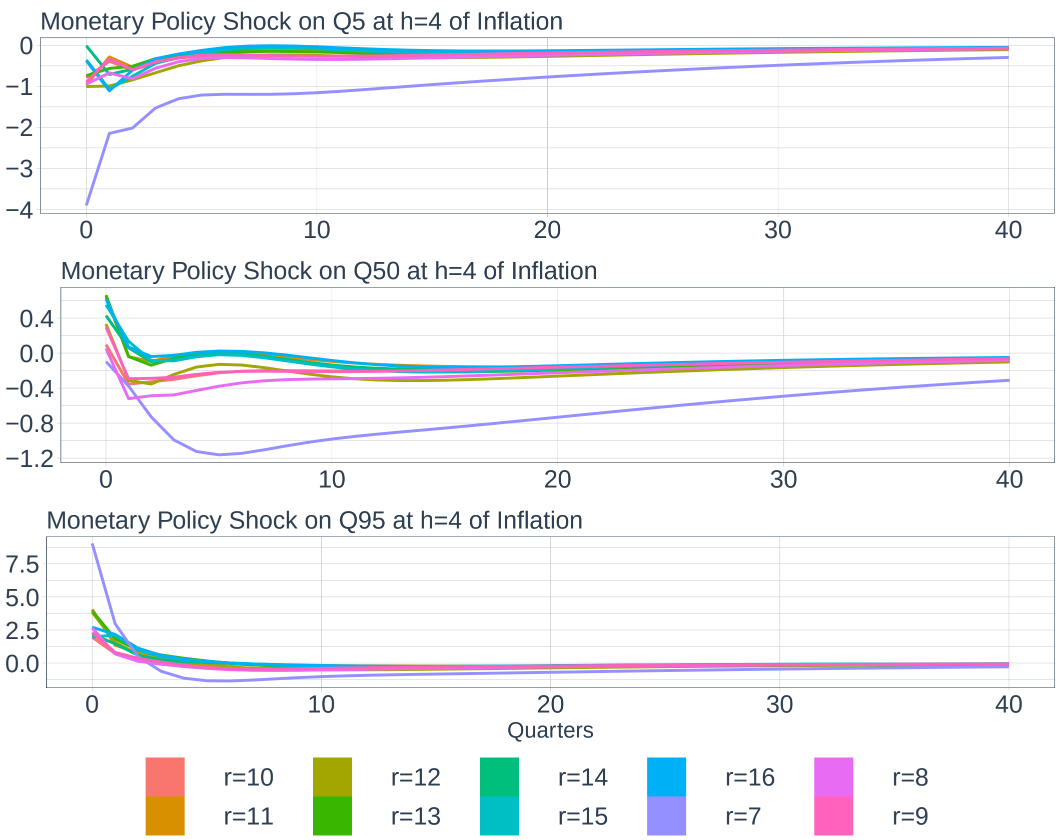


Figure (13) shows the quantile IRFs that obtain using different numbers of static factors. Again, the IRFs are similar to the baseline case of $r = 8$, the GDP growth distribution shifts to the left as both tails are pushed in that direction. An exception occurs when $r = 7$, which suggests that this specification does not cover enough information to yield correct monetary policy shocks. In the case of inflation, for all numbers of factors the left tail and median shift to the left whereas the right tail moves out further to the right. For all specifications, the expected distribution spreads out. Overall, the spread of the expected inflation distribution shows good robustness towards changing the number of static factors used as regressors for the quantile regression and for the identification of the monetary policy shock.

Figure 13: Quantile IRFs with differing numbers of static factors.



(a) QR-IRFs ΔGDP h=4

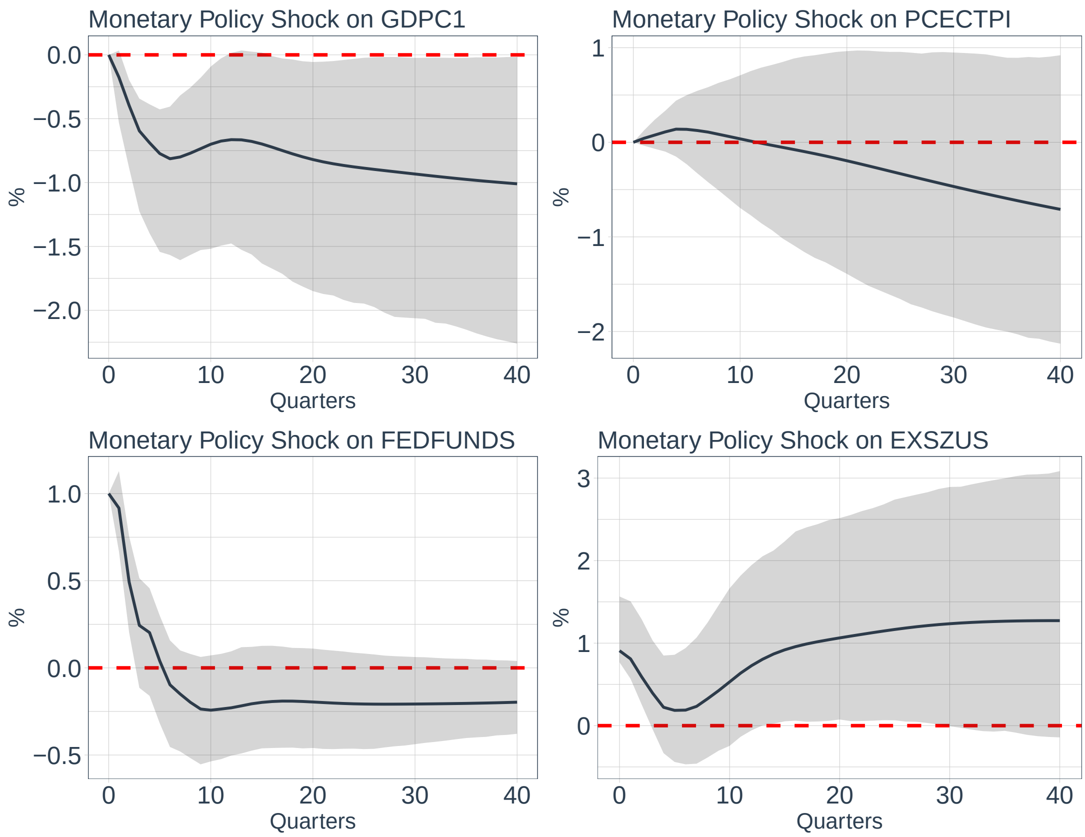


(b) QR-IRFs PCE inflation h=4

B.0:.2 Changing the number of lags

The AIC criterion suggests $p = 3$ lags for the VAR of the factors given by equation (4). In the main exercise I prefer the more parsimonious specification suggested by the HQ criterion. Figure (14) reports the impulse responses of the identifying variables to the monetary policy shock when $p = 3$. There are no major qualitative differences relative to the baseline case. Therefore, I refrain from reporting the resulting quantile IRFs as the choice of p does not impact the choice of regressors. As the structural IRF is largely unaffected, so are the results for the quantile IRFs.

Figure 14: Responses of identifying variables to 100 basis points monetary policy shock with $r = 8$ and $p = 3$.



Appendix C: Bootstrap procedure

To compute the confidence intervals for the impulse responses I broadly follow the procedure outlined in J. H. Stock & Watson (2016). First, I use a Wild bootstrap with changing sign to bootstrap the factor VAR in equation (5). The bootstrapped factors are then multiplied by the originally estimated matrix of loadings, $\hat{A}$ to obtain the common component χ_b (the subscript b stands for "bootstrapped"). Second, I add the original idiosyncratic component ξ_o to the bootstrapped common component. This step is owed to the fact that the idiosyncratic components are cross-sectionally weakly correlated. The proposed method in J. H. Stock & Watson (2016) is to run indivudual AR(p) models on each idiosyncratic series and bootstrap from there. However, this neglects the cross-sectional correlation. Third, I multiply the sum of common component χ_b and ξ_o by the standard deviations of the original data σ_o and add the mean of the original data μ_o. This rescales the bootstrapped data X_b to the scale of the original data.

$$\chi_b = \hat{A}F_b$$
$$X_b = \sigma_o(\chi_b + \xi_o) + \mu_0$$

Once this new data set is constructed I run it through the machinery of estimating the DFM and identifying the structural shocks as described in section 2. To compute the quantile IRFs I use the initial estimates of the β_τ coefficients in combination with the bootstrapped versions of $D(L)^{-1}, S, H$. The procedure is repeated 500 times and I take the 95$^{\text{th}}$ and 5$^{\text{th}}$ percentiles of the empirical distributions of the IRFs as the confidence bounds.

Appendix D: Variables, Transformations and Goodness of Fit

The variables are taken from FRED-QD. For more detailed information on the variables names see `https://s3.amazonaws.com/files.research.stlouisfed.org/fred-md/FRED-QD_appendix.pdf`.

Transformation code 1 means no transformation, 2 means taking first differences, 5 implies difference of the logs, 6 means double difference in the logs, 7 means first difference of percentage changes. The R^2 measure is from a linear regression of the transformed variable on the $r = 8$ and $r = 16$ static factors and a constant.

Variable	Trans.	ADF p-val.	$r = 8$	$r = 16$
GDPC1	5	0.01	0.86	0.94
pcecc96	5	0.01	0.65	0.74
PCDGx	5	0.01	0.46	0.53
PCESVx	5	0.01	0.56	0.67
PCNDx	5	0.01	0.46	0.59
GPDIC1	5	0.01	0.71	0.79
FPIx	5	0.01	0.75	0.81
Y033RC1Q027SBEAx	5	0.01	0.67	0.73
PNFIx	5	0.01	0.70	0.76
PRFIx	5	0.01	0.62	0.70
A014RE1Q156NBEA	1	0.01	0.63	0.77
GCEC1	5	0.01	0.20	0.72
A823RL1Q225SBEA	1	0.01	0.15	0.61
FGRECPTx	5	0.01	0.37	0.40
SLCEx	5	0.01	0.39	0.53
EXPGSC1	5	0.01	0.35	0.41
IMPGSC1	5	0.01	0.48	0.54
DPIC96	5	0.01	0.45	0.57
OUTNFB	5	0.01	0.87	0.93

Variable	Trans.	ADF p-val.	$r = 8$	$r = 16$
OUTBS	5	0.01	0.85	0.93
OUTMS	5	0.01	0.96	0.98
INDPRO	5	0.01	0.92	0.95
IPFINAL	5	0.01	0.89	0.92
IPCONGD	5	0.01	0.78	0.87
IPMAT	5	0.01	0.81	0.84
IPDMAT	5	0.01	0.79	0.83
IPNMAT	5	0.01	0.56	0.68
IPDCONGD	5	0.01	0.80	0.85
IPB51110SQ	5	0.01	0.66	0.73
IPNCONGD	5	0.01	0.40	0.65
IPBUSEQ	5	0.01	0.82	0.84
IPB51220SQ	5	0.01	0.06	0.60
TCU	2	0.01	0.93	0.94
CUMFNS	2	0.01	0.91	0.93
PAYEMS	5	0.01	0.96	0.97
USPRIV	5	0.01	0.96	0.97
MANEMP	5	0.01	0.90	0.93
SRVPRD	5	0.01	0.93	0.96
USGOOD	5	0.01	0.93	0.95
DMANEMP	5	0.01	0.90	0.92
NDMANEMP	5	0.01	0.78	0.84
USCONS	5	0.01	0.73	0.82
USEHS	5	0.01	0.74	0.76
USFIRE	5	0.01	0.68	0.69
USINFO	5	0.01	0.40	0.43
USPBS	5	0.01	0.84	0.87
USLAH	5	0.01	0.78	0.85
USSERV	5	0.01	0.83	0.85
USMINE	5	0.01	0.27	0.39
USTPU	5	0.01	0.93	0.94
USGOVT	5	0.01	0.66	0.71
USTRADE	5	0.01	0.88	0.90
USWTRADE	5	0.01	0.86	0.88
CES9091000001	5	0.01	0.11	0.22
CES9092000001	5	0.01	0.66	0.68
CES9093000001	5	0.01	0.70	0.73
CE16OV	5	0.01	0.85	0.88
CIVPART	2	0.01	0.59	0.62
UNRATE	2	0.01	0.89	0.93
UNRATESTx	2	0.01	0.86	0.91
UNRATELTx	2	0.01	0.70	0.79
LNS14000012	2	0.01	0.50	0.56
LNS14000025	2	0.01	0.89	0.91
LNS14000026	2	0.01	0.74	0.81

Variable	Trans.	ADF p-val.	$r = 8$	$r = 16$
UEMPLT5	5	0.01	0.41	0.50
UEMP5TO14	5	0.01	0.68	0.76
UEMP15T26	5	0.01	0.56	0.68
UEMP27OV	5	0.01	0.54	0.70
LNS13023621	5	0.01	0.87	0.91
LNS13023557	5	0.01	0.28	0.37
LNS13023705	5	0.01	0.22	0.35
LNS13023569	5	0.01	0.26	0.35
LNS12032194	5	0.01	0.64	0.68
HOABS	5	0.01	0.92	0.94
HOAMS	5	0.01	0.91	0.95
HOANBS	5	0.01	0.93	0.95
AWHMAN	2	0.01	0.74	0.76
AWHNONAG	2	0.01	0.52	0.56
AWOTMAN	2	0.01	0.68	0.72
HWIx	5	0.01	0.76	0.80
HOUST	5	0.01	0.68	0.88
HOUST5F	5	0.01	0.29	0.48
PERMIT	5	0.01	0.71	0.87
HOUSTMW	5	0.01	0.26	0.51
HOUSTNE	5	0.01	0.26	0.42
HOUSTS	5	0.01	0.52	0.63
HOUSTW	5	0.01	0.51	0.60
splice_CMRMTSPL	5	0.01	0.88	0.88
splice_retail	5	0.01	0.84	0.90
AMDMNOx	5	0.01	0.67	0.70
ACOGNOx	5	0.01	0.90	0.92
AMDMUOx	5	0.01	0.48	0.51
ANDENOx	5	0.01	0.34	0.40
INVCQRMTSPL	5	0.01	0.63	0.80
PCECTPI	5	0.01	0.96	0.97
PCEPILFE	5	0.01	0.96	0.97
GDPCTPI	5	0.01	0.93	0.94
GPDICTPI	5	0.01	0.77	0.84
IPDBS	5	0.01	0.91	0.93
DGDSRG3Q086SBEA	5	0.01	0.92	0.95
DDURRG3Q086SBEA	5	0.01	0.78	0.89
DSERRG3Q086SBEA	5	0.01	0.91	0.92
DNDGRG3Q086SBEA	5	0.01	0.87	0.92
DHCERG3Q086SBEA	5	0.01	0.90	0.91
DMOTRG3Q086SBEA	5	0.01	0.46	0.68
DFDHRG3Q086SBEA	5	0.01	0.69	0.73
DREQRG3Q086SBEA	5	0.01	0.75	0.79
DODGRG3Q086SBEA	5	0.01	0.52	0.60
DFXARG3Q086SBEA	5	0.01	0.49	0.65

Variable	Trans.	ADF p-val.	$r=8$	$r=16$
DCLORG3Q086SBEA	5	0.01	0.59	0.64
DGOERG3Q086SBEA	5	0.01	0.80	0.86
DONGRG3Q086SBEA	5	0.01	0.71	0.72
DHUTRG3Q086SBEA	5	0.01	0.73	0.79
DHLCRG3Q086SBEA	5	0.01	0.78	0.83
DTRSRG3Q086SBEA	5	0.01	0.59	0.60
DRCARG3Q086SBEA	5	0.01	0.63	0.65
DFSARG3Q086SBEA	5	0.01	0.75	0.81
DIFSRG3Q086SBEA	5	0.01	0.22	0.29
DOTSRG3Q086SBEA	5	0.01	0.61	0.65
CPIAUCSL	5	0.01	0.94	0.95
CPILFESL	5	0.01	0.89	0.92
WPSFD49207	5	0.01	0.88	0.93
PPIACO	5	0.01	0.85	0.91
WPSFD49502	5	0.01	0.86	0.92
WPSFD4111	5	0.01	0.31	0.49
PPIIDC	5	0.01	0.85	0.90
WPSID61	5	0.01	0.86	0.89
WPU0531x	5	0.01	0.21	0.54
WPU0561x	5	0.01	0.70	0.73
OILPRICEx	5	0.01	0.82	0.86
AHETPIx	5	0.01	0.70	0.76
CES2000000008x	5	0.01	0.34	0.52
CES3000000008x	5	0.01	0.47	0.58
COMPRMS	5	0.01	0.79	0.92
COMPRNFB	5	0.01	0.57	0.78
RCPHBS	5	0.01	0.55	0.76
OPHMFG	5	0.01	0.66	0.81
OPHNFB	5	0.01	0.67	0.83
OPHPBS	5	0.01	0.66	0.81
ULCBS	5	0.01	0.58	0.91
ULCMFG	5	0.01	0.82	0.90
ULCNFB	5	0.01	0.59	0.90
UNLPNBS	5	0.01	0.37	0.89
FEDFUNDS	1	0.39	0.92	0.95
tb3ms	1	0.41	0.92	0.95
tb6ms	1	0.43	0.93	0.96
gs1	1	0.45	0.94	0.96
GS10	1	0.61	0.92	0.96
MORTGAGE30US	1	0.67	0.95	0.97
AAA	1	0.63	0.92	0.96
BAA	1	0.62	0.92	0.96
BAA10YM	1	0.01	0.70	0.80
mrtggs10	1	0.01	0.58	0.73
tb6m3m	1	0.01	0.11	0.42

Variable	Trans.	ADF p-val.	$r=8$	$r=16$
gs1tb3m	1	0.01	0.39	0.66
gs10tb3m	1	0.01	0.34	0.83
cpf3mtb3m	1	0.02	0.86	0.94
BOGMBASEREALx	5	0.01	0.30	0.55
M1REAL	5	0.01	0.53	0.70
M2REAL	5	0.01	0.62	0.75
BUSLOANSx	5	0.01	0.55	0.62
consumerx	5	0.01	0.49	0.64
NONREVSLx	5	0.01	0.53	0.75
REALLNx	5	0.01	0.43	0.56
REVOLSLx	5	0.01	0.63	0.72
TOTALSLx	5	0.01	0.65	0.83
DRIWCIL	1	0.01	0.62	0.84
TABSHNOx	5	0.01	0.61	0.86
TLBSHNOx	5	0.01	0.41	0.61
LIABPI	5	0.01	0.29	0.58
TNWBSHNOx	5	0.01	0.59	0.85
NWPI	1	0.92	0.59	0.79
TARESA	5	0.01	0.53	0.83
HNOREMQ027Sx	5	0.01	0.36	0.60
TFAABSHNOx	5	0.01	0.53	0.83
vxoclsx	1	0.01	0.39	0.61
USSTHPIx	5	0.01	0.46	0.64
SPCS10RSAx	6	0.01	0.72	0.82
SPCS20RSAx	6	0.01	0.81	0.90
TWEXAFEGSMTHx	5	0.01	0.60	0.97
EXUSEU	5	0.01	0.63	0.91
EXSZUS	5	0.01	0.09	0.49
EXJPUS	5	0.01	0.03	0.43
EXUSUK	5	0.01	0.25	0.48
EXCAUS	5	0.01	0.47	0.50
UMCSENT	1	0.02	0.66	0.73
USEPUINDXM	2	0.01	0.70	0.81
B020RE1Q156NBEA	2	0.01	0.56	0.64
B021RE1Q156NBEA	2	0.01	0.62	0.75
GFDEGDQ188S	2	0.01	0.56	0.75
GFDEBTNx	2	0.01	0.39	0.50
IPMANSICS	5	0.01	0.93	0.96
IPB51222S	5	0.01	0.06	0.62
IPFUELS	5	0.01	0.38	0.44
UEMPMEAN	2	0.01	0.55	0.70
CES0600000007	2	0.01	0.61	0.65
TOTRESNS	5	0.01	0.25	0.51
NONBORRES	7	0.01	0.25	0.32
GS5	2	0.01	0.26	0.41

Variable	Trans.	ADF p-val.	$r = 8$	$r = 16$
TB3SMFFM	1	0.01	0.65	0.73
T5YFFM	1	0.01	0.42	0.83
AAAFFM	1	0.02	0.60	0.88
WPSID62	5	0.01	0.65	0.83
PPICMM	5	0.01	0.41	0.47
CPIAPPSL	5	0.01	0.63	0.70
CPITRNSL	5	0.01	0.83	0.87
CPIMEDSL	5	0.01	0.71	0.75
CUSR0000SAC	5	0.01	0.90	0.94
CUSR0000SAD	5	0.01	0.66	0.77
CUSR0000SAS	5	0.01	0.83	0.86
CPIULFSL	5	0.01	0.91	0.92
CUSR0000SA0L2	5	0.01	0.94	0.96
CUSR0000SA0L5	5	0.01	0.94	0.95
CES0600000008	5	0.01	0.76	0.82
DTCOLNVHFNM	5	0.01	0.19	0.48
DTCTHFNM	5	0.01	0.37	0.63
INVEST	5	0.01	0.33	0.48
hwium	2	0.01	0.67	0.81
claims	5	0.01	0.78	0.84
splice_BUSINV	5	0.01	0.74	0.82
splice_ISRATIO	2	0.01	0.79	0.86
conspi	2	0.01	0.34	0.72
cp3m	2	0.01	0.72	0.89
compapff	1	0.01	0.87	0.91
PERMITNE	5	0.01	0.29	0.41
PERMITMW	5	0.01	0.49	0.69
PERMITS	5	0.01	0.61	0.74
PERMITW	5	0.01	0.53	0.65
NIKKEI225	5	0.01	0.23	0.36
NASDAQCOM	5	0.01	0.51	0.77
CUSR0000SEHC	5	0.01	0.81	0.89
TLBSNNCBx	5	0.01	0.18	0.34
TLBSNNCBBDIx	1	0.01	0.59	0.70
TTAABSNNCBx	5	0.01	0.35	0.63
TNWMVBSNNCBx	5	0.01	0.35	0.57
TNWMVBSNNCBBDIx	2	0.01	0.34	0.71
TLBSNNBx	5	0.01	0.31	0.50
TLBSNNBBDIx	1	0.01	0.49	0.67
TABSNNBx	5	0.01	0.50	0.80
TNWBSNNBx	5	0.01	0.45	0.75
TNWBSNNBBDIx	2	0.01	0.26	0.70
CNCFx	5	0.01	0.24	0.74
S_P_500	5	0.01	0.62	0.86
S_P__indust	5	0.01	0.59	0.84

Variable	Trans.	ADF p-val.	$r = 8$	$r = 16$
S_P_div_yield	2	0.01	0.60	0.76
S_P_PE_ratio	5	0.01	0.45	0.64