Who should *not* share? The merits of withholding unused vehicles

Preliminary and incomplete

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Abstract

People repeatedly demand travel, using available vehicles scattered around space. What can justify vehicle withholding (i.e. preventing others from using it, for own future use) from the social welfare perspective? This paper investigates heterogeneity in the potential cost of search for alternative vehicles as such justification. It is shown that travellers whose search cost is substantially higher than that of others (e.g. limited-mobility people) can optimally withhold a vehicle. The heterogeneity of search costs should be sufficiently strong, e.g. a uniform distribution is not variable enough to justify withholding by anyone. In an example calibrated for car use in London, it is shown that at most 41% of car users should withhold their vehicles under the most extreme modelling assumptions, while all others should share.
Keywords: Vehicle sharing, Transportation demand, Spatial search frictions

JEL codes: D61, L92, O18, R40

1. Introduction

Vehicle sharing is a technology that allows the same vehicle to be used by different people at different points in time, reducing the number of vehicles that are needed to meet transportation demand. Today, shared vehicles (SV) remain a fringe transportation option and serve only a fraction of travel demand, while exclusive-use vehicles continue to be viewed by most people as the default option. This paper turns the comparison on its head and

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assumes vehicle sharing is the default option, while vehicle *withholding* is an alternative option. Here withholding is defined as prevention of other people from legally using the vehicle, for the purpose of own future use. This paper asks the question: what can justify, from the social welfare perspective, exclusive use of vehicles by some people?

Historically, exclusive nature of vehicle use was driven by lack of technology for effective sharing. For example although carsharing was first attempted in 1948 (Shaheen et al., 1998), it has reached commercial success only in the 21st century, after new communication technologies allowed to remotely grant control of the vehicle to a specific individual. Today, the technological challenge of vehicle sharing is successfully solved with minimal additional equipment required for the vehicle.

Another potential motivation for vehicle exclusive access is the moral hazard problem: users may show more care about the vehicle if they expect they will continue to use the same vehicle in the future. Modern carsharing companies, however, are increasingly able to track vehicle movements and detect the driving style, reducing the moral hazard problem. The rest of this paper does not consider this problem.

The rest of the paper is focused on analysing whether vehicle withholding by some people travelling (*travellers* henceforth) can be justified by heterogeneity of such people. Most of the paper is focused on one dimension of such heterogeneity, while other dimensions are discussed in the end.

While vehicle sharing allows to meet the same travel demand with fewer vehicles, it also creates spatial search frictions. As available vehicles are scattered around space, they have to be searched (usually via a mobile app) and then the traveller normally has to walk to the vehicle of choice.

This paper explores whether traveller heterogeneity in walking costs (i.e. in costs of physical effort or in the opportunity cost of time) can justify vehicle withholding by some travellers. The answer is yes, but only if the walking cost heterogeneity is exceptionally high, and only for a minority of travellers with the highest walking cost. In particular, it is shown that a uniform distribution of walking costs is not variable enough and does not justify withholding by anyone. In an automobile travel demand example calibrated for the city of London, the maximum share of travellers who should optimally withhold their vehicles is 41% in the most adverse scenario, and is much lower than that under more plausible scenarios. All remaining travellers should optimally share automobiles. The result is proven for arbitrary distribution of walking costs with a calibrated upper bound.

It should be emphasised that it is not the absolute level of walking cost that matters for optimal withholding, but its comparison with that of other travellers. When the walking cost is homogenous, regardless of its level, the first-come-first-serve basis for vehicle use maximises welfare, so withholding is not justified. Withholding by traveler A can be optimal only if other travellers typically have much lower walking costs, so the cost of withholding (i.e. increased search cost for other travellers, and longer period of inactivity of the withheld vehicle) is below its benefit (allowing traveller A to avoid walking in the future).

2. The model

The model is based on Zakharenko (2023b) with some modifications. There is a continuum of individuals who live in infinite continuous time and travel between two infinite linear streets repeatedly, using a vehicle. The precise location of the next origin is the same as the location of the previous destination. All vehicles are (potentially) shared and are free-floating, i.e. can be released for the use by others at any time and location. At the end of each trip, travellers decide whether to release the vehicle or withhold it for future use.

Departure for the next trip is a Poisson process that cannot be planned in advance, such that the expected duration of stay between trips is τ. The mass of travelers who depart, per unit of time per unit of linear street space, is exogenously given by L. The exact location at the destination street is also random and uniformly distributed across space,

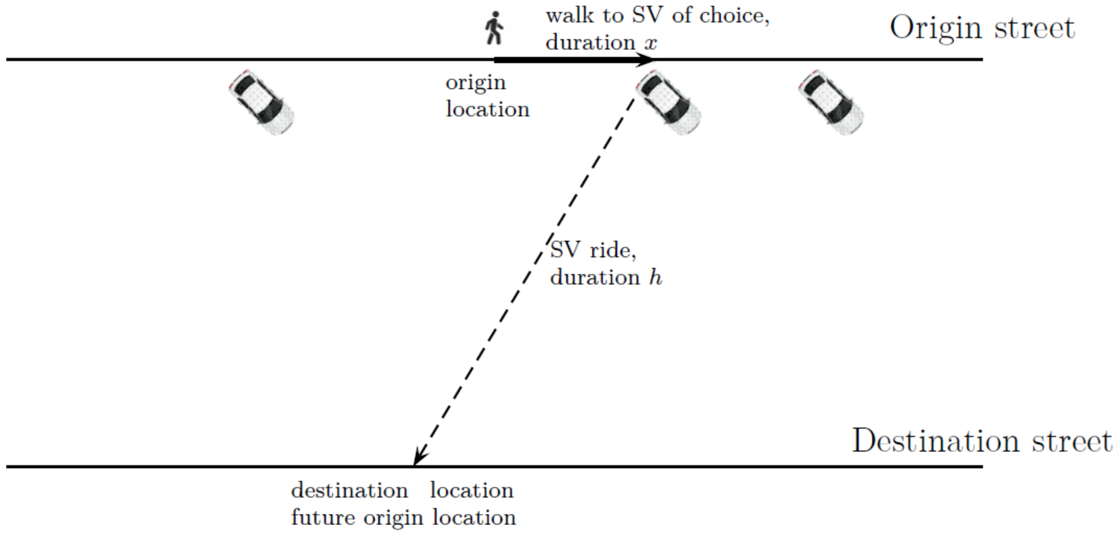

Figure 1: Illustration of a typical trip. Vehicle image courtesy of Macrovector/Freepik.

which also means that the distribution of vehicles along each street is also uniform. Given these assumptions, the total amount of vehicle-kilometers travelled, as well as the cost of such travel, are an exogenous part of social transportation cost and are not modelled explicitly.

The social cost of parked vehicles is ϕ per unit of time; this includes the vehicle standing costs and the social cost of parking.[2] Parked vehicles can be in one of the three states: withheld, vacant, and reserved, explained below.

When a traveller chooses to withhold a vehicle at the end of the previous trip, the vehicle remains in this state until the next trip, that is, for τ units of time in expectation. When the vehicle is released, it becomes vacant and available for anyone. The endogenous density of vacant vehicles is denoted by μ.

Immediately before the next trip, travellers who previously released their vehicles search for a vehicle again. With positive probability, previously used vehicle is still available at the same location, so the traveller can use it with zero walking cost involved. With the remaining probability, the previous vehicle is no longer available, and another one has to be found. In

[2]In previous variants of this model, Zakharenko (2022) and Zakharenko (2023b), these two elements were modelled separately.

Notation	Description	Units
λ	fraction of travellers sharing vehicles	
μ	density of vacant vehicles	veh*/ kmS**
τ	mean duration of stay at destination	h
ϕ	vehicle standing cost	\$/h/veh
C	Social cost of transportation	\$/h/kmS
L	travel demand	per***/ h/ kmS
$\bar{w}$	mean cost of walking	\$/h

Table 1: Notational glossary.*per vehicle or number of vehicles; **km of street space; *** number of persons.

social optimum, travellers will always use the nearest available vehicle, which is $\frac{1}{2\mu}$ units of time (and also units of space, assuming unitary walking speed) away in expectation.[3] The vehicle has to be reserved while the traveller is walking towards it. The incurred walking cost $w \geq 0$ is heterogenous with c.d.f. $F(\cdot)$ and p.d.f. $f(\cdot)$.

It can be shown that the socially optimal decision to withhold a vehicle is increasing in the traveller's walking cost. That is, there exists $\hat{w} \in [0, \infty)$ such that all travellers with $w < \hat{w}$ release their vehicle and those with $w > \hat{w}$ withhold it.[4] Denote by $\lambda = F(\hat{w})$ the endogenous fraction of traveller population who release their vehicle after each trip. Denote also by $\bar{w}(\lambda)$ the mean walking cost among those who release their vehicles:

$$\bar{w}(\lambda) = \frac{1}{\lambda} \int_0^{F^{-1}(\lambda)} w f(w) \mathrm{d}w. \tag{1}$$

The social transportation cost function can then be shown to be equal to

$$C = \left[\frac{\lambda L}{2\mu} - \frac{1}{2\tau}, 0 \right] (\phi + \bar{w}) + \mu\phi + (1 - \lambda)\tau\phi L \to \min_{\lambda, \mu} \tag{2}$$

Denote by *quasi-shared* state the environment saturated with vehicles, so that a released

[3]The coefficient 2 here is because vehicles can be searched in both directions along the street.

[4]Those with $w = \hat{w}$ are indifferent and may play a mixed strategy, which will affect aggregate outcomes only if $\hat{w}$ is a mass point of the walking cost distribution.

vehicle cannot be picked up by anyone other than last customer. It is characterised by $\mu = \lambda\tau L$. In such state, vehicle use does not depend on whether it is withheld or released, and (2) becomes $\phi\tau L$, i.e. a constant.

In a truly shared state

$$\mu < \lambda\tau L, \tag{3}$$

where there are fewer vehicles than travellers so the former have to be actually shared, the first-order conditions of local optimum are:

$$\frac{\partial C}{\partial \lambda} = \frac{L}{2\mu}(\phi + \hat{w}) - \frac{1}{2\tau}\frac{\hat{w} - \bar{w}}{\lambda} - \tau\phi L \begin{cases} = 0, \lambda < 1, \\ \leq 0, \lambda = 1, \end{cases}, \tag{4}$$

$$\frac{\partial C}{\partial \mu} = -\frac{\lambda L}{2\mu^2}(\phi + \bar{w}) + \phi = 0. \tag{5}$$

The elements of the Hessian H are

$$\frac{\partial^2 C}{\partial \lambda^2} = \frac{1}{2}\left[\frac{L}{\mu} - \frac{1}{\lambda\tau}\right]\frac{1}{f(\hat{w})} + \frac{\hat{w} - \bar{w}}{\lambda^2\tau} > 0, \tag{6}$$

$$\frac{\partial^2 C}{\partial \lambda \mathrm{d}\mu} = -\frac{L}{2\mu^2}(\phi + \hat{w}) < 0, \tag{7}$$

$$\frac{\partial^2 C}{\partial \mu^2} = \frac{\lambda L}{\mu^3}(\phi + \bar{w}) > 0. \tag{8}$$

Because the diagonal elements of the Hessian are positive, the curve defined by (4) held with equality is optimal (cost-minimising) λ for given μ and is labeled *the demand curve*, $\lambda_D(\mu)$. Likewise, the curve defined by (5) is optimal μ for a given λ and is labeled *the supply curve*, $\lambda_S(\mu)$. The intersection of demand and supply is an interior local optimum if the Hessian is positive-definite, and is a saddle point otherwise.

Lemma 1. *Both demand and supply are non-negatively sloped. Their intersection is a local minimum if the demand is flatter (with respect to μ) than supply, and is a saddle point otherwise.*

Proof. From the implicit function theorem, $\frac{\mathrm{d}\lambda_D}{\mathrm{d}\mu} = -\frac{\partial^2 C \backslash \partial\lambda\partial\mu}{\partial^2 C \backslash \partial\lambda^2}$. Likewise, the slope of the supply curve is $\frac{\mathrm{d}\lambda_S}{\mathrm{d}\mu} = -\frac{\partial^2 C \backslash \partial\mu^2}{\partial^2 C \backslash \partial\lambda\partial\mu}$. Both are positive due properties of the Hessian. Multiplying both ratios by positive quantity $-\frac{\partial^2 C}{\partial\lambda \mathrm{d}\mu}\frac{\partial^2 C}{\partial\lambda^2}$, we conclude that $\frac{\mathrm{d}\lambda_D}{\mathrm{d}\mu} < \frac{\mathrm{d}\lambda_S}{\mathrm{d}\mu}$ iff $\left(\frac{\partial^2 C}{\partial\lambda \mathrm{d}\mu}\right)^2 < \frac{\partial^2 C}{\partial\lambda^2}\frac{\partial^2 C}{\partial\mu^2}$, i.e. the Hessian is positive-definite and the intersection point is a local minimum. The reverse inequality $\frac{\mathrm{d}\lambda_D}{\mathrm{d}\mu} > \frac{\mathrm{d}\lambda_S}{\mathrm{d}\mu}$ implies the Hessian is negative-definite. ■

Zakharenko (2023b) describes a special case of the above model with homogenous waking cost w. In the notation of the current paper, homogenous w means $\hat{w} = \bar{w} = w$, meaning that the demand curve (4) becomes $\frac{L}{2\mu}(\phi + w) - \tau\phi L \begin{cases} = 0, \lambda < 1, \\ \leq 0, \lambda = 1, \end{cases}$ In other words, the demand curve is vertical (λ_D takes any value) at $\mu = \frac{\phi+w}{2\tau\phi}$, while $\lambda_D(\mu) = 1, \forall\mu > \frac{\phi+w}{2\tau\phi}$.[5]

The fully shared local optimum is where $\lambda = 1$, while μ is defined by the supply curve.

2.1. Quasi-shared critical points

What is the relationship between the quasi-shared curve $\mu = \lambda\tau L$, demand, and supply curves? By inserting this equation for μ into (4,5), we find that the demand and supply curves intersect the quasi-shared line at the same point(s) given by

$$\phi + \bar{w}(\lambda) = 2\lambda\tau^2\phi L. \tag{9}$$

In other words, the equations $\lambda_D(\lambda\tau L) = \lambda$ and $\lambda_S(\lambda\tau L) = \lambda$ have the same solution(s), if any. Because the left-hand side of (9) is strictly positive, such point(s), if they exist, correspond to positive values of λ and μ.

Furthermore, the determinant of the Hessian at such points is given by

$$\frac{1}{\lambda^4\tau^4L^2}\left[(\hat{w} - \bar{w})(\phi + \bar{w}) - \frac{1}{4}(\phi + \hat{w})^2\right].$$

[5]This result is repeated in formula (7) in Zakharenko (2023b), after setting to zero the parameters not introduced in the current paper.

This a non-positive quantity, and is strictly negative as long as $\bar{w} \neq \frac{1}{2}(\hat{w} - \phi)$. Thus, any quasi-shared critical point is a saddle point of $C(\lambda, \mu)$.

2.2. Truly shared critical points

This section studies interior ($\lambda < 1$) Truly Shared (satisfying (3)) Critical Points, or TSCP. First, solve (5) for $\bar{w}$ as follows: $\bar{w} = \frac{2\mu^2\phi}{\lambda L} - \phi$. Next, insert this solution into (4) (for interior λ), to obtain

$$\frac{L}{2\mu}(\phi + \hat{w})\left(1 - \frac{\mu}{\lambda\tau L}\right) - \phi\tau L\left(1 - \frac{\mu^2}{\lambda^2\tau^2 L^2}\right) = 0.$$

Next, divide both sides by term $1 - \frac{\mu}{\lambda\tau L}$ (which is positive due to (3)):

$$\frac{L}{2\mu}(\phi + \hat{w}) - \tau\phi L\left(1 + \frac{\mu}{\lambda\tau L}\right) = 0.$$

Given the latter expression, we can replace the term $\frac{L}{2\mu}(\phi + \hat{w})$ in (4) (for interior λ) by $\tau\phi L\left(1 + \frac{\mu}{\lambda\tau L}\right) = \tau\phi L + \frac{\mu\phi}{\lambda}$, to arrive at the following property of a TSCP:

$$2\mu\tau\phi = \hat{w} - \bar{w}. \tag{10}$$

By solving (10) for μ and inserting into (5), we arrive at a univariate equation as necessary condition of a TSCP:

$$2\lambda\tau^2\phi(\phi + \bar{w}(\lambda))L = (\hat{w}(\lambda) - \bar{w}(\lambda))^2. \tag{11}$$

Furthermore, (3) and (10) jointly imply $\hat{w} - \bar{w} < 2\lambda\tau^2\phi L$, which together with (11) implies $\hat{w} - \bar{w} > \bar{w} + \phi$, or

$$\hat{w} - 2\bar{w} > \phi, \tag{12}$$

as a second necessary condition of a TSCP. (11) and (12) are jointly sufficient for a TSCP.

Note that (12) is quite restrictive and requires a sufficiently large variation in walking

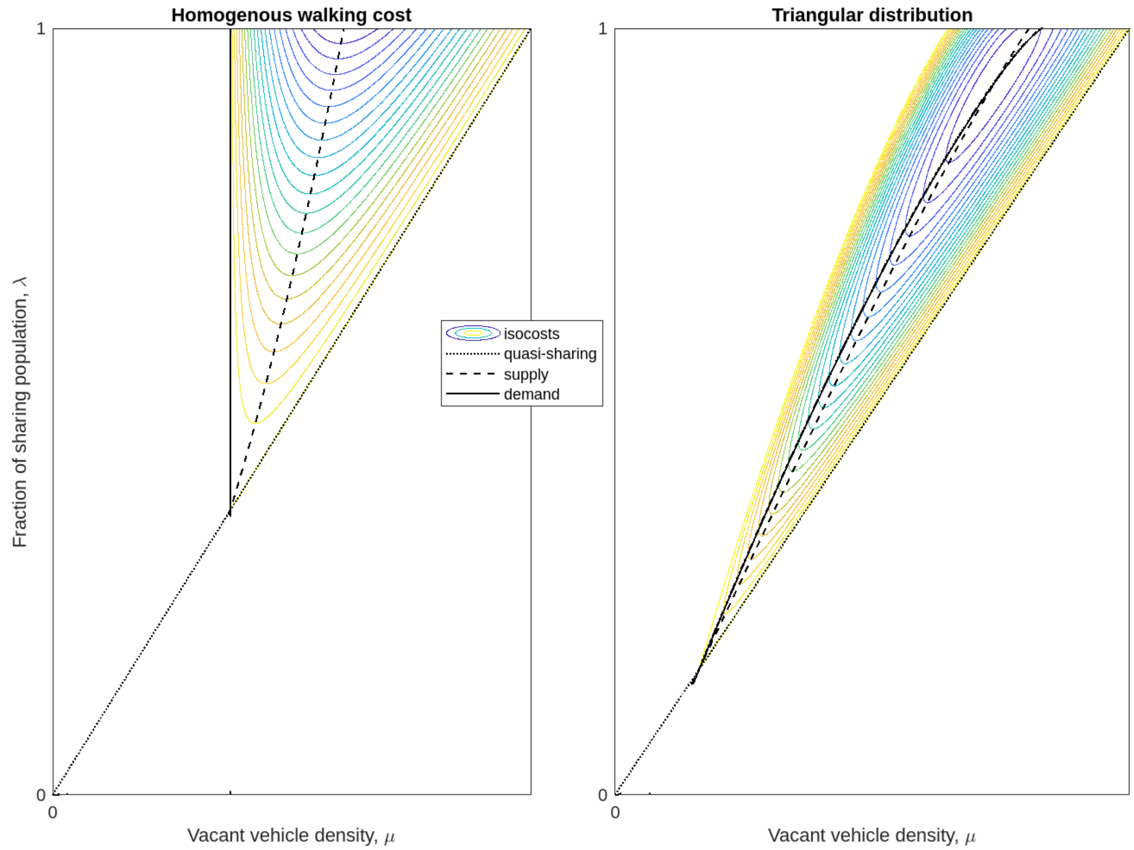

Figure 2: Examples of social cost of transportation as function of μ and λ.

costs w. For example, any uniform distribution with a positive support does not satisfy this property, as $\hat{w} - 2\bar{w} \leq 0$ (with equality if the lower bound is zero, and inequality if it is positive). This means that a uniform distribution of walking costs is not variable enough to justify withholding a vehicle by anyone.

2.3. Maximum withholding

This section plays devil's advocate to construct a social optimum that maximises the share of travelers who should withhold their vehicle. Note that in a quasi-shared state, the number of vehicles matches the number of travellers, meaning all of them are indifferent between withholding and not [explain why!]. This section focuses on cases where withholding is strictly preferred for some travelers, which is only possible when some vehicles are truly shared. The objective of this section is therefore to characterise the lowest possible λ in a truly shared global minimum of the social cost function (2).

With unbounded support of walking cost w, one can construct an example with vanishingly small fraction λ of travelers sharing in the first-best equilibrium. It is quite plausible to assume however that there exists an upper bound on the walking cost, because travelers can use taxi or other means of transportation to reach the shared vehicle. Some SV operators in Moscow, for example, connect their customers to a taxi service to do so, helping to reduce the transaction costs of finding transportation to their SV. We will therefore assume an upper bound of w_H. Playing devil's advocate again, we assume the lower bound of the distribution is zero, as large variance in w is essential for socially optimal vehicle withholding. Intuitively, some people may walk certain distance per day anyway, so for them there is no opportunity cost of walking towards an SV.

2.3.1. Binomial distribution

To maximise the chance that some pair $\{\lambda_L, \mu_L\}$ satisfying (3) and $\lambda_L < 1$ is a global minimum of social cost (2), one wants to minimise $\bar{w}(\lambda_L)$ in (2). That is equivalent to assuming that fraction λ_L of travelers have the lowest possible walking cost of zero. Furthermore, we also want the social cost to be as *high* as possible at all alternative local minima with $\lambda > \lambda_L$. For that, we need to maximise $\bar{w}(\lambda), \lambda > \lambda_L$, by assuming that the remaining fraction $1 - \lambda_L$ of travelers have the highest possible walking cost w_H. Such binary distribution maximises the fraction of travelers who can withhold their vehicle in the social optimum.

The demand curve for such binary distribution is as follows.

- For $\lambda < \lambda_L$, we have that $\hat{w} = \bar{w} = 0$; the demand curve (4) is vertical at $\mu = \frac{1}{2\tau}$. For candidate social optimum at λ_L to be truly shared (i.e. for (3) to hold at λ_L), the following must be true:

$$\lambda_L > \frac{1}{2\tau^2 L}. \tag{13}$$

- At $\lambda = \lambda_L$, $\hat{w}$ increases from zero to w_H, while $\bar{w}$ remains at zero. The demand curve is horizontal, with μ ranging from $\frac{1}{2\tau}$ to $\mu_1^D \equiv \frac{\lambda_L \tau L(\phi + w_H)}{w_H + 2\lambda_L \tau^2 \phi L}$. This whole segment is truly shared, as long as (13) is true.

- For $\lambda > \lambda_L$, $\hat{w} = w_H$ and $\bar{w} = \left(1 - \frac{\lambda_L}{\lambda}\right) w_H$; the demand curve takes form

$$\lambda_D(\mu)^2 = \begin{cases} \frac{\lambda_L \mu w_H}{\tau L(\phi + w_H - 2\mu\tau\phi)}, & \mu \in \left[\mu_1^D, \mu_2^D\right); \\ 1, & \mu > \mu_2^D, \end{cases} \tag{14}$$

  where $\mu_2^D \equiv \frac{\tau L(\phi + w_H)}{\lambda_L w_H + 2\tau^2 \phi L}$.

Using the above expressions for $\bar{w}(\lambda)$, we can also find the supply curve (5):

$$\lambda_S(\mu) = \begin{cases} \frac{2\mu^2}{L}, & \mu \leq \mu_1^S \equiv \sqrt{\frac{1}{2}\lambda_L L}; \\ \frac{2\mu^2\phi}{L(\phi + w_H)} + \frac{\lambda_L w_H}{\phi + w_H}, & \mu > \mu_1^S. \end{cases} \tag{15}$$

2.3.2. Local optimum with withholding

For λ_L to be indeed the social optimal λ, it is necessary that the demand and supply curves intersect at λ_L; from (15), such intersection must occur at μ equal to μ_1^S. Since the demand curve has a horizontal segment at λ_L, the supply curve should cross through that segment: $\frac{1}{2\tau} \leq \mu_1^S \leq \mu_1^D$. The first inequality $\frac{1}{2\tau} \leq \mu_1^S$ follows from (13), while the second inequality $\mu_1^S \leq \mu_1^D$ requires that

$$w_H \geq \sqrt{2\lambda_L L}\phi\tau \tag{16}$$

as a necessary condition of local optimum existence. Note that the right-hand side of (16) is greater than ϕ by (13): vehicle withholding is not optimal for anyone when the maximum walking cost w_H is lower or equal than the vehicle cost ϕ.

If the above conditions are met, the point $\{\mu_1^S, \lambda_L\}$ is indeed a local optimum, by Lemma 1 and by the fact that demand is horizontal while supply is upward-sloping.

Next, we investigate other local optima at higher values of λ, and specify conditions under which the point $\{\mu_1^S, \lambda_L\}$ is the global optimum.

Proposition 1. *Any combination $\{\lambda, \mu\}$ for $\lambda \in (\lambda_L, 1)$ cannot be a local minimum of (2).*

Proof. Since a local interior minimum is a TSCP, it must satisfy conditions (10) and (12). For the bilateral distribution of walking costs studied in the current section, for $\lambda > \lambda_L$, these two conditions are

$$2\mu\tau\phi = \frac{\lambda_L}{\lambda} w_H, \tag{17}$$

$$\left(2\frac{\lambda_L}{\lambda} - 1\right) w_H > \phi. \tag{18}$$

Because the relationship between λ and μ in (17) is negative, while in both demand (14) and supply (15) such relationship is positive, all these conditions can be simultaneously met in at most one point satisfying $\lambda \in (\lambda_L, 1)$. We will refer to this as the "candidate point".

Next, we calculate the Hessian and its determinant at the candidate point. Due atomic distribution of the walking cost, $f(\hat{w}) = f(w_H) = \infty$ in (6) for $\lambda \in (\lambda_L, 1)$, hence (6,7,8) can be rewritten as

$$\frac{\partial^2 C}{\partial \lambda^2} = \frac{\lambda_L w_H}{\lambda^3 \tau},$$

$$\frac{\partial^2 C}{\partial \lambda \mathrm{d}\mu} = -\frac{L}{2\mu^2}(\phi + w_H),$$

$$\frac{\partial^2 C}{\partial \mu^2} = \frac{\lambda L}{\mu^3}\left(\phi + w_H - \frac{\lambda_L}{\lambda} w_H\right).$$

The determinant of the Hessian at the candidate point is then equal to

$$\begin{aligned}
\det H &= \frac{\lambda_L w_H}{\lambda^3 \mu^3 \tau}\left(\lambda L(\phi + w_H) - \lambda_L L w_H\right) - \left(\frac{L}{2\mu^2}(\phi + w_H)\right)^2 \\
&\underbrace{=}_{(15)\text{ for }\mu > \mu_1^S} \frac{2\lambda_L w_H \phi}{\lambda^3 \mu \tau} - \left(\frac{L}{2\mu^2}(\phi + w_H)\right)^2 \\
&\underbrace{=}_{(17)} \left(\frac{2\phi}{\lambda}\right)^2 - \left(\frac{L}{2\mu^2}(\phi + w_H)\right)^2.
\end{aligned}$$

The sign of $\det H$ is therefore the same as the sign of

$$\frac{2\phi}{\lambda} - \frac{L}{2\mu^2}(\phi + w_H) \underbrace{=}_{(15)\text{ for }\mu > \mu_1^S} \frac{2\phi}{\lambda} - \frac{\phi}{\lambda - \frac{\lambda_L w_H}{\phi + w_H}} \underbrace{<}_{(18)} \frac{2\phi}{\lambda} - \frac{\phi}{\lambda - \frac{1}{2}\lambda} = 0.$$

Thus, the determinant of the Hessian at the candidate point is negative, meaning that such point, if it exists, is a saddle point rather than a local minimum of social cost. ■

2.3.3. The global optimum and maximum withholding

The only alternative local optimum is therefore a fully shared one (with no one withholding vehicles), where $\lambda = 1$ and $\mu = \mu_2^S$ that solves $\lambda_S(\mu_2^S) = 1$. Then, any withholding can exist in global social optimum if and only if the local optimum defined in section 2.3.2 delivers a lower social transportation cost than that of a fully shared optimum:

$$C(\lambda_L, \mu_1^S) \leq C(1, \mu_2^S). \tag{19}$$

The aim of this section is to find the lowest λ_L that satisfies (19), which in turn ensures that the maximal fraction $1 - \lambda_L$ of travellers are withholding their vehicles.

The value of minimal λ_L depends on exogenous model parameters L and τ, among others. Both of these positively affect the demand for shared mobility: a higher demand density L means there are more people who can share, while a higher duration of stay between trips

τ increases the cost of withholding. Exposition of results that follow can be simplified and made more intuitive by replacing these two parameters by new notation. Specifically, denote by *sharers* the fraction λ_L of travellers actually sharing vehicles in the [low] local optimum. Denote by n the socially optimal customer-to-vehicle ratio among the sharers; it is defined as ratio of $\lambda_L L\tau$ (the density of parked vehicles if the sharers withheld their vehicles) to μ_1^S (the actual vehicle density), $n\mu_1^S \equiv \lambda_L L\tau$. By comparing the latter against the definition of μ_1^S in (15), we can backtrack the demand density L that results in that specific value of n, and also find the socially optimal μ, as follows: $\mu_1^S = \frac{n}{2\tau}$ and $L = \frac{n^2}{2\lambda_L\tau}$. By substituting the latter into (13) and (16), these constraints can be rewritten as $n > 1$ and $w_H \geq n\phi$, respectively, as necessary conditions of existence of a local optimum with vehicle sharing by some travellers and withholding by others. The two inequalities impose bounds on the traveller-to-vehicle ratio: $n \in \left(1, \frac{w_H}{\phi}\right]$.

Next, we can calculate the social transportation cost at the designated global optimum, as follows (cf.(2), recalling that $\bar{w}(\lambda_L) = 0$):

$$C_0(\lambda_L, n) = C(\lambda_L, \mu_1^S) = \frac{1}{2}\left[\frac{\lambda_L L}{\mu_1^S} - \frac{1}{\tau}\right]\phi + \mu_1^S\phi + (1-\lambda_L)\tau\phi L = \frac{\phi}{\tau}\left(n - \frac{1}{2} + \frac{1-\lambda_L}{\lambda_L}\frac{n^2}{2}\right). \tag{20}$$

The social transportation cost at the competing full-sharing local optimum is (noting that $\mu_2^S = \frac{n}{2\tau}\sqrt{\frac{\phi + w_H - \lambda_L w_H}{\lambda_L \phi}}$ and recalling that $\bar{w}(1) = (1-\lambda_L)w_H$)

$$C_1(\lambda_L, n) = C(1, \mu_2^S) = \frac{1}{2}\left[\frac{L}{\mu_2^S} - \frac{1}{\tau}\right](\phi + (1-\lambda_L)w_H) + \mu_2^S\phi = \frac{n}{\tau}\sqrt{\frac{\phi}{\lambda_L}(\phi + (1-\lambda_L)w_H)} - \frac{\phi + (1-\lambda_L)w_H}{2\tau}. \tag{21}$$

Finally, we need to find the minimal λ_L that satisfies (cf.(19))

$$C_0(\lambda_L, n) \leq C_1(\lambda_L, n). \tag{22}$$

Note that $C_0(1,n) = C_1(1,n)$, by construction. Also note that, as $\lambda_L \to 0$, $C_0(\lambda_L, n) = O\left(\frac{1}{\lambda_L}\right)$ while $C_1(\lambda_L, n) = O\left(\frac{1}{\sqrt{\lambda_L}}\right)$,[6] meaning that (22) is violated for sufficiently small λ_L, so the minimal λ_L is bounded away from zero.

One can show that, as n increases from 1 to $\frac{w_H}{\phi}$, the minimal λ_L that satisfies (22) rises from $\frac{\phi}{w_H}$ to 1.

2.3.4. Calibration

[Incomplete]

$$w_H = \frac{\text{Total cost of search session}}{\text{Walking time}} - \phi. \tag{23}$$

Calibration is done for the case of London. For maximum total cost, assume travellers get to the vehicle by taxi. Taxi tariff 1:[7] £3.8 for initial 190.8m, and £0.2 for every additional 95.4m. Assuming the SV is 2km away, the ride will cost £7.6. Assuming the trip takes 12min (5min wait for taxi and 7min ride), with maximal value of time of £1/min, the total cost of a search session is 12+7.6=£19.6.

The walking time for 2km is assumed to be 24 min. The vehicle cost standing cost ϕ is calibrated at £540/month (based on a typical cost of compact-size "vehicle subscription" cost plus insurance). Assuming non-trivial transportation demand exists for 10 hours per day, this monthly standing cost corresponds to £1.8/h or £0.03/min. In addition to the capital cost of the vehicle *per se*, we also add the social cost of parking, as unused vehicles create negative congestion externalities.[8] Zakharenko (2016) develops a method to calculate the social cost of parking; this method has been applied to Melbourne (van Ommeren et al., 2021) and Stockholm (Eliasson and Börjesson, 2022). Because no similar estimate was done

[6] Here $O(x)$ means the same order of magnitude as x.

[7] https://tfl.gov.uk/modes/taxis-and-minicabs/taxi-fares/tariffs, accessed on 29.01.2024.

[8] Zakharenko (2023a) and Zakharenko (2023b) use a separate notation for parking social cost; in the current paper, it is integrated with ϕ for notational brevity.

for London, I proxy it (on the conservative side) by the minimal fee of £0.02/min.[9] Thus, $\phi =$£0.05/min.

Based on above calibration, the value of maximal "walking cost" is calibrated at $w_H = (19.6 + .05 \times 12)/24 - .05 =$£0.84/min. The ratio $\frac{w_H}{\phi}$ is then $0.84/0.05 = 16.8$.

The minimal λ_L also depends on equilibrium n. Jochem et al. (2020), based on stated consumer preferences, estimate that a shared car replaces $n = 13.3$ private cars in London. Then, the minimal λ_L that satisfies (22) is 0.94. Zakharenko (2023a), by comparing observed usage intensity of shared vs. private vehicles, makes a more conservative estimate of $n = 6$, which corresponds to a minimal λ_L of 0.59. The latter estimate implies that at most 41% of London automobile users, those with highest walking costs, should use their vehicles exclusively, while the remaining 59% should share. This estimate is based on the assumption of extreme walking cost inequality, so that the 41% have such high walking cost that they would need to use taxi to reach a vacant shared vehicle, while the other 59% have zero walking cost. Under any other distribution of walking cost, the share of those withholding a vehicle would be lower than 41%, given the above calibration. In particular, increasing the share of high-walking-cost population beyond 41% would mean that they should optimally stop withholding their vehicles, and share them alongside others.

3. Conclusion

This paper investigates the theoretical justifications for vehicle withholding, i.e. preventing a vacant vehicle from being used by others, for the purpose of own future use. It is shown that inequality in the cost of search for alternative vehicles can justify withholding by a fraction of individuals with the highest search cost; a theoretical upper bound on such

[9] This is based on the £1.1/h minimal fee, as published at https://www.justpark.com/uk/parking/london/, accessed on 30.01.2024.

fraction is established, and calibrated for the city of London.

Future research may explore other potential justifications for vehicle withholding, such traveller inequality in frequency of vehicle use, spatial inequality, and others.

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