Voluntary Certification
A Multi Stage Analysis

Preliminary and Incomplete Draft

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March 14, 2024

Abstract

Voluntary certification serves as a strategic tool for firms in markets characterized by asymmetric information regarding quality. This paper investigates the timing of certification decisions, considering how heterogeneous firm types strategically choose when to certify. I complement existing models by introducing a temporal dimension, where firms decide whether to certify in the first period, second period, or abstain entirely. I incorporate multiple firm types (low, middle, high) and two time periods. I find a partial-pooling equilibrium, where high and middle types certify in the same period, and furthermore a separating equilibrium, where different types adopt distinct certification strategies across periods.

Keywords: Market Transparency, Certification, Information and Product Quality, Asymmetric Information, Repeated Games
JEL classification: L15, D82, C73, G24

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1 Introduction

Voluntary certification can be used by firms to improve their reputation when the true quality is unobservable. In a market, where sellers have private information about the quality of their product but cannot credibly disclose this information on their own, there is demand for a third party that reveals the true quality through a certificate. The availability of certificates modifies market beliefs about uncertified firms. As certificates act as a form of signalling device in this context, both the presence and absence of a certificate can be informative for the consumer. Since information about quality is only provided by the certifier if the firm invests in the certificate, the expected quality of the firm's product depends on the market's expectation of when certain types of a firm should certify. Certification can operate as a threat by forcing firms to pay high certification fees to avoid the penalty the market applies to uncertified firms. On the other hand, a certification standard in an industry can reduce the effects of asymmetric information and moral hazard in a market with unobservable quality.

A firm can not only decide whether to buy a certificate at all but also when to certify. If certification is voluntary, the availability of a certification does not necessarily imply that every firm purchases the certificate. While some firms will certify right away others wait before applying for a certification. Good performance by firms, along with sectoral and regional characteristics, can have an influence on when a firm decides to get certified[1].

My model is adapted from the static seller-certification model in the analysis of seller versus buyer certification from Stahl and Strausz (2017). I extend the model by adding a second period and an additional type of firm. The quality of the firm, and thereby its product, is given exogenously. I focus only on the timing of the certification and the certification fee that is chosen by the certifier. There is no public signal about firm quality. The certifier is an external party who credibly certify the quality of the firm for a fee. Product quality becomes public information at the time of the certification. There are two periods and the firm can decide whether to certify in the first period, in the second period, or never. By considering multiple periods and allowing firms to choose when to certify, my model captures the interplay between heterogeneous properties of a firm in the dimensions of quality and production costs with the cost of buying a certificate from a third party player and how this transfers to the timing of certification. Alongside identifying the equilibria of the game, I am particularly interested in time-dependent separating equilibria in which different firm types decide to certify in different periods. I find that if players are patient enough and production induces mid-ranged costs for the

[1] DeCanio and Watkins (1998) find that firm-specific variables such as firm size, earnings, and insider shareholding are significant determinants of U.S. firms' participation in voluntary pollution prevention programs. Similarly, Nakamura, Takahashi, and Vertinsky (2001) state that Japanese firms were originally reluctant in adopting ISO certification standards in quality introduced by the European Union. They find that Japanese certification rates of ISO 14001 are found to be significantly affected by firm size, the average age of firm employees, export ratio, and debt ratio.

high and middle type of the firm, in the pure strategy equilibrium the high type certifies the product in the first period, the middle type certifies the product in the second period and the low type never certifies the product.

Related Literature

The utilization of voluntary certification by firms to enhance their reputation in situations where true quality is unobservable has drawn significant attention in economic literature. Certification serves as a mechanism through which firms signal their commitment to quality, influencing market perceptions and behavior.

My paper is adapted from the seller-certification specification of the model by Stahl and Strausz (2017). In their model firms face a trade-off between revealing their true quality and maximizing short-term profit. It is a static framework and firms are categorized as high or low quality, with certification enabling high-quality firms to demand higher prices. The high quality firm faces production cost whereas a low quality firm produces for free. The true quality of the firm is unknown to the buyer who has a belief about the firm's type. In the case of seller-certification, the firm decides whether to sell the product, at which price to sell the product and whether to buy a certificate from a third party that will truthfully reveal the firm's quality. If the firm owns a certification for high quality it thereby is able to demand a higher price.

Building upon this framework, I extend the analysis to incorporate the timing of certification decisions, introducing a dynamic element to the strategic interaction between firms and consumers. The main trade off the firm in the static seller-certification model by Stahl and Strausz (2017) is that high type wants to set itself apart from low type and set a higher price. The authors find that in the seller certification equilibrium the high type firm will certify if the certification fee is not too high and the low type firm does never certify. They disregard long-term effects of certification. In my model, a firm that certifies in the first period it also is certified in the second period. This is an additional trade-off factor: if the firm certifies in the first period it also has to bear the cost in the first period. On the other hand, if the firm is interested in the additional profit from certification but does not want to bear the cost in this period it asks a smaller price in the current period, but still wants to be recognized as the high type and ask for a higher price in the later period.

To catch the dynamics I am interested in, it is not sufficient to only extend the model by adding a period. Therefore, I also add an additional type of firm (low, middle, high). My main contribution to the literature is the analysis of equilibria in which different types of the firm certify in different periods. I find different types of the equilibria: a partial-pooling equilibrium in which certain types of the firm certify in the same period and others never certify. This replicates the results of Stahl and Strausz (2017) in my dynamic setting with more types. More interestingly, I also find a separating equilibrium in which the different types all choose different strategies in different periods in equilibrium. This is a new result and enriches the base model.

The model introduced in my paper builds on a strand of literature that analyzes markets in which a certification intermediary can reveal otherwise private information to the buyers. Albano and Lizzeri (2001) find that a certification intermediary improves information about quality and increases incentives to provide high quality in an environment with severe information asymmetries. Efficiency is increased but quality is still underprovided in equilibrium relative to full information. Dranove and Jin (2010) provide an overview over theoretical and empirical literature on quality disclosure and certification with a focus on quality assurance mechanisms, the accuracy of certificates and voluntary versus mandatory seller disclosure. They review empirical findings regarding quality measurement, the effect of certifiers on consumer choice and seller behavior, as well as the economics of certifiers.

Both sellers and buyers may demand certification from a third party to raise market transparency. Stahl and Strausz (2017) compare seller and buyer certification regarding its effect on market transparency or effort. They find that seller certification is more effective in raising transparency than buyer certification.

Certifiers may not only strategically choose their certification fee but also the information they disclose. Lizzeri (1999) discusses strategic manipulation of information gathered from privately informed parties by a certification intermediary. It can be optimal for a monopoly certifier to reveal only whether quality is above a minimal standard. Information may be fully revealed if their is competition among certifiers. Hvide (2009) develops a simple theory of segmentation and fee-setting in certification markets. Given the test standards, certifiers compete for customers via their fee-setting. In equilibrium sellers with low unobservable quality self-select to an easy test and pay a lower endogenous certification fee and sellers with high unobservable quality self-select to a hard test and pay a higher fee. Verrecchia (1983) analyses a manager's decision to disclose or withhold information regarding a risky asset. Strausz (2005) derives conditions under which reputation enables certifiers to resist capture and maintain their honesty in a dynamic model with monopolistic producers and a, not necessarily honest, certification intermediaries. Honest certification requires high prices that may even exceed the static monopoly price and exhibits features of a natural monopoly. Similarly, incentives to manipulate the certification process are studied by Mathis, McAndrews, and Rochet (2009), or, in the context of rating agencies, for example by Faure-Grimaud, Peyrache, and Quesada (2009), Bar-Isaac and Shapiro (2011), Bolton, Freixas, and Shapiro (2012), C. Opp, M. Opp, and Harris (2013) and Skreta and Veldkamp (2009). Moral hazard problems of certification are furthermore considered by Ozerturk (2014) and Bizzotto and Vigier (2021).

Miao (2009) studies certification in markets with adverse selection and finds that competition among certifiers can increase information provided through quality ratings but is not always welfare improving. Farhi, Lerner, and Tirole (2013) analyze why strategic product quality certifiers may not reveal the identity of sellers who unsuccessful apply for a certification. Mandating transparency benefits the sellers but buyers become less informed about product quality. Pollrich and Strausz (2024) show that restricting certifiers' fee structures is irrelevant for maximizing their profits and trade efficiency. Restrictions in the fee structure can be accounted for by adjusting the disclosure rule. This does affect market transparency but not economic efficiency or rent distributions. Further research that focus on the market transparency effect of certification was conducted by Pollrich and Wagner (2016), Harbaugh and Rasmusen (2018), and Kattwinkel and Knoepfle (2023).

Results related to my paper are also presented in the analysis of certification in dynamic models. In a dynamic model with fixed quality Van Der Schaar and Zhang (2015) consider markets where learning occurs simultaneously through reputation and certification. They find that market learning through reputation alone can be inefficient and that certification allows good agents to compensate bad signals. Certification and reputational learning can act as complementary forces.

Several authors furthermore study the influence of endogenous quality. Marinovic, Skrzypacz, and Varas (2018) study firms' incentives to build and maintain reputation for quality, when endogenous quality is persistent and can be certified at a cost. The certifier is not modeled as an additional player. The optimal equilibria allow firms to maintain high quality forever, once it is reached for the first time. Bizzotto and Harstad (2023) study the optimal and the equilibrium certifier from the long-run perspective. Firms enter the market and can invest to improve product quality. The socially optimal certifier identity is determined by the importance of externalities, firm's investment and entry.

Furthermore, my paper contributes to a strand of literature that seeks to reveal conditions under which sellers disclose information about their product depending on heterogeneous characteristics. More general setups on the revelation of verifiable information have been studied previously, for example by Grossman (1981), Milgrom (1981), Grossman and Hart (1980) and Jovanovic (1982).

As certification (or the absence of a certificate) acts as a quality signal, my research is as well related to the literature on signaling and, in particular, on signaling of unobservable quality through prices Wolinsky (1983).

Lastly, there have been several empirical studys on certification in different industries. Fore example, Jin (2005) in an empirical study finds that health maintenance organizations (HMOs) use voluntary disclosure to differentiate from competitors. She shows that early disclosers are more likely to operate in highly competitive markets but the average disclosure rate tends to be lower in such markets. Xiao (2010) empirically tests the value of certification for childcare centers and suggests that endogenous accreditation choices by firms can impact the effectiveness of certification. Beaver, Shakespeare, and Soliman (2006) examine how ratings from from certified bond-rating agencies differ from those of non-certified agencies. Houde (2022) provides insights on who should pay for voluntary environmental certification programs by conducting a case study. The author shows that firms are highly strategic with respect to this certification and extract consumer surplus associated with certified products.

The remainder of this paper is organized as follows. In Section 2, the model is introduced and I solve a benchmark case without certification. Section 3 derives the equilibrium conditions and analytic results for the certification model. Section 4 concludes. All proofs and further auxiliary lemmas are relegated to the Appendix.

2 The Model

I analyze a two period game between a long-lived firm that sells one product of a certain quality, a long-lived certifier and a myopic consumer in each period. All agents in the game know if the current period is the first or the second period. The firm's type is $i \in \{H, M, L\}$ and determined exogenously before the beginning of the game. The type is private information of the firm. With probability λ_i the firm's type is i and $\lambda_H + \lambda_M + \lambda_L = 1$. A firm of type i can produce a product of quality q_i with production costs c_i. I assume $q_H > q_M > q_L$ and $c_H \geq c_M \geq c_L$. The firm can decide whether to pay the certifier to certify the product. In that case, the certifier learns the true type of the firm and reveals the true quality of the product to the consumer. In each period, the firm can decide whether to sell the product to a myopic consumer and for which price. Furthermore, it is profitable for the firm type i with quality q_i to sell the uncertified product for price q_i, i.e. $q_i \geq c_i$ for $i \in \{H, M, L\}$. The certifier ex ante does not know the true type of the firm and sets a fee that has to be payed by the firm to the certifier to reveal the true quality to the consumer. The myopic consumer knows the price of the product in the current period and the probability with which the firm's type is i.

The consumer's willingness to pay is equal to the expected quality of the product. If the firm decided to buy a certificate, the consumer learns not only the price of the product but also that the product is certified and the true quality that is revealed truthfully by the certifier in the period of the certification and in the following period if there is one.

The firm and the certifier discount future profits with discount factor $\delta \in (0, 1)$. The firm maximizes its profit. It faces a trade-off between selling the good for a price that is smaller or equal to the expected quality or paying for the costly certification and being able to sell the good for a price equal to the certified (true) quality. Furthermore, the firm can decide between certifying in the first period and being able to sell the good for the higher price in both periods. Or the firm can postpone the certification and the induced cost to the next period but has to sell the good for the cheaper price in the current period.

The certifier chooses the certification fee for the firm to maximize expected profit.

First, I consider a benchmark without the certifying party. The firm cannot decide whether or not to pay a certifier to reveal the true quality of its product to the consumer. But if the firm sells the product in the first period, the consumer from the first period reveals the true quality of the product to the potential consumer in the following period.

Benchmark Model: Customer Testimonial

If the consumer in the first period buys the product, the quality of the product is revealed to the consumer in the second period[2]. The firm discounts future profits with discount factor $\delta \in (0, 1)$. Before the beginning of the game, the type of firm is drawn by nature. With probability λ_i the type is $i = H, M, L$ and $\lambda_H + \lambda_M + \lambda_L = 1$. The type is private information to the firm. In the first period, the firm learns its type and decides whether to sell the product and sets the price if it decides to sell. The consumer observes the price and decides whether to buy the product.

In the second period, the firm again sets the price. The consumer observes the price and learns if the product was bought in the previous period. If the product was bought in the first period, the consumer learns the true type of the firm. The consumer decides whether to buy the product.

In equilibrium, if the production cost is sufficiently small, all types of the firm offer the product in the first period in equilibrium. In the second period, the consumer then learns about the quality of the product before deciding whether to buy the product. If the production cost is too high, in equilibrium, only the types of the firm for which the production costs are still sufficiently small sell the product in the first period. The following proposition sums up the necessary conditions for the different types of the equilibrium.

Proposition 1. *There are three different (not necessarily unique) equilibria:*

1. *Separating Equilibrium Low Quality: If $c_H > \frac{q_L + \delta q_H}{1+\delta}$ and $c_M > \frac{q_L + \delta q_M}{1+\delta}$, in equilibrium, type L sells the product for price q_L in both periods and no other type sells the product.*

2. *Partial-Pooling Equilibrium: If $c_H > \frac{\lambda_L q_L + \lambda_M q_M + q_H}{1+\delta}$ and $c_i \leq \frac{\lambda_L q_L + \lambda_M q_M + \delta q_i}{1+\delta}$ for $i = M, L$, in equilibrium, type L and M sell for the expected quality $\lambda_L q_L + \lambda_M q_M$ in the first period and for the testified quality respectively in the second period while H does not sell the product.*

3. *Pooling Equilibrium: If $c_i \leq \frac{\lambda_H q_H + \lambda_M q_M + \lambda_L q_L + \delta q_i}{1+\delta}$ for $i = H, M, L$, in equilibrium, all three types of the firm sell the product for the expected quality $\lambda_L q_L + \lambda_M q_M + \lambda_H q_H$*

[2] The consumer in the first period does not have to reveal the true quality. As long as the consumer in the second period believes that the quality that is revealed to him is the true quality, the results hold.

in the first period, and all types sell the good for the testified quality in the second period.

The good will be traded in both periods.

The formal proof is given in appendix A.

Depending on the production costs of the different types of the firm, different types of the firm sell the product in equilibrium. If the production costs of type H and M are too high, they will not sell the product for the price associated with the expected quality in the first period. However, if the production costs are sufficiently small, the higher types will sell the product in the first period and benefit from the higher price of a certified product in the second period. [3]

Note that by using the terms partial-pooling and pooling equilibrium I refer to the property of the equilibria that several types make the same decision reagrding selling the product (in the benchmark model) or selling the product (un-)certified (in the certification model). Still, the price for which the product is sold, in the benchmark model, specifically, in the second period, can be different for different types of the firm.

The next section introduces a model in which the consumer in one period cannot reveal the quality of the product to the consumer in the next period. Instead, a certifier offers to reveal the true quality of a product to the consumer for a fee that the firm has to pay.

3 Certification Model

The firm's type is private information of the firm. The firm can pay a fee to a certifier to certify the quality of the good. In this case, the certifier learns the true type of the firm and reveals the true quality of the product to the consumer. In each period, the myopic consumer learns the price of the good, and if the firm is certified also the quality of the good. In equilibrium, if the consumer is offered an uncertified good, he will update his belief about the expected quality of the good accordingly. For example, if in equilibrium only a firm of type H sells the good certified in the first period, a consumer who is offered an uncertified good in the first period expects that the expected quality of the good is given by $q_1^e = \lambda_L q_L + \lambda_M q_M$. If no firm certifies in equilibrium, the consumer believes that the expected quality of the good is given by $\tilde{q}_1^e = \lambda_L q_L + \lambda_M q_M + \lambda_H q_H$. The assumptions from section 2 remain.

[3] For some intuition on the beliefs of the consumer note that if the consumers expect that a product of quality q_L or q_M is offered in the first period at price q^e but the consumer in the second period did not learn about the product's quality, the consumer concludes that the good was not offered in the first period. Therefore the consumer expects that the quality of the product is not q_M or q_L but q_H. But this would be an incentive for firm types L and M to not offer the product in the first period and try to sell the product in the second period for a price of q_H. Therefore, the consumer should expect that the quality if an unknown good in the second period is q^e as well.

The timing of the game is as follows:Iin the first period the certifier sets the certification fee. The firm learns about its own type and decides whether to certify its good or not. If the firm wants to sell the product it sets the price of the good. Afterwards, the consumer observes the price of the product, if the good is for sale, and learns if the good is certified. In that case he also learns the true quality of the good. The consumer decides whether to buy the product.

In the second period the firm can decide again whether to certify its product, if it did not already buy a certificate in the first period. The firm decides whether to sell the product and sets the price of the good. The consumer observes the price of the product, if the good is for sale. Again, if the good is certified the consumer also learns the true quality of the good. The consumer decides whether to buy the product.

The analysis of the game proceeds as follows. Using backwards induction, I start with the decision of the consumer. Note that in any period a consumer buys the product if the expected quality of the product is greater than or equal to the price of the product. Consequently, the firm chooses the highest possible price for which the consumer still buys the product. If the consumer learns the true quality of the product, a firm which sells a product of quality q chooses the price q. If the consumer does not know the true quality of the product, the firm sells the product for a price that is equal to the expected quality in that period if the good is traded. The following lemma summarizes these statements.

Lemma 1. *Consider a firm that produces a product of quality q. If in some period the firm sells a certified good, the good is traded for a price equal to q. If the firm sells the good uncertified, the good is traded for a price equal to the expected quality.*

Next, I analyze the certification decisions of the different types of the firm.

Note first that a firm of type L will never certify its product. In any period the L's profit from selling the good uncertified and not having to pay the certification fee will always be at least as high as the profit from selling the good uncertified.

Lemma 2. *It is never profitable for a firm of type L to certify its product.*

The formal proof is given in the appendix A.

Consequently, it is sufficient to analyze when it is profitable for type H and M to certify their product. Next, conditions are derived under which the different types of the firm certify the product. A firm certifies the product if the profit from selling the good certified (and being able to charge a higher price than uncertified but having to bear the certification cost) is higher than the profit from selling the good uncertified.

Assume the certification fee is given by $f > 0$. A firm of type q with production cost c at least weakly prefers certifying the good in the first period over certifying the good in the second period if the profit from selling the certified good in both periods is greater or equal to selling the uncertified good for a price equal to the expected quality q_1^e in the first period and selling the certified good for a price equal to quality q in the second period.

$$\begin{aligned} (q-c)(1+\delta) - f &\geq q_1^e + \delta q - c(1+\delta) - \delta f \\ \frac{q-q_1^e}{1-\delta} &\geq f. \end{aligned} \tag{1}$$

A firm of type q with production cost c at least weakly prefers certifying the good in the first period over selling the good uncertified in both periods if the profit from selling the certified good in both periods is greater or equal to selling the uncertified good for a price equal to the expected quality q_1^e in the first period and q_2^e in the second period.

$$\begin{aligned} (q-c)(1+\delta) - f &\geq q_1^e + \delta q_2^e - c(1+\delta) \\ q - q_1^e + \delta(q - q_2^e) &\geq f. \end{aligned} \tag{2}$$

A firm of type q with production cost c at least weakly prefers selling the good uncertified in the first period and selling the certified good in the second period over selling the good uncertified in both periods if the profit from selling the certified good in both periods is greater or equal to selling the uncertified good for a price equal to the expected quality q_1^e in the first period and q_2^e in the second period.

$$\begin{aligned} q_1^e + \delta q - (1+\delta)c - \delta f &\geq q_1^e + \delta q_2^e - c(1+\delta) \\ q - q_2^e &\geq f. \end{aligned} \tag{3}$$

Furthermore, a firm sells the product if the profit is greater or equal to zero. First of all, note that if in equilibrium a firm of type M certifies in any of the two periods, type H will not sell the good uncertified.

Lemma 3. *It is never profitable for a firm of type H to sell the good uncertified if type M certifies the good in any of the two periods.*

The formal proof is given in appendix A.

Depending on the discount factor $\delta \in (0,1)$, different strategies are optimal for the different types of the firms.

There are different types of equilibrium. In a separating equilibrium no two types of the firm choose the same period in which they buy the certificate. And in a partial-pooling equilibrium, type H and M choose the same period in which they buy their certificate. In a pooling equilibrium, all firm types make the same certification decision. Note that because of lemma 2, there is no pure pooling equilibrium in which all firm types certify in the same period.

In equilibrium, the certifier chooses the certification fee that maximizes his expected profit. Depending on the level of the fee, different types of the firm prefer to certify in the first or in the second period of the game. For example, lemma 7 (derived in appendix A.1) shows that if H and M certify in the first period and L never certifies and the certification fee is given by $f = (q_M - q_L)(1 + \delta)$, the expected profit of the certifier is given by

$$\Pi^C = (\lambda_H + \lambda_M)(q_M - q_L)(1 + \delta) \tag{4}$$

because with probability λ_M a type M firm is drawn and with probability λ_H a type H firm is drawn. Both of these types certify in the first period and pay the certification fee f. With probability λ_L a type L firm is drawn which never certifies.

Several auxiliary results are derived in appendix A.1. Lemma 6 lists conditions under which firm type H prefers to certify in the first period and firm type M certifies in the second period. Lemma 7 states conditions such that types H and M of the firm both prefer certifying in the first period. In lemma 8 conditions are given such that firm types H and M certify in the second period. Lemma 9 and lemma 10 explore under which conditions only firm H certifies. Lemma 11 gives an overview over the dominated strategies of the certifier. Summarizing those findings and solving for the respective certification fees that maximizes the expected profit of the certifier yields the following propositions.

If the production costs of H and M are sufficiently small and the discount factor is suitable, in the unique equilibrium in pure strategies, both firm types H and M certify the product right away, accepting the certification fee, and take advantage of the higher price they can ask for their product. The equilibrium certification fee reflects the quality difference between type M and type L.

Proposition 2. *Denote $q_1^e = \lambda_L q_L + \lambda_M q_M$ and $\tilde{q_1^e} = \lambda_H q_H + q_1^e$. Let $\tilde{q_1^e} < q_H - q_M + q_L$. If $\delta \geq \frac{q_M - q_H - q_L + q_1^e}{q_M - q_L}$, $q_H - (q_M - q_L) \geq c_H$ and $q_L \geq c_M$ then the equilibrium certification fee is given by $f = (q_M - q_L)(1+\delta)$. The unique equilibrium is the partial-pooling equilibrium, H and M certify in the first period and L sells uncertified. The good is traded in both periods.*

The formal proof is given in appendix A.1.

If $\delta \geq \frac{q_M - q_H - q_L + q_1^e}{q_M - q_L}$ and the production costs of both type H and type M are sufficiently small, both types certify in the first period in the partial-pooling equilibrium. The intuition behind the certifier's choice of the fee is that the certifier extracts the additional profit that a higher type firm can make by buying the certificate to distinguish itself from a lower type firm. As both the high and the middle type are revealing their quality in the first period, the certification fee is influenced the quality difference between the lower of the two types, M, and type L who sells the good uncertified, and the discount factor. Both types value the higher price they can demand for their product by certifying it enough that they buy the certificate in the first period. The production costs are small enough that both firm types do not have an incentive to delay paying the certification fee to the second period.

If the production costs of either firm type H exceed $q_H - q_M + q_L$ or type M exceed q_L (or both) but are still sufficiently small, the different types of the firm certify in different periods in the separating equilibrium.

Proposition 3. *Recall* $q_1^e = \lambda_M q_M + \lambda_L q_L$ *and* $\tilde{q_1^e} = \lambda_H q_H + \lambda_M q_M + \lambda_L q_L$. *Let* $\tilde{q_1^e} < q_H - q_M + q_L$. *If* $\frac{q_1^e - q_L}{q_M - q_L} > \delta \geq \frac{q_M - q_L - q_H + q_1^e}{q_M - q_L}$, $\frac{q_H(1+\delta) - (q_M - q_L)}{1+\delta} \geq c_H$ *and* $\frac{q_1^e + \delta q_L}{1+\delta} \geq c_M$ *and* $c_H \geq q_H - q_M + q_L$ *or* $c_M \geq q_L$, *the unique equilibrium in pure strategies is a separating equilibrium in which the certification fee is* $f = q_M - q_L$, *type* H *certifies in the first period, type* M *certifies in the second period and type* L *never certifies. The good is traded in both periods.*

The proof is given in appendix A.1.

In this second case the production costs of firm type H and M are still bound by a threshold but are higher than in the first case for at least one of the types. Then, firm type M prefers to demand a price smaller than its true quality (i.e., equal to the expected quality) in the first period and only certifies in the second period. Thereby this type also delays paying the certification fee to the second period but consequently can also only demand the higher price that comes with certification in the second period. Accordingly, the certification fee is again influenced by the quality difference between the middle and the low type firm but is also smaller than in the first case where both types certify in the first period.

Figure 1 visualizes the two equilibrium regions for $\delta \in (\frac{q_M - q_L - q_H + q_1^e}{q_M - q_L}, \frac{q_1^e - q_L}{q_M - q_L}]$ in dependence of the cost c_M and c_H.

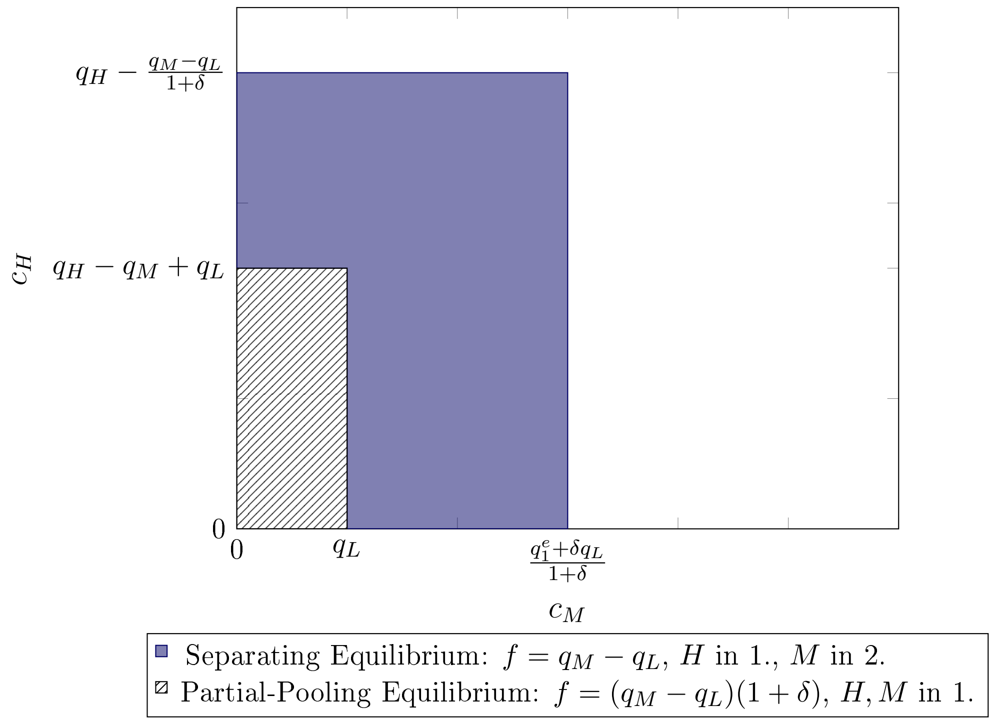

Figure 1: Equilibrium regions if $\frac{q_1^e - q_l}{q_M - q_L} > \delta \geq \frac{q_M - q_L - q_H + q_1^e}{q_M - q_L}$ depending on the cost c_M and c_H.

As illustrated in figure 1, the partial pooling equilibrium in which type H and M certify in the first period is the equilibrium if the production costs of both types are small. If the production is more costly for at least one of the firms (but not too costly), type H still certifies in the first period but M waits and certifies in the second period instead. Regarding this equilibrium region, note that the higher the discount factor δ, the greater the difference $q_H - \frac{q_M - q_L}{1+\delta}$ but also the smaller the difference between $\frac{q_1^e + \delta q_l}{1+\delta}$ and q_L. But the difference $(q_H - \frac{q_M - q_L}{1+\underline{\delta}}) - (q_H - \frac{q_M - q_L}{1+\delta})$ is greater than the difference $(\frac{q_1^e + \delta q_l}{1+\underline{\delta}} - q_L) - (\frac{q_1^e + \delta q_l}{1+\delta} - q_L)$ if $\underline{\delta} < \delta$. Consequently, the equilibrium region of the separating equilibrium in which each type of the firm certifies in a different period increases with higher values of the discount factor δ.

4 Conclusion

This paper presents a model of heterogeneous firm types that can decide whether to buy a costly certifcate from a third-party certifier to reveal the true quality of their product and about the timing of such a certification. I demonstrate that different types of the firm prefer to buy the certificate at different points in time. I find unique equilibria in pure strategies if the agents of the game are patient enough. The introduction of multiple firm types and a second period further allows for a more nuanced examination of certification equilibria. If there are only a high and a low type of the firm, the low type firm never buys the certificate. Therefore, the insights are limited to understanding the conditions that induce the high type to buy the certificate to differentiate itself from the low type. In addition to high and low-quality firms, the inclusion of middle-quality firms introduces partial-pooling and separating equilibria in the two-period game. Of particular interest are the findings regarding the separating equilibrium in the time dimension, wherein different types of firms adopt distinct certification strategies across periods. I thereby can better disentangle the effects of heterogeneous quality and production costs of the firm types on the certification decision.

If the production costs are sufficiently small, the higher type H and M of the firm certify the product as soon as possible to set themselves apart from the low quality type L and be able to charge a higher price. When the production costs increase but are not too high, the unique equilibrium is a separating equilibrium in which the high quality type certifies in the first period, the medium quality type certifies in the second period and the low type never certifies. My model thereby gives theoretical insights on how properties of a firm influence the decision when to buy certificate. It provides an explanation on why some firms wait to get certified but others immediately certify.

Compared to the benchmark model, where the firm cannot certify its product, the middle and high quality firm can set a higher price for their product in the certification model. Furthermore, the high and middle type firm are able to sell their product in the certification model even if the production costs are high enough that they would not sell the product in the benchmark model. Voluntary certification acts as a strategic signaling device through which the firm can communicate its quality to the consumer. Thereby, the availability of certification alters market perceptions of an uncertified firm, imposing an implicit penalty in terms of a lower price that consumers are willing to pay. Production costs and the quality difference between different types of the firm influence the level of the certification fee and the timing of certification decisions.

The introductory goal of this paper was to prove the existence of an equilibrium featuring separation along the time dimension. I characterize the necessary conditions for such an equilibrium, along with a partial pooling equilibrium. I illustrate the influence of heterogeneous quality and costs on the strategic timing of certification decisions by a firm. It remains to extend the results presented in this paper, building on my preliminary findings. Moving forward, my research aims to comprehensively characterize all remaining equilibria in my certification model to provide a more nuanced understanding and insights that can inform policy decisions and strategic planning in markets with certification.

A Appendix

Proof of Proposition 1. Recall that $c_i \leq q_i$ for $i = L, M, H$. In the third equilibrium case, the consumer expects that the quality of a product offered in the first period is $q_1^e = \lambda_L q_L + \lambda_M q_M + \lambda_L q_L$ and buys the product if the price is smaller or equal to q_1^e. It is not profitable for any type of of the firm to deviate to a higher price because the consumer will not buy the product in that case. If $c_i \leq \frac{\lambda_H q_H + \lambda_M q_M + \lambda_L q_L + \delta q}{1+\delta}$ for $i = H, M, L$ it is also not profitable for the firm to deviate to not selling the product and earning zero profit. It is not profitable for any type to deviate to a smaller price and earn less while paying the same costs.

In the second equilibrium case, the consumer expects that the quality of a product offered in the first period is $q_1^e = \lambda_L q_L + \lambda_M q_M$ and buys the product if the price is smaller or equal to q_1^e. It is not profitable for type M or L of of the firm to deviate to a higher price because the consumer will not buy the product in that case. If $c_i \leq \frac{\lambda_M q_M + \lambda_L q_L + \delta q}{1+\delta}$ for $i = M, L$ it is also not profitable for on of those two types of the firm to deviate to not selling the product and earning zero profit. It is not profitable for any type to deviate to a smaller price and earn less while paying the same costs. Lastly, because $c_H > \frac{\lambda_L q_L + \lambda_M q_M + \delta q_H}{1+\delta}$, it is not profitable for the type H firm to deviate from not selling the product to selling the product because the production costs would be higher than the revenue.

If the product was not offered in the first period or the consumer did not buy the product in the first period, the consumer in the second period cannot learn about the product's quality. □

Proof of Lemma 2. If firm L sells the product uncertified the profit is

$$q_1^e + \delta q_2^e - c_L(1+\delta).$$

Because q_L is the lowest possible quality the expected quality of the product in both periods is bound below by q_L. Consequently, type L's profit is greater or equal to

$$(1+\delta)(q_L - c_L). \tag{5}$$

If a certification induces costs $f > 0$, type L's profit from certifiying in the first period is given by

$$(1+\delta)(q_L - c_L) - f$$

which is smaller than (5). And type L's profit from certifiying in the second period is given by

$$q_1^e + \delta q_L - (1+\delta)c_L - \delta f$$

which is is smaller than (5) as well. □

Proof of Lemma 3. 1 Denote $q^e = \lambda_H q_H + \lambda_L q_L$ the expected quality if M certifies in the first period and L and H sell the good uncertified. If M certifies in the first period, (2) implies that

$$q_M - q^e + \delta(q_M - q^e) \geq f.$$

But for H to prefer selling the good uncertified instead of certifying it in the first period the converse of (2) implies

$$q_H - q^e + \delta(q_H - q^e) < f.$$

Combing both conditions poses a contradiction as $q_H > q_M$. If it is profitable for M to certify the product in the first period, it is as well profitable for H to certify the product in the first period instead if selling it uncertified.

Next, note that if M certifies the product in the second period and the other types sell the good uncertifed, the expected quality in the second period is equal to q^e from above. Using (3), M prefers certifying the product in the second period over selling the good uncertified if

$$q_M - q^e \geq f.$$

But, using the converse of (2), for H to prefer selling the good uncertified over certifying the good in the second period it must hold that

$$q_H - q^e < f.$$

Combing both conditions poses again a contradiction as $q_H > q_M$. If it is profitable for M to certify the product in the second period, it is as well profitable for H to certify the product in the second period instead if selling it uncertified. □

Note that if the difference $q_H - c_H$ or $q_M - c_M$ is sufficiently big, in equilibrium, at least one type of the firm will certify its product in one of the periods.

Lemma 4. *Denote the expected quality if no firm certiefies by* $\tilde{q}_1^e = \lambda_H q_H + \lambda_M q_M + \lambda_L q_L$. *If* $q_H - c_H > q_1^e$ *or* $q_M - c_M > q_1^e$, *all types of the firm selling uncertified is no equilibrium outcome of the game.*

Proof of Lemma 4. Firstly, consider the case $q_H - c_H > \lambda_H q_H + \lambda_M q_M + \lambda_L q_L$. If in equilibrium no firm certifies, a consumers expects quality $\tilde{q}_1^e = \lambda_H q_H + \lambda_M q_M + \lambda_L q_L$ from an uncertified good in any period. It is always more profitable for the certifier to set $f = q_H - \tilde{q}_1^e$ such that firm H is indifferent between certifying in the second period and earning $q_H - c_H - f$ and not certifying and earning $\tilde{q}_1^e - c_H$. Instead of not making any profit because no firm buys the certificate. To see this, check that the profit of firm type H in the second period when buying the certifcate for $q_H - \tilde{q}_1^e$ is equal to the profit in the second period when selling the good uncertified:

$$q_H - c_H - f = q_H - c_H - q_H + q^e = q^e - c_H$$

Analogously, if $q_M - c_M > \lambda_H q_H + \lambda_M q_M + \lambda_L q_L$, it is always more profitable for the certifier to set $f = q_M - q^e$ such that firm M is indifferent between certifying in the second period and earning $q_M - c_M - (q_M - q^e) = q^e - c_M$ and not certifiying and earning $q^e - c_M$ instead of not making any profit because no firm buys the certificate. □

Lemma 5. *Denote* $\tilde{q}_1^e = \lambda_H q_H + \lambda_M q_M + \lambda_L q_L$ *the expected quality in period one and two of an uncertified product. If the certification fee is given by* $f > (q_H - \tilde{q}_1^e)(1 + \delta)$ *and* $\tilde{q}_1^e \geq c_H$ *all three types of the firm prefer to sell the good uncertified.*

Proof. If in equilibrium no type of the firm certifies the product, the expected quality in both periods is given by $\tilde{q}_1^e$. It must be more profitable for every type to sell the product uncertified in both periods instead of buying a certificate in either the first or the second period. Form the converse of equation (2) follows that

$$(q_H - \tilde{q}_1^e)(1 + \delta) < f$$

and

$$(q_M - \tilde{q}_1^e)(1 + \delta) < f$$

for H and M to prefer selling the good uncertified over certifying it in the first period. Furthermore, for H and M to also prefer selling the good uncertified over deviating to certifying the good in the second period it must hold that

$$(q_H - \tilde{q_1^e}) < f$$

and

$$(q_M - \tilde{q_1^e}) < f.$$

Lastly, both types of the firm must prefer to sell the good uncertified over not selling the good at all. Form this follows that

$$\tilde{q_1^e} \geq c_H$$

and

$$\tilde{q_1^e} \geq c_M.$$

Because $(q_H - \tilde{q_1^e})(1+\delta) > q_H - \tilde{q_1^e} > q_M - \tilde{q_1^e}$ and $(q_H - \tilde{q_1^e})(1+\delta) > (q_M - \tilde{q_1^e})(1+\delta)$ holds, lemma 5 follows. Note that if the certifier can choose a fee that does not fulfill the conditions given above, the case that no firm certifies is never an equilibrium. □

A.1 Equilibrium Regions

To prepare for the proof of proposition 2 and proposition 3 some lemmas are derived. Lemma 6 lists conditions under which firm type H prefers to certify in the first period and firm type M certifies in the second period. Lemma 7 states conditions such that types H and M of the firm both prefer certifying in the first period. In lemma 8 conditions are given such that firm types H and M certify in the second period. Lemma 9 and lemma 10 explore under which conditions only firm H certifies. Lemma 11 gives an overview over the dominated strategies of the certifier. The conditions are combined to derive the results of proposition 2 and for proposition 3.

Lemma 6. *Denote $q_1^e = \lambda_L q_L + \lambda_M q_M$. H certifies in the first period, M certifies in the second period and L sells uncertified if*

1. $\frac{q_1^e - q_L}{q_M - q_L} > \delta \geq \frac{q_M - q_L - q_H + q_1^e}{q_M - q_L}$*, the certification fee is given by* $f \leq q_M - q_L$ *and if* $q_H - \frac{f}{1+\delta} \geq c_H$ *and* $\frac{q_1^e + \delta q_M - \delta f}{1+\delta} \geq c_M$*, or*
2. *if* $\frac{q_M - q_L - q_H + q_1^e}{q_H - q_L} > \delta$, $q_1^e > q_H - q_M + q_L$*, the certification fee is given by* $f \leq \frac{q_H - q_1^e}{1-\delta}$ *and if* $q_H - \frac{f}{1+\delta} \geq c_H$ *and* $\frac{q_1^e + \delta q_M - \delta f}{1+\delta} \geq c_M$*,*

the good is traded in every period.

Proof of Lemma 6. If in equilibrium H certifies in the first period, M certifies in the second period and L is uncertified in both periods a consumer expects quality $q_1^e = \lambda_L q_L + \lambda_M q_M$ from an uncertified good in period 1 and $q_2^e = q_L$ from an uncertified good in the second period. The consumer's valuation for an uncertified good in the first period is q_1^e and q_2^e for an uncertified good in the second period. The valuation for a certified good is its revealed quality.

Using (1), it is not profitable for H to deviate from the equilibrium strategy of certifying the product in the first period and selling it at price q_H to selling the good uncertified in the first period and certify in the second period if

$$\frac{q_H - q_1^e}{1-\delta} \geq f. \tag{6}$$

Using (2) it is not profitable for H to deviate to selling the good uncertified in both periods if

$$q_H - q_1^e + \delta(q_H - q_L) \geq f. \tag{7}$$

Lastly, it must be feasible for H to sell the good instead of not selling it

$$(1+\delta)(q_H - c_H) - f \geq 0$$
$$\frac{q_H(1+\delta) - f}{1+\delta} \geq c_H.$$

Using the converse of (1), it is not profitable for M to deviate from the equilibrium strategy of certifying the product in the second period to certifying the good in the first period if

$$\frac{q_M - q_1^e}{1-\delta} < f. \tag{8}$$

Using (3), it is not profitable for M to deviate to selling the good uncertified in both periods if

$$q_M - q_L \geq f \tag{9}$$

and lastly, it is not profitable for M to deviate from the equilibrium strategy to not selling the good if

$$q_1^e + \delta q_M - c_M(1+\delta) - \delta f \geq 0$$
$$\frac{q_1^e + \delta(q_M - f))}{1+\delta} \geq c_M.$$

It is not profitable for L to deviate from selling the good uncertified to certifiying the

good in the second period as stated in lemma 2.
Summarizing, the conditions for a feasible certifaction fee f for which H certifies in the first period and M certifies in the second period are

$$\min\left\{\frac{q_H - q_1^e}{1-\delta}, q_H(1+\delta) - q_1^e - q_L, q_M - q_L\right\} \geq f > \frac{q_M - q_1^e}{1-\delta} \tag{10}$$

and two conditions for the production costs

$$q_H - \frac{f}{1+\delta} \geq c_H \text{ and } \frac{q_1^e + \delta q_M - \delta f}{1+\delta} \geq c_M.$$

If $\frac{q_H - q_1^e}{1-\delta} \geq q_M - q_L$, then also $q_H - q_1^e + \delta(q_H - q_L) \geq q_M - q_L$ and vice versa, because

$$\begin{aligned} q_H - q_1^e + \delta(q_H - q_L) &\geq q_M - q_L \\ \delta &\geq \frac{q_M - q_L - q_H + q_1^e}{q_H - q_L}. \end{aligned}$$

And for $\frac{q_H - q_1^e}{1-\delta} \geq q_M - q_L$

$$\begin{aligned} \frac{q_H - q_1^e}{1-\delta} &\geq q_M - q_L \\ q_H - q_1^e &\geq (1-\delta)(q_M - q_L) \\ \delta(q_M - q_L) &\geq q_M - q_L - q_H + q_1^e \\ \delta &\geq \frac{q_M - q_L - q_H + q_1^e}{q_M - q_L}. \end{aligned}$$

There are three cases that have to be examined

1. $\min\left\{\frac{q_H - q_1^e}{1-\delta}, q_H(1+\delta) - q_1^e - q_L, q_M - q_L\right\} = q_M - q_L$
2. $\min\left\{\frac{q_H - q_1^e}{1-\delta}, q_H(1+\delta) - q_1^e - q_L, q_M - q_L\right\} = \frac{q_H - q_1^e}{1-\delta}$
3. $\min\left\{\frac{q_H - q_1^e}{1-\delta}, q_H(1+\delta) - q_1^e - q_L, q_M - q_L\right\} = q_H - q_1^e + \delta(q_H - q_L)$

In the first case it must hold that $\frac{q_H - q_1^e}{1-\delta} \geq q_M - q_L$ which is equivalent to

$$\delta \geq \frac{q_M - q_L - q_H + q_1^e}{q_M - q_L}.$$

Furthermore, because $q_H - q_1^e + \delta(q_H - q_L) \geq q_M - q_L$ must hold, this implies

$$\delta \geq \frac{q_M - q_L - q_H + q_1^e}{q_H - q_L}.$$

If $\frac{q_H - q_1^e}{1-\delta} \geq q_M - q_L$ and $q_H - q_1^e + \delta(q_H - q_L) \geq q_M - q_L$ and $f \leq q_M - q_L$, then, because of (10) it must furthermore hold that $f > \frac{q_M - q_1^e}{1-\delta}$. This is the case if

$$\begin{aligned} \frac{q_M - q_1^e}{1-\delta} &< q_M - q_L \\ q_M - q_1^e &< (q_M - q_L)(1-\delta) \\ \delta(q_M - q_L) &< q_1^e - q_L \\ \delta &< \frac{q_1^e - q_L}{q_M - q_L}. \end{aligned}$$

Note, that because $q_H > q_M$ holds, it is true that $\frac{q_1^e - q_L}{q_M - q_L} > \frac{q_M - q_L - q_H + q_1^e}{q_M - q_L}$. Summarizing, the case is given by

1. $f \leq q_M - q_L$, if $\frac{q_1^e - q_L}{q_M - q_L} > \delta \geq \frac{q_M - q_L - q_H + q_1^e}{q_M - q_L}$.

This leaves case 2 and 3 as the remaining cases.

First look at the second case. Note that $q_M - q_L \geq \frac{q_H - q_1^e}{1-\delta}$ implies

$$\delta \leq \frac{q_M - q_L - q_H + q_1^e}{q_M - q_L} \tag{11}$$

which, for $\delta > 0$ implies $q_1^e > q_H - q_M + q_L$. And, because $q_H - q_1^e + \delta(q_H - q_L) \geq \frac{q_H - q_1^e}{1-\delta}$ must hold, this requires

$$\frac{q_1^e - q_L}{q_H - q_L} \geq \delta. \tag{12}$$

And for $\delta > 0$ this implies $q_1^e > q_L$. Furthermore, it is easy to see that $\frac{q_H - q_1^e}{1-\delta} > \frac{q_M - q_1^e}{1-\delta}$ is always true because $q_H > q_M$. Check that $\frac{q_1^e - q_L}{q_H - q_L} > \frac{q_M - q_L - q_H + q_1^e}{q_M - q_L}$ if $q_H > q_1^e$:

$$\begin{aligned} \frac{q_M - q_L - q_H + q_1^e}{q_M - q_L} &< \frac{q_1^e - q_L}{q_H - q_L} \\ (q_1^e - q_L)\left(\frac{1}{q_M - q_L} - \frac{1}{q_H - q_L}\right) &< \frac{q_H - q_M}{q_M - q_L} \\ \frac{(q_1^e - q_L)(q_H - q_M)}{q_H - q_L} &< q_H - q_M. \\ q_1^e - q_L &< q_H - q_L. \end{aligned}$$

Summarizing, the second case is given by

2. $f \leq \frac{q_H - q_1^e}{1-\delta}$, if $\delta \leq \frac{q_M - q_L - q_H + q_1^e}{q_H - q_L}$

Lastly, check the third case. First of all, $\frac{q_H - q_1^e}{1-\delta} \geq q_H - q_1^e + \delta(q_H - q_L)$ implies

$$\delta \geq \frac{q_1^e - q_L}{q_H - q_L}.$$

Furthermore, $q_M - q_L > q_H - q_1^e + \delta(q_H - q_L)$ implies

$$\delta < \frac{q_M - q_L - q_H + q_1^e}{q_H - q_L}.$$

Summarizing, this means that $\frac{q_1^e - q_L}{q_H - q_L} \leq \delta < \frac{q_M - q_L - q_H + q_1^e}{q_H - q_L}$ would have to hold. But because $q_H > q_1^e$ holds, it follows that

$$\begin{aligned}
\frac{q_1^e - q_L}{q_H - q_L} &> \frac{q_M - q_L - q_H + q_1^e}{q_M - q_L} \\
(q_1^e - q_L)\left(\frac{1}{q_H - q_L} - \frac{1}{q_M - q_L}\right) &> \frac{q_M - q_H}{q_M - q_L} \\
(q_1^e - q_L)\frac{q_M - q_L - q_H + q_L}{(q_H - q_L)(q_M - q_L)} &> \frac{q_M - q_H}{q_M - q_L} \\
q_H - q_M > (q_1^e - q_L)\frac{q_H - q_M}{q_H - q_L} \\
q_H - q_L > q_1^e - q_L
\end{aligned}$$

is always true. Consequently, this third case poses a contradiction and can be excluded.
□

Below, more conditions are derived under which, type H and M prefer to certify in the first period.

Lemma 7. *If the certification fee is given be $f \leq (q_M - q_L)(1+\delta)$ and if $q_H - \frac{f}{1+\delta} \geq c_H$, $q_M - \frac{f}{1+\delta} \geq c_M$, a firm of type H or M certifies in the first period and type L never certifies. The good is traded in every period.*

Proof of lemma 7. If in equilibrium H and M certify in the first period and L never certifies a buyer expects that the quality of an uncertified good in either period is given by q_L. Using (1), it is not profitable for H to deviate from certifying in the first period to certifying in the second period if

$$\frac{q_H - q_L}{1 - \delta} \geq f$$

and using (2), it is not profitable for H to deviate from certifying in the first period to selling the good uncertified in both periods if

$$\begin{aligned}
q_H - q_L + \delta(q_H - q_L) &\geq f \\
(q_H - q_L)(1 + \delta) &\geq f.
\end{aligned}$$

And the same must be true for M:

$$\frac{q_M - q_L}{1-\delta} \geq f$$

and

$$\begin{aligned} q_M - q_L + \delta(q_M - q_L) &\geq f \\ (q_M - q_L)(1+\delta) &\geq f. \end{aligned}$$

Consequently, as $\frac{q_H - q_L}{1-\delta} \geq (q_H - q_L)(1+\delta) \geq (q_M - q_L)(1+\delta)$ and $\frac{q_M - q_L}{1-\delta} \geq (q_M - q_L)(1+\delta)$ it follows that $(q_M - q_L)(1+\delta)$ is an upper bound for the certification fee f.

In equilibrium, it must not be profitable neither for H nor M to deviate to not selling the product:

$$\begin{aligned} (q_H - c_H)(1+\delta) - f &\geq 0 \\ q_H - \frac{f}{1+\delta} &\geq c_H \end{aligned}$$

and

$$\begin{aligned} (q_M - c_M)(1+\delta) - f &\geq 0 \\ q_M - \frac{f}{1+\delta} &\geq c_M. \end{aligned}$$

It is again not profitable for L to deviate to certify the product in any period of the game. As the expected quality of an uncertified good in equilibrium is q_L, L cannot demand a higher price for a certified product but would on the other hand have to pay the certification fee. Furthermore, as $q_L > c_L$, it is always more profitable for L to sell the product at price q_L instead of not selling the product. □

Lemma 8. *Denote $\tilde{q_1^e} = \lambda_H q_H + \lambda_M q_M + \lambda_L q_L$. If $\delta < \frac{q_M - q_L - q_H + \tilde{q_1^e}}{q_M - q_L}$, the certification fee is given by $f \leq q_M - q_L$ and $\frac{\tilde{q_1}^e + \delta(q_H - f)}{1+\delta} \geq c_H$ and $\frac{\tilde{q_1}^e + \delta(q_M - f)}{1+\delta} \geq c_M$, types H and M of the firm certify in the second period and type L never certifies. The good is traded in every period.*

Proof of lemma 8. From the converse of (1) follows for H

$$\frac{q_H - \tilde{q_1^e}}{1-\delta} < f$$

and for M

$$\frac{q_M - \tilde{q_1^e}}{1-\delta} < f.$$

From (3) for the two firm types follows

$$q_H - q_L \geq f$$

and

$$q_M - q_L \geq f.$$

Summarizing, it must hold that $q_M - q_L \geq f > \frac{q_H - \tilde{q_1^e}}{1-\delta}$ and this implies

$$\delta < \frac{q_M - q_L - q_H + \tilde{q_1^e}}{q_M - q_L}$$

to ensure that $q_M - q_L > \frac{q_H - \tilde{q_1^e}}{1-\delta}$. Note that this is only feasible for $\delta \in (0,1)$ if $\tilde{q_1^e} > q_H - q_M + q_L$. Lastly, both type H and M do not have an incentive to deviate to not selling the good if

$$\tilde{q_1}^e + \delta q_H - \delta f \geq (1+\delta) c_H$$

and

$$\tilde{q_1}^e + \delta q_M - \delta f \geq (1+\delta) c_M.$$

□

Lemma 9. *Recall* $q_1^e = \lambda_L q_L + \lambda_M q_M$. *For all* $\delta \in (0,1)$, *if* $f \leq (q_H - q_1^e)(1+\delta)$ *and* $q_1^e \geq c_H$, *a firm of type* H *certifies in the first period and no other type of firm certifies the good. The good is traded in every period.*

Proof. If in equilibrium only type H certifies the good in the first period and no other type of firm buys a certificate, the consumer expects the quality of an uncertified good to be q_1^e in both periods. Form (1) follows that

$$\frac{q_H - q_1^e}{1-\delta} \geq f$$

must hold and from (2) follows that also

$$
\begin{aligned}
q_H - q_1^e + \delta(q_H - q_1^e) &\geq f \\
(q_H - q_1^e)(1+\delta) &\geq f
\end{aligned}
$$

must hold for H to not have an incentive to deviate. Furthermore, from the converse of (2) and (3) it follows that for M to not have an incentive to deviate from never certifying it must hold that

$$q_M - q_1^e < f \text{ and } (q_M - q_1^e)(1+\delta) < f.$$

Combining, the certification fee f must satisfy

$$(q_H - q_1^e)(1+\delta) \geq f > (q_M - q_1^e)(1+\delta)$$

where $(q_H - q_1^e)(1+\delta) > (q_M - q_1^e)(1+\delta)$ is always fulfilled for all $f \leq (q_H - q_1^e)(1+\delta)$ because $q_H > q_M$. □

Lemma 10. *Recall* $q_1^e = \lambda_M q_M + \lambda_L q_L$ *and* $\tilde{q_1^e} = \lambda_H q_H + q_1^e$. *If* $\frac{\tilde{q_1^e} - q_1^e}{q_H - q_M} > \delta$, *the certification fee is given by* $f \leq q_H - q_1^e$ *and* $\frac{\tilde{q_1^e} + \delta q_H}{1+\delta} \geq c_H$, *type* H *certifies in the second period and no other type certifies.The good is traded in every period.*

Proof. If in equilibrium only type H certifies the good in the second period and no other type of firm buys a certificate, the consumer expects the quality of an uncertified good to be $\tilde{q_1^e}$ in the first period and q_1^e in the second period. From from the converse of (1) follows that

$$\frac{q_H - \tilde{q_1^e}}{1-\delta} < f$$

must hold for H to not have an incentive to deviate to certifying in the first period. Also,

$$q_H - q_1^e \geq f$$

must hold because of (3).

Furthermore, from the converse of (2) and (3) it follows that for M to never certify it must hold that

$$q_M - q_1^e < f \text{ and } q_M + \tilde{q_1^e} + \delta(q_M - q_1^e) < f.$$

Combining, the certification fee f must satisfy $q_H - q_1^e \geq f > \max\{\frac{q_H - \tilde{q_1^e}}{1-\delta}, q_M - q_1^e, q_M + \tilde{q_1^e} + \delta(q_M - q_1^e)\}$.

From $q_H > q_M$ follows that $q_H - q_1^e > q_M - q_1^e$. Furthermore, $q_H - q_1^e > \frac{q_H - \tilde{q_1^e}}{1-\delta}$ if

$$\begin{aligned} \frac{q_H - q_1^e - q_H + \tilde{q_1^e}}{q_H - q_1^e} &> \delta \\ \frac{\tilde{q_1^e} - q_1^e}{q_H - q_1^e} &> \delta \end{aligned}$$

and $q_H - q_1^e > q_M - \tilde{q_1^e} + \delta(q_M - q_1^e)$ if

$$\frac{q_H - q_1^e - q_M + \tilde{q_1^e}}{q_M - q_1^e} > \delta.$$

Note that $\frac{\tilde{q_1^e} - q_1^e}{q_H - q_1^e} < \frac{q_H - q_1^e - q_M + \tilde{q_1^e}}{q_M - q_1^e}$ because

$$(\tilde{q_1^e} - q_1^e) \cdot (q_M - q_1^e) < (\underbrace{q_H - q_1^e - q_M + \tilde{q_1^e}}_{> \tilde{q_1^e} - q_1^e}) \cdot (\underbrace{q_H - q_1^e}_{> q_M - q_1^e})$$

is true (because $q_H > q_M$ and $\tilde{q_1^e} > q_1^e$).

Lastly, for $f \leq q_H - q_1^e$ H must prefer to sell the good uncertified in the first period and certified in the second period over deviating to not selling the good, $\tilde{q_1^e} + \delta q_H - \delta f \geq (1+\delta)c_H$:

$$\begin{aligned} \tilde{q_1^e} + \delta q_H - \delta(q_H - q_1^e) &\geq (1+\delta)c_H \\ \frac{\tilde{q_1^e} - \delta q_1^e}{1+\delta} &\geq c_H \end{aligned}$$

and M must prefer to sell the good uncertified in both periods over not selling the good:

$$\begin{aligned} \tilde{q_1^e} + \delta q_1^e &\geq (1+\delta)c_M \\ \frac{\tilde{q_1^e} + \delta q_1^e}{1+\delta} &\geq c_M. \end{aligned}$$

Summarizing, if $\delta < \frac{\tilde{q_1^e} - q_1^e}{q_H - q_1^e}$ and $\frac{\tilde{q_1^e} + \delta q_1^e}{1+\delta} \geq c_H$, H certifies in the second period and no other type certifies. □

Lemma 11. *Recall* $q_1^e = \lambda_L q_L + \lambda_M q_M$. *If* $\delta < \frac{q_M - q_L - q_H + q_1^e}{q_M - q_L}$, *the profit maximizing certifier always prefers to set* $f = (q_M - q_L)(1+\delta)$ *or* $f = \frac{q_H - q_1^e}{1-\delta}$ *over* $f = (q_H - q_1^e)(1+\delta)$, *if all three options are feasible. Furthermore, the certifier always prefers to choose* $f = (q_H - q_1^e)(1+\delta)$ *over choosing* $f = q_H - q_1^e$, *if both options are feasible.*

Proof. Assume all conditions are satisfied such that the certifier can choose between the different certification fees that induce either of the equilibrium outcomes discussed in this lemma. Assume first that $\delta \geq \frac{q_M - q_L - q_H + q_1^e}{q_M - q_L}$. If H certifies in the first period and no other type certifies, the expected profit of the certifier is given by

$$\Pi^c((q_H - q_1^e)(1+\delta)) = (q_H - q_1^e)(1+\delta)\lambda_H.$$

If H and M both certify in the first period, the expected profit of the certifier is given by

$$\Pi^c((q_M - q_L)(1+\delta)) = (q_M - q_L)(1+\delta)(\lambda_H + \lambda_M)$$

and because $q_M - q_L > q_H - q_1^e$ if $\delta \geq \frac{q_M - q_L - q_H + q_1^e}{q_M - q_L}$, this expected profit is in this case greater than $\Pi^c((q_H - q_1^e)(1+\delta))$.

Note furthermore that if $\delta < \frac{q_M - q_L - q_H + q_1^e}{q_M - q_L}$

$$\Pi^c\left(\frac{q_H - q_1^e}{1-\delta}\right) = \frac{q_H - q_1^e}{1-\delta}(\lambda_H + \delta\lambda_M) > (q_H - q_1^e)(1+\delta)(\lambda_H + \lambda_M) > \Pi^c\left((q_H - q_1^e)(1+\delta)\right).$$

This implies that the certifier also prefers to choose the fee such that H certifies in the first period and M certifies in the second period over a fee for which H certifies in the first period and no other type certifies.

□

Proof of Proposition 2. First, consider H certifying in the first period and M certifying in the second period. As shown in the proof of A.1, if $min\{\frac{q_H - q_1^e}{1-\delta}, q_H - q_1^e + \delta(q_H - q_L), q_M - q_L\} = q_M - q_L$ it is type $H's$ best response to certify in the first period, and $M's$ best response to certify in the second period, while L never certifies, if $f = q_M - q_L$. If the certifier chooses $f = q_M - q_L$, the expected profit is given by

$$\Pi^C(q_M - q_L) = (q_M - q_L)(\lambda_H + \delta\lambda_M).$$

The certifier chooses the firm such that it maximizes the expected profit. But the profit from choosing the certification $f = (q_M - q_L)(1+\delta)$ for which H and M certify in the first period and L never certifies is given by

$$\Pi^C = (q_M - q_L)(1+\delta)(\lambda_H + \lambda_M)$$

and is greater than $\Pi^C = (q_M - q_L)(\lambda_H + \delta\lambda_M)$:

$$
\begin{aligned}
(\lambda_H + \delta\lambda_M)(q_M - q_L) &< (q_M - q_L)(1+\delta)(\lambda_H + \lambda_M) \\
0 &< (q_M - q_L)((1+\delta-1)\lambda_H + (1+\delta-\delta)\lambda_M) \\
0 &< (q_M - q_L)(\delta\lambda_H + \lambda_M).
\end{aligned}
$$

Consequently, the certifier will choose $f = (q_M - q_L)(1+\delta)$ in equilibrium. A firm of type H or M certifies in the first period and a firm of type L never certifies. A firm of type i sells the product for price q_i in both periods. Consumers buy the product in both periods.

Recall next from the proof of lemma 8 that H and M only prefer to both certify in the second period if $\tilde{q_1^e} > q_H - q_M + q_L$ contradicting the assumption on $\tilde{q_1^e}$ in proposition 2. Therefore, H and M certifying in the second period cannot be an equilibrium here.

Lastly, check that because of $q_H - q_M + q_L > \tilde{q_1^e}$ it holds that

$$
(q_M - q_L)(1+\delta) \leq (q_H - \tilde{q_1^e})(1+\delta)
$$

and therefore, recalling the conditions in the proof of lemma 5, firm H would always prefer to deviate from not certifying to certifying the product in the first period if no other type of firms certifies. Because $\delta \in (0,1)$, $q_H - q_M + q_L > \tilde{q_1^e}$ is only feasible in the case of $\delta \geq \frac{q_M - q_H - q_L + q_1^e}{q_M - q_L}$. □

Proof of Prop 3. Recall $q_1^e = \lambda_M q_M + \lambda_L q_L$ and $\tilde{q_1^e} = \lambda_H q_H + \lambda_M q_M + \lambda_L q_L$. Let $\delta \geq \frac{q_M - q_L - q_H + q_1^e}{q_M - q_L}$ and $q_1^e > q_H - q_M + q_L$.

The proof of lemma 6 states that if $\frac{q_1^e - q_L}{q_M - q_L} > \delta$ and $f = q_M - q_L$, H certifying in the first period and M certifying in the second period is a candidate for the equilibrium if $\frac{q_H(1+\delta) - (q_M - q_L)}{1+\delta} \geq c_H$ and $\frac{q_1^e + \delta q_L}{1+\delta} \geq c_M$. If $c_H > \tilde{q_1^e}$, then $c_H > \frac{q_1^e + \delta q_1^e}{1+\delta}$ because $\tilde{q_1^e} > q_1^e$. Consequently, type H does not find it profitable to certify if type M does not certify in any of the two periods (see lemma 9 and lemma 10). Furthermore, if $c_M > \tilde{q_1^e}$ this implies $c_M > q_1^e$ and $c_M > \frac{\tilde{q_1^e} + \delta q_1^e}{1+\delta}$. Therefore, type M does not find it profitable to sell the uncertified good if type H certifies the product in any of the two periods.

Lastly, case 1 in the proof of lemma 6 states that if $f = q_M - q_L$, H certifies in the first period and M certifies in the second period if $\frac{q_1^e - q_L}{q_M - q_L} > \delta \geq \frac{q_M - q_L - q_H + q_1^e}{q_M - q_L}$ and if $\frac{q_H(1+\delta) - (q_M - q_L)}{1+\delta} \geq c_H$ and $\frac{q_1^e + \delta q_L}{1+\delta} \geq c_M$, i.e. type H and type M prefer to not deviate to selling the good uncertified instead. To see that this equilibrium is the unique equilibrium in pure strategies if $\delta \geq \frac{\tilde{q_1^e} - q_H - q_M + q_L}{q_H - \tilde{q_1^e}}$ recall that for all types of the firm to prefer selling the good uncertified in equilibrium it must hold that $\tilde{q_1^e} \geq c_H$. But

$$\begin{aligned}\frac{q_H(1+\delta)-(q_M-q_L)}{1+\delta} &\geq \tilde{q_1^e} \\ \delta(q_H-\tilde{q_1^e}) &\geq \tilde{q_1^e}-q_H+q_M-q_L \\ \delta &\geq \frac{\tilde{q_1^e}-q_H+q_M-q_L}{q_H-\tilde{q_1^e}}.\end{aligned}$$

If this inequality is fulfilled, then all types selling the good uncertified cannot be an equilibrium. And because $\tilde{q_1^e} < q_H - q_M + q_L$ it follows that $\tilde{q_1^e} - q_H - (q_M - q_L) < 0$ and therefore, all $\delta > 0$ fulfill $\delta \geq \frac{\tilde{q_1^e}-q_H-q_M+q_L}{q_H-\tilde{q_1^e}}$. Lastly, H and M certifying in the first period is no equilibrium if $c_H > q_H - q_M + q_L$ or $c_L > q_L$. This is feasible for $q_H - \frac{q_M-q-L}{1+\delta} \geq c_H$ because in this case

$$\delta > \frac{q_M - q_L - q_H + \tilde{q_1^e}}{q_H - \tilde{q_1^e}}$$

must hold, which is true for all $\delta > 0$ if $\tilde{q_1^e} < q_H - q_M + q_l$. And it is feasible for $\frac{q_1^e+\delta q_L}{1+\delta}$ because

$$q_1^e + \delta q_L > q_L + \delta q_L$$

as $q_1^e > q_L$.

□

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