Abstract

We consider a model of collective persuasion, in which members of an advisory committee receive private continuous signals and then vote on a policy change. A decision maker then decides whether to adopt the change upon observing each vote. The conflict of interest between the decision maker and the committee creates a pressure toward unanimity. If a decision maker cannot be persuaded to adopt the policy change under unanimous voting, she can never be persuaded. If a decision maker cannot be persuaded under a $k$-rule, she can never be persuaded under a lower $k$-rule. Our results thus provide a rationale for the use of unanimity rule, in spite of its poor performance in information aggregation (Feddersen and Pesendorfer 1998). We further discuss why our continuous-signal model produces results different from discrete-signal models (Battaglini 2017; Gradwohl and Feddersen 2018).
1 Introduction

A manager is considering whether to hire a candidate or leave the post open. To evaluate the candidate, she consults a committee of experts who have private information about the competence of the candidate. However, committee members often have their own interest in the decision. For example, the committee may be more eager to have a new hire, while the manager is more concerned with the salary involved. A common practice is for each committee member to make a binary recommendation, either for the candidate or against the candidate, to the manager and the manager then makes the final decision.

In this case, the group of experts serves as an advisory committee and provides decision-relevant information to the manager through voting. This paper studies how the committee transmits information to the manager when there is a conflict of interest between the committee and the manager. Our model applies to a lot of real-world situations. Examples include the Federal Advisory Council, which advises the Federal Reserve Board, the Investor Advisory Committee, which advises the US Securities and Exchange Commission, advisory committees for the US Food and Drug Administration and expert panels appointed in WTO dispute settlement proceedings. Despite their importance in real-world decision making, such committees, usually called advisory committees, are understudied in the literature (Gradwohl and Feddersen 2018).\footnote{There are also committees that are usually regarded as decision-making committees but face a decision-making problem similar to the one we describe. For example, the Federal Open Market Committee independently determines monetary policy, but the economic consequences of the policy depend on the market response. In the United States, the Congress collectively determines whether to pass a bill, but the Article I of the Constitution requires that every bill passed by the Congress must be presented to the president for approval, and the president can veto the passed bill within 10 days. Presidential vetoes do happen sometimes.}

We study the behavior of such an advisory committee with a standard voting framework. There is an unknown state of the world, and the decision maker needs to choose between a policy change and the status quo. Each member of the committee receives a continuous signal about the state, and each casts a vote, i.e., whether to adopt the policy change. The decision maker is informed of each vote, and then chooses between the policy change and the status quo. All committee members share the same preference. Their preference is completely aligned with the decision maker’s preference when the state is known, but the decision maker is more conservative. Thus, when information is imperfect, there are situations, in which the decision maker prefers
the status quo, while the committee prefers the policy change.

Recent work on advisory committees (Levit and Malenko 2011; Battaglini 2017; Gradwohl and Feddersen 2018) shows that information transmission between the committee and the decision maker is impossible when the conflict of interest is too large. This is because an advisory committee differs from a decision-making committee in one key aspect. In a decision-making committee, votes are aggregated by an exogenous voting rule that selects one of the two options. In an advisory committee, the decision rule is endogenous and chosen by the decision maker. In equilibrium, the decision maker adopts a certain decision (voting) rule and the committee members cast their votes in anticipation of the equilibrium decision rule and the other committee members’ equilibrium voting behaviors. As a result, a given voting rule may not be sustainable if the decision maker is not willing to follow it. When the conflict of interest is large enough, no voting rule can persuade the decision maker into adopting the policy change. As a result, the committee’s information is never utilized.

While existing work focuses on the impossibility of information transmission, we focus instead on situations in which information transmission is possible and ask which voting rule can sustain information transmission for a wider range of scenarios. We show the following result in Section 5:

**Proposition** If a decision maker can be persuaded (into adopting the policy change) by any voting rule, then she can be persuaded by unanimity rule.

Ever since Feddersen and Pesendorfer (1998) pointed out the inferiority of unanimity rule in a jury voting model, it remains puzzling why unanimity rule is so prevailing in many decision-making process despite its poor performance in information aggregation from a theoretical point of view. Moreover, unanimity rule is also the uniquely bad voting rule if deliberation is allowed (Austen-Smith and Feddersen 2006; Gerardi and Yariv 2007). Our result provides a rationale for the use of unanimity rule. When the conflict of interest is large, unanimity rule may be the only decision rule that could persuade a decision maker to take a certain action. Although other voting rules may also be used to persuade the decision maker when the conflict of interest is moderate, our result shows that whenever some other voting rules can sustain communication,
so does unanimity rule. This means that unanimity rule can be used to transmit information for a wider range of scenarios than all other voting rules.

We can further rank all $k$-rules, i.e., rules that only adopt the policy change when the number of votes recommending the policy change exceeds a threshold $k$, according to their ability to persuade a conservative decision maker.

**Proposition** If a decision maker cannot be persuaded when the committee members vote according to $k$-rule, she can never be persuaded when the committee members vote according to $(k - 1)$-rule.

The clear ranking among all $k$-rules in terms of persuasiveness is again different from the comparison between different $k$-rules according to the information aggregation criterion. For example, Persico (2004) shows that the optimal voting rule for a committee is the statistical rule, which depends on the preference of the committee. For an advisory committee, voting behavior depends not only on the preference of the committee, but also on the degree of the conflict of interest between the decision maker and the committee members. When the decision maker is more conservative, and reluctant to adopt the policy change, the committee members have to aim for a higher consensus level, a higher $k$-rule, for example, to persuade her.

Our baseline results thus confirm the intuition that more affirmative votes make the case for a policy change more convincing. Therefore, a higher $k$-rule is more persuasive than a lower $k$-rule. Such reasoning, however, ignores the fact that the committee members adjust their votes strategically and vote more aggressively for a policy change when facing a higher $k$-rule (Austen-Smith and Banks 1996). While we show that such naïve reasoning leads to the right conclusion when signals are continuous under fairly general conditions, in Section 6, we consider a version of our model with binary signals (Gradwohl and Feddersen 2018) and show that this often-used specification leads to drastically different results. In particular, unanimity rule is the least persuasive among the supermajority rules. This suggests that the assumption of discrete signals is not innocuous. We further discuss how the differences arise and how our results from the continuous case and the binary case generalize to discrete signals that take more than two values.
2 Literature review

Our model builds on the voting model pioneered by Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1997). These two seminal papers model the voting of a decision-making committee as non-cooperative strategic games. Austen-Smith and Banks (1996) show that truthful voting is generally not an equilibrium. Feddersen and Pesendorfer (1997) show that in spite of that, private information held by the committee members can be fully aggregated in a large election. Thereafter, the literature has been mainly focusing on studying the performance of decision-making committees in information aggregation. The literature studies the effect of voting rules adopted by the committees on information aggregation (Feddersen and Pesendorfer 1998; Duggan and Martinelli 2001; Li et al. 2001), the efficiency of information aggregation when pre-voting communication is allowed (Coughlan 2000; Austen-Smith and Feddersen 2006; Gerardi and Yariv 2007), and costly information acquisition (Li 2001; Persico 2004; Martinelli 2006). The committees in this literature are all decision-making committees, but the committee we consider in the current paper is an advisory committee. Thus, the voting rules in these papers are exogenously given while the voting rule in our paper is endogenous.

This paper is also related to the cheap-talk literature initiated by Crawford and Sobel (1982). In the cheap-talk literature, the papers closest to the current paper are the ones with multiple senders (Krishna and Morgan 2001; Battaglini 2002; Ambrus and Takahashi 2008). Papers in the cheap-talk literature, different from the current paper, mostly focus on whether full revelation is obtainable. In this paper, we restrict our attention to a binary state space and action space, and focus on the feasibility of information transmission rather than the possibility of full revelation.

The papers most closely related to this paper are Austen-Smith (1993), Wolinsky (2002), Levit and Malenko (2011), Battaglini (2017), Gradwohl and Feddersen (2018) and Battaglini, Morton and Patacchini (2020). Austen-Smith (1993) considers a heterogeneous two-expert committee and a decision maker who does not commit to a decision rule. He compares simultaneous voting and sequential voting, and examines the information properties of these two mechanisms. Levit and Malenko (2011), Battaglini (2017) and Gradwohl and Feddersen (2018) consider simultaneous voting models similar to ours but with discrete signals. They find that information transmission is

\footnote{Li and Suen (2009) and Gerling et al. (2005) provide excellent surveys of the earlier works in this literature.}
impossible if the conflict of interest between the committee and the decision maker is large enough, regardless of the size of the committee. Wolinsky (2002) also reaches a similar impossibility result in a model with verifiable information but with a different information and payoff structure. Ekmekci and Lauermann (2020) introduce costly participation and noise to Battaglini (2017). In contrast to the previous literature, they show that if there are only costs and no noise, information is fully aggregated when the size of the committee is large enough. Battaglini, Morton and Patacchini (2020) test the predictions of Battaglini (2017) experimentally. We go beyond these papers by not only deriving conditions under which information transmission is impossible for all voting rules but also studying the corresponding condition for a given voting rule in detail.

The rest of the paper is organized as follows. Section 3 introduces the model. Section 4 characterizes the equilibria. In Section 5, we present the main results of the paper. Section 6 discusses the model with discrete signals. Section 7 concludes. Most of the proofs are relegated to the Appendix.

3 Model

3.1 Setup

There is a committee of $N$ homogeneous members. Each member $i$ receives information about the state of the world $\theta \in \{y, n\}$, and then votes simultaneously on two options, status quo $N$, or nay, alternative $Y$, or yay, $v_i \in \{Y, N\}$. The decision maker (DM) is first informed of each vote, and then makes a final decision between status quo $N$ and alternative $Y$, $D \in \{Y, N\}$. The common prior probability that the state is $y$ is $p \in (0, 1)$.

**Payoffs.** The payoffs of the committee members and the DM, denoted by $u_C(D, \theta)$ and $u_{DM}(D, \theta)$, respectively, depend on DM’s choice $D$ and the state $\theta$. The payoffs of status quo $N$ are normalized to zero in both states for both parties, i.e., $u_C(N, \theta) = u_{DM}(N, \theta) = 0$ for both $\theta \in \{y, n\}$. The payoffs of alternative $Y$ depend on the state, i.e., $u_C(Y, n) = -1/2$, $u_C(Y, y) = 1/2$, $u_{DM}(Y, n) = -\alpha$, and $u_{DM}(Y, y) = 1 - \alpha$. The parameter $\alpha \in (\frac{1}{2}, 1)$ measures the conflict of interest between the DM and the committee members. Under perfect information, all players have the same preference, i.e., all players strictly prefer alternative $Y$ in state $y$ and
status quo $N$ in state $n$. For every interior belief about the state, the DM’s expected payoff of adopting alternative $Y$ is less than that of the committee members. Moreover, we assume that the optimal uninformed decision for the DM is status quo $N$, i.e., $\alpha > p$.\(^3\)

\[
\begin{array}{ccc}
\theta = n & \theta = y \\
D = N & 0 & 0 & D = N & 0 & 0 \\
D = Y & -\alpha & 1 - \alpha & D = Y & -\frac{1}{2} & \frac{1}{2}
\end{array}
\]

Table 1: The DM’s (left) and committee member’s (right) payoffs.

**Information.** Before voting, each member $i$ receives a private signal $s_i$ regarding the state $\theta$. Signal $s_i$ is distributed on $(a, b)$ according to distribution function $F(.)$ if the state is $y$ and $G(.)$ if the state is $n$, where $a, b \in \mathbb{R} \cup \{-\infty, +\infty\}$. The distributions $F(.)$ and $G(.)$ admit continuous density functions $f(.)$ and $g(.)$ respectively, on $(a, b)$. All signals are independently distributed conditional on the true state.

**Strategies.** A voting strategy of a committee member is a (measurable) function that maps his signal into the probability of voting for alternative $Y$. For committee member $i$, it is a function $m_i : (a, b) \to [0, 1]$. A voting strategy $m_i$ is partisan if committee member $i$ votes for one of the two options with probability 1, i.e., $\Pr(v_i = Y) = 1$ or $\Pr(v_i = N) = 1$. Otherwise, $m_i$ is nonpartisan. A voting strategy $m_i$ is increasing if $m_i(s_i) \geq m_i(s'_i)$ for all $s_i \geq s'_i$. A voting strategy $m_i$ is a cutoff strategy if there exists an $s^*_i \in (a, b)$ such that $m_i(s_i) = I$ for all $s_i > s^*_i$ and $m_i(s_i) = J$ for all $s_i < s^*_i$, where $I, J \in \{0, 1\}$ and $I \neq J$.

A decision rule of the DM is a function $d : \{Y, N\}^N \to \{Y, N\}$ that maps a vote profile into one of the two options. Denote the sets $\{v \in \{Y, N\}^N : d(v) = Y\}$ and $\{v \in \{Y, N\}^N : d(v) = N\}$ by $V^+$ and $V^-$, respectively. A decision rule $d$ is fully characterized by $V^+$ or $V^-$. A decision rule $d$ is constant in $m_i$, if, for all $v_{-i} \in \{Y, N\}^{N-1}$, $d(Y, v_{-i}) = d(N, v_{-i})$. A decision rule $d$ is constant, if the DM always chooses one of the two options, i.e., for all $v \in \{Y, N\}^N$, $d(v) = Y$ or $N$. Or equivalently, either $V^+$ or $V^-$ is empty. Denote the number of yay votes $\{#i|v_i = Y\}$

\(^3\)This assumption is inessential for our results. If $\alpha < p$, then the DM will choose alternative $Y$ instead of status quo $N$ in a nonresponsive equilibrium. Our characterizations for the responsive equilibria remain valid.
by $|v|$. A decision rule $d$ is a $k$-rule if there exists a threshold $k \in \{1, 2, ..., N\}$ such that, for all $v \in \{Y, N\}^N$, $d(v) = Y$ if $|v| \geq k$, and $d(v) = N$ if $|v| < k$. Each $k$-rule is uniquely characterized by the corresponding threshold $k$. For example, a $k$-rule is simple majority rule if $k = \frac{N+1}{2}$. Finally, a decision rule $d$ is a weighted voting rule, if there exists a weight profile $w = (w_1, w_2, \ldots, w_N) \in \mathbb{R}_+^N$ and a quota $Q \in \mathbb{R}_+$ such that $d(v) = Y$ if $\sum_{i=1}^{N} w_i 1_{\{v_i=Y\}} \geq Q$ and $d(v) = N$ if $\sum_{i=1}^{N} w_i 1_{\{v_i=Y\}} < Q$, where $1$ is the indicator function.\footnote{See Shapley and Shubik (1954) and Felsenthal and Machover (1998), for example, for the study of these voting rules in cooperative game theory.}

**Equilibrium.** An equilibrium consists of an equilibrium voting strategy profile $m$ and an equilibrium decision rule $d$. We assume that the DM always chooses the policy change $Y$ when indifferent. An equilibrium is *responsive*, if the DM chooses both options with positive probability in equilibrium. We say that a DM, characterized by her preference parameter $\alpha$, *can be persuaded*, if there exists a responsive equilibrium. A DM *can be persuaded by a decision rule* $d$ or a decision rule $d$ *is persuasive*, if there exists a responsive equilibrium in which the decision rule is $d$. A decision rule $d$ is *more persuasive* than another decision rule $d'$, if $d'$ is persuasive for a given DM implies that $d$ is persuasive for that DM.\footnote{The relation is strict if $d$ is more persuasive than $d'$ and $d'$ is not more persuasive than $d$.} Persuasiveness for voting strategy profile $m$ is similarly defined. An equilibrium is *symmetric*, if all committee members use the same voting strategy, and *asymmetric* otherwise. Finally, two equilibria $(m, d)$ and $(m', d')$ are *outcome-equivalent*, if for all signal profiles $(s_1, s_2, \ldots, s_N) \in (a, b)^N$, $\Pr[d(v) = Y | (s_1, s_2, \ldots, s_N)] = \Pr[d'(v) = Y | (s_1, s_2, \ldots, s_N)]$.

### 3.2 Assumptions on information structure

Let $h_F(s) := \frac{f(s)}{1-F(s)}$ and $h_G(s) := \frac{g(s)}{1-G(s)}$ be the hazard functions for distributions $F(.)$ and $G(.)$, respectively. Define the hazard ratio at signal $s$ as the ratio of the hazard functions at signal $s$. We impose the following three assumptions on $F(.)$ and $G(.)$.

**Assumption 1 (MLRP)** $F(.)$ and $G(.)$ satisfy the monotone likelihood ratio property (MLRP), i.e., $f(s)/g(s)$ is strictly increasing in $s$. 
Assumption 2 (Unbounded likelihood ratio) As $s$ approaches $a$, $f(s)/g(s)$ approaches zero. As $s$ approaches $b$, $f(s)/g(s)$ approaches positive infinity.

Assumption 3 (IHRP) $F(.)$ and $G(.)$ satisfy the increasing hazard ratio property (IHRP), i.e., $h_F(s)/h_G(s)$ is strictly increasing in $s$.

Assumption 1 is a standard assumption in the literature, which guarantees that a higher signal indicates that it is more likely for the state to be $y$. In Lemma 5 in the Appendix, we summarize some useful properties of distributions that satisfy MLRP.

Assumption 2 implies unbounded posterior odds, which means that signals can be arbitrarily precise. This assumption guarantees that given any $k$-rule, an informative equilibrium exists in the corresponding decision-making committee. Moreover, it implies that a large committee is arbitrarily persuasive (See Corollary 1 in Section 5.2).

Assumption 3 is a regularity assumption that has been studied by Duggan and Martinelli (2001) in the context of a decision-making committee. Kalashnikov and Rachev (1986) first introduced this property in the statistical literature. It was also shown to be an important condition for the absence of information cascades in observational learning models (Herrera and Hörner 2011, 2013). In survival analysis, IHRP is referred to as the “ageing faster property”. This is because the hazard function represents the instantaneous probability of death. Thus, the probability of death for an agent whose lifetime distribution function is given by $F(.)$ increases faster as $s$ increases than the probability of death for an agent whose lifetime distribution function is given by $G(.)$. Most but not all distributions commonly used in economics and political science satisfy IHRP. For example, if both $F(.)$ and $G(.)$ are normal distributions that satisfy MLRP, then $h_F(.)/h_G(.)$ is strictly increasing. See Lemma 6 in the Appendix for a proof of this.

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6 A weaker assumption that guarantees this is $\frac{f(a)}{g(a)} < \frac{1-p}{p} < \frac{f(b)}{g(b)}$. This assumption is employed by Duggan and Martinelli (2001). It makes sure that a committee member who behaves “naively” (i.e., as if his vote alone determines the outcome) will vote for status quo $N$ (alternative $Y$) after receiving a signal that is low (high) enough.

7 See Herrera and Hörner (2011) for a discussion and a list of distributions that satisfy IHRP. A notable case that fails IHRP is the exponential distribution, whose hazard ratio is a constant (Duggan and Martinelli 2001; Herrera and Hörner 2011). It is thus a knife edge case.
4 Equilibrium characterization

In this section, we show that it is without loss to restrict our attention to a specific class of equilibria. We have

**Proposition 1** For any equilibrium \((m, d)\) of our model, there exists an equilibrium \((\hat{m}, \hat{d})\) such that

1. Each member \(i\)’s voting strategy \(\hat{m}_i\) is either an increasing cutoff strategy or a partisan strategy;

2. The DM’s decision rule \(\hat{d}\) is a weighted voting rule;

3. \((\hat{m}, \hat{d})\) is outcome-equivalent to \((m, d)\).

From this point on, we consider only equilibria in which the committee consists of two groups of members - informative voters who use increasing cutoff strategies and partisans who always vote for one of the two options. Moreover, since partisans are ignored by the DM as well as the other committee members, the presence of partisans in effect reduces the size of the committee. Therefore, one can further focus on equilibria in which all the committee members use increasing cutoff strategies. In that case, the cutoff profile \(s^* = (s_1^*, s_2^*, \ldots, s_N^*)\) fully characterizes the equilibrium voting strategy profile, so we also use \(m\) and \(s^*\) interchangeably.

5 Persuasiveness

In this section, we first consider symmetric voting and characterize the set of DMs that can be persuaded. When voting is symmetric, we show that if a DM cannot be persuaded under unanimity rule, she can never be persuaded. Moreover, if a DM cannot be persuaded under a specific \(k\)-rule, she can never be persuaded under a lower \(k\)-rule. Then we show that there is no loss in restricting attention to symmetric voting.

5.1 Symmetric voting

We first consider responsive equilibria in which voting is symmetric. Symmetric equilibrium is of great interest to us, because later we will show that if a DM cannot be persuaded by symmetric
voting, then she can never be persuaded.

By Proposition 1, in a symmetric responsive equilibrium, there is no loss to assume the committee members use an increasing cutoff strategy. Since the committee members all use the same cutoff strategy, vote identity provides no extra information to the DM in addition to the number of yay votes $|v|$. Thus, the equilibrium decision rule must be a $k$-rule.

Suppose the decision rule is a $k$-rule. In this case, the voting problem is standard, and equivalent to the one faced by a decision-making committee with continuous signals under the corresponding $k$-rule. In a symmetric responsive equilibrium, a committee member is indifferent between voting for alternative $Y$ and status quo $N$ conditional on being pivotal after receiving the cutoff signal $s^*$, that is,

$$
p f(s^*) (1 - F(s^*))^{k-1} F(s^*)^{N-k} - (1 - p) g(s^*) (1 - G(s^*))^{k-1} G(s^*)^{N-k} = 0,
$$

which means that the cutoff $s^*$ is the solution to

$$
\frac{p}{1 - p} (1 - F(s))^{k-1} F(s)^{N-k} = \frac{g(s)}{f(s)}.
$$

(1)

By Lemma 5 in the Appendix, both $\frac{1 - F(s)}{1 - G(s)}$ and $\frac{F(s)}{G(s)}$ are strictly increasing in $s$. By MLRP, $\frac{g(s)}{f(s)}$ is strictly decreasing in $s$. By Assumption 2, this means (1) always has a unique solution.

**Definition 1** $s(k,N)$ is defined as the unique solution to (1).

The voting problem in a decision-making committee under a given $k$-rule is a standard voting problem that has been studied in the literature (Duggan and Martinelli 2001). Now, we are ready to move on to our collective decision-making problem, in which the decision is made by a DM instead of the committee and therefore not every $k$-rule can be supported in a responsive equilibrium.

**Definition 2** $\alpha(k,N)$ is defined as the unique solution to

$$
\frac{\alpha}{1 - \alpha} = \frac{h_G(s(k,N))}{h_F(s(k,N))}.
$$

11
The following lemma shows that a given $k$-rule can be supported in a responsive equilibrium when the DM is not too conservative, i.e., $\alpha \leq \alpha (k, N)$.

**Lemma 1** A symmetric responsive equilibrium is characterized by a $k \in \{1, 2, ..., N\}$ such that

1. the equilibrium voting strategy is an increasing cutoff strategy with cutoff $s (k, N)$;
2. the equilibrium decision rule is a $k$-rule.

We refer to such an equilibrium as a $k$-equilibrium. For each $k \in \{1, 2, ..., N\}$, a $k$-equilibrium exists if and only if $\alpha \leq \alpha (k, N)$.

To understand the existence condition, note that in equilibrium, optimality of the $k$-rule requires that the DM chooses alternative $Y$ if $|v| \geq k$, and status quo $N$ if $|v| < k$. Given $s^* = s (k, N)$, this means

\[
\frac{p}{1 - p} \left( \frac{1 - F (s^*)}{1 - G (s^*)} \right)^{k-1} \left( \frac{F (s^*)}{G (s^*)} \right)^{N-k+1} < \frac{\alpha}{1 - \alpha} \leq \frac{p}{1 - p} \left( \frac{1 - F (s^*)}{1 - G (s^*)} \right)^k \left( \frac{F (s^*)}{G (s^*)} \right)^{N-k},
\]

which, by (1), can be rewritten as

\[
\frac{F (s^*) g (s^*)}{G (s^*) f (s^*)} < \frac{\alpha}{1 - \alpha} \leq \frac{(1 - F (s^*)) g (s^*)}{(1 - G (s^*)) f (s^*)}.
\]

By Lemma 5 in the Appendix, MLRP implies that $\frac{g (s)}{f (s)} \frac{F (s)}{G (s)} < 1$ for all $s$. Therefore, for a given $k$, the second inequality in (3) is necessary and sufficient for the existence of the corresponding $k$-equilibrium.

In the following example, we apply Lemma 1 to compare the conditions for existence of two $k$-equilibria.

**Example 1** Suppose voting is symmetric, and $F (.)$ and $G (.)$ are two normal distributions satisfying MLRP.

1. When $N = 1$, a DM can be persuaded if and only if $\alpha \leq \alpha (1, 1)$;
2. When $N = 2\kappa + 1$, simple majority rule is persuasive if and only if $\alpha \leq \alpha (\kappa + 1, 2\kappa + 1)$;
3. \( \alpha(1,1) > \alpha(\kappa + 1, 2\kappa + 1) \) for all \( \kappa \geq 1 \) when \( p > \frac{1}{2} \).

This example illustrates an interesting case in which a single expert can be more persuasive than simple majority rule even when the committee is arbitrarily large. In other word, a poorly organized committee may fail to be more persuasive than a single expert, and interestingly it happens when the prior favors alternative \( Y \), rather than the opposite.

We are now ready to state our first result on persuasiveness, when voting is symmetric, for a committee of fixed size.

**Proposition 2 (Symmetric voting)** Suppose voting is symmetric. For all \( k' \leq k \), \( k \)-rule is strictly more persuasive than \( k' \)-rule. In particular, unanimity rule is the most persuasive among all \( k \)-rules.

The first part of Proposition 2 states that if a DM cannot be persuaded by a given \( k \)-rule, then she can never be persuaded by a lower \( k \)-rule. The second part states that if a DM cannot be persuaded by unanimous voting, then she can never be persuaded by symmetric voting. This provides a rationale for the pressure for unanimity, as well as the pressure for a higher level of consensus.

From (2), we see that increasing \( k \) has two opposite effects on persuasiveness. The first effect is a direct consensus effect. By Lemma 5 in the Appendix, MLRP implies that \( F(s^*) < G(s^*) \). Thus, increasing \( k \) means that the right-hand side of (2) increases. Intuitively, more yay votes suggests that the state is more likely to be \( y \). The second effect is an indirect strategic effect. Because of strategic considerations, committee members are more willing to vote for alternative \( Y \) under a higher \( k \)-rule, i.e., \( s(k + 1, N) < s(k, N) \). By Lemma 5 in the Appendix, this decreases the right-hand side of (2). IHRP makes sure that the direct effect dominates the indirect effect. As a result, \((k + 1)\)-rule is always more persuasive than \( k \)-rule.

Figure 1 illustrates \( \alpha(k, N) \) as a function of \( k \) for an example with normally distributed signals.
Our next result considers voting under unanimity rule and ranks persuasiveness of unanimity rule for committees of different sizes.

**Lemma 2** Voting under unanimity rule is always symmetric in a responsive equilibrium. Moreover, the persuasiveness of unanimity rule strictly increases with the size of the committee, i.e., \( \alpha (N, N) \) is strictly increasing in \( N \).

Duggan and Martinelli (2001) prove the first part of Lemma 2 for a decision-making committee. To understand the second part of Lemma 2, note that \( s (N, N) \) is the solution to

\[
\frac{p}{1-p} \left( \frac{1 - F(s)}{1 - G(s)} \right)^{N-1} = \frac{g(s)}{f(s)}.
\]

By MLRP, \( \frac{g(s)}{f(s)} \) is a strictly decreasing function of \( s \). By Lemma 5 in the Appendix, \( \left( \frac{1-F(s)}{1-G(s)} \right)^{N-1} \) is a strictly increasing function of \( s \) and \( N \). Thus, \( s (N, N) \) must be decreasing in \( N \). By IHRP, \( \alpha (N, N) \) is increasing in \( N \). This means that under unanimity, a larger committee is always strictly more persuasive than a smaller committee. Moreover, since unanimity rule is the most persuasive \( k \)-rule, a larger committee is always strictly more persuasive than a smaller one, when voting is symmetric.\(^9\)

\[^{8}\Phi (s) := \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{s} e^{-t^2/2} dt \text{ is the cumulative distribution function of the standard normal distribution.}\
\[^{9}\text{When the signals are binary, a larger committee is not always strictly more persuasive than a smaller one. See the discussion in Section 6.} \]
5.2 Asymmetric voting

In this subsection, we consider asymmetric voting. Can a committee be more persuasive when asymmetric voting is allowed? Proposition 3 says the answer is no, and unanimous voting is still the most persuasive. Thus, in order to be more persuasive, committee members should make recommendations using the same criterion.

**Proposition 3**  *Unanimous voting is the most persuasive.*

The message of Proposition 3 is very clear: if a DM cannot be persuaded by unanimous voting, she can never be persuaded. Since voting under unanimity rule is always symmetric, this means that allowing asymmetric voting cannot make a committee more persuasive.

To understand Proposition 3, consider a pair of equilibrium voting strategy profile and decision rule \((m, d)\) and suppose in equilibrium all committee members use nonpartisan strategies and are pivotal with positive probability. Denote the likelihood ratio conditional on a vote profile \(v\) by \(L(v)\), i.e., \(L(v) := \frac{\Pr(v|\theta=y)}{\Pr(v|\theta=n)}\). When \((m, d)\) constitutes a responsive equilibrium, by optimality of the equilibrium decision rule \(d\), we must have

\[
\frac{p}{1-p} \max_{v \in V^-} L(v) < \frac{\alpha}{1-\alpha} \leq \frac{p}{1-p} \min_{v \in V^+} L(v). \tag{4}
\]

When voting is symmetric, it is easy to find the vote profile that minimizes \(L(v)\) in \(V^+\). Consider a \(k\)-equilibrium, the vote profile, which has exactly \(k\) yay votes and \(n-k\) nay votes, minimizes \(L(v)\) in \(V^+\). Therefore, \(\frac{p}{1-p} \min_{v \in V^+} L(v) = h_{\mathcal{G}(s(N),N)} \) in the \(k\)-equilibrium. When voting is asymmetric, it is however not as easy to figure out which vote profile in \(V^+\) minimizes \(L(v)\). However, while it is not so easy to figure out the persuasiveness of an arbitrary cutoff strategy profile, it is relatively easy to find an upper-bound of persuasiveness for *any* equilibrium cutoff strategy profile. Instead of looking for \(\arg\min_{v \in V^+} L(v)\), we consider the pivotal events \(\text{piv}_i\) of committee member \(i\) whose cutoff is the highest. Then we show that, this member must choose a cutoff higher than \(s(N,N)\) and there must be some profile \(v_{-i} \in \text{piv}_i\) such that

\[
\frac{p}{1-p} L(Y, v_{-i}) < \frac{\alpha(N,N)}{1-\alpha(N,N)}. \tag{4}
\]

Intuitively, asymmetric voting leads to some vote profiles that favor alternative \(Y\) more, and some other vote profiles that favor status quo \(N\) more. These vote
profiles cannot average out, because the DM can tell these vote profiles apart as she observes every single vote.

Next, suppose in equilibrium some committee members use partisan strategies or are never pivotal. By Proposition 1, in this case the committee in effect becomes a committee of a smaller size. By Lemma 2, this can only reduce persuasiveness.

If we focus on the class of equilibria in which the equilibrium decision rule is a \( k \)-rule, we can also show that symmetric voting is more persuasive than asymmetric voting for any specific \( k \)-rule.

**Proposition 4** Given a \( k \)-rule, symmetric voting is strictly more persuasive than asymmetric voting. Moreover, for all \( k' \leq k \), \( k \)-rule is strictly more persuasive than \( k' \)-rule.

Proposition 4 states that allowing asymmetric voting does not make a \( k \)-rule more persuasive. Therefore, the ranking of \( k \)-rules in terms of persuasiveness in Proposition 2 still holds.

The idea of the proof is similar to that of Proposition 3. Fixing a specific \( k \)-rule, we can show that \( \frac{p}{1-p} \min_{v \in V} L(v) \leq \frac{\alpha(k,N)}{1-\alpha(k,N)} \), which implies that symmetric voting is more persuasive than asymmetric voting for a given \( k \)-rule.

The last result of this section is a corollary of Proposition 3, which provides the upper-bound of the conflict of interest between the DM and the committee members below which the DM can be persuaded.

**Corollary 1 (Existence of a responsive equilibrium)** A DM can be persuaded if and only if \( \alpha \leq \alpha(N,N) \). Moreover, for any \( \alpha < 1 \), there exists an \( N(\alpha) \) such that a DM can be persuaded when the committee has at least \( N(\alpha) \) members.

The corollary has two parts. The first part provides a tight upper-bound for the degree of the conflict of interest between the DM and the committee members above which the DM cannot be persuaded. This upper-bound can be achieved by unanimous voting. Gradwohl and Feddersen (2018) also derive an upper-bound in a binary-signal model, but it is not achievable by unanimous voting. For more general discrete-signal models, Battaglini (2017) derives an upper-bound that is not tight. In the next section, we will discuss the difference between the continuous-signal
model and the discrete-signal model, with a special focus on the binary case. The second part of the corollary states that a committee can be arbitrarily persuasive when the committee is large enough. This result relies on Assumption 2.

6 Discrete signals

In the baseline model, we assume that each committee member receives a continuous signal. In this section, we consider the situation in which each member receives a discrete signal. We first focus on the binary case and then discuss briefly the general case.

6.1 Binary signals

We first consider the binary-signal model, which has been studied by Gradwohl and Feddersen (2018). The assumption of binary signal is a common one in the collective decision-making literature. It is often thought to be an innocuous assumption that provides a good approximation of models with more general information structures. As in the previous section, we first analyze the persuasiveness of \( k \)-rules under symmetric voting, which is not discussed by Gradwohl and Feddersen (2018), then we discuss whether asymmetric voting can make a committee more persuasive and how persuasive a committee can be.

Before voting, each committee member \( i \) receives a private signal \( s_i \) about the state \( \theta \). The prior of the state being \( y \) is \( \frac{1}{2} \). The signal is binary, i.e., \( s_i \in \{Y, N\} \). The signal is informative and symmetric, i.e.,

\[
\Pr (s_i = Y | \theta = y) = \Pr (s_i = N | \theta = n) = q,
\]

where \( q \in \left( \frac{1}{2}, 1 \right) \). The specification so far is identical to the one in Gradwohl and Feddersen (2018). For simplicity, we further assume that \( N \) is odd.

When the signals are binary, a voting strategy of committee member \( i \) is a pair of probability \((\rho_N^i, \rho_Y^i)\), where \( \rho_N^i \) is the probability of voting for alternative \( Y \) after receiving an \( N \)-signal and \( \rho_Y^i \) is the probability of voting for alternative \( Y \) after receiving a \( Y \)-signal. Voting is truthful, if voting is according to one’s own signal, i.e., \((\rho_N^i, \rho_Y^i) = (0, 1)\); and voting is informative, if voting is responsive to the signal, i.e., \( \rho_Y^i \neq \rho_N^i \).
We first restrict our attention to symmetric voting. That is, for all committee member \( i \), \( (\rho^i_N, \rho^i_Y) = (\rho_N, \rho_Y) \). In a responsive equilibrium, if voting is symmetric and increasing, the equilibrium decision rule must be a \( k \)-rule. Given a \( k \)-rule, there exists a unique symmetric voting strategy that is optimal and informative. When \( k = \frac{N+1}{2} \), voting is truthful. When \( k > \frac{N+1}{2} \), the committee members vote for both options with positive probability after receiving an \( N \)-signal, i.e., \( \rho_N \in (0,1) \) and \( \rho_Y = 1 \). When \( k < \frac{N+1}{2} \), the committee members vote for both options with positive probability after receiving a \( Y \)-signal, i.e., \( \rho_N = 0 \) and \( \rho_Y \in (0,1) \).

For all \( k > \frac{N+1}{2} \), since a committee members are indifferent between voting for alternative \( Y \) and status quo \( N \) conditional on being pivotal after receiving an \( N \)-signal, we must have

\[
\frac{1}{2} (1-q) (q + (1-q) \rho_N)^{k-1} ((1-q) (1-\rho_N))^{N-k} - \frac{1}{2} q (q \rho_N + (1-q))^{k-1} (q (1-\rho_N))^{N-k} = 0,
\]

which means that \( \rho_N \) is the solution to

\[
\left( \frac{q \rho_N + (1-q)}{q + (1-q) \rho_N} \right)^{k-1} \left( \frac{q}{1-q} \right)^{N-k+1} = 1. \tag{5}
\]

Since the left-hand side of (5) is smaller than 1 when \( \rho_N = 0 \), larger than 1 when \( \rho_N = 1 \), and strictly increasing in \( \rho_N \), (5) has a unique solution in \((0,1)\).

**Definition 3** For \( k > \frac{N+1}{2} \), \( \rho_N (k, N) \) is defined as the unique solution to (5).

Given the definition of \( \rho_N (k, N) \), define

**Definition 4** \( \alpha_2 (k,N) \) is defined by

\[
\alpha_2 (k,N) := \begin{cases} 
\frac{1}{2} & \text{if } k < \frac{N+1}{2}, \\
q & \text{if } k = \frac{N+1}{2}, \\
\frac{q(q+(1-q)\rho_N(k,N))}{1-2q(1-q)(1-\rho_N(k,N))} & \text{if } k > \frac{N+1}{2}.
\end{cases}
\]

Note that, for \( k > \frac{N+1}{2} \), \( \alpha_2 (k,N) \) is the solution to

\[
\frac{\alpha}{1-\alpha} = \frac{q}{1-q} \frac{q + (1-q) \rho_N (k,N)}{q \rho_N (k,N) + (1-q)}. \tag{6}
\]
The right-hand side of (6) is the likelihood ratio of a \( Y \)-signal and a yay vote, given the voting strategy \( \rho_N (k, \mathcal{N}, 1) \). Since \( \rho_N (k, \mathcal{N}) \in (0, 1) \), the right-hand side of (6) is strictly larger than \( \frac{q}{1-q} \) and smaller than \( \left( \frac{q}{1-q} \right)^2 \). The following lemma shows when a DM can be persuaded.

**Lemma 3** When voting is symmetric,

1. A DM can never be persuaded by a \( k \)-rule with \( k < \frac{\mathcal{N}+1}{2} \);
2. A DM can be persuaded by a \( k \)-rule with \( k \geq \frac{\mathcal{N}+1}{2} \) if and only if \( \alpha \leq \alpha_2 (k, \mathcal{N}) \).

We also refer to such a symmetric responsive equilibrium as a \( k \)-equilibrium. For a \( k \)-rule to be supported in a responsive equilibrium, it must be the case that the DM finds it optimal to follow the given \( k \)-rule, that is,

\[
\frac{\Pr (|v| = k - 1 | \theta = y)}{\Pr (|v| = k - 1 | \theta = n)} < \frac{\alpha}{1 - \alpha} \leq \frac{\Pr (|v| = k | \theta = y)}{\Pr (|v| = k | \theta = n)}.
\] (7)

There are three cases. In a \( k \)-equilibrium where \( k = \frac{\mathcal{N}+1}{2} \), voting is truthful. Observing \( \frac{\mathcal{N}+1}{2} \) yay votes and \( \frac{\mathcal{N}-1}{2} \) nay votes is equivalent to observing \( \frac{\mathcal{N}+1}{2} \) \( Y \)-signals and \( \frac{\mathcal{N}-1}{2} \) \( N \)-signals. Because the signals are symmetric, it is also equivalent to observing a single \( Y \)-signal. Therefore, a DM is willing to choose alternative \( Y \) after observing \( \frac{\mathcal{N}+1}{2} \) yay votes and \( \frac{\mathcal{N}-1}{2} \) nay vote given truthful voting if \( \alpha \leq q \).

In a \( k \)-equilibrium where \( k > \frac{\mathcal{N}+1}{2} \), the committee members vote for both options with positive probability after receiving an \( N \)-signal. Therefore, committee member \( i \) is indifferent between alternative \( Y \) and status quo \( N \) conditional on being pivotal after receiving an \( N \)-signal, which implies that, conditioning on \( k - 1 \) yay votes and \( \mathcal{N} - k \) nay votes is equivalent to observing a \( Y \)-signal, by signal symmetry. For the DM, observing \( k \) yay votes and \( \mathcal{N} - k \) nay votes is equivalent to observing a \( Y \)-signal and a yay vote. Because the committee members cast yay votes with positive probability after receiving an \( N \)-signal, the DM’s posterior belief is strictly lower than that conditional on two \( Y \)-signals. The more often the committee members vote for alternative \( Y \) after receiving an \( N \)-signal, the lower the DM’s posterior belief conditional on a \( Y \)-signal and a yay vote is. Therefore, a DM is willing to choose alternative \( Y \) after observing \( k \) yay votes and \( \mathcal{N} - k \) nay vote if \( \alpha \leq \alpha_2 (k, \mathcal{N}) \).
In a $k$-equilibrium where $k < \frac{N+1}{2}$, the committee members vote for both options with positive probability after receiving a $Y$-signal. Similar to the logic in the previous case, for the DM, observing $k$ yay votes and $N-k$ nay votes is equivalent to observing an $N$-signal and a yay vote. Because the committee members cast yay votes only after receiving a $Y$-signal, the DM’s posterior belief is exactly equal to that conditional on observing a $Y$-signal and an $N$-signal. Due to signal symmetry, the DM’s posterior is exactly $\frac{1}{2}$, after observing $k$ yay votes and $N-k$ nay votes. Since $\alpha > \frac{1}{2}$, she is never willing to choose alternative $Y$. Therefore, no DM can be persuaded by a minority rule.

Given the equilibrium characterization, we immediately have

**Proposition 5** When voting is symmetric,

1. Simple majority rule is the least persuasive among all $k$-rules satisfying $k \geq \frac{N+1}{2}$;

2. For all $k > \frac{N+1}{2}$ and $k' \geq k$, $k$-rule is strictly more persuasive than $k'$-rule. In particular, unanimity rule is the least persuasive among all $k$-rules such that $k > \frac{N+1}{2}$.

Figure 2 illustrates Proposition 5 by plotting $\alpha_2(k,N)$ against $k$.

![Graph showing the persuasiveness of $k$-rules under symmetric voting in the binary case. Parameters: $N = 21$, $q = 0.7$.](image)

Figure 2: Persuasiveness of $k$-rules under symmetric voting in the binary case.

Parameters: $N = 21$, $q = 0.7$.

It is easy to see that $\alpha_2(k,N) > q$ for all $k > \frac{N+1}{2}$, so if a DM can be persuaded by simple majority rule, then she can also be persuaded by any other majority rules. For all $k > \frac{N+1}{2}$, because $\rho_N(k,N)$ is strictly increasing in $k$, $\alpha_2(k,N)$ is strictly decreasing in $k$. Thus, if a DM
can be persuaded by unanimity rule, then she can also be persuaded by any other supermajority rules.

To understand why persuasiveness decreases with the consensus level \( k \) in the binary-signal model but increases in the continuous-signal model, consider the continuous-signal model and suppose the equilibrium voting cutoff is \( s \). Assuming \( p = \frac{1}{2} \), the likelihood ratio of \( k \) yay votes and \( n - k \) nay votes is given by

\[
\frac{\Pr (|v| = k|\theta = y)}{\Pr (|v| = k|\theta = n)} = \frac{g(s)}{f(s)} \times \frac{1 - F(s)}{1 - G(s)}.
\]

For a given signal \( s \), consider a hypothetical signal \( s^- \), such that \( \frac{f(s^-)}{g(s^-)} = \frac{g(s)}{f(s)} \). A signal \( s^- \) cancels a signal \( s \) exactly, that is, the posterior conditional on a signal \( s^- \) and a signal \( s \) is exactly the prior. We call \( s^- \) the anti-signal of signal \( s \). Therefore, the likelihood ratio of observing \( k \) yay votes and \( N - k \) nay votes is equal to observing an anti-signal \( s^- \) and a yay vote, which indicates a signal that is larger than the cutoff \( s \). By MLRP, when \( s \) decreases, the likelihood ratio of the anti-signal \( s^- \) increases, while that of a yay vote decreases. However, by IHRP, the likelihood ratio of the anti-signal \( s^- \) increases “faster” than the decrease of that of a yay vote. Therefore, the likelihood ratio of \( k \) yay votes and \( N - k \) nay votes decreases with \( s \). A higher level of consensus \( k \) reduces the equilibrium cutoff \( s(k, N) \), which means that persuasiveness increases with \( k \).

Consider next the binary-signal model. Suppose \( k > \frac{N+1}{2} \). The likelihood ratio of \( k \) yay votes and \( N - k \) nay votes is equal to the likelihood ratio of a \( Y \)-signal (the anti-signal of the \( N \)-signal) and a yay vote, i.e.,

\[
\frac{\Pr (|v| = k|\theta = y)}{\Pr (|v| = k|\theta = n)} = \frac{q}{1-q} \times \frac{q + (1 - q) \rho_N (k, N)}{q \rho_N (k, N) + (1 - q)}.
\]

The likelihood ratio of a \( Y \)-signal is constant, and the likelihood ratio of a yay vote depends on how often the committee members vote for alternative \( Y \) after receiving an \( N \)-signal. When the equilibrium consensus level increases, the committee members vote for alternative \( Y \) more
often after receiving an $N$-signal, which means that the likelihood ratio of a yay vote is lower. Unlike in the continuous-signal model, the decrease in the likelihood ratio of the yay vote is not compensated by an increase in the likelihood ratio of a $Y$-signal. As a result, when the committee members vote for alternative $Y$ more often after receiving an $N$-signal, the vote profile with $k$ yay votes and $N - k$ nay votes leads to a lower posterior belief.

Now, we will discuss another difference between the binary-signal model and the continuous-signal model, when asymmetric voting is allowed. Consider

**Definition 5** $\bar{\alpha}$ is defined by

$$\bar{\alpha} := \frac{\left(\frac{q}{1-q}\right)^2}{1 + \left(\frac{q}{1-q}\right)^2}.$$ 

We have

**Proposition 6** Suppose $N \in \{3, 5, 7, ..\}$. A DM can be persuaded if and only if $\alpha \leq \bar{\alpha}$. Moreover, asymmetric voting is strictly more persuasive than symmetric voting.

Gradwohl and Feddersen (2018) derive this result.\superscript{10} We include it here for completeness. Intuitively, if a DM strictly prefers status quo $N$ given two $Y$-signals, then she can never be persuaded by any committee. As Lemma 3 suggests, a DM can be persuaded by a $k$-rule only if $\alpha \leq \alpha_2(k, N)$, and $\alpha_2(k, N)$ is always strictly smaller than $\bar{\alpha}$. From the discussion of Proposition 5, the persuasiveness of a committee depends on the likelihood of a $Y$-signal and a yay vote, which can never be higher than that of two $Y$-signals. This implies that symmetric voting cannot achieve the upperbound of persuasiveness $\bar{\alpha}$. When asymmetric voting is allowed, the upperbound is achieved by an asymmetric voting strategy profile, in which one committee member always votes for status quo $N$ while all other members vote truthfully.

To summarize, there are four main differences between the binary-signal model considered in this section and the continuous-signal model in the previous sections. First, unanimity rule is the most persuasive when the signals are continuous, but the least persuasive among all the supermajority rules when the signals are binary. Second, restricting to supermajority rules, when

\superscript{10}Gradwohl and Feddersen (2018) state the “only if” part of our Proposition 6 as a separate lemma (Lemma 1) while noting in the text that if $\alpha \leq \bar{\alpha}$, the asymmetric voting strategy profile we use here is always persuasive.
voting is symmetric, a higher $k$-rule is more persuasive when the signals are continuous, but less persuasive when the signals are binary. Third, when the signals are binary, asymmetric voting can make a committee more persuasive. This is not possible in the continuous-signal model. Finally, any DM can be persuaded by a committee that is large enough in the continuous-signal model but not in the binary-signal model.

The first three differences arise because of the assumption of binary signals. We have deliberately chosen a set of parameters that makes the comparison easiest and sharpest. More generally, these results would depend on the prior probability $p$ (which could be nonuniform), the signals (which could be asymmetric) and the size of the committee (which could be even). However, the point remains that the binary assumption generates results that are very different from those of the continuous model. In the next subsection, we will discuss what the ordering of $k$-rules looks like in a general discrete model. The last difference comes from Assumption 2, and has little to do with whether the signals are continuous or not. If Assumption 2 is relaxed, the last difference disappears, and we get a result similar to both Battaglini (2017) and Gradwohl and Feddersen (2018) in the continuous-signal model.

### 6.2 General discrete signals

Now we discuss briefly the case when the committee members receive discrete signals that can take more than two values, similar to the one considered in Battaglini (2017), who does not discuss the ranking of $k$-rules in terms of their persuasiveness.\footnote{Battaglini (2017) models the voting problem using the Poisson game approach introduced by Myerson (1998a, 1998b, 2000). Therefore, his model is, strictly speaking, different from ours. However, the two approaches often produce similar results.} Suppose member $i$ receives a private signal $s_i \in \{t_1, t_2, \ldots t_M\}$, where $M \geq 2$ is the number of possible signal realizations. Let $q_F(t_m)$ and $q_G(t_m)$ be the probabilities that $s_i = t_m$ when the state is $y$ and $n$, respectively. Denote $h_F(t_m) := q_F(t_m) / \sum_{l=m}^M q_F(t_l)$ and $h_G(t_m) := q_G(t_m) / \sum_{l=m}^M q_G(t_l)$. MLRP and IHRP become:

**Assumption 4 (MLRP)** $F(.)$ and $G(.)$ satisfy the monotone likelihood ratio property (MLRP), i.e., $q_F(t_m) / q_G(t_m)$ is strictly increasing in $m$.\footnote{Battaglini (2017) models the voting problem using the Poisson game approach introduced by Myerson (1998a, 1998b, 2000). Therefore, his model is, strictly speaking, different from ours. However, the two approaches often produce similar results.}
Assumption 5 (IHRP) $F(.)$ and $G(.)$ satisfy the increasing hazard ratio property (IHRP), i.e., $h_{F}(t_{m}) / h_{G}(t_{m})$ is strictly increasing in $m$.

Notice that when the signals are discrete, there is no loss to assume that MLRP holds. This is because we can simply reorder the signals and combine signals that have the same likelihood ratio.

We focus on symmetric voting. When the signals are discrete, the ordering of $k$-rules in terms of their persuasiveness follows patterns that combine features of the binary case and the continuous case.

To see that, consider a symmetric responsive equilibrium in which the committee members are indifferent between the two options after receiving signal $t_{m}$. Then, in equilibrium, the posterior odds from observing $k$ yay votes and $N - k$ nay votes is given by

$$\frac{p \ Pr (|v| = k | \theta = y)}{1 - p \ Pr (|v| = k | \theta = n)} = \frac{q_{G}(t_{m})}{q_{F}(t_{m})} \left( q_{F}(t_{m}) \rho_{m}(k, N) + \sum_{l=m+1}^{M} q_{F}(t_{l}) \right) \left( q_{G}(t_{m}) \rho_{m}(k, N) + \sum_{l=m+1}^{M} q_{G}(t_{l}) \right)$$

where $\rho_{m}(k, N)$ is the probability that a committee member votes for alternative $Y$ after receiving signal $t_{m}$.

As $k$ increases, the committee members become more ready to vote for alternative $Y$. If the committee members remain indifferent at $t_{m}$, this means that $\rho_{m}(k, N)$ increases. By Lemma 7 in the Appendix, MLRP implies that the likelihood ratio of a yay vote decreases. Thus, the persuasiveness of a $k$-rule decreases with $k$, provided that it does not change the committee members’ indifferent signal.

On the other hand, since $\rho_{m}(k, N) \leq 1$, when the committee members are indifferent at signal $t_{m}$, the posterior odds is bounded below by the inverse of the hazard ratio, i.e.,

$$\frac{p \ Pr (|v| = k | \theta = y)}{1 - p \ Pr (|v| = k | \theta = n)} \geq \frac{q_{G}(t_{m})}{q_{F}(t_{m})} \left( q_{F}(t_{m}) \sum_{l=m+1}^{M} q_{F}(t_{l}) \right) \left( q_{G}(t_{m}) \sum_{l=m+1}^{M} q_{G}(t_{l}) \right)^{-1} = \frac{h_{G}(t_{m})}{h_{F}(t_{m})},$$

12 This bound is in fact valid whenever the committee members weakly prefer voting for alternative $Y$ after receiving signal $t_{m}$.
By IHRP, \( h_G(t_m) / h_F(t_m) \) is strictly decreasing in \( m \). As a result, as \( k \) increases and the signal at which the committee members are indifferent decreases, the lower bound rises and persuasiveness may rise.

Figure 3 illustrates how \( \alpha_M(k,N) \) varies with \( k \) in an example where the signals take four different values.

![Figure 3: Persuasiveness of k-rules under symmetric voting in the discrete case.](image)

Parameters: \( p = \frac{1}{2}, N = 21, q_F = \left( \frac{1}{8}, \frac{3}{16}, \frac{1}{4}, \frac{7}{16} \right), q_G = \left( \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4} \right) \).

When \( k \leq 7 \), the committee members vote for both options with positive probability after receiving signal \( t_4 \). Since \( t_4 \) is the highest possible signal, we have

\[
\Pr(|v| = k| \theta = y) = \frac{q_G(t_4) q_F(t_4) \rho_4(k,N)}{q_F(t_4) q_G(t_4) \rho_4(k,N)} = 1.
\]

This means that the committee cannot persuade any DM with \( \alpha > \frac{1}{2} \). When \( 8 \leq k \leq 12 \), the committee members vote for both options with positive probability after receiving signal \( t_3 \). The persuasiveness of the committee first jumps above \( \frac{1}{2} \) and then decreases. The same is true for \( 14 \leq k \leq 17 \) and \( 19 \leq k \leq 21 \), when the committee members are indifferent after receiving signals \( t_2 \) and \( t_1 \), respectively. On the other hand, compare \( k = 12, k = 17 \), and \( k = 21 \), for example, we have \( \alpha_M(12,21) < \alpha_M(17,21) < \alpha_M(21,21) \). The persuasiveness of the committee increases with \( k \) when the change in \( k \) is large enough. Note that unanimity rule is not the most persuasive in this example.

To summarize, when the signals are discrete and can take more than two values, MLRP implies that persuasiveness decreases with \( k \) if the committee members remain indifferent after
receiving the same signal, while IHRP produces an upward trend in persuasiveness as the number of signals given which the committee members vote for alternative \( Y \) increases.

Finally, a bound on the committee’s ability to persuade similar to the one in Lemma 1 in Battaglini (2017) can be obtained by bounding the two terms in (8) individually by choosing \( t_1 \) for the anti-signal and \( t_M \) for the yay vote. Consider

\[ \bar{\alpha}_M \text{ is defined by} \]

\[ \bar{\alpha}_M := \frac{\frac{q_G(t_1) q_F(t_M)}{q_F(t_1) q_G(t_M)}}{1 + \frac{q_G(t_1) q_F(t_M)}{q_F(t_1) q_G(t_M)}}. \]

If \( \alpha > \bar{\alpha}_M \), then the DM can never be persuaded. Notice, however, that this bound is not tight. In the example presented in Figure 3, \( \bar{\alpha}_M = \frac{7}{9} \approx 0.778 \).

7 Conclusion

We consider a voting model in which members of an advisory committee receive private continuous signals and vote on a policy change before a decision maker makes the final decision. Our approach differs from existing works on voting rules in that we treat the voting rule as a part of an equilibrium. We show that, in order to persuade a conservative DM, the committee should vote symmetrically and unanimously. Our results offer a new perspective on an old debate between Feddersen and Pesendorfer (1998) and Coughlan (2000). In our setting, the committee must first be able to persuade the DM before information could be put into use. We suggest that a more stringent majority requirement could be adopted to overcome the persuasiveness hurdle, even though it may result in a suboptimal use of information.

While we think that in many situations modeling information with continuous signals is a more natural and realistic choice, we also study the model with discrete signals. We show that this simplification is not innocuous. When the number of possible signal realizations is small, it significantly alters much of the results. Modelers, therefore, should be heedful of such a possibility when applying voting models. Discrete-signal models should be used when it better approximates reality, but not as an approximation of the continuous-signal model.
Finally, we have made several assumptions on the environment: 1) the committee is homogenous, 2) the DM observes each vote, 3) there is no pre-voting communication, 5) information is exogenous. Relaxing these assumptions could lead to a better understanding of how advisory committees work.

References


Appendix

Lemma 4 Suppose in equilibrium member $i$ uses a nonpartisan strategy and is pivotal with positive probability. Then, member $i$ uses a cutoff strategy.

Proof of Lemma 4. Suppose in equilibrium member $i$ uses a nonpartisan strategy and is pivotal with positive probability. Denote the set of vote profiles of the other committee members, given which member $i$ is pivotal by $\text{piv}_i$. Since $\Pr(\text{piv}_i) > 0$, there exists $v_{-i} \in \text{piv}_i$ such that $\Pr(v_{-i}) > 0$. Since the committee member $i$ uses a nonpartisan strategy in equilibrium, for all $v_{-i} \in \text{piv}_i$, $\Pr(v_{-i}) > 0$ implies that $\Pr(v = (Y, v_{-i})) > 0$ and $\Pr(v = (N, v_{-i})) > 0$. This means that the DM can update her belief using Bayes’ rule after seeing the vote profiles $(Y, v_{-i})$ or $(N, v_{-i})$. Denote the likelihood ratio conditional on a yay vote from member $i$ by $L_i := \frac{\Pr(v = Y \mid \theta = y)}{\Pr(v = Y \mid \theta = n)}$. We must have either $L_i > 1$ or $L_i < 1$. Otherwise, member $i$’s vote is uninformative. Since the DM always chooses $Y$ in the case of indifference by assumption, this means that for all $v_{-i} \in \text{piv}_i$ such that $\Pr(v_{-i}) > 0$, the DM takes the same action after seeing $(Y, v_{-i})$ or $(N, v_{-i})$. This contradicts the assumption that member $i$ is pivotal with positive probability.

If $L_i > 1$ ($L_i < 1$), casting a yay vote can only change the final outcome from $N$ to $Y$ ($Y$ to $N$). In the first case, given the others’ voting strategies $m_{-i}$ and the DM’s decision rule $d$, member $i$ with signal $s_i$ voting for alternative $Y$ gets

$$\frac{pf(s_i)}{pf(s_i) + (1-p)g(s_i)} \Pr(\text{piv}_i \mid \theta = y) - \frac{(1-p)g(s_i)}{pf(s_i) + (1-p)g(s_i)} \Pr(\text{piv}_i \mid \theta = n).$$

If he votes for status quo $N$, he gets

$$-\frac{pf(s_i)}{pf(s_i) + (1-p)g(s_i)} \Pr(\text{piv}_i \mid \theta = y) + \frac{(1-p)g(s_i)}{pf(s_i) + (1-p)g(s_i)} \Pr(\text{piv}_i \mid \theta = n).$$

When member $i$ with signal $s_i$ is indifferent between voting for alternative $Y$ and status quo $N$, we have

$$\frac{pf(s_i)}{(1-p)g(s_i)} = \frac{\Pr(\text{piv}_i \mid \theta = n)}{\Pr(\text{piv}_i \mid \theta = y)},$$

where the left-hand side is strictly increasing in $s_i$ by MLRP and the right-hand side is strictly
positive and independent of $s_i$. Therefore, by Assumption 2, a solution to (9) exists and is unique. Member $i$ must use an increasing cutoff strategy in equilibrium. Similarly, in the second case, member $i$ must use a decreasing cutoff strategy in equilibrium.

**Proof of Proposition 1.** Consider an arbitrary equilibrium $(m, d)$ of our model. In equilibrium, the committee members can be classified into three types. Type I members use partisan strategies. Type II members use nonpartisan strategies, but are never pivotal. Type III members use nonpartisan strategies and are pivotal with positive probability. We construct the desired equilibrium $(\hat{m}, \hat{d})$ by going through the committee members one by one. Let $(m^0, d^0) = (m, d)$ and re-order the committee members according to their types so that type I members appear first, type II second, and type III last. For each step $i$, we modify committee member $i$’s strategy and the DM’s decision rule and show that $(m^i, d^i)$ remains an equilibrium and is outcome-equivalent to $(m^{i-1}, d^{i-1})$. The final product $(m^N, d^N)$ will satisfy the desired properties. Consider step $i$. There are three cases.

1. Suppose in equilibrium $(m^{i-1}, d^{i-1})$ member $i$ is a type I member. We consider only the case when member $i$ always votes for $Y$. The other case is similar. We modify the decision rule such that, for all $v_{-i} \in \{Y, N\}^{N-1}$, $d^i(N, v_{-i}) = d^{i-1}(Y, v_{-i})$, and leave the strategy profile unchanged, that is, $m^i = m^{i-1}$. After the modification, $d^i$ is constant in $m^i$. Since $d^{i-1}$ is an equilibrium decision rule given $m^{i-1}$, for all $v_{-i} \in \{Y, N\}^{N-1}$, $d^i(N, v_{-i}) = d^{i-1}(Y, v_{-i})$ is optimal. For all $v_{-i} \in \{Y, N\}^{N-1}$, $(N, v_{-i})$ does not occur under $m^i$, so $d^i(N, v_{-i}) = d^i(Y, v_{-i})$ is also optimal, if we simply assign the DM the same belief as $(Y, v_{-i})$. Under the new decision rule $d^i$, member $i$ is never pivotal. Thus, the partisan strategy is optimal for member $i$. For member $j \neq i$, $d^i$ assigns the same outcome to any vote profile that occurs with positive probability as $d^{i-1}$. Thus, $m^i_j$ is optimal for member $j$. Hence, $(m^i, d^i)$ is an equilibrium. Finally, since $d^i$ assigns the same outcome to any vote profile that occurs with positive probability as $d^{i-1}$, $(m^i, d^i)$ is clearly outcome-equivalent to $(m^{i-1}, d^{i-1})$.

2. Suppose in equilibrium $(m^{i-1}, d^{i-1})$ member $i$ is a type II member. We modify the strategy profile such that $m^i_j(.) = 1$, and the decision rule such that, for all $v_{-i} \in \{Y, N\}^{N-1}$,
\[ d^i (Y; v_{-i}) = d^i (N; v_{-i}) = d^{i-1} (Y; v_{-i}). \] After the modification, member \( i \) uses a partisan strategy and the decision rule \( d^i \) is constant in \( m_i^i \). For all \( v_{-i} \in \{Y, N\}^{N-1} \) such that \( \Pr (v_{-i}) > 0 \) given \( m^{i-1}_i \), since member \( i \) is never pivotal, we must have \( d^{i-1} (Y; v_{-i}) = d^{i-1} (N; v_{-i}) \). Thus, \( d^i \) assigns the same outcomes to all these profiles \( v_{-i} \) as \( d^{i-1} \). Therefore, \( d^i \) is optimal. For all \( v_{-i} \in \{Y, N\}^{N-1} \) such that \( \Pr (v_{-i}) = 0 \), \( d^i \) is also optimal, if we assign the DM the proper off-the-equilibrium-path belief. Under the new decision rule \( d^i \), member \( i \) is never pivotal. Thus, the partisan strategy is optimal for him. For member \( j \neq i \), if \( m^{i-1}_j \) is partisan, then \( m^i_j \) is still optimal because, by Part 1 and Part 2 of this proof, the decision rule \( d^i \) is constant in \( m^i_j \); if \( m^{i-1}_j \) is nonpartisan, \( m^i_j \) is still optimal because, under \( (m^{i-1}, d^{i-1}) \), the probability that \( Y \) is chosen given member \( j \)'s vote and any signal profile for members \( -j \) is the same as before. This also means that \( (m^i_j, d^i) \) is outcome-equivalent to \( (m^{i-1}, d^{i-1}) \).

3. Suppose in equilibrium \( (m^{i-1}, d^{i-1}) \) member \( i \) is a type III member. By Lemma 4, \( m^{i-1}_i \) is a cutoff strategy, which can be either increasing or decreasing. If \( m^{i-1}_i \) is increasing, we leave the strategy profile and the decision rule unchanged, i.e., \( (m^i, d^i) = (m^{i-1}, d^{i-1}) \). If \( m^{i-1}_i \) is decreasing, we modify the strategy profile such that \( m^i_i (.) = 1 - m^{i-1}_i (.) \), and the decision rule such that for all \( v_{-i} \in \{Y, N\}^{N-1} \), \( d^i (Y; v_{-i}) = d^{i-1} (N; v_{-i}) \) and \( d^i (N; v_{-i}) = d^{i-1} (Y; v_{-i}) \). This in effect relabels member \( i \)'s a yay vote as a nay vote and a nay vote as a yay vote. Obviously, \( d^i \) is optimal for the DM and \( m^i_j \) is optimal for member \( j \). \( (m^i, d^i) \) remains an equilibrium and is outcome-equivalent to \( (m^{i-1}, d^{i-1}) \).

It remains to show that \( \hat{d} = d^N \) is a weighted voting rule. We would like to find a pair of \( w \) and \( Q \) that represents \( \hat{d} \). For player \( i \) who uses a cutoff strategy \( s^*_i \), denote the likelihood ratio conditional on a yay vote from player \( i \) by \( L^+_i \), i.e., \( L^+_i = \frac{1-F(s^*_i)}{1-G(s^*_i)} \), and the likelihood ratio conditional on a nay vote from player \( i \) by \( L^-_i \), i.e., \( L^-_i = \frac{F(s^*_i)}{G(s^*_i)} \). Let the weight of a yay vote from member \( i \) be \( \ln L^+_i - \ln L^-_i \), i.e.,

\[
  w_i := \ln L^+_i - \ln L^-_i.
\]

For player \( i \) who uses a partisan strategy, set \( w_i := 0 \).
Denote the total weight of vote profile \( v \) by \( W(v) := \sum_{i=1}^{N} w_i 1\{v_i = Y\} \). Now we show that for all vote profiles \( v \) and \( v' \) that occur with positive probability in equilibrium, the total weight of \( v \) is larger than the total weight of \( v' \) if and only if \( \Pr (\theta = y|v) \geq \Pr (\theta = y|v') \).

Let \( C \) be the set of committee members who use cutoff strategies. Consider a vote profile \( v \), the likelihood ratio conditional on \( v \) is

\[
\prod_{i \in C \cap \{i: v_i = Y\}} \frac{1 - F(s_i^*)}{1 - G(s_i^*)} \prod_{i \in C \cap \{i: v_i = N\}} \frac{F(s_i^*)}{G(s_i^*)}
\]

\[
= \exp \left( \sum_{i \in C \cap \{i: v_i = Y\}} \ln L_i^+ + \sum_{i \in C \cap \{i: v_i = N\}} \ln L_i^- \right)
\]

\[
= \exp \left( \sum_{i \in C \cap \{i: v_i = Y\}} \ln L_i^+ - \sum_{i \in C \cap \{i: v_i = N\}} \ln L_i^- + \sum_{i \in C} \ln L_i^- \right)
\]

\[
= \exp (W(v) + c),
\]

where \( c \) is a constant. This implies that the total weight is strictly increasing in \( \Pr (\theta = y|v) \).

Now we find the quota \( Q \). Given the equilibrium decision rule \( \hat{d} \) and voting strategy \( \hat{m} \), consider the corresponding \( V^+ \). There exists a \( \underline{v} \in V^+ \) that such that \( \Pr (\underline{v}) > 0 \) and for all \( v \in V^+ \) that occur with positive probability in equilibrium, we have

\[
\Pr (\theta = y|\underline{v}) \leq \Pr (\theta = y|v).
\]

Set \( Q \) to be the total weight of vote profile \( \underline{v} \), that is,

\[
Q := W(\underline{v}).
\]

This proves that \( \hat{d} \) is a weighted voting rule. \( \blacksquare \)

Lemma 5 summarizes some basic properties of distributions that satisfy MLRP that will be useful later in the proofs.

**Lemma 5** Suppose \( F(\cdot) \) and \( G(\cdot) \) satisfy MLRP. Then,
1. For all \( s \in (a, b) \),
\[
\frac{f(s)}{1 - F(s)} < \frac{g(s)}{1 - G(s)} \quad \text{and} \quad \frac{f(s)}{F(s)} > \frac{g(s)}{G(s)};
\]

2. For all \( s \in (a, b) \),
\[
F(s) < G(s);
\]

3. \( \frac{1 - F(s)}{1 - G(s)} \) and \( \frac{F(s)}{G(s)} \) are strictly increasing in \( s \);

**Proof of Lemma 5.** Part 1 follows from the proofs that likelihood ratio dominance implies hazard rate dominance and reverse hazard rate dominance (See Shaked and Shanthikumar (2007)). It is enough to note that the proofs apply when the weak inequalities are replaced by strict inequalities.

Part 2 follows from the proof that likelihood ratio dominance implies first-order stochastic dominance. Again, we note that the proof goes through when the weak inequality are replaced by strict inequality.

Part 3:
\[
\frac{\partial}{\partial s} \left( \frac{1 - F(s)}{1 - G(s)} \right) = \frac{1 - F(s)}{1 - G(s)} \left( \frac{g(s)}{1 - G(s)} - \frac{f(s)}{1 - F(s)} \right) > 0,
\]
where the inequality follows from Part 1.

\[
\frac{\partial}{\partial s} \left( \frac{F(s)}{G(s)} \right) = \frac{F(s)}{G(s)} \left( \frac{f(s)}{F(s)} - \frac{g(s)}{G(s)} \right) > 0,
\]
where the inequality follows from Part 1. ■

**Lemma 6 (Normal distribution)** Suppose \( F(.) \) and \( G(.) \) are two normal distributions satisfying MLRP. Then,

1. \( F(.) \) and \( G(.) \) have the same variance, that is, \( f(s) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(s-\mu_F)^2}{2\sigma^2}} \), and \( g(s) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(s-\mu_G)^2}{2\sigma^2}} \) for some \( \mu_F > \mu_G \) and \( \sigma > 0 \);

2. \( F(.) \) and \( G(.) \) satisfy IHRP, that is, \( h_F(s) / h_G(s) \) is strictly increasing in \( s \).

**Proof of Lemma 6.** Suppose \( g(.) = \frac{1}{\sigma_G \sqrt{2\pi}} e^{-\frac{(s-\mu_G)^2}{2\sigma_G^2}} \) and \( f(.) = \frac{1}{\sigma_F \sqrt{2\pi}} e^{-\frac{(s-\mu_F)^2}{2\sigma_F^2}} \).
1. \( f(s)/g(s) = \frac{\sigma_F}{\sigma_G} \exp \left( \frac{(s-\mu_F)^2}{2\sigma_F^2} - \frac{(s-\mu_G)^2}{2\sigma_G^2} \right) \). Consider \( \ln (f(s)/g(s)) = \ln f(s) - \ln g(s) \).

\[
\frac{\partial}{\partial s} \left( \ln f(s) - \ln g(s) \right) = s \left( \frac{1}{\sigma_G^2} - \frac{1}{\sigma_F^2} \right) + \left( \frac{\mu_F}{\sigma_F^2} - \frac{\mu_G}{\sigma_G^2} \right).
\]

By MLRP, \( \frac{\partial}{\partial s} (\ln f(s) - \ln g(s)) > 0 \) for all \( s \). However, if \( \sigma_G^2 < \sigma_F^2 \), the above expression goes to negative infinity when \( s \) goes to negative infinity. If \( \sigma_G^2 > \sigma_F^2 \), it goes to negative infinity when \( s \) goes to positive infinity. Hence, we must have \( \sigma_G^2 = \sigma_F^2 = \sigma^2 \). Thus,

\[
\frac{\partial}{\partial s} (\ln f(s) - \ln g(s)) = \frac{1}{\sigma^2} (\mu_F - \mu_G).
\]

MLRP then implies that \( \mu_F > \mu_G \).

2. Taking derivative of \( h_F(.) / h_G(.) \), we have

\[
\frac{\partial}{\partial s} \left( \frac{h_F(s)}{h_G(s)} \right) = \frac{\partial}{\partial s} \left( \frac{f(s)(1 - G(s))}{g(s)(1 - F(s))} \right)
= \frac{1}{\sigma^2} (\mu_F - \mu_G) \frac{f(s)}{g(s)} \frac{1 - G(s)}{1 - F(s)} + \frac{f(s) - g(s)(1 - F(s))}{g(s)(1 - F(s))^2}
= \frac{h_F(s)}{h_G(s)} \left( \frac{f(s)}{1 - F(s)} - \frac{g(s)}{1 - G(s)} + \frac{1}{\sigma^2} (\mu_F - \mu_G) \right)
= \frac{h_F(s)}{h_G(s)} \left( h_F(s) - h_F(s + (\mu_F - \mu_G)) + \frac{1}{\sigma^2} (\mu_F - \mu_G) \right),
\]

where the last equality follows from the observation that \( h_G(s) = h_F(s + (\mu_F - \mu_G)) \). 

\( \frac{\partial}{\partial s} \left( \frac{h_F(s)}{h_G(s)} \right) \) has the same sign as \( h_F(s) - h_F(s + (\mu_F - \mu_G)) + \frac{1}{\sigma^2} (\mu_F - \mu_G) \).

By the mean value theorem, there exists an \( s' \in (s, s + (\mu_F - \mu_G)) \) such that

\[
h_F(s) - h_F(s + (\mu_F - \mu_G)) = -h_F'(s') (\mu_F - \mu_G).
\]

Note that \( h_{N(\mu,\sigma^2)}(s) = \frac{1}{\sigma} h_{N(0,1)} \left( \frac{s-\mu}{\sigma} \right) \). Thus, \( h_{N(\mu,\sigma^2)}'(s) = \frac{1}{\sigma^2} h_{N(0,1)}' \left( \frac{s-\mu}{\sigma} \right) \). Sampford
(1952) shows that $0 < h'_{N(0,1)}(.) < 1$. Hence, $0 < h'_{G}(s') < \frac{1}{\sigma^2}$. Therefore,

$$h'_{F}(s') (\mu_F - \mu_G) < \frac{1}{\sigma^2} (\mu_F - \mu_G),$$

which implies that $\frac{\partial}{\partial s} \left( \frac{h_F(s)}{h_G(s)} \right) > 0$.

\[\Box\]

**Proof of Lemma 1.** In text. \[\Box\]

**Proof of Part 3 of Example 1.** Suppose both $G(.)$ and $F(.)$ are normal distributions satisfying MLRP. Then, by Part 1 of Lemma 6, $G = N(\mu_G, \sigma^2)$ and $F(\mu_F, \sigma^2)$, where $\mu_F > \mu_G$. $s(1,1)$ is defined by

$$\frac{(1-p) g(s(1,1))}{pf(s(1,1))} = 1.$$

Since $f\left(\mu_G + \mu_F\right) = g\left(\mu_G + \mu_F\right)$, $p > \frac{1}{2}$ implies that $s(1,1) < \frac{\mu_G + \mu_F}{2}$. Moreover, when $s = \frac{\mu_G + \mu_F}{2}$,

$$\frac{(1-F(s))F(s)}{(1-G(s))G(s)} = 1.$$ By Part 3 of Lemma 5, $s(1,1) < \frac{\mu_G + \mu_F}{2}$ implies that

$$\left(\frac{(1 - F(s(1,1)))F(s(1,1))}{(1 - G(s(1,1)))G(s(1,1))}\right)^\kappa < 1 = \frac{(1-p) g(s(1,1))}{pf(s(1,1))}.$$ At $s^* = s(\kappa + 1, 2\kappa + 1),$

$$\left(\frac{(1 - F(s^*))F(s^*)}{(1 - G(s^*))G(s^*)}\right)^\kappa = \frac{(1-p) g(s^*)}{pf(s^*)}.$$ By MLRP, the right-hand side is strictly decreasing in $s$. By Part 3 of Lemma 5, the left-hand side is strictly increasing in $s$. Hence, $s(1,1) < s(\kappa + 1, 2\kappa + 1)$. By Definition 2 and IHRP, $\alpha(1,1) > \alpha(\kappa + 1, 2\kappa + 1)$ for all $\kappa$. \[\Box\]

**Proof of Proposition 2.** Consider $k$-rule and its corresponding equilibrium cutoff $s(k,N)$.

$$\frac{(1 - F(s))^{k-1} F(s)^{N-k}}{(1 - G(s))^{k-1} G(s)^{N-k}} = \frac{(1-F(s))^k F(s)^{N-(k+1)} (1-G(s)) F(s)}{(1-G(s))^k G(s)^{N-(k+1)} (1-F(s)) G(s)} < \frac{(1-F(s))^k F(s)^{N-(k+1)}}{(1-G(s))^k G(s)^{N-(k+1)}},$$
where the inequality follows from MLRP and Part 2 of Lemma 5. Thus, the left-hand side of (1) is strictly increasing in \(k\). Moreover, by Part 3 of Lemma 5, the left-hand side of (1) is strictly increasing in \(s\). By MLRP, \(\frac{g(s)}{f(s)}\) is strictly decreasing in \(s\). Thus, to satisfy (1), it must be the case that 
\[ s(k + 1, N) < s(k, N). \]
The proposition then follows immediately from IHRP.

**Proof of Lemma 2.** We only show here that in a responsive equilibrium voting under unanimity rule is always symmetric. The proof for the second part of Lemma 2 is in the main text. In a responsive equilibrium, we have 
\[ \Pr (|v| = N) > 0 \]
and 
\[ \Pr (|v| < N) > 0. \]
This means that all committee members use nonpartisan strategies and are pivotal with positive probability. Thus, by Lemma 4, all committee members use cutoff strategies. The optimality condition for the cutoff \(s^*_i\) is
\[
\frac{p}{1-p} \prod_{k \neq i} \left( \frac{1-F(s^*_k)}{1-G(s^*_k)} \right) = \frac{g(s^*_i)}{f(s^*_i)}.
\]
This means that for all \(i, j \in \{1, 2, ..., N\}, \)
\[
\frac{1-F(s^*_i)}{1-G(s^*_i)} = \frac{1-F(s^*_j)}{1-G(s^*_j)}.
\]
By IHRP, this implies that for all \(i, j \in \{1, 2, ..., N\}, s^*_i = s^*_j.\)

**Proof of Proposition 3.** Consider a responsive asymmetric equilibrium. Suppose in equilibrium all committee members use nonpartisan strategies and are pivotal with positive probability. By Lemma 4 and Proposition 1, it is without loss to think of the equilibrium as a pair \((s^*, d)\), where \(s^* \in (a, b)^N\) is a cutoff profile and \(d\) is a weighted voting rule. In an asymmetric equilibrium, there exists a member \(i\) such that 1) for all \(j, s^*_i \geq s^*_j\), and 2) for some \(j, s^*_i > s^*_j\). Let \(s^*_\text{max} := s^*_i\). The optimality condition for the cutoff \(s^*_\text{max}\) is
\[
\frac{p}{1-p} \Pr (\text{piv}_i | \theta = y) = \frac{g(s^*_\text{max})}{f(s^*_\text{max})}.
\]
By Part 3 of Lemma 5, for any \(v_{-i} \in \text{piv}_i, \)
\[
\frac{\Pr (v_{-i} | \theta = y)}{\Pr (v_{-i} | \theta = n)} < \left( \frac{1-F(s^*_\text{max})}{1-G(s^*_\text{max})} \right)^{N-1},
\]

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which implies that
\[
\frac{g(s_{\text{max}}^*)}{f(s_{\text{max}}^*)} < p \left( \frac{1 - F(s_{\text{max}}^*)}{1 - G(s_{\text{max}}^*)} \right)^{N-1}.
\]

Consider a symmetric responsive equilibrium in which the equilibrium decision rule is unanimity rule, we have
\[
\frac{p}{1 - p} \left( \frac{1 - F(s(N,N))}{1 - G(s(N,N))} \right)^{N-1} = \frac{g(s(N,N))}{f(s(N,N))}.
\]

By MLRP, \( \frac{g(s)}{f(s)} \) is a strictly decreasing function of \( s \). By Part 3 of Lemma 5, \( \left( \frac{1 - F(s)}{1 - G(s)} \right)^{N-1} \) is a strictly increasing function of \( s \). Therefore, \( s_{\text{max}}^* > s(N,N) \).

In the asymmetric responsive equilibrium \((s^*, d)\), since the DM can tell every single vote profile apart, \( s^* \) being a responsive equilibrium implies that
\[
\frac{\alpha}{1 - \alpha} \leq \min_i \min_{v_{-i} \in \text{piv}_i} \frac{p}{1 - p} \frac{\Pr(v_{-i}|\theta = y) (1 - F(s^*_i))}{\Pr(v_{-i}|\theta = n) (1 - G(s^*_i))}.
\]

For all \( i \), the optimality of the cutoff \( s^*_i \) implies that \( \exists v_{-i} \in \text{piv}_i, \)
\[
\frac{p}{1 - p} \frac{\Pr(v_{-i}|\theta = y)}{\Pr(v_{-i}|\theta = n)} \leq \frac{g(s^*_i)}{f(s^*_i)}.
\]

Therefore,
\[
\frac{\alpha}{1 - \alpha} \leq \min_i \frac{g(s^*_i) (1 - F(s^*_i))}{f(s^*_i) (1 - G(s^*_i))}.
\]

Since \( \frac{h_F(s)}{h_G(s)} \) is strictly increasing,
\[
\frac{\alpha}{1 - \alpha} \leq \min_i \frac{g(s^*_i) (1 - F(s^*_i))}{f(s^*_i) (1 - G(s^*_i))} = \frac{g(s_{\text{max}}^*) (1 - F(s_{\text{max}}^*))}{f(s_{\text{max}}^*) (1 - G(s_{\text{max}}^*))} < \frac{g(s(N,N)) (1 - F(s(N,N)))}{f(s(N,N)) (1 - G(s(N,N)))},
\]

which implies that a \( k \)-equilibrium in which voting is unanimous exists.

Finally, suppose in equilibrium some committee members use partisan strategies or are never pivotal. As noted in the main text, in this case the committee in effect becomes a smaller committee. By Lemma 2 and the first part of this proof, this can only reduce persuasiveness.
Proof of Proposition 4. We first show that in a responsive (symmetric or asymmetric) equilibrium in which the equilibrium decision rule is a $k$-rule, all committee members must use cutoff strategies. Since the equilibrium is responsive, we have $\Pr(|v| \geq k) > 0$ and $\Pr(|v| < k) > 0$. Independence of committee members’ signals then implies that all committee members are pivotal with positive probability. Given that the decision rule is $k$-rule and that the realized signals of the committee members can be arbitrarily precise, they cannot use partisan strategies in equilibrium. By Lemma 4, all committee members must use cutoff strategies.

Consider a responsive asymmetric equilibrium $(s^*, d)$ in which the equilibrium decision rule $d$ is a $k$-rule. There exists a member $i$ such that 1) for all $j$, $s_i^* \geq s_j^*$, and 2) for some $j$, $s_i^* > s_j^*$. Let $s_{\text{max}}^* := s_i^*$. Consider committee member $i$. We have

$$\frac{p}{1-p} \Pr(|v_{-i}| = k-1|\theta = y) = \frac{g(s_{\text{max}}^*)}{f(s_{\text{max}}^*)}$$

By Part 3 of Lemma 5, for all $v_{-i}$ such that $|v_{-i}| = k-1$,

$$\frac{\Pr(v_{-i}|\theta = y)}{\Pr(v_{-i}|\theta = n)} < \left( \frac{1 - F(s_{\text{max}}^*)}{1 - G(s_{\text{max}}^*)} \right)^{k-1} \left( \frac{F(s_{\text{max}}^*)}{G(s_{\text{max}}^*)} \right)^{N-k},$$

which implies that

$$\frac{g(s_{\text{max}}^*)}{f(s_{\text{max}}^*)} < \frac{p}{1-p} \left( \frac{1 - F(s(k,N))}{1 - G(s(k,N))} \right)^{k-1} \left( \frac{F(s(k,N))}{G(s(k,N))} \right)^{N-k}.$$

Consider a $k$-equilibrium, we have

$$\frac{p}{1-p} \left( \frac{1 - F(s(k,N))}{1 - G(s(k,N))} \right)^{k-1} \left( \frac{F(s(k,N))}{G(s(k,N))} \right)^{N-k} = \frac{g(s(k,N))}{f(s(k,N))}.$$

By MLRP, $\frac{g(s)}{f(s)}$ is a strictly decreasing function of $s$. By Part 3 of Lemma 5, $\frac{1-F(s)}{1-G(s)}$ and $\frac{F(s)}{G(s)}$ are strictly increasing functions of $s$. Therefore, $s_{\text{max}}^* > s(k,N)$.

In the asymmetric responsive equilibrium $(s^*, d)$, since all vote profiles occur with positive probability in equilibrium and the DM can tell every single vote profile apart, we have

$$\frac{\alpha}{1-\alpha} \leq \min_{i} \min_{\{v_{-i}|v_{-i}|=k-1\}} \frac{p}{1-p} \frac{\Pr(v_{-i}|\theta = y) (1 - F(s_i^*))}{\Pr(v_{-i}|\theta = n) (1 - G(s_i^*))}.$$
For all $i$, $\exists v_{-i}$ such that $|v_{-i}| = k - 1$ and

$$\frac{p}{1 - p} \frac{\Pr(v_{-i} | \theta = y)}{\Pr(v_{-i} | \theta = n)} \leq \frac{g(s_i^*)}{f(s_i^*)}.$$ 

Therefore,

$$\frac{\alpha}{1 - \alpha} \leq \min_i \frac{g(s_i^*) (1 - F(s_i^*))}{f(s_i^*) (1 - G(s_i^*))}.$$ 

Since $\frac{h_F(\cdot)}{h_G(\cdot)}$ is strictly increasing,

$$\frac{\alpha}{1 - \alpha} \leq \min_i \frac{g(s_i^*) (1 - F(s_i^*))}{f(s_i^*) (1 - G(s_i^*))} = \frac{g(s_{\text{max}}) (1 - F(s_{\text{max}}))}{f(s_{\text{max}}) (1 - G(s_{\text{max}}))} < \frac{g(s(k,N)) (1 - F(s(k,N)))}{f(s(k,N)) (1 - G(s(k,N)))},$$

which implies that a $k$-equilibrium exists. The second part of the proposition then follows immediately from Proposition 2. 

**Proof of Corollary 1.** By Proposition 3, if there does not exist a $k$-equilibrium in which $k = N$, then there does not exist any responsive equilibrium. By Lemma 1, there exists a $k$-equilibrium in which $k = N$ if and only if $\alpha \leq \alpha(N,N)$. Consider the optimality condition for the committee members,

$$\frac{p}{1 - p} \left( \frac{1 - F(s(N,N))}{1 - G(s(N,N))} \right)^{N-1} = \frac{g(s(N,N))}{f(s(N,N))}.$$ 

If $\lim_{N \to \infty} s(N,N) = g \in (a, b)$, then $\lim_{N \to \infty} \frac{g(s(N,N))}{f(s(N,N))} = \frac{g(g)}{f(g)} < \infty$, and

$$\lim_{N \to \infty} \left( \frac{1 - F(s(N,N))}{1 - G(s(N,N))} \right)^{N-1} = \lim_{N \to \infty} \left( \frac{1 - F(s)}{1 - G(s)} \right)^{N-1} = \infty.$$ 

Therefore, $\lim_{N \to \infty} s(N,N) = a.$

$$\lim_{N \to \infty} \frac{h_G(s(N,N))}{h_F(s(N,N))} = \lim_{s \to a} \frac{h_G(s)}{h_F(s)} = \lim_{s \to a} \frac{g(s)}{f(s)} \frac{1 - F(s)}{1 - G(s)} = \lim_{s \to a} \frac{g(s)}{f(s)} = \infty,$$

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where the last equality follows from Assumption 2. This implies $\alpha(N,N) \to 1$ as $N \to \infty$. ■

**Proof of Lemma 3.** In any symmetric responsive equilibrium, the DM finds it optimal to follow a given $k$-rule if and only if (7) holds. There are three cases.

1. Suppose $k < \frac{N+1}{2}$. If a $k$-equilibrium exists, the voting strategy $(\rho_N, \rho_Y)$ must satisfy $\rho_N = 0$ and $\rho_Y \in (0,1)$, which implies that member $i$ must be indifferent between alternative $Y$ and status quo $N$ conditional on being pivotal after receiving a $Y$-signal, i.e.,

$$\frac{\Pr(piv_i | \theta = y)}{\Pr(piv_i | \theta = n)} = \frac{\Pr(|v_{-i}| = k-1 | \theta = y)}{\Pr(|v_{-i}| = k-1 | \theta = n)} = 1 - q.$$

Given the voting strategy, we have

$$\frac{\Pr(|v| = k-1 | \theta = y)}{\Pr(|v| = k-1 | \theta = n)} = \frac{1 - q - q(1 - \rho_Y) + (1 - q)}{q (1 - q)(1 - \rho_Y) + q} < 1,$$

and

$$\frac{\Pr(|v| = k | \theta = y)}{\Pr(|v| = k | \theta = n)} = \frac{1 - q}{q} = \frac{1}{1 - \alpha},$$

as $\alpha > \frac{1}{2}$. Thus, (7) can never be satisfied. Therefore, there does not exist a symmetric responsive equilibrium in which the equilibrium decision rule is a minority rule.

2. Suppose $k = \frac{N+1}{2}$. If a $k$-equilibrium exists, voting is truthful. We have

$$\frac{\Pr(piv_i | \theta = y)}{\Pr(piv_i | \theta = n)} = \frac{\Pr(|v_{-i}| = k-1 | \theta = y)}{\Pr(|v_{-i}| = k-1 | \theta = n)} = \frac{\frac{N-1}{2} (1-q) \frac{N-1}{2} q}{(1-q) \frac{N-1}{2} q} = 1.$$

Given truthful voting, we have

$$\frac{\Pr(|v| = k-1 | \theta = y)}{\Pr(|v| = k-1 | \theta = n)} = \frac{q \frac{N-1}{2} (1-q) \frac{N-1}{2} q}{(1-q) \frac{N-1}{2} q} = \frac{1 - q}{q} < 1,$$

and

$$\frac{\Pr(|v| = k | \theta = y)}{\Pr(|v| = k | \theta = n)} = \frac{q \frac{N-1}{2} (1-q) \frac{N-1}{2} q}{(1-q) \frac{N-1}{2} q} = \frac{q}{1 - q}.$$

Therefore, (7) is equivalent to $\alpha \leq q = \alpha_2 \left( \frac{N+1}{2}, N \right)$.
3. Suppose \( k > \frac{N+1}{2} \). If a \( k \)-equilibrium exists, the voting strategy \((\rho_N, \rho_Y)\) must satisfy
\( \rho_N \in (0,1) \) and \( \rho_Y = 1 \), which implies that member \( i \) is indifferent between alternative \( Y \) and status quo \( N \) conditional on being pivotal after receiving an \( N \)-signal, i.e.,
\[
\frac{\Pr (\text{piv}_i | \theta = y)}{\Pr (\text{piv}_i | \theta = n)} = \frac{\Pr (|v_{-i}| = k-1 | \theta = y)}{\Pr (|v_{-i}| = k-1 | \theta = n)} = \frac{q}{1-q}.
\]
Given the voting strategy, we have
\[
\frac{\Pr (|v| = k-1 | \theta = y)}{\Pr (|v| = k-1 | \theta = n)} = \frac{q}{1-q} \frac{1-q}{q} = 1,
\]
and
\[
\frac{\Pr (|v| = k | \theta = y)}{\Pr (|v| = k | \theta = n)} = \frac{q}{1-q} \frac{q+(1-q)\rho_N}{q\rho_N + (1-q)} = \frac{\alpha_2 (k,N)}{1-\alpha_2 (k,N)}.
\]
Therefore, \((7)\) is equivalent to \( \alpha \leq \alpha_2 (k,N) \).

Proof of Proposition 5. In text.

Proof of Proposition 6. Consider a responsive equilibrium. In equilibrium, there must be at least one committee member who uses a nonpartisan strategy and is pivotal with positive probability. Let member \( i \) be that committee member. Moreover, it is without loss to assume that the equilibrium decision rule is increasing.

By pivotal consideration, we have
\[
\frac{1-q}{q} \leq \frac{\Pr (\text{piv}_i | \theta = y)}{\Pr (\text{piv}_i | \theta = n)} \leq \frac{q}{1-q},
\]
which implies
\[
\min_{v_{-i} \in \text{piv}_i, \Pr (v_{-i}) > 0} \frac{\Pr (v_{-i} | \theta = y)}{\Pr (v_{-i} | \theta = n)} \leq \frac{q}{1-q}.
\]
Therefore,
\[
\min_{v \in V^+, \Pr (v) > 0} \frac{\Pr (v | \theta = y)}{\Pr (v | \theta = n)} \leq \min_{v_{-i} \in \text{piv}_i, \Pr (v_{-i}) > 0} \frac{\Pr (v_{-i} | \theta = y)}{\Pr (v_{-i} | \theta = n)} \frac{q}{1-q} \leq \left( \frac{q}{1-q} \right)^2.
\]
Thus, we must have $\alpha \leq \overline{\alpha}$ in a responsive equilibrium.

Next, we consider $\alpha = \overline{\alpha}$ and construct a responsive equilibrium which involves the committee members voting asymmetrically. Consider the following asymmetric voting strategy profile and decision rule: 1) committee member 1 always votes for status quo $N$, and 2) committee member $i \in \{2, ..., N\}$ votes truthfully; the DM chooses alternative $Y$ when there are at least $\frac{N+1}{2}$ votes for alternative $Y$ from committee members $\{2, ..., N\}$ and status quo $N$ otherwise.

To check that this is an equilibrium, consider committee member 1. Since he is never pivotal, it is optimal for him to vote for status quo $N$.

Next, consider committee member $i \in \{2, ..., N\}$. Since

$$\frac{q}{1-q} \frac{\Pr (|v_{-i}| = \frac{N-1}{2} | \theta = y)}{\Pr (|v_{-i}| = \frac{N-1}{2} | \theta = n)} = \frac{q^{\frac{N+1}{2}} (1-q)^{\frac{N-3}{2}}}{(1-q) \frac{N+1}{2} q^{\frac{N-3}{2}}} = \left( \frac{q}{1-q} \right)^2 > 1,$$

he strictly prefers voting for alternative $Y$ after receiving a $Y$-signal. Moreover, he is indifferent between the two options after receiving an $N$-signal, since

$$\frac{1-q}{q} \frac{\Pr (|v_{-i}| = \frac{N-1}{2} | \theta = y)}{\Pr (|v_{-i}| = \frac{N-1}{2} | \theta = n)} = \frac{q^{\frac{N-1}{2}} (1-q)^{\frac{N-3}{2}}}{(1-q) \frac{N-1}{2} q^{\frac{N-3}{2}}} = 1.$$

Finally, since

$$1 < \frac{\overline{\alpha}}{1-\overline{\alpha}} = \left( \frac{q}{1-q} \right)^2,$$

the DM also finds it optimal to follow the decision rule. ■

**Lemma 7** Suppose the signals are discrete. If $F(\cdot)$ and $G(\cdot)$ satisfy MLRP, then, for $m \in \{1, 2, ..., M-1\}$, the function

$$H(\rho) := \frac{q_{F}(t_m) \rho + \sum_{l=m+1}^{M} q_{F}(t_l)}{q_{G}(t_m) \rho + \sum_{l=m+1}^{M} q_{G}(t_l)}$$

is strictly decreasing.
Proof of Lemma 7. Differentiating $H(\cdot)$, we get

$$H'(\rho) = \frac{q_F(t_m) \left( \sum_{l=m+1}^{M} q_G(t_l) \right) - q_G(t_m) \left( \sum_{l=m+1}^{M} q_F(t_l) \right)}{\left( q_G(t_m) \rho + \sum_{l=m+1}^{M} q_G(t_l) \right)^2} < 0.$$ 

To see why the inequality holds, note that by MLRP, for all $l \in \{m + 1, \ldots, M\}$,

$$\frac{q_F(t_m)}{q_G(t_m)} q_G(t_l) < q_F(t_l),$$

which means

$$\frac{q_F(t_m)}{q_G(t_m)} \left( \sum_{l=m+1}^{M} q_G(t_l) \right) < \sum_{l=m+1}^{M} q_F(t_l).$$