# Time-varying preferences for storable goods*

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**Abstract**

We investigate a storable good market where a firm faces consumers with time-varying preferences that differ between the purchase stage and the consumption stage. Consumers hold (possibly biased) beliefs at the purchase stage. The quantity purchased for storage — i.e., ex ante storage — stimulates current sales, whereas the quantity actually stored — i.e., ex post storage — depresses future sales. We identify a trade-off such that the firm may resort to price cuts to stimulate ex ante storage or price rises to dampen ex post storage. Ex post discrimination emerges across ex ante identical consumers. Our analysis provides testable predictions and policy implications.

KEYWORDS: dynamic pricing, naïve consumers, rational consumers, time-varying preferences, sophisticated consumers, storable goods, storage.
JEL CLASSIFICATION: D21, D42, L12.

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# 1 Introduction

A large portion of products traded in upstream or downstream markets are perishable in consumption but can be stored for future consumption. The relevance of consumer storage has been empirically documented, especially in anticipation of higher future prices (e.g., Erdem et al. 2003; Hendel and Nevo 2004, 2006a, 2006b; Osborne 2018; Perrone 2017; Pesendorfer 2002; Pires 2016; Wang 2015). This suggests that a firm selling a storable good is inclined to devote significant attention to future changes in market conditions. Natural examples studied in the economic literature are intertemporal fluctuations in demand (Dudine et al. 2006) or in production costs (Antoniou and Fiocco 2023).

In this paper, we characterize a firm's dynamic pricing policy and consumers' storage incentives in the presence of consumers whose preferences vary over time. Departing from the extant literature, we consider time-varying preferences that differ between the purchase stage and the consumption stage. For a wide range of products, consumers can only form beliefs about their tastes at the time of purchase and learn their actual valuation at the time of consumption. This is typically the case of experience goods (e.g., wine, medications, health care and beauty products). As emphasized in the literature on consumer naïveté, some consumers hold biased beliefs about their preferences and revise their decisions over time for different reasons, such as present-focus preferences, preference reversals and time inconsistency (e.g., Ericson and Laibson 2019; Laibson 1997; O'Donoghue and Rabin 1999; Strack and Taubinsky 2022).

Time-varying preferences for storable goods imply that the quantity purchased for storage, referred to as *ex ante storage*, may depart from the quantity actually stored, referred to as *ex post storage*. The firm benefits from ex ante storage, which stimulates current sales. Conversely, ex post storage is detrimental to the firm because it depresses future sales. We show that the firm faces a trade-off when promoting ex ante storage. To suitably identify the forces behind this trade-off, we first examine a setting characterized by consumer naïveté. A fraction of the consumer population consists of 'sophisticated' consumers that hold unbiased beliefs and perfectly know their preferences at the purchase stage. The remaining fraction is formed by 'naïve' consumers that hold biased beliefs at the purchase stage and learn their actual preferences at the consumption stage. Naïfs may either underestimate or overestimate their preferences when purchasing the good.[1] With naïve underestimating consumers, the firm benefits from promoting ex ante storage because it anticipates that naïfs shall consume a larger quantity after learning their higher valuation for the good and thus the amount of ex ante storage translates into further consumption. The presence of consumers that become aware of their stronger preferences inflates future demand and leads to rising prices over time. However, the promotion of ex ante storage comes at the firm's cost that sophisticates accumulate this amount in the form of ex post storage, which is consumed without being bought at a higher price in the future. The firm prefers to stimulate ex ante storage for sufficiently small values of the storage cost. The price is set below the static monopoly level to encourage ex ante storage, especially as long as static monopoly prices are feasible. Conversely, when naïve consumers overestimate

[1] Our insights qualitatively carry over to a setting where underestimating consumers coexist with overestimating consumers. We refer to the discussion in Section 4 for details.

their preferences, ex ante storage is detrimental to the firm because it cannot induce further consumption. Demand falls in the future and prices decline over time. As consumers do not have any incentives to buy for storage purposes in anticipation of lower future prices, ex ante storage disappears. However, naïfs still prefer to store ex post by keeping some leftovers for future consumption after learning their lower valuation for the good, provided that the storage cost is sufficiently small. Naïfs' ex post storage is such that the level of consumption equalizes their marginal utilities over time. To dampen the amount of ex post storage, which depresses future sales, the firm increases the price above the static monopoly level.

Equipped with the results under consumer naïveté, we turn to a setting where all consumers are rational and hold unbiased beliefs but they are uncertain about their preferences at the purchase stage. This allows us to endogenize the firm's choice between allowing ex ante storage (which leads to ex post storage as well) and only ex post storage. Notably, under consumer naïveté, this choice is crucially driven by whether naïfs underestimate or overestimate their preferences, which inflates or curbs future demand, respectively. Furthermore, with naïve underestimating consumers, only sophisticates store ex post and, with naïve overestimating consumers, ex ante storage vanishes. With rational consumers, the levels of ex ante and ex post storage are simultaneously determined in a non-trivial manner. As buyers that discover high valuation for the good are induced to consume the entire quantity bought, a crucial condition for the identification of ex ante and ex post storage is that the level of consumption by buyers with low evaluation equalizes their marginal utilities over time. Remarkably, this condition is directly satisfied for all consumers in a classical setting where preferences are invariant between the purchase and the consumption stage. When promoting ex ante storage, the firm benefits from further consumption by buyers that discover high valuation for the good. However, buyers with low valuation accumulate in the form of ex post storage not only the entire amount of ex ante storage but also the difference between their expected and actual demand. We show that, as a result of this trade-off, the firm prefers to stimulate ex ante storage when future demand is sufficiently high. Otherwise, the firm opts for allowing only ex post storage. Interestingly, we find that allowing only ex post storage can be more profitable for the firm even when ex ante storage is feasible. As ex ante storage stimulates current sales and ex post storage depresses future sales, this result might appear prima facie surprising. To gain some insights, it is helpful to note that allowing only ex post storage reduces ceteris paribus the quantity bought and stored for future consumption. Furthermore, as prices tend to decrease over time, buyers are more inclined to consume rather than store the good. The firm's dynamic pricing policy and the associated distortions from the static monopoly level resemble those derived under consumer naïveté. Although consumers are ex ante identical, the firm implements ex post discrimination by favoring consumption by buyers with high valuation for the good.

Our analysis is conducted in a fairly general setting and can be extended in different directions. Specifically, we consider the case where the firm offers a nonlinear pricing policy in the form of a contract menu that screens consumers according to their actual valuation for the good, once it has been realized. In addition to the standard informational rents stemming from superior information, consumers can gain from postponing their purchases in anticipation of a discount that the firm provides in the future because they can resort to the quantity stored. Consumers that discover high valuation for the good are better off than in the static solution, whereas consumers that learn low valuation incur larger losses. Furthermore, we compare our results derived under the firm's lack of commitment to future prices with those under full commitment. When the firm prefers to promote ex ante storage, price comparisons crucially depend on the shape of demand. With only ex post storage, prices move in different directions across periods. As argued in Section 6, our results deliver a range of potentially significant testable predictions and policy implications in markets where consumers exhibit preferences that vary over time.

**Structure of the paper.** The rest of the paper is organized as follows. Section 2 describes the related literature. Section 3 sets out the formal model and derives the equilibrium features of dynamic pricing and storage under consumer naïveté. Section 4 turns to a setting with rational consumers. Section 5 examines different extensions. Section 6 concludes the analysis with testable predictions and policy implications. The Appendix collects the main formal proofs. The Supplementary Appendix provides additional formal results and associated proofs.

## 2 Related literature

Our paper belongs to the extensive literature on storable goods. The influential work of Bénabou (1989) characterizes a firm's optimal pricing policy in an inflationary environment where a continuum of speculators engage in storage activities detrimental to the firm. Jeuland and Narasimhan (1985) identify price discrimination across consumers as an explanation for temporary discounts. In a setting where a share of consumers can store, Hong et al. (2002) show that consumer storage generates equilibrium price dispersion. Our study is closely related to the seminal paper of Dudine et al. (2006), which considers a monopoly market for storage goods where demand fluctuates deterministically over time. Differently from our framework, consumer preferences do not change between the purchase stage and the consumption stage, which implies that ex ante storage coincides with ex post storage. In a setting à la Dudine et al. (2006), Antoniou and Fiocco (2019) show that a firm unable to commit to future prices exhibits strategic incentives to hold inventories when facing the possibility of buyer stockpiling. Adopting a supply side perspective, Antoniou and Fiocco (2023) find that with intertemporal cost variations price dynamics hinge upon the curvature of demand and the magnitude of the consumer storage cost. Hendel et al. (2014) study nonlinear pricing of storable goods and find cyclical patterns in prices and sales. Heterogeneity in the consumers' ability to store implies that larger bundles are more likely to be on sale. Adding consumer storage into Su's (2007) analysis of a seller's dynamic pricing in the presence of strategic buyers, Su (2010) shows that the seller may set a constant fixed price or provide periodic price promotions at predictable time intervals. Hendel and Nevo (2013) investigate intertemporal price discrimination across consumers with heterogeneous storage abilities and derive temporary price reductions. Mitraille and Thille (2009) show that, when production is controlled by a monopolist, speculators' competitive storage affects both the level and the volatility of price. The economic effects of storability have also been studied in competitive environments. In a market where two firms compete in quantities, Anton and Das Varma (2005) show that competition for consumer storage leads to price cuts and equilibrium prices rise over time. Under price competition with differentiated goods, Guo and Villas-Boas (2007) find that preference heterogeneity generates differential propensity for consumer storage, which strengthens future competition and may prevent consumer storage from occurring in equilibrium. Nava and Schiraldi (2014) examine firms' incentives to implement periodic price reductions in order to enforce collusion. In a market characterized by imperfectly competitive production, Mitraille and Thille (2014) find that firms' incentives to sell to speculators can be quite strong, which may lead to sufficiently high prices to drive consumers out of the market. In a subsequent work, Mitraille and Thille (2016) show that in an infinite horizon game producers' incentives to sell to speculators are non-monotonic in the number of producers.

Our work can also contribute to the fast-growing literature on consumer naïveté. The pioneering work of Strotz (1955) recognizes that individuals may exhibit time inconsistent behavior because the discount applied to a future utility depends on the time distance from the present date. In a setting where consumers exhibit hyperbolic discount functions (which induce dynamically inconsistent preferences) and may invest in an illiquid asset as an imperfect commitment device, Laibson (1997) shows that consumption tracks income and provides a rationale for asset-specific marginal propensities to consume. O'Donoghue and Rabin (1999) consider present-biased preferences (which give stronger relative weight to the earlier moment as it gets closer) and find that naïve people, who fail to foresee their self-control problems, procrastinate immediate-cost activities but preproperate (do too soon) immediate-reward activities. In a two-period model where an ex ante inferior choice may tempt the decision-maker in the second period, Gul and Pesendorfer (2001) characterize a range of axioms that allow both a preference for commitment and self-control. Eliaz and Spiegler (2006) derive the menu of contracts that the principal offers to an agent with dynamically inconsistent preferences in order to screen the agent's types, which differ in their degree of sophistication, and show that the principal can exploit more naïve types to a larger extent. Fudenberg and Levine (2006) develop a 'dual-self' model where in each stage game the long-run self chooses a self-control action and then the short-run (completely myopic) self takes the final decision. Heidhues and Kőszegi (2009) investigate costly yet futile attempts at self-control for consumption of a harmful product and find that higher sophistication often decreases welfare. Incorporating uncertainty about the nature of temptation into the classical Strotz (1955) model, Dekel and Lipman (2012) find a connection with the self-control model of Gul and Pesendorfer (2001). Based on limited attention, implementation errors or costly decision making, Fudenberg and Strzalecki (2015) characterize a generalization of discounted logistic choice, where the attractiveness of menus is adjusted to reflect the agent's choice aversion. Heidhues and Kőszegi (2017) investigate naiveté-based price discrimination through which offers are conditioned on external information about consumer naïveté and find that the exploitation of consumer's mistakes can lead to Pareto-inferior outcomes and often reduces total welfare. In a market for durable goods with positive network externalities where consumers can be sophisticated or naïve in terms of beliefs about future network sizes, Hattori and Zennyo (2018) show that the firm may charge the sequential-diffusion pricing that makes sophisticated consumers early adopters. Heidhues and Strack (2021) demonstrate that, when a partially-naive quasi-hyperbolic discounter repeatedly chooses whether to complete a task, the probability of completing it conditional on not having done so earlier increases towards the deadline. Strack and Taubinsky (2022) characterize the conditions under which time inconsistency can be identified and its degree can be estimated. We refer to Ericson and Laibson (2019) for an excellent review of models about intertemporal choice, which devotes special attention to present-focus preferences and preference reversals. Consumer naïveté has also been experimentally and empirically estimated. Read and van Leeuwen (1998) perform a field experiment where participants are asked to choose a snack to be delivered after seven days and at that designated time they have the (unexpected) opportunity to revise their choices. Participants turn out to be dynamically inconsistent because they choose far more unhealthy snacks for immediate choice than for advance choice. DellaVigna and Malmendier (2004) show that the optimal contract between profit maximizing firms and consumers with time inconsistent preferences and naïve beliefs matches the empirical observations in a range of industries, such as credit card, gambling, health club, life insurance, mail order, mobile phone, and vacation time-sharing. Analyzing a data set from three US health clubs, DellaVigna and Malmendier (2006) find that consumer behavior is difficult to reconcile with standard preferences and beliefs, which suggests that consumers are overconfident about future self-control or about future efficiency. The experimental evidence in Halevy (2015) challenges the view that present-bias preferences are the main source of time inconsistent choices. Acland and Levy (2015) experimentally document the presence of partial naïveté with respect to present bias and of habit formation in gym attendance. In Fedyk's (2022) online laboratory experiment, participants exhibit significant present bias, naïveté about their present bias, and sophisticated beliefs about others' present bias.

# 3 Consumer naïveté

## 3.1 The model

**Consumers.** We consider a two-period market for a storable good characterized by a continuum of consumers normalized to unity. A fraction $\lambda \in (0,1)$ of the consumer population consists of 'sophisticated' consumers and the remaining fraction $1-\lambda$ is formed of 'naïve' consumers. Specifically, sophisticated consumers hold unbiased beliefs and perfectly know their preferences at the purchase stage. They have a quasi-linear utility function $U^s(x_\tau, y_\tau) = u^s(x_\tau) + y_\tau$ that depends on the consumption level $x_\tau$ and money $y_\tau$ in period $\tau \in \{1,2\}$. The continuously differentiable function $u^s(\cdot)$ is increasing and concave in $x_\tau$, i.e., $u^{s\prime}(\cdot) > 0$ and $u^{s\prime\prime}(\cdot) < 0$, with the standard normalization $u^s(0) = 0$. Naïve consumers hold biased beliefs about their preferences. At the first period purchase stage naïfs believe that they share the same utility $U^s(x_\tau, y_\tau)$ as of sophisticates. However, at the consumption stage they realize that their actual preferences are characterized by $U^n(x_\tau, y_\tau) = u^n(x_\tau) + y_\tau$, where $u^n(\cdot)$ differs from $u^s(\cdot)$. This implies that consumers are ex ante identical but ex post heterogeneous.[2]

[2]Our results qualitatively carry over to a more complicated setting where naïfs also differ ex ante from sophisticates in terms of beliefs. As it will clear below, what matters for our analysis is that (at least some) consumers exhibit preferences that vary between the purchase and the consumption stage.

The continuously differentiable function $u^n(\cdot)$ is increasing and concave in $x_\tau$, i.e., $u^{n\prime}(\cdot) > 0$ and $u^{n\prime\prime}(\cdot) < 0$, with the standard normalization $u^n(0) = 0$. We identify two main classes of naïve consumers according the direction of their belief biases.

*Underestimating consumers*: Naïfs that underestimate their preferences exhibit $u^{n\prime}(\cdot) > u^{s\prime}(\cdot)$ and $|u^{n\prime\prime}(\cdot)| \geq |u^{s\prime\prime}(\cdot)|$ for any consumption level $x_\tau$.

*Overestimating consumers*: Naïfs that overestimate their preferences exhibit $u^{n\prime}(\cdot) < u^{s\prime}(\cdot)$ and $|u^{n\prime\prime}(\cdot)| \leq |u^{s\prime\prime}(\cdot)|$ for any consumption level $x_\tau$.

Underestimating consumers anticipate a lower utility than the actual one for any consumption level, whereas overestimating consumers expect a higher utility than what eventually materializes.[3]

Consumers can store the good in the first period for consumption in the second period at a unit cost $c \geq 0$. Without loss of insights, we assume no discounting on the second period. Given any price sequence $\{p_\tau\}_{\tau=1}^{2}$, consumers purchase $q_\tau$ and then consume $x_\tau$ in each period $\tau \in \{1, 2\}$. They also choose the storage level $z$ in the first period, which constitutes the amount of the good to be stored for consumption in the second period. At the first period purchase stage all consumers maximize the aggregate net utility $\Psi^s(x_\tau, z) \triangleq \sum_{\tau=1}^{2} [U^s(x_\tau, y_\tau) - q_\tau p_\tau] - cz$. Sophisticated consumers allocate the quantity bought in the first period between consumption and storage, i.e., $q_1 = x_1 + z$, whereas the quantity bought in the second period satisfies $q_2 = x_2 - z$ because in the second period consumers can enjoy the quantity stored in the first period. However, after buying in the first period, naïve consumers learn their actual preferences and maximize $\Psi^n(\overline{x}_\tau, \overline{z}) \triangleq \sum_{\tau=1}^{2} U^n(\overline{x}_\tau, y_\tau) - q_1 p_1 - \overline{q}_2 p_2 - c\overline{z}$. Given the quantity $q_1$ bought in the first period (which coincides with the one of sophisticates), naïfs adjust the levels of consumption and storage according to their actual preferences, i.e., $q_1 = \overline{x}_1 + \overline{z}$, whereas the quantity bought in the second period satisfies $\overline{q}_2 = \overline{x}_2 - \overline{z}$. For any consumer of type $i = s, n$ (where $s$ stands for sophisticates and $n$ for naïfs), we define by $D^i(p_\tau) \triangleq \arg\max_q \{u^i(q) - qp_\tau\}$ the static demand function associated with utility $U^i(\cdot)$, which is continuously differentiable and decreasing with the price $p_\tau$, i.e., $D^{i\prime}(p_\tau) < 0$. For later purposes, we also define by $\sigma(p_\tau) \triangleq \lambda D^s(p_\tau) + (1 - \lambda) D^n(p_\tau)$ the aggregate demand in period $\tau \in \{1, 2\}$ when all consumers know their actual preferences. Ex ante storage $S(p_1, p_2^e)$ represents the amount of the good that consumers buy for storage purposes at the first period purchase stage when facing the first period price $p_1$ and expecting the second period price $p_2^e$. Ex post storage $\overline{S}(p_1, p_2^e)$ denotes the level of storage that consumers actually carry in the second period. As naïfs have incentives to revise their consumption and storage decisions after learning their preferences, the amount of ex post storage $\overline{S}(p_1, p_2^e)$ may differ from the amount of ex ante storage $S(p_1, p_2^e)$.

**Firm.** A monopolistic firm operates in the market. The firm's aggregate profit is $\Pi \triangleq \Pi_1 + \Pi_2$, where the first period profit $\Pi_1$ and the second period profit $\Pi_2$ are respectively given by

$$\Pi_1 = p_1 [D^s(p_1) + S(p_1, p_2^e)] \quad \text{and} \quad \Pi_2 = p_2 \left[\sigma(p_2) - \overline{S}(p_1, p_2^e)\right]. \tag{1}$$

[3]The relation between $u^{n\prime}(\cdot)$ and $u^{s\prime}(\cdot)$ ensures that the difference in the corresponding static demand functions exhibits the same sign for any price. The relation between $u^{n\prime\prime}(\cdot)$ and $u^{s\prime\prime}(\cdot)$ implies that such difference does not decrease (in absolute value) with the price. This assumption allows a sharper characterization of our results and is imposed only for technical tractability.

When purchasing the good in the first period, naïfs believe that their preferences coincide with those of sophisticates. The first period demand for consumption $D^s(p_1)$ is inflated by the level of ex ante storage $S(p_1, p_2^e)$. In the second period, however, all consumers are aware of their actual preferences and the demand for consumption becomes $\sigma(p_2)$. Ex post storage $\overline{S}(p_1, p_2^e)$ curbs the second period demand because consumers resort to this quantity stored in the first period in order to satisfy their consumption needs in the second period. Due to the lack of commitment to future prices, the firm maximizes its continuation profit in each period. Production costs are normalized to zero. The firm's profit $\Pi_\tau$ in period $\tau$ satisfies the following standard assumption.

*Profit concavity* $\Pi''_\tau(p_\tau) < 0$, $\tau \in \{1, 2\}$.

This ensures that the second-order conditions for profit maximization are fulfilled.

**Timing and equilibrium concept.** The first period of the game includes the following three stages. First, the firm sets the price for the good. Second, consumers purchase a quantity of the good. Third, consumers learn their actual preferences and consumption takes place. In the second period the game evolves as in the first period, with the only difference that all consumers know their actual preferences. The solution concept is the subgame perfect Nash equilibrium.

### 3.2 Two relevant benchmarks

We start our analysis with two benchmark cases. First, we consider the case where all consumers are perfectly aware of their actual preferences. The firm's static profit maximization problem in period $\tau \in \{1, 2\}$ writes as

$$\max_{p_\tau} p_\tau \sigma(p_\tau).$$

As the firm faces the same problem in each period, the equilibrium static monopoly price $p_\tau^m$ remains constant over time. Hence, consumers do not exhibit any incentives to store and the firm's dynamic profit maximization problem reduces to a replica of the static problem.

Now, we turn to the case where a fraction $1 - \lambda$ of the consumer population consists of naïfs believing that they have the same preferences as sophisticates but consumers cannot store. This identifies the static solution in the presence of consumer naïveté. In the first period, the firm's profit maximization problem is given by

$$\max_{p_1} p_1 D^s(p_1), \tag{2}$$

which determines the first period equilibrium static monopoly price $p_1^{sm}$. In the second period, all consumers know their actual preferences and the firm's profit maximization problem becomes

$$\max_{p_2} p_2 \sigma(p_2), \tag{3}$$

which yields the second period equilibrium static monopoly price $p_2^m$. The sequence of static monopoly prices evolves over time according to the direction of naïfs' belief biases. When naïfs underestimate their preferences at the first period purchase stage, demand grows in the second period and thus the static monopoly prices rise over time, i.e., $p_1^{sm} < p_2^m$. The firm increases the price in the second period because naïfs have learned their higher valuation for the good. As described in Section 5, for sufficiently small values of the storage cost, consumers' eagerness to store prevents the firm from achieving the static solution. Conversely, when naïfs overestimate their preferences at the first period purchase stage, demand falls in the second period and thus the static monopoly prices decline over time, i.e., $p_1^{sm} > p_2^m$. Anticipating a lower future price, consumers do not have any incentives to store at the purchase stage. However, after learning their actual preferences, naïfs are inclined to revise their consumption and storage decisions. This can generate some amount of ex post storage that affects the firm's pricing policy. Below, we disentangle our analysis according to naïfs' belief biases.

### 3.3 Underestimating consumers

As all consumers are convinced of having utility $U^s(\cdot)$ at the first period purchase stage, they choose the same level of ex ante storage, namely, the amount of the good bought for storage purposes and intended for future consumption before their actual preferences are realized. Differentiating the aggregate net utility $\Psi^s(\cdot)$ (defined in Section 3) with respect to the storage level $z$ subject to the associated constraints yields $\partial\Psi^s/\partial z = -p_1 - c + p_2^e$. When consumers store a unit of the good in the first period for consumption in the second period, they incur a cost equal to the first period price inflated by the storage cost, i.e., $p_1 + c$. Buying in the second period, consumers expect to pay the second period price $p_2^e$. Then, if $p_1 + c < p_2^e$ (i.e., $\partial\Psi^s/\partial z > 0$), consumers prefer to buy the amount of the good that allows storing the entire quantity consumed in the second period. If $p_1 + c = p_2^e$ (i.e., $\partial\Psi^s/\partial z = 0$), consumers are indifferent between buying for storage purposes and waiting until the second period to purchase the good. If $p_1 + c > p_2^e$ (i.e., $\partial\Psi^s/\partial z < 0$), consumers prefer to buy only for current consumption. Ex ante storage $S(p_1, p_2^e)$ reflects the storage level $z$ that maximizes the aggregate net utility $\Psi^s(\cdot)$ subject to the associated constraints. In line with the main literature (e.g., Anton and Das Varma 2005; Dudine et al. 2006), ex ante storage is given by

$$S(p_1, p_2^e) = \begin{cases} \sigma(p_1 + c) & \text{if} \quad p_1 + c < p_2^e \\ [0, \sigma(p_1 + c)] & \text{if} \quad p_1 + c = p_2^e \\ 0 & \text{if} \quad p_1 + c > p_2^e. \end{cases} \tag{4}$$

As shown below, the last two cases are the only relevant outcomes in equilibrium.[4] Throughout the paper, we refer to $p_1 + c \geq p_2$ as the *storability constraint*.[5]

Three pricing policies are potentially at the firm's disposal. The first pricing policy is such that the storability constraint is strictly satisfied, i.e., $p_1 + c > p_2$. This removes consumers'

[4] In anticipation that actual preferences will be realized before the second period starts, for $p_1 + c < p_2^e$ the level of ex ante storage corresponds to the entire second period demand $\sigma(p_1 + c)$, provided that arbitrage is allowed. Otherwise, consumers intend to store $D^s(p_1 + c)$. This can be incorporated into our model without affecting the qualitative results.

[5] For expositional convenience, we sometimes refer to $p_2$ instead of $p_2^e$, because the expected price coincides with the equilibrium price in the second period (under perfect forsight and no uncertainty).

storage incentives and leads to the static monopoly prices, provided that they can be implemented, as described below.[6] The second pricing policy makes the storability constraint binding, i.e., $p_1 + c = p_2$. At the purchase stage, consumers are indifferent between storing the good in the first period and purchasing it in the second period. The third pricing policy, i.e., $p_1 + c < p_2$, induces consumers to buy the good in order to store the entire demand in the second period. Yet, this pricing policy is not implementable because the firm succumbs to the temptation to reduce the price in the second period, which enhances its profit through higher sales.[7]

As the firm cannot commit to future prices, we find from the second period profit in (1) that the firm's dynamic profit maximization problem is subject to the following *sequential optimality constraint*

$$p_2\left(\overline{S}\left(p_1, p_2^e\right)\right) \triangleq \arg\max_{\widetilde{p}_2} \widetilde{p}_2\left[\sigma\left(\widetilde{p}_2\right) - \overline{S}\left(p_1, p_2^e\right)\right]. \tag{5}$$

The second period price $p_2\left(\cdot\right)$ maximizes the firm's second period profit for any level of ex post storage $\overline{S}\left(p_1, p_2^e\right) \geq 0$ inherited from the first period. As argued in Section 3.1, ex post storage $\overline{S}\left(p_1, p_2^e\right)$ may differ from ex ante storage $S\left(p_1, p_2^e\right)$ because naïve consumers have incentives to adjust their consumption and storage decisions after learning their actual preferences. Notably, if the storability constraint is binding, i.e., $p_1 + c = p_2$, consumers are indifferent about the amount of ex ante storage but the sequential optimality constraint (5) dictates the amount of ex post storage.

As discussed in Section 3.2, when naïve consumers underestimate their preferences, the first period static monopoly price $p_1^{sm}$ is lower than the second period static monopoly price $p_2^m$. The static solution is still implementable under the possibility of storage as long as consumers do not have any (strict) incentives to store, i.e., $p_1^{sm} + c \geq p_2^m$. There exists a threshold $\widetilde{c}^u \triangleq p_2^m - p_1^{sm} > 0$ for the storage cost $c$ such that the static solution is implementable if and only if it satisfies the following *static feasibility constraint*

$$c \geq \widetilde{c}^u. \tag{6}$$

The storage cost $c$ must be sufficiently large to dissuade consumers from buying in the first period to store for consumption in the second period at the static monopoly prices. Otherwise, consumers are inclined to store and the static solution cannot be achieved.

The firm's dynamic problem is affected by the possibility of storage at least when the static feasibility constraint (6) fails to hold. As a lower storage cost makes consumers more eager to store, we find that there exists a threshold $\overline{c}^u > 0$ for the storage cost $c$ below which storage can emerge in equilibrium.[8] The dynamic storage solution must satisfy the following *dynamic feasibility constraint*

$$c < \overline{c}^u. \tag{7}$$

[6]Intuitively, after discovering their higher valuation for the good, a fortiori naïve consumers do not intend to engage in storage activities.

[7]We refer to the proof of Lemma 1 for technical details.

[8]Further details can be found in Proposition 1.

For $c \geq \bar{c}^u$, storage definitely harms the firm because it is too costly. Notably, the firm faces the following trade-off. On the one hand, although consumers behave identically at the first period purchase stage, the firm benefits from storage by anticipating that naïve consumers shall discover their higher valuation for the good and prefer to consume in the first period more than what they initially planned. On the other hand, storage is detrimental to the firm because prices are linked through the binding storability constraint, i.e., $p_1 + c = p_2$, and thus the firm forgoes $c$ for each unit of the good bought at $p_1$ and stored in the first period, which is consumed without being bought at $p_2$ in the second period.

We can now proceed with the equilibrium characterization. Given the firm's profit $\Pi \triangleq \Pi_1 + \Pi_2$, where $\Pi_1$ and $\Pi_2$ are described in (1), and the sequential optimality constraint (5), the firm's dynamic profit maximization problem writes as

$$\max_{\{p_1, p_2\}} \Pi(p_1, p_2) \quad s.t. \ (5). \tag{8}$$

Before solving for the equilibrium, in the following lemma we provide the main features of the static and dynamic solution.

**Lemma 1** *A. The static solution is feasible if and only if $c \geq \tilde{c}^u$. Prices are $p_1^{sm}$ and $p_2^m$, where $p_1^{sm} + c \geq p_2^m$.*

*B. The dynamic storage solution is feasible if and only if $c < \bar{c}^u$. Prices are $p_1^{*u}$ and $p_2^{*u} = p_1^{*u} + c$, with ex ante storage $S^{*u} > 0$ and ex post storage $\overline{S}^{*u} = \lambda S^{*u} > 0$. For $c \geq \bar{c}^u$, the dynamic solution exhibits no storage. Prices are $p_1^{**u} = p_2^{**u} - c$ and $p_2^{**u} = p_2^m$.*

The results in point A of Lemma 1 directly follow from the static feasibility constraint (6). When the storage cost is large enough, i.e., $c \geq \tilde{c}^u$, the firm can charge the first period static monopoly price $p_1^{sm}$ at which consumers do not have any (strict) incentives to store, i.e., $p_1^{sm} + c \geq p_2^m$. Hence, the second period static monopoly price $p_2^m$ satisfies the sequential optimality constraint (5). A rise in the fraction $\lambda$ of sophisticates (with lower valuation for the good) curbs the second period demand $\sigma(\cdot)$ (defined in Section 3) and leads to a reduction in the second period static monopoly price $p_2^m$. This implies that a higher $\lambda$ reduces the threshold $\tilde{c}^u$, which relaxes the static feasibility constraint (6).

Point B of Lemma 1 formalizes the dynamic solution. In this case, the possibility of storage affects the firm's dynamic pricing policy. We know from the dynamic feasibility constraint (7) that the firm can induce consumers to store when the storage cost is relatively small, i.e., $c < \bar{c}^u$. Prices are linked through the binding storability constraint, i.e., $p_2^{*u} = p_1^{*u} + c$, which makes consumers indifferent about storage. The firm can adjust the first period price to manipulate storage through the sequential optimality constraint (5). Specifically, the firm can promote storage by cutting the first period price. A reduction in the first period price leads to a lower price in the second period as well (due to the binding storability constraint) and thus requires a higher amount of storage to dampen the second period demand net of storage so that the sequential optimality constraint (5) is satisfied. As previously discussed, the firm benefits from inducing consumers to store at the first period purchase stage, because it anticipates that naïfs shall consume a larger quantity after learning their higher valuation for the good.

This yields the level of ex ante storage $S^{*u} > 0$. Notably, the firm prefers to set prices such that only sophisticates store some quantity for future consumption, which leads to the level of ex post storage $\overline{S}^{*u} = \lambda S^{*u} > 0$. Any price that induces naïfs to carry some quantity in the second period would be unduly low for the firm, which would lose $c$ for each unit sold at $p_1$ and stored in the first period rather than sold at $p_2 = p_1 + c$ in the second period. Under certain circumstances, ex post storage $\overline{S}^{*u}$ declines with the fraction $\lambda$ of sophisticates. The reason is that a higher $\lambda$ translates into a lower number of naïfs, which makes storage less attractive to the firm. In this case, the threshold $\bar{c}^u$ declines with $\lambda$ as well, which tightens the dynamic feasibility constraint (7).[9] When the storage cost is large enough, i.e., $c \geq \bar{c}^u$, the dynamic storage solution cannot be implemented because the dynamic feasibility constraint (7) is violated. Prices are still linked through the binding storability constraint but the second period price is set at the static monopoly level, as implied by the sequential optimality constraint (5).

Given the static feasibility constraint (6) and the dynamic feasibility constraint (7), we find from Lemma 1 that for $\tilde{c}^u < c < \bar{c}^u$ the static solution and the dynamic storage solution are both implementable. In the following lemma, we establish the condition under which this interval is non-empty.

**Lemma 2** *There exists a threshold $\widetilde{\lambda}$ such that $\bar{c}^u > \tilde{c}^u$ if and only if $\lambda < \widetilde{\lambda}$.*

Lemma 2 shows that the threshold for the dynamic storage solution is higher than the one for the static solution, i.e., $\bar{c}^u > \tilde{c}^u$, if and only if the fraction $\lambda$ of sophisticated consumers is small enough, i.e., $\lambda < \widetilde{\lambda}$.[10] In this case, there exists a range for the storage cost, i.e., $\tilde{c}^u < c < \bar{c}^u$, such that the firm can choose between the static solution and the dynamic storage solution. As shown in the proof of Lemma 2, the condition $\lambda < \widetilde{\lambda}$ corresponds to the case where for $c = \tilde{c}^u$ (the threshold for the feasibility of the static solution) the firm benefits from storage at the static monopoly prices and consumers are indifferent about it (because the storability constraint is binding), i.e., $\partial \Pi \left(p_1^{sm}, p_1^{sm} + c\right) / \partial S|_{c=\tilde{c}^u} > 0$, with $p_1^{sm} + \tilde{c}^u = p_2^m$. Thus, the threshold for the dynamic storage solution exceeds the one for the static solution, i.e., $\bar{c}^u > \tilde{c}^u$. The idea is that, when the number of sophisticates is small enough, i.e., $\lambda < \widetilde{\lambda}$, storage becomes particularly profitable for the firm because a large portion of consumers consists of naïfs that consume in the first period the entire quantity bought. Conversely, a relatively large number of sophisticates, i.e., $\lambda > \widetilde{\lambda}$, is equivalent to the case where for $c = \tilde{c}^u$ storage is detrimental to the firm at the static monopoly prices and consumers are indifferent about it, i.e., $\partial \Pi \left(p_1^{sm}, p_1^{sm} + c\right) / \partial S|_{c=\tilde{c}^u} < 0$, with $p_1^{sm} + \tilde{c}^u = p_2^m$. In this case, the threshold for the dynamic storage solution lies below the one for the static solution, i.e., $\bar{c}^u < \tilde{c}^u$, which implies that the two solutions are mutually exclusive.

We are now in a position to characterize the levels of prices and storage that emerge in equilibrium.

[9] As shown in the proof of Lemma 1, a sufficient (albeit not necessary) condition for $\overline{S}^{*u}$ to decrease with $\lambda$ is that $p_1^{*u}$ (weakly) rises with $\lambda$. This mitigates consumers' storage incentives even further. Under linear demand functions (with parallel shift), we find that $\overline{S}^{*u}$ and $\bar{c}^u$ unambiguously decrease with $\lambda$.

[10] In the proof of Lemma 2, we characterize an arguably mild condition for the uniqueness of the threshold $\widetilde{\lambda}$, according to which the second period profit function must be relatively concave (i.e., $\Pi_2''' \left(p_2^m\right) \leq 0$). Notably, this constitutes a sufficient (albeit not necessary) condition.

**Proposition 1** *A. Let $\lambda \leq \widetilde{\lambda}$. In equilibrium, (i) for $c < \widehat{c}^u$, where $\widehat{c}^u \in [\widetilde{c}^u, \overline{c}^u]$, the dynamic storage solution arises, and (ii) for $c \geq \widehat{c}^u$, the static solution arises.*

*B. Let $\lambda > \widetilde{\lambda}$. In equilibrium, (i) for $c < \overline{c}^u$, the dynamic storage solution arises, (ii) for $\overline{c}^u \leq c < \widetilde{c}^u$, the dynamic solution without storage arises, and (iii) for $c \geq \widetilde{c}^u$, the static solution arises.*

Proposition 1 indicates that the equilibrium solution and the associated values of prices and storage crucially depend on the fraction $\lambda$ of sophisticates and the magnitude of the storage cost $c$. We know from Lemma 2 that, if the number of sophisticates is relatively small, i.e., $\lambda < \widetilde{\lambda}$, the firm can choose between the static solution and the dynamic storage solution for intermediate values of the storage cost, i.e., $\widetilde{c}^u < c < \overline{c}^u$. As shown in point A of Proposition 1, in this case there exists a threshold $\widehat{c}^u \in [\widetilde{c}^u, \overline{c}^u]$ such that for $c < \widehat{c}^u$ storage emerges in equilibrium, despite the static solution being feasible for $c \geq \widetilde{c}^u$.[11] Intuitively, the firm promotes storage as long as its cost is relatively small. It follows from Lemma 1 that the equilibrium prices are linked through the binding storability constraint and only sophisticates store ex post, whereas naïfs consume the entire quantity bought. For $c \geq \widehat{c}^u$, the firm sets the static monopoly prices in equilibrium.

We know from Lemma 2 that, when the number of sophisticates is relatively large, i.e., $\lambda > \widetilde{\lambda}$, the static solution and the dynamic storage solution are mutually exclusive. As point B of Proposition 1 indicates, in this case the firm implements the dynamic storage solution whenever it is feasible, i.e., $c < \overline{c}^u$. For $\overline{c}^u \leq c < \widetilde{c}^u$, storage is definitely harmful to the firm but the static monopoly prices cannot be implemented. The dynamic solution without storage characterized in Lemma 1 emerges in equilibrium, with prices being linked through the binding storability constraint. The firm raises the first period price such that the second period price is at the static monopoly level, which satisfies the sequential optimality constraint (5) in the absence of storage. The static monopoly prices are charged in equilibrium whenever possible, i.e., for $c \geq \widetilde{c}^u$.

To better appreciate the main features of the firm's equilibrium pricing policy, we characterize the price distortions from the static monopoly level. Our findings are summarized in the following proposition.

**Proposition 2** *A. In the first period, the equilibrium price exhibits the following features: (i) for $c < \widetilde{c}^u$ it is higher than the static monopoly price if $\lambda > \overline{\lambda}$, (ii) for $\widetilde{c}^u \leq c < \widehat{c}^u$, it is lower than the static monopoly price, and (iii) otherwise, i.e., for $c \geq \max\{\widetilde{c}^u, \widehat{c}^u\}$, it coincides with the static monopoly price.*

*B. In the second period, the equilibrium price is lower than the static monopoly price in the presence of storage. Otherwise, it coincides with the static monopoly price.*

The results in point A of Proposition 2 identify the price distortions in the first period. When the storage cost is sufficiently small that the static feasibility constraint (6) is violated, i.e., $c < \widetilde{c}^u$, the firm raises the first period price above the static monopoly level if the number of sophisticates is relatively large, i.e., $\lambda > \overline{\lambda}$. As these consumers carry the quantity stored in the second period, storage is detrimental to the firm and the first period price is distorted

[11] It holds $\widehat{c}^u = \widetilde{c}^u = \overline{c}^u$ if and only if $\lambda = \widetilde{\lambda}$.

upward in order to mitigate consumers' storage incentives. When the dynamic solution without storage emerges in equilibrium, the firm exacerbates the distortion of the first period price above the static monopoly level in order to remove storage. For $\widetilde{c}^u \leq c < \widehat{c}^u$, we know from Proposition 1 that storage emerges in equilibrium, despite the static solution being feasible. This interval is non-empty for $\lambda < \widetilde{\lambda}$. The firm benefits from storage because a relatively large share of consumers is naïve and thus resorts to the quantity intended for storage in order to satisfy the first period consumption needs. The first period price is distorted below the static monopoly level in order to stimulate storage. When the storage cost is large enough, i.e., $c \geq \max\{\widetilde{c}^u, \widehat{c}^u\}$, it follows from Proposition 1 that the static monopoly prices are implemented.[12] Notably, we find that, under linear demand functions (with parallel shift), in the first period the equilibrium price in the presence of storage is lower than the static monopoly price if and only if the storage cost is large enough. When storage is relatively costly for consumers, the firm reduces the first period price below the static monopoly level to promote storage.

Point B of Proposition 2 shows that in the second period the equilibrium price cannot exceed the static monopoly level. This result directly follows from the sequential optimality constraint (5). As storage curbs the second period demand faced by the firm, the second period price in the presence of storage lies below the static monopoly level. When storage vanishes, the firm sets the static monopoly price in the second period.

Finally, we investigate the impact of the share $\lambda$ of sophisticates in the consumer population on the firm's equilibrium pricing policy in the presence of storage. Our results are formalized in the following proposition.

**Proposition 3** *In the presence of storage, the equilibrium price increases with $\lambda$ in each period if it is sufficiently lower than the static monopoly price in the first period.*

Proposition 3 shows that in each period a higher number of sophisticates can inflate the equilibrium price with storage. Put differently, instead of magnifying consumers' exploitation and exacerbating the firm's market power, consumer naïveté can lead to lower prices. To appreciate the rationale for this result, it is helpful to recall that naïfs end up consuming the quantity intended for storage. Thus, a higher number of naïfs strengthens the firm's incentives for price cuts in order to stimulate storage. A sufficient (albeit not necessary) condition for this result is that the equilibrium price is sufficiently lower than the static monopoly price in the first period. In this case, the firm significantly benefits from promoting storage that translates into additional consumption by naïfs.[13] Notably, we find that, under linear demand functions (with parallel shift), the equilibrium price in the presence of storage increases with the number of sophisticates in each period if and only if it is lower than the static monopoly price in the

[12] Recall from Proposition 1 that, for $\lambda \leq \widetilde{\lambda}$, the threshold $\widehat{c}^u$ is such that the static solution is more profitable than the dynamic storage solution if and only if $c > \widehat{c}^u$. As $\widehat{c}^u \in [\widetilde{c}^u, \overline{c}^u]$, we have $\max\{\widetilde{c}^u, \widehat{c}^u\} = \widehat{c}^u$. For $\lambda > \widetilde{\lambda}$, the static solution is implemented whenever possible, i.e., for $c \geq \widetilde{c}^u$, which implies that $\max\{\widetilde{c}^u, \widehat{c}^u\} = \widetilde{c}^u$. We refer to the proof of Proposition 2 for further details.

[13] We know from Lemma 1 that, in the dynamic solution without storage, the second period price coincides with the static monopoly level. A higher number of sophisticates (with lower valuation for the good) mitigates the second period demand and the corresponding price. It follows from the binding storability constraint that the first period price declines as well (see the proof of Proposition 3 for technical details). In the static solution, the first period price clearly does not depend on the share of sophisticates because all consumers behave identically at the purchase stage.

first period. As discussed after Proposition 2, this is the case if and only if the storage cost is large enough.

### 3.4 Overestimating consumers

When overestimating their preferences, at the first period consumption stage naïve consumers realize that their actual valuation for the good is lower than what they believed. As demand falls in the second period, consumers expect that prices decrease over time, i.e., $p_2^e > p_1$, which removes their incentives to buy the good in the first period for storage purposes. In other terms, ex ante storage vanishes, i.e., $S(p_1, p_2^e) = 0$. Sophisticated consumers do not revise their storage decisions. However, after discovering their lower valuation for the good, naïfs may choose to leave some quantity for future consumption. Thus, although prices decline over time, ex post storage can emerge, i.e., $\overline{S}(p_1, p_2^e) > 0$. Intuitively, this occurs if naïfs' actual preferences for the good are sufficiently weaker than what they believed. After buying the amount of the good that corresponds to the demand for consumption $D^s(p_1)$ driven by the believed utility $U^s(\cdot)$, naïfs learn their actual preferences and choose the consumption level in the first period in order to maximize their actual utility $U^n(\cdot)$, given the first period price $p_1$ and the second period expected price $p_2^e$. Let $\bar{s}^i(p_1, p_2^e)$ be the amount of ex post storage per consumer of type $i = s, n$. We have $\bar{s}^s(p_1, p_2^e) = 0$ because sophisticated consumers do not store ex post. The level of consumption $D^s(p_1) - \bar{s}^n(p_1, p_2^e)$ per naïve consumer equalizes marginal utilities over time, i.e., $u^{n\prime}(D^s(p_1) - \bar{s}^n(p_1, p_2^e)) + c = u^{n\prime}(D^n(p_2^e))$, where the first period marginal utility from consumption is inflated by the storage cost $c$ forgone for each unit consumed. As the marginal utility from consumption in the second period reflects the corresponding price, i.e., $u^{n\prime}(D^n(p_2^e)) = p_2^e$, ex post storage per naïve consumer writes as

$$\bar{s}^n(p_1, p_2^e) = \max\{0, D^s(p_1) - D^n(p_2^e - c)\}. \tag{9}$$

After learning their actual preferences, naïve consumers store the difference (if positive) between the quantity bought $D^s(p_1)$, which corresponds to the demand for consumption with the believed utility $U^s(\cdot)$, and the quantity consumed $D^n(p_2^e - c)$ according to the actual utility $U^n(\cdot)$. Substituting the static monopoly prices $p_1^{sm}$ and $p_2^m$ (derived in Section 3.2) into the expression for $\bar{s}^n(\cdot)$ in (9), we find that $\bar{s}^n(\cdot)$ decreases with the storage cost $c$, i.e., $\partial\bar{s}^n(p_1^{sm}, p_2^m)/\partial c = D^{n\prime}(p_2^m - c) < 0$. Solving $\bar{s}^n(p_1^{sm}, p_2^m) = D^s(p_1^{sm}) - D^n(p_2^m - c) = 0$ for $c$, we obtain that there exists a threshold $\tilde{c}^o \triangleq p_2^m - D^{n-1}(D^s(p_1^{sm}))$ such that the static solution is feasible if and only if $c \geq \tilde{c}^o$. For sufficiently small values of the storage cost, i.e., $c < \tilde{c}^o$, naïve consumers intend to store some quantity of the good for future consumption after learning their lower valuation. In this case, the static solution cannot be implemented. As demand falls in the second period, this occurs despite the fact that the static monopoly prices decline over time, i.e., $p_1^{sm} > p_2^m$. The firm's dynamic problem is described in (8), where ex ante storage vanishes and ex post storage stems from (9). In the following proposition, we derive the equilibrium prices and storage.

**Proposition 4** *In equilibrium, (i) for $c < \bar{c}^o$, the dynamic storage solution arises, which yields prices $p_1^{*o}$ and $p_2^{*o}$, with $p_1^{*o} > p_2^{*o}$, as well as ex post storage $\overline{S}^{*o} > 0$, (ii) for $\bar{c}^o \leq c < \tilde{c}^o$, the dynamic* *solution without storage arises, which yields prices $p_1^{**o}$and $p_2^{**o} = p_2^m$, with $p_1^{**o} > p_2^m$, and (iii) for $c \geq \widetilde{c}^o$, the static solution arises, which yields prices $p_1^{sm}$ and $p_2^m$, with $p_1^{sm} > p_2^m$.*

Proposition 4 shows that, when naïve consumers overestimate their preferences, storage arises in equilibrium for relatively small values of the storage cost, i.e., $c < \overline{c}^o$, in line with the case of underestimating consumers. This occurs especially when the fraction $\lambda$ of sophisticated consumers is large enough, because their stronger preferences for the good inflate the second period demand and the corresponding price, which stimulates storage. Differently from the case of underestimating consumers, naïfs engage in storage activities. After finding the good less valuable than at the purchase stage, naïfs decide to keep some leftovers for future consumption, which generates the aggregate level of ex post storage $\overline{S}^{*o} > 0$. It follows from the previous analysis that $\overline{S}^{*o}$ equalizes naïfs' intertemporal marginal utilities. For any given first period price, the level of ex post storage $\overline{s}^{*o}$ per naïve consumer increases with $\lambda$. Anticipating a rise in the second period price driven by a higher number of sophisticates (with stronger preferences for the good), naïve consumers are more inclined to store. In this case, the threshold $\overline{c}^o$ increases with $\lambda$ as well, which makes storage more likely to emerge in equilibrium.[14] Furthermore, the equilibrium prices are not linked through the binding storability constraint but through the amount of storage. As shown in Proposition 4, for intermediate values of the storage cost, i.e., $\overline{c}^o \leq c < \widetilde{c}^o$, storage cannot emerge any longer but the static solution is still unfeasible. The first period equilibrium price is such that ex post storage vanishes, whereas the second period equilibrium price reflects the static monopoly level, as implied by the sequential optimality constraint (5). Proposition 4 also indicates that the static solution is implemented whenever it is feasible. This occurs if and only if the expression in (9) is non-positive at the static monopoly prices and thus consumers do not have any (strict) incentives to store, i.e., $c \geq \widetilde{c}^o$. Notably, the threshold $\widetilde{c}^o$ increases with the fraction $\lambda$ of sophisticated consumers, thereby making the static solution more difficult to achieve. As previously discussed, a higher $\lambda$ inflates the second period demand and the corresponding price, which magnifies naïfs' incentives to store. Notably, we find from Proposition 4 the equilibrium prices always decrease over time.

To better appreciate the impact of overestimating consumers on the firm's equilibrium pricing policy, we identify the price distortions with respect to the static monopoly level. Our results are formalized in the following proposition.

**Proposition 5** *A. In the first period, the equilibrium price exhibits the following features: (i) for $c < \widetilde{c}^o$, it is higher than the static monopoly price, and (ii) for $c \geq \widetilde{c}^o$, it coincides with the static monopoly price.*

*B. In the second period, the equilibrium price is lower than the static monopoly price in the presence of storage. Otherwise, it coincides with the static monopoly price.*

Point A of Proposition 5 indicates that the first period price distortions substantially differ from the case of underestimating consumers. Storage is definitely detrimental to the firm because it cannot stimulate further consumption in the first period and curbs demand in the

[14] A sufficient (albeit not necessary) condition for $\overline{s}^{*o}$ to increase with $\lambda$ is that $p_1^{*o}$ (weakly) decreases with $\lambda$, which encourages storage to a further extent. Under linear demands (with parallel shift), we find that $\overline{s}^{*o}$ and $\overline{c}^o$ definitely increase with $\lambda$. Technical details can be found in the proof of Proposition 4.

second period. When the static solution is not feasible, i.e., $c < \tilde{c}^o$, the firm mitigates consumers' storage incentives by raising the first period price above the static monopoly level. The upward price distortion persists when storage cannot emerge and the static solution is still unfeasible. For $c \geq \tilde{c}^o$, it follows from Proposition 4 that the static solution applies. In line with the case of underestimating consumers, the results in point B of Proposition 5 directly follow from the sequential optimality constraint (5).

Finally, we investigate how the share $\lambda$ of sophisticates in the consumer population shapes the firm's equilibrium pricing policy in the presence of storage. We obtain the following results.

**Proposition 6** *In the presence of storage, the equilibrium price decreases with $\lambda$ in the first period and increases with $\lambda$ in the second period if $\lambda$ is high enough.*

Proposition 6 indicates that the equilibrium prices with storage move in different directions across periods in response to an increase in the fraction $\lambda$ of sophisticates, especially when $\lambda$ is high enough. In particular, the first period equilibrium price tends to decrease with $\lambda$. Given that a higher number of sophisticated consumers (with stronger preferences for the good) inflates the second period demand and the corresponding price, one might expect that the firm should raise the first period price in order to alleviate naïfs' stronger storage incentives. To appreciate the rationale for our findings, it is helpful to note that ceteris paribus a larger portion of sophisticates reduces the impact of naïfs' aggregate storage on the second period demand, which allows the firm to mitigate the price distortion from the static monopoly level. The second period equilibrium price tends to increase with the number of sophisticates because the second period demand becomes higher.[15]

To gain further insights, we also consider linear demand functions (with parallel shift). We find that, when sophisticates' willingness to pay is not too high, the first period equilibrium price unambiguously decreases with $\lambda$, in line with our previous analysis. Otherwise, it is concave in $\lambda$. Thus, for sufficiently small values of $\lambda$, a higher $\lambda$ increases the first period equilibrium price. For any given first period price, a rise in $\lambda$ induces the firm to inflate the second period price because of higher demand. This makes more attractive to the firm to sell in the second period and discourage storage through a higher first period price. As the magnitude of the responsiveness of the second period price to $\lambda$ rises with sophisticates' willingness to pay, the positive price effect of $\lambda$ dominates the (previously discussed) negative effect and thus the first period equilibrium price increases with $\lambda$ when sophisticates' willingness to pay is high enough. For sufficiently large values of $\lambda$, the negative effect is relatively pronounced due to the limited impact of naïfs' aggregate storage on the second period demand, which implies that the first period equilibrium price decreases with $\lambda$. We also find that the second period equilibrium price definitely increases with $\lambda$, because the positive effect stemming from higher demand outweighs the negative effect of higher storage per naïve consumer. It follows from our discussion that, when the share of naïfs in the consumer population is relatively large (i.e.,

[15] In the dynamic solution without storage characterized in Proposition 4, the equilibrium price increases with the number of sophisticates in each period. A higher $\lambda$ inflates the second period demand and the corresponding static monopoly price that the firm charges in equilibrium. The first period equilibrium price rises as well in order to ensure that storage vanishes in equilibrium (see the proof of Proposition 6 for technical details). In the static solution, the first period equilibrium price clearly does not depend on the fraction of sophisticates because all consumers behave in the same manner at the purchase stage.

for $\lambda$ low enough), the firm reduces the price in each period in response to a higher number of naïfs, which makes consumers better off.

## 4 Rational consumers

We now consider a setting where all consumers are uncertain about their preferences at the first period purchase stage and hold unbiased beliefs. In other words, all consumers are rational. At the first period purchase stage consumers assign a probability $\theta \in (0,1)$ of having high utility $U^h(x_\tau, y_\tau) = u^h(x_\tau) + y_\tau$ and a complementary probability $1-\theta$ of having low utility $U^l(x_\tau, y_\tau) = u^l(x_\tau) + y_\tau$ in period $\tau \in \{1,2\}$. The utility $U^k(x_\tau, y_\tau)$, with $k = h, l$, depends on the consumption level $x_\tau$ and money $y_\tau$. The continuously differentiable function $u^k(\cdot)$ is increasing and concave in $x_\tau$, i.e., $u^{k\prime}(\cdot) > 0$ and $u^{k\prime\prime}(\cdot) < 0$, with the standard normalization $u^k(0) = 0$. Furthermore, it holds $u^{h\prime}(\cdot) > u^{l\prime}(\cdot)$ for any consumption level $x_\tau$. Let $\widetilde{D}(p_\tau) \triangleq \arg\max_q \left\{\theta u^h(q) + (1-\theta) u^l(q) - q p_\tau\right\}$ the static demand function stemming from the consumer expected utility maximization problem, which is continuously differentiable and decreasing with the price $p_\tau$, i.e., $\widetilde{D}'(p_\tau) < 0$.

Our analysis can be decomposed into two main cases according to whether consumers buy the good in the first period for storage purposes or not. Intuitively, this crucially depends on the evolution of prices over time. In particular, when prices are expected to increase in the second period, ex ante storage $S(p_1, p_2^e)$ can emerge in equilibrium. It follows from the analysis in Section 3 that it can hold $S(p_1, p_2^e) > 0$ only if prices are linked through the binding storability constraint, i.e., $p_1 + c = p_2$. After learning their preferences, buyers choose the amount of consumption that determines for the residual part the level of ex post storage $\overline{S}(p_1, p_2^e)$ carried in the second period. If prices move over time such that the storability constraint is not binding, i.e., $p_1 + c > p_2$, which occurs (at least) with a declining price sequence, ex ante storage $S(p_1, p_2^e)$ vanishes and only ex post storage $\overline{S}(p_1, p_2^e)$ can emerge in equilibrium. Notably, the evolution of prices over time is driven by the magnitude of demand faced by the firm. In the first period, the demand for consumption $\widetilde{D}_1(p_1)$ at the purchase stage corresponds to the (previously derived) static demand function $\widetilde{D}(p_1)$ stemming from the consumer expected utility maximization problem. In the second period, after consumers have learned their preferences, the demand for consumption $\widetilde{D}_2(p_2)$ consists of high demand $D^h(p_2)$ for the share $\theta$ of consumers with high valuation for the good and low demand $D^l(p_2)$ for the residual share $1-\theta$ of consumers with low valuation, where $D^k(p_2) \triangleq \arg\max_q \left\{u^k(q) - q p_2\right\}$, for $k = h, l$.[16] In addition, we allow for a general magnitude of the second period demand for consumption $\widetilde{D}_2(\cdot)$ according to a parameter $\zeta$, where $\partial \widetilde{D}_2 / \partial \zeta > 0$. The role of $\zeta$ will become clear in the subsequent analysis.

For any level of ex post storage $\overline{S}(p_1, p_2^e) \geq 0$ inherited from the first period, the firm's dynamic profit maximization problem is subject to the following *sequential optimality constraint*

$$p_2\left(\overline{S}(p_1, p_2^e)\right) \triangleq \arg\max_{\widetilde{p}_2} \widetilde{p}_2 \left[\widetilde{D}_2(\widetilde{p}_2, \zeta) - \overline{S}(p_1, p_2^e)\right]. \tag{10}$$

[16]Notably, we adopt a general formulation for $\widetilde{D}_2(\cdot)$ that may also stem from consumer expected utility maximization problem when consumers are uncertain about their preferences in the second period as well.

The firm does not have any incentives to reduce the first period price to the extent that even consumers with high valuation store ex post in equilibrium. As these consumers could be inclined to store only if ex ante storage is promoted and thus the storability constraint is binding, i.e., $p_1 + c = p_2$, the firm would sell all consumers some units at $p_1$ instead of $p_2$, thereby incurring a cost equal to $c$.[17] For any given first period price $p_1$ and second period expected price $p_2^e$, we denote by $s\left(p_1, p_2^e\right)$ the level of ex ante storage for each consumer (which is identical across consumers) and by $\bar{s}^k\left(p_1, p_2^e\right)$ the level of ex post storage per consumer of type $k = h, l$. We have $\bar{s}^h\left(p_1, p_2^e\right) = 0$ because consumers with high valuation do not store ex post. In line with the analysis in Section 3.4, after learning their preferences, in the first period consumers with low valuation choose the amount of consumption that equalizes their marginal utilities over time. Given the quantity $\widetilde{D}_1\left(p_1\right) + s\left(p_1, p_2^e\right)$ purchased in the first period, the level of consumption $\widetilde{D}_1\left(p_1\right) + s\left(p_1, p_2^e\right) - \bar{s}^l\left(p_1, p_2^e\right)$ per consumer with low valuation is such that $u^{l\prime}\left(\widetilde{D}_1\left(p_1\right) + s\left(p_1, p_2^e\right) - \bar{s}^l\left(p_1, p_2^e\right)\right) + c = u^{l\prime}\left(D^l\left(p_2^e\right)\right)$, where the first period marginal utility from consumption is inflated by the storage cost $c$ forgone for each unit consumed.[18] As the marginal utility from consumption in the second period reflects the corresponding price, i.e., $u^{l\prime}\left(D^l\left(p_2^e\right)\right) = p_2^e$, the *condition for the equalization of marginal utilities* writes as

$$\bar{s}^l\left(p_1, p_2^e\right) = \max\left\{0, \widetilde{D}_1\left(p_1\right) + s\left(p_1, p_2^e\right) - D^l\left(p_2^e - c\right)\right\}. \tag{11}$$

Intuitively, consumers with low valuation store the difference (if positive) between the quantity purchased $\widetilde{D}_1\left(p_1\right) + s\left(p_1, p_2^e\right)$ and the quantity consumed $D^l\left(p_2^e - c\right)$ after learning their preferences. If the storability constraint is binding, i.e., $p_1 + c = p_2$, the sequential optimality constraint (10) and the condition for the equalization of marginal utilities in (11) jointly determine the levels of (aggregate) ex ante storage $S\left(p_1, p_2^e\right) = s\left(p_1, p_2^e\right)$ and (aggregate) ex post storage $\bar{S}\left(p_1, p_2^e\right) = (1-\theta)\,\bar{s}^l\left(p_1, p_2^e\right)$. If the storability constraint is not binding, i.e., $p_1 + c > p_2$, the sequential optimality constraint (10) identifies the second period price $p_2$ and the condition for the equalization of marginal utilities in (11) dictates the level of ex post storage $\bar{S}\left(p_1, p_2^e\right) = (1-\theta)\,\bar{s}^l\left(p_1, p_2^e\right)$, whereas ex ante storage vanishes, i.e., $S\left(p_1, p_2^e\right) = 0$.

The static solution is feasible as long as all consumers, after learning their preferences, abstain from storing at the static monopoly prices $p_1^m$ and $p_2^m$, where $p_1^m = \arg\max_{p_1}\ p_1\widetilde{D}_1\left(p_1\right)$ and $p_2^m = \arg\max_{p_2}\ p_2\widetilde{D}_2\left(p_2, \zeta\right)$. This occurs if and only if, irrespective of consumers' valuation for the good, the first period marginal utility from consuming the entire quantity, inflated by the storage cost $c$, is (weakly) higher than the second period marginal utility from consumption, i.e., $u^{k\prime}\left(\widetilde{D}_1\left(p_1^m\right)\right) + c \geq u^{k\prime}\left(D^k\left(p_2^m\right)\right) = p_2^m$, for $k = h, l$. As consumers with low valuation are more eager to store ex post, this condition reduces to $\widetilde{D}_1\left(p_1^m\right) - D^l\left(p_2^m - c\right) \leq 0$. Hence, there exists a threshold $\widetilde{c} \triangleq p_2^m - D^{l-1}\left(\widetilde{D}_1\left(p_1^m\right)\right)$ such that the static solution is feasible if and only if $c \geq \widetilde{c}$. Remarkably, this condition is more stringent than in the case where consumers perfectly know their preferences, i.e., $p_1^m + c \geq p_2^m$. Even in the absence of storage incentives at the purchase stage, consumers that discover low valuation for the good may be

[17] A fortiori, consumers with high valuation do not store ex post when the storability constraint is not binding. We refer to the proof of Proposition 7 for technical details.

[18] The condition that consumers with high valuation do not store ex post, i.e., $u^{h\prime}\left(\widetilde{D}_1\left(p_1\right) + s\left(p_1, p_2^e\right)\right) \geq u^{h\prime}\left(D^h\left(p_2^e\right)\right)$, is to be checked ex post.

inclined to store for the following period. This complicates the firm's problem by making static monopoly prices more difficult to implement.

The firm's aggregate profit is given by $\Pi \triangleq \Pi_1 + \Pi_2$, where

$$\Pi_1 = p_1 \left[ \widetilde{D}_1 (p_1) + S (p_1, p_2^e) \right] \text{ and } \Pi_2 = p_2 \left[ \widetilde{D}_2 (p_2, \zeta) - \overline{S} (p_1, p_2^e) \right].$$

Given the sequential optimality constraint (10), the firm's dynamic profit maximization problem writes as

$$\max_{\{p_1, p_2\}} \Pi (p_1, p_2) \ \ s.t. \ (10). \tag{12}$$

To convey our main results in a more compelling manner, we focus on the relevant case where the storage cost $c$ is sufficiently small that storage emerges in equilibrium. In the following proposition, we establish the main equilibrium features of prices and storage.

**Proposition 7** *A. When $\zeta$ is sufficiently high, ex ante storage $S^* > 0$ and ex post storage $\overline{S}^* > 0$ emerge in equilibrium for $c < \overline{c}^*$. Prices are $p_1^*$ and $p_2^* = p_1^* + c$.*

*B. Otherwise, only ex post storage $\overline{S}^{**} > 0$ emerges in equilibrium for $c < \overline{c}^{**}$. Prices are $p_1^{**}$ and $p_2^{**}$, with $p_1^{**} + c > p_2^{**}$.*

To better appreciate the rationale for the results in Proposition 7, it is helpful to consider the difference between the level of ex ante storage $S (p_1, p_2^e)$ at the first period purchase stage and the level of ex post storage $\overline{S} (p_1, p_2^e)$ that the firm expects to arise in the second period. It follows from the condition for the equalization of marginal utilities in (11) that

$$\Delta S (p_1, p_2^e) \triangleq S (p_1, p_2^e) - \overline{S} (p_1, p_2^e) = \theta S (p_1, p_2^e) - (1 - \theta) \left[ \widetilde{D}_1 (p_1) - D^l (p_2^e - c) \right]. \tag{13}$$

If $\Delta S (p_1, p_2^e) > (<) \, 0$, storage stimulates (depresses) the firm's sales. Promoting ex ante storage can be beneficial to the firm only if it induces additional sales that translate into further consumption. Otherwise, consumers carry these sales in the second period and, as prices are linked through the binding storability constraint, i.e., $p_2 = p_1 + c$, the firm loses $c$ for each unit bought at $p_1$ instead of $p_2$. As the expression for $\Delta S (\cdot)$ in (13) reveals, the firm faces a trade-off when promoting ex ante storage. On the one hand, ex ante storage stimulates the firm's current sales that translate into further consumption by buyers with high valuation for the good (occurring with probability $\theta$) because they consume the entire quantity bought. On the other hand, buyers with low valuation for the good (occurring with probability $1 - \theta$) accumulate in the form of ex post storage not only the entire amount of ex ante storage but also the difference between their expected and actual demand, which depresses the firm's future sales.

When the firm removes ex ante storage through prices such that the storability constraint is not binding, i.e., $p_1 + c > p_2$, ex post storage still persists. As ex ante storage stimulates the firm's current sales but ex post storage depresses the firm's future sales, one might think that the firm should promote ex ante storage whenever possible. To gain some insights, it is important to note that, for any given prices, preventing ex ante storage allows the firm to mitigate consumers' incentives to store ex post. For any first period price $p_1$, consumers buy a lower quantity compared to the case where ex ante storage is promoted. Furthermore, as prices with only ex post storage are not linked through the binding storability constraint, i.e., $p_1 + c > p_2$, the firm charges a relatively low second period price, which encourages consumption to a further extent. In particular, given the condition for the equalization of marginal utilities in (11), we find from $p_1 + c > p_2$ that the demand for consumption $D^l(p_2^e - c)$ of buyers with low valuation outweighs the corresponding demand $D^l(p_2^e - c) = D^l(p_1)$ in the presence of ex ante storage.

Proposition 7 shows that the choice between promoting ex ante storage and only ex post storage crucially depends on the magnitude of the second period demand $\widetilde{D}_2(\cdot)$, as driven by the parameter $\zeta$. When ex ante storage is allowed, a rise in the second period demand $\widetilde{D}_2(\cdot)$ magnifies consumers' incentives to buy the good for storage purposes in anticipation of a higher second period price. Given that ex ante storage is gathered by all consumers but ex post storage is only accumulated by consumers with low valuation (with probability $1-\theta$), a higher second period demand makes ex ante storage more profitable for the firm. When only ex post storage emerges, an increase in the second period demand $\widetilde{D}_2(\cdot)$ leads to a higher second period price, which merely encourages storage by consumers with low valuation. Thus, promoting only ex post storage becomes less attractive for the firm. As shown in point A of Proposition 7, when the second period demand $\widetilde{D}_2(\cdot)$ is relatively high (as implied by a sufficiently high $\zeta$), the firm promotes ex ante storage $S^* > 0$, which translates into ex post storage $\overline{S}^* > 0$, and charges prices linked through the binding storability constraint, i.e., $p_2^* = p_1^* + c$, provided that the storage cost is relatively small. Otherwise, as point B of Proposition 7 indicates, the firm allows only ex post storage $\overline{S}^{**} > 0$ by setting prices such that the storability constraint is not binding, i.e., $p_1^{**} + c > p_2^{**}$, as long as storage is relatively cheap.[19] Using linear demands (with parallel shift), we find that there exists a threshold $\overline{\zeta}$ such that promoting ex ante storage is profit superior if and only if $\zeta > \overline{\zeta}$. As $\overline{\zeta} > 0$, this occurs when the second period demand $\widetilde{D}_2(\cdot)$ is sufficiently higher than the first period demand $\widetilde{D}_1(\cdot)$.

It is worth noting that the results about ex ante storage in point A of Proposition 7 reflect those with naïve underestimating consumers derived in Section 3.3. Such buyers are more inclined to consume after learning their higher valuation for the good and inflate the second period demand. The firm prefers to implement a pricing policy that yields ex ante storage in order to stimulate their consumption. In the same vein, the results about only ex post storage in point B of Proposition 7 correspond to those with naïve overestimating consumers derived in Section 3.4. Such buyers are more reluctant to consume after learning their lower valuation for the good and dampen the second period demand. The firm opts for a pricing policy that allows only ex post storage in order to mitigate consumers' storage incentives.

As in the setting with consumer naïveté, we now examine the price distortions from the static monopoly level.

**Proposition 8** *A. In the first period, in the presence of ex ante storage, the equilibrium price exhibits the following features: (i) for $c < \widetilde{c}$, it is lower (higher) than the static monopoly price if $\zeta$ is low (high)*

[19]When $\widetilde{D}_2(\cdot)$ is significantly high, the second period price $p_2$ rises to the extent that the condition $p_1 + c > p_2$ is violated and thus the firm cannot resort to only ex post storage. Conversely, when $\widetilde{D}_2(\cdot)$ is relatively low, ex ante storage cannot be promoted.

*enough and $\theta$ is high (low) enough, and (ii) for $c \geq \widetilde{c}$, it is lower than the static monopoly price. In the second period, it is lower than the static monopoly price.*

*B. In the first period, in the presence of only ex post storage, the equilibrium price is higher than the static monopoly price. In the second period, it is lower than the static monopoly price.*

The results in point A of Proposition 8 indicate that, when the static solution is not feasible, i.e., $c < \widetilde{c}$, the first period price distortions in the presence of ex ante storage are driven by the magnitude of demand, as captured by the parameter $\zeta$, and by the probability $\theta$ that consumers exhibit high valuation for the good. It follows from the discussion after Proposition 7 that, in the presence of ex ante storage, a decrease in the second period demand $\widetilde{D}_2(\cdot)$, as implied by a lower $\zeta$, mitigates the level of ex ante storage because consumers expect a lower second period price. Facing a lower first period demand, the firm prefers to reduce the corresponding price. Furthermore, a rise in the probability $\theta$ of high valuation for the good makes it more attractive for the firm to lower the first period price in order to stimulate ex ante storage. As a result, when $\zeta$ is sufficiently low and $\theta$ is sufficiently high, the firm prefer to distort the first period price below the static monopoly level. An upward price distortion occurs for $\zeta$ high enough and $\theta$ low enough. When the static solution is feasible, i.e., $c \geq \widetilde{c}$, the first period price is definitely lower than the static monopoly level. In order to promote ex ante storage, the firm implements a downward price distortion. As the results in point B of Proposition 8 reveal, in the presence of only ex post storage, the firm prefers to distort the first period price above the static monopoly level in order to curb the amount of the good bought in the first period and stored for the second period. Given that ex post storage emerges irrespective of whether ex ante storage is promoted or not, we find from the sequential optimality constraint (10) that the second period price unambiguously lies below the static monopoly level.

The results about price distortions in a setting with rational consumers formalized in point A of Proposition 8 are significantly related to those with naïve underestimating consumers derived in Section 3.3. Specifically, we know from Proposition 2 that, for sufficiently small values of the storage cost, if the number of sophisticates is large enough, the first period price exceeds the static monopoly level. As such consumers have lower valuation for the good, this corresponds to the condition of $\theta$ low enough with rational consumers. Given that a lower $\theta$ curbs demand also in the first period, the condition of $\zeta$ high enough ensures that the amount of ex ante storage is sufficiently large to induce the firm to charge a first period price above the static monopoly level. Differently from consumer naïveté, in a setting with rational consumers we can also characterize the conditions under which the first period price lies below the static monopoly level. The reason is that, with naïve underestimating consumers, the second period demand rises with the number of naïfs, which corresponds to a higher $\theta$ with rational consumers. The condition of $\zeta$ low enough ensures that the amount of ex ante storage is sufficiently small to induce the firm to charge a first period price below the static monopoly level. Along the same lines, the results about price distortions formalized in point B of Proposition 8 correspond to those with naïve overestimating consumers derived in Section 3.4. As Proposition 5 shows, the first period price is unambiguously distorted above the static monopoly level in order to mitigate ex post storage by naïve consumers that exhibit lower valuation for the good.

# 5 Robustness and extensions

Our work is robust and can be extended in different directions. We refer to the Supplementary Appendix for the formal proofs of our descriptive claims.

## 5.1 Nonlinear pricing

It is interesting to explore the case of a nonlinear pricing policy (e.g., Antoniou and Fiocco 2019; Hendel et al. 2014). Intuitively, the firm has incentives to design a contract menu that discriminates across consumers in order to extract their surplus. As consumers share the same beliefs about their preferences at the first period purchase stage, a contract menu can be offered only in the second period. By the revelation principle, this menu constitutes an incentive compatible mechanism that induces consumers to select the contract intended for their type. Standard arguments indicate that consumers with high valuation for the good receive some informational rents associated with the gains from selecting the contract intended for consumers with low valuation. This is the case of naïfs in the presence of naïve underestimating consumers (analyzed in Section 3.3) and of sophisticates in the presence of naïve overestimating consumers (analyzed in Section 3.4). Notably, the opportunity to consume the quantity stored provides consumers with some additional surplus in the second period, which leads the firm to offer a discount on the second period payment. The firm intends to recoup such a discount by charging a higher upfront payment in the first period. Anticipating this, consumers have incentives to skip their purchases in the first period and enjoy the discount in the second period at the cost of no consumption (and no storage) in the first period. We find that, with naïve underestimating consumers, naïfs' reluctance to accept a high upfront payment allows them to retain some surplus, which can be augmented by informational rents in the second period. Conversely, when overestimating their preferences, naïfs may even end up with losses because they are willing to accept an excessively high upfront payment. The firm is able to fully extract the surplus of sophisticated consumers unless they benefit from postponing their purchases or exploiting their superior information in the second period.

The pricing policy in the static solution is rather straightforward. As consumers cannot resort to the quantity stored for second period consumption, the firm does not offer any discount in the second period, which removes consumers' incentives to postpone their purchases. In the first period, sophisticates are always left with zero surplus. Naïfs make gains (losses) when underestimating (overestimating) their preferences because the firm chooses an excessively low (high) payment with respect to their actual valuation for the good. Irrespective of being sophisticated or naïve, consumers with high valuation can obtain some informational rents in the second period.

The comparison in terms of consumer surplus between the dynamic storage solution and the static solution delivers results of some interest. To identify the major effects at play in a tractable manner, we consider the case where the number of consumers with high valuation is large enough. This sterilizes the additional standard effects arising from consumers' informational advantage because the firm does not serve consumers with low valuation and thus fully captures the informational rents of consumers with high valuation. To fix ideas, suppose that storage is costless.[20] We find that in the dynamic storage solution the firm prefers to sell in bulk the entire quantity in the first period such that buyers with high valuation consume efficiently in the second period. This removes consumers' incentives to skip their purchases in the first period. Consequently, sophisticates obtain zero surplus in the dynamic storage solution. As the firm can fully extract their surplus in the static solution as well, sophisticates are indifferent between the two solutions.[21] When underestimating their preferences, naïfs are better off compared to static solution because they (partially) retain the gains from consuming the quantity stored. Conversely, the firm exploits naïve overestimating consumers by extracting higher utility from storage than what eventually materializes. This imposes higher losses on naïfs with respect to the static solution.

### 5.2 Price commitment

To identify the price effects attributable to the firm's lack of commitment, we examine the situation where the firm can credibly commit to future prices. This allows us to compare the results under limited commitment derived in our analysis with those under full commitment. When the firm prefers to promote ex ante storage — as in the case of naïve underestimating consumers (Section 3.3) or a sufficiently high future demand with rational consumers (Section 4) — the equilibrium prices under full commitment are still linked through the binding storability constraint, which makes consumers willing to buy for storage purposes. Intuitively, this emerges for sufficiently small values of the storage cost. Under full commitment, the firm is able to induce consumers to store the entire second period demand. Under limited commitment, however, consumers anticipate that, in response to a large amount of storage, the firm succumbs to the temptation to charge a relatively low price in the second period in order to stimulate its sales. Ex ante and ex post storage coexist also under full commitment because consumers with low valuation for the good still carry some quantity in the second period. Notably, the price comparison between the two commitment regimes ultimately depends on the price responsiveness of the extra gain from storage accruing to the firm under full commitment compared to limited commitment. We show that the equilibrium prices under full commitment are higher than under limited commitment if and only if the firm's extra gain from storage under full commitment increases with the price. As at the same prices the level of storage is higher under full commitment, a price rise enhances the firm's extra gain from storage under full commitment for any given storage difference between the two commitment regimes. Put differently, limited commitment magnifies the firm's incentives for a price reduction in order to promote storage. A sufficient (albeit not necessary) condition for the firm's extra gain from storage under full commitment to increase with the price and thus for limited commitment to yield lower prices is that the storage difference between the two commitment regimes (weakly) increases with the price. Equivalently, the price impact on storage must be larger (in absolute terms) under limited commitment than under full commitment. Given that the amount of storage coincides with the second period demand under full commitment and

[20] Clearly, our qualitative results carry over to sufficiently small values of the storage cost.

[21] The static solution could make sophisticates better off in the (unlikely) situation where they receive some informational rents only in the static solution.

stems from sequential optimality under limited commitment, this condition crucially depends on the shape of demand. In particular, it is satisfied as long as the second period demand is not too convex.

When the firm prefers to induce only ex post storage — as in the case of naïve overestimating consumers (Section 3.4) or a sufficiently low future demand with rational consumers (Section 4) — the equilibrium prices under full commitment are such that the storability constraint is no longer binding, in line with limited commitment. Consumers with low valuation store ex post for sufficiently small values of the storage cost. We find that the comparison between the equilibrium prices under the two commitment regimes now varies across periods. Being ex post storage identical at the same prices (as implied by the condition for the equalization of marginal utilities), a rise in the first period price is definitely more effective at reducing storage under full commitment. This is because under limited commitment consumers are more reluctant to decrease their amount of storage in anticipation of the firm's opportunistic behavior. Thus, for any given second period price, full commitment strengthens the firm's incentives to inflate the first period price in order to dampen storage. Furthermore, the opportunity to credibly announce future prices implies that, for any given first period price, the firm prefers to set a lower second period price than under limited commitment in order to stimulate first period consumption by buyers with low valuation, which leads to a reduction in their amount of storage. Such insights are corroborated in a framework with linear demands (and parallel shift), where full commitment generates a higher equilibrium price in the first period and a lower equilibrium price in the second period with respect to limited commitment.

## 6 Concluding remarks

We examine a market where a firm sells a storable good to consumers with time-varying preferences that differ between the purchase stage and the consumption stage. Time-varying preferences induce consumers to revise their consumption and storage decisions. Hence, the amount purchased for storage, i.e., ex ante storage, which stimulates the firm's current sales, may depart from the quantity actually stored, i.e., ex post storage, which depresses the firm's future sales. We show that the firm's dynamic pricing policy and consumers' storage incentives crucially hinge upon on the trade-off that the firm faces when promoting ex ante storage. For the sake of exposition, we first consider a setting where some consumers are naïve and hold biased beliefs about their preferences at the purchase stage. When naïve consumers underestimate their preferences, demand grows over time and the firm may resort to price cuts in order to encourage ex ante storage, which translates into further consumption. Conversely, with naïve overestimating consumers, demand falls over time and the firm may prefer to raise prices in order to dampen ex post storage, whereas ex ante storage is removed. Then, we turn to a setting with all rational consumers that hold unbiased beliefs and are uncertain about their preferences at the purchase stage. Endogenizing the firm's choice between allowing ex ante storage and only ex post storage, we find that the firm promotes ex ante storage when future demand is high enough. Otherwise, only ex post storage emerges.

Our work delivers different predictions that lend themselves to an empirically testable validation. As a firm selling a storable good may prefer to cut prices with growing demand and raise prices with declining demand, our results suggest that storability introduces some form of counter-cyclicality of prices. This translates into relatively low markups in growing markets and high markups in shrinking markets. Our analysis also provides some insights into the impact of uncertainty on price levels. As with stable deterministic demand the firm clearly charges the same price over time but with expected constant random demand the firm increases the current price to dampen ex post storage (at least with linear demands and parallel shift), uncertainty tends to inflate prices for storable goods in the short run. An additional result that deserves empirical investigation is the presence of storage even when prices decline over time. Notably, our analysis delivers potentially significant policy implications, especially in terms of welfare consequences of consumer naïveté. Contrary to what common wisdom suggests, we find that, instead of magnifying consumers' exploitation and exacerbating the firm's market power, consumer naïveté can reduce prices and make consumers better off. This unveils unintended anticompetitive effects of regulatory interventions that are meant to 'educate' consumers and simplify their decision problems by enlarging the scope for mandatory disclosure of different aspects of prices and product characteristics. An accurate evaluation of consumers' expectations and biases about their preferences is therefore of paramount importance in markets for storable goods.

# Appendix

**Proof of Lemma 1.** The firm faces the following three pricing options: (I) $p_1 + c > p_2$; (II) $p_1 + c = p_2$; (III) $p_1 + c < p_2$.

(I) Suppose $p_1 + c > p_2$. It follows from (4) that ex ante storage is $S\left(p_1, p_2^e\right) = 0$. The firm's dynamic maximization problem in (8) can be decomposed in two static problems. In the first period, the firm's maximization problem is given by (2). Differentiating the maximand in (2) with respect to $p_1$ yields

$$\phi_1\left(p_1\right) \triangleq D^s\left(p_1\right) + p_1 D^{s\prime}\left(p_1\right), \tag{A1}$$

where $\phi_1'\left(\cdot\right) < 0$ (by profit concavity). In the second period, the firm's maximization problem is given by (3). Differentiating the maximand in (3) with respect to $p_2$ yields

$$\phi_2\left(p_2\right) \triangleq \sigma\left(p_2\right) + p_2 \sigma'\left(p_2\right), \tag{A2}$$

where $\phi_2'\left(\cdot\right) < 0$ (by profit concavity). The first period static monopoly price $p_1^{sm}$ satisfies $\phi_1\left(p_1^{sm}\right) = 0$ and the second period static monopoly price $p_2^m$ satisfies $\phi_2\left(p_2^m\right) = 0$. We find that

$$p_1^{sm} = -\frac{D^s\left(p_1^{sm}\right)}{D^{s\prime}\left(p_1^{sm}\right)} \text{ and } p_2^m = -\frac{\sigma\left(p_2^m\right)}{\sigma'\left(p_2^m\right)}. \tag{A3}$$

The static solution is feasible if and only if $p_1^{sm} + c \geq p_2^m$ or, equivalently, $c \geq \widetilde{c}^u$, where $\widetilde{c}^u \triangleq p_2^m - p_1^{sm}$. Applying the implicit function theorem to $\phi_2(p_2^m) = 0$, where $\phi_2(\cdot)$ is defined by (A2), yields

$$\frac{\partial p_2^m}{\partial \lambda} = \frac{D^n\left(p_2^m\right) - D^s\left(p_2^m\right) + p_2^m\left[D^{n\prime}\left(p_2^m\right) - D^{s\prime}\left(p_2^m\right)\right]}{\Pi_2''\left(p_2^m\right)}. \tag{A4}$$

With naïve underestimating (overestimating) consumers, it holds $\partial p_2^m/\partial\lambda < (>)\, 0$ (by profit concavity). As $p_1^{sm} \to p_2^m$ at $\lambda \to 1$ and $p_1^{sm}$ does not depend on $\lambda$, we find that $\widetilde{c}^u \triangleq p_2^m - p_1^{sm} > 0$ and $\partial\widetilde{c}^u/\partial\lambda < 0$.

(II) Suppose $p_1 + c = p_2$. It follows from (4) that ex ante storage is $S\left(p_1, p_2^e\right) \in [0, \sigma\left(p_1 + c\right)]$. The firm's dynamic maximization problem is given by (8). Proceeding backward and using $p_2 = p_1 + c$, we find from the first-order condition for the firm's second period maximization problem associated with the sequential optimality constraint (5) that ex post storage is $\overline{S}\left(p_1\right) = \max\left\{0, \phi_2\left(p_1 + c\right)\right\}$, where $\phi_2\left(\cdot\right)$ is defined by (A2). The following two cases emerge.

(IIa) Let $\overline{S}\left(p_1\right) > 0$. This case identifies the dynamic storage solution. As sophisticated consumers know their actual preferences at the purchase stage, their level of ex ante storage coincides with their level of ex post storage. If naïve consumers exhaust the good in the first period after learning their actual preferences, we obtain that $\overline{S}\left(p_1\right) = \phi_2(p_1 + c) = \lambda S\left(p_1\right)$. Otherwise, naïve consumers also carry some quantity in the second period and $\overline{S}\left(p_1\right) = \phi_2\left(p_1 + c\right) \in \left(\lambda S\left(p_1\right), S\left(p_1\right)\right]$. First, we consider $\overline{S}\left(p_1\right) = \phi_2\left(p_1 + c\right) = \lambda S\left(p_1\right)$, which implies $S\left(p_1\right) = \phi_2\left(p_1 + c\right)/\lambda$. The firm's maximization problem becomes

$$\max_{p_1} p_1\left[D^s\left(p_1\right) + S\left(p_1\right)\right] + \left(p_1 + c\right)\left[\sigma\left(p_1 + c\right) - \lambda S\left(p_1\right)\right]. \tag{A5}$$

Using (A1) and (A2), the first-order condition for $p_1$ can be written as

$$\lambda\phi_1\left(p_1\right) + \phi_2\left(p_1 + c\right) + \left[p_1\left(1 - \lambda\right) - c\lambda\right]\phi_2'\left(p_1 + c\right) = 0. \tag{A6}$$

The equilibrium prices with storage are

$$p_1^{*u} = \frac{\lambda}{1 - \lambda}c - \frac{\lambda\phi_1\left(p_1^{*u}\right) + \phi_2\left(p_1^{*u} + c\right)}{\left(1 - \lambda\right)\phi_2'\left(p_1^{*u} + c\right)} \;\text{ and }\; p_2^{*u} = p_1^{*u} + c. \tag{A7}$$

The equilibrium ex ante storage is $S^{*u} \triangleq S\left(p_1^{*u}\right) = \phi_2\left(p_1^{*u} + c\right)/\lambda$ and the equilibrium ex post storage is $\overline{S}^{*u} \triangleq \overline{S}\left(p_1^{*u}\right) = \phi_2\left(p_1^{*u} + c\right)$. We now characterize the condition for the feasibility of the dynamic storage solution, i.e., $\overline{S}^{*u} > 0$. Differentiating $\overline{S}^{*u}$ with respect to $c$ yields $\partial\overline{S}^{*u}/\partial c = \left(\partial p_2^{*u}/\partial c\right)\phi_2'\left(p_2^{*u}\right) < 0$, where the inequality follows from $\partial p_2^{*u}/\partial c > 0$ and $\phi_2'\left(\cdot\right) < 0$ (by profit concavity). To show $\partial p_2^{*u}/\partial c > 0$, we compute the derivative of the left-hand side of the first-order condition for $p_2^{*u}$ — obtained by replacing $p_1$ with $p_2 - c$ in (A6) — with respect to $c$. This yields after some manipulation $-\lambda\phi_1'\left(p_2^{*u} - c\right) - \phi_2'\left(p_2^{*u}\right) > 0$, where the inequality follows from $\phi_\tau'\left(\cdot\right) < 0$, $\tau \in \{1, 2\}$ (by profit concavity). It follows from the implicit function theorem that $\partial p_2^{*u}/\partial c > 0$. Now, we demonstrate that $\overline{S}^{*u} > 0$ at $c = 0$. This corresponds to $\sigma\left(p_1^{*u}\right) + p_1^{*u}\sigma'\left(p_1^{*u}\right) > 0$. Using (A2), this is equivalent to $p_1^{*u} = p_2^{*u} < p_2^m$. Substituting $\phi_2\left(p_2^m\right) = 0$ into the left-hand side of the first-order condition for $p_1^{*u}$ in (A6) evaluated at $c = 0$ yields $\lambda\phi_1(p_2^m) + p_2^m\left(1 - \lambda\right)\phi_2'(p_2^m) < 0$, where the inequality follows from $\phi_1\left(p_2^m\right) < 0$ (as $p_2^m > p_1^{sm}$) and $\phi_2'\left(\cdot\right) < 0$ (by profit concavity). This implies that $p_1^{*u} = p_2^{*u} <$

$p_2^m$ and thus $\overline{S}^{*u} > 0$ at $c = 0$. As $\overline{S}^{*u} < 0$ for $c$ arbitrarily large, we find from $\partial \overline{S}^{*u} / \partial c < 0$ (see above) that there exists a unique threshold $\overline{c}^u > 0$ such that $\overline{S}^{*u} > 0$ if and only if $c < \overline{c}^u$.

Now, we derive the condition ensuring that $\overline{S}^{*u}$ decreases with $\lambda$, which implies that $\overline{c}^u$ decreases with $\lambda$ as well. Taking the derivative of $\overline{S}^{*u} = \overline{S}\left(p_1^{*u}(\lambda), \lambda\right)$ with respect to $\lambda$ yields

$$\frac{d\overline{S}^{*u}}{d\lambda} = \frac{\partial \overline{S}\left(p_1^{*u}\right)}{\partial p_1} \frac{\partial p_1^{*u}}{\partial \lambda} + \frac{\partial \overline{S}^{*u}}{\partial \lambda}.$$

Given that $\overline{S}^{*u}$ follows from the first-order condition for $p_2$, as implied by the sequential optimality constraint (5), we find that

$$\frac{\partial \overline{S}^{*u}}{\partial \lambda} = D^s\left(p_2^{*u}\right) - D^n\left(p_2^{*u}\right) + p_2^{*u}\left[D^{s\prime}\left(p_2^{*u}\right) - D^{n\prime}\left(p_2^{*u}\right)\right] < 0,$$

where the inequality follows from the assumptions about naïve underestimating consumers. Furthermore, we obtain that

$$\frac{\partial \overline{S}\left(p_1^{*u}\right)}{\partial p_1} = 2\sigma'\left(p_1^{*u} + c\right) + \left(p_1^{*u} + c\right)\sigma''\left(p_1^{*u} + c\right) < 0,$$

where the inequality follows from profit concavity. Thus, a sufficient (albeit not necessary) condition for $d\overline{S}^{*u}/d\lambda < 0$ is that $\partial p_1^{*u}/\partial \lambda \geq 0$. It follows from $d\overline{S}^{*u}/d\lambda < 0$ that $\partial \overline{c}^u / \partial \lambda < 0$. Using linear demand functions of the form $D^s(p_\tau) = \alpha^s - \beta p_\tau$ and $D^n(p_\tau) = \alpha^n - \beta p_\tau$, where $\alpha^n > \alpha^s$, we find from (A7) that the equilibrium prices with storage are $p_1^{*u} = \left[(1-\lambda)(\alpha^n - 2\beta c) + 2\alpha^s \lambda\right] / (4\beta)$ and $p_2^{*u} = p_1^{*u} + c$. We also obtain that the equilibrium ex post storage is $\overline{S}^{*u} = \left[\alpha^n(1-\lambda) - 2\beta c(1+\lambda)\right]/2$, which yields $d\overline{S}^{*u}/d\lambda < 0$, and that $\overline{c}^u = \alpha^n(1-\lambda) / \left[2\beta(1+\lambda)\right]$, which yields $\partial \overline{c}^u / \partial \lambda < 0$.

Now, suppose that at the equilibrium prices in (A7) naïfs carry some quantity in the second period, i.e., $\overline{S}\left(p_1^{*u}\right) = \phi_2\left(p_1^{*u} + c\right) \in \left(\lambda S\left(p_1^{*u}\right), S\left(p_1^{*u}\right)\right]$. Intuitively, when the preferences of sophisticates and naïfs are sufficiently similar, or the storage cost is small enough, naïfs may also be inclined to store some amount of the good after learning their actual preferences. This occurs if and only if naïfs' marginal utility from consuming the entire quantity bought in the first period, inflated by the storage cost $c$, is lower than their marginal utility in the second period, which coincides with the price, i.e., $u^{n\prime}\left(D^s\left(p_1^{*u}\right) + S\left(p_1^{*u}\right)\right) + c < u^{n\prime}\left(D^n\left(p_2^{*u}\right)\right) = p_2^{*u}$ or, equivalently, $u^{n\prime}\left(D^s\left(p_1^{*u}\right) + S\left(p_1^{*u}\right)\right) < p_1^{*u}$. Note that the firm's profit is maximized in the benchmark case (described in Section 3.2) where all consumers are perfectly aware of their preferences before purchasing in the first period. As $\partial \overline{S}(\cdot)/\partial p_1 < 0$ (see above), the firm has an incentive to reduce the first period price from the benchmark case of perfectly aware consumers up to the level where only sophisticates store ex post. Any lower price would induce naïfs to store ex post as well. It follows from the objective function of the firm's maximization problem in (A5) that each unit of ex post storage would impose a cost $c$ on the firm. This implies that, if at the equilibrium prices in (A7) naïfs store ex post, the firm prefers to set higher prices that mitigate the distortion from the benchmark case of perfectly aware consumers. The equilibrium prices become $\widetilde{p}_1^{*u}$ and $\widetilde{p}_2^{*u} = \widetilde{p}_1^{*u} + c$, where $\widetilde{p}_\tau^{*u} > p_\tau^{*u}$, $\tau \in \{1,2\}$. Specifically, $\widetilde{p}_1^{*u}$ and $S\left(\widetilde{p}_1^{*u}\right)$ are such that only sophisticates store ex post, i.e.,

$\overline{S}\left(\widetilde{p}_1^{*u}\right) = \phi_2\left(\widetilde{p}_1^{*u} + c\right) = \lambda S\left(\widetilde{p}_1^{*u}\right)$, and naïfs' marginal utility from consuming the entire quantity bought in the first period is equal to the price, i.e., $u^{n\prime}\left(D^s\left(\widetilde{p}_1^{*u}\right) + S\left(\widetilde{p}_1^{*u}\right)\right) = \widetilde{p}_1^{*u}$. With linear demands $D^s\left(p_\tau\right) = \alpha^s - \beta p_\tau$ and $D^n\left(p_\tau\right) = \alpha^n - \beta p_\tau$, where $\alpha^n > \alpha^s$, we find that there exists a unique threshold $\widetilde{\alpha}^s \triangleq \left[\alpha^n\left(3\lambda - 1\right) + 2\beta c\left(1 + \lambda\right)\right] / \left(2\lambda\right)$ such that for $\alpha^s \leq \widetilde{\alpha}^s$ the prices with storage $p_1^{*u}$ and $p_2^{*u}$ previously characterized emerge in equilibrium. For $\alpha^s > \widetilde{\alpha}^s$, the equilibrium prices become $\widetilde{p}_1^{*u} = \left[\alpha^n\left(1 - 2\lambda\right) + 2\alpha^s\lambda - 2\beta c\right] / \left(2\beta\right)$ and $\widetilde{p}_2^{*u} = \widetilde{p}_1^{*u} + c$. To complete our proof, we show that naïfs' ex post storage cannot be sustained in equilibrium. It follows from the sequential optimality constraint (5), where ex post storage $\overline{S}\left(p_1\right)$ is replaced by sophisticates' ex post storage $\overline{S}^s\left(p_1\right)$, weighted by $\lambda$, and by naïfs' ex post storage $\overline{S}^n\left(p_1\right)$, weighted by $1 - \lambda$, that the first-order condition for the firm's second period maximization problem is $\sigma\left(p_2\right) - \lambda\overline{S}^s\left(p_1\right) - \left(1 - \lambda\right)\overline{S}^n\left(p_1\right) + p_2\sigma'\left(p_2\right) = 0$. Naïfs' ex post storage $\overline{S}^n\left(p_1\right)$ is such that their marginal utility from consumption in the first period, inflated by the storage cost $c$, is equal to their marginal utility in the second period, which coincides with the price, i.e., $u^{n\prime}\left(D^s\left(p_1\right) + \overline{S}^s\left(p_1\right) - \overline{S}^n\left(p_1\right)\right) + c = u^{n\prime}\left(D^n\left(p_2\right)\right) = p_2$. Note that the quantity intended for storage at the purchase stage $\overline{S}^s\left(p_1\right)$ is the same as for sophisticates (because all consumers behave identically) and corresponds to sophisticates' ex post storage (because they do not revise their storage decisions). This implies from $p_1 + c = p_2$ that $\overline{S}^n\left(p_1\right) = D^s\left(p_1\right) + \overline{S}^s\left(p_1\right) - D^n\left(p_1\right)$. Using the first-order condition for the firm's second period maximization problem yields $\overline{S}^s\left(p_1\right) = \sigma\left(p_1 + c\right) + \left(p_1 + c\right)\sigma'\left(p_1 + c\right) + \left(1 - \lambda\right)\left[D^n\left(p_1\right) - D^s\left(p_1\right)\right]$ and $\overline{S}^n\left(p_1\right) = \sigma\left(p_1 + c\right) + \left(p_1 + c\right)\sigma'\left(p_1 + c\right) - \lambda\left[D^n\left(p_1\right) - D^s\left(p_1\right)\right]$. The firm's maximization problem writes as

$$\max_{p_1} p_1\left[D^s\left(p_1\right) + \overline{S}^s\left(p_1\right)\right] + \left(p_1 + c\right)\left[\sigma\left(p_1 + c\right) - \lambda\overline{S}^s\left(p_1\right) - \left(1 - \lambda\right)\overline{S}^n\left(p_1\right)\right].$$

Using the envelope theorem, the first-order condition for $p_1$ is given by

$$D^s\left(p_1\right) + \overline{S}^s\left(p_1\right) + p_1\left[D^{s\prime}\left(p_1\right) + \overline{S}^{s\prime}\left(p_1\right)\right] - \left(p_1 + c\right)\left[\lambda\overline{S}^{s\prime}\left(p_1\right) + \left(1 - \lambda\right)\overline{S}^{n\prime}\left(p_1\right)\right] = 0.$$

Using the expressions for $\overline{S}^{s\prime}\left(p_1\right)$ and $\overline{S}^{n\prime}\left(p_1\right)$, along with $\overline{S}^n\left(p_1\right) = D^s\left(p_1\right) + \overline{S}^s\left(p_1\right) - D^n\left(p_1\right)$, the first-order condition for $p_1$ can be rewritten after some manipulation as

$$\overline{S}^n\left(p_1\right) + D^n\left(p_1\right) + p_1\left[\lambda D^{s\prime}\left(p_1\right) + \left(1 - \lambda\right)D^{n\prime}\left(p_1\right)\right] - c\left[2\sigma'\left(p_1 + c\right) + \left(p_1 + c\right)\sigma''\left(p_1 + c\right)\right] = 0.$$

Substituting this condition into $\phi_2\left(\cdot\right)$, where $\phi_2\left(\cdot\right)$ is defined by (A2), we obtain $-\overline{S}^n\left(p_1\right) - \lambda\left[D^n\left(p_1\right) - D^s\left(p_1\right)\right] + c\left[2\sigma'\left(p_1 + c\right) + \left(p_1 + c\right)\sigma''\left(p_1 + c\right)\right]$. Given that the expression in the first square brackets is positive (with naïve underestimating consumers) and the expression in the second square brackets is negative (by profit concavity), we find that for $\overline{S}^n\left(p_1\right) > 0$ it holds $p_2 = p_1 + c \geq p_1 > p_2^m$. This violates the sequential optimality constraint (5) and thus the solution with naïfs' ex post storage cannot be sustained in equilibrium.

(IIb) For $c \geq \overline{c}^u$, consumers do not store. This identifies the dynamic solution without storage. We find from the sequential optimality constraint (5) that the equilibrium prices are

$$p_1^{**u} = p_2^{**u} - c \text{ and } p_2^{**u} = p_2^m. \tag{A8}$$

As the firm's maximization problem in (A5) allows for any $S(\cdot)$, option (IIa) dominates option (IIb) whenever it is feasible, i.e., $\overline{S}^{*u} > 0$.

(III) Suppose $p_1 + c < p_2$. It follows from (4) that ex ante storage is $S(p_1) = \sigma(p_1 + c)$. Given that at least sophisticates carry their entire ex ante storage in the second period, ex post storage cannot be lower than $\lambda\sigma(p_1 + c)$. For any first period price $p_1$, this level of ex post storage is (weakly) higher than the one in option (IIa), where $\overline{S}(p_1) = \lambda S(p_1)$ and $S(p_1) \in [0, \sigma(p_1 + c)]$. It follows from the sequential optimality constraint (5) that the firm prefers to set a second period price $p_2$ that does not exceed the level $p_1 + c$ in option (IIa). This contradicts the supposition $p_1 + c < p_2$ and thus option (III) is not implementable. ∎

**Proof of Lemma 2.** Set $c = \widetilde{c}^u$, where $\widetilde{c}^u \triangleq p_2^m - p_1^{sm}$. It follows from the static feasibility constraint (6) that the static solution is feasible. As $p_1^{sm} + \widetilde{c}^u = p_2^m$, substituting $\phi_1(p_1^{sm}) = 0$ and $\phi_2(p_2^m) = 0$, where $\phi_1(\cdot)$ is defined by (A1) and $\phi_2(\cdot)$ by (A2), into the left-hand side of the first-order condition for $p_1^{*u}$ in (A6) at $c = \widetilde{c}^u$ yields $[p_1^{sm}(1-\lambda) - \widetilde{c}^u\lambda]\,\phi_2'(p_2^m)$. As $\phi_2'(\cdot) < 0$ (by profit concavity), we find that at $c = \widetilde{c}^u$ it holds $p_1^{sm} > p_1^{*u}$ if and only if $p_1^{sm}(1-\lambda) - \widetilde{c}^u\lambda > 0$. As $\partial\overline{S}(\cdot)/\partial p_1 < 0$ (see the proof of Lemma 1), it holds $\overline{c}^u > \widetilde{c}^u$ if and only if $p_1^{sm}(1-\lambda) - \widetilde{c}^u\lambda > 0$. To see this, note that, if $p_1^{sm}(1-\lambda) - \widetilde{c}^u\lambda > 0$, we have $p_1^{sm} > p_1^{*u}$ and thus a price decrease from $p_1^{sm}$ to $p_1^{*u}$ at $c = \widetilde{c}^u$ leads from zero storage to positive storage (as $\partial\overline{S}(\cdot)/\partial p_1 < 0$), which implies that $\overline{c}^u > \widetilde{c}^u$. Furthermore, if $\overline{c}^u > \widetilde{c}^u$, a price change from $p_1^{sm}$ to $p_1^{*u}$ at $c = \widetilde{c}^u$ leads from zero storage to positive storage, which implies $p_1^{sm} > p_1^{*u}$ (as $\partial\overline{S}(\cdot)/\partial p_1 < 0$) and thus $p_1^{sm}(1-\lambda) - \widetilde{c}^u\lambda > 0$. As $\widetilde{c}^u \triangleq p_2^m - p_1^{sm}$, there exists a threshold $\widetilde{\lambda} \triangleq p_1^{sm}/p_2^m > 0$ such that $p_1^{sm}(1-\lambda) - \widetilde{c}^u\lambda = p_1^{sm} - \lambda p_2^m > 0$, or equivalently $\overline{c}^u > \widetilde{c}^u$, if and only if $\lambda < \widetilde{\lambda}$. Note from the firm's maximization problem in (A5) that $p_1^{sm}(1-\lambda) - \widetilde{c}^u\lambda > 0$ is equivalent to $\partial\Pi(p_1^{sm}, p_1^{sm} + c)/\partial S|_{c=\widetilde{c}^u} > 0$, where $p_1^{sm} + \widetilde{c}^u = p_2^m$.

We now characterize the condition under which the threshold $\widetilde{\lambda}$ is unique. To this aim, define $\Upsilon(\lambda) \triangleq p_1^{sm} - \lambda p_2^m$. As $\Upsilon(\lambda) \to p_1^{sm} > 0$ for $\lambda \to 0$ and $\Upsilon(\lambda) \to 0$ for $\lambda \to 1$, a sufficient (albeit not necessary) condition for a unique threshold $\widetilde{\lambda} > 0$ such that $\Upsilon(\lambda) > 0$ if and only if $\lambda < \widetilde{\lambda}$ is that $\Upsilon(\lambda)$ is (weakly) convex in $\lambda$, i.e., $\partial^2\Upsilon/\partial\lambda^2 = -2(\partial p_2^m/\partial\lambda) - \lambda\left(\partial^2 p_2^m/\partial\lambda^2\right) \geq 0$. It follows from (A4) that

$$
\begin{aligned}
\frac{\partial^2 p_2^m}{\partial\lambda^2} = {} & \frac{\partial p_2^m}{\partial\lambda}\frac{2\left[D^{n\prime}(p_2^m) - D^{s\prime}(p_2^m)\right] + p_2^m\left[D^{n\prime\prime}(p_2^m) - D^{s\prime\prime}(p_2^m)\right]}{\Pi_2''(p_2^m)} \\
& + \frac{D^n(p_2^m) - D^s(p_2^m) + p_2^m\left[D^{n\prime}(p_2^m) - D^{s\prime}(p_2^m)\right]}{\left[\Pi_2''(p_2^m)\right]^2} \\
& \times \left\{2\left[D^{n\prime}(p_2^m) - D^{s\prime}(p_2^m)\right] + p_2^m\left[D^{n\prime\prime}(p_2^m) - D^{s\prime\prime}(p_2^m)\right] - \frac{\partial p_2^m}{\partial\lambda}\Pi_2'''(p_2^m)\right\}.
\end{aligned}
$$

Then, it holds $\partial^2\Upsilon/\partial\lambda^2 \geq 0$ if and only if

$$
\begin{aligned}
& \lambda\frac{\partial p_2^m}{\partial\lambda}\left\{2\left[D^{n\prime}(p_2^m) - D^{s\prime}(p_2^m)\right] + p_2^m\left[D^{n\prime\prime}(p_2^m) - D^{s\prime\prime}(p_2^m)\right]\right\}\Pi_2''(p_2^m) \\
& + 2\left[\Pi_2''(p_2^m)\right]^2\frac{\partial p_2^m}{\partial\lambda} + \lambda\left\{D^n(p_2^m) - D^s(p_2^m) + p_2^m\left[D^{n\prime}(p_2^m) - D^{s\prime}(p_2^m)\right]\right\} \\
& \qquad \times \left\{2\left[D^{n\prime}(p_2^m) - D^{s\prime}(p_2^m)\right] + p_2^m\left[D^{n\prime\prime}(p_2^m) - D^{s\prime\prime}(p_2^m)\right] - \frac{\partial p_2^m}{\partial\lambda}\Pi_2'''(p_2^m)\right\} \leq 0.
\end{aligned}
$$

Using (A4), we find after some manipulation that $\partial^2 Y/\partial\lambda^2 \geq 0$ if and only if

$$2\left[2D^{n\prime}\left(p_2^m\right)+p_2^m D^{n\prime\prime}\left(p_2^m\right)\right]-\lambda\frac{\partial p_2^m}{\partial\lambda}\Pi_2^{\prime\prime\prime}\left(p_2^m\right)\leq 0.$$

As the expression in square brackets is negative (by profit concavity) and $\partial p_2^m/\partial\lambda < 0$ (see (A4) with naïve underestimating consumers), we find that a sufficient (albeit not necessary) condition for $\partial^2 Y/\partial\lambda^2 \geq 0$ is that $\Pi_2^{\prime\prime\prime}\left(p_2^m\right) \leq 0$. Thus, if $\Pi_2^{\prime\prime\prime}\left(p_2^m\right) \leq 0$, there exists a unique threshold $\widetilde{\lambda}$ such that $\bar{c}^u > \widetilde{c}^u$ if and only if $\lambda < \widetilde{\lambda}$, where $\widetilde{\lambda} \triangleq p_1^{sm}/p_2^m > 0$. Using linear demand functions $D^s\left(p_\tau\right) = \alpha^s - \beta p_\tau$ and $D^n\left(p_\tau\right) = \alpha^n - \beta p_\tau$, where $\alpha^n > \alpha^s$, we obtain from (A1) and (A2) that $p_1^{sm} = \alpha^s/\left(2\beta\right)$ and $p_2^m = \left[\alpha^n\left(1-\lambda\right)+\alpha^s\lambda\right]/\left(2\beta\right)$. This implies that $Y\left(\lambda\right) \triangleq p_1^{sm} - \lambda p_2^m > 0$ if and only if $\lambda < \widetilde{\lambda}$, where $\widetilde{\lambda} = \alpha^s/\left(\alpha^n-\alpha^s\right) > 0$ (it holds $\widetilde{\lambda} < 1$ if and only if $\alpha^n > 2\alpha^s$). ■

**Proof of Proposition 1.** First, suppose $\lambda \leq \widetilde{\lambda}$. It follows from Lemma 2 that $\bar{c}^u \geq \widetilde{c}^u$, where $\bar{c}^u = \widetilde{c}^u$ if and only if $\lambda = \widetilde{\lambda}$. Let $\Pi^m$ be the firm's profit in the static solution (derived in point (I) of Lemma 1) and $\Pi^{*u}$ be the firm's profit in the dynamic storage solution (derived in point (IIa) of Lemma 1). At $c = \widetilde{c}^u$ it holds $\Pi^{*u} \geq \Pi^m$, where $\Pi^{*u} = \Pi^m$ if and only if $\bar{c}^u = \widetilde{c}^u$. To see this, note that at $c = \widetilde{c}^u$ the firm obtains the static monopoly profit by charging the prices $p_1^{sm}$ and $p_2^m = p_1^{sm} + c$. As $\overline{S}^{*u} > 0$, a revealed preference argument shows that the firm is better off by implementing the dynamic storage solution (which is feasible) through the prices $p_1^{*u}$ and $p_2^{*u} = p_1^{*u} + c$. At $c = \bar{c}^u$ it holds $\Pi^m \geq \Pi^{*u}$, where $\Pi^m = \Pi^{*u}$ if and only if $\bar{c}^u = \widetilde{c}^u$, because the dynamic solution yields no storage (as shown in point (IIb) of Lemma 1) and the static solution arises from an unconstrained maximization problem. Note that $\Pi^m$ is independent of $c$, whereas $\partial\Pi^{*u}/\partial c = \left[p_1^{*u}\left(1-\lambda\right)-c\lambda\right]\phi_2'\left(p_1^{*u}+c\right)/\lambda$. As $\phi_2'\left(\cdot\right) < 0$ (by profit concavity), we have $\partial\Pi^{*u}/\partial c < 0$ if and only if $p_1^{*u}\left(1-\lambda\right)-c\lambda > 0$. For $c > \widetilde{c}^u$, it follows from the static feasibility constraint (6) that $p_2^m < p_1^{sm} + c$. As the sequential optimality constraint (5) implies $p_2^{*u} < p_2^m$, we find that $p_2^{*u} = p_1^{*u} + c < p_1^{sm} + c$, which implies $p_1^{*u} < p_1^{sm}$. Using (A1) and (A2), we obtain from $p_1^{*u} < p_1^{sm}$ and $p_2^{*u} < p_2^m$ that $\lambda\phi_1\left(p_1^{*u}\right)+\phi_2\left(p_1^{*u}+c\right) > 0$. As $\phi_2'\left(\cdot\right) < 0$ (by profit concavity), it follows from the first-order condition for $p_1^{*u}$ in (A6) that $p_1^{*u}\left(1-\lambda\right)-c\lambda > 0$ and thus $\partial\Pi^{*u}/\partial c < 0$. Recalling that at $c = \widetilde{c}^u$ it holds $\Pi^{*u} \geq \Pi^m$ and at $c = \bar{c}^u$ it holds $\Pi^m \geq \Pi^{*u}$ (where the equalities follow if and only if $\widetilde{c}^u = \bar{c}^u$), we find from the intermediate value theorem that there exists a unique threshold $\widehat{c}^u \in \left[\widetilde{c}^u, \bar{c}^u\right]$ such that for $c < \widehat{c}^u$ it holds $\Pi^{*u} > \Pi^m$ and for $c \geq \widehat{c}^u$ it holds $\Pi^m \geq \Pi^{*u}$, with $\Pi^m = \Pi^{*u}$ if and only if $c = \widehat{c}^u$. If follows from Lemma 1 that in equilibrium (i) for $c < \widehat{c}^u$ the dynamic storage solution (characterized in point (IIa) of Lemma 1) arises, and (ii) for $c \geq \widehat{c}^u$ the static solution (characterized in point (I) of Lemma 1) arises. Now, suppose $\lambda > \widetilde{\lambda}$. It follows from Lemma 2 that $\widetilde{c}^u > \bar{c}^u$. It follows from Lemma 1 that in equilibrium (i) for $c < \bar{c}^u$ the dynamic storage solution (characterized in point (IIa) of Lemma 1) arises, (ii) for $\bar{c}^u \leq c < \widetilde{c}^u$ the dynamic solution without storage (characterized in point (IIb) of Lemma 1) arises, and (iii) for $c \geq \widetilde{c}^u$ the static solution (characterized in point (I) of Lemma 1) arises. ■

**Proof of Proposition 2.** First, we show the price comparisons in the first period. For $c < \widetilde{c}^u$, the static feasibility constraint (6) is violated. We know from Proposition 1 that either the dynamic storage solution or the dynamic solution without storage is implemented in equilibrium. First, we consider the dynamic storage solution. Substituting $\phi_1\left(p_1^{sm}\right) = 0$, where $\phi_1\left(\cdot\right)$ is defined by (A1), into the first-order condition for $p_1^{*u}$ in (A6) yields $\left[p_1^{sm}\left(1-\lambda\right) - c\lambda\right]\phi_2'\left(p_1^{sm}+c\right) + \phi_2\left(p_1^{sm}+c\right)$. It follows from $c < \widetilde{c}^u$ or, equivalently, $p_1^{sm} + c < p_2^m$ that $\phi_2\left(p_1^{sm}+c\right) > 0$. As $\phi_2'\left(\cdot\right) < 0$ (by profit concavity), we find that, for $c < \widetilde{c}^u$, there exists a unique threshold $\overline{\lambda} \triangleq p_1^{sm} / \left(p_1^{sm}+c\right)$ such that a sufficient (but not necessary) condition for $p_1^{*u} > p_1^{sm}$ is that $\lambda > \overline{\lambda}$. Now, we consider the dynamic solution without storage. Using (A8), we obtain from $c < \widetilde{c}^u$ or, equivalently, $p_1^{sm} + c < p_2^m = p_2^{**u}$ that $p_1^{**u} = p_2^{**u} - c > p_1^{sm}$. For $\widetilde{c}^u \leq c < \widehat{c}^u$, where $\widehat{c}^u \in \left[\widetilde{c}^u, \overline{c}^u\right]$, we know from Proposition 1 that the dynamic storage solution emerges in equilibrium. This interval is non-empty for $\lambda < \widetilde{\lambda}$. We find from $c \geq \widetilde{c}^u$ or, equivalently, $p_1^{sm} + c \geq p_2^m$ that $p_1^{*u} < p_1^{sm}$. To see this, we proceed by contradiction. Note that $p_1^{*u} \geq p_1^{sm}$ implies $p_2^{*u} = p_1^{*u} + c \geq p_1^{sm} + c \geq p_2^m$. This violates $p_2^{*u} < p_2^m$, as implied by the sequential optimality constraint (5). For $c \geq \max\left\{\widetilde{c}^u, \widehat{c}^u\right\}$, it follows from Proposition 1 that the static solution applies in equilibrium. Specifically, if $\lambda \leq \widetilde{\lambda}$, the static solution arises in equilibrium for $c \geq \widehat{c}^u$, with $\widehat{c}^u \in \left[\widetilde{c}^u, \overline{c}^u\right]$, which implies $\widehat{c}^u \geq \widetilde{c}^u$ or, equivalently, $\widehat{c}^u = \max\left\{\widetilde{c}^u, \widehat{c}^u\right\}$. If $\lambda > \widetilde{\lambda}$, the static solution arises in equilibrium whenever it is feasible, i.e., for $c \geq \widetilde{c}^u$, which implies $\widetilde{c}^u \geq \widehat{c}^u$ or, equivalently, $\widetilde{c}^u = \max\left\{\widetilde{c}^u, \widehat{c}^u\right\}$. Turning to the price comparisons in the second period, we find from the sequential optimality constraint (5) that in the presence of storage it holds $p_2^{*u} < p_2^m$. In the absence of storage, the second period equilibrium price is $p_2^m$. Using linear demand functions $D^s\left(p_\tau\right) = \alpha^s - \beta p_\tau$ and $D^n\left(p_\tau\right) = \alpha^n - \beta p_\tau$, where $\alpha^n > \alpha^s$, we obtain that $p_1^{*u} < p_1^{sm}$ (where $p_1^{*u}$ and $p_1^{sm}$ are derived in the proofs of Lemmas 1 and 2) if and only if $c > \left(\alpha^n - 2\alpha^s\right) / \left(2\beta\right)$. ■

**Proof of Proposition 3.** Taking the derivative of the left-hand side of the first-order condition for $p_1^{*u}$ in (A6) with respect to $\lambda$ yields

$$\Omega \triangleq \phi_1\left(p_1^{*u}\right) + \frac{\partial\phi_2\left(p_1^{*u}+c\right)}{\partial\lambda} - \left(p_1^{*u}+c\right)\phi_2'\left(p_1^{*u}+c\right) + \left[p_1^{*u}\left(1-\lambda\right) - c\lambda\right]\frac{\partial\phi_2'\left(p_1^{*u}+c\right)}{\partial\lambda}.$$

It follows from the implicit function theorem (and the binding storability constraint) that $\partial p_\tau^{*u}/\partial\lambda > 0$, $\tau \in \{1,2\}$, if and only if $\Omega > 0$. Let $p_1^{*u} < p_1^{sm}$. This implies from (A1) that $\phi_1\left(p_1^{*u}\right) > 0$. Two cases emerge. First, suppose $\partial\phi_2'\left(p_1^{*u}+c\right)/\partial\lambda \geq 0$. As $p_1^{*u} + c = p_2^{*u} < p_2^m$ due to the sequential optimality constraint (5), we obtain that $\lambda\phi_1\left(p_1^{*u}\right) + \phi_2\left(p_1^{*u}+c\right) > 0$. This implies from (A6) that $p_1^{*u}\left(1-\lambda\right) - c\lambda > 0$. As $\phi_2'\left(\cdot\right) < 0$ (by profit concavity) and $\partial\phi_2'\left(\cdot\right)/\partial\lambda \geq 0$ (by supposition), we find from $\partial\phi_2\left(\cdot\right)/\partial\lambda < 0$ (with naïve underestimating consumers), where $\phi_2\left(\cdot\right)$ is given by (A2), that a sufficient (albeit not necessary) condition for $\Omega > 0$ or, equivalently, $\partial p_\tau^{*u}/\partial\lambda > 0$, $\tau \in \{1,2\}$, is that $\phi_1\left(p_1^{*u}\right) > \left|\partial\phi_2\left(p_1^{*u}+c\right)/\partial\lambda\right|$. Now, suppose $\partial\phi_2'\left(p_1^{*u}+c\right)/\partial\lambda < 0$. The expression for $\Omega$ can be rewritten as

$$\Omega \triangleq \phi_1\left(p_1^{*u}\right) + \frac{\partial\phi_2\left(p_1^{*u}+c\right)}{\partial\lambda} - \left(p_1^{*u}+c\right)\left[\phi_2'\left(p_1^{*u}+c\right) + \lambda\frac{\partial\phi_2'\left(p_1^{*u}+c\right)}{\partial\lambda}\right] + p_1^{*u}\frac{\partial\phi_2'\left(p_1^{*u}+c\right)}{\partial\lambda}.$$

Given that $\phi_2'\left(\cdot\right) < 0$ (by profit concavity) and $\partial\phi_2'\left(\cdot\right)/\partial\lambda < 0$ (by supposition), the expression in square brackets is negative. It follows from $\partial\phi_2\left(\cdot\right)/\partial\lambda < 0$ (with naïve underestimating consumers), where $\phi_2\left(\cdot\right)$ is given by (A2), that a sufficient (albeit not necessary) condition for $\Omega > 0$ or, equivalently, $\partial p_\tau^{*u}/\partial\lambda > 0$, $\tau \in \{1,2\}$, is that $\phi_1\left(p_1^{*u}\right) >$

$|\partial\phi_2(p_1^{*u}+c)/\partial\lambda + p_1^{*u}[\partial\phi_2'(p_1^{*u}+c)/\partial\lambda]|$. Hence, we can conclude that $\partial p_\tau^{*u}/\partial\lambda > 0$, $\tau \in \{1,2\}$, if $\phi_1(p_1^{*u})$ is large enough or, equivalently, if $p_1^{*u}$ is sufficiently lower than $p_1^{sm}$. Using linear demand functions of the form $D^s(p_\tau) = \alpha^s - \beta p_\tau$ and $D^n(p_\tau) = \alpha^n - \beta p_\tau$, where $\alpha^n > \alpha^s$, we obtain that $\partial p_\tau^{*u}/\partial\lambda < 0$, $\tau \in \{1,2\}$, if and only if $c > (\alpha^n - 2\alpha^s)/(2\beta)$ (where $p_\tau^{*u}$ is derived in the proof of Lemma 1). It follows from the proof of Proposition 2 that this condition is equivalent to $p_1^{*u} < p_1^{sm}$. Now, we consider the case where the dynamic solution without storage emerges in equilibrium. Using (A8), we find from (A4) that $\partial p_\tau^{**u}/\partial\lambda < 0$, $\tau \in \{1,2\}$. Finally, we consider the case where the static solution emerges in equilibrium. Using (A3), we obtain that $p_1^{sm}$ is independent of $\lambda$ and $\partial p_2^m/\partial\lambda < 0$. ■

**Proof of Proposition 4.** We know from Section 3.4 that the static solution is feasible if and only if $c \geq \tilde{c}^o$. As at $p_1^{sm} = \lim_{\lambda\to 1} p_2^m$ we have $p_2^m = u^{n\prime}(D^n(p_2^m)) > u^{n\prime}(D^s(p_1^{sm}))$ (due to $u^{n\prime\prime}(\cdot) < 0$ and the assumptions about naïve overestimating consumers), it holds $\tilde{c}^o > 0$ for $\lambda$ high enough. Taking the derivative of $\tilde{c}^o$ with respect to $\lambda$ yields $\partial\tilde{c}^o/\partial\lambda = \partial p_2^m/\partial\lambda > 0$, where the inequality follows from (A4). Using linear demand functions of the form $D^s(p_\tau) = \alpha^s - \beta p_\tau$ and $D^n(p_\tau) = \alpha^n - \beta p_\tau$, where $\alpha^s > \alpha^n$, we find from (A3) that $\tilde{c}^o = (\alpha^s - \alpha^n)(1+\lambda)/(2\beta) > 0$. For $c < \tilde{c}^o$, the static solution is not feasible. The firm's dynamic profit maximization problem is given by (8), where (aggregate) ex post storage $\overline{S}(p_1, p_2^e) = (1-\lambda)\overline{s}^n(p_1, p_2^e)$ in the sequential optimality constraint (5) stems from (9). Proceeding backward, the firm's second period maximization problem writes as

$$\max_{p_2} p_2\left[\sigma(p_2) - \overline{S}(p_1, p_2^e)\right].$$

The first-order condition for $p_2$ is given by

$$\sigma(p_2) + p_2\sigma'(p_2) - \overline{S}(p_1, p_2^e) = 0. \quad \text{(A9)}$$

As $p_2^e = p_2$, the firm's first period maximization problem becomes

$$\max_{p_1} p_1 D^s(p_1) + p_2\left[\sigma(p_2) - \overline{S}(p_1, p_2)\right] \quad \textit{s.t.}\ (9)\ \text{and (A9).} \quad \text{(A10)}$$

It follows from the envelope theorem that the first-order condition for $p_1$ is

$$D^s(p_1) + p_1 D^{s\prime}(p_1) - p_2\frac{d\overline{S}(p_1, p_2)}{dp_1} = 0. \quad \text{(A11)}$$

Applying the implicit function theorem to (9) and (A9) yields

$$\frac{d\overline{S}(p_1, p_2)}{dp_1} = \frac{(1-\lambda)D^{s\prime}(p_1)\left[2\sigma'(p_2) + p_2\sigma''(p_2)\right]}{2\sigma'(p_2) + p_2\sigma''(p_2) + (1-\lambda)D^{n\prime}(p_2 - c)} < 0, \quad \text{(A12)}$$

where the inequality follows from $\phi_2'(\cdot) < 0$ (by profit concavity). Using (A9) and (A11), we find that the equilibrium prices with storage are given by

$$p_1^{*o} = -\frac{D^s(p_1^{*o}) - p_2^{*o}\left[d\overline{S}(p_1^{*o}, p_2^{*o})/dp_1\right]}{D^{s\prime}(p_1^{*o})} \quad \text{and} \quad p_2^{o*} = -\frac{\sigma(p_2^{o*}) - \overline{S}(p_1^{*o}, p_2^{*o})}{\sigma'(p_2^{o*})}. \quad \text{(A13)}$$

Given that $p_1^{*o} > p_1^{sm}$ and $p_2^{o*} < p_2^{m}$ (see the proof of Proposition 5) as well as $p_1^{sm} > p_2^{m}$ (see Section 3.2), it holds $p_1^{*o} > p_2^{o*}$. Inserting (A13) into (9), the equilibrium (aggregate) ex post storage is $\overline{S}^{*o} \triangleq \overline{S}\left(p_1^{*o}, p_2^{*o}\right) = (1-\lambda)\,\overline{s}^{*o} = (1-\lambda)\left[D^s\left(p_1^{*o}\right) - D^n\left(p_2^{*o} - c\right)\right]$. Using linear demands $D^s\left(p_\tau\right) = \alpha^s - \beta p_\tau$ and $D^n\left(p_\tau\right) = \alpha^n - \beta p_\tau$, where $\alpha^s > \alpha^n$, we find from (A13) that the equilibrium prices are $p_1^{*o} = \left[\alpha^s\left(7 - 3\lambda^2\right) + 4\alpha^n\left(1-\lambda\right)^2 + 2\beta c\left(1-\lambda\right)^2\right] / \left[8\beta\left(2-\lambda\right)\right]$ and $p_2^{*o} = (3-\lambda)\left[\alpha^s\left(3\lambda - 1\right) + 4\alpha^n\left(1-\lambda\right) + 2\beta c\left(1-\lambda\right)\right] / \left[8\beta\left(2-\lambda\right)\right]$. Inserting $p_1^{*o}$ and $p_2^{*o}$ into (9), the equilibrium ex post storage is $\overline{S}^{*o} = (1-\lambda)\left[\alpha^s\left(3+\lambda\right) - 4\alpha^n - 2\beta c\left(3-\lambda\right)\right] / \left[4\left(2-\lambda\right)\right]$.

Substituting $\phi_1\left(p_1^{sm}\right) = 0$, with $\phi_1(\cdot)$ defined by (A1), for $c = \widetilde{c}^o$ (where the static solution is feasible) into the right-hand side of the first-order condition for $p_1^{*o}$ in (A11) yields $-p_2\left[d\overline{S}\left(p_1^{sm}, p_2\right)/dp_1\right] > 0$, where the inequality follows (A12). Thus, we obtain that $p_1^{*o} > p_1^{sm}$, which implies from (A12) that ex post storage should become negative for $c = \widetilde{c}^o$. As this is not feasible, there exists a threshold $\overline{c}^o < \widetilde{c}^o$ such that for $\overline{c}^o \leq c < \widetilde{c}^o$ the dynamic storage solution and the static solution are both unfeasible. In this case, the dynamic solution without storage emerges in equilibrium. The first period equilibrium price is such that storage vanishes, i.e., $D^s\left(p_1\right) - D^n\left(p_2 - c\right) = 0$, and the second period equilibrium price reflects the static monopoly price due to the sequential optimality constraint (5). Thus, the equilibrium prices are given by

$$p_1^{**o} = D^{s-1}\left(D^n\left(p_2^m - c\right)\right) \;\text{ and }\; p_2^{**o} = p_2^m. \tag{A14}$$

Given that $p_1^{**o} > p_1^{sm}$ (see the proof of Proposition 5) and $p_1^{sm} > p_2^{m}$ (see Section 3.2), it holds $p_1^{**o} > p_2^{m}$.

Now, we characterize the conditions for the uniqueness of the threshold $\overline{c}^o$ below which the dynamic storage solution applies. Taking the derivative of $\overline{S}^{*o} = \overline{S}\left(p_1^{*o}(c), c\right)$ with respect to $c$ yields

$$\frac{d\overline{S}^{*o}}{dc} = \frac{d\overline{S}\left(p_1^{*o}, p_2^{*o}\right)}{dp_1}\frac{\partial p_1^{*o}}{\partial c} + \frac{\partial \overline{S}^{*o}}{\partial c}.$$

Applying the implicit function theorem to (9) and (A9) yields

$$\frac{\partial \overline{S}^{*o}}{\partial c} = \frac{(1-\lambda)\,D^{n\prime}\left(p_1^{*o}\right)\left[2\sigma'\left(p_2^{*o}\right) + p_2^{*o}\sigma''\left(p_2^{*o}\right)\right]}{2\sigma'\left(p_2^{*o}\right) + p_2^{*o}\sigma''\left(p_2^{*o}\right) + (1-\lambda)\,D^{n\prime}\left(p_2^{*o} - c\right)} < 0,$$

where the inequality follows from $\phi_2'(\cdot) < 0$ (by profit concavity). As $d\overline{S}\left(p_1^{*o}, p_2^{*o}\right)/dp_1 < 0$ (see (A12)), we find that a sufficient (albeit not necessary) condition for $d\overline{S}^{*o}/dc < 0$ is that $\partial p_1^{*o}/\partial c > 0$. Taking the derivative of the right-hand side of the first-order condition for $p_1^{*o}$ in (A11) with respect to $c$ yields

$$-\frac{\partial p_2\left(p_1^{*o}\right)}{\partial c}\frac{d\overline{S}\left(p_1^{*o}, p_2^{*o}\right)}{dp_1} - p_2^{*o}\frac{\partial^2 \overline{S}\left(p_1^{*o}, p_2^{*o}\right)}{\partial p_1 \partial c}.$$

Applying the implicit function theorem to (9) and (A9) yields

$$\frac{\partial p_2\left(p_1^{*o}\right)}{\partial c} = \frac{(1-\lambda)\,D^{n\prime}\left(p_1^{*o}\right)}{2\sigma'\left(p_2^{*o}\right) + p_2^{*o}\sigma''\left(p_2^{*o}\right) + (1-\lambda)\,D^{n\prime}\left(p_2^{*o} - c\right)} > 0,$$

where the inequality follows from $\phi_2'(\cdot) < 0$ (by profit concavity). As $d\overline{S}(p_1^{*o}, p_2^{*o})/dp_1 < 0$ (see (A12)), we find from the implicit function theorem that a sufficient (albeit not necessary) condition for $\partial p_1^{*o}/\partial c > 0$ is that $\partial^2 \overline{S}(p_1^{*o}, p_2^{*o})/\partial p_1 \partial c \leq 0$. Taking the derivative of $d\overline{S}(p_1, p_2)/dp_1$ in (A12) with respect to $c$ yields after some manipulation

$$\frac{\partial^2 \overline{S}(p_1^{*o}, p_2^{*o})}{\partial p_1 \partial c} = \frac{(1-\lambda)^2 D^{s\prime}(p_1^{*o})}{\left[\Pi_2''(p_2^{*o}) + (1-\lambda) D^{n\prime}(p_2^{*o} - c)\right]^2} \left\{ \Pi_2'''(p_2^{*o}) D^{n\prime}(p_2^{*o} - c) \frac{\partial p_2(p_1^{*o})}{\partial c} \right.$$
$$\left. - \Pi_2''(p_2^{*o}) D^{n\prime\prime}(p_2^{*o} - c) \left[ \frac{\partial p_2(p_1^{*o})}{\partial c} - 1 \right] \right\}.$$

As $\partial p_2(p_1^{*o})/\partial c \in (0,1)$, sufficient (albeit not necessary) conditions for $\partial^2 \overline{S}(p_1^{*o}, p_2^{*o})/\partial p_1 \partial c \leq 0$ and thus for $\partial p_1^{*o}/\partial c > 0$ are that $\Pi_2'''(p_2^{*o}) \leq 0$ and $D^{n\prime\prime}(p_2^{*o} - c) \leq 0$. Such conditions ensure that $d\overline{S}^{*o}/dc < 0$, which implies that there exists a unique threshold $\overline{c}^o$ below which the dynamic storage solution applies in equilibrium. With linear demands (see above), we find that $d\overline{S}^{*o}/dc < 0$ and $\overline{c}^o = \left[\alpha^s(3+\lambda) - 4\alpha^n\right] / \left[2\beta(1-\lambda)(3-\lambda)\right]$, where $\overline{c}^o > 0$ either if $\alpha^s \geq (4/3)\alpha^n$ or if $\alpha^s < (4/3)\alpha^n$ and $\lambda > (4\alpha^n - 3\alpha^s)/\alpha^s$. Finally, we derive the condition ensuring that $\overline{s}^{*o}$ increases with $\lambda$, which implies that $\overline{c}^o$ increases with $\lambda$ as well. Taking the derivative of $\overline{s}^{*o} = \overline{s}^n(p_1^{*o}(\lambda), \lambda)$ with respect to $\lambda$ yields

$$\frac{d\overline{s}^{*o}}{d\lambda} = \frac{d\overline{s}^n(p_1^{*o}, p_2^{*o})}{dp_1} \frac{\partial p_1^{*o}}{\partial \lambda} + \frac{\partial \overline{s}^{*o}}{\partial \lambda}.$$

Applying the implicit function theorem to (9) and (A9) yields

$$\frac{\partial \overline{s}^{*o}}{\partial \lambda} = D^{n\prime}(p_2^{*o} - c) \frac{D^s(p_2^{*o}) - D^n(p_2^{*o}) + p_2^{*o}\left[D^{s\prime}(p_2^{*o}) - D^{n\prime}(p_2^{*o})\right]}{2\sigma'(p_2^{*o}) + p_2^{*o}\sigma''(p_2^{*o}) + (1-\lambda) D^{n\prime}(p_2^{*o} - c)} > 0,$$

where the inequality follows from $\phi_2'(\cdot) < 0$ (by profit concavity) and the assumptions about naïve overestimating consumers. As $d\overline{s}^n(p_1^{*o}, p_2^{*o})/dp_1 < 0$ (see (A12)), we find that a sufficient (albeit not necessary) condition for $d\overline{s}^{*o}/d\lambda > 0$ is that $\partial p_1^{*o}/\partial \lambda \leq 0$. With linear demands (see above), we find that $d\overline{s}^{*o}/d\lambda > 0$ and $\partial \overline{c}^o/\partial \lambda > 0$. ■

**Proof of Proposition 5.** First, we show the price comparisons in the first period. For $c < \overline{c}^o$, we know from Proposition 4 that the dynamic storage solution applies in equilibrium. Substituting $\phi_1(p_1^{sm}) = 0$, with $\phi_1(\cdot)$ defined by (A1), into the right-hand side of the first-order condition for $p_1^{*o}$ in (A11) yields $-p_2\left[d\overline{S}(p_1^{sm}, p_2)/dp_1\right] > 0$, where the inequality follows (A12). It follows from the implicit function theorem that $p_1^{*o} > p_1^{sm}$. For $\overline{c}^o \leq c < \widetilde{c}^o$, we know from Proposition 4 that the dynamic solution without storage applies in equilibrium. As the second period equilibrium price reflects the static monopoly price due to the sequential optimality constraint (5) but the static solution would induce naïfs' ex post storage and thus cannot be implemented, naïfs' first period marginal utility $u^{n\prime}(D^s(p_1))$ must be higher than in the static solution in order to deter storage. It follows from $D^{s\prime}(p_1) < 0$ and $u^{n\prime\prime}(D^s(p_1)) < 0$ that $p_1^{**o} > p_1^{sm}$. For $c \geq \widetilde{c}^o$, we know from Proposition 4 that the static solution applies in equilibrium. Turning to the second period equilibrium price, we find from the sequential optimality constraint (5) that in the presence of storage it holds $p_2^{*o} < p_2^m$. In the absence of storage, the second period equilibrium price is $p_2^m$. ■

**Proof of Proposition 6.** Taking the derivative of the left-hand side of the first-order condition for $p_1^{*o}$ in (A11) with respect to $\lambda$ and recalling $\overline{S}(p_1, p_2^e) = (1-\lambda)\,\overline{s}^n(p_1, p_2^e)$ yields

$$p_2^{*o}\frac{d\overline{s}^n(p_1^{*o}, p_2^{*o})}{dp_1} - (1-\lambda)\left[\frac{\partial p_2(p_1^{*o})}{\partial\lambda}\frac{d\overline{s}^n(p_1^{*o}, p_2^{*o})}{dp_1} + p_2^{*o}\frac{\partial^2\overline{s}^n(p_1^{*o}, p_2^{*o})}{\partial p_1\partial\lambda}\right].$$

It follows from the implicit function theorem that the sign of $\partial p_1^{*o}/\partial\lambda$ corresponds to the sign of this expression. As $d\overline{s}^n(p_1^{*o}, p_2^{*o})/dp_1 < 0$ (see (A12)) but the sign of the expression in square brackets is a priori ambiguous, we find that a sufficient (albeit not necessary) condition for $\partial p_1^{*o}/\partial\lambda < 0$ is that $\lambda$ is high enough. Taking the derivative of the left-hand side of the first-order condition for $p_2^{*o}$ in (A9) with respect to $\lambda$ and recalling $\overline{S}(p_1, p_2^e) = (1-\lambda)\,\overline{s}^n(p_1, p_2^e)$ yields

$$D^s(p_2^{*o}) - D^n(p_2^{*o}) + p_2^{*o}\left[D^{s\prime}(p_2^{*o}) - D^{n\prime}(p_2^{*o})\right] + \overline{s}^n(p_1^{*o}, p_2^{*o}) - (1-\lambda)\frac{d\overline{s}^{*o}}{d\lambda}.$$

It follows from the implicit function theorem that the sign of $\partial p_2^{*o}/\partial\lambda$ corresponds to the sign of this expression. Given that the difference between the first two terms is positive and the expression in square brackets is also positive (with naïve overestimating consumers) as well as $\overline{s}^n(p_1^{*o}, p_2^{*o}) > 0$ but the sign of the last term is a priori ambiguous, a sufficient (albeit not necessary) condition for $\partial p_2^{*o}/\partial\lambda > 0$ is that $\lambda$ is high enough. Using linear demand functions of the form $D^s(p_\tau) = \alpha^s - \beta p_\tau$ and $D^n(p_\tau) = \alpha^n - \beta p_\tau$, where $\alpha^s > \alpha^n$, we find that for $\alpha^s \leq (6/7)(2\alpha^n + \beta c)$ it holds $\partial p_1^{*o}/\partial\lambda \leq 0$ (where $p_1^{*o}$ is derived in the proof of Proposition 4), with $\partial p_1^{*o}/\partial\lambda = 0$ if and only if $\alpha^s = (6/7)(2\alpha^n + \beta c)$. For $\alpha^s > (6/7)(2\alpha^n + \beta c)$, there exists a unique threshold $\lambda^o \triangleq 2 - (5\alpha^s - 4\alpha^n - 2\beta c)/\sqrt{(5\alpha^s - 4\alpha^n - 2\beta c)(3\alpha^s - 4\alpha^n - 2\beta c)} \in (0,1)$ such that $\partial p_1^{*o}/\partial\lambda > 0$ if and only if $\lambda < \lambda^o$. Furthermore, we find that $\partial p_2^{*o}/\partial\lambda > 0$ (where $p_2^{*o}$ is derived in the proof of Proposition 4). Now, we consider the case where the dynamic solution without storage arises in equilibrium. Using (A14), we obtain from $\partial p_2^m/\partial\lambda > 0$ (see (A4)) that $\partial p_1^{**o}/\partial\lambda = [1/D^{s\prime}(p_1^{**o})]\,D^{n\prime}(p_2^m - c)(\partial p_2^m/\partial\lambda) > 0$ and from $p_2^{**o} = p_2^m$ that $\partial p_2^{**o}/\partial\lambda > 0$. Finally, we consider the case where the static solution emerges in equilibrium. Using (A3), we find that $p_1^{sm}$ is independent of $\lambda$ and $\partial p_2^m/\partial\lambda > 0$. ■

**Proof of Proposition 7.** It follows from the sequential optimality constraint (10) that the first-order condition for $p_2$ is given by

$$\widetilde{D}_2(p_2, \zeta) + p_2\widetilde{D}_2^\prime(p_2, \zeta) - \overline{S}(p_1, p_2^e) = 0. \tag{A15}$$

First, suppose that the storability constraint is binding, i.e., $p_1 + c = p_2$. Below, we show that this emerges in equilibrium for $\zeta$ high enough. Given that $S(p_1, p_2^e) = s(p_1, p_2^e)$ and $\overline{S}(p_1, p_2^e) = (1-\theta)\,\overline{s}^l(p_1, p_2^e)$, using (11) and (A15) as well as $p_1 + c = p_2$ yields

$$S(p_1) = \frac{\widetilde{D}_2(p_1 + c, \zeta) + (p_1 + c)\,\widetilde{D}_2^\prime(p_1 + c, \zeta)}{1-\theta} + D^l(p_1) - \widetilde{D}_1(p_1) \tag{A16}$$

and

$$\overline{S}(p_1) = \widetilde{D}_2(p_1 + c, \zeta) + (p_1 + c)\,\widetilde{D}_2^\prime(p_1 + c, \zeta). \tag{A17}$$

It follows from (12) that the firm's first-period maximization problem writes as

$$\max_{p_1} p_1 \left[\widetilde{D}_1(p_1) + S(p_1)\right] + (p_1 + c)\left[\widetilde{D}_2(p_1 + c, \zeta) - \overline{S}(p_1)\right] \quad s.t. \text{ (11) and (A15).}$$

By the envelope theorem, the first-order condition for $p_1$ is given by

$$\widetilde{D}_1(p_1) + S(p_1) + p_1\left[\widetilde{D}_1'(p_1) + S'(p_1)\right] - (p_1 + c)\overline{S}'(p_1) = 0. \tag{A18}$$

Using (A16) and (A17) yields

$$S'(p_1) = \frac{2\widetilde{D}_2'(p_1 + c, \zeta) + (p_1 + c)\widetilde{D}_2''(p_1 + c, \zeta)}{1 - \theta} + D^{l\prime}(p_1) - \widetilde{D}_1'(p_1)$$

and

$$\overline{S}'(p_1) = 2\widetilde{D}_2'(p_1 + c, \zeta) + (p_1 + c)\widetilde{D}_2''(p_1 + c, \zeta).$$

Using (11), we find after some manipulation that the first-order condition for $p_1$ in (A18) can be rewritten as

$$D^l(p_1) + p_1 D^{l\prime}(p_1) + \overline{S}(p_1) + \left[2\widetilde{D}_2'(p_1 + c, \zeta) + (p_1 + c)\widetilde{D}_2''(p_1 + c, \zeta)\right]\frac{\theta p_1 - (1 - \theta)c}{1 - \theta} = 0. \tag{A19}$$

The equilibrium prices with ex ante storage (where the storability constraint is binding) are given by

$$p_1^* = \frac{1 - \theta}{\theta}c - \frac{(1 - \theta)\left[D^l(p_1^*) + p_1^* D^{l\prime}(p_1^*)\right] + \widetilde{D}_2(p_1^* + c, \zeta) + (p_1^* + c)\widetilde{D}_2'(p_1^* + c, \zeta)}{\theta\left[2\widetilde{D}_2'(p_1^* + c, \zeta) + (p_1^* + c)\widetilde{D}_2''(p_1^* + c, \zeta)\right]} \text{ and } p_2^* = p_1^* + c. \tag{A20}$$

It follows from (A16) and (A17) that the equilibrium ex ante storage is $S^* \triangleq S(p_1^*)$ and the equilibrium ex post storage is $\overline{S}^* \triangleq \overline{S}(p_1^*)$. A sufficient (albeit not necessary) condition for $S^* > 0$, which ensures the feasibility of the solution with ex ante storage (as $S^* < \overline{S}^*$), is that $\zeta$ is high enough (as $\partial\widetilde{D}_2/\partial\zeta > 0$). There exists a threshold $\widehat{c}^*$, where $\widehat{c}^* \leq \widetilde{c}$, such that for $c < \widehat{c}^*$ the dynamic solution with ex ante storage emerges in equilibrium (the static solution is unfeasible). This solution must also satisfy the condition that consumers with high valuation for the good do not store ex post, i.e., $\widetilde{D}_1(p_1^*) + S(p_1^*) - D^h(p_1^*) \leq 0$. Otherwise, the price would be so low that consumers with high valuation would store ex post as well. In line with the rationale adopted in the proof of Lemma 1, each unit of ex post storage would impose a cost $c$ on the firm. Thus, the firm prefers to set the first period equilibrium price $\widetilde{p}_1^*$ such that $\widetilde{D}_1(\widetilde{p}_1^*) + S(\widetilde{p}_1^*) - D^h(\widetilde{p}_1^*) = 0$. Using linear demand functions of the form $D^h(p_\tau) = \alpha^h - \beta p_\tau$ and $D^l(p_\tau) = \alpha^l - \beta p_\tau$, which yield $\widetilde{D}_1(p_1) = \theta\alpha^h + (1 - \theta)\alpha^l - \beta p_1$ and $\widetilde{D}_2(p_2, \zeta) = \theta(\alpha^h + \zeta) + (1 - \theta)(\alpha^l + \zeta) - \beta p_2$, where $\alpha^h > \alpha^l$ and $\zeta \in \mathbb{R}$, we find from (A20) that the equilibrium prices are $p_1^* = \left[\theta\alpha^h + 2(1 - \theta)\alpha^l - 2\theta\beta c + \zeta\right]/(4\beta)$ and $p_2^* = p_1^* + c$.

Evaluating the expressions in (A16) and (A17) at $p_1^*$, we find that the equilibrium ex ante storage is $S^* = \left[\theta\left(2\theta-1\right)\alpha^h + 2\theta\left(1-\theta\right)\alpha^l + \zeta - 2\left(2-\theta\right)\beta c\right] / \left[2\left(1-\theta\right)\right]$ and the equilibrium ex post storage is $\overline{S}^* = \left[\theta\alpha^h + \zeta - 2\left(2-\theta\right)\beta c\right]/2$. This solution is feasible for $S^* > 0$ or, equivalently, $c < \overline{c} = \left[\theta\left(2\theta-1\right)\alpha^h + 2\theta\left(1-\theta\right)\alpha^l + \zeta\right] / \left[2\beta\left(2-\theta\right)\right]$. Furthermore, we find from Section 4 that the static solution, which yields prices $p_1^m = \left[\theta\alpha^h + \left(1-\theta\right)\alpha^l\right] / \left(2\beta\right)$ and $p_2^m = \left[\theta\alpha^h + \left(1-\theta\right)\alpha^l + \zeta\right] / \left(2\beta\right)$, is feasible if and only if $c \geq \widetilde{c} = \left[2\theta\left(\alpha^h - \alpha^l\right) + \zeta\right] / \left(2\beta\right)$. As the firm's profit in the static solution is always higher than in the dynamic solution with ex ante storage, the firm prefers to implement the static solution whenever possible. Thus, the dynamic solution with ex ante storage emerges in equilibrium for $c < \widehat{c}^* = \min\left\{\overline{c}, \widetilde{c}\right\}$.

Now, suppose that the storability constraint is not binding, i.e., $p_1 + c > p_2$. Below, we show that this emerges in equilibrium for $\zeta$ low enough. As $p_2 = p_2^e$ and $S\left(\cdot\right) = 0$, we find from (12) that the firm's first-period maximization problem writes as

$$\max_{p_1} p_1 \widetilde{D}_1\left(p_1\right) + p_2\left[\widetilde{D}_2\left(p_2, \zeta\right) - \overline{S}\left(p_1, p_2\right)\right] \quad s.t. \;\; \text{(11) and (A15).}$$

It follows from the envelope theorem that the first-order condition for $p_1$ is given by

$$\widetilde{D}_1\left(p_1\right) + p_1\widetilde{D}_1'\left(p_1\right) - p_2\frac{d\overline{S}\left(p_1, p_2\right)}{dp_1} = 0. \tag{A21}$$

Using (A15) and (A21), we obtain that the equilibrium prices with only ex post storage (where the storability constraint is not binding) are given by

$$p_1^{**} = -\frac{\widetilde{D}_1\left(p_1^{**}\right) - p_2^{**}\left[d\overline{S}\left(p_1^{**}, p_2^{**}\right)/dp_1\right]}{\widetilde{D}_1'\left(p_1^{**}\right)} \quad \text{and} \quad p_2^{**} = -\frac{\widetilde{D}_2\left(p_2^{**}, \zeta\right) - \overline{S}\left(p_1^{**}, p_2^{**}\right)}{\widetilde{D}_2'\left(p_2^{**}, \zeta\right)}. \tag{A22}$$

It follows from (11), where $S\left(\cdot\right) = 0$, that the equilibrium ex post storage is $\overline{S}^{**} \triangleq \overline{S}\left(p_1^{**}, p_2^{**}\right) = \left(1-\theta\right)\left[\widetilde{D}_1\left(p_1^{**}\right) - D^l\left(p_2^{**} - c\right)\right]$. There is a threshold $\widehat{c}^{**}$, where $\widehat{c}^{**} \leq \widetilde{c}$, such that for $c < \widehat{c}^{**}$ the dynamic solution with only ex post storage arises in equilibrium (the static solution is unfeasible). With linear demands (see above), we find from (A22) that the equilibrium prices are $p_1^{**} = \left[\theta\left(9 - 4\theta - \theta^2\right)\alpha^h + \left(1-\theta\right)\left(11 - 6\theta - \theta^2\right)\alpha^l + 2\left(1-\theta\right)^2\beta c + 2\left(1-\theta\right)\zeta\right] / \left[8\beta\left(2-\theta\right)\right]$ and $p_2^{**} = \left(3-\theta\right)\left[\theta\left(1+\theta\right)\alpha^h + \left(3+\theta\right)\left(1-\theta\right)\alpha^l + 2\left(1-\theta\right)\beta c + 2\zeta\right] / \left[8\beta\left(2-\theta\right)\right]$. Evaluating the expression in (11), where $S\left(\cdot\right) = 0$, at $p_1^{**}$ and $p_2^{**}$, we find that the equilibrium ex post storage is $\overline{S}^{**} = \left(1-\theta\right)\left[\theta\left(5-\theta\right)\alpha^h - \left(1 + 4\theta - \theta^2\right)\alpha^l + 2\zeta - 2\left(3-\theta\right)\beta c\right] / \left[4\left(2-\theta\right)\right]$, where $\overline{S}^{**} > 0$ for $c < \overline{c}^{**} = \left[\theta\left(5-\theta\right)\alpha^h - \left(1 + 4\theta - \theta^2\right)\alpha^l + 2\zeta\right] / \left[2\beta\left(3-\theta\right)\right]$. As the firm's profit in the static solution is always higher than in the dynamic solution with only ex post storage, the firm prefers to implement the static solution whenever possible. Thus, the dynamic solution with only ex post storage arises in equilibrium for $c < \widehat{c}^{**} = \min\left\{\overline{c}^{**}, \widetilde{c}\right\}$.

Now, we compare the two dynamic storage solutions. In the solution with ex ante storage, we find from the sequential optimality constraint (5) that, for any $p_1$ and $p_2$ (where $p_2 = p_1 + c$), a rise in $\widetilde{D}_2\left(\cdot\right)$, driven by a higher $\zeta$ (due to $\partial\widetilde{D}_2/\partial\zeta > 0$), requires an increase in $\overline{S}\left(\cdot\right)$, which emerges with probability $1-\theta$. It follows from the condition for the equalization of marginal utilities in (11) that each additional unit of $\overline{S}\left(\cdot\right)$ allows for an additional unit of $S\left(\cdot\right)$. As this is bought by all consumers, the solution with ex ante storage becomes more profitable for the firm. Conversely, in the solution with only ex post storage, we find from the sequential optimality constraint (5) that, for any $p_1$, a rise in $\widetilde{D}_2(\cdot)$ leads to a higher $p_2$. The condition for the equalization of marginal utilities in (11) implies that $\overline{S}(\cdot)$ increases, which makes this solution less profitable. Consequently, when $\widetilde{D}_2(\cdot)$ is sufficiently high, the solution with ex ante storage emerges in equilibrium for $c < \widehat{c}^*$. Conversely, when $\widetilde{D}_2(\cdot)$ is sufficiently low, the solution with only ex post storage emerges in equilibrium for $c < \widehat{c}^{**}$. Note that, when $\widetilde{D}_2(\cdot)$ is significantly high, the second period price $p_2$ is also relatively high, which violates the storability constraint $p_1 + c > p_2$ and makes the solution with only ex post storage unfeasible. When $\widetilde{D}_2(\cdot)$ is significantly low, the solution with ex ante storage cannot be implemented, because the expression for ex ante storage would be negative. Using linear demands (see above), we find that the difference in the firm's profits between the solution with ex ante storage and the solution with only ex post storage increases with $\zeta$. Hence, there exists a unique threshold $\widehat{\zeta} \triangleq \sqrt{2\left[\theta(1+\theta)\alpha^h + (1-\theta^2)\alpha^l - 2(1-\theta)\beta c\right]^2(2-\theta)(1-\theta)}/(2\theta) - \left(1+\theta^2-\theta\right)\alpha^h - \left(1+\theta^2\right)(1-\theta)\alpha^l/\theta + 2\beta c/\theta$ such that the solution with ex ante storage is more profitable than the solution with only ex post storage if and only if $\zeta > \widehat{\zeta}$, where $\widehat{\zeta} > 0$. ■

**Proof of Proposition 8.** First, we consider the solution with ex ante storage. We start with the case $c < \widetilde{c}$. Evaluating the first-order condition for $p_1$ in (A18) at $p_1^m$ and using (A17), we obtain after multiplying by $1-\theta$ that

$$\begin{aligned}
&(1-\theta)\left[D^l(p_1^m) + p_1^m D^{l\prime}(p_1^m)\right] + \widetilde{D}_2(p_1^m + c, \zeta) + (p_1^m + c)\widetilde{D}_2'(p_1^m + c, \zeta) \\
&+ \left[2\widetilde{D}_2'(p_1^m + c, \zeta) + (p_1^m + c)\widetilde{D}_2''(p_1^m + c, \zeta)\right]\left[\theta p_1^m - (1-\theta)c\right].
\end{aligned}$$

It holds $p_1^* < (>)\, p_1^m$ when this expression is negative (positive). As the expression in the first square brackets in the second line is negative (by profit concavity), we find that the expression in the second line is negative if $\theta$ is high enough. Furthermore, for $\zeta$ low enough, we have $p_2^m < p_1^m + c$, which implies that the sum of the last two terms in the first line is negative. As for $\theta$ high enough the first term in the first line is relatively small, we find that, for $\zeta$ low enough and $\theta$ high enough, the entire expression is negative and thus $p_1^* < p_1^m$.[22] Conversely, if $\theta$ is low enough, the expression in the second line is positive. Furthermore, for $\zeta$ high enough, we have $p_2^m > p_1^m + c$, which implies that the sum of the last two terms in the first line is positive. Given that there exist sufficiently large values of $\zeta$ such that the expression in the first line is positive, we find that, for $\zeta$ high enough and $\theta$ low enough, the entire expression is positive and thus $p_1^* > p_1^m$. Using linear demand functions of the form $D^h(p_\tau) = \alpha^h - \beta p_\tau$ and $D^l(p_\tau) = \alpha^l - \beta p_\tau$, where $\alpha^h > \alpha^l$, we have $p_1^* < p_1^m$ (where $p_1^*$ and $p_1^m$ are derived in the proof of Proposition 7) if and only if $\zeta < \overline{\zeta} \triangleq \theta\left(\alpha^h + 2\beta c\right)$, with $\partial\overline{\zeta}/\partial\theta > 0$. Turning to the case where $c \geq \widetilde{c}$, we show that $p_1^* < p_1^m$. It follows from the sequential optimality constraint (10) that $p_2^* = p_1^* + c < p_2^m$. To show $p_1^* < p_1^m$, it suffices to demonstrate that $p_2^m < p_1^m + c$. Proceeding by contradiction, suppose that $p_2^m \geq p_1^m + c$ or, equivalently, $c \leq p_2^m - p_1^m$. Using the definition of $\widetilde{c}$ (see Section 4), we find that $c \geq \widetilde{c}$ if and only if $c \geq p_2^m - D^{l-1}\left(\widetilde{D}_1(p_1^m)\right)$. It follows from $\widetilde{D}_1(p_1) > D^l(p_1)$ that $p_2^m - D^{l-1}\left(\widetilde{D}_1(p_1^m)\right) > p_2^m - p_1^m$. This implies that,

[22] A sufficient (albeit not necessary) condition for the expression in square brackets in the first line to be negative is that $D^{h\prime}(\cdot) \geq D^{l\prime}(\cdot)$, which reinforces this result.

for $c \geq \widetilde{c}$, we cannot have $c \leq p_2^m - p_1^m$. Consequently, we conclude that, for $c \geq \widetilde{c}$, it holds $p_1^* < p_1^m$.

Now, we consider the solution with only ex post storage. Evaluating the first-order condition for $p_1$ in (A21) at $p_1^m$ yields $-p_2 \left[ d\overline{S}\left(p_1^m, p_2\right) / dp_1 \right] > 0$, where the inequality holds if and only if $d\overline{S}\left(p_1^m, p_2\right) / dp_1 < 0$. Applying the implicit function theorem to (11), where $S(\cdot) = 0$, and (A15) yields

$$\frac{d\overline{S}\left(p_1, p_2\right)}{dp_1} = \frac{\left(1-\theta\right) \widetilde{D}_1'\left(p_1\right) \left[2\widetilde{D}_2'\left(p_2, \zeta\right) + p_2 \widetilde{D}_2''\left(p_2, \zeta\right)\right]}{2\widetilde{D}_2'\left(p_2, \zeta\right) + p_2 \widetilde{D}_2''\left(p_2, \zeta\right) + \left(1-\theta\right) D^{l\prime}\left(p_2 - c\right)} < 0,$$

where the inequality follows from profit concavity. Then, it holds $p_1^{**} > p_1^m$. Using the sequential optimality constraint (10), we find that $p_2^* < p_2^m$ and $p_2^{**} < p_2^m$. ■

# References


Acland, D., Levy, M. R. (2015). Naïveté, projection bias, and habit formation in gym attendance. Management Science, 61(1), 146-160.

Anton, J. J., Das Varma, G. (2005). Storability, market structure, and demand-shift incentives. Rand Journal of Economics, 36(3), 520-543.

Antoniou, F., Fiocco, R. (2019). Strategic inventories under limited commitment. Rand Journal of Economics, 50(3), 695-729.

Antoniou, F., Fiocco, R. (2023). Storable good market with intertemporal cost variations. International Economic Review, 64(1), 361-385.

Bénabou, R. (1989). Optimal price dynamics and speculation with a storable good. Econometrica, 57(1), 41-80.

Dekel, E., Lipman, B. L. (2012). Costly self-control and random self-indulgence. Econometrica, 80(3), 1271-1302.

DellaVigna, S., Malmendier, U. (2004). Contract design and self-control: Theory and evidence. Quarterly Journal of Economics, 119(2), 353-402.

DellaVigna, S., Malmendier, U. (2006). Paying not to go to the gym. American Economic Review, 96(3), 694-719.

Dudine, P., Hendel, I., Lizzeri, A. (2006). Storable good monopoly: The role of commitment. American Economic Review, 96(5), 1706-1719.

Eliaz, K., Spiegler, R. (2006). Contracting with diversely naive agents. Review of Economic Studies, 73(3), 689-714.

Erdem, T., Imai, S., Keane, M. P. (2003). Brand and quantity choice dynamics under price uncertainty. Quantitative Marketing and Economics, 1(1), 5-64.

Ericson, K. M., Laibson, D. (2019). Intertemporal choice. In: Bernheim, B. D., DellaVigna, S., Laibson, D. (Eds.), Handbook of behavioral economics — Foundations and applications, Volume 2, 1-67. Elsevier, Amsterdam.

Fedyk, A. (2022). Asymmetric naivete: Beliefs about self-control. Available at https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2727499 (last retrieved in February 2024).

Fudenberg, D., Levine, D. K. (2006). A dual-self model of impulse control. American Economic Review, 96(5),1449-1476.

Fudenberg, D., Strzalecki, T. (2015). Dynamic logit with choice aversion. Econometrica, 83(2), 651-691.

Gul, F., Pesendorfer, W. (2001). Temptation and self-control. Econometrica, 69(6), 1403-1435.

Guo, L., Villas-Boas, J. M. (2007). Consumer stockpiling and price competition in differentiated markets. Journal of Economics and Management Strategy, 16(4), 827-858.

Halevy, Y. (2015). Time consistency: Stationarity and time invariance. Econometrica, 83(1), 335-352.

Hattori, K., Zennyo, Y. (2018). Heterogeneous consumer expectations and monopoly pricing for durables with network externalities. Available at https://mpra.ub.uni-muenchen.de/89893/9/MPRA_paper_89893.pdf (last retrieved in February 2024).

Heidhues, P., Kőszegi, B. (2009). Futile attempts at self-control. Journal of the European Economic Association, 7(2-3), 423-434.

Heidhues, P., Kőszegi, B. (2017). Naïvete-based discrimination. Quarterly Journal of Economics, 132(2), 1019-1054.

Heidhues, P., Strack, P. (2021). Identifying present-bias from the timing of choices. American Economic Review, 111(8), 2594-2622.

Hendel, I., Lizzeri, A., Roketskiy, N. (2014). Nonlinear pricing of storable goods. American Economic Journal: Microeconomics, 6(3), 1-34.

Hendel, I., Nevo, A. (2004). Intertemporal substitution and storable products. Journal of the European Economic Association, 2(2-3), 536-547.

Hendel, I., Nevo, A. (2006a). Sales and consumer inventory. Rand Journal of Economics, 37(3), 543-561.

Hendel, I., Nevo, A. (2006b). Measuring the implications of sales and consumer inventory behavior. Econometrica, 74(6), 1637-1673.

Hendel, I., Nevo, A. (2013). Intertemporal price discrimination in storable goods markets. American Economic Review, 103(7), 2722-2751.

Hong, P., McAfee, R. P., Nayyar, A. (2002). Equilibrium price dispersion with consumer inventories. Journal of Economic Theory, 105(2), 503-517.

Jeuland, A. P., Narasimhan, C. (1985). Dealing-temporary price cuts-by seller as a buyer discrimination mechanism. Journal of Business, 58(3), 295-308.

Laibson, D. (1997). Golden eggs and hyperbolic discounting. Quarterly Journal of Economics, 112(2), 443-477.

Mitraille, S., Thille, H. (2009). Monopoly behaviour with speculative storage. Journal of Economic Dynamics and Control, 33(7), 1451-1468.

Mitraille, S., Thille, H. (2014). Speculative storage in imperfectly competitive markets. International Journal of Industrial Organization, 35, 44-59.

Mitraille, S., Thille, H. (2016). Speculative constraints on oligopoly. In: Thuijsman, F., Wagener, F. (Eds.), Advances in dynamic and evolutionary games: Theory, applications, and numerical methods, 229-248. Birkhäuser, Cham.

Nava, F., Schiraldi, P. (2014). Sales and collusion in a market with storage. Journal of the European Economic Association, 12(3), 791-832.

O'Donoghue, T., Rabin, M. (1999). Doing it now or later. American Economic Review, 89(1), 103-124.

Osborne, M. (2018). Approximating the cost-of-living index for a storable good. American Economic Journal: Microeconomics, 10(2), 286-314.

Perrone, H. (2017). Demand for nondurable goods: A shortcut to estimating long-run price elasticities. Rand Journal of Economics, 48(3), 856-873.

Pesendorfer, M. (2002). Retail sales: A study of pricing behavior in supermarkets. Journal of Business, 75(1), 33-66.

Pires, T. (2016). Costly search and consideration sets in storable goods markets. Quantitative Marketing and Economics, 14(3), 157-193.

Read, D., van Leeuwen, B. (1998). Predicting hunger: The effects of appetite and delay on choice. Organizational Behavior and Human Decision Processes, 76(2), 189-205.

Strack, P., Taubinsky, D. (2022). Dynamic preference 'reversals' and time inconsistency. Available at `https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3875128` (last retrieved in February 2024).

Strotz, R. H. (1955). Myopia and inconsistency in dynamic utility maximization. Review of Economic Studies, 23(3), 165-180.

Su, X. (2007). Intertemporal pricing with strategic customer behavior. Management Science, 53(5), 726-741.

Su, X. (2010). Intertemporal pricing and consumer stockpiling. Operations Research, 58(4-part-2), 1133-1147.

Wang, E. Y. (2015). The impact of soda taxes on consumer welfare: Implications of storability and taste heterogeneity. Rand Journal of Economics, 46(2), 409-441.

# Time-varying preferences for storable goods

Supplementary Appendix

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## 1 Introduction

This Supplementary Appendix complements the paper and proceeds as follows. Section 2 extends our model to nonlinear pricing. Section 3 examines the role of the firm's commitment to future prices.

## 2 Nonlinear pricing

In the following remark, we characterize the main features of the dynamic storage solution with nonlinear pricing. We also compare the dynamic storage solution and the static solution in terms of consumer surplus.

**Remark 1** *A. With naïve underestimating consumers, in the dynamic storage solution, sophisticated consumers obtain nonnegative surplus and naïve consumers obtain positive surplus. If the number of naïve consumers is large enough, sophisticated consumers are as well off and naïve consumers are better off with respect to the static solution for sufficiently small values of the storage cost.*

*B. With naïve overestimating consumers, in the dynamic storage solution, sophisticated consumers obtain nonnegative surplus and naïve consumers may obtain negative surplus. If the number of sophisticated consumers is large enough, sophisticated consumers are at most as well off and naïve consumers are worse off with respect to the static solution for sufficiently small values of the storage cost.*

**Proof of Remark 1.** The firm offers consumers a nonlinear pricing policy in the form of a contract menu $\left\{Q_\tau^i, P_\tau^i\right\}$ that specifies a quantity $Q_\tau^i$ and a payment $P_\tau^i$ for consumers of type $i = s, n$ (where $s$ stands for sophisticates and $n$ for naïfs) in period $\tau \in \{1, 2\}$. As in the baseline model, a fraction $\lambda \in (0, 1)$ of the consumer population consists of sophisticated

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consumers and the remaining fraction $1-\lambda$ is formed of naïve consumers. We denote the utility of type $i$'s consumers by $U^i(x_\tau, y_\tau) = u^i(x_\tau) + y_\tau$ that depends on the consumption level $x_\tau$ and money $y_\tau$. The continuously differentiable utility function $u^i(\cdot)$ is increasing and concave, i.e., $u^{i\prime}(\cdot) > 0$ and $u^{i\prime\prime}(\cdot) < 0$, with the standard normalization $u^i(0) = 0$. To ensure the concavity of the firm's profit function, we introduce a (constant) unit cost of production $\gamma > 0$.[1] First, we consider the case of naïve underestimating consumers, where $u^{n\prime}(\cdot) > u^{s\prime}(\cdot)$. By the revelation principle, in the second period the firm offers the pricing policy $\{(Q_2^s, P_2^s), (Q_2^n, P_2^n)\}$, where $(Q_2^s, P_2^s)$ is designed for sophisticates and $(Q_2^n, P_2^n)$ for naïfs. Proceeding backward, the firm's second period maximization problem writes as

$$\max_{\{(Q_2^s, P_2^s),(Q_2^n, P_2^n)\}} \lambda(P_2^s - \gamma Q_2^s) + (1-\lambda)(P_2^n - \gamma Q_2^n) \tag{S1}$$

subject to the following participation and incentive constraints

$$u^s\left(Q_2^s + \overline{S}^s\right) - P_2^s \geq u^s\left(\overline{S}^s\right) \tag{S2}$$

$$u^n\left(Q_2^n + \overline{S}^n\right) - P_2^n \geq u^n\left(\overline{S}^n\right) \tag{S3}$$

$$u^s\left(Q_2^s + \overline{S}^s\right) - P_2^s \geq u^s\left(Q_2^n + \overline{S}^s\right) - P_2^n \tag{S4}$$

$$u^n\left(Q_2^n + \overline{S}^n\right) - P_2^n \geq u^n\left(Q_2^s + \overline{S}^n\right) - P_2^s, \tag{S5}$$

where $\overline{S}^i$ denotes the amount of ex post storage inherited from the first period by consumers of type $i = s, n$. In line with our main analysis, as all consumers believe that they have the same preferences at first period purchase stage but naïfs discover higher valuation for the good, naïfs consume (weakly) more than sophisticates in the first period, which implies that their level of ex post storage is (weakly) lower, i.e., $\overline{S}^n \leq \overline{S}^s$. The participation constraints (S2) and (S3) ensure that consumers of type $i = s, n$ are willing to accept the second period pricing policy when obtaining at least the utility $u^i\left(\overline{S}^i\right)$ associated with consumption of the quantity stored $\overline{S}^i$. The incentive constraints (S4) and (S5) induce consumers to select the contract intended for their type instead of the one intended for the other type. Following a standard approach, we show that, as naïfs have higher valuation for the good, the participation constraint (S2) for sophisticates and the incentive constraint (S5) for naïfs are binding in equilibrium, whereas the participation constraint (S3) for naïfs and the incentive constraint (S4) for sophisticates are slack. Combining (S2) and (S5) yields $u^n\left(Q_2^n + \overline{S}^n\right) - P_2^n - u^n\left(\overline{S}^n\right) \geq u^n\left(Q_2^s + \overline{S}^n\right) - P_2^s - u^n\left(\overline{S}^n\right) \geq u^s\left(Q_2^s + \overline{S}^s\right) - P_2^s - u^s\left(\overline{S}^s\right) \geq 0$, where the second inequality follows from $u^{n\prime}(\cdot) > u^{s\prime}(\cdot)$ and $u^{i\prime\prime}(\cdot) < 0$, for $i = s, n$ (recall $\overline{S}^n \leq \overline{S}^s$). Thus, the constraint (S3) is implied by the constraints (S2) and (S5). Furthermore, the constraints (S2) and (S5) must be binding in equilibrium, otherwise the firm could increase payments and be better off.[2] Substituting the binding constraints (S2) and (S5) into the firm's second period maximization problem in (S1) and taking the first-order conditions for $Q_2^s \geq 0$ and $Q_2^n \geq 0$ yields

$$u^{s\prime}\left(Q_2^s + \overline{S}^s\right) - (1-\lambda)\, u^{n\prime}\left(Q_2^s + \overline{S}^n\right) - \lambda\gamma \leq 0 \tag{S6}$$

[1] Our findings are qualitatively unaffected in the absence of any cost of production, with the main difference that the quantity offered can be at the highest possible level (which may stem from capacity constraints).

[2] The constraint (S4) will be checked ex post.

and

$$u^{n\prime}\left(Q_2^n+\overline{S}^n\right)-\gamma\leq 0, \tag{S7}$$

where the equalities hold for $Q_2^s>0$ and $Q_2^n>0$, respectively. In equilibrium, it holds $Q_2^n\geq Q_2^s$. As $u^{n\prime}(\cdot)>u^{s\prime}(\cdot)$, the quantity $Q_2^s$ for sophisticates is such that their consumption $Q_2^s+\overline{S}^s$ is distorted below the efficient level, i.e., $u^{s\prime}\left(Q_2^s+\overline{S}^s\right)-\gamma>0$. The quantity $Q_2^n$ for naïfs ensures that for $Q_2^n>0$ their consumption $Q_2^n+\overline{S}^n$ is efficient, i.e., $u^{n\prime}\left(Q_2^n+\overline{S}^n\right)-\gamma=0$. Notably, sophisticates' consumption can be efficient even for $Q_2^n=0$ because they resort to the quantity stored.[3] Hence, given the first period pricing policy and consumers' storage decisions, the second period pricing policy is $\{(Q_2^s,P_2^s),(Q_2^n,P_2^n)\}$, where $Q_2^s$ and $Q_2^n$ respectively satisfy (S6) and (S7), whereas $P_2^s$ and $P_2^n$ follow from the binding constraints (S2) and (S5). It holds $Q_2^s=P_2^s=0$ for $\lambda\leq\left[u^{n\prime}\left(\overline{S}^n\right)-u^{s\prime}\left(\overline{S}^s\right)\right]/\left[u^{n\prime}\left(\overline{S}^n\right)-\gamma\right]$.

Given the first period pricing policy $(Q_1^s,P_1^s)$ and the second period (expected) pricing policy $(Q_2^s,P_2^s)$, sophisticates' ex ante and ex post storage $S^s=\overline{S}^s\geq 0$ is given by

$$S^s=\overline{S}^s=\arg\max_{\widetilde{S}^s} u^s\left(Q_1^s-\widetilde{S}^s\right)+u^s\left(Q_2^s+\widetilde{S}^s\right)-P_1^s-P_2^s-c\widetilde{S}^s. \tag{S8}$$

This yields $u^{s\prime}(Q_2^s+S^s)-u^{s\prime}(Q_1^s-S^s)-c\leq 0$, where the equality holds for $S^s>0$.[4] Naïfs' ex ante storage is also equal to $S^s$ because naïfs believe that they have the same preferences as sophisticates at the first period purchase stage. Given the first period pricing policy $(Q_1^s,P_1^s)$ and the second period (expected) pricing policy $(Q_2^n,P_2^n)$, naïfs' ex post storage $\overline{S}^n\geq 0$ is

$$\overline{S}^n=\arg\max_{\widetilde{S}^n} u^n\left(Q_1^s-\widetilde{S}^n\right)+u^n\left(Q_2^n+\widetilde{S}^n\right)-P_2^n-c\widetilde{S}^n. \tag{S9}$$

This yields $u^{n\prime}\left(Q_2^n+\overline{S}^n\right)-u^{n\prime}\left(Q_1^s-\overline{S}^n\right)-c\leq 0$, where the equality holds for $\overline{S}^n>0$.

In anticipation of the second period pricing policy $\{(Q_2^s,P_2^s),(Q_2^n,P_2^n)\}$ (as implied by (S1) with $u^{n\prime}(\cdot)>u^{s\prime}(\cdot)$) and consumers' storage decisions (as implied by (S8) and (S9)), the firm's first period maximization problem writes as

$$\max_{\{Q_1^s,P_1^s\}} P_1^s-\gamma Q_1^s+\lambda\left(P_2^s-\gamma Q_2^s\right)+(1-\lambda)\left(P_2^n-\gamma Q_2^n\right) \tag{S10}$$

subject to the following participation and non-skipping constraints

$$u^s(Q_1^s-S^s)-cS^s-P_1^s+u^s(Q_2^s+S^s)-P_2^s\geq 0 \tag{S11}$$

$$u^s(Q_1^s-S^s)-cS^s-P_1^s+u^s(Q_2^s+S^s)-P_2^s\geq u^s(Q_2^s)-P_2^s. \tag{S12}$$

The participation constraint (S11) ensures that all consumers, believing that they share the same utility $U^s(\cdot)$, are willing to accept the first period pricing policy $\{Q_1^s,P_1^s\}$, in anticipation of the second period pricing policy $\{Q_2^s,P_2^s\}$. The non-skipping constraint (S12) ensures that consumers do not have any incentives to remain idle in the first period by skipping their purchases

[3] Substituting the binding constraints (S2) and (S5) into (S4) yields after some manipulation $u^n\left(Q_2^n+\overline{S}^n\right)-u^n\left(Q_2^s+\overline{S}^n\right)\geq u^s\left(Q_2^n+\overline{S}^s\right)-u^s\left(Q_2^s+\overline{S}^s\right)$, where the inequality follows from $Q_2^n\geq Q_2^s$ as well as $u^{n\prime}(\cdot)>u^{s\prime}(\cdot)$ and $u^{i\prime\prime}(\cdot)<0$, for $i=s,n$ (recall $\overline{S}^n\leq\overline{S}^s$).

[4] As consumers' expectations are correct in equilibrium, with a small abuse of notation we omit the expectation operator in the second period pricing policy.

and buying in the second period. We show that the non-skipping constraint (S12) is binding in equilibrium.[5] Proceeding by contradiction, suppose that the constraint (S12) is slack and thus $u^s(Q_2^s) - P_2^s < 0$. As the participation constraint (S2) yields $P_2^s \leq u^s(Q_2^s + S^s) - u^s(S^s)$, it must hold $u^s(Q_2^s) < u^s(Q_2^s + S^s) - u^s(S^s)$. However, this violates $u^s(Q_2^s) + u^s(S^s) \geq u^s(Q_2^s + S^s)$ (by subadditivity of $u^i(\cdot)$, for $i = s, n$). Hence, the non-skipping constraint (S12) is binding in equilibrium. Taking the first-order condition for $Q_1^s$ associated with the firm's maximization problem in (S10) yields after some manipulation

$$\begin{aligned}
&u^{s\prime}(Q_1^s - S^s) + Q_2^{s\prime}(Q_1^s)\left[u^{s\prime}(Q_2^s + S^s) - u^{s\prime}(Q_2^s)\right] + S^{s\prime}(Q_1^s)\left[u^{s\prime}(Q_2^s + S^s) - u^{s\prime}(Q_1^s - S^s) - c\right] \\
&\quad + \lambda\left\{u^{s\prime}(Q_2^s + S^s)\left[Q_2^{s\prime}(Q_1^s) + S^{s\prime}(Q_1^s)\right] - u^{s\prime}(S^s)S^{s\prime}(Q_1^s) - \gamma Q_2^{s\prime}(Q_1^s)\right\} \\
&\quad + (1-\lambda)\left\{u^{n\prime}\left(Q_2^n + \overline{S}^n\right)\left[Q_2^{n\prime}(Q_1^s) + \overline{S}^{n\prime}(Q_1^s)\right] - u^{n\prime}\left(Q_2^s + \overline{S}^n\right)\left[Q_2^{s\prime}(Q_1^s) + \overline{S}^{n\prime}(Q_1^s)\right]\right. \\
&\quad \left. + u^{s\prime}(Q_2^s + S^s)\left[Q_2^{s\prime}(Q_1^s) + S^{s\prime}(Q_1^s)\right] - u^{s\prime}(S^s)S^{s\prime}(Q_1^s) - \gamma Q_2^{n\prime}(Q_1^s)\right\} \\
&\quad - \mu\left\{u^{s\prime\prime}(Q_2^s + S^s)\left[Q_2^{s\prime}(Q_1^s) + S^{s\prime}(Q_1^s)\right] - u^{s\prime\prime}(Q_1^s - S^s)\left[1 - S^{s\prime}(Q_1^s)\right]\right\} \\
&\quad - \nu\left\{u^{n\prime\prime}\left(Q_2^n + \overline{S}^n\right)\left[Q_2^{n\prime}(Q_1^s) + \overline{S}^{n\prime}(Q_1^s)\right] - u^{n\prime\prime}\left(Q_1^s - \overline{S}^n\right)\left[1 - \overline{S}^{n\prime}(Q_1^s)\right]\right\} \\
&\quad - \varphi\left\{u^{s\prime\prime}(Q_2^s + S^s)\left[Q_2^{s\prime}(Q_1^s) + S^{s\prime}(Q_1^s)\right] - (1-\lambda)u^{n\prime\prime}\left(Q_2^s + \overline{S}^n\right)\left[Q_2^{s\prime}(Q_1^s) + \overline{S}^{n\prime}(Q_1^s)\right]\right\} \\
&\quad - \psi u^{n\prime\prime}\left(Q_2^n + \overline{S}^n\right)\left[Q_2^{n\prime}(Q_1^s) + \overline{S}^{n\prime}(Q_1^s)\right] - \gamma = 0,
\end{aligned} \tag{S13}$$

where $\mu \geq 0$ and $\nu \geq 0$ are the Kuhn-Tucker multipliers associated with consumers' storage decisions (see (S8) and (S9)) as well as $\varphi \geq 0$ and $\psi \geq 0$ are the Kuhn-Tucker multipliers associated with the first-order conditions (S6) and (S7). Thus, with naïve underestimating consumers, in the dynamic storage solution, the pricing policy consists of the first period pricing policy $(Q_1^{s*u}, P_1^{s*u})$, where $Q_1^{s*u}$ and $P_1^{s*u}$ respectively satisfy the first-order condition (S13) and the binding non-skipping constraint (S12), and of the second period pricing policy $\{(Q_2^{s*u}, P_2^{s*u}), (Q_2^{n*u}, P_2^{n*u})\}$ associated with the firm's maximization problem in (S1) (with $u^{n\prime}(\cdot) > u^{s\prime}(\cdot)$) that yields the first-order conditions (S6) and (S7). The levels of storage are $S^{s*u}$ for sophisticates and $\overline{S}^{n*u}$ for naïfs (as implied by (S8) and (S9)). The dynamic storage solution requires that (at least) sophisticates store, i.e., $S^{s*u} > 0$, which occurs for sufficiently small values of the storage cost, i.e., for $c < u^{s\prime}\left(Q_2^s|_{S^s=0}\right) - u^{s\prime}\left(Q_1^s|_{S^s=0}\right)$.

Let $\Psi^i \triangleq \sum_{\tau=1}^{2}\left[U^i(x_\tau, y_\tau) - P_\tau\right] - c\overline{S}^i$ be the aggregate net utility of consumers of type $i = s, n$. Ignoring money $y_\tau$ without loss of generality, it follows from the binding constraint (S12) that sophisticated consumers obtain

$$\Psi^{s*u} = u^s(Q_2^{s*u}) - P_2^{s*u} = u^s(Q_2^{s*u}) + u^s(S^{s*u}) - u^s(Q_2^{s*u} + S^{s*u}) \geq 0,$$

where the equality holds if and only if $Q_2^{s*u} = P_2^{s*u} = 0$. Naïve consumers obtain

$$\begin{aligned}
\Psi^{n*u} &= u^n\left(Q_1^{s*u} - \overline{S}^{n*u}\right) - c\overline{S}^{n*u} - P_1^{s*u} + u^n\left(Q_2^{n*u} + \overline{S}^{n*u}\right) - P_2^{n*u} \\
&= u^n\left(Q_1^{s*u} - \overline{S}^{n*u}\right) + u^n\left(Q_2^{s*u} + \overline{S}^{n*u}\right) - u^s(Q_1^{s*u} - S^{s*u}) \\
&\quad - 2u^s(Q_2^{s*u} + S^{s*u}) + u^s(S^{s*u}) + u^s(Q_2^{s*u}) + c\left(S^{s*u} - \overline{S}^{n*u}\right) > 0.
\end{aligned}$$

[5] Clearly, one constraint must be binding in equilibrium, otherwise the firm could increase the first period payment and be better off.

In the static solution, the pricing policy consists of the first period pricing policy $(Q_1^{sm}, P_1^{sm})$, where $Q_1^{sm}$ is such that $u^{s\prime}(Q_1^{sm}) - \gamma = 0$ and $P_1^{sm} = u^s(Q_1^{sm})$, and of the second period pricing policy $\{(Q_2^{sm}, P_2^{sm}), (Q_2^{nm}, P_2^{nm})\}$ associated with the firm's maximization problem in (S1) (with $u^{n\prime}(\cdot) > u^{s\prime}(\cdot)$) that yields the first-order conditions (S6) and (S7), where $S^s = \overline{S}^n = 0$. The static solution can only emerge in the absence of storage, which occurs for sufficiently large values of the storage cost, i.e., for $c \geq u^{s\prime}\left(Q_2^s|_{S^s=0}\right) - u^{s\prime}\left(Q_1^s|_{S^s=0}\right)$ (see above). Sophisticated consumers obtain $\Psi^{sm} = 0$ and naïve consumers obtain $\Psi^{nm} = u^n(Q_1^{sm}) + u^n(Q_2^{sm}) - u^s(Q_1^{sm}) - u^s(Q_2^{sm}) > 0$.

We now compare the dynamic storage solution and the static solution in terms of consumer surplus. For the sake of convenience, we focus our attention on the case where the number of naïfs is large enough, i.e., $\lambda \leq \widetilde{\lambda}_{nl}^u \triangleq \left[u^{n\prime}\left(\overline{S}^{n*u}\right) - u^{s\prime}(S^{s*u})\right] / \left[u^{n\prime}\left(\overline{S}^{n*u}\right) - \gamma\right]$ (see the discussion after (S7)). This implies that $Q_2^{s*u} = P_2^{s*u} = 0$. Then, it holds $\Psi^{s*u} = \Psi^{sm} = 0$. Furthermore, we show that it holds $\Psi^{n*u} > \Psi^{nm}$, provided that the storage cost $c$ is small enough. To proceed in a insightful manner, we fix $c = 0$. Our results apply for $c$ small enough. First, we prove that the firm cannot benefit from $Q_2^n > 0$. It follows from the first-order condition for $Q_1^s$ in (S13) that the impact of $Q_1^s$ on the firm's profit $\Pi$ through $Q_2^n$ evaluated at $Q_2^n = 0$ is

$$\left.\frac{\partial \Pi}{\partial Q_1^s}\right|_{Q_2^n=0} = (1-\lambda)\left\{u^{n\prime}\left(\overline{S}^n\right)\left[Q_2^{n\prime}(Q_1^s) + \overline{S}^{n\prime}(Q_1^s)\right] - u^{n\prime}\left(\overline{S}^n\right)\overline{S}^{n\prime}(Q_1^s) - \gamma Q_2^{n\prime}(Q_1^s)\right\}.$$

Given that $Q_2^{n\prime}(Q_1^s) = -1$ and $\overline{S}^{n\prime}(Q_1^s) = 1$ (as implied by (S7) and (S9)), it follows from (S7) that $\partial\Pi/\partial Q_1^s|_{Q_2^n=0} \geq 0$, which means that the firm does not have any incentives to decrease $Q_1^s$ in order to get $Q_2^n > 0$. Thus, it holds $Q_2^{s*u} = Q_2^{n*u} = 0$. We obtain from consumers' storage decisions (see (S8) and (S9)) that $S^{s*u} = \overline{S}^{n*u} = Q_1^{s*u}/2$, which yields $\Psi^{n*u} = 2\left[u^n(Q_1^{s*u}/2) - u^s(Q_1^{s*u}/2)\right]$. As $u^{s\prime}(Q_1^{sm}) - \gamma = 0$ and $u^{n\prime}(Q_1^{s*u}/2) - \gamma \leq 0$ (see (S7)), we find from $u^{n\prime}(\cdot) > u^{s\prime}(\cdot)$ that $Q_1^{s*u}/2 > Q_1^{sm}$, which implies that $u^n(Q_1^{s*u}/2) - u^s(Q_1^{s*u}/2) > u^n(Q_1^{sm}) - u^s(Q_1^{sm})$. Furthermore, it follows from $Q_2^{sm} < Q_1^{sm} < Q_1^{s*u}/2$ that $u^n(Q_1^{s*u}/2) - u^s(Q_1^{s*u}/2) > u^n(Q_2^{sm}) - u^s(Q_2^{sm})$. Thus, it holds $\Psi^{n*u} > \Psi^{nm}$. Using quadratic utility functions of the form $u^s = \alpha^s Q_\tau - (\beta/2)Q_\tau^2$ and $u^n = \alpha^n Q_\tau - (\beta/2)Q_\tau^2$, where $\alpha^n > \alpha^s$, we find that in the dynamic storage solution the first period quantity is $Q_1^{s*u} = 2(\alpha^n - \gamma)/\beta$ and storage is $S^{s*u} = \overline{S}^{n*u} = (\alpha^n - \gamma)/\beta$. In the second period, we have $Q_2^{s*u} = Q_2^{n*u} = 0$. In the static solution, the first period quantity is $Q_1^{sm} = (\alpha^s - \gamma)/\beta$. In the second period, we obtain $Q_2^{nm} = (\alpha^n - \gamma)/\beta$ as well as $Q_2^{sm} = 0$ for $\lambda \leq (\alpha^n - \alpha^s)/(\alpha^n - \gamma)$ and $Q_2^{sm} = \left[\alpha^n(\lambda - 1) + \alpha^s - \lambda\gamma\right]/(\beta\lambda)$, otherwise. Comparing consumer surplus between the two solutions yields $\Psi^{s*u} = \Psi^{sm} = 0$ and $\Psi^{n*u} > \Psi^{nm} > 0$.

Now, we turn to the case of naïve overestimating consumers, where $u^{s\prime}(\cdot) > u^{n\prime}(\cdot)$. As all consumers believe that they have the same preferences at first period purchase stage and naïfs discover lower valuation for the good, sophisticates consume (weakly) more than naïfs in the first period, which implies that their level of storage is (weakly) lower, i.e., $S^s \leq \overline{S}^n$. Following the same logic as for the case of naïve underestimating consumers, it can be shown that the participation constraint (S3) for naïfs and the incentive constraint (S4) for sophisticates are binding in equilibrium, whereas the participation constraint (S2) for sophisticates and the incentive constraint (S3) for naïfs are slack. Substituting the binding constraints (S3) and (S4) into the firm's second period maximization problem in (S1) and taking the first-order conditions for $Q_2^s \geq 0$ and $Q_2^n \geq 0$ yields

$$u^{s\prime}\left(Q_2^s + \overline{S}^s\right) - \gamma \leq 0 \tag{S14}$$

and

$$u^{n\prime}\left(Q_2^n + \overline{S}^n\right) - \lambda u^{s\prime}\left(Q_2^n + \overline{S}^s\right) - (1-\lambda)\gamma \leq 0, \tag{S15}$$

where the equalities hold for $Q_2^s > 0$ and $Q_2^n > 0$, respectively. In equilibrium, it holds $Q_2^s \geq Q_2^n$. Hence, given the first period pricing policy and consumers' storage decisions, the second period pricing policy is $\{(Q_2^s, P_2^s), (Q_2^n, P_2^n)\}$, where $Q_2^s$ and $Q_2^n$ respectively satisfy (S14) and (S15), whereas $P_2^s$ and $P_2^n$ follow from the binding constraints (S3) and (S4). It holds $Q_2^n = P_2^n = 0$ for $\lambda \geq \left[u^{n\prime}\left(\overline{S}^n\right) - \gamma\right] / \left[u^{s\prime}\left(\overline{S}^s\right) - \gamma\right]$.

In anticipation of the second period pricing policy $\{(Q_2^s, P_2^s), (Q_2^n, P_2^n)\}$ (as implied by (S1) with $u^{s\prime}(\cdot) > u^{n\prime}(\cdot)$) and consumers' storage decisions (as implied by (S8) and (S9)), the firm's first period maximization problem is given by (S10) subject to the participation constraint (S11) and the non-skipping constraint (S12). As before, the non-skipping constraint (S12) is binding in equilibrium. Taking the first-order condition for $Q_1^s$ yields after some manipulation

$$\begin{aligned}
&u^{s\prime}(Q_1^s - S^s) + Q_2^{s\prime}(Q_1^s)\left[u^{s\prime}(Q_2^s + S^s) - u^{s\prime}(Q_2^s)\right] + S^{s\prime}(Q_1^s)\left[u^{s\prime}(Q_2^s + S^s) - u^{s\prime}(Q_1^s - S^s) - c\right]\\
&\quad +\lambda\left\{u^{s\prime}(Q_2^s + S^s)\left[Q_2^{s\prime}(Q_1^s) + S^{s\prime}(Q_1^s)\right] - u^{s\prime}(Q_2^n + S^s)\left[Q_2^{n\prime}(Q_1^s) + S^{s\prime}(Q_1^s)\right]\right.\\
&\quad \left. +u^{n\prime}\left(Q_2^n + \overline{S}^n\right)\left[Q_2^{n\prime}(Q_1^s) + \overline{S}^{n\prime}(Q_1^s)\right] - u^{n\prime}\left(\overline{S}^n\right)\overline{S}^{n\prime}(Q_1^s) - \gamma Q_2^{s\prime}(Q_1^s)\right\}\\
&\quad +(1-\lambda)\left\{u^{n\prime}\left(Q_2^n + \overline{S}^n\right)\left[Q_2^{n\prime}(Q_1^s) + \overline{S}^{n\prime}(Q_1^s)\right] - u^{n\prime}\left(\overline{S}^n\right)\overline{S}^{n\prime}(Q_1^s) - \gamma Q_2^{n\prime}(Q_1^s)\right\}\\
&\quad -\mu\left\{u^{s\prime\prime}(Q_2^s + S^s)\left[Q_2^{s\prime}(Q_1^s) + S^{s\prime}(Q_1^s)\right] - u^{s\prime\prime}(Q_1^s - S^s)\left[1 - S^{s\prime}(Q_1^s)\right]\right\}\\
&\quad -\nu\left\{u^{n\prime\prime}\left(Q_2^n + \overline{S}^n\right)\left[Q_2^{n\prime}(Q_1^s) + \overline{S}^{n\prime}(Q_1^s)\right] - u^{n\prime\prime}\left(Q_1^s - \overline{S}^n\right)\left[1 - \overline{S}^{n\prime}(Q_1^s)\right]\right\}\\
&\quad -\varphi u^{s\prime\prime}(Q_2^s + S^s)\left[Q_2^{s\prime}(Q_1^s) + S^{s\prime}(Q_1^s)\right]\\
&-\psi\left\{u^{n\prime\prime}\left(Q_2^n + \overline{S}^n\right)\left[Q_2^{n\prime}(Q_1^s) + \overline{S}^{n\prime}(Q_1^s)\right] - \lambda u^{s\prime\prime}(Q_2^n + S^s)\left[Q_2^{n\prime}(Q_1^s) + S^{s\prime}(Q_1^s)\right]\right\} - \gamma = 0,
\end{aligned} \tag{S16}$$

where $\mu \geq 0$ and $\nu \geq 0$ respectively represent the Kuhn-Tucker multipliers associated with consumers' storage decisions (see (S8) and (S9)) as well as $\varphi \geq 0$ and $\psi \geq 0$ respectively represent the Kuhn-Tucker multipliers associated with the first-order conditions (S14) and (S15). Consequently, we find that, with naïve overestimating consumers, in the dynamic storage solution, the pricing policy consists of the first period pricing policy $(Q_1^{s*o}, P_1^{s*o})$, where $Q_1^{s*o}$ and $P_1^{s*o}$ respectively satisfy the first-order condition (S16) and the binding non-skipping constraint (S12), and of the second period pricing policy $\{(Q_2^{s*o}, P_2^{s*o}), (Q_2^{n*o}, P_2^{n*o})\}$ associated with the firm's maximization problem in (S1) (with $u^{s\prime}(\cdot) > u^{n\prime}(\cdot)$) that yields the first-order conditions (S14) and (S15). The levels of storage are $S^{s*o}$ for sophisticated consumers and $\overline{S}^{n*o}$ for naïve consumers (as implied by (S8) and (S9)). The dynamic storage solution requires that (at least) naïve consumers store, i.e., $\overline{S}^{n*o} > 0$, which occurs for sufficiently small values of the storage cost, i.e., for $c < u^{n\prime}\left(Q_2^n|_{\overline{S}^n=0}\right) - u^{n\prime}\left(Q_1^s|_{\overline{S}^n=0}\right)$.

Let $\Psi^i \triangleq \sum_{\tau=1}^{2}\left[U^i(x_\tau, y_\tau) - P_\tau\right] - c\overline{S}^i$ be the aggregate net utility of consumers of type

$i = s, n$. Ignoring money $y_\tau$ without loss of generality, it follows from the binding constraint (S12) that sophisticated consumers obtain

$$\begin{aligned}\Psi^{s*o} &= u^s\left(Q_2^{s*o}\right) - P_2^{s*o} \\ &= u^s\left(Q_2^{s*o}\right) + u^s\left(Q_2^{n*o} + S^{s*o}\right) + u^n\left(\overline{S}^{n*o}\right) - u^s\left(Q_2^{s*o} + S^{s*o}\right) - u^n\left(Q_2^{n*o} + \overline{S}^{n*o}\right) \geq 0,\end{aligned}$$

where the equality holds if and only if $Q_2^{s*o} = P_2^{s*o} = 0$. Naïve consumers obtain

$$\begin{aligned}\Psi^{n*o} &= u^n\left(Q_1^{s*o} - \overline{S}^{n*o}\right) - c\overline{S}^{n*o} - P_1^{s*o} + u^n\left(Q_2^{n*o} + \overline{S}^{n*o}\right) - P_2^{n*o} \\ &= -u^s\left(Q_1^{s*o} - S^{s*o}\right) - u^s\left(Q_2^{s*o} + S^{s*o}\right) + u^n\left(Q_1^{s*o} - \overline{S}^{n*o}\right) \\ &\qquad + u^s\left(Q_2^{s*o}\right) + u^n\left(\overline{S}^{n*o}\right) - c\left(\overline{S}^{n*o} - S^{s*o}\right) \lesseqgtr 0.\end{aligned}$$

In the static solution, the pricing policy consists of the first period pricing policy $\left(Q_1^{sm}, P_1^{sm}\right)$, where $Q_1^{sm}$ is such that $u^{s\prime}\left(Q_1^{sm}\right) - \gamma = 0$ and $P_1^{sm} = u^s\left(Q_1^{sm}\right)$, and of the second period pricing policy $\left\{\left(Q_2^{sm}, P_2^{sm}\right), \left(Q_2^{nm}, P_2^{nm}\right)\right\}$ associated with the firm's maximization problem in (S1) (with $u^{s\prime}(\cdot) > u^{n\prime}(\cdot)$) that yields the first-order conditions (S14) and (S15), where $S^s = \overline{S}^n = 0$. The static solution can only emerge in the absence of storage, which occurs for sufficiently large values of the storage cost, i.e., for $c \geq u^{n\prime}\left(Q_2^n|_{\overline{S}^n=0}\right) - u^{n\prime}\left(Q_1^s|_{\overline{S}^n=0}\right)$ (see above). Sophisticated consumers obtain $\Psi^{sm} = u^s\left(Q_2^{nm}\right) - u^n\left(Q_2^{nm}\right) \geq 0$ and naïve consumers obtain $\Psi^{nm} = -u^s\left(Q_1^{sm}\right) + u^n\left(Q_1^{sm}\right) < 0$.

We now compare the dynamic storage solution and the static solution in terms of consumer surplus. For the sake of convenience, we focus our attention on the case where the number of sophisticates is large enough, i.e., $\lambda \geq \widetilde{\lambda}_{nl}^o \triangleq \left[u^{n\prime}\left(\overline{S}^{n*o}\right) - \gamma\right] / \left[u^{s\prime}\left(\overline{S}^{s*o}\right) - \gamma\right]$ (see the discussion after (S15)). This implies that $Q_2^{n*o} = P_2^{n*o} = 0$. We show that $\Psi^{s*o} \leq \Psi^{sm}$ and $\Psi^{n*o} < \Psi^{nm}$, provided that the storage cost $c$ is small enough. To proceed in a insightful manner, we fix $c = 0$. Our results apply for $c$ small enough. First, we prove that the firm cannot benefit from $Q_2^s > 0$. It follows from the first-order condition for $Q_1^s$ in (S16) that the impact of $Q_1^s$ on the firm's profit $\Pi$ through $Q_2^s$ evaluated at $Q_2^s = 0$ is

$$\begin{aligned}\left.\frac{\partial \Pi}{\partial Q_1^s}\right|_{Q_2^s=0} &= u^{s\prime}\left(Q_1^s - S^s\right) + Q_2^{s\prime}\left(Q_1^s\right)\left[u^{s\prime}\left(S^s\right) - u^{s\prime}(0)\right] + S^{s\prime}\left(Q_1^s\right)\left[u^{s\prime}\left(S^s\right) - u^{s\prime}\left(Q_1^s - S^s\right)\right] \\ &\quad + \lambda\left\{u^{s\prime}\left(S^s\right)\left[Q_2^{s\prime}\left(Q_1^s\right) + S^{s\prime}\left(Q_1^s\right)\right] - u^{s\prime}\left(S^s\right)S^{s\prime}\left(Q_1^s\right) - \gamma Q_2^{s\prime}\left(Q_1^s\right)\right\}.\end{aligned}$$

We find from consumers' storage decisions (see (S8) and (S9)) that $S^s$ is such that the expression in the last square brackets in the first line vanishes, which implies $S^s = Q_1^s/2$. As $Q_2^{s\prime}\left(Q_1^s\right) = -1$ and $S^{s\prime}\left(Q_1^s\right) = 1$, it follows from (S14) that $\partial\Pi/\partial Q_1^s|_{Q_2^s=0} > 0$, which means that the firm does not have any incentives to decrease $Q_1^s$ in order to get $Q_2^s > 0$. Thus, it holds $Q_2^{s*o} = Q_2^{n*o} = 0$. As $\Psi^{s*o} = 0$, we have $\Psi^{s*o} \leq \Psi^{sm}$. Furthermore, we obtain from consumers' storage decisions (see (S8) and (S9)) that $S^{s*o} = \overline{S}^{n*o} = Q_1^{s*o}/2$, which yields $\Psi^{n*o} = -2\left[u^s\left(Q_1^{s*o}/2\right) - u^n\left(Q_1^{s*o}/2\right)\right] < 0$. As $u^{s\prime}\left(Q_1^{sm}\right) - \gamma = 0$ and $u^{s\prime}\left(Q_1^{s*o}/2\right) - \gamma \leq 0$ (see (S14)), we find that $Q_1^{s*o}/2 \geq Q_1^{sm}$, which implies that $u^s\left(Q_1^{s*o}/2\right) - u^n\left(Q_1^{s*o}/2\right) \geq u^s\left(Q_1^{sm}\right) - u^n\left(Q_1^{sm}\right)$, where the inequality follows from $u^{s\prime}(\cdot) > u^{n\prime}(\cdot)$. Thus, it holds $\Psi^{n*o} < \Psi^{nm}$. Using quadratic utility functions of the form $u^s = \alpha^s Q_\tau - (\beta/2)\, Q_\tau^2$ and $u^n = \alpha^n Q_\tau - (\beta/2)\, Q_\tau^2$, where $\alpha^s > \alpha^n$, we find that in the dynamic storage solution the first period quantity is $Q_1^{s*o} = 2\left(\alpha^s - \gamma\right)/\beta$

and storage is $S^{s*o} = \overline{S}^{n*o} = (\alpha^s - \gamma)/\beta$. In the second period, we have $Q_2^{s*o} = Q_2^{n*o} = 0$. In the static solution, the first period quantity is $Q_1^{sm} = (\alpha^s - \gamma)/\beta$. In the second period, we obtain $Q_2^{sm} = (\alpha^s - \gamma)/\beta$ as well as $Q_2^{nm} = [\alpha^n - \lambda\alpha^s - (1-\lambda)\gamma] / [\beta(1-\lambda)]$ for $\lambda < (\alpha^n - \gamma)/(\alpha^s - \gamma)$ and $Q_2^{nm} = 0$ otherwise. Comparing consumer surplus between the two solutions yields $0 = \Psi^{s*o} \leq \Psi^{sm}$ and $\Psi^{n*o} < \Psi^{nm} < 0$. ■

# 3 Price commitment

In the following remark, we characterize the main features of the dynamic storage solution when the firm can commit to future prices.

**Remark 2** *A. With naïve underestimating consumers, under full commitment, the dynamic storage solution arises in equilibrium for $c < \widehat{c}^{cu}$. This yields prices $p_1^{cu}$ and $p_2^{cu} = p_1^{cu} + c$ as well as ex ante storage $S^{cu} > 0$ and ex post storage $\overline{S}^{cu} = \lambda S^{cu} > 0$.*

*B. With naïve overestimating consumers, under full commitment, the dynamic storage solution arises in equilibrium for $c < \overline{c}^{co}$. This yields prices $p_1^{co}$ and $p_2^{co}$, with $p_1^{co} > p_2^{co}$, as well as ex post storage $\overline{S}^{co} > 0$.*

**Proof of Remark 2.** First, we consider the case of naïve underestimating consumers. Following the same rationale as in the proof of Lemma 1, the firm does not have any incentives to promote storage by naïve consumers. As storage can emerge only if the storability constraint is binding, i.e., $p_1 + c = p_2$, the firm's maximization problem writes as

$$\max_{p_1} p_1 [D^s(p_1) + S(p_1)] + (p_1 + c)[\sigma(p_1 + c) - \lambda S(p_1)].$$

The firm's profit increases with $S(\cdot)$ if and only if $p_1(1-\lambda) - c\lambda > 0$. In this case, the firm prefers to promote full storage, i.e., $S(p_1) = \sigma(p_1 + c)$. Otherwise, no storage emerges in equilibrium. For $p_1(1-\lambda) - c\lambda > 0$, the firm's maximization problem reduces to

$$\max_{p_1} p_1 D^s(p_1) + [(2-\lambda)p_1 + (1-\lambda)c]\,\sigma(p_1 + c).$$

The first-order condition for $p_1$ is

$$D^s(p_1) + p_1 D^{s\prime}(p_1) + (2-\lambda)\sigma(p_1 + c) + [(2-\lambda)p_1 + (1-\lambda)c]\,\sigma'(p_1 + c) = 0. \tag{S17}$$

The equilibrium full commitment prices with storage are given by

$$p_1^{cu} = -\frac{D^s(p_1^{cu}) + (2-\lambda)\sigma(p_1^{cu} + c) + (1-\lambda)\sigma'(p_1^{cu} + c)\,c}{D^{s\prime}(p_1^{cu}) + (2-\lambda)\sigma'(p_1^{cu} + c)} \quad \text{and} \quad p_2^{cu} = p_1^{cu} + c. \tag{S18}$$

The equilibrium full commitment levels of ex ante and ex post storage are respectively $S^{cu} \triangleq S(p_1^{cu}) = \sigma(p_1^{cu} + c)$ and $\overline{S}^{cu} = \lambda S^{cu} = \lambda\sigma(p_1^{cu} + c)$. As $p_1^{cu}(1-\lambda) - c\lambda > 0$ for $c$ small enough, there exists a threshold $\widehat{c}^{cu}$ such that for $c < \widehat{c}^{cu}$ the dynamic storage solution arises in equilibrium. In line with the proof of Lemma 1 of the paper, we find that, if at the equilibrium prices in (S18) naïfs store ex post, the firm prefers to set higher prices that mitigate the distortion from the benchmark case of perfectly aware consumers. The equilibrium full commitment prices become $\widetilde{p}_1^{cu}$ and $\widetilde{p}_2^{cu} = \widetilde{p}_1^{cu} + c$, where $\widetilde{p}_\tau^{cu} > p_\tau^{cu}$, $\tau \in \{1, 2\}$.

Specifically, $\widetilde{p}_1^{cu}$ and $S\left(\widetilde{p}_1^{cu}\right)$ are such that only sophisticates store ex post, i.e., $\overline{S}\left(\widetilde{p}_1^{cu}\right) = \lambda S\left(\widetilde{p}_1^{cu}\right) = \lambda\sigma\left(\widetilde{p}_1^{cu} + c\right)$, and naïfs' marginal utility from consuming the entire quantity bought in the first period is equal to the price, i.e., $u^{n\prime}\left(D^s\left(\widetilde{p}_1^{cu}\right) + S\left(\widetilde{p}_1^{cu}\right)\right) = \widetilde{p}_1^{cu}$. Using linear demand functions of the form $D^s\left(p_\tau\right) = \alpha^s - \beta p_\tau$ and $D^n\left(p_\tau\right) = \alpha^n - \beta p_\tau$, where $\alpha^n > \alpha^s$, we find from (S18) that in equilibrium the full commitment prices with storage are $p_1^{cu} = \left[(2-\lambda)(1-\lambda)\alpha^n + \left(1+2\lambda-\lambda^2\right)\alpha^s - (3-2\lambda)\beta c\right] / \left[2\beta(3-\lambda)\right]$ and $p_2^{cu} = p_1^{cu} + c$, with ex ante storage $S^{cu} = \left[(4-\lambda)(1-\lambda)\alpha^n - \left(1+\lambda^2-4\lambda\right)\alpha^s - 3\beta c\right] / \left[2(3-\lambda)\right]$. The dynamic storage solution is more profitable than the dynamic solution without storage if and only if $c < \widehat{c}_1^{cu} = \left\{6(1-\lambda)\alpha^n + 4\lambda\alpha^s + \left[\left(1-2\lambda^2\right)\left(\alpha^n - \alpha^s\right) + \lambda\left(\alpha^n + \alpha^s\right)\right]\sqrt{2(3-2\lambda)}\right\} / \left[2\beta(3+2\lambda)\right]$. As the firm's profit in the dynamic storage solution decreases with $c$ when the static solution is feasible, i.e., for $c \geq \widetilde{c}$ (see Section 3.3 of the paper), the firm's profit is higher in the dynamic storage solution than in the static solution if and only if $c < \widehat{c}_2^{cu} = \frac{\lambda(1-\lambda)\alpha^n + \left(3+\lambda^2-2\lambda\right)\alpha^s}{\beta(4\lambda-3)} - \frac{\sqrt{3-\lambda}}{\beta(4\lambda-3)}\sqrt{\left(1+2\lambda^2-3\lambda\right)^2\alpha^{n2} + \left(1+4\lambda-3\lambda^2-4\lambda^3-4\lambda^4\right)\alpha^{s2} + 2(1-\lambda)\left(2-\lambda-4\lambda^2+4\lambda^3\right)\alpha^n\alpha^s}$. Thus, the dynamic storage solution arises in equilibrium if and only if $c < \widehat{c}^{cu} = \widehat{c}_1^{cu}$ for $\widehat{c}_1^{cu} < \widetilde{c}$ and if and only if $c < \widehat{c}^{cu} = \widehat{c}_2^{cu}$, otherwise.

Now, we move to the case of naïve overestimating consumers. As $p_2 = p_2^e$, the firm's maximization problem writes as

$$\max_{\{p_1, p_2\}} p_1 D^s\left(p_1\right) + p_2\left[\sigma\left(p_2\right) - \overline{S}\left(p_1, p_2\right)\right],$$

where $\overline{S}\left(p_1, p_2\right) = (1-\lambda)\,\overline{s}^n\left(p_1, p_2\right)$ and $\overline{s}^n\left(p_1, p_2\right) = D^s\left(p_1\right) - D^n\left(p_2 - c\right)$ (see the expression (9) of the paper). The first-order conditions for $p_1$ and $p_2$ are respectively given by

$$D^s\left(p_1\right) + p_1 D^{s\prime}\left(p_1\right) - (1-\lambda)\,p_2 D^{s\prime}\left(p_1\right) = 0 \tag{S19}$$

and

$$\sigma\left(p_2\right) + p_2\left[\sigma'\left(p_2\right) + (1-\lambda)\,D^{n\prime}\left(p_2 - c\right)\right] - (1-\lambda)\left[D^s\left(p_1\right) - D^n\left(p_2 - c\right)\right] = 0. \tag{S20}$$

The equilibrium full commitment prices are $p_1^{co}$ and $p_2^{co}$ that simultaneously satisfy the first-order conditions (S19) and (S20). The equilibrium full commitment ex post storage is $\overline{S}^{co} \triangleq \overline{S}\left(p_1^{co}, p_2^{co}\right) = (1-\lambda)\,\overline{s}^{co} = (1-\lambda)\left[D^s\left(p_1^{co}\right) - D^n\left(p_2^{co} - c\right)\right]$. As $\lim_{\lambda\to1} p_1^{co} = \lim_{\lambda\to1} p_2^{co}$, we have $\overline{S}^{co} > 0$ for $\lambda$ high enough and $c$ low enough. Thus, there exists a threshold $\overline{c}^{co}$ such that the dynamic storage solution arises in equilibrium for $c < \overline{c}^{co}$. Substituting $p_1^m$ into the left-hand side of the first-order condition for $p_1^{co}$ in (S19) yields $-(1-\lambda)\,p_2 D^{s\prime}\left(p_1^m\right) > 0$, which implies that $p_1^{co} > p_1^m$. Substituting $p_2^m$ into the left-hand side of the first-order condition for $p_2^{co}$ in (S20) yields $-(1-\lambda)\left[D^s\left(p_1\right) - D^n\left(p_2^m - c\right) - p_2 D^{n\prime}\left(p_2^m - c\right)\right] < 0$, which implies that $p_2^{co} < p_2^m$. As $p_1^m > p_2^m$, we obtain that $p_1^{co} > p_2^{co}$. Using linear demand functions $D^s\left(p_\tau\right) = \alpha^s - \beta p_\tau$ and $D^n\left(p_\tau\right) = \alpha^n - \beta p_\tau$, where $\alpha^s > \alpha^n$, we find from (S18) that in equilibrium the full commitment prices with storage are $p_1^{co} = \left[\left(3+\lambda-2\lambda^2\right)\alpha^s + 2(1-\lambda)^2\alpha^n + (1-\lambda)^2\beta c\right] / \left[\beta\left(7-2\lambda-\lambda^2\right)\right]$ and $p_2^{co} = \left[(3\lambda-1)\alpha^s + 4(1-\lambda)\alpha^n + 2(1-\lambda)\beta c\right] / \left[\beta\left(7-2\lambda-\lambda^2\right)\right]$, with ex post storage $\overline{S}^{co} = (1-\lambda)\left[\left(3+\lambda^2\right)\alpha^s - \left(5+\lambda^2-2\lambda\right)\alpha^n - 2(3-\lambda)\beta c\right] / \left(7-2\lambda-\lambda^2\right)$, where $\overline{S}^{co} > 0$ if and only if $c < \overline{c}^{co} = \left[\left(3+\lambda^2\right)\alpha^s - \left(5+\lambda^2-2\lambda\right)\alpha^n\right] / \left[2\beta(3-\lambda)\right]$. As $\overline{c}^{co} < \widetilde{c}$, we find that the dynamic storage solution arises in equilibrium if and only if $c < \overline{c}^{co}$.

In the following remark, we compare the equilibrium prices under full commitment and limited commitment in the most relevant case where the storage cost $c$ is sufficiently low that storage emerges in equilibrium irrespective of the commitment regime. To this aim, with naïve underestimating consumers, we introduce the extra gain from storage accruing to the firm under full commitment at the price $p_1$, i.e., $\Gamma(p_1) \triangleq [p_1(1-\lambda) - c\lambda][S^{cu}(p_1) - S^{*u}(p_1)]$.[6] ■

**Remark 3** *A. With naïve underestimating consumers, in the presence of storage, the equilibrium prices under full commitment are higher than under limited commitment,* $p_\tau^{cu} > p_\tau^{*u}$, $\tau \in \{1,2\}$, *if and only if the extra gain from storage under full commitment increases with the price in equilibrium,* $\partial\Gamma(p_1^{cu})/\partial p_1 > 0$.

*B. With naïve overestimating consumers, in the presence of storage, under full commitment the firm has an incentive to charge a higher first period price and a lower second period price compared to limited commitment.*

**Proof of Remark 3.** First, we consider the case of naïve underestimating consumers. Substituting the first-order condition for $p_1^{cu}$ in (S17) into the left-hand side of the first-order condition for $p_1^{*u}$ in (A6) of the paper yields after some manipulation

$$\begin{aligned}
&\frac{1-\lambda}{\lambda}\left[(1-\lambda)\sigma(p_1^{cu}+c) + (p_1^{cu}+c)\sigma'(p_1^{cu}+c)\right] \\
&+\frac{p_1^{cu}(1-\lambda)-c\lambda}{\lambda}\left[(2-\lambda)\sigma'(p_1^{cu}+c) + (p_1^{cu}+c)\sigma''(p_1^{cu}+c)\right] \\
&= -\frac{\partial\Gamma(p_1^{cu})}{\partial p_1}.
\end{aligned}$$

Thus, it holds $p_\tau^{cu} > p_\tau^{*u}$, $\tau \in \{1,2\}$, if and only if $\partial\Gamma(p_1^{cu})/\partial p_1 > 0$. Note that $S^{*u}(p_1) = [\sigma(p_1+c) + (p_1+c)\sigma'(p_1+c)]/\lambda < S^{cu}(p_1) = \sigma(p_1+c)$ (as storage cannot exceed the second period demand), where $S^{*u}(p_1)$ and $S^{cu}(p_1)$ are respectively derived in the proofs of Lemma 1 of the paper and of Remark 2. This implies that the expression in the first square brackets is negative. Hence, as $p_1^{cu}(1-\lambda) - c\lambda > 0$, a sufficient (albeit not necessary) condition for $\partial\Gamma(p_1^{cu})/\partial p_1 > 0$ is that the expression in the second square brackets is nonpositive, which occurs if and only if $\partial S^{*u}(p_1^{cu})/\partial p_1 \leq \partial S^{cu}(p_1^{cu})/\partial p_1$. This is satisfied when $\sigma(\cdot)$ is not too convex. Consequently, we have $p_\tau^{cu} > p_\tau^{*u}$, $\tau \in \{1,2\}$, as long as $\sigma(\cdot)$ is not too convex.

Now, we move to the case of naïve overestimating consumers. Substituting $p_1^{cu}(p_2)$ in the first-order condition (S19) into the left-hand side of (A11) of the paper for a given $p_2$ yields $p_2\left\{(1-\lambda)D^{s\prime}(p_1^{cu}(p_2)) - \left[d\overline{S}^{*o}(p_1^{cu}(p_2),p_2)/dp_1\right]\right\} < 0$, where the inequality follows from (A12) of the paper.[7] Thus, for a given $p_2$, the firm has an incentive to set a higher price under full commitment than under limited commitment. Substituting $p_2^{cu}(p_1)$ in the first-order condition (S19) into the left-hand side of (A9) of the paper for a given $p_1$ yields $-(1-\lambda)p_2^{cu}(p_1)D^{n\prime}(p_2^{cu}(p_1)-c) > 0$. Thus, for a given $p_1$, the firm has an incentive to set a lower price under full commitment than under limited commitment. Using linear demands (see the proof of Remark 2), we find that $p_1^{co} > p_1^{*o}$ and $p_2^{co} < p_2^{*o}$. ■

[6] For the sake of clarity, with a small abuse of notation we denote by $S^{cu}(p_1)$ and $S^{*u}(p_1)$ the level of storage as a function of $p_1$ under full commitment and limited commitment, respectively.

[7] In line with the proof of Remark 2, for the sake of clarity, with a small abuse of notation we denote by $p_1^{cu}(p_2)$ the price under full commitment as a function of $p_2$ (and $p_2^{cu}(p_1)$ mutatis mutandis). Furthermore, $\overline{S}^{*o}(p_1,p_2)$ identifies the level of storage under limited commitment as a function of $p_1$ and $p_2$.