Which product to advertise? Optimal price advertising with heterogeneous consumers

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Abstract

Retailers advertise the price of some goods in order to attract customers. In a setting with incomplete information and a multiproduct monopolist selling two goods, we show that, when the retailer can advertise only one good, the optimal advertising strategy depends on the heterogeneity of both consumers' tastes and shopping costs. When consumers have relatively high shopping costs, it becomes profitable to advertise the product for which valuations are more heterogeneous, because it induces more complementarities. Such an advertising strategy lowers prices and can even yield a higher consumer surplus than under perfect information. Thus, we describe conditions under which policies aimed at promoting price transparency or preventing price obfuscation may be sub-optimal for consumers.

1 Introduction

While manufacturers advertise to boost a specific product against its competitors, retailers usually sell hundreds or thousands of different products, and their advertising strategy primarily

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aims at attracting more customers into their stores. In a setting where consumers only have imperfect information about prices before visiting a store, and where they must incur shopping costs to learn those prices, retailers can advertise the price of a small subset of the products they sell in order to attract customers.

Such marketing practices, of which weekly price leaflets are the embodiment, raise several questions. First, from a retailer's standpoint, it is important to define which products to advertise: is it more profitable to commit on low prices on products that are equally valued by most consumers, such as bread or toilet paper, or is it a better strategy to advertise more polarising goods, such as soy milk or coconuts, for which consumers' valuations are more heterogeneous. Then, do consumers benefit equally from all kinds of advertising strategies? Eventually, from a regulator's standpoint, it is critical to understand when and why it may be optimal to regulate price advertising and price disclosure strategies.

To answer these questions, we investigate the advertising and pricing behavior of a multi-product monopolist. In our model, a monopolist retailer sells two goods, denoted by A and B. It can inform consumers about the price of one of these goods by sending an advertisement, which corresponds to a commitment on the price of one good. Consumers exhibit homogeneous valuations for good A, while their valuations for good B are heterogeneous. They are interested in purchasing at most one unit of each good, are privately informed about their valuations for good B, and are unaware of unadvertised prices before they reach the store. Based on the advertisement they receive, they form rational expectations regarding the price of the other good. Consumers can pay a shopping cost to visit the store and learn all prices. An important assumption of our model is that consumers differ in their shopping costs. We document how this model relates to important product markets such as food retail.

We first investigate a benchmark with no information. We show there cannot be any market without price information, because consumers incur shopping costs before visiting the store, so that they are held-up to the point where they would rather stay home. This result is consistent with the Diamond paradox, and it shows that selling one good for which consumer's valuations are homogeneous is enough to cancel Rhodes (2015)'s findings that two goods can be enough to ensure the existence of a search equilibrium in the absence of price information.

We then study a benchmark model with full information. We first investigate a case in which consumers have homogeneous shopping costs, and we then analyse another case in which consumers have heterogeneous shopping costs. This allows us to describe an important feature of heterogeneous shopping costs: any increase in shopping costs imply an increase in shopping costs' heterogeneity, so that consumers can be better off when shopping costs increase. Indeed, when shopping costs increase, the monopolist must leave a rent to some consumers in order to keep attracting the optimal amount of demand.

We then solve our advertising model to analyse the impact of partial information. We characterize subgame equilibria depending on whether the retailer advertises good A or good B. Once again, to disentangle the role of shopping costs' heterogeneity, we first consider a case in which consumers have homogeneous shopping costs, and we then investigate the case in which consumers have heterogeneous shopping costs.

We exhibit that good B, for which consumers have heterogeneous valuations, begets more complementarities than good A, because all consumers who visit the store after having received an advert for good B end up purchasing good A as well, whereas only some of the consumers who visited the store after having received an advert for good A end up purchasing good B as well. As a consequence, the advertised price of good B is always lower than the advertised price of good A, so that consumers prefer the situation in which good B is advertised over the situation in which good A is advertised. Even more, we show this complementarity effect is so potent consumers may end up being better off when the monopolist advertises good B than when there is full information.

The importance of this complementarity effect can be traced back to Ramsey (1927), who analyzes the optimal pricing of a multiproduct monopoly. The model shows that complementarities among products affect the optimal price-cost margins, such that the more complementary a product is to the others, the lower its margin should be. This is because a lower price for a complementary product increases the demand and surplus for the other products. The pro-competitive impact of the complementarity generated by one-stop shoppers has been empirically assessed by Thomassen et al. (2017).[1]

Eventually, we endogenize the retailer's optimal advertising strategy. We show that accounting for shopping costs heterogeneity has very strong implications. Indeed, we first demonstrate that, when shopping costs are the same for all consumers, the monopolist is able to perfectly

[1] Thomassen et al. (2017) find that supermarkets have more market power when they lower their prices than when they raise them, because they can attract more consumers who also buy complementary products. The authors empirically quantify how pro-competitive the complementarity effect is: they show that, through the complementarities they beget, one-stop shoppers drive competition between retailers more than multi-stop shoppers, even though the latter cherry-pick the best price for each good.

mimic the full information equilibria, so that partial information does not change anything compared to the full information benchmark. However, we prove that, when shopping costs vary across consumers, and when they are sufficiently high, the monopolist may find it optimal to advertise good B for some ranges of parameters, and we show it makes consumers better off than under full information.

We contribute to the literature on price advertisement and partial information markets by modeling two types of goods: one with constant valuations, following Lal and Matutes (1994), and one with heterogeneous valuations, following Rhodes (2015). We also introduce heterogeneous shopping costs among consumers, which affect their decisions to visit and buy from the retailer.

Our model and results differ significantly from Lal and Matutes (1994), who assume that consumers have the same valuation H for each product sold by firms. In their model, a firm's advertised price for one product does not reveal any information about its price for the other product. In contrast, we allow consumer valuations for one of the two goods to be heterogeneous and drawn from a continuous distribution. This implies that a change in the advertised price affects the composition of the searchers, and thus the optimal pricing strategy of the firm for the other product.

In this regard, our model is closer to Rhodes (2015) than to Lal and Matutes (1994), because we also consider multiproduct retailing with heterogeneous valuations. However, we obtain a different result from Rhodes (2015), who shows that a lower advertised price attracts more price-sensitive consumers, and thus lowers the firm's incentive to charge high prices for the other products. In our model, a lower advertised price attracts consumers with a higher willingness to pay, and thus increases the firm's incentive to charge high prices for the other good. This is because we assume heterogeneous shopping costs rather than homogeneous ones.

Our model and results also relate to and differ from Ellison (2005), who shows partial information usually benefits firms rather than consumers. This is because Ellison (2005) focuses on a model with competing firms who can restrict consumers' knowledge of prices through add-on pricing. Their results rely on firms being able to use partial information to discriminate between two kinds of consumers. Our results, however, are in line with some papers showing how partial information may benefit consumers more than full information (See Zapechelnyuk (2020), Luco (2019)).[2]

[2]Zapechelnyuk (2020) studies a problem of quality certification in a moral hazard setting, where a producer

Through complementarity, we put forward a novel mechanism to explain why partial information may benefit consumers more than full information. This has important policy implications. Indeed, we show that, provided consumers have sufficiently high shopping costs, and provided these shopping costs are heterogeneous enough, it is detrimental to consumers to promote full information. In other words, a regulator should not facilitate full price information, as partial information incentivizes retailers to offer very large discounts which benefit consumers.

The paper is organized as follows. Section 2 presents our mainline model, Section 3 analyses a benchmark without information, Section 4 characterizes a benchmark with full information, and Section 5 considers advertising. Eventually, Section 6 discusses some policy implications, and Section 7 concludes.

2 Model

A monopolist retailer sells two goods, A and B, at a marginal cost normalized to zero. It is located at 0 on a segment of length 1. A mass 1 of consumers is uniformly distributed on the segment, and a consumer located at a distance x from the retailer incurs shopping costs tx to visit the store. Shopping costs can either be interpreted as transportation costs, as in a Hotelling model à la Lal and Matutes (1994), or as search costs, as in Rhodes (2015). We assume all consumers have an equal valuation $v_A = a$ for good A, while they differ in their valuations v_B for good B. More precisely, v_B is uniformly distributed between $a - z$ and $a + z$, so that the average valuation for each good is the same. We assume $z > \frac{a}{3}$.

The utility of each consumer is additively separable. The utility of a consumer located in x is the sum of her valuations for each good she buys, minus the prices she pays and the shopping costs. We normalize the outside option to 0. As a result, the purchasing behavior of a consumer located in x stems from the following maximization program:

chooses a quality and a price of a product, and a consumer observes the price and a rating assigned by a certifier. The paper shows that partial information may be better than full information for two reasons. First, partial information may induce more efficient price-quality trade-offs, as the producer faces less price competition from lower-quality products. Second, partial information may create more incentives for quality improvement. Conducing an empirical evaluation of a price disclosure obligation for Chilean petrol stations, Luco (2019) show that, in some areas, perfect information was detrimental to consumers. The authors resort to tacit collusion as the most likely theoretical rationale behind their results.

$$U(x, v) = \max(v_A + v_B - p_A - p_B - tx, v_A - p_A - tx, v_B - p_B - tx, 0)$$

Goods A and B are neither substitutes nor complements, but shopping costs induce some complementarities, because a consumer who visits the store incurs tx regardless of the number of products she buys.

Before they reach the store, consumers are uninformed about prices, unless advertised. The price of an advertised good is known by all, and the price of each good is observed by consumers once they are in the store. We assume the retailer can advertise at most one good, and we normalize advertising costs to 0. This corresponds to the situation in which retailers only advertise a subset of the products they sell.

The timing of the game is as follows:

Stage 1: the monopolist decides which good to advertise and sets both prices.

Stage 2: consumers observe the price of the advertised good, and form rational expectations on the price of the unadvertised good. They decide whether or not to visit the store and pay the shopping cost.

Stage 3: Once in the store, consumers observe the price of the unadvertised good and choose which goods to purchase.

We look for subgame perfect Bayesian-Nash equilibria.

We believe this model corresponds to several important product markets, such as food retail or consumer staples. Indeed, these are markets in which the number of available goods is so large the literature has shown consumers usually display limited to no price recall (see Monroe and Lee, 1999). These are also markets in which consumers' taste heterogeneity itself varies across products: there is less consumer heterogeneity for peanut butter or salt than for Jerusalem artichokes or costly olive oil. Moreover, as shown by Bagwell (2007), retailers' advertising mostly contains information about prices, rather than information about products' characteristics. We thus focus on a setting where valuations are known in advance, and where search is a sunk cost spent in order to learn prices.[3].

Furthermore, using consumer panel data, Smith and Thomassen (2012) show that consumers visit on average 1.71 shops per week, but that the standard deviation is 1.31. This is coherent with two features from our model: on the one hand, that consumers do not visit more shops

[3]Other articles instead assume that prices are known in advance but the value of the match between a consumer and a product is learned after search has occurred (see for instance Chen, Li, and Zhang (2022))

imply that they incur some sort of shopping costs. These shopping costs can be interpreted as transportation costs, that is, the physical cost of going to the shop in terms of gasoline, or as search costs, that is, the time and energy taken for visiting a store. On the other hand, that consumers do not all visit the same number of shops imply they incur heterogeneous shopping costs. This can be due to consumers living closer or further away from the store, or to individuals having more or less available time.

Eventually, our timing corresponds to the fact that price advertisement is decided a few weeks in advance, whereas the prices of unadvertised goods can be reset on a daily basis.

3 Benchmark model with no advertising

We first show that there is no market when firms cannot advertise prices, that is, when consumers learn all prices once in the store. Such a result relates to a large literature that can be traced back to the so-called "Diamond paradox". Diamond (1971) analyzed a market for a single good with a large number of identical sellers who have different reservation prices, and a large number of buyers who have a common valuation of the good. He showed that, in the presence of positive search costs, the unique equilibrium price is equal to the highest reservation price of the sellers, which implies that no trade occurs. This is because buyers have no incentive to search for lower prices, since they expect to face the same price everywhere, and sellers have no incentive to lower their prices, since they expect to face no demand.

However, whether the Diamond paradox arises with multiproduct retailers is an intricate question. Lal and Matutes (1994) show that the Diamond paradox occurs when the retailers do not advertise any product, and consumers face a positive search cost. In this case, consumers expect to pay the highest possible price for any product, and therefore do not search. Retailers anticipate this and charge the monopoly price for both products. No trade occurs, even though there are potential gains from trade. Yet, Rhodes (2015) shows this result holds because Lal and Matutes (1994) disregard consumer taste heterogeneity. Rhodes shows there can still be a market absent advertisement by introducing heterogeneity in consumers' valuations for different products. He assumes consumers have a common ranking of products, but differ in their willingness to pay for each product. He also assumes consumers have a reservation utility for each product, which is the minimum utility they need to buy it. He then derives the conditions under which consumers will search and buy from a multiproduct retailer, even if they do not know any of its prices beforehand. He finds that consumers will search if their expected utility from visiting the store is greater than their reservation utility for the most preferred product. This depends on the distribution of consumers' valuations, the number of products, and the search cost. He also shows that a larger product range increases the probability of search, because it attracts more consumers who have low reservation utilities for some products, but that there can still be an equilibrium without information when the monopolist only sells two goods.

We show that, when a multiproduct retailer sells one good for which consumers have constant valuations (as in Lal and Matutes (1994)) and another good for which consumers have heterogeneous valuations (as in Rhodes (2015)), the Diamond paradox still prevails.

Without advertising, we denote $p_A^{\emptyset}$ and $p_B^{\emptyset}$ the prices of goods A and B, where $\emptyset$ stresses the absence of information.

Lemma 1. *Without advertising, $p_A^{\emptyset} = a$ and $p_B^{\emptyset} = a + z$, so that no consumer ever visits the store.*

Proof. See proof in the appendix A.1. □

The intuition is that, in Rhodes (2015), the key mechanism that allows for search without advertisement is the heterogeneity of consumers' preferences and valuations for different products. This implies that consumers can rationally expect a price below the reservation utility for each product. However, if consumers have homogeneous valuations for good A, then they all have the same reservation utility for that product. This implies that the retailer can charge a high price for good A without losing any customers, and therefore has no incentive to lower its prices on good B either. Another way to think about it is to note that, since all consumers have the same valuation for good A, the monopolist will price it to its customers' reservation utility, so that the problem eventually boils down to the classic single-good case. Thus, the Diamond paradox reappears, and no trade occurs in the absence of advertisement.

4 Benchmark model with perfect information

In order to describe the effect of advertising, we first solve a benchmark model with perfect information. This setting corresponds to the standard case in which consumers are informed about all prices before visiting the store. Within our framework, it can also be interpreted as the situation in which the monopolist advertises all prices. We first consider the case in which consumers display homogeneous shopping costs, and then investigate the case in which they have heterogeneous shopping costs. These costs can either be interpreted as transportation costs, or as search costs. Homogeneous shopping costs thus correspond to all consumers living at the same distance from the store, or having a similar distaste for searching prices. Heterogeneous shopping costs are accounted for by consumers inhabiting more or less distant neighborhoods, or having heterogeneous tastes for searching prices. Lal and Matutes (1994) investigate a setting à la Hotelling, and thus assume heterogeneous shopping costs, which they interpret as transportation costs. Conversely, Rhodes (2015) analyses a setting with homogeneous search costs. Our two perfect information benchmarks aim at disentangling how shopping costs' modelling choices may impact advertising strategies and consumer surplus.

4.1 Homogeneous shopping costs

There are two differences with the main model.

First, consumers are assumed to be perfectly informed about prices. The timing of the game is thus as follows: in stage 1, the monopolist sets both prices. In stage 2, consumers learn their valuations for good B, learn both prices, and decide whether or not to incur shopping costs. In stage 3, consumption occurs.

Second, consumers have homogeneous shopping costs. We assume they have to pay $\frac{t}{2}$ to visit the store, no matter their location x. Alternatively, this is equivalent to assuming all consumers are located on $x = \frac{1}{2}$. Formally, the purchasing behavior of a consumer now stems from the following maximization program:

$$U(v) = \max(v_A + v_B - p_A - p_B - \frac{t}{2}, v_A - p_A - \frac{t}{2}, v_B - p_B - \frac{t}{2}, 0)$$

Equilibrium prices depend on shopping costs t. We denote by p_A^f and p_B^f the prices of goods A and B under full information. p_A^{f0} and p_B^{f0} are the optimal prices absent transportation costs. We also distinguish p_A^{fl} and p_B^{fl}, the optimal prices under full information when shopping costs are low, from p_A^{fm} and p_B^{fm}, the optimal prices under full information when shopping costs are medium, and from p_A^{fh} and p_B^{fh}, the optimal prices under full information when shopping costs are high. Eventually, overlined prices (e.g. $\overline{p}_A^{f0}$) correspond to homogeneous shopping costs.

Lemma 2. *When shopping costs are null, the monopolist charges $\overline{p}_A^{f0} = a$ and $\overline{p}_B^{f0} = \frac{a+z}{2}$.*

Proof. See proof in the appendix A.2. □

This first benchmark corresponds to the case where there are no shopping-cost-induced complementarities between good A and good B.

Lemma 3. *When shopping costs are homogeneous, under perfect information, there exists a threshold* $\hat{t} \equiv 2(a + \frac{(a+z)^2-(2a+z-\frac{t}{2})^2}{8z})$ *such that:*

- *when* $t \leq \hat{t}$*:* $\overline{p}_A^{fl} = a - \frac{t}{2}$ *and* $\overline{p}_B^{fl} = \frac{a+z}{2}$*. All consumers visit the store.*
- *when* $t \geq \hat{t}$*:* $\overline{p}_A^{fh} + \overline{p}_B^{fh} = \frac{1}{2}(2a + z - \frac{t}{2})$*. Only consumers whose taste for good* B *is above* $v_0 \equiv \overline{p}_A^{fh} + \overline{p}_B^{fh} + \frac{t}{2} - a$ *visit the store.*

Proof. See proof in the appendix A.3. □

The main difference with the free shopping equilibrium is that, although both goods are independent, shopping costs induce some complementarities. This is because consumers incur shopping costs only once, whether they decide to buy one or two goods. As a consequence, the monopolist sells the basket of goods at a discount in order to subsidize transportation costs.

With $a = z = 0.5$, the blue line on Figure 3 shows consumer surplus as a function of shopping costs t. Consumer surplus is equal to 0.125. It does not vary with t as the prices adjust for the shopping cost of the consumers who visit the store. With those parameters, the monopolist always finds it profitable to attract all consumers to its store, and consumers do not loose any surplus as shopping costs increase. Indeed, the monopolist fully subsidises shopping costs.

4.2 Heterogeneous shopping costs

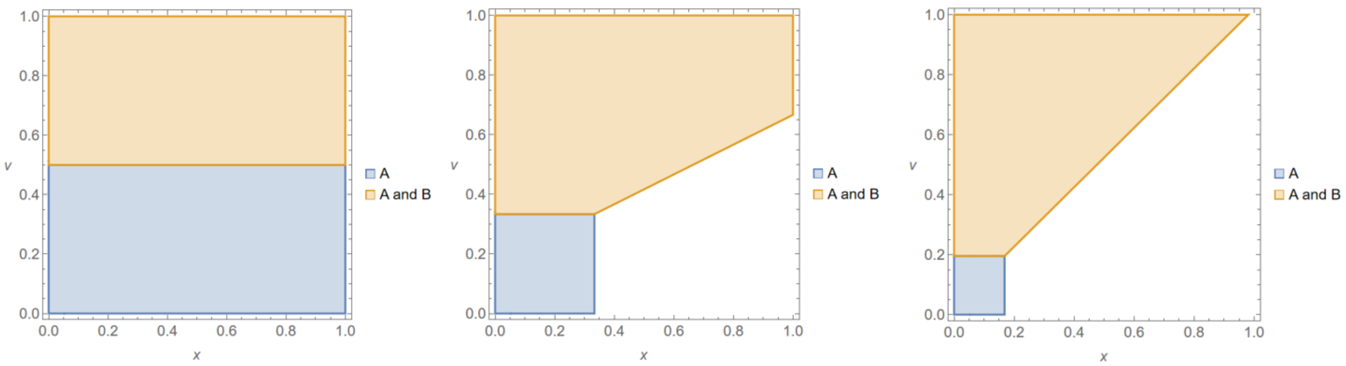
We keep the same setting. The only difference with the previous section is that, as in our main model, consumers are now uniformly distributed on the unit segment, and a consumer located at a distance x from the retailer incurs shopping costs tx to visit the store. As a result, the purchasing behavior of a consumer located in x stems from the following maximization program:

$$U(x, v) = \max(v_A + v_B - p_A - p_B - tx, v_A - p_A - tx, v_B - p_B - tx, 0)$$

Lemma 4. *When shopping costs are heterogeneous, under perfect information, there exists two thresholds* $\underline{t}^f$ *and* $\overline{t}^f \equiv 2a + z - p_A - p_B$ *that characterize the equilibrium:*

- *when* $t \leq \underline{t}^f$*: all consumers visit the store. The price of good* A *is* $p_A^{fl} = a - t$ *and the price of good* B *is* $p_B^{fl} = \frac{a+z}{2}$.
- *when* $\underline{t}^f \leq t \leq \overline{t}^f$*: all consumers whose location* x *is close to* 0 *visit the store. Among consumers whose location* x *is close to* 1*, only those whose valuation* v_B *for good* B *is high enough visit the store.*
- *when* $t \geq \overline{t}^f$*: all consumers whose location* x *is close to* 0 *visit the store. No consumer whose location* x *is close to* 1 *visits the store.*

Proof. See proof in the appendix A.4. □



1) Low shopping costs (t=0.1) 2) Medium shopping costs (t=0.5) 3) High shopping costs (t=0.99)

Figure 1: Equilibrium purchasing decisions of consumers depending on x, v, and t
Full information benchmark with heterogeneous shopping costs

With $a = z = 0.5$, Figure 1 shows these three cases. In the first case, the monopolist decreases p_A to attract the consumer located in x whose taste v for good B is equal to $a - z$. Since all consumers visit the store to buy good A, the price of good B is kept to its monopoly price $\frac{a+z}{2}$. In the next two cases, though, the monopolist discounts both p_A and p_B to keep attracting consumers. While discounting good A appeals to all consumers, discounting good B enables the retailer to reach consumers that live further away from the store, but whose valuation for good B is very high. Graphically, it enlarges the yellow area on Figure 1. This has important implications for optimal prices and the associated consumer surplus. With $a = z = 0.5$, Figure 2 shows the prices of goods A and B as a function of the shopping cost parameter t, and Figure 3 contrasts consumer surplus as a function of t when consumers have heterogeneous shopping costs, and when consumers have homogeneous shopping costs. There is an important difference that stresses why it is important to take the heterogeneity of consumers' shopping costs into account. With heterogeneous shopping costs, the consumer surplus is increasing in shopping costs as long as the monopolist does not exclude any consumers. With homogeneous shopping costs, the consumer surplus is constant in shopping costs as long as the monopolist does not exclude any consumers. This is because, with heterogeneous shopping costs, an increase in the parameter t corresponds to an increase in shopping costs' heterogeneity along the x axis, so that when the monopolist subsidises the shopping costs of the furthest consumer, any increase in t raises the economic rent of all other consumers. In other words, in a world of heterogeneous shopping costs, an increase in t makes all consumers who still visit the store better off. That is why the consumer surplus increases in t in the equilibrium where all consumers visit the store.

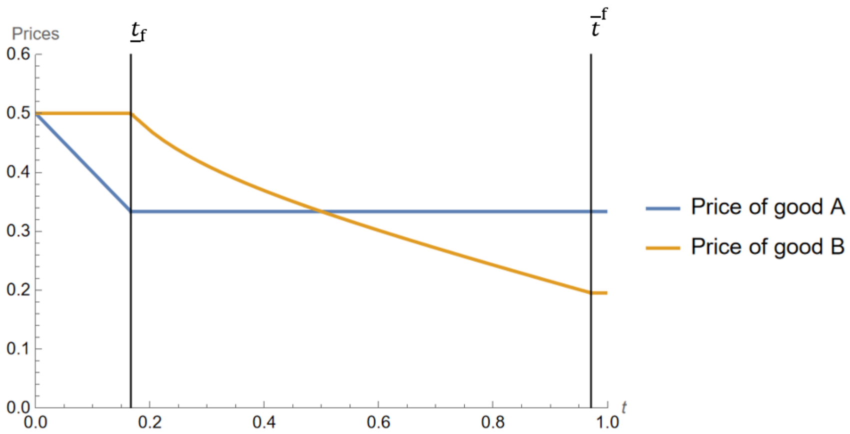


Figure 2: Equilibrium prices as a function of t
Full information benchmark with heterogeneous shopping costs

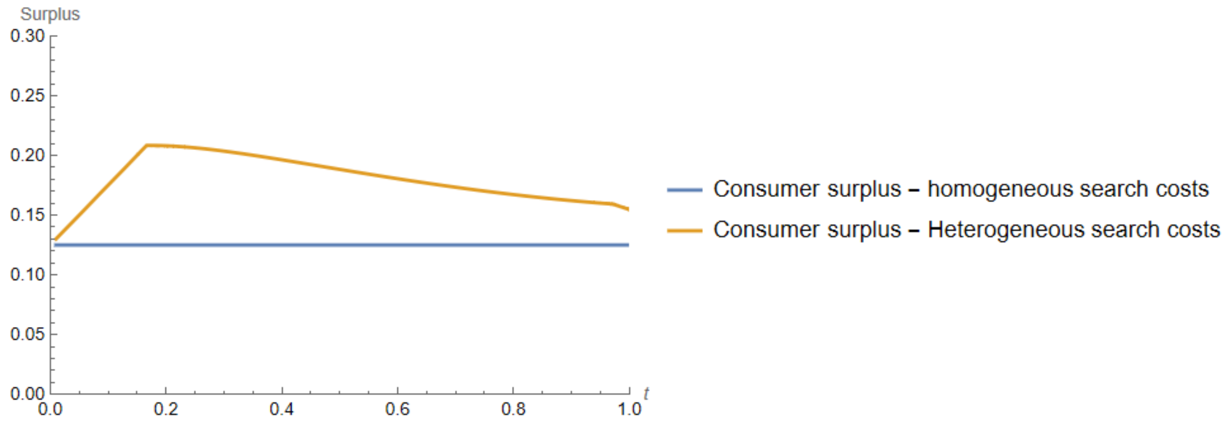


Figure 3: Consumer surplus as a function of t
Full information benchmark

5 Advertising

We now focus on partial information. This corresponds to our main model. We first describe the subgame equilibria that prevail when the monopolist advertises good A, and then describe the subgame equilibria that prevail when the monopolist advertises good B. In both cases, we first solve the case in which consumers have homogeneous shopping costs, and then solve another case in which consumers have heterogeneous shopping costs. The aim is to precisely analyse how the consequences of heterogeneity displayed by the full information benchmark may affect the partial information equilibria. Eventually, we endogenize the retailer's advertising strategy.

5.1 The monopolist advertises good A

5.1.1 Homogeneous shopping costs

First, we consider the case in which the monopolist advertises good A and consumers have homogeneous shopping costs $\frac{t}{2}$. We use the same notations as before, and we denote by p_A^A and p_B^A (resp. p_A^B and p_B^B) the prices of goods A and B when the monopolist advertises good A (resp. good B).

Lemma 5. *There exists a threshold* $\hat{t} \equiv 2(a + \frac{(a+z)^2}{8z})$ *such that:*

- *when* $t \leq \hat{t}$: $\overline{p}_A^{Al} = \overline{p}_A^{fl} = a - \frac{t}{2}$ *and* $\overline{p}_B^{Al} = \overline{p}_B^{fl} = \frac{a+z}{2}$. *All consumers visit the store.*

- *when $t \geq \hat{t}$: $\overline{p}_A^{Ah} = p_A^{\emptyset} = a$ and $\overline{p}_B^{Ah} = p_B^{\emptyset} = a + z$. No consumer ever visits the store.*

Proof. See proof in the appendix A.5. □

This result rests on the same set of intuitions we described through the no-information benchmark: when it advertises good A, the monopolist can only commit on the price of good A, so that it has an incentive to hold consumers up on their shopping costs through the price of good B. As long as consumers are sure to recover their shopping costs by purchasing good A, there is no market failure. However, when they need to recover part of their shopping costs through good B to visit the store, the hold-up effect is too acute and nobody visits the store. Indeed, for any anticipated price p_B^a, the retailer knows it only faces consumers whose taste for good B is high enough to compensate for a fraction of the shopping costs. As such, it has an incentive to set p_B above p_B^a in order to exploit this excess of taste for good B among in-store consumers. Since anticipations are rational, the only possible outcome is market collapse.

5.1.2 Heterogeneous shopping costs

We now turn to heterogeneous shopping costs.

Proposition 1. *When shopping costs are heterogeneous, under partial information, there exists two thresholds $\underline{t}^A = \underline{t}^f$ and $\overline{t}^A \in [\underline{t}^f, \overline{t}^f]$*

- *when $t \leq \underline{t}^A$: all consumers visit the store. Prices are equal to the equivalent full information benchmark: $p_A^{Al} = p_A^{fl} = a - t$ and $p_B^{Al} = p_B^{fl} = \frac{a+z}{2}$.*
- *when $\underline{t}^A \leq t \leq \overline{t}^A$: all consumers whose location x is close to 0 visit the store. Among consumers whose location x is close to 1, only those whose valuation v_B for good B is close to $a + z$ visit the store. $p_A^{Am} < p_A^f$ and $p_B^{Am} > p_B^f$*
- *when $t \geq \overline{t}^A$: all consumers whose location x is close to 0 visit the store. No consumer whose location x is close to 1 visits the store. $p_A^{Ah} < p_A^f$ and $p_B^{Ah} > p_B^f$*

Proof. See proof in the appendix A.6. □

Proposition 1 stems from the fact that advertising A may distort up the valuations for good B of in-store consumers with respect to the general population. As consumers do not observe the price of the unadvertised good before visiting the store, everything happens as if the monopolist set the price of the unadvertised good once consumers are in the store, taking advantage of this distortion by setting a higher p_B: the monopolist sets p_B to maximise $p_B * q_B(p_B)$. Through q_B, the optimal p_B depends on the distribution of in-store consumers' valuations for good B. At stage 2, consumers whose valuation for good B is below the anticipated price p_B^a of good B visit the store when $a - p_A - tx \geq 0$, and consumers whose valuation for good B is above p_B^a visit the store when $a + v - p_A - p_B^a - tx \geq 0$. Two cases occur: either the price of product A is low enough for the most remote consumers ($x = 1$) to pay the shopping cost to purchase product A only, that is, $a - p_A - t \geq 0$, or there are consumers who are too far to pay the shopping cost to purchase product A only, but who are willing to do so to purchase both goods, that is $\exists\{x, v\}, a - p_A - tx < 0 \quad \text{and} \quad a + v - p_A - p_B^a - tx \geq 0$. In the first case, in-store consumers' valuations for B are uniformly distributed between $a - z$ and $a + z$, so that the equilibrium price p_B is equal to $\frac{a+z}{2}$. In the latter case, consumers with a large valuation for good B are over-represented in the store, so that the average valuation for good B of in-store consumers is distorted up. Hence, the optimal price p_B is above $\frac{a+z}{2}$.

For $a = z = 0.5$, Figure 5 shows the equilibrium purchasing decisions of consumers depending on which good is advertised, on x, and on v_B: whenever there are consumers who do not visit the store, there is an over-representation of consumers with high valuations for good B. Contrasting Figure 5 with Figure 1 shows the extent of the distortion: whereas the full information monopolist sets a very low p_B when shopping costs are high (around 0.2 when $t = 0.99$), the advertising monopolist always deviates towards an inefficiently high p_B when shopping costs are high (around 0.6 when $t = 0.9$).

With $a = z = \frac{1}{2}$ and $t = 0.4$ (to ensure $\underline{t}^A \leq t$), Figure 4 provides a graphical interpretation for this deviation. Under full information, equilibrium prices are represented by the blue dot, which is the intersection of the best-response pricing strategies $p_B^f(p_A)$ and $p_A^f(p_B)$. For clarity, we do not represent $p_A^f(p_B)$ on the graph. When the monopolist advertises good A, the blue line represents the optimal price of good B as a function of the price p_A and the anticipated price p_B^a. Since anticipations are rational, imperfect equilibrium prices must necessarily be on the blue line. The red dot is the favorite imperfect information equilibrium of the monopolist, because it corresponds to the only system of prices such that p_B is the best-response to the observed p_A. In other words, the red dot equilibrium minimises the damages the monopolist incurs because of the lack of information. On the graph, one can notice $p_B^A > p_B^f$ and $p_A^A < p_A^f$.

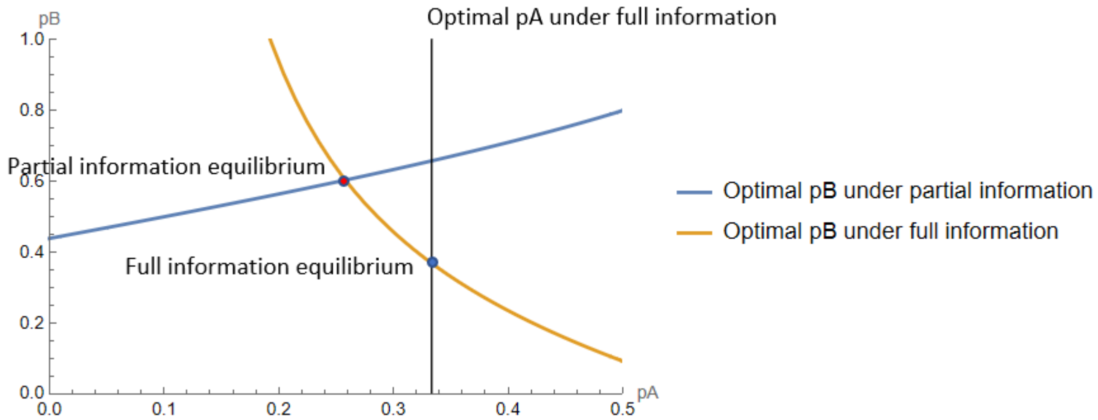


Figure 4: Best-response prices for good B under full information and under partial information (advertising good A)

The extent of this deviation must be contrasted against the case in which consumers have homogeneous shopping costs. In both cases, the monopolist raises the price of the unadvertised good B to take advantage of the fact that the average valuation for good B of in-store consumers is higher than the average valuation for good B of the general population. However, in the homogeneous shopping costs setting, this deviation prevents the existence of an equilibrium as soon as it arises, whereas it is compatible with an equilibrium in the heterogeneous shopping costs setting. The reason is that, with heterogeneous shopping costs, there are always consumers who visit the store to purchase good A, whether or not they are interested in good B. As a consequence, the monopolist retailer has an incentive to keep p_B sufficiently low to sell good B to consumers who visit the store whatever p_B^a. Thus, whereas the average valuation for good B of in-store consumers is uniformly distributed over $[p_B^a + t, a + z]$ when consumers have homogeneous shopping costs, so that the optimal price p_B is always above p_B^a, it is still distributed over $[a - z, a + z]$ when consumers have heterogeneous shopping costs. Note that it is no longer uniformly distributed over the interval, though, which explains why the optimal price p_B is above $\frac{a+z}{2}$. However, the fact there are in-store consumers whose valuation for good B lies below p_B^a creates an incentive for the monopolist to offer a lower price p_B. There are now two strengths at play: on the one hand, the monopolist has an incentive to raise p_B to take advantage of the distorted-up distribution of in-store consumers' valuations for good B; on the other hand, the monopolist has an incentive to maintain p_B low enough to keep selling good B

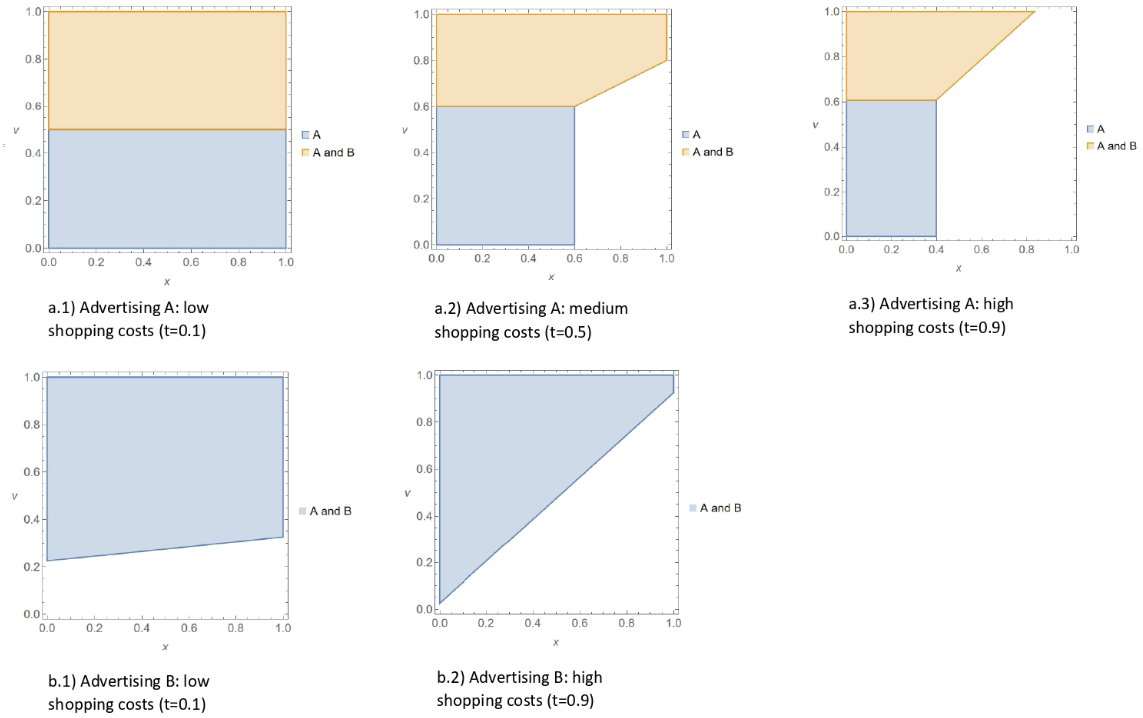


Figure 5: Equilibrium purchasing decisions of consumers depending on x, v, and the advertised good

to consumers who decided to visit the store to purchase good A. The optimal p_B is the price that equates the costs and benefits of raising p_B.

5.2 The monopolist advertises good B

In the same setting, we now characterize the subgame equilibria when the monopolist advertises good B.

5.2.1 Homogeneous shopping costs

First, we consider the case in which the monopolist advertises good B and consumers have homogeneous shopping costs $\frac{t}{2}$.

Lemma 6. *When the retailer advertises good B, it always sets $\overline{p}_A^B = a$ and $\overline{p}_B^B = \frac{1}{4}(2z - t)$. There exists a threshold $\hat{v} \equiv \frac{1}{4}(2z + t)$ such that only consumers whose valuations v_B for good B lie above $\hat{v}$ visit the store.*

Proof. See proof in the appendix A.7. □

The intuition is that, since all consumers have the same valuations for good A, there is no reason for the monopolist to set an unadvertised price p_A below $v_A = a$. Consumers anticipate that, and thus only consider v_B and p_B to decide whether or not to incur the shopping cost.

The main consequence of this is that advertising good B begets more complementarities than advertising good A. Indeed, when the monopolist advertises good B, all consumers who come to buy good B end up buying good A as well, whereas when it advertises good A, there are consumers who visit the store and buy good A only. The consequences of this differential complementarity of advertisement on profits and consumer surplus shall be more thoroughly investigated in the next section.

5.2.2 Heterogeneous shopping costs

We now turn to heterogeneous shopping costs.

Lemma 7. *When the retailer advertises good B, it always sets $p_A^B = a$. Moreover, there exists a threshold $\underline{v}$ and a threshold $\bar{v}$ such that:*

- *no consumer whose valuation v_B for good B lies below $\underline{v}$ ever visits the store.*
- *all consumers whose valuations v_B for good B lies above $\bar{v}$ visit the store.*

Proof. See proof in the appendix A.8. □

As with the homogeneous shopping costs case, advertising good B begets more complementarities than advertising good A. With $a = z = \frac{1}{2}$, Figure 5 gives an illustration of the equilibrium purchasing decisions of consumers. When shopping costs are low (subgraph b.1), $\underline{v}$ is slightly above 0.2, and $\bar{v}$ is slightly below 0.4.

5.3 Optimal advertising strategy

In this section, we endogenize the retailer's advertising strategy. We first show that, with homogeneous shopping costs, the monopolist is able to replicate perfect information, so that consumers cannot benefit from partial information. Then, we show that introducing heterogeneity in shopping costs sometimes makes it profitable for the monopolist to advertise good B, so that consumers end up being better off than under perfect information.

5.3.1 Homogeneous shopping costs

We first look at the optimal advertising strategy of the monopolist when consumers have homogeneous shopping costs.

We show that the monopolist optimal advertising strategy allows it to perfectly mimic its full information pricing strategy. Under full information, Lemma 3 states that the retailer can either set $\overline{p}_A^{fl} = a - \frac{t}{2}$ to attract all consumers to the store, and $\overline{p}_B^{fl} = \frac{a+z}{2}$ to extract consumer surplus, or it can set $\overline{p}_A^{fh} + \overline{p}_B^{fh} = \frac{1}{2}(2a + z - \frac{t}{2})$ so that only consumers who value good B enough visit the store. Under imperfect information, Lemma 5 states that, when the monopolist advertises good A, it sets $\overline{p}_A^{Al} = \overline{p}_A^{fl}$ and $\overline{p}_B^{Al} = \overline{p}_B^{fl}$, provided it makes a positive profit. Lemma 6 states that, when the monopolist advertises good B, it sets $\overline{p}_A^{B} = a$ and $\overline{p}_B^{B} = \frac{1}{4}(2z - t)$, so that $\overline{p}_A^{B} + \overline{p}_B^{B} = \overline{p}_A^{fh} + \overline{p}_B^{fh} = \frac{1}{2}(2a + z - \frac{t}{2})$.

The monopolist is thus able to reproduce perfectly both perfect information equilibria, so that partial information impacts neither the profit nor the consumer surplus.

5.3.2 Heterogeneous shopping costs

We now turn to heterogeneous shopping costs.

Lemma 8. *For any given shopping cost t, the monopolist advertises a lower price for good B than for good A.*

Proof. See proof in the appendix A.9. □

There are two main drivers behind this result, and both relate to a key feature of our model: while the advertised good is used to attract customers, the unadvertised good enables the monopolist to extract profit from visiting consumers. The first driver is that, for any given discount relative to a price a, advertising A allows to attract more consumers than advertising B, because all customers are potentially interested in purchasing good A for any price below a, while only customers whose v is above p_B are sensitive to an advertisement for good B. Thus, to reach any amount of consumers, there is no need to discount A as much as B. The second driver stems from the fact that advertising B begets more complementarities. Indeed, when the retailer advertises B, $p_A^B = a$, hence all consumers who decide to visit the store because of an advertisement for B end up purchasing A, whereas when the retailer advertises A, consumers who decide to visit the store only purchase B when $p_B^A \leq v$. Thus, while an additional discount on the advertised price is always compensated for by higher sales of the other good, this effect is stronger when the monopolist advertises B. Since the monopolist internalizes this cross-product externality, the optimal advertised price is lower for good B than for good A.

For $a = z = 0.5$, Figure 6 illustrates Lemma 8. It shows the equilibrium advertised prices as a function of the shopping costs t. For any given t, the optimal p_B when good B is advertised is below the optimal p_A when good A is advertised.

Lemma 9. *The advertised price of good A p_A^A is convex and decreases in t.*

Proof. See proof in the appendix A.10. □

As shopping costs increase, the advertised price of good A p_A^A decreases more slowly. Each change in the slope corresponds to the switch from one kind of equilibrium to the next, as displayed on Figure 5. That p_A decreases more slowly stems from the fact that when t increases, the marginal amount of consumers any additional discount on p_A attracts decreases as well. Indeed, not only does a higher t means higher shopping costs, it also reflects a stronger heterogeneity in shopping costs: a consumer located in x spends tx to visit the store, so that the further away from the store a consumer is, the more she is affected by an increase in t. As a consequence, when t increases, a monopolist who maintains a constant advertised price p_A loses a disproportionate amount of far-away customers. As t increases, the proportion of far-away consumers decreases, so that any further increase in t requires a smaller additional discount on the advertised price p_A.

We then analyse the effect of advertisement on consumer surplus. More precisely, we compare consumer surpluses when the monopolist advertises A, when the monopolist advertises B, and under the standard perfect information setting, in which the monopolist commits on both prices before consumers decide whether or not to visit the store.

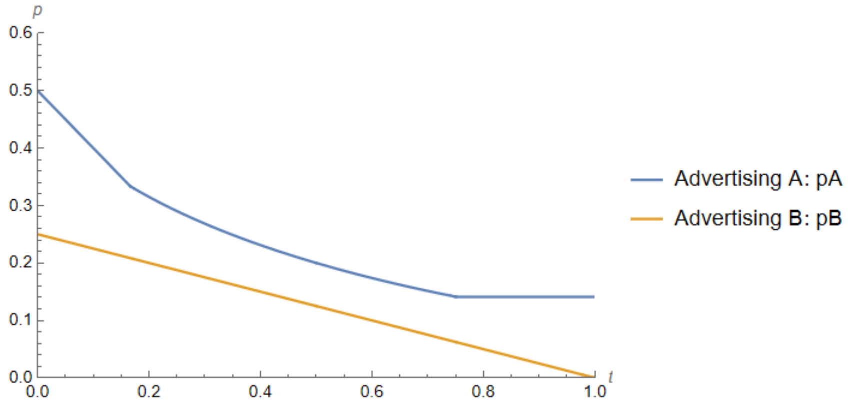


Figure 6: Equilibrium advertised prices as a function of the shopping costs t
Heterogeneous shopping costs

Proposition 2. *Consumers are better off when the monopolist advertises good B than under perfect information, and they are better off under perfect information than when the monopolist advertises good A.*

Proof. See proof in the appendix A.11. □

For $a = z = 0.5$, Figure 7 shows consumer surplus as a function of shopping costs, when good A is advertised, when good B is advertised, and under perfect information. When the monopolist advertises B, both the advertised price and the unadvertised price are lower than when the monopolist advertises A. This is because there is no profitable upward deviation on the price of the unadvertised good, and because complementarities foster a low advertised price p_B^B. As a consequence, consumers benefit a lot from the monopolist advertising good B rather than good A.

Because the monopolist who advertises good B cannot commit on the price of good A, it must offer a very low price for good B in order to attract enough demand. Moreover, advertising good B begets a lot of complementarities between goods A and B: all consumers who come to purchase good B end up purchasing good A as well. Since the monopolist sells good A for a very high price, it has an interest in advertising B aggressively to attract more customers. For $a = z = \frac{1}{2}$, contrasting Figure 5 with Figure 1 shows that, whereas the full information monopolist sets $p_B^f h = 0.2$ when shopping costs are high ($t = 0.99$), the advertising monopolist advertises $p_B^B h = 0.02$ when shopping costs are high ($t = 0.9$). Consumers benefit from these

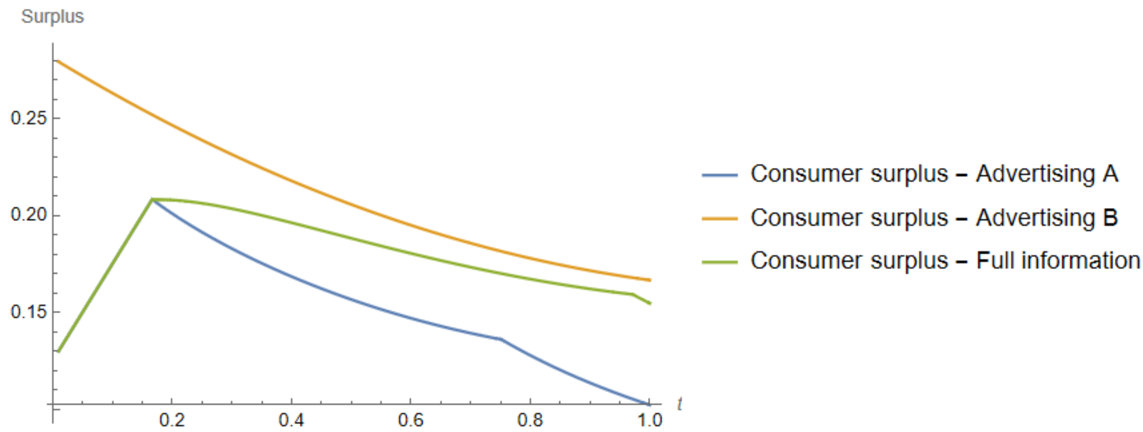


Figure 7: Consumer surplus as a function of shopping costs, when good A is advertised, when good B is advertised, and under perfect information
Heterogeneous shopping costs

complementarities: there are very few consumers who would have visited the store in a perfect information world but who abstain in the partial information one, and those who visit the store end up paying less. With $a = z = \frac{1}{2}$, Figure 7 shows that the additional surplus consumers can expect from partial information is particularly large when shopping costs are small. That is because, under perfect information, the monopolist uses p_A to attract consumers while keeping prices high, whereas it must advertise very low a p_B to reach enough consumers and exploit the complementarity effect in the case of partial information with advertisement for good B.

Anticipating stages 2 and 3, the monopolist chooses which good to advertise. Formally, it advertises good A when $\Pi(p_A^A, p_B^a) \geq \Pi(p_A^a, p_B^B)$ where $\Pi(p_A^A, p_B^a)$ is the profit expected from advertising good A, taking into account consumers' anticipations of the unadvertised price p_B^a.

The monopolist faces the following trade-offs: on the one hand, advertising A is a stronger incentive for consumers to visit the store, because all consumers are potentially interested by the discount, while only those whose valuation is above p_B are potentially interested by an advertisement for good B. Thus, for a given discount, advertising A brings more customers to the store. On the other hand, advertising B begets more complementarity, so that fewer in-store customers is compatible with higher aggregated sales.

Proposition 3. *There exists a threshold $\bar{\bar{t}}$ such that:*

- *When $t \leq \bar{\bar{t}}$ the monopolist advertises good A.*

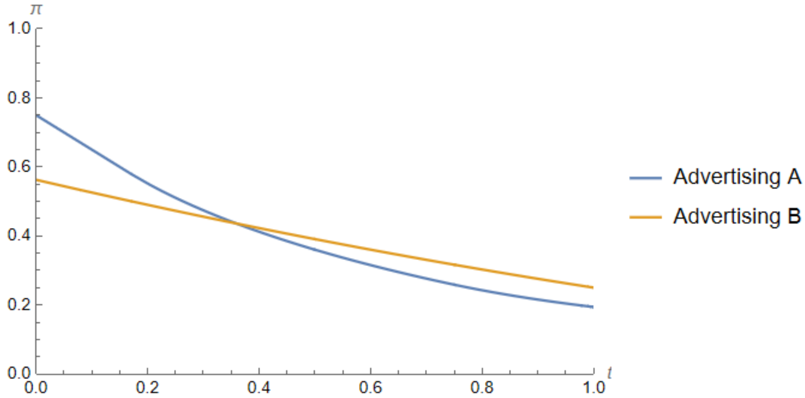


Figure 8: Profit as a function of shopping costs t, depending on which good is advertised
Heterogeneous shopping costs

- *When* $t \geq \bar{\bar{t}}$ *the monopolist advertises good* B.

Proof. See proof in the appendix A.12. □

Intuitively, this result stems from the fact that advertising good B begets more complementarity than advertising good A: while shopping costs are below $\bar{\bar{t}}$, advertising A allows the monopolist to set higher prices while attracting enough demand. However, when shopping costs increase above $\bar{\bar{t}}$, the complementarity effect of advertising good B enables the monopolist to keep the aggregated demand for goods A and B higher than when it advertises good A. Thus, in spite of lower average prices, selling more is profitable. With $a = z = 0.5$, Figure 8 shows the profit of the monopolist as a function of the shopping costs, depending on whether it advertises good A or good B. With those parameters, $\bar{\bar{t}}$ is close to 0.35. With $a = z = 0.5$, Figure 9 shows aggregated sales as a function of shopping costs, depending on whether the monopolist advertises good A or good B. When the ad strategy switch occurs, aggregated sales are already higher when the monopolist advertises good B. As seen in Proposition 2, consumers benefit from the strategic switch, because they can purchase more for a lower aggregated price.

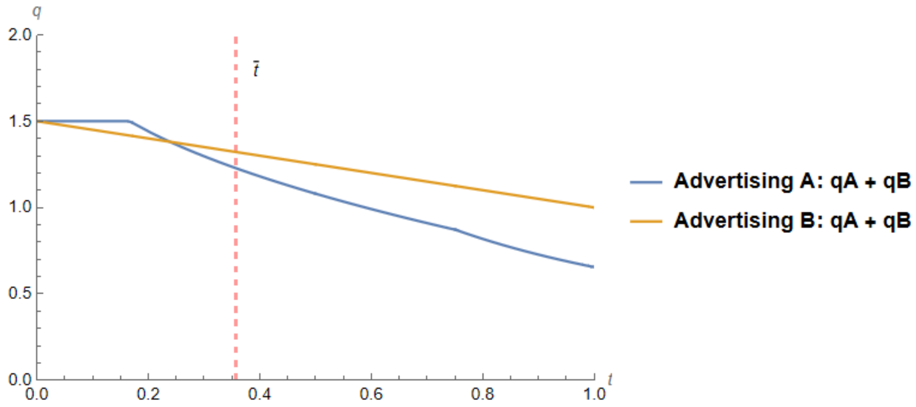


Figure 9: Aggregated sales as a function of shopping costs t, depending on which good is advertised
Heterogeneous shopping costs

6 Policy implications

We now describe some policy implications of our findings.

Our foremost result is that consumers benefit from a lack of information, as long the retailer advertises the price of the good for which valuations are heterogeneous. This relies on several conditions: there must exist a monopolist multiproduct retailer facing consumers who must have high enough shopping costs, these shopping costs must be sufficiently heterogeneous, the monopolist retailer must not easily be able to beget a situation of perfect information, and it must be able to advertise price discounts without restrictions.

We think this corresponds to several important product markets, a simple example being supermarkets. In their rulings on competition cases, Competition Authorities usually define the relevant food retail market at the local scale, so that there is a non-negligible amount of consumers facing actual monopolies. Due to the number of products they sell, these supermarkets cannot easily commit on all prices, and consumers differ in their shopping costs because of their distance to the store, as well as their taste for shopping and availability.

Focusing on such markets, our model and findings have two important policy implications.

First, the regulator should not necessarily push for full price disclosure. Mandates for full price disclosure usually take the form of an online platform where retailers are to report prices. Such policies have already been implemented with mixed results: regarding Israeli supermarkets,Ater and Rigbi (2023) find evidence that the mandate resulted in lower prices, considering Chilean petrol stations, Luco (2019) find that price disclosure made petrol stations improve their margins. Within our framework, a more efficient way to protect consumers facing a monopoly is to incentivize retailers to advertise the goods they sell for which consumers' valuations are the most heterogeneous.

Second, limiting the ability to advertise important price discounts harm consumers. For instance, France has long had laws banning loss-leading pricing, and it has more recently been implementing laws to prevent retailers from offering more than 34 %-discounts on the price of most goods. Within our framework, limiting price discounts is likely to be particularly detrimental to consumers. Indeed, faced with a limited discount capacity, our model shows the retailer always prefers to advertise good A, because advertising good B is profitable only when the discount consented on good B is very large. Such bans imply that the consumer-surplus-enhancing advertising strategy is never adopted by the retailer, so that consumers end up worse off than under full information.

Other policy implications are left for further research. In particular, it may be fruitful to model upstream producers. For instance, the advertising strategy of the retailer has an impact on the profits of the producers, so that regulators could gain valuable insights into potentially anti-competitive practices. Indeed, through the bargaining process, the retailer may be able to use price advertising as a threat to make producers compete away their margins.

7 Conclusion

Retailers typically offer a vast assortment of products to their customers. As a result, consumers are unable to display perfect recall of the prices of each item. In addition, there are products that elicit diverse valuations from different consumers, and products that generate similar valuations across consumers. Moreover, there are various factors that cause consumers to have different shopping costs, whether we regard them as transportation costs or as search costs. In a setup in which a monopolist retailer sells two products to consumers who have heterogeneous valuations for one good and homogeneous valuations for the other good, the aim of this paper is to examine the impact of price advertising on the advertising strategy of the retailer, and on the welfare of consumers.

Our main finding is that consumers can benefit from having incomplete information, provided they have heterogeneous shopping costs. The key mechanism that drives this result is the differential complementarity effect, that is, the fact that advertising the good for which consumers' valuations are heterogeneous makes all consumers who visit the store buy the unadvertised good as well, whereas advertising the good for which consumers' valuations are homogeneous only makes some consumers who visit the store buy the unadvertised good as well. When the monopolist cannot commit on both prices, and provided the shopping costs are high enough, it exploits the complementarity effect by setting a very low advertised price. This benefits consumers, who end up paying less for the two goods than under perfect information.

This paper points out how the heterogeneity in consumer valuations impacts the optimal advertising and pricing strategies of a multiproduct monopolist, as well as the associated consumer surplus. More specifically, we contribute to the literature by modelling both the heterogeneity of consumers' valuations heterogeneity, and the heterogeneity of consumers' shopping costs.

Eventually, our model and results have two main policy implications. On the one hand, we provide a theoretical rationale to not systematically push for full price disclosure. On the other hand, we document how the ability to advertise important price discounts benefit consumers, so that regulating discounts or advertisement may harm consumers.

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Appendix

A Proofs

A.1 Proof Lemma 1

Let p_A^a and p_B^a be the prices anticipated by consumers. Without loss of generality, we assume $p_A^a \leq a$ and $a - z \leq p_B^a \leq a + z$.

First, assume $p_A^a < a$ and $p_B^a \leq a+z$. Then, all consumers whose location x and taste v_B for good B are such that $tx \leq a - p_A^a + v_B - p_B^a$ visit the store. Let D be the mass of consumers who visit the store. Everything happens as if, when the monopolist sets p_A and p_B, consumers had already paid the shopping cost, so that they make separate purchase decisions for each good: they buy good A provided $p_A \leq a$ and they buy good B provided $p_B \leq v_B$. Regarding good A, the monopolist maximises $\Pi_A = D * p_A * \mathbb{1}_{p_A \leq a}$. The optimal price p_A is thus equal to a, independently of the anticipated price for good B. Since consumer make rational anticipations, $p_A^a = a$.

Now, assume $p_A^a = a$ and $a - z < p_B^a < a + z$. Then, all consumers whose location x and taste v_B for good B are such that $tx \leq v_B - p_B^a$ visit the store. Regarding good B, the monopolist maximises $\Pi_B(p_B) = p_B \int_0^{\min(1, \frac{a+z-p_B^a}{t})} \int_{tx+p_B^a}^{a+z} \mathbb{1}_{p_B \leq v_B} \, dv_B \, dx$. Note that the double integral represents demand for good B as a function of the price p_B, given that only consumers whose $v_B \geq p_B^a + tx$ are potential buyers. The double integral is needed to account for both the individual location x and the individual taste v_B for good B.

The proof consists in showing there always exists a profitable upward price deviation. To do that, we show the derivative of the profit function is always strictly positive. Let $\epsilon > 0$. On the one hand, we have:

$$\Pi_B(p_B^a) = p_B^a \int_0^{\min(1, \frac{a+z-p_B^a}{t})} \int_{tx+p_B^a}^{a+z} 1 \, dv_B \, dx$$

$$= \begin{cases} p_B^a \frac{(a+z-p_B^a)^2}{4tz} & \text{if } \min(1, \frac{a+z-p_B^a}{t}) = \frac{a+z-p_B^a}{t} \\ p_B^a \frac{a+z-p_B^a - \frac{t}{2}}{2z} & \text{if } \min(1, \frac{a+z-p_B^a}{t}) = 1 \end{cases}$$

On the other hand, as consumers decide to visit the store based on the price they anticipate, but make their final purchase decisions based on actual prices, the profit of the retailer were it to set $p_B = p_B^a + \epsilon$ is given by:

$$\Pi_B(p_B^a + \epsilon) = (p_B^a + \epsilon) * \int_0^{\min(\frac{a+z-p_B^a}{t},1)} \int_{tx+p_B^a}^{a+z} \mathbb{1}_{p_B^a+\epsilon \leq v_B} \, dv_B \, dx$$

because only consumers whose v_B is above the observed price $p_B^a + \epsilon$ buy good B

$$= (p_B^a + \epsilon) * (\int_0^{\frac{\epsilon}{t}} \int_{p_B^a+\epsilon}^{a+z} 1 \, dv_B \, dx + \int_{\frac{\epsilon}{t}}^{\min(\frac{a+z-p_B^a}{t},1)} \int_{tx+p_B^a}^{a+z} 1 \, dv_B \, dx)$$

because in-store consumers are consumers whose $v_B \geq tx + p_B^a$ and:

when $x \leq \dfrac{\epsilon}{t}$, in-store consumers only buy good B when $v_B \geq p_B^a + \epsilon$

when $x \geq \dfrac{\epsilon}{t}$, all in-store consumers verify $v_B \geq p_B^a + \epsilon$

$$= \begin{cases} (p_B^a + \epsilon)(\frac{(a+z-p_B^a)^2}{4tz} - \frac{\epsilon^2}{4tz}) & \text{if } \min(1, \frac{a+z-p_B^a}{t}) = \frac{a+z-p_B^a}{t} \\ (p_B^a + \epsilon)\frac{a+z-p_B^a - \frac{t}{2} - \frac{\epsilon^2}{2t}}{2z} & \text{if } \min(1, \frac{a+z-p_B^a}{t}) = 1 \end{cases}$$

As a consequence:

$$\lim_{\epsilon \to 0} \frac{\Pi_B(p_B^a + \epsilon) - \Pi_B(p_B^a)}{\epsilon} > 0$$

$$\iff \begin{cases} \lim_{\epsilon \to 0} \frac{(a+z-p_B^a)^2 - \epsilon p_B^a - \epsilon^2}{4tz} > 0 & \text{if } \min(1, \frac{a+z-p_B^a}{t}) = \frac{a+z-p_B^a}{t} \\ \lim_{\epsilon \to 0} \frac{a+z-p_B^a - \frac{t}{2} - \frac{\epsilon}{2t}(p_B^a + 2\epsilon)}{2z} > 0 & \text{if } \min(1, \frac{a+z-p_B^a}{t}) = 1 \end{cases}$$

$$\iff \begin{cases} (a+z-p_B^a)^2 > 0 & \text{if } \min(1, \frac{a+z-p_B^a}{t}) = \frac{a+z-p_B^a}{t} \\ \frac{a+z-p_B^a}{t} > \frac{1}{2} & \text{if } \min(1, \frac{a+z-p_B^a}{t}) = 1 \end{cases}$$

Thus, the retailer always has an incentive to charge ϵ-more once consumers are in the store. As a consequence, the only rationally anticipated prices are $p_A^{\emptyset} = a$ and $p_B^{\emptyset} = a + z$, so that no consumer ever visits the store.

A.2 Proof Lemma 2

When $t = 0$, all consumers visit the store, and buy a good whenever their valuation for this good is above its price. As a consequence, the monopolist sets the price of each good independently.

All consumers buy good A as long as $p_A \leq a$. The optimal price is thus $\overline{p}_A^{f0} = a$.

Each consumer buys good B as long as $p_B \leq v_B$. The profit made on good B is thus equal to

$$\Pi_B(p_B) = \mathbb{P}(v_B \geq p_B) * p_B = (1 - \frac{p_B - (a-z)}{a+z-(a-z)})p_B$$

Hence $\overline{p}_B^{f0} = \frac{a+z}{2}$. Note that we have assumed $z > \frac{a}{3}$ to ensure $\frac{a+z}{2} \geq a - z$.

A.3 Proof Lemma 3

To solve for equilibrium prices, we begin by noting there are only two possible cases: either all consumers visit the store, or only those who value good B enough.

When the monopolist decides to sell to all consumers, p_A is set to its maximum level such that all consumers still visit the store: $\overline{p}_A^{fl} = a - \frac{t}{2}$. Since all consumers visit the store to buy good A, p_B is set to its monopoly level: $\overline{p}_B^{fl} = \overline{p}_B^{f0} = \frac{a+z}{2}$. This strategy leaves the retailer with a profit $\overline{\Pi^{fl}} \equiv a - \frac{t}{2} + \frac{(a+z)^2}{8z}$.

Regarding the second strategy, the monopolist maximises $\Pi(p_A, p_B) = \mathbb{P}(v_B \geq v_0) * (p_A + p_B) = \frac{a+z-v_0}{2z}(p_A + p_B)$ where $v_0 = p_A + p_B + \frac{t}{2} - a$ is the valuation for good B of the consumer who is indifferent between staying at home and visiting the store. Optimal prices are all combinations of p_A and p_B such that $\overline{p}_A^{fh} + \overline{p}_B^{fh} = \frac{1}{2}(2a + z - \frac{t}{2})$. This strategy leaves the retailer with a profit $\overline{\Pi^{fh}} \equiv \frac{(2a+z-\frac{t}{2})^2}{8z}$.

In equilibrium, the monopolist implements the strategy that maximises its profit:

$$\overline{\Pi^{fl}} \leq \overline{\Pi^{fh}} \iff t \geq 2(a + \frac{(a+z)^2 - (2a+z-\frac{t}{2})^2}{8z})$$

For instance, when $a = z = \frac{1}{2}$, $\hat{t} = 1$.

When shopping costs t lie above this $\hat{t}$, the monopolist prefers the strategy that excludes some consumers from the market.

A.4 Proof Lemma 4

To solve for equilibrium prices, we begin by noting there are only two possible cases: either all consumers visit the store, or only those whose live close enough to the store and/or who enjoy good B enough visit the store.

When the monopolist decides to sell to all consumers, p_A is set to its maximum level such that all consumers still visit the store. In other words, the consumer who is the least likely to visit the store, that is, the one who lives in $x = 1$ and does not value good B at all ($v_B = 0$) must be indifferent between visiting the store and staying home: $p_A^{fl} = a - t$. Since all consumers visit the store to buy good A, p_B is set to its monopoly level: $p_B^{fl} = \frac{a+z}{2}$. This generates a profit:

$$\Pi(p_A^{fl}, p_B^{fl}) = p_A^{fl} * 1 + p_B^{fl} * \mathbb{P}(v_B \geq p_B^{fl}) = a - t + \frac{a+z}{2} * \frac{a+z}{4z} = a + \frac{(a+z)^2}{8z}$$

Regarding the second strategy, the monopolist maximises

$$\Pi(p_A, p_B) = (p_A + p_B)\mathbb{P}(v_A + v_B - p_A - p_B - tx \geq 0) + p_A\mathbb{P}(v_A - p_A - tx \geq 0)\mathbb{P}(v_B - p_B < 0)$$
$$= \begin{cases} (p_A + p_B)\frac{2a+z-p_A-p_B-\frac{t}{2}}{2z} + p_A\frac{(p_B-a+z)(a-p_A)}{2tz} & \text{if } t \leq 2a + z - p_A - p_B \\ (p_A + p_B)\frac{(2a+z-p_A-p_B)^2}{4tz} + p_A\frac{(p_B-a+z)(a-p_A)}{2tz} & \text{if } t \geq 2a + z - p_A - p_B \end{cases}$$

In both cases, only those consumers whose combination of location x and valuation for good B v_B is good enough visit the store. The difference between the two sub-cases is that, When $t \leq 2a + z - p_A - p_B$, there are consumers located in $x = 1$ who visit the store, whereas when $t \geq 2a+z-p_A-p_B$, no consumer located in $x = 1$ ever visits the store, no matter her valuation for good B.

In equilibrium, the monopolist implements the strategy that maximises its profit: $\underline{t}^f$ corresponds to the threshold above which it is no longer profitable to attract all consumers to the store, and $\overline{t}^f = 2a + z - p_A - p_B$ corresponds to the threshold above which consumers located in $x = 1$ whose valuation for good B is $v_B = a + z$ no longer visit the store.

The equilibrium prices p_A^{fm}, p_A^{fh},p_B^{fm}, and p_B^{fh} are too intricate to be written in their closed-form but Figure 2 provides an illustration when $t = z = \frac{1}{2}$.

A.5 Proof Lemma 5

To solve for equilibrium prices, we begin by noting there are two *a priori* possible cases: either all consumers visit the store, or only those who value good B enough.

In the first case, p_A is set to its maximum level such that all consumers still visit the store: $\overline{p}_A^{Al} = \overline{p}_A^{fl} = a - \frac{t}{2}$. Since all consumers visit the store to buy good A, p_B is set to its monopoly level: $\overline{p}_B^{Al} = \overline{p}_B^{fl} = \frac{a+z}{2}$. This strategy leaves the retailer with a profit $\overline{\Pi^{Al}} \equiv a - \frac{t}{2} + \frac{(a+z)^2}{8z}$.

Regarding the second case, we need to show that, when only consumers who value good B enough decide to incur shopping costs to visit the store, there is no market. The proof is similar to the benchmark case without information, and consists in showing that, for any anticipated price $p_B^a < a + z$, the monopolist has an incentive to unexpectedly raise p_B above p_B^a.

Thus, $\hat{t}$ is such that the monopolist retailer makes no profit when all consumers visit the store:

$$\overline{\Pi^{Al}} = 0 \Longleftrightarrow \hat{t} = 2(a + \frac{(a+z)^2}{8z})$$

A.6 Proof Proposition 1

To solve for equilibrium prices, we begin by noting there are only two possible cases: either all consumers visit the store, or only those whose live close enough to the store and/or who enjoy good B enough visit the store.

When the monopolist decides to sell to all consumers, p_A is set to its maximum level such that all consumers still visit the store. In other words, the consumer who is the least likely to visit the store, that is, the one who lives in $x = 1$ and does not value good B at all ($v_B = 0$) must be indifferent between visiting the store and staying home: $p_A^{Al} = a - t$. Since all consumers visit the store to buy good A, p_B is set to its monopoly level: $p_B^{Al} = \frac{a+z}{2}$. Up to this point, this is similar to perfect information (Lemma 4).

However, whenever some consumers cease to visit the store, the monopolist chooses p_A and p_B to maximise:

$$\begin{aligned}\Pi(p_A, p_B) =& (p_A + p_B)\mathbb{P}(v_B \geq p_B | v_A + v_B - p_A - p_B^a - tx \geq 0)\mathbb{P}(v_A + v_B - p_A - p_B^a - tx \geq 0) \\ &+ p_A \mathbb{P}(v_A - p_A - tx \geq 0)\mathbb{P}(v_B - p_B < 0)\end{aligned}$$

where p_B^a is the price of good B as anticipated by consumers.

To be completed.

A.7 Proof Lemma 6

When the monopolist advertises B, everything happens as if the monopolist set the price of good A once consumers are already in the store. Since all consumers have the same valuation $v_A = a$ for good A, all consumers who are in the store purchase A when $p_A \leq a$, and no consumer purchases A when $p_A > a$. The optimal price for good A is thus $\overline{p}_A^B = a$, and all in-store consumers purchase A.

Anticipating this price, consumers decide to visit the store when $v_B - \overline{p}_B^B - \frac{t}{2} \geq 0$. As a consequence, there exists a threshold $\hat{v} \equiv \overline{p}_B^B + \frac{t}{2}$ such that only consumers whose valuations v_B for good B are above $\hat{v}$ visit the store.

Taking that into account, the monopolist sets $\overline{p}_B^B$ to maximise its profit:

$$\Pi(\overline{p}_B^B) = (\overline{p}_A^B + \overline{p}_B^B)\mathbb{P}(v_B \geq \hat{v}) = (a + \overline{p}_B^B)\frac{a + z - \overline{p}_B^B - \frac{t}{2}}{2z}$$

The solution is $\overline{p}_B^B = \frac{1}{4}(2z - t)$.

A.8 Proof Lemma 7

When the monopolist advertises B, everything happens as if the monopolist set the price of good A once consumers are already in the store. Since all consumers have the same valuation $v_A = a$ for good A, all consumers who are in the store purchase A when $p_A \leq a$, and no consumer purchases A when $p_A > a$. The optimal price for good A is thus $p_A^B = a$, and all in-store consumers purchase A.

Anticipating this price, consumers decide to visit the store when $v_B - p_B^B - tx \geq 0$. This allows us to define $\underline{v}$ and $\bar{v}$.

$\underline{v}$ is given by $\underline{v} - p_B - t * x \leq 0$ when $x = 0$, that is $\underline{v} \equiv p_B^B$.

$\bar{v}$ is given by $\bar{v} - p_B - t * x \geq 0$ when $x = 1$, that is $\bar{v} \equiv p_B^B + t$.

A.9 Proof Lemma 8

To be completed.

A.10 Proof Lemma 9

To be completed.

A.11 Proof Proposition 2

To be completed.

A.12 Proof Proposition 3

To be completed.