Upstream Collusion with Downstream Compensation

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Abstract

Successful upstream collusion must satisfy both the incentive constraint (keeping cartel members in line) and the compensation constraint (preventing lawsuits from downstream firms). Our analysis shows that there is a non-monotonic and inverted U-shaped relationship between cartel incidence and upstream market concentration when both constraints are taken into account. The inverted U-shaped relationship becomes more pronounced as compensation for downstream firms increases. Although compensation constraint reduces the harm of upstream collusion to social welfare and consumer surplus, it facilitates upstream collusion when compensation is small. These findings have important implications for antitrust policy and market regulation.

Keywords: upstream collusion, downstream compensation, successive oligopoly, repeated game

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1 Introduction

Cartels exist in supply chains.[1] When upstream firms collude to raise prices, downstream firms will be hurt directly. They therefore have the power, motivation, and information to report the cartel and pursue antitrust damage claims. To avoid litigation, upstream cartels must compensate downstream firms for their losses, creating a compensation constraint for cartel enforcement. This brings about two key questions: How does this additional constraint affect the sustainability of tacit agreements? And, what are the conditions or market characteristics that favor the formation of cartels in supply chains?

In a seminal paper, Schinkel et al. (2008) first study the stability of upstream collusion that must compensate downstream firms. Gu et al. (2019) complement Schinkel et al. (2008) by identifying the condition under which upstream collusion with compensation is profitable, setting aside the usual incentive constraint. Of course, a more realistic setting is to consider both constraints simultaneously. Such a model will better predict cartel incidence and inform antitrust policies. This paper attempts to accomplish the mission. Specifically, it considers a model of successive oligopolies with multiple upstream and downstream firms. It identifies the conditions for upstream collusion with compensation to be self-enforcing, and analyzes the impacts of market concentration and compensation constraint.

Our findings show that the relationship between cartel incidence and market concentration in the upstream industry is non-monotonic, and takes the form of an inverted U-shape: The threshold discount factor for upstream collusion to be self-enforcing decreases with an increasing number of upstream firms until a certain point, after which it begins to increase. The non-monotonic relationship can be explained by two opposing forces. On the one hand, as demonstrated by Gu et al. (2019), the profit of upstream collusion, after compensating downstream firms, increases in competition in the upstream market and thus makes it more attractive for cartel members. This strengthens the incentives for cooperation. On the other hand, a larger number of firms also increases the potential gain from defection, which weakens the incentive to cooperate, as predicted by standard cartel theory.[2] The balance between

[1] There are many related antitrust cases, among which a well-known and highly controversial one is *Illinois Brick Co. v. Illinois* (Schinkel et al., 2008). A recent notable example is the antitrust case against Japanese auto parts makers (Gu et al., 2019). More cases can be found in Richman and Murray (2007).

[2] The standard result, established under no compensation constraint, is that the relationship between

these two forces results in a non-monotonic relationship.

In addition, we find that the larger the compensation is, the more pronounced the non-monotonic relationship. This is because a larger compensation for downstream firms makes the upstream collusion less profitable or even unprofitable. Then an increase in the number of upstream firms will increase the collusive profit, and thus facilitate the collusion.

Finally, although a larger compensation hinders upstream collusion, the presence of a compensation constraint facilitates upstream collusion if the compensation is small, compared to the no-compensation case. On the one hand, the compensation constraint makes upstream collusion less profitable by reducing collusion profit, which hinders upstream collusion. This is the "profit-dampening effect". On the other hand, with compensation constraint, the cartel will choose a larger collusive output as it needs to take downstream firms' profit into account. This "output-amplifying effect" makes deviation less tempting and thus tends to facilitate upstream collusion. Moreover, an increase in compensation strengthens the former effect while leaving the latter one unchanged. Therefore, the output-amplifying effect overpowers the profit-dampening effect and makes upstream collusion more sustainable when the compensation is small enough.

This paper helps resolve a discrepancy in previous empirical research on the relationship between market concentration and collusion. Some studies have found that higher concentration leads to an increased likelihood of collusion (Hay and Kelley, 1974; Frass and Greer, 1977), while others suggest the opposite (Asch and Seneca, 1976). According to our theory, the different findings may result from different collusive requirements (with or without compensation) and different concentration degrees of the examined markets (the non-monotonic relationship given the compensation requirement). Indeed, our theoretical finding has empirical support: Symeonidis (2003) shows evidence of an inverted U-shape relationship between market concentration and the possibility of collusion.

This paper contributes to the tacit collusion literature. Standard theory on tacit collusion focuses on the incentives for firms to collude (Tirole, 1988). Most studies on upstream collusion also focus on collusion incentives and identify different factors facilitating collusion (Choe and Matsushima, 2013; Jullien and Rey, 2007; Piccolo and Reisinger, 2011; Nocke and

cartel incidence and market concentration is monotonic (Bain, 1956; Tirole, 1988; Ivaldi et al., 2003).

White, 2007, 2010). Our paper sheds light on the impact of the compensation constraint on collusion and shows this additional constraint facilitates upstream collusion when compensation is small.

This paper also adds to the studies which examine the impact of the *Illinois Brick rule* on collusion in antitrust laws. In 1977, the U.S. Supreme Court ruled in *Illinois Brick Co. v. Illinois* that only direct purchasers have the right to seek damages from companies that violate federal antitrust laws. While this ruling has been supported by some scholars (Landes and Posner, 1979; Lopatka and Page, 2003), who argue that it can encourage private antitrust action and help create a private channel of enforcement alongside public enforcement, others challenge it. Schinkel et al. (2008) point out that the *Illinois Brick rule* can actually facilitate collusion by creating a barrier between upstream cartels and indirect customers. Using a vertical supply chain model, they demonstrate that upstream cartels can effectively avoid damage claims by compensating downstream firms. A companion work by Schinkel and Tuinstra (2005) shows the existence of cartels in alternative and competitive market structures in which either upstream or downstream firms' profits are zero in competition. Motivated by the antitrust case of Japanese auto parts makers (DOJ, 2013), Gu et al. (2019) echo Schinkel et al. (2008) and study the conditions under which upstream collusion is profitable after compensating direct purchasers. This paper builds on Schinkel et al. (2008) and Gu et al. (2019) by examining the conditions that are favorable to cartel formations and the extent to which antitrust authorities should be concerned.

The remainder of the paper is organized as follows: Section 2 sets up the successive Cournot model. Section 3 presents our main results with linear demand. Section 4 analyzes the Cournot model and presents our main results with general demand. Section 5 considers a Bertrand model, and Section 6 concludes. All proofs are collected in the Appendix.

2 Model

Consider two vertically related industries with $m(> 1)$ identical upstream firms producing homogeneous input, and $n(\geq 1)$ identical downstream firms producing homogeneous final product.[3] Let $P = P(Q)$ be the inverse demand for the final product, where P and Q are price and quantity, respectively.

Firms in both industries compete à la Cournot, and the two industries interact through an endogenous demand for input. Upstream firms produce input at constant marginal cost κ and sell to downstream firms. The price of the inputs is determined by $t = t(\sum_{j=1}^{m} q_{u,j})$, where $q_{u,j}$ is the quantity produced by upstream firm j, $j \in \{1, 2, ...m\}$. The profit of upstream firm j is $\pi_{u,j} = q_{u,j}\,[t(\sum_{k=1}^{m} q_{u,k}) - \kappa]$, and the total upstream profit is $\Pi_u = \sum_{j=1}^{m} \pi_{u,j}$.

Downstream firms use the input to produce the final product at constant marginal cost c, on a one-for-one basis. The output of each downstream firm is given by $q_{d,i}$, $i \in \{1, 2...n\}$. The profit of downstream firm i is given by $\pi_{d,i} = q_{d,i}\,[P(\sum_{k=1}^{m} q_{d,k}) - c - t]$, and the total downstream profit is $\Pi_d = \sum_{i=1}^{n} \pi_{d,i}$. In equilibrium, $\sum_{j=1}^{m} q_{u,j} = \sum_{i=1}^{n} q_{d,i} = Q$. Since firms in either the upstream or the downstream industry are symmetric, we will neglect the subscripts i or j when there is no risk of confusion.

To make the analysis tractable, we impose the following restrictions on the demand (Lopez and Vives, 2019):

A1. The demand function $P(Q)$ is twice continuously differentiable with (1) $P(Q) > 0$ and $P'(Q) < 0$ for any $Q > 0$; and (2) the demand concavity

$$\rho(Q) \equiv \frac{P''(Q)Q}{P'(Q)}$$

is constant and equal to ρ.

The parameter ρ captures the degree of demand concavity.[4] This assumption is very mild as many commonly used demand functions, including linear demand and constant-elastic demand, share this property. To ensure that the successive oligopoly equilibrium behaves properly, we also assume the demand concavity satisfies:

A2. $\rho + 1 > 0$.

A2 is common in Cournot competition models (Greenhut and Ohta, 1979; Novshek, 1985; Salinger, 1988; Nocke and White, 2010), which implies that output quantities of downstream (upstream) firms are strategic substitutes, and each firm's profit is strictly concave in its

[3] In our analysis, we will treat m and n as continuous variables, rather than integers, to take derivatives.

[4] Mrazova and Neary (2017) use the term "demand convexity" for $-\frac{p''(Q)Q}{p'(Q)}$.

quantity choice. Under this assumption, there exists a unique Nash equilibrium in quantities in each industry (Gu et al., 2019).

Firms are engaged in repeated interaction with an infinite horizon and the future payoff is discounted at a common discount factor δ. The stage game is a sequential move game. Upstream firms move first and choose their quantities, either collusively or competitively. Knowing the choices of upstream firms, downstream firms simultaneously and independently choose their own quantities. If there is no upstream collusion, the stage game ends. If there is upstream collusion, at the end of the stage game, each upstream firm pays a lump sum transfer $\tau_u (> 0)$ to downstream firms to eliminate their incentives of filing a lawsuit against upstream collusion. The stage game is repeated in every period. Monitoring is perfect: all past actions will be known to all firms at the end of each stage. Collusion is sustained by implementing *grim trigger strategy*: any deviation is followed by infinitely repeated play of competitive equilibrium.

Notice, to avoid litigation, the lump sum transfer $\tau_d (> 0)$ received by each downstream firm should be large enough, where $n\tau_d = m\tau_u \equiv \tau$. More specifically, let B be the downstream-industry threshold profit level for no litigation, then each downstream firm's profit after compensation should be no less than $\frac{B}{n}$.

Some discussions on τ and B are warranted here. First, the lump sum transfer τ has many different interpretations, and thus can be delivered in more subtle ways in business practices, such as the relation-specific investment (Spencer and Qiu, 2001), the fixed entrance fees in the sense of two-part tariff, and so on. Second, besides the lump sum transfer τ, there are many other mechanisms to compensate the downstream, such as transfer pricing with rationing (Schinkel et al., 2008), or passive ownership (Hunold and Stahl, 2016). With this said, our main results will not be affected by the specific compensation scheme if it does not alter cartel stability.[5] The legality of the compensation scheme will determine its specific choice. In the discussion below, we will focus on the size of B, rather than the specific compensation scheme.

[5]For example, B can result either from a lump sum transfer τ to downstream firms, with $B = Q(P(Q) - t(Q) - c) + \tau$, or from a rationing scheme that supplies each downstream firm $\frac{Q}{n}$ of input at price $\widehat{t}$, with $B = Q(P(Q) - \widehat{t} - c)$. When $\tau = Q(t(Q) - \widehat{t}\ )$, the incentives for cartel stability are the same, regardless of the compensation scheme used. In light of this, either a lump sum transfer or a rationing scheme can be employed, as long as they result in the same B for downstream firms.

Third, a large enough B prevents downstream firms from pursuing antitrust claims. But how large? A natural candidate is the downstream firms' profit when there is no collusion. But not necessarily. The key factor determining B is the method used to calculate damages in private antitrust lawsuits, which can vary by jurisdictions.[6] For example, in some jurisdictions, damages are often calculated as the difference between the collusive price and the competitive price multiplied by the quantity purchased. This method together with a low pass-through rate, tends to underestimate the profit loss of downstream firms, resulting in a value of B that is lower than their profit in competition. However, in the US, victims of anti-competitive behavior can claim treble damages under Section 4 of the Clayton Acts, resulting in a higher value of B. Therefore, we consider a more general case to account for all these variations. That is, B can be higher than, equal to, or lower than the downstream firms' profit when there is no collusion.

3 An example: linear demand

To begin with, we use an example to highlight our main results. In the next section, we will show that these findings hold with the general demand function.

Consider a linear market demand for final consumption goods as $P(Q) = a - bQ$, with $a > c + \kappa$ and $b > 0$. To simplify notation, normalize $\frac{(a-c-\kappa)^2}{b} \equiv 1$.

When there is no collusion, the total output and profits for upstream and downstream firms are $Q^* = \frac{mn(a-c-\kappa)}{b(n+1)(m+1)}$, $\Pi_u^* = \frac{mn}{(n+1)(m+1)^2}$, and $\Pi_d^* = \frac{m^2 n}{(n+1)^2(m+1)^2}$.

When there is upstream collusion without compensation, the total output is $Q^o = \frac{n(a-c-\kappa)}{2b(n+1)}$. To ensure no deviation, the discount factor should be $\delta \geq \underline{\delta}^o = \frac{(m+1)^2}{(m+1)^2+4m}$.

When there is upstream collusion with compensation, the upstream firms conspire to maximize the joint profits of two industries so that the upstream collusive profit is maximized after compensation. Then, the collusion output is $Q^w = \frac{(a-c-\kappa)}{2b}$. To ensure no deviation, the discount factor should be large enough, or

$$\delta \geq \underline{\delta}^w = \frac{\frac{[2mn-(m-1)(n+1)]^2}{4mn(n+1)} - (1-4B)}{\frac{[2mn-(m-1)(n+1)]^2}{4mn(n+1)} - \frac{4mn}{(n+1)(m+1)^2}}.$$

[6]For a detailed discussion on damage measurement, please see Hovenkamp (2021).

Then, we have the following results (see also Figure 1 for an illustration):

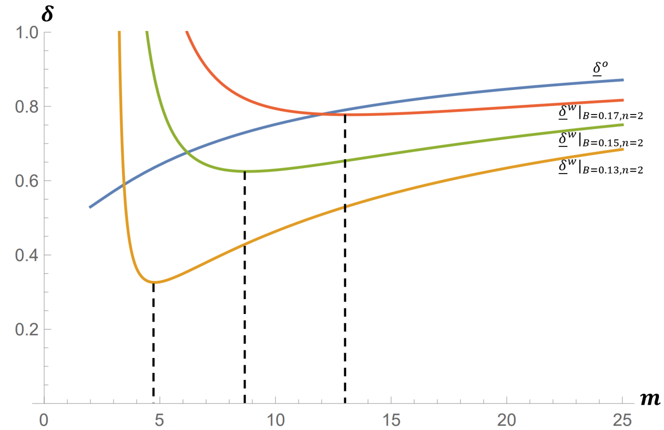

Figure 1: Threshold discount factor under linear demand

First, the relationship between cartel formation (with compensation) and upstream market concentration follows an inverted U-shaped curve. That is, there exists $\bar{m}(B) > 2$, such that $\underline{\delta}^w$ decreases in m when $m < \bar{m}(B)$, but $\underline{\delta}^w$ increases in m when $m \geq \bar{m}(B)$.

Second, although a larger compensation hinders upstream collusion, it increases the likelihood that more upstream firms facilitates collusion. That is, $\frac{\partial \underline{\delta}^w}{\partial B} > 0$ and $\frac{\partial \bar{m}(B)}{\partial B} > 0$.

Third, the compensation constraint facilitates upstream collusion if the downstream firms' required compensation is small. That is, $\underline{\delta}^o > \underline{\delta}^w$ if $B < \frac{(3n-1)m^2+2(3n+1)m-(n+1)}{4n(n+1)(m^2+6m+1)} = \widetilde{B}$.[7]

4 Analysis

The analysis will be unfolded as follows. First, we characterize the successive Cournot equilibrium to show the competition outcome. Then, we show the collusive outcome and derive the threshold discount factor for collusive agreements to be self-enforcing. By investigating the threshold discount factor, we can identify the properties of upstream collusion with compensation constraint. After that, we compare the threshold discount factors of upstream collusion with and without compensation to isolate the impact of compensation constraint on welfare and collusion stability.

[7]For a given B, Figure 1 also implies this condition is more likely to hold if m is large, which guarantees a larger downstream competition profit.

4.1 Competition

For a given input price t and constant marginal cost c, a downstream firm chooses q_d to maximize its profit $\pi_d = (P(Q) - t - c)\, q_d$. Summing up all downstream firms' first-order conditions (FOCs), we have a unique mapping between t and Q:

$$t(Q) = P(Q) + \frac{1}{n} P'(Q)Q - c. \tag{1}$$

The profit for an upstream firm is $\pi_u = (t(Q) - \kappa) q_u$, where $t(Q)$ is characterized by (1). Summing up upstream firms' FOCs, we get the unique equilibrium output Q^* determined by the following condition:

$$\frac{(\rho + 2) - (m-1)(n-1)}{mn} P'(Q^*)Q^* + P'(Q^*)Q^* + P(Q^*) = c + \kappa. \tag{2}$$

The profits for representative downstream and upstream firms are, respectively,

$$\pi_u^* = -\frac{\rho + n + 1}{nm^2} P'(Q^*) \left(Q^*\right)^2, \quad \pi_d^* = -\frac{1}{n^2} P'(Q^*) \left(Q^*\right)^2.$$

4.2 Upstream collusion

When there is compensation constraint, the upstream cartel selects an output Q^w to maximize the joint profits of upstream firms after compensation:

$$\max_Q \Pi_u (Q) - \tau,$$

$$s.t., \ \Pi_d(Q) + \tau \geq B, \tag{3}$$

$$\Pi_u (Q) - \tau > \Pi_u(Q^*). \tag{4}$$

The *compensation constraint* (3) ensures no less than a threshold no-litigation profit of B for downstream firms. The *participation constraint* (4) guarantees the upstream collusion is profitable. The compensation constraint should be binding so the optimization problem can be rewritten as

$$\max_{Q} \Pi(Q) - B,$$
$$s.t., \ \Pi(Q) - B > \Pi_u(Q^*)$$

where $\Pi(Q) \equiv \Pi_u(Q) + \Pi_d(Q)$ is the joint profit of the two industries. The collusive output Q^w maximizes the two industries' joint profit, and is determined by:

$$P'(Q^w)Q^w + P(Q^w) = c + \kappa. \tag{5}$$

Notice, the condition (4) indicates upstream collusion is profitable if and only if $B \in (\underline{B}, \overline{B})$, where $\underline{B} \equiv \Pi_d(Q^w)$, representing the profit for downstream firms without any compensation, and $\overline{B} \equiv \Pi(Q^w) - \Pi_u(Q^*)$, meaning that downstream firms obtain all of the collusive profit. To ensure $\overline{B} > \underline{B}$, we need $\Pi_u(Q^w) > \Pi_u(Q^*)$, or $Q^w < Q^*$, which is a necessary but not sufficient condition for upstream collusion to be profitable.[8]

Moreover, since $\Pi(Q^w)$ and $\Pi_d(Q^w)$ are independent of m and $\Pi_u(Q^*)$ is decreasing in m, $\overline{B}$ is increasing in m. Therefore, for a given B, there exists $\underline{m}(B)$, such that the participation constraint (4) is equivalent to:[9]

$$m > \underline{m}(B). \tag{6}$$

Hence, we have the following result:

Lemma 1. *For a given B, upstream collusion with compensation is profitable if and only if $m > \underline{m}(B)$.*

Lemma 1 states that the profitability of upstream collusion depends on the number of upstream firms. When there are many upstream firms, competition is intense and the profit for upstream firms is low. By cooperating to raise input prices, upstream firms can significantly increase their profit even after compensating downstream firms.

[8] In Gu et al. (2019), this is the sufficient and necessary condition for profitable collusion as they assume $B = \Pi_d(Q^*)$. Since we allow a more general B, the condition will be more stringent when $B > \Pi_d(Q^*)$.

[9] To simplify our analysis, we restrict attention to the case where B is independent of m. Alternatively, we can take B as a function of m and n. As long as the speed of B increases on m is not so large, it does not change our main results qualitatively while complicating the analysis.

From now on, we assume condition (6) holds so the upstream collusion is profitable. We next analyze upstream firms' incentives for cooperation in the collusive equilibrium. Upstream firms each produce the collusive output $q_u^w = \frac{1}{m}Q^w$ until deviation happens, in which case they revert to Cournot equilibrium forever. When there is no deviation, the collusive profit for each upstream firm is:

$$\pi_u^w = \frac{1}{m}\left[\Pi\left(Q^w\right) - B\right].$$

Conditional on the other upstream firms are cooperating, the problem for the deviant firm becomes: $\max_q \pi_u = \left[t(\frac{m-1}{m}Q^w + q) - \kappa\right] q$. Hence, its profit-maximizing output is determined by the first-order condition:

$$\frac{\left(\rho + n + 1\right)\widehat{q}_u^w + \widehat{Q}^w}{n} P'(\widehat{Q}^w) + P(\widehat{Q}^w) = \kappa + c,$$

where $\widehat{Q}^w = \frac{m-1}{m}Q^w + \widehat{q}_u^w$. And the deviant firm's profit in the current period would be:

$$\widehat{\pi}_u^w = \left[t\left(\widehat{Q}^w\right) - \kappa\right]\widehat{q}_u^w.$$

Thus, by defection, the deviant firm receives $\widehat{\pi}_u^w$ in the current period but loses future collusion profit from next period on. The IC constraint for no deviation is equivalent to:

$$\delta \geq \underline{\delta}^w \equiv \frac{\widehat{\pi}_u - \pi_u^w}{\widehat{\pi}_u^w - \pi_u^*} = 1 - \frac{\Pi_u^w - \Pi_u^*}{\widehat{\Pi}_u^w - \Pi_u^*}, \tag{7}$$

where $\widehat{\Pi}_u^w = m\widehat{\pi}_u^w$, and $\Pi_u^w = m\pi_u^w = \Pi\left(Q^w\right) - B$.

The stability of the cartel is connected to its profitability through the condition in (7). The upstream cartel is not sustainable unless it is profitable, meaning $\Pi_u^w > \Pi_u^*$; otherwise, we would have $\underline{\delta}^w > 1$, which is impossible.

When m is small enough ($m \leq \underline{m}(B)$), an increase of m (so that $m > \underline{m}(B)$) makes upstream collusion switch from unprofitable to profitable so $\underline{\delta}^w$ decreases in m. This indicates that there is a region in which the probability of upstream collusion is higher when there are more upstream firms. On the other hand, when m is large enough, Π_u^* approaches zero, and so $\underline{\delta}^w$ is approximately equal to $\frac{(m-1)\Pi(Q^w)+B}{m\Pi(Q^w)}$, which is increasing in the number of upstream firms. We summarize the result below.

Proposition 1. *The relationship between the formation of upstream cartels with compensation and the number of upstream firms is non-monotonic. To be more specific,*

(i) When the number of upstream firms is small, an increase in the number of upstream firms makes cartel more likely (i.e. there exists $\overline{m}(B) > \underline{m}(B)$ such that $\frac{\partial \underline{\delta}^w}{\partial m} < 0$ if $m \in (\underline{m}(B), \overline{m}(B))$);

(ii) When the number of upstream firms is large, an increase in the number of upstream firms makes cartel less likely (i.e. $\frac{\partial \underline{\delta}^w}{\partial m} > 0$ if m is large enough).

The reasoning behind the non-monotonic relationship between cartel formation and upstream competition can be explained as follows. In deciding whether to defect, an upstream firm balances between short-term gain in the present period and long-term loss of collusive profit in the future. Collusion is maintained when the long-term loss outweighs the short-term gain. When the upstream market is highly concentrated, the benefits from collusion are low, resulting in lower long-term loss from defection relative to the short-term gain and making it difficult to sustain the tacit agreement. In extreme cases, such as when there are $\underline{m}(B)$ upstream firms, firms would receive the same profit in Cournot equilibrium as in collusion. Thus, the long-term loss would become zero, making defection impossible to punish. However, as m increases above $\underline{m}(B)$, upstream collusion becomes more profitable, increasing the long-term loss. Defection will then be punished, making the incentive constraint easier to satisfy. This explains why the threshold discount factor $\underline{\delta}^w$ may decrease in the number of upstream firms when the number is small.

On the other hand, increasing the number of firms in a competitive upstream market raises the collusive profit, but it also raises the gain from defection disproportionately, making it harder to maintain the incentives. As a result, a further increase in the number of firms in a highly competitive upstream market reduces the likelihood of cartel formation.

From (7), where the threshold discount factor can be rewritten as $\underline{\delta}^w = \frac{\widehat{\Pi}_u^w - \Pi(Q^w)+B}{\widehat{\Pi}_u^w - \Pi_u^*}$, one can see that $\underline{\delta}^w$ is increasing in B. So a larger compensation makes upstream collusion less likely. An interesting question is how the compensation constraint affects the non-monotonic relationship. Taking derivative of $\frac{\partial \underline{\delta}^w}{\partial m}$ with respect to B, we have $\frac{\partial^2 \underline{\delta}^w}{\partial m \partial B} < 0$. Together with $\frac{\partial \underline{\delta}^w}{\partial m} \mid_{m=\overline{m}(B)} = 0$, we conclude that $\frac{\partial \overline{m}(B)}{\partial B} > 0$. Therefore, we have the following result.

Proposition 2. *The more compensation the downstream firms require, the more likely an increase in the number of upstream firms facilitates the upstream collusion.*

This proposition 2 shows that, as the compensation goes up, the inflection point of $\underline{\delta}^w$ becomes larger, making the non-monotonic relationship between collusion and the number of upstream firms more salient. This is because greater compensation reduces the profitability of upstream collusion, and thus it takes more upstream firms to make the collusion sustainable, leading to an increase in the inflection point.

4.3 Impacts of compensation constraint

Below we compare the results of upstream collusion with and without the compensation constraint to isolate its impacts on welfare and cartel stability.

The equilibrium derivation of upstream collusion without compensation is quite standard. When there is no compensation constraint, the collusive output, Q^o that maximizes the cartel's profit is determined by:

$$\frac{\rho + 2}{n} P'(Q^o)Q^o + P'(Q^o)Q^o + P(Q^o) = c + \kappa. \tag{8}$$

Conditional on all other upstream firms producing at collusion output $\frac{1}{m}Q^o$, the upstream deviant firm's output $\widehat{q}_u^o$ is chosen to maximize $\pi_u = \left[t(\frac{m-1}{m}Q^o + q) - \kappa\right] q$. Therefore, the upstream firm's profits from collusion and defection are, respectively:

$$\pi_u^o = -\frac{\rho + n + 1}{nm} P'(Q^o) \left(Q^o\right)^2; \quad \widehat{\pi}_u^o = \widehat{q}_u^o \left[t\left(\widehat{Q}^o\right) - \kappa\right],$$

where $\widehat{Q}^o = \frac{m-1}{m}Q^o + \widehat{q}_u^o$.

As a result, upstream collusion without compensation is sustainable only if:

$$\delta \geq \underline{\delta}^o = \frac{\widehat{\pi}_u^o - \pi_u^o}{\widehat{\pi}_u^o - \pi_u^*} = 1 - \frac{\Pi_u^o - \Pi_u^*}{\widehat{\Pi}_u^o - \Pi_u^*},$$

where $\widehat{\Pi}_u^o = m\widehat{\pi}_u^o$ and $\Pi_u^o = m\pi_u^o$.

Now, we are ready to compare the upstream collusion with and without compensation.

First, we compare the welfare. Let CS^o (CS^w) and SW^o (SW^w) be the consumer surplus and social welfare under collusion without (with) compensation constraint, respectively. As firms are identical in upstream and downstream markets, the welfare comparison is pinned down by the comparison of total output Q. Comparing (2), (8) and (5) indicates that $Q^o < Q^w < Q^*$. Since consumer surplus and social welfare are increasing in output, this then implies that compensation constraint reduces the harm of upstream collusion to both consumer surplus and social welfare, i.e. $SW^o < SW^w < SW^*$ and $CS^o < CS^w < CS^*$.

Next, we compare the two threshold discount factors, $\underline{\delta}^o(Q^o)$ versus $\underline{\delta}^w(Q^w)$. As discussed above, the compensation constraint raises collusive output and reduces the cartel profit. To fully understand the effects of this constraint, we examine the two effects separately:

$$\underline{\delta}^o(Q^o) - \underline{\delta}^w(Q^w) = \underbrace{\underline{\delta}^o(Q^w) - \underline{\delta}^w(Q^w)}_{\textit{Profit-dampening Effect } (-)} + \underbrace{\underline{\delta}^o(Q^o) - \underline{\delta}^o(Q^w)}_{\textit{Output-amplifying Effect } (+)}$$

For the same collusive output, the compensation constraint will reduce the cartel's profit. The impact of such profit difference is referred to as the "Profit-dampening Effect". Similarly, the impact of output difference is referred to as the "Output-amplifying Effect".

To investigate the profit-dampening effect, we begin with the two kinds of collusion choosing the same collusive output. For any $Q^c \in [Q^w, Q^*)$, we have $\pi_u^o(Q^c) - \pi_u^w(Q^c) = \frac{1}{m}[B - \Pi_d(Q^c)]$, and $\widehat{\pi}_u^w(Q^c) = \widehat{\pi}_u^o(Q^c)$. The former equation indicates that, on the same collusive output, the difference between two collusive profits is just the compensation to downstream firms. The latter indicates that the deviant firm obtains the same profit under both types of collusion when other cartel members produce the same output $\frac{1}{m}Q^c$. That is, compensation increases the short-term gain and reduces the long-term loss of defection. Hence, we have:

Lemma 2 (Profit-dampening Effect)**.** *Fixing the collusive output at $Q^c \in [Q^w, Q^*)$, the compensation constraint hinders upstream collusion (i.e. $\underline{\delta}^w(Q^c) > \underline{\delta}^o(Q^c)$).*

The intuition for Lemma 2 is straightforward. If the compensation constraint does not change the collusive output, it will make upstream collusion less likely because the compensation reduces the benefits from collusion but leaves the gain from defection unchanged.

As for the output-amplifying effect, we have the following result.

Lemma 3 (Output-amplifying Effect)**.** *A larger collusive output facilitates upstream collusion without compensation (i.e.* $\frac{\partial \underline{\delta}^o(Q^c)}{\partial Q^c} < 0$ *for* $Q^c \in [Q^o, Q^*)$*, therefore* $\underline{\delta}^o(Q^o) > \underline{\delta}^o(Q^w)$*).*

According to Lemma 3, upstream collusion without compensation is more likely to occur when the collusive output Q^c is closer to the Cournot output Q^*. When the collusive output is closer to the competitive output, both the short-term gain and long-term loss of defection will go down, but the short-term gain will decrease at a faster rate since all other cartel members will produce more output. Because $Q^o < Q^w$, we have $\underline{\delta}^o(Q^o) > \underline{\delta}^o(Q^w)$, and thus the output-amplifying effect tends to facilitate upstream collusion.

Combining Lemmas 2 and 3, we find that the profit-dampening effect tends to hinder upstream collusion, while the output-amplifying effect tends to facilitate it. The question that remains is: which effect will prevail? On the one hand, $\underline{\delta}^w$ increases as B increases. On the other hand, we have $\underline{\delta}^w(Q^w)\mid_{B\to\underline{B}} = \underline{\delta}^o(Q^w) < \underline{\delta}^o(Q^o)$ and $\underline{\delta}^o(Q^o) < 1 = \underline{\delta}^w(Q^w)\mid_{B\to\overline{B}}$. Hence, we conclude:

Proposition 3. *There exists* $\widetilde{B} \in \left(\underline{B}, \overline{B}\right)$ *such that,* $\underline{\delta}^w < \underline{\delta}^o$ *for* $B < \widetilde{B}$*;* $\underline{\delta}^w = \underline{\delta}^o$ *for* $B = \widetilde{B}$*;* $\underline{\delta}^w > \underline{\delta}^o$ *for* $B > \widetilde{B}$*.*

Proposition 3 shows that the compensation constraint facilitates upstream collusion when the compensation is small, which is counter-intuitive. This is because the profit-dampening effect increases as the compensation goes up, while the output-amplifying effect is independent of the compensation. When the compensation is small, the output-amplifying effect dominates the profit-dampening effect and leads to $\underline{\delta}^w < \underline{\delta}^o$. However, when the required compensation is large, the result reverses.

5 Bertrand competition

In the Cournot model, we show there is a non-monotonic relationship between cartel incidence and upstream market competition, and the compensation constraint facilitates upstream collusion if compensation is small. In this section, we show the results remain valid in Bertrand model with *asymmetric* upstream firms and downstream *product differentiation*.

Consider the scenario where both upstream and downstream firms engage in Bertrand competition. For compensation to be relevant, the final consumption goods cannot be perfect substitutes so downstream firms will have positive profit in Bertrand competition. We therefore assume the final products are differentiated. Each downstream firm faces symmetric demands $q_i(p_i, \mathbf{p}_{-i})$ with $\frac{\partial q_i(p_i,\mathbf{p}_{-i})}{\partial p_i} < 0$ and $\frac{\partial q_i(p_i,\mathbf{p}_{-i})}{\partial p_h} > 0$ for $i, h \in \{1, .., n\}$ and $h \neq i$.[10] They produce the final products using homogeneous inputs supplied by upstream firms with varying marginal costs κ_j, where $j \in \{1, 2, ..., m\}$. Without loss of generality, we assume $\kappa_1 \leq \kappa_2 \leq \cdots \leq \kappa_m$. The other settings remain the same as the Cournot model.

In the Bertrand competition, the most efficient upstream firm(s), those with the lowest marginal cost, supplies the market at a price of $t^* = \kappa_2$. Prices for the final consumption goods are characterized by downstream firms' first-order conditions (FOCs):

$$q_i(p_i, \mathbf{p}_{-i}) + (p_i - t^* - c)\frac{\partial q_i(p_i, \mathbf{p}_{-i})}{\partial p_i} = 0, \;\; i \in \{1, .., n\}.$$

The profits of upstream and downstream firms are, respectively,

$$\pi^*_{u,j} = \begin{cases} (\kappa_2 - \kappa_1)Q(\kappa_2), & j = 1; \\ 0, & j \neq 1; \end{cases}$$
$$\pi^*_{d,i} = (p^*_i - c - \kappa_2)\, q_i(p^*_i, \mathbf{p}^*_{-i}),$$

where $Q(\kappa_2) = \sum_{i=1}^n q_i(p^*_i, \mathbf{p}^*_{-i})$ is the total output.

When there is upstream collusion with compensation, let the collusive input price be t^w, where $t^w > \kappa_m$ to ensure that all upstream firms produce a positive output.[11] Then, the total output is $Q(t^w) = \sum_{i=1}^n q_i(p^w_i, \mathbf{p}^w_{-i})$, where prices are characterized by downstream firms' FOCs at collusive input price. Each downstream firm's profit is $\pi^w_{d,i} = (p^w_i - c - t^w)\, q_i(p^w_i, \mathbf{p}^w_{-i})$. An upstream firm j supplies s_j share of the market, with $s_j \geq 0$ and $\sum_{j=1}^m s_j = 1$. Upstream firms compensate downstream firms with a lump sum transfer, $\tau_j = r_j \tau$, with $r_j \geq 0$ and

[10] Regular restrictions on demands are assumed to ensure existence and uniqueness of Bertrand equilibrium.
[11] To ensure this happens, we assume the cost difference $\kappa_m - \kappa_1$ is not too large.

$\sum_{j=1}^{m} r_j = 1$; $\tau \equiv B - \sum_{i=1}^{n} \pi_{d,i}^{w} \geq 0$ indicating the total transfer. In a market where transfers among horizontally competing firms are illegal, each upstream firm produces its share of input s_j at cost κ_j, resulting in a profit of $\pi_{u,j}^{w} = (t^w - \kappa_j)s_j Q(t^w) - \tau_j$.

5.1 Optimal collusive scheme

Following the literature of collusion among firms with asymmetric marginal costs (Patinkin, 1947; Rothschild, 1999; Collie et al., 2004), we focus on the optimal collusive scheme, which assumes collusive price is chosen to maximize the joint profits of the two industries, while the allocation scheme is to minimize the threshold discount factor.

Therefore, the optimal collusive input price is determined by:

$$t^w = \arg\max_t \sum_{j=1}^{m} (t - \kappa_j) s_j Q(t) + \sum_{i=1}^{n} (p_i - c - t)\, q_i(p_i, \mathbf{p}_{-i}). \tag{9}$$

The upstream cartel is sustainable if[12]

$$\delta \geq \underline{\delta}^w \equiv \frac{m - 1 + \frac{\tau}{(t^w - \kappa_1)Q(t^w)}}{m - \frac{\pi_{u,1}^{*}}{(t^w - \kappa_1)Q(t^w)}}. \tag{10}$$

The corresponding optimal allocation scheme $\{s_j, r_j\}$ is:

$$\{s_j, r_j\} = \begin{cases} \{\frac{1-(m-m_e)(1-\underline{\delta}^w)}{m_e}, \frac{1}{m_e}\}, & \forall\ j \text{ with } \ \kappa_j = \kappa_1; \\ \{1 - \underline{\delta}^w, 0\}, & \forall\ j \text{ with } \ \kappa_j > \kappa_1. \end{cases} \tag{11}$$

Here m_e indicates the number of the most efficient firms, i.e., $\kappa_j = \kappa_1$ for $j \in \{1, .., m_e\}$.

The optimal allocation scheme implies the most efficient firms bear the burden of compensation while less efficient ones do not make any compensations. This is because, although the compensation tightens a firm's IC constraint, it has a smaller impact on a more efficient firm as a more efficient firm obtains a larger profit for a given market share. As a result, the threshold discount factor will be minimized if we let the most efficient firms do all compensation. Moreover, for all less efficient firms with $\kappa_j > \kappa_1$, they are endowed with the

[12] Please see Appendix B.1 for a detailed derivation of the optimal collusive scheme.

same market share as they earn zero profit in competition.

Nevertheless, if the required compensation is too large, collusion is not profitable and thus not sustainable. This occurs when[13]

$$B > \sum_{i=1}^{n} (p_i^w - c - \kappa_1) q_i(p_i^w, \mathbf{p}_{-i}^w) - \pi_{u,1}^* = \bar{\Pi}(t^w) - \pi_{u,1}^*. \tag{12}$$

Recall B is downstream firms' profit in collusive equilibrium after compensation, and so the right side of (12) characterizes the upper bound for feasible B, which is the maximized collusion profit given t^w.[14] This suggests that increasing upstream competition will facilitate cartel formation only if it increases the maximized collusion profit. When does this happen? Intuitively, if there is an increased number of upstream *efficient* firms, the maximized collusion profit increases, and thus has the tendency to make collusion profitable and sustainable. To illustrate this insight, consider the following example.

Example. *Consider the demand system* $q_i = a - p_i + \lambda \sum_{l=1}^{n}(p_l - p_i)$ *for* $i \in \{1, .., n\}$, *where a larger value of* λ *indicates a less differentiated final product. There are* $m(\geq 2)$ *upstream firms, out of which* $m-1$ *are efficient (i.e.,* $\kappa_j = 0$ *for* $j \in \{1, .., m-1\}$*) and one is inefficient (i.e.,* $\kappa_m = 1$*). The Figure 2 is plotted with* $B = \Pi_d^*$, $a = 10$, $c = 0$, *and* $\lambda = 0.3$, $n = 2$ *unless specified otherwise.*[15]

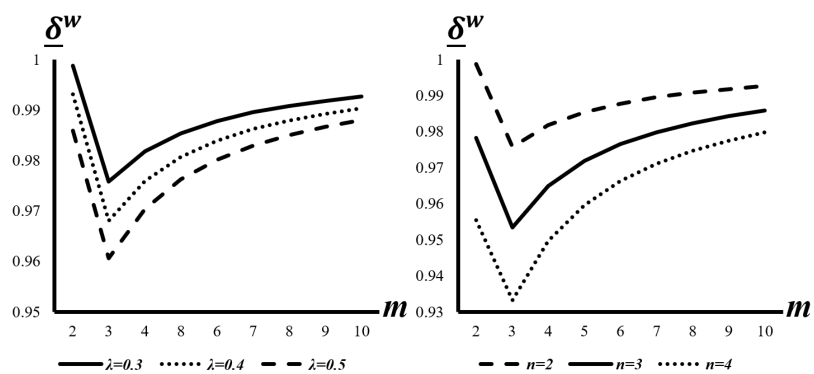

Figure 2: Threshold discount factor with price competition

Figure 2 shows there is a non-monotonic relationship between the cartel incidence and

[13]Note $\underline{\delta}^w \leq 1$ implies $(t^w - \kappa_1)Q(t^w) - \pi_{u,1}^* \geq \tau$. Adding Π_d^w to both sides produces $\bar{\Pi}(t^w) - \pi_{u,1}^* \geq B$. Also note that $\Pi_u^* = \pi_{u,1}^*$.

[14]Given t^w, the collusion profit will be maximized if all inputs are supplied by the most efficient firm(s).

[15]See Appendix B.3 for detailed calculations example of $\underline{\delta}^w$.

upstream market competition. The intuition is similar to that for the Cournot case: The collusive profit for upstream firms after compensation increases in competition in the upstream market, thus making cooperation more attractive for cartel members. This strengthens the incentives for cooperation. Meanwhile, a larger number of firms increases the potential gain from defection, which weakens the incentive to cooperate. The balance between these two forces results in a non-monotonic relationship.

Moreover, Figure 2 provides insights into how product differentiation and downstream competition impact cartel incidence. The left and right panels illustrate the effects of product differentiation and downstream competition, respectively. As can be seen from the figure, reducing product differentiation (i.e. λ is larger) or strengthening downstream competition (i.e. n is larger) makes upstream collusion easier to sustain. This is because both smaller product differentiation and stronger downstream competition reduce downstream firms' profit and their required compensation, which makes upstream collusion more profitable and thus easier to sustain.

5.2 Impacts of compensation constraint

We examine the impact of compensation constraint on optimal collusion by comparing the above result with the case with no compensation constraint. In the latter case, let the optimal collusive input price and market share be $\{t^o, s_j^o\}$, then collusion can be sustained when[16]

$$\delta \geq \underline{\delta}^o \equiv \frac{m-1}{m - \frac{\pi^*_{u,1}}{(t^o-\kappa_1)Q(t^o)}}, \tag{13}$$

where $Q(t^o) = \sum_{i=1}^{n} q_i(p_i^o, \mathbf{p}_{-i}^o)$ and the final goods prices are characterized by downstream firms' FOCs at collusive input price t^o.

Comparing (10) with (13), we observe two differences. The first difference is the additional term in the numerator which captures the profit-dampening effect, indicating that the compensation constraint tends to hinder upstream collusion. The second difference is the collusive input price which captures the output-amplifying effect, indicating that the compensation constraint tends to facilitate upstream collusion. Specifically, for a given market share, we

[16] See Appendix B.2 for detailed calculations.

have $t^o > t^w$ and $(t^o - \kappa_1)Q(t^o) < (t^w - \kappa_1)Q(t^w)$. This is because the collusion involves the production of inefficient firms, and the collusive input price is too high from the perspective of maximizing $(t - \kappa_1)Q(t)$. As a result, we have $\underline{\delta}^o(t^o) > \underline{\delta}^o(t^w)$. If $\tau \to 0$, the first effect tends to vanish and will be dominated by the second one, meaning that the compensation constraint facilitates upstream collusion. However, if τ is large enough, the result reverses as $\underline{\delta}^o(t^o) < 1 = \underline{\delta}^w(t^w)$. Therefore, in the Bertrand model, the compensation constraint facilitates upstream collusion if and only if the compensation is small.

6 Conclusion

Cartels are prevalent in supply chains and their success depends not only on preventing member firms from defecting but also on compensating direct victims to avoid legal action or antitrust scrutiny. The compensation constraint plays a crucial role in determining the stability of upstream collusion. Our research reveals that, with compensation constraint, the relationship between cartel prevalence and upstream market concentration is likely to be non-monotonic and follow an inverted U-shaped curve. Although a larger compensation hinders upstream collusion, it makes this inverted U-shaped relationship more salient. What's more, as compared to upstream collusion without compensation, upstream collusion with compensation is easier to sustain if the amount of compensation is small.

For simplicity, we use successive oligopoly and uniform input price to model vertical relation. Alternatively, the vertical relation can be modeled as vertical hierarchies with exclusive dealing, as well as with both inter- and intra-brand competition. The uniform input price can be also replaced by two-part tariffs or ad-valore pricing. Intuitively, the different setups might strengthen (or weaken) our results if it strengthens (or mitigates) double markups. For example, exclusive dealing tends to weaken horizontal competition and strengthen double marginalization. Then, the upstream collusion is more likely to be unprofitable. As compared to uniform pricing, ad-valore pricing tends to mitigate double markups (Shy and Wang, 2011; Gu et al., 2022). Consequently, the upstream collusion is more likely to be profitable and non-monotonic relationship will be less likely to exist. These alternative setups deserve thorough investigations in the near future.

Appendix

Appendix A. Proofs of main results

Proof of Proposition 1. Taking derivative of $\underline{\delta}$ with respect to m, we have

$$\frac{\partial \underline{\delta}^w}{\partial m} = \frac{\frac{\partial \Pi^*}{\partial m}\left(\widehat{\Pi}_u^w - \Pi_u^*\right) + \left(\frac{\partial \widehat{\Pi}_u^w}{\partial m} - \frac{\partial \Pi_u^*}{\partial m}\right)\left(\Pi_u^w - \Pi_u^*\right)}{\left(\widehat{\Pi}_u^w - \Pi_u^*\right)^2}.$$

As $\frac{\partial \Pi_u^*}{\partial m} = \frac{m-1}{m}\frac{t'(Q^o)(Q^o)^2}{m(\rho+m+1)} < 0$, $\widehat{\Pi}_u^w - \Pi_u^* > 0$, $\frac{\partial \widehat{\Pi}_u^w}{\partial m} = \frac{1-ms}{m}t'(\widehat{Q}^w)(\widehat{Q}^w)^2 > 0$, and $\Pi_u^w - \Pi_u^* > 0$, where $s = \frac{\widehat{q}_u^w}{\widehat{Q}^w} > \frac{1}{m}$, in numerator, the first term is negative, and the second term is positive.

When m is sufficiently large, Π_u^* approaches zero so an additional upstream firm makes little effect on total profit, i.e., $\frac{\partial \Pi_u^*}{\partial m} \to 0$. But both $\frac{\partial \widehat{\Pi}_u^w}{\partial m} - \frac{\partial \Pi_u^*}{\partial m}$ and $\Pi_u^w - \Pi_u^*$ are strictly positive. Thus, more upstream firm makes upstream collusion less stable when m is sufficiently large.

When m is sufficiently small, i.e. $m \to \underline{m}(B)$, we have $\underline{\delta}^w|_{m \to \underline{m}(B)} = 1$ as the profitability condition also implies $\Pi_u^w = \Pi_u^*$. When m is slightly above $\underline{m}(B)$ (e.g. $\underline{m}(B) + \varepsilon$ with $\varepsilon > 0$), collusion becomes profitable, i.e. $\Pi_u^w > \Pi_u^*$, and thus $\underline{\delta}^w|_{m=\underline{m}(B)+\varepsilon} < 1 = \underline{\delta}^w|_{m \to \underline{m}(B)}$. Therefore, there exists a non-empty region for m in which $\frac{\partial \underline{\delta}^w}{\partial m} < 0$ when m is small.

Proof of Lemma 2. As $\pi_u^w(Q^c) = \pi_u^o(Q^c) - \frac{1}{m}[\Pi_d(Q^*) - \Pi_d(Q^c)]$, $\widehat{\pi}_u^w(Q^c) = \widehat{\pi}_u^o(Q^c)$, $\underline{\delta}(Q^c) = \frac{\widehat{\pi}_u^w(Q^c) - \pi_u^w(Q^c)}{\widehat{\pi}_u^w(Q^c) - \pi_u^*} = \frac{\widehat{\pi}_u^w(Q^c) - \pi_u^o(Q^c) + \frac{1}{m}[\Pi_d(Q^*) - \Pi_d(Q^c)]}{\widehat{\pi}_u^w(Q^c) - \pi_u^*} > \frac{\widehat{\pi}_u^w(Q^c) - \pi_u^o(Q^c)}{\widehat{\pi}_u^w(Q^c) - \pi_u^*} = \frac{\widehat{\pi}_u^o(Q^c) - \pi_u^o(Q^c)}{\widehat{\pi}_u^o(Q^c) - \pi_u^*(Q^c)} = \underline{\delta}^o(Q^c)$.

Proof of Lemma 3. For any given $Q^c \in [Q^o, Q^*)$, as Π_u^* is independent of Q^c, we can show

$$\underline{\delta}^o(Q^c) = \frac{\widehat{\Pi}_u^o(Q^c) - \Pi_u^o(Q^c)}{\widehat{\Pi}_u^o(Q^c) - \Pi_u^*} = 1 - \frac{\Pi_u^o(Q^c) - \Pi_u^*}{\widehat{\Pi}_u^o(Q^c) - \Pi_u^*} = 1 - \int_{Q^*}^{Q^c} \overline{\delta}\frac{\frac{\partial \widehat{\Pi}_u^o}{\partial Q}}{\int_{Q^*}^{Q^c}\frac{\partial \widehat{\Pi}_u^o}{\partial Q}dQ}dQ,$$

where $\overline{\delta} = \frac{\frac{\partial \Pi_u^o}{\partial Q}}{\frac{\partial \widehat{\Pi}_u^o}{\partial Q}}$. Therefore, provided $\overline{\delta}$ is increasing in Q, $\underline{\delta}^o$ must be decreasing.

Now, we need to show $\overline{\delta} = \frac{\frac{\partial \Pi_u^o}{\partial Q}}{\frac{\partial \widehat{\Pi}_u^o}{\partial Q}}$ is increasing in Q.

Because $\Pi_u^o(Q) = (t(Q) - k)Q$, we have $\frac{\partial \Pi_u^o}{\partial Q} = \frac{\rho+n+2}{n}P'Q + P - c - k < 0$ and $\frac{\partial^2 \Pi_u^o}{\partial Q^2} = \frac{(\rho+2)(\rho+n+1)}{n}P' < 0$. The first sign is because $Q \geq Q^o$, and so increasing Q reduces $\Pi_u^o(Q)$.

On the other hand, $\widehat{\Pi}_u^o(Q) = m(t(\widehat{Q}) - k)\widehat{q}$, where $\widehat{Q} = \frac{m-1}{m}Q + \widehat{q}$, and $\widehat{q}$ is chosen to maximize the deviated firm's profit. From the FOC, we have $\frac{\partial \widehat{Q}}{\partial Q} = \frac{m-1}{m}\frac{1}{\rho s+2}$, where $s = \frac{\widehat{q}}{\widehat{Q}} > \frac{1}{m}$. Given demand concavity ρ, we can show that $\frac{\partial \widehat{\Pi}_u^o}{\partial Q} = \frac{(m-1)(\rho+n+1)}{n}P'\widehat{q} < 0$ and $\frac{\partial^2 \widehat{\Pi}_u^o}{\partial Q^2} = -\frac{(m-1)^2(\rho+n+1)}{nm(\rho s+2)}P' > 0$.

Obviously, taking derivative of $\bar{\delta}$ with respect to Q yields $\frac{\partial \bar{\delta}}{\partial Q} = \frac{\frac{\partial^2 \Pi_u^o}{\partial Q^2}\frac{\partial \widehat{\Pi}_u^o}{\partial Q} - \frac{\partial^2 \widehat{\Pi}_u^o}{\partial Q^2}\frac{\partial \widehat{\Pi}_u^o}{\partial Q}}{\left(\frac{\partial \widehat{\Pi}_u^o}{\partial Q}\right)^2} > 0$.

Appendix B. Result for Bertrand competition

B.1. Upstream collusion with compensation

When upstream firms collude with compensation constraint, upstream firm j's collusive profit is $\pi_{u,j}^w = (t^w - \kappa_j)s_j Q(t^w) - \tau_j$ and profit from defection is $\widehat{\pi}_{u,j}^w = (t^w - \kappa_j)Q(t^w)$. The IC constraint is $\frac{\delta}{1-\delta}(\pi_{u,j}^w - \pi_{u,j}^*) \geq \widehat{\pi}_{u,j}^w - \pi_{u,j}^w$. Dividing both sides by $(t^w - \kappa_j)Q(t^w)$ we have

$$\frac{\delta}{1-\delta}\left[s_j - \frac{\tau_j}{(t^w-\kappa_j)Q(t^w)} - \frac{\pi_{u,j}^*}{(t^w-\kappa_j)Q(t^w)}\right] \geq (1-s_j) + \frac{\tau_j}{(t^w-\kappa_j)Q(t^w)}$$

Summing up all firms' IC constraints and eliminating s_j result in

$$\frac{\delta}{1-\delta}\left[1 - \sum_{j=1}^{m}\frac{\tau_j}{(t^w-\kappa_j)Q(t^w)} - \frac{\pi_{u,1}^*}{(t^w-\kappa_1)Q(t^w)}\right] \geq (m-1) + \sum_{j=1}^{m}\frac{\tau_j}{(t^w-\kappa_j)Q(t^w)}.$$

Hence, collusion can be sustained if $\delta \geq \frac{m-1+\sum_{j=1}^{m}\frac{r_j\tau}{(t^w-\kappa_j)Q(t^w)}}{m-\frac{\pi_{u,1}^*}{(t^w-\kappa_1)Q(t^w)}}$.

Since r_j only appears in the numerator and $\kappa_1 \leq \kappa_2 \leq ... \leq \kappa_m$, minimizing the threshold discount factor requires $r_1 = ... = r_{m_e} = \frac{1}{m_e}$ and $r_j = 0$ for $j > m_e$, and so we have the threshold discount factor as given in (10). By substituting r_j into every firm's IC constraint, we have the optimal market share as shown in (11).

B.2. Upstream collusion without compensation

When no compensation is needed, the collusive profit of upstream firm j is $\pi_{u,j}^o = (t^o - \kappa_j)s_j Q(t^o)$. The profit from defection for firm j is $\widehat{\pi}_{u,j}^o = (t^o - \kappa_j)Q(t^o)$. The collusive input price is determined by:

$$t^o = \arg\max_t \sum_{j=1}^{m} (t - \kappa_j) s_j Q(t).$$

The IC constraint is $\frac{\delta}{1-\delta}(\pi^o_{u,j} - \pi^*_{u,j}) \geq \widehat{\pi}^o_{u,j} - \pi^o_{u,j}$. Summing up all firms' IC constraints and eliminating s_j produce

$$\frac{\delta}{1-\delta}\left[1 - \frac{\pi^*_{u,1}}{(t^o - \kappa_j)Q(t^o)}\right] \geq (m-1).$$

Then, the threshold discount factor is $\underline{\delta}^o = \frac{m-1}{m - \frac{\pi^*_{u,1}}{(t^o-\kappa_1)Q(t^o)}}$. The optimal collusive scheme is

$$s_j = \begin{cases} \frac{1-(m-m_e)(1-\underline{\delta}^o)}{m_e}, & \forall\ j \text{ with } \ \kappa_j = \kappa_1; \\ 1 - \underline{\delta}^o, & \forall\ j \text{ with } \ \kappa_j > \kappa_1. \end{cases}$$

B.3. Calculation example

Consider a market with two downstream firms. The downstream firms compete in price and have a demand function as follows: $q_i = 10 - p_i + 0.3\,(p_h - p_i)$, for $i, h \in \{1, 2\}$ and $i \neq h$.

First, let's consider a case of two upstream firms with different marginal costs of $\kappa_1 = 0$ and $\kappa_2 = 1$. Then, in competition equilibrium, prices, output, and industrial joint profit are

$$t^{2*} = 1; \quad p^{2*} = 4.913; \quad Q^{2*} = 10.17913; \quad \Pi^{2*} = 49.985.$$

To solve the two upstream firms collusive equilibrium, we first solve the optimization problem in (9), which results in

$$3 - 2.6t + 2.3s_2 = 0. \tag{14}$$

Assume the compensation is large enough to ensure $B = \Pi^{2*}_d$. In this case, firm 1 provides the compensation. When firm 2's IC constraint holds with equality, we have $\underline{\delta}^{2w} = 1 - s_2$. When $\delta = \underline{\delta}^{2w} = 1 - s_2$, firm 1's IC constraint is satisfied with equality, which gives

$$(4.6s_1 - 2.3)t(10 - t) - (19 - t)(t - 1) = 20.7s_1. \tag{15}$$

Solving the two-equation system (14) and (15) we have

$$t^{2w} = 1.15488; \quad s_1^{2w} = 0.99883; \quad s_2^{2w} = 0.00117.$$

The threshold discount factor is $\underline{\delta}^{2w} = 0.99883$, and joint profits becomes $\Pi^{2w} = 49.988$.

Second, let's consider a case with three upstream firms. The additional firm, firm $1'$ is identical to firm 1, i.e., $\kappa_{1'} = 0$. Then, the competition equilibrium outcomes are

$$t^{3*} = 0; \quad p^{3*} = 4.3478261; \quad Q^{3*} = 11.304348; \quad \Pi^{3*} = 49.149.$$

In three upstream firms collusive equilibrium, we should have $r_1 = r_{1'}$ and $s_1 = s_{1'}$. Solving for the equilibrium, we have

$$t^{3w} = 1.17523; \quad s_1^{3w} = s_{1'}^{3w} = 0.4879112; \quad s_2^{3w} = 0.0241776.$$

Industrial joint profits becomes $\Pi^{3w} = 49.997$. The threshold discount factor is $\underline{\delta}^{3w} = 0.9758224 < \underline{\delta}^{2w}$, indicating a more efficient upstream firm raises the likelihood of collusion.

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