On the Elasticity of Substitution between Labor and ICT and IP Capital and Traditional Capital

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September 7, 2022

Abstract

I estimate CES aggregate production functions for the US, the UK, Japan, Germany, and Spain using data from the EU KLEMS database. I distinguish between three types of capital: information and communication technologies (ICT), intellectual property (IP) capital, and traditional capital. I assume that the aggregate output is produced using labor and these three types of capital and allow for differences in the elasticities of substitution between labor, an aggregate of ICT and IP capital, and traditional capital. The estimated elasticities of substitution between ICT and IP capital are strictly below one for all sample countries implying gross-complementarity. ICT and IP capital together are gross-substitutes for labor while traditional capital is a gross-complement.

Keywords: CES Production Function; Elasticities of Substitution; System of Equations; ICT; IP Capital; Traditional Capital

JEL classification: E22; E25; J23; O33

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1 Introduction

Macro and growth models commonly use explicit production technologies that combine labor and capital. The appropriateness of these models depends on the assumptions regarding the production technology including the elasticity of substitution between labor and capital and the direction of technological change.

The values of the elasticity of substitution and the direction of technological change are important for explaining, for example, movements in factor income shares (e.g., Caballero and Hammour, 1998, Karabarbounis and Neiman, 2013). A large number of studies that focus on labor share documents that it has fallen. The literature offers competing explanations for this. Karabarbounis and Neiman (2013) use cross-country data and find that labor and capital are gross-substitutes. They attribute the fall in labor income share to the rapid fall in prices of capital and to capital deepening. Glover and Short (2020) use similar data and challenge these estimates showing that they can be upward biased because of omitted variables. Their estimates indicate that labor and capital are gross-complements.[1] The estimates of Glover and Short (2020) suggest that alternative explanations might be in order for the fall in labor income share such as, for example, the rise in product-market concentration and import competition (Autor, Dorn, Katz, Patterson, and Reenen, 2017, Grossman, Helpman, Oberfield, and Sampson, 2017).

Two recent studies provide an in-depth analysis of the fall in labor income share in the US by differentiating types of capital (Eden and Gaggl, 2018, Koh, Santaeulàlia-Llopis, and Zheng, 2020). Eden and Gaggl (2018) attribute the fall in labor income share to the uptake of information and communication technologies (ICT) and potential high substitutability of these technologies with labor because of, for example, the ease that routine tasks yield to automation (e.g., Acemoglu and Autor, 2011, Autor, Levy, and Murnane, 2003, Autor and Dorn, 2013, Jerbashian, 2019). In contrast, Koh et al. (2020) in an accounting exercise attribute the fall in labor income share to the capitalization and to the rise of compensation of intellectual property (IP) capital, R&D before 1980

[1] Herrendorf, Herrington, and Valentinyi (2015) also estimate a below one elasticity of substitution between labor and capital. Gechert, Havranek, Irsova, and Kolcunova (2022) corroborate this evidence in their meta analysis of 121 studies.

and software after 1980.

I use data from the EU KLEMS database for the US, the UK, Japan, Germany, and Spain and estimate a normalized CES production function for total industrial value added together with the corresponding, normalized first order conditions. In these estimations, I follow the approach developed and implemented by Grandville (1989), Klump and de La Grandville (2000), Klump, McAdam, and Willman (2007) and León-Ledesma, McAdam, and Willman (2010). The normalization is motivated by the observation that the elasticity of substitution is defined as a point elasticity and its identification needs benchmark values for the level of production and factor inputs and incomes. It basically represents the production function in a consistent indexed number form and facilitates the identification of parameters. León-Ledesma et al. (2010) use Monte Carlo simulations to provide comprehensive evidence regarding the superiority of this estimation method for identifying elasticities of substitution together with factor-biased technological change as compared to, for example, the estimation of first order conditions only and a translog function. The use of this estimation method then can be especially relevant for this study because it attempts to identify these parameters for ICT that has been subject to exceptionally rapid technological progress.

I assume that the CES production technology utilizes labor L, an aggregate of ICT and IP capital IK, and traditional capital TK. The assumption that ICT and IP capital enter into production jointly is motivated by, for example, that computers and software have a joint use. The CES technology has the following form:

$$Y_t = \left[\varpi_{LIK}^{\frac{1}{\varepsilon_1}} LIK_t^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + \varpi_{TK}^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_{TK} t} TK_t \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} \right]^{\frac{\varepsilon_1}{\varepsilon_1 - 1}}, \tag{1}$$

where

$$LIK_t = \left(\varpi_L^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_L t} L_t \right)^{\frac{\varepsilon_2 - 1}{\varepsilon_2}} + \varpi_{IK}^{\frac{1}{\varepsilon_2}} IK_t^{\frac{\varepsilon_2 - 1}{\varepsilon_2}} \right)^{\frac{\varepsilon_2}{\varepsilon_2 - 1}},$$

$$IK_t = \left[\varpi_{ICT}^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_{ICT} t} K_{ICT,t} \right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}} + \varpi_{IP}^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_{IP} t} K_{IP,t} \right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}} \right]^{\frac{\varepsilon_3}{\varepsilon_3 - 1}},$$

and ϖ-s are share parameters, ε-s are elasticity of substitution parameters, γ-s are technological progress parameters, correspondingly.

The first order conditions that follow from a standard profit maximization problem are given by

$$\frac{r_{ICT,t}K_{ICT,t}}{Y_t} = \frac{\varpi_{LIK}^{\frac{1}{\varepsilon_1}} LIK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}}}{\varpi_{LIK}^{\frac{1}{\varepsilon_1}} LIK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}} + \varpi_{TK}^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_{TK}t} TK_t\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}}} \times \frac{\varpi_{IK}^{\frac{1}{\varepsilon_2}} IK_t^{\frac{\varepsilon_2-1}{\varepsilon_2}}}{\varpi_L^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_L t} L_t\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}} + \varpi_{IK}^{\frac{1}{\varepsilon_2}} IK_t^{\frac{\varepsilon_2-1}{\varepsilon_2}}} \times \frac{\varpi_{ICT}^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_{ICT}t} K_{ICT,t}\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}}}{\varpi_{ICT}^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_{ICT}t} K_{ICT,t}\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}} + \varpi_{IP}^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_{IP}t} K_{IP,t}\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}}}, \tag{2}$$

$$\frac{r_{IP,t}K_{IP,t}}{Y_t} = \frac{\varpi_{LIK}^{\frac{1}{\varepsilon_1}} LIK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}}}{\varpi_{LIK}^{\frac{1}{\varepsilon_1}} LIK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}} + \varpi_{TK}^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_{TK}t} TK_t\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}}} \times \frac{\varpi_{IK}^{\frac{1}{\varepsilon_2}} IK_t^{\frac{\varepsilon_2-1}{\varepsilon_2}}}{\varpi_L^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_L t} L_t\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}} + \varpi_{IK}^{\frac{1}{\varepsilon_2}} IK_t^{\frac{\varepsilon_2-1}{\varepsilon_2}}} \times \frac{\varpi_{IP}^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_{IP}t} K_{IP,t}\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}}}{\varpi_{ICT}^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_{ICT}t} K_{ICT,t}\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}} + \varpi_{IP}^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_{IP}t} K_{IP,t}\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}}}, \tag{3}$$

$$\frac{w_t L_t}{Y_t} = \frac{\varpi_{LIK}^{\frac{1}{\varepsilon_1}} LIK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}}}{\varpi_{LIK}^{\frac{1}{\varepsilon_1}} LIK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}} + \varpi_{TK}^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_{TK}t} TK_t\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}}} \times \frac{\varpi_L^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_L t} L_t\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}}}{\varpi_L^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_L t} L_t\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}} + \varpi_{IK}^{\frac{1}{\varepsilon_2}} IK_t^{\frac{\varepsilon_2-1}{\varepsilon_2}}}, \tag{4}$$

$$\frac{r_{TK,t}TK_t}{Y_t} = \frac{\varpi_{TK}^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_{TK}t} TK_t\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}}}{\varpi_{LIK}^{\frac{1}{\varepsilon_1}} LIK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}} + \varpi_{TK}^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_{TK}t} TK_t\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}}}, \tag{5}$$

where r_{IP}, r_{ICT}, and r_{TK} are rates of return on IP capital, ICT capital, and TK capital, respectively, and w is the wage rate.

There are data for labor and total capital compensation and total hours of employment. This allows to recover the rate of return on total capital, r, and wages. The EU KLEMS database also provides data on the stocks of various types of capital, the corresponding investment prices and depreciation rates. This allows to obtain the stocks of ICT, IP capital, and traditional capital, the prices of investments in these types of capital and their rates of depreciation. I use data for the prices of investment in ICT, IP capital, and traditional capital and their depreciation rates, together with the rate of return on total capital, to obtain the values of r_{ICT}, r_{IP}, and r_{TK}. In particular, I assume a non-arbitrage condition that the rate of return on total capital (investment) is equal to the rate of return on a unit of capital of type $i \in \{ICT, IP, TK\}$, which was purchased at the price $p_{i,t-1}$, rented out for a period and resold:

$$1 + r_t = \frac{r_{i,t} + (1 - \delta_i)\, p_{i,t}}{p_{i,t-1}}, \tag{6}$$

where δ_i is the depreciation rate of capital type i.[2]

I follow León-Ledesma et al. (2010) and Herrendorf et al. (2015) and normalize equations (1) and (2)-(5) using the geometric means of the variables. I take the logarithm of the normalized equations and estimate these transformed equations using the feasible generalized non-linear least-squares method.[3] Table 1 offers the estimation results for sample countries.

The estimated elasticity of substitution between labor and traditional capital is (statistically significantly) below 1 for all sample countries implying that labor and traditional capital are gross-complements. It attains the lowest value in the UK, 0.590, and the highest value in Spain, 0.774. These minimum and maximum values are statistically significantly different. However, the estimates of ε_1 in descending/acceding order are not

[2]The Technical Appendix shows that this non-arbitrage condition can be derived from the problem of a firm that invests in IP, ICT, and TK. Similar a conditions are used, for example, by Caselli and Feyrer (2007) and Eden and Gaggl (2018). Figure 2 and Table I in the Data Appendix: Figures and Descriptive Tables use equation (6) and offer the shares of compensation of labor, ICT, IP capital, TK, total capital, and capital without IP in sample countries. They corroborate the accounting results of Koh et al. (2020) that the compensation share of capital without IP is virtually flat in the US.

[3]The Technical Appendix describes the normalization of equations (1), (2)-(5). The Data Appendix: Figures and Descriptive Tables provides further details about the data.

Table 1: Main Results

A. Estimates

Parameter	(1) US	(2) UK	(3) Japan	(4) Germany	(5) Spain
ε_1	0.729***	0.590***	0.744***	0.694***	0.774***
	(0.028)	(0.039)	(0.039)	(0.058)	(0.013)
ε_2	1.761***	1.304***	1.293***	1.122***	1.940***
	(0.208)	(0.066)	(0.055)	(0.039)	(0.139)
ε_3	0.911***	0.935***	0.901***	0.934***	0.938***
	(0.020)	(0.011)	(0.017)	(0.014)	(0.008)
γ_L	0.015***	0.018***	-0.007***	0.009***	0.012***
	(0.001)	(0.002)	(0.002)	(0.002)	(0.001)
γ_{ICT}	0.413***	0.440***	0.367***	0.790***	0.611***
	(0.111)	(0.101)	(0.066)	(0.170)	(0.085)
γ_{IP}	-0.095***	-0.273***	-0.202***	-0.158***	-0.164***
	(0.023)	(0.053)	(0.025)	(0.027)	(0.032)
γ_{TK}	-0.022***	0.001	0.063***	-0.005	-0.026***
	(0.004)	(0.001)	(0.008)	(0.003)	(0.003)
Sample Years	1998–2019	1996–2019	1996–2018	1996–2019	1996–2018
Obs. (per eq.)	22	24	23	24	23

B. Measures of Fit

Log Likelihood	288.016	242.386	241.07	294.107	278.839
AIC	-562.032	-470.772	-468.141	-574.214	-543.678
BIC	-554.394	-462.525	-460.192	-565.968	-535.730
R2					
Eq. (1)	0.988	0.987	0.181	0.956	0.974
Eq. (2)	0.986	0.937	0.912	0.975	0.857
Eq. (3)	0.765	-0.301	-1.012	-0.235	0.695
Eq. (4)	0.905	0.972	0.955	0.996	0.824
Eq. (5)	-0.038	0.698	-1.165	0.373	0.775

Note: This table offers the results from the estimation of normalized and logarithmed equations (1), (2)-(5). Panel *A* offers the estimates of the parameters for sample countries, sample years, and the corresponding number of observations in each equation. Sample years are given by the availability of data in the EU KLEMS database. Panel *B* offers various measures of fit including Log Likelihood, Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and R-squared of each equation. Negative R-squared indicates a negative correlation between the predicted variable (right-hand side) and the original data (left-hand side) at least for some sample years. All regressions use the feasible generalized non-linear least-squares estimation method. Standard errors are in parentheses and are robust to arbitrary heteroscedasticity and serial correlation. *** indicates significance at the 1% level, ** at the 5% level, and * at the 10% level. The Technical Appendix offers the explicit system of estimated equations (23), (25)-(28). The Data Appendix: Figures and Descriptive Tables offers further details about the data.

statistically distinguishable. The estimated elasticity of substitution between labor and ICT and IP capital is significantly above 1 for all sample countries implying that labor and ICT and IP capital are gross-substitutes.[4] It attains the lowest value in Germany, 1.122, and the highest value in Spain, 1.940. Similarly to the traditional capital, the minimum and maximum values of the estimate of ε_2 are statistically significantly different. However, the estimates of ε_2 in descending/acceding order are not statistically distinguishable. In turn, the estimates of the elasticity of substitution between ICT and IP, ε_3, are significantly below 1 implying that these are gross-complements. The estimates of ε_3 are statistically indistinguishable across sample countries.

The estimates of the labor augmenting technical change parameter γ_L are positive in all sample countries except Japan, where it is negative but very close to 0.[5] The estimates of the ICT capital augmenting technical change parameter γ_{ICT} are positive for all sample countries and γ_{ICT} is several orders of magnitude larger than γ_L. The high value of γ_{ICT} can reflect the fast technological progress in information and communication technologies. The estimates of IP capital augmenting technical change parameter γ_{IP} is negative in all sample countries and their absolute values tend to be smaller than γ_{ICT}. In turn, the estimates of the traditional capital augmenting technical change parameter γ_{TK} are negative and statistically significant for the US and Spain. They are statistically insignificant and very close to 0 in the UK and Germany and positive and statistically significant in Japan.

The negative values of γ_{IP} and γ_{TK} are not straightforward to justify in a neoclassical setting (Herrendorf et al., 2015, also estimate a negative technical change parameter for capital in the US). A potential explanation can be that some part of the accumulated IP capital and traditional capital cannot be put in proper use in the near term though returns on these types of capital continue adhering the non-arbitrage condition (6). Admittedly, equations (1) and (2)-(5) do not facilitate such a justification.

[4]Eden and Gaggl (2018) have similar a finding for ICT using US data and single equation/first order condition with no biased technological progress parameters. Antràs (2004) shows that this can introduce an upward bias in the estimates of the elasticity of substitution.

[5]Output has fallen in Japan during the sample years which can explain the negative value of γ_L. Figure 4 in the Data Appendix: Figures and Descriptive Tables offers normalized value added in sample countries together with predicted values.

1.1 Labor Share in the US

Many papers document fall in the labor income share in the US. This fall is visible in Figure 1 which offers the variation in labor income share in the EU KLEMS data as well as the predicted labor income share using equation (4). The main/benchmark prediction results use parameter estimates from Table 1. These results slightly over-predict the fall in labor share but are very close to the original data. Panel A of Table 2 provides the exact numbers.

Figure 1 and Panel B of Table 2 also offer counterfactual predictions for cases when there is no technological progress and changes in (1) ICT and IP capital, (2) traditional capital, (3) ICT, and (4) IP capital. I fix the corresponding trend index to its sample initial value to have no technological progress and set the value of capital stock equal to its sample initial value to have no changes in it. A rough interpretation of the counterfactual exercise, for example, for ICT is that it corresponds to fixing the number of computers and their productivity.[6]

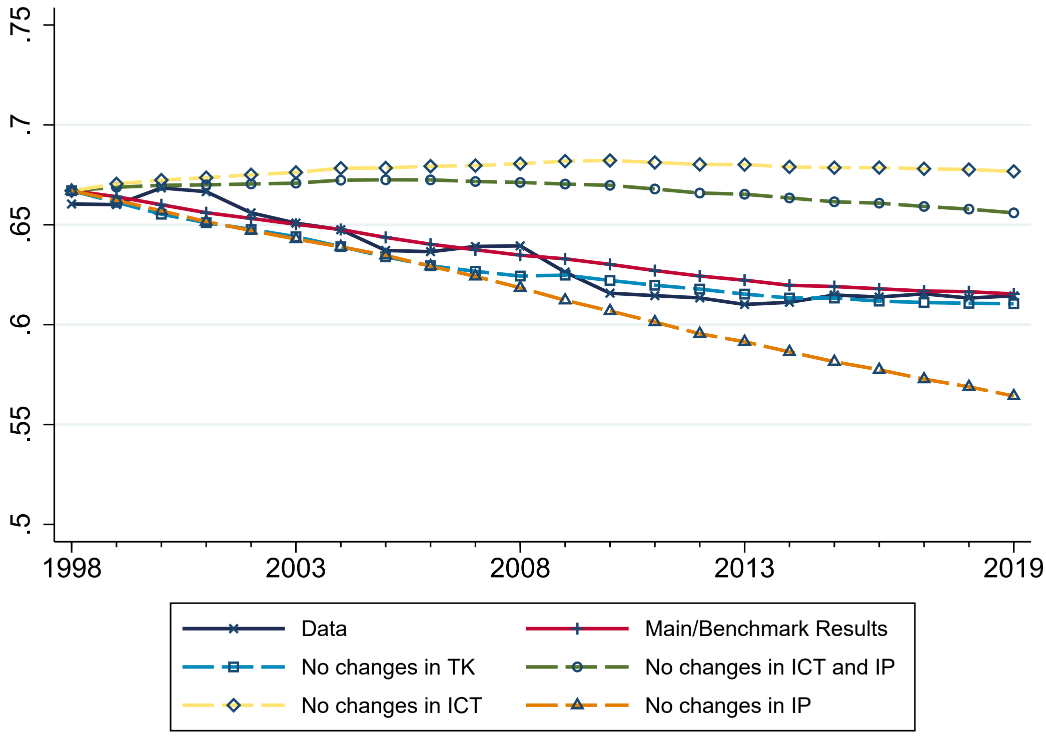
These counterfactual exercises suggest that both ICT and IP capital and traditional capital play a role for the fall in labor income share in the US. About 80 percent of the fall in labor income share can be attributed to the substitutability between labor and ICT and IP capital and the rapid technological progress in ICT and accumulation of IP capital according to columns 1 and 2 of Panel B, column 3 of Panel C, and column 4 of Panel D in Table 2. This is consistent with the juxtaposition of the results of Karabarbounis and Neiman (2013), Eden and Gaggl (2018), and Koh et al. (2020).

According to columns 1 and 2 of Panel B of Table 2, the remainder of the fall in the labor share in the US can be attributed to the complementarity between labor and traditional capital and the higher value of labor augmenting technological change than traditional capital augmenting technological change and the speed of its accumulation.[7] This is inline with the evidence presented by Lawrence (2015) that the effective capital-labor ratios have fallen in industries that account for the largest portion of the fall in

[6] These counterfactual exercises do not take into account potential adjustments in the supply of the factors of production that are not fixed in the exercise.

[7] The counterfactual exercise that removes technological progress and accumulation of TK over-predicts the fall in labor share according to column 1 of Panel B of Table 2.

Figure 1: Labor Income Share in the US: Data, Predicted, and Counterfactual



Note: This figure illustrates the labor income share in the US computed using the data from the EU KLEMS database as well as the predicted labor income share using equation (4). The counterfactual predictions are for cases when there is no technological progress and changes in (1) ICT and IP capital, (2) traditional capital, (3) ICT, and (4) IP capital. The corresponding trend index is fixed to its sample initial value to have no technological progress in type of capital and the value of capital stock is set equal to its sample initial value to have no changes in its level. All prediction results use parameter estimates from Table 1.

labor share.

Table 2: Labor Income Share in the US in 1998 and 2019: Data, Predicted, and Counterfactual

A. Data and Main/Benchmark Prediction

Year	(1) Data	(2) Main/Benchmark
1998	0.660	0.667
2019	0.614	0.615

B. Counterfactual: No Technological Progress and Capital Accumulation

Year	(1) TK	(2) ICT and IP	(3) ICT	(4) IP
2019	0.610	0.656	0.677	0.564
% of Predicted $\Delta_{2019-1998}$	1.096	0.212	-0.192	1.981

C. Counterfactual: No Technological Progress

Year	(1) TK	(2) ICT and IP	(3) ICT	(4) IP
2019	0.645	0.613	0.664	0.519
% of Predicted $\Delta_{2019-1998}$	0.423	1.038	0.058	2.846

D. Counterfactual: No Capital Accumulation

Year	(1) TK	(2) ICT and IP	(3) ICT	(4) IP
2019	0.579	0.651	0.626	0.643
% of Predicted $\Delta_{2019-1998}$	1.692	0.308	0.788	0.462

Note: This table offers sample initial and end values of labor income share in the US computed using the EU KLEMS data. It also offers the predicted labor income share using equation (4) in Panel A. The counterfactual predictions are for (1) ICT and IP capital, (2) traditional capital, (3) ICT, and (4) IP capital. The counterfactual exercise in Panel B removes technological progress and changes in the level of corresponding capital. The counterfactual exercise in Panel C removes technological progress, and Panel D removes changes in the level of capital types. The trend index is fixed to its sample initial value to have no technological progress and the value of capital stock is set equal to its sample initial value to have no changes in it. The initial value in panels B-D is the same as the initial value in column 2 of Panel A because of this. The second rows in panels B-D offer the change in labor income share as compared to the change predicted in column 2 of Panel A. The prediction results use parameter estimates from Table 1.

2 Concluding Remarks

This study explores the elasticities of substitution between labor, ICT and IP capital, and traditional capital. It uses data from the EU KLEMS database and the estimation methodology developed and applied by Grandville (1989), Klump and de La Grandville

(2000), Klump et al. (2007) and León-Ledesma et al. (2010). The estimates of the elasticity of substitution between labor and ICT and IP capital are coherently above 1 in all sample countries implying that labor and ICT and IP capital are gross-substitutes. The estimates of the elasticity of substitution between labor and traditional capital are below 1 implying that labor and traditional capital are gross-complements. Similarly, estimates of the elasticity of substitution between ICT and IP capital are below 1.

These results help to explain the fall in the share of labor income, for example, in the US. In particular, most of the fall in labor share in the US can be attributed to the fast technological progress in ICT and extensive IP capital accumulation according to the results in Table 2.

Can the estimates of the elasticities of substitution in Table 1 imply that the elasticity of substitution between labor and total capital changes over time in the light of increased investments in ICT and IP capital (see, e.g., Koh et al., 2020, and Figure 5 in the Data Appendix: Figures and Descriptive Tables)? I present estimates of elasticities of substitution between labor, total capital, ICT and IP capital, and traditional capital from various alternative CES aggregates for the production function in the Appendix - Further Results. On one hand, the estimates of the elasticity of substitution between labor and total capital in sample countries do not give support to such an inference notwithstanding large differences in the share of ICT and IP capital levels and investments between sample countries. On the other hand, the estimates from the various alternative CES aggregates suggest that the estimates presented in Table 1 are empirically plausible.

Acknowledgements

I gratefully acknowledge the financial support from the Spanish Government through grant PID2021-126549NB-I00, MCIU/AEI/FEDER, UE. All errors remain my own.

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A Appendix - Further Results

This section presents further results. I do not differentiate between capital types and assume that output is produced using a CES technology that combines labor and capital:

$$Y_t = \left[\varpi_L^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_L t} L_t \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + \varpi_K^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_K t} K_t \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} \right]^{\frac{\varepsilon_1}{\varepsilon_1 - 1}}, \tag{7}$$

where

$$K = K_{ICT} + K_{IP} + TK.$$

I estimate parameters ε_1, γ_L, and γ_K for all sample countries using normalized and logarithmed Y_t from equation (7) and the corresponding normalized and logarithmed first order conditions as given by equations (31), (32) and (33) in the Technical Appendix. Table 3 reports the results.

In all sample countries, the estimated elasticity of substitution between labor and capital, ε_1, is statistically significantly below 1 which rules out the Cobb-Douglas production function assumption. However, the estimate of ε_1 varies across countries. It is the lowest in the UK and the highest in Germany attaining values 0.689 and 0.978, correspondingly. All the estimates are statistically different from each other with exception of the estimates for Japan and the US. The estimates of ε_1 are also significantly higher than the estimates of the elasticity of substitution between labor and traditional capital in Table 1 with the exception of the estimate for Spain where these estimates are not statistically distinguishable. This can be because K includes K_{ICT} and K_{IP} pair that together are substitutes for L according to the results in Table 1. Nevertheless, the country-level variation in the estimates of ε_1 in Table 3 has almost no correlation with the share of ICT and IP capital in total capital countries. This share can be computed using data from Table III. This result holds because of very low share of ICT and IP capital in total capital in Germany but a relatively high estimated ε_1.[8]

[8]Herrendorf et al. (2015) use data from the US for 1947-2010 period and estimate a below 1 elasticity of substitution between labor and capital. I use their data and estimation algorithm and find a below

Table 3: One Type of Capital

A. Estimates

Parameter	(1) US	(2) UK	(3) Japan	(4) Germany	(5) Spain
ε_1	0.924***	0.689***	0.885***	0.978***	0.811***
	(0.012)	(0.004)	(0.010)	(0.009)	(0.009)
γ_L	0.063***	0.005***	-0.026***	0.053***	0.025***
	(0.010)	(0.001)	(0.005)	(0.016)	(0.002)
γ_K	-0.084***	0.011***	0.059***	-0.081**	-0.041***
	(0.017)	(0.002)	(0.010)	(0.034)	(0.003)
Sample Years	1998–2019	1996–2019	1996–2018	1996–2019	1996–2018
Obs. (per eq.)	22	24	23	24	23

B. Measures of Fit

Log Likelihood	239.958	244.377	208.902	256.778	228.828
AIC	-473.917	-482.753	-411.805	-507.557	-451.656
BIC	-470.643	-479.219	-408.398	-504.022	-448.25
R2					
Eq. (31)	0.990	0.970	0.119	0.983	0.989
Eq. (32)	0.954	0.989	0.853	0.911	0.907
Eq. (33)	0.858	0.290	0.883	0.666	0.892

Note: This table offers the results from the estimation of normalized and logarithmed equations (31), (32) and (33). Panel *A* offers the estimates of the parameters for sample countries, sample years, and the corresponding number of observations in each equation. Sample years correspond to the availability of data in the EU KLEMS database. Panel *B* offers various measures of fit including Log Likelihood, Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and R-squared of each equation. Negative R-squared indicates a negative correlation between the predicted variable (right-hand side) and the original data (left-hand side) at least for some sample years. All regressions use the feasible generalized non-linear least-squares estimation method. Standard errors are in parentheses and are robust to arbitrary heteroscedasticity and serial correlation. *** indicates significance at the 1% level, ** at the 5% level, and * at the 10% level. The Technical Appendix offers the exact system of estimated equations, and Table III in the Data Appendix: Figures and Descriptive Tables offers further description of the data.

Is it possible to gauge whether this estimation provides a better explanation of the data than the one presented in Table 1 in the main text? The production function in equation (1) allows the elasticity of substitution between labor and traditional capital to be different from the elasticity of substitution between labor and ICT and IP capital. This is motivated by the results of a growing literature that studies the effects of automation on wages and employment and shows that the ICT and IP have competed away employment and earnings (e.g., Autor et al., 2003, Autor and Dorn, 2013, Acemoglu and Autor, 2011).[9] Eden and Gaggl (2018) also estimate a production function that allows the elasticity of substitution between labor and traditional capital to be different from the elasticity of substitution between labor and ICT capital. They use US data from the BEA and restrict the elasticity of substitution between labor and traditional capital to be equal to 1 which seems to be a more accurate representation of their data. They also estimate the resulting first order conditions without technological change parameters. The estimation results in Table 1 use EU KLEMS data and show that there are significant differences between these elasticities. The elasticity of substitution between labor and traditional capital is significantly below 1 in sample countries, whereas the elasticity of substitution between labor and ICT and IP capital is significantly above 1.

A way to proceed further is to compare the measures of fit, albeit discerning between models solely on these is problematic since they are purely statistical measures. The estimation results presented in 3 under perform the estimation results presented in Table 1 for the US, Japan, Germany and Spain in terms of Log Likelihood, Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC). However, they have higher Log Likelihood, AIC and BIC measures for the UK and tend to have higher values of R-squared in individual equations in all sample countries.

1 elasticity of substitution in 1947-1980 period, before the "computer era", and above 1 in 1981-2010. Sample split, however, matters for the value of the latter estimate. For example, I also obtain above 1 elasticity of substitution for 1985-2010 but not for initial years starting on 1980 and between 1981 and 1985.

[9] This literature particularly focuses on the high substitutability between occupations performing routine tasks and ICT and IP capital. I abstract from such considerations because of data limitations in the EU KLEMS database and the implied necessity for further nests in the production function. For the latter reason, I also abstract from differences in substitutability between labor and capital across skill-levels as considered, for example, by Krusell, Ohanian, Ríos-Rull, and Violante (2000).

These estimations are not easily and directly comparable in these statistical measures given the differences in the equations and their number. Arguably, a more comparable approach assumes that the production function has the following form:

$$Y_t = \Big[\varpi_L^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_L t} L_t\right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + \varpi_{ICT}^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_{ICT} t} K_{ICT,t}\right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + \varpi_{IP}^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_{IP} t} K_{IP,t}\right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + \varpi_{TK}^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_{TK} t} TK_t\right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}\Big]^{\frac{\varepsilon_1}{\varepsilon_1 - 1}}. \tag{8}$$

When equation (1) allows for differences in substitutability between labor, ICT, IP capital, and traditional capital, the substitutability between labor and different types of capital is the same in equation (8). This elasticity of substitution is also equal to the elasticity of substitution between different types of capital. In contrast, ICT, IP capital, and traditional capital are perfect substitutes in equation (7).

I estimate parameters ε_1, γ_L, γ_{ICT}, γ_{IP}, and γ_{TK} for all sample countries using normalized and logarithmed Y_t from equation (8) and the corresponding normalized and logarithmed first order conditions. The estimated system is given by equations (35), (36), (37), (38) and (39) in the Technical Appendix. Table 4 reports the results.

The estimated elasticity of substitution between labor, ICT, IP capital, and traditional capital, ε_1, is significantly below 1 in all sample countries. It is higher than the estimate of the elasticity between labor and traditional capital in Table 3 in all countries except Spain. The higher value of this elasticity can stem from fixing the elasticity of substittution between labor and ICT and IP capital to be equal to the elasticity of substittution between labor and traditional capital in equation (8). The signs of directed technological change parameters coincide in Table 1 and Table 4 though there are a few differences in the magnitudes and statistical significance.

The estimation results presented in Table 4 under perform the estimation results presented in Table 1 on almost all measures of fit with the exception of R-squared of a few equations and Log Likelihood, AIC, and BIC measures for the UK. Most notably, the R-squared of the first order conditions for IP capital is lower for the UK and Germany in Table 1 as compared to the R-squared in Table 4. The R-squared of the first order

Table 4: Single Elasticity of Substitution

A. Estimates

Parameter	(1) US	(2) UK	(3) Japan	(4) Germany	(5) Spain
ε_1	0.845***	0.662***	0.907***	0.932***	0.639***
	(0.017)	(0.014)	(0.013)	(0.015)	(0.006)
γ_L	0.037***	0.005***	-0.038***	0.026***	0.015***
	(0.004)	(0.001)	(0.008)	(0.006)	(0.001)
γ_{ICT}	0.109***	0.074***	0.418***	0.685***	0.021***
	(0.026)	(0.021)	(0.071)	(0.166)	(0.006)
γ_{IP}	-0.143***	-0.011***	-0.211***	-0.276***	-0.108***
	(0.022)	(0.003)	(0.040)	(0.066)	(0.006)
γ_{TK}	-0.040***	0.001	0.151***	-0.026*	-0.019***
	(0.008)	(0.002)	(0.020)	(0.014)	(0.002)
Sample Years	1998–2019	1996–2019	1996–2018	1996–2019	1996–2018
Obs. (per eq.)	22	24	23	24	23

B. Measures of Fit

	(1) US	(2) UK	(3) Japan	(4) Germany	(5) Spain
Log Likelihood	280.849	250.834	225.389	284.665	259.744
AIC	-551.698	-491.669	-440.777	-559.33	-509.488
BIC	-546.243	-485.779	-435.1	-553.44	-503.81
R2					
Eq. (35)	0.987	0.986	-1.022	0.943	0.978
Eq. (36)	0.991	0.893	0.792	0.905	0.791
Eq. (37)	-0.718	0.483	-1.923	0.617	0.091
Eq. (38)	0.945	0.990	0.872	0.905	0.827
Eq. (39)	0.153	0.708	-0.647	0.376	0.658

Note: This table offers the results from the estimation of equations (35)-(39). Panel A offers the estimates of the parameters for sample countries, sample years, and the corresponding number of observations in each equation. Sample years correspond to the availability of data in the EU KLEMS database. Panel B offers various measures of fit including Log Likelihood, Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and R-squared of each equation. Negative R-squared indicates a negative correlation between the predicted variable (right-hand side) and the original data (left-hand side) at least for some sample years. All regressions use the feasible generalized non-linear least-squares estimation method. Standard errors are in parentheses and are robust to arbitrary heteroscedasticity (and serial correlation). *** indicates significance at the 1% level, ** at the 5% level, and * at the 10% level. The Technical Appendix offers the exact system of estimated equations, and Table III in the Data Appendix: Figures and Descriptive Tables offers further description of the data.

conditions for traditional capital is also lower for the US in Table 1 as compared to the R-squared in Table 4.

A.A Two Alternative Nests

I follow the literature on automation and labor demand and write the CES nests in production function in equation (1) so that the production function permits for a difference in the elasticities of substitution between labor and traditional capital and labor and ICT and IP capital. By construction, the elasticity of substitution between labor and traditional capital and the elasticity of substitution between ICT and IP capital and traditional capital are the same in equation (1) and traditional capital is either a complement or a substitute for the combination of labor and ICT and IP capital. A potential rationale for this is that the aggregate of ICT and IP capital, being a substitute for labor, is used in similar tasks as labor. Nevertheless, various CES nests are possible, and I explore two alternatives in this section.

First, I assume that the elasticity of substitution between labor and ICT and IP capital is equal to the elasticity of substitution between traditional capital and ICT and IP capital. Moreover, the aggregate of ICT and IP capital is either a complement or a substitute for the combination of labor and traditional capital. I further assume that the production function is given by

$$Y_t = \left(\varpi_{LTK}^{\frac{1}{\varepsilon_1}} LTK_t^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + \varpi_{IK}^{\frac{1}{\varepsilon_1}} IK_t^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} \right)^{\frac{\varepsilon_1}{\varepsilon_1 - 1}}, \tag{9}$$

where

$$LTK_t = \left[\varpi_L^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_L t} L_t \right)^{\frac{\varepsilon_2 - 1}{\varepsilon_2}} + \varpi_{TK}^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_{TK} t} TK_t \right)^{\frac{\varepsilon_2 - 1}{\varepsilon_2}} \right]^{\frac{\varepsilon_2}{\varepsilon_2 - 1}},$$

$$IK_t = \left[\varpi_{ICT}^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_{ICT} t} K_{ICT,t} \right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}} + \varpi_{IP}^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_{IP} t} K_{IP,t} \right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}} \right]^{\frac{\varepsilon_3}{\varepsilon_3 - 1}}.$$

I estimate the parameters for all sample countries using normalized and logarithmed Y_t from equation (9) and the corresponding normalized and logarithmed first order conditions as given by equations (45)-(49) in the Technical Appendix. Table 5 reports the results.

The estimation of normalized and logarithmed Y_t from equation (9) and the corresponding normalized and logarithmed first order conditions (31)-(33) does not seem to yield coherent results across sample countries. The estimates of parameters vary significantly across countries in this case. The estimate of ε_1, the elasticity of substitution between labor and ICT and IP capital, is below 1 in the US, the UK, and Spain. It is above 1 in Germany. Moreover, the feasible generalized non-linear least-squares estimation method fails to identify it for Japan and assigns an unreasonable negative value to it. The estimate of ε_2, the elasticity of substitution between labor and traditional capital, is above 1 in the US, Japan, Germany, and Spain. It seems though unreasonably high in Japan. Moreover, it is below 1 for Germany. The estimates of ε_3 display firmer coherency across countries. They are below 1 in all countries except Japan. The estimate of ε_3 is above 1 and seems unreasonably high. The estimates of labor augmenting technical change, γ_L, seem to be unreasonably low in all countries. The estimate of this parameter is not distinguishable from 0 in the US and Germany. It is below 0 in Spain. The estimates of γ_{ICT} and γ_{TK} also vary significantly. The estimate of γ_{ICT} attains negative value in Japan, and the estimate of γ_{TK} attains a large positive value in Spain. These estimated values seem to be hard to justify. Nevertheless, the estimation of normalized and logarithmed Y_t in equation (9) and the corresponding normalized and logarithmed first order conditions attains higher values of statistical measures of fit according to Panel B of Table 1 and Table 5.

Another specification of the production function nests first the types of capital and then nests these with labor. It is given by

$$Y_t = \left[\varpi_L^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_L t} L_t \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + \varpi_{TKIK}^{\frac{1}{\varepsilon_1}} IKTK_t^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} \right]^{\frac{\varepsilon_1}{\varepsilon_1 - 1}}, \tag{10}$$

Table 5: Separate Nests for Labor and Traditional Capital and for ICT and IP

A. Estimates

Parameter	(1) US	(2) UK	(3) Japan	(4) Germany	(5) Spain
ε_1	0.338***	0.417***	-279.987	1.138***	0.306**
	(0.083)	(0.082)	(0.000)	(0.040)	(0.123)
ε_2	1.381***	0.727***	5.659***	1.160***	1.029***
	(0.065)	(0.026)	(1.172)	(0.073)	(0.006)
ε_3	0.833***	0.779***	5.528***	0.932***	0.867***
	(0.020)	(0.040)	(1.740)	(0.016)	(0.043)
γ_L	0.000	0.006***	0.010***	-0.003	-0.110***
	(0.002)	(0.001)	(0.001)	(0.007)	(0.017)
γ_{ICT}	0.133***	0.141***	-0.069***	0.758***	0.248***
	(0.036)	(0.040)	(0.005)	(0.174)	(0.092)
γ_{IP}	-0.073***	-0.055***	0.010***	-0.165***	-0.154***
	(0.012)	(0.010)	(0.002)	(0.035)	(0.040)
γ_{TK}	0.028***	0.004*	0.001	0.025*	0.226***
	(0.004)	(0.003)	(0.001)	(0.013)	(0.034)
Sample Years	1998–2019	1996–2019	1996–2018	1996–2019	1996–2018
Obs. (per eq.)	22	24	23	24	23

B. Measures of Fit

Log Likelihood	326.709	272.057	313.719	288.303	318.062
AIC	-639.418	-530.113	-615.438	-562.605	-622.124
BIC	-631.781	-521.867	-608.625	-554.359	-614.175
R2					
Eq. (45)	0.996	0.986	0.765	0.963	0.991
Eq. (46)	0.989	0.931	0.945	0.971	0.866
Eq. (47)	0.383	0.587	0.626	-0.049	0.824
Eq. (48)	0.964	0.989	0.966	0.920	0.917
Eq. (49)	0.493	0.718	0.673	0.353	0.827

Note: This table offers the results from the estimation of equations (45)-(49). Panel *A* offers the estimates of the parameters for sample countries, sample years, and the corresponding number of observations in each equation. Sample years correspond to the availability of data in the EU KLEMS database. Panel *B* offers various measures of fit including Log Likelihood, Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and R-squared of each equation. Negative R-squared indicates a negative correlation between the predicted variable (right-hand side) and the original data (left-hand side) at least for some sample years. All regressions use the feasible generalized non-linear least-squares estimation method. Standard errors are in parentheses and are robust to arbitrary heteroscedasticity (and serial correlation). *** indicates significance at the 1% level, ** at the 5% level, and * at the 10% level. The Technical Appendix offers the exact system of estimated equations, and Table III in the Data Appendix: Figures and Descriptive Tables offers further description of the data.

where

$$IKTK_t = \left[\varpi_{IK}^{\frac{1}{\varepsilon_2}} IK_t^{\frac{\varepsilon_2-1}{\varepsilon_2}} + \varpi_{TK}^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_{TK}t} TK_t \right)^{\frac{\varepsilon_2-1}{\varepsilon_2}} \right]^{\frac{\varepsilon_2}{\varepsilon_2-1}},$$

$$IK_t = \left[\varpi_{ICT}^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_{ICT}t} K_{ICT,t} \right)^{\frac{\varepsilon_3-1}{\varepsilon_3}} + \varpi_{IP}^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_{IP}t} K_{IP,t} \right)^{\frac{\varepsilon_3-1}{\varepsilon_3}} \right]^{\frac{\varepsilon_3}{\varepsilon_3-1}}.$$

This production function can be thought to be an extension of the production function in equation (7). It permits imperfect substitutability between traditional capital and ICT and IP capital.

I estimate the parameters for all sample countries using normalized and logarithmed Y_t from equation (10) and the corresponding normalized and logarithmed first order conditions as given by equations (55)-(59) in the Technical Appendix. Table 6 reports the results.

The elasticity of substitution between labor and capital ε_1 is strictly below 1 in all countries. Similarly to the results reported in Table 3, it is higher than the estimates in Table 1 in all countries except Spain. The estimate of the elasticity of substitution between traditional capital and ICT and IP capital is significantly above 1 in all countries but the UK and Japan where it is statistically indistinguishable from 1. Importantly, these estimates can effectively rule out perfect substitutability between traditional capital and ICT and IP capital as assumed in the production function in equation 7. The estimates of ε_3 are below 1 in all countries. The estimates of labor augmenting technical change, γ_L, seem to fall in a reasonable ballpark in all countries. Nevertheless, the estimates γ_{ICT}, γ_{IP}, and γ_{TK} seem to be either rather high or low in a few countries. For example, the estimate of γ_{ICT} is 5.434 in Japan. Such an estimate implies extremely strong technological progress in ICT. In turn, the estimate of γ_{IP} is -0.578 in the US, and the estimate of γ_{TK} is -0.392 in Japan. It can be hard to justify such estimates.

The results from comparison of the measures of statistical fit are more mixed in this case as compared to the estimations presented in Table 5. The Log Likelihood, AIC, and BIC measures are higher in the US, Germany, and Spain in estimations presented in Table 1 as compared to Table 6. They are somewhat lower in the UK and Japan in Table

Table 6: One Nest for Capital Types

A. Estimates

Parameter	(1) US	(2) UK	(3) Japan	(4) Germany	(5) Spain
ε_1	0.960***	0.710***	0.933***	0.629***	0.577***
	(0.006)	(0.053)	(0.035)	(0.043)	(0.047)
ε_2	1.914***	1.558***	1.019***	1.183***	1.648***
	(0.243)	(0.359)	(0.010)	(0.059)	(0.209)
ε_3	0.972***	0.902***	0.993***	0.935***	0.966***
	(0.004)	(0.018)	(0.004)	(0.012)	(0.006)
γ_L	0.035***	0.007***	-0.056*	0.011***	0.012***
	(0.002)	(0.001)	(0.034)	(0.000)	(0.002)
γ_{ICT}	1.220***	0.334***	5.434**	0.820***	1.135***
	(0.115)	(0.060)	(2.687)	(0.167)	(0.252)
γ_{IP}	-0.578***	-0.125***	-0.082*	-0.223***	-0.449***
	(0.047)	(0.019)	(0.048)	(0.042)	(0.095)
γ_{TK}	0.004***	0.002	-0.392*	-0.001	-0.016***
	(0.002)	(0.001)	(0.227)	(0.003)	(0.001)
Sample Years	1998–2019	1996–2019	1996–2018	1996–2019	1996–2018
Obs. (per eq.)	22	24	23	24	23

B. Measures of Fit

Log Likelihood	279.795	254.987	243.876	279.711	248.342
AIC	-545.590	-495.973	-473.752	-545.422	-482.684
BIC	-537.953	-487.727	-465.803	-537.175	-474.735
R2					
Eq. (55)	0.997	0.985	0.728	0.934	0.939
Eq. (56)	0.989	0.945	0.878	0.962	0.855
Eq. (57)	0.493	-0.324	-3.817	0.338	0.822
Eq. (58)	0.815	0.982	0.883	0.817	0.732
Eq. (59)	0.527	0.869	0.492	0.938	0.934

Note: This table offers the results from the estimation of equations (55)-(59). Panel *A* offers the estimates of the parameters for sample countries, sample years, and the corresponding number of observations in each equation. Sample years correspond to the availability of data in the EU KLEMS database. Panel *B* offers various measures of fit including Log Likelihood, Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and R-squared of each equation. Negative R-squared indicates a negative correlation between the predicted variable (right-hand side) and the original data (left-hand side) at least for some sample years. All regressions use the feasible generalized non-linear least-squares estimation method. Standard errors are in parentheses and are robust to arbitrary heteroscedasticity (and serial correlation). *** indicates significance at the 1% level, ** at the 5% level, and * at the 10% level. The Technical Appendix offers the exact system of estimated equations, and Table III in the Data Appendix: Figures and Descriptive Tables offers further description of the data.

1 than in Table 6. The differences in the values of R-squared are also mixed. For example, the R-squared tends to be somewhat higher for value added and wages equations in Table 1 than in Table 6. It is lower in the equation for the compensation rate of traditional capital.

All in all, the results of this exploration suggest that equations (1) and (2)-(5) provide economically and statistically sound explanation for the data.

A.B Results for Additional Countries

The EU KLEMS database has the necessary data for estimation of the parameters in equations (1) and (2)-(5) also for Austria, Belgium, Czechia, Finland, France, Italy, the Netherlands, and Sweden. I present the results of estimation of normalized and logarithmed equations in Table 7 for these countries.

The estimate of the elasticity of substitution between labor and traditional capital ε_1 is statistically significantly below 1 in all countries except France where it is above 1. In turn, the estimate of the elasticity of substitution between labor and ICT and IP capital ε_2 is statistically significantly above 1 in all countries. It is very large in France because the estimation algorithm has failed to identify it. Taken together, these results support the inference in the main text that the substitutability between labor and ICT and IP capital is larger than the substitutability between labor and traditional capital. Moreover, labor and ICT and IP capital are gross-substitutes and labor and traditional capital are gross-complements $\hat{\varepsilon}_2 > 1 > \hat{\varepsilon}_1$.

The estimates of ε_3, γ_L, γ_{ICT}, γ_{IP}, and γ_{TK} display a larger variability across countries. Most notably, the estimate of ε_3 is below 1 in Austria, Finland, Italy, Portugal, and Sweden and above one in the remainder of countries. The estimates of γ_{ICT} are positive and the estimates of γ_{IP} are negative in countries where $\hat{\varepsilon_3} < 1$. Positive and large estimates of γ_{ICT} are relatively straightforward to motivate given fast technological change in ICT.

The initial values of ε_1, ε_2, and ε_3 are set to 0.9, 1.1, and 0.9, correspondingly, in the feasible generalized non-linear least-squares estimation algorithm in all estimations. The estimation algorithm properly identifies all parameters for France with estimated point values $\hat{\varepsilon}_1 = 0.947$, $\hat{\varepsilon}_2 = 1.047$, and $\hat{\varepsilon}_3 = 0.991$ when the initial values of ε_1, ε_2, and ε_3 are set to 0.5, 1.5, and 0.5 instead of 0.9, 1.1, and 0.9. In general, there is a tendency that the estimates of parameters depend on the specified set of initial values because of high non-linearity of the estimated system of equations. The initial values for $\varepsilon_1 = 0.9$ and $\varepsilon_2 = 1.1$ are selected based on the evidence on complementary between total capital and labor and (a higher) substitutability between ICT and labor (e.g., Autor et al., 2003, Autor and Dorn, 2013, Gechert et al., 2022, Glover and Short, 2020). The motivation for the initial value of $\varepsilon_3 = 0.9$ is that ICT and IP capital, that includes software, databases, and patents, tend to be complementary in their use.

Table 7: Results for Additional Countries

A. Estimates

Parameter	(1) Austria	(2) Belgium	(3) Czechia	(4) Finland	(5) France	(6) Italy	(7) Netherlands	(8) Portugal	(9) Sweden
ε_1	0.360***	0.379***	0.836***	0.631***	1.470***	0.616***	0.740***	0.657***	0.598***
	(0.032)	(0.039)	(0.011)	(0.029)	(0.006)	(0.042)	(0.045)	(0.035)	(0.040)
ε_2	1.389***	3.411***	1.198***	1.235***	770.159	1.030***	1.289***	1.376***	1.332***
	(0.045)	(0.711)	(0.017)	(0.018)		(0.004)	(0.061)	(0.071)	(0.040)
ε_3	0.940***	1.111***	1.035***	0.992***	1.655***	0.990***	1.041***	0.941***	0.933***
	(0.007)	(0.022)	(0.004)	(0.002)	(0.081)	(0.001)	(0.005)	(0.017)	(0.006)
γ_L	0.008***	0.006***	-0.003	0.003**	0.014***	-0.011***	0.002	0.023***	0.013***
	(0.001)	(0.001)	(0.005)	(0.002)	(0.001)	(0.003)	(0.002)	(0.002)	(0.002)
γ_{ICT}	0.594***	-0.257***	-0.744***	2.927***	-0.066***	1.224***	-0.800***	0.753***	0.460***
	(0.062)	(0.047)	(0.085)	(0.545)	(0.007)	(0.230)	(0.096)	(0.205)	(0.064)
γ_{IP}	-0.191***	0.111***	0.426***	-0.430***	0.000	-0.032	0.193***	-0.423***	-0.107***
	(0.023)	(0.018)	(0.043)	(0.088)	(0.001)	(0.065)	(0.026)	(0.147)	(0.015)
γ_{TK}	-0.004***	-0.001***	0.035***	0.003	-0.016***	-0.009***	0.008***	-0.040***	-0.002
	(0.001)	(0.000)	(0.007)	(0.002)	(0.003)	(0.001)	(0.003)	(0.006)	(0.003)
Sample Years	1996–2019	1997–2019	1996–2019	1996–2019	1996–2019	1996–2019	1996–2019	2001–2018	1996–2017
Obs. (per eq.)	24	23	24	24	24	24	24	18	22

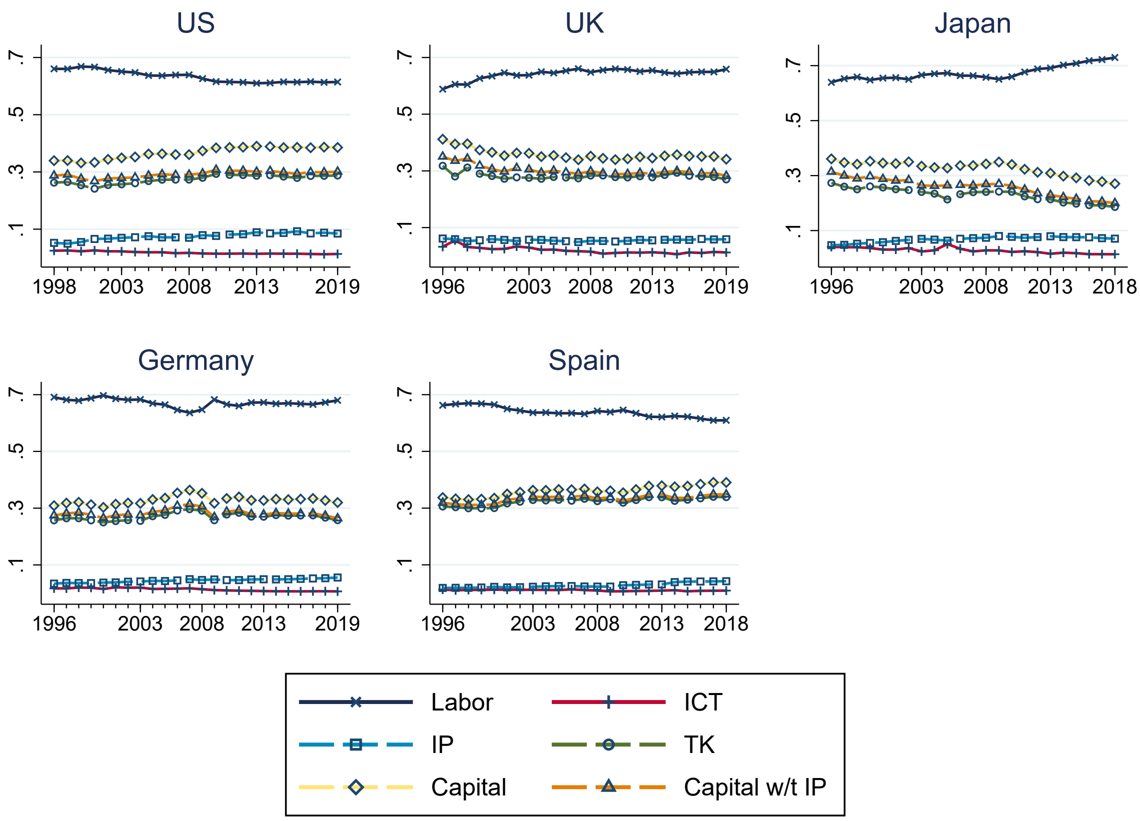
B. Measures of Fit

	(1) Austria	(2) Belgium	(3) Czechia	(4) Finland	(5) France	(6) Italy	(7) Netherlands	(8) Portugal	(9) Sweden
Log Likelihood	321.585	317.682	293.035	286.167	336.806	371.948	324.230	228.320	250.167
AIC	-629.169	-621.364	-572.071	-558.334	-661.612	-729.897	-634.460	-442.640	-486.334
BIC	-620.923	-613.415	-563.824	-550.088	-654.543	-721.650	-626.214	-436.407	-478.697
R2									
Eq. (1)	0.983	0.986	0.970	0.914	0.977	0.858	0.968	0.488	0.977
Eq. (2)	0.880	0.941	0.964	0.947	0.905	0.858	0.992	0.967	0.922
Eq. (3)	0.733	0.596	0.965	0.928	0.906	0.724	0.728	0.068	0.855
Eq. (4)	0.989	0.795	0.992	0.981	0.883	0.986	0.949	0.711	0.984
Eq. (5)	0.673	0.781	0.780	0.672	0.831	0.942	0.045	0.878	0.804

Note: This table offers the results from the estimation of normalized and logarithmed equations (1), (2)-(5). Panel *A* offers the estimates of the parameters for sample countries, sample years, and the corresponding number of observations in each equation. Sample years are given by the availability of data in the EU KLEMS database. Panel *B* offers various measures of fit including Log Likelihood, Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and R-squared of each equation. Negative R-squared indicates a negative correlation between the predicted variable (right-hand side) and the original data (left-hand side) at least for some sample years. All regressions use the feasible generalized non-linear least-squares estimation method. Standard errors are in parentheses and are robust to arbitrary heteroscedasticity and serial correlation. *** indicates significance at the 1% level, ** at the 5% level, and * at the 10% level. The Technical Appendix offers the explicit system of estimated equations (23), (25)-(28).

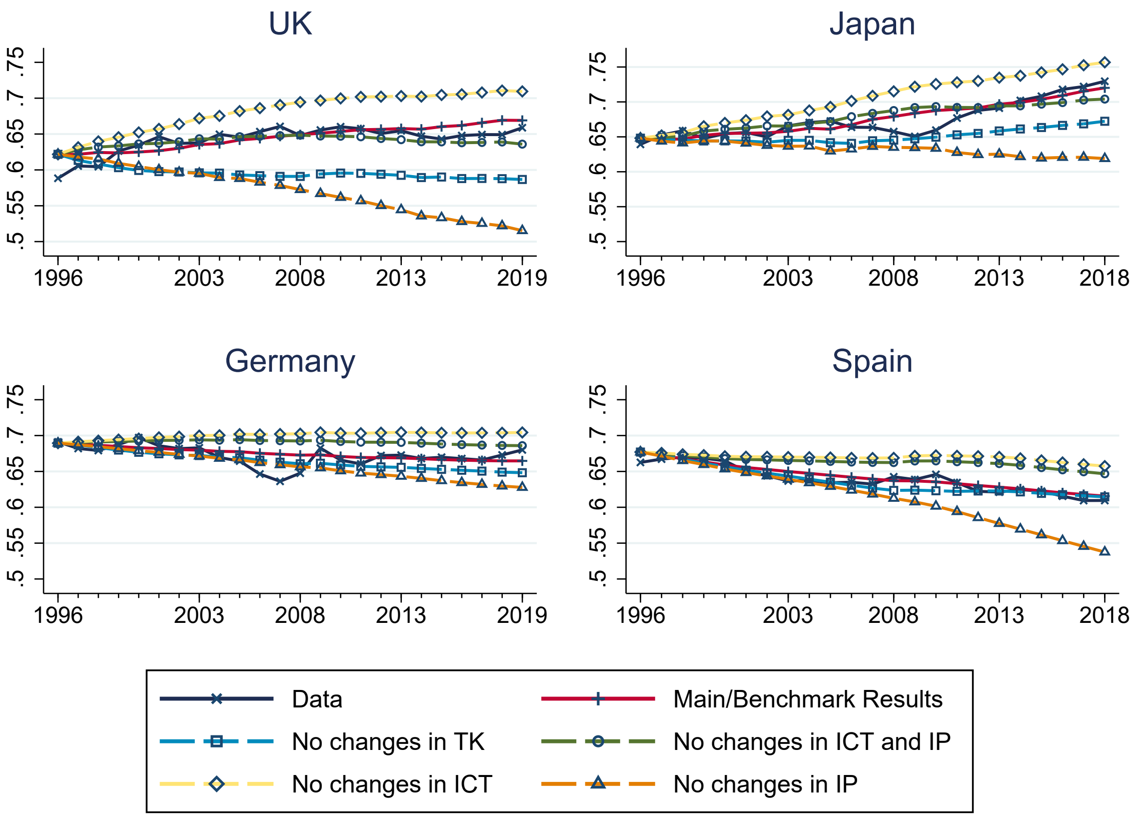
B Data Appendix: Figures and Descriptive Tables

Figure 2: The Shares of Compensation of Labor, ICT, IP Capital, and Traditional Capital



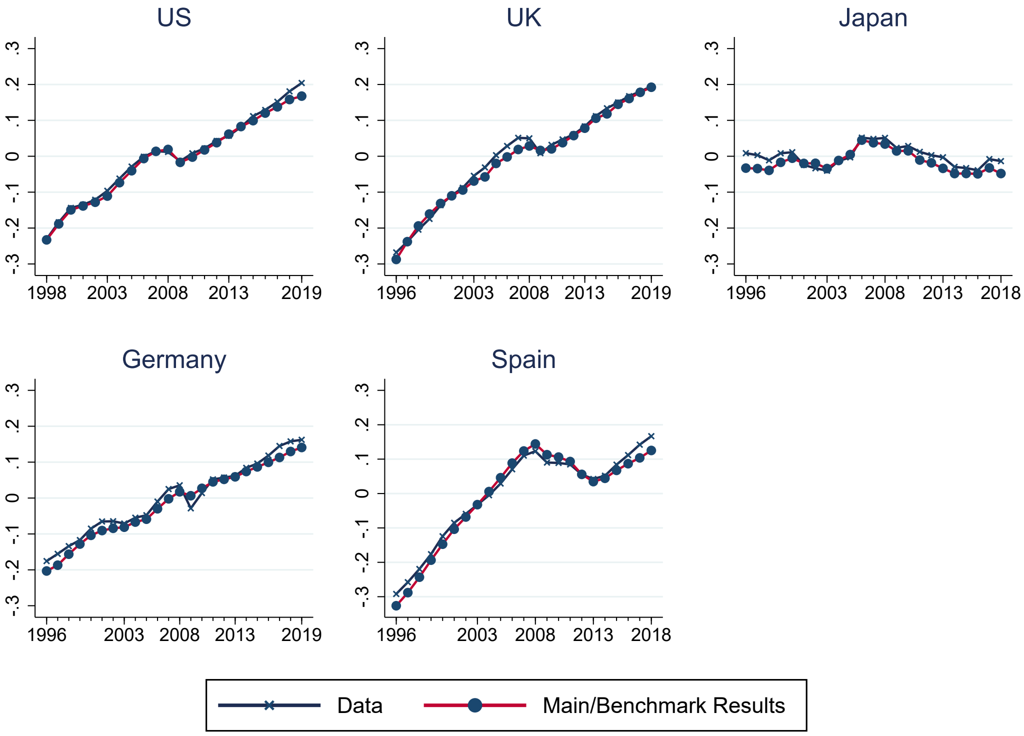
Note: This figure illustrates the shares of compensation of labor, ICT, IP capital, traditional capital, total capital, and capital without IP capital out of value added in sample countries. Table I offers sample initial and end values.

Figure 3: Labor Income Share: Data, Predicted, and Counterfactual



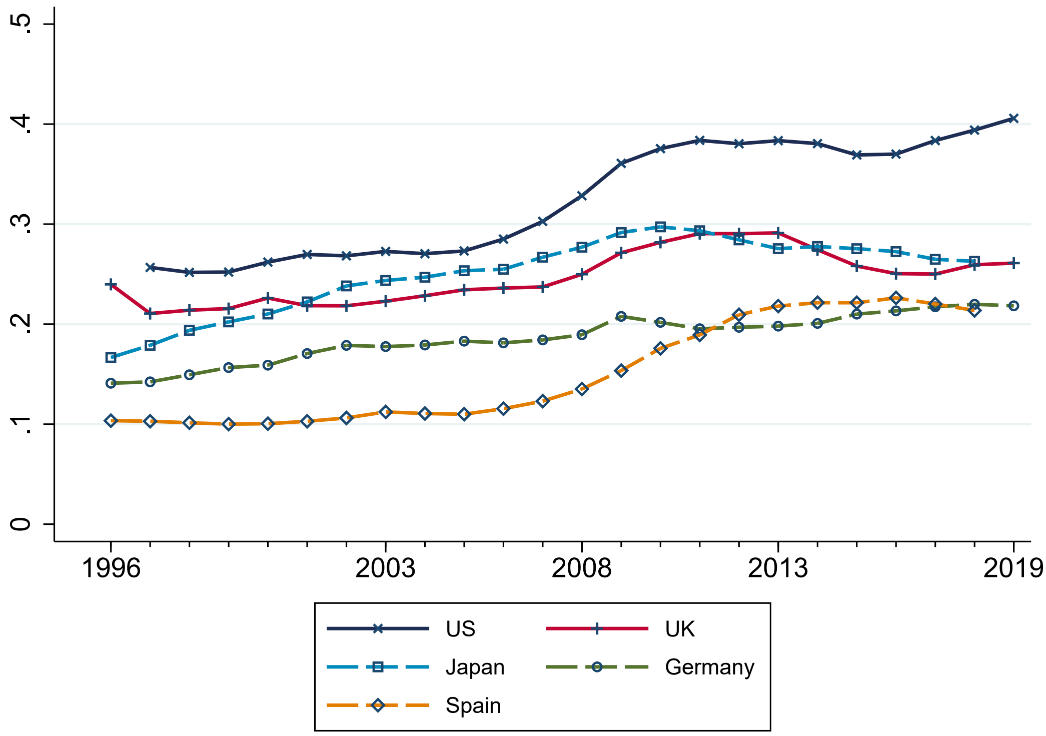
Note: This figure illustrates the labor income share in the UK, Japan, Germany and Spain computed using the data from the EU KLEMS database as well as the predicted labor income share using equation (4). The counterfactual predictions are for cases when there is no technological progress and changes in (1) ICT and IP capital, (2) traditional capital, (3) ICT, and (4) IP capital. The corresponding trend index is fixed to its sample initial value to have no technological progress in type of capital and the value of capital stock is set equal to its sample initial value to have no changes in its level. All prediction results use parameter estimates from Table 1.

Figure 4: Normalized and Logarithmed Real Value Added: Data and Prediction



Note: This figure illustrates the evolution of the normalized and logarithmed real value added in sample countries computed using data from the EU KLEMS database. It also illustrates the predicted values of this variable using equation (1) and parameter estimates from Table 1.

Figure 5: Investments in ICT and IP Capital out of Total Investments



Note: This figure illustrates the evolution of real investments in ICT and IP capital out of total real investments in sample countries.

Table I: The Shares of Compensation of Labor, ICT, IP Capital, and Traditional Capital in Sample Countries

	US		UK		Japan		Germany		Spain	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Share of Compensation	1998	2019	1996	2019	1996	2018	1996	2019	1996	2018
Labor	0.660	0.614	0.588	0.659	0.639	0.730	0.691	0.680	0.663	0.610
ICT	0.025	0.013	0.032	0.012	0.041	0.014	0.017	0.007	0.013	0.009
IP	0.052	0.085	0.061	0.059	0.047	0.071	0.034	0.056	0.019	0.042
Traditional Capital	0.263	0.288	0.318	0.270	0.273	0.186	0.257	0.258	0.306	0.339
Capital	0.340	0.386	0.412	0.341	0.361	0.270	0.309	0.320	0.337	0.390
Capital w/t IP	0.288	0.301	0.350	0.282	0.314	0.200	0.274	0.265	0.319	0.348

Note: This table offers sample initial and end values of the shares of compensation of Labor, ICT, IP capital, and traditional capital out of value added. It also offers the shares of compensation of capital and capital without IP capital.

Table II: Derived Parameters

Derived Parameter	(1) US	(2) UK	(3) Japan	(4) Germany	(5) Spain
α_1	0.726	0.717	0.769	0.730	0.676
α_2	0.872	0.895	0.876	0.920	0.944
α_3	0.187	0.231	0.272	0.210	0.270
δ_K	0.051	0.047	0.110	0.041	0.035
δ_{ICT}	0.167	0.223	0.207	0.207	0.187
δ_{IP}	0.183	0.253	0.240	0.208	0.230
δ_{TK}	0.034	0.034	0.068	0.032	0.029

Note: This table offers the values of parameters α_1, α_2, and α_3 and the values of depreciation rate parameters in sample countries. The values of α-s are derived using equations (20), (21), and (22). The values of depreciation rate parameters are derived using information about the values of depreciation rate parameters of different types of capital from the 2017 release of the EU KLEMS database accessible at this link (last accessed: 29.08.2022). To construct these values, I take the averages of depreciation rates across industries (there is almost no variation across industries) and weighted averages using real capital stocks as weights. The values of depreciation rates of different types of capital are: information technology (IT) capital $\delta_{IT} = 0.315$, communications technology (CT) capital $\delta_{CT} = 0.115$, software and databases $\delta_{Soft_D B} = 0.315$, transport equipment $\delta_{TraEq} = 0.170$, other machinery $\delta_{OMach} = 0.129$, other buildings and structures $\delta_{OCon} = 0.024$, dwellings $\delta_{RStruc} = 0.011$, other intellectual property $\delta_{OIPP} = 0.129$, and research and development IP $\delta_{RD} = 0.2$.

Table III: Basic Statistics

Variable	US			UK			Japan			Germany			Spain		
	(1) Mean	(2) 1998	(3) 2019	(4) Mean	(5) 1996	(6) 2019	(7) Mean	(8) 1996	(9) 2018	(10) Mean	(11) 1996	(12) 2019	(13) Mean	(15) 1996	(14) 2018
Value Added, NAC, current, bn	13753	8459	20016	1360	824	1977	420200	457100	404400	2293	1742	3106	828	452	1090
Total Labor Income, NAC, current, bn	8646	5587	12298	878	485	1303	282700	292200	295000	1539	1204	2112	525	299	664
Total Hours Worked, mn	252	243	274	49	45	55	116	125	114	59	58	63	31	24	34
Capital Stock, NAC, current, bn	44122	24472	64992	2991	1577	4622	137500	164900	109400	8246	6331	11444	3344	1644	4272
ICT Capital Stock, NAC, current, bn	684	526	874	57	49	75	9564	13460	5080	87	103	72	31	20	37
IP Capital Stock, NAC, current, bn	4188	2031	6259	204	115	321	25418	19838	25677	408	262	637	86	30	154
TK Stock, NAC, current, bn	39249	21915	57860	2729	1413	4226	102500	131600	78603	7750	5966	10735	3227	1594	4081
Value Added Price Index	0.930	0.735	1.126	0.887	0.719	1.087	1.037	1.118	1.011	0.916	0.840	1.068	0.898	0.673	1.025
Investment Price Index	0.954	0.816	1.095	0.876	0.722	1.104	1.023	1.096	1.007	0.926	0.881	1.093	0.968	0.732	1.041
ICT Investment Price Index	1.559	3.615	0.718	1.283	3.725	1.137	1.691	3.387	0.980	1.673	3.379	0.951	1.154	1.189	0.994
IP Investment Price Index	0.936	0.774	1.038	0.895	0.790	1.061	1.002	0.994	1.024	0.920	0.825	1.061	0.878	0.664	1.024
TK Investment Price Index	0.945	0.755	1.178	0.859	0.663	1.115	0.998	1.034	1.004	0.908	0.849	1.107	0.974	0.726	1.047

Note: This table offers the averages and sample initial and end values of the variables used in estimations. Nominal variables are in national currency units. Price indices are normalized to 1 in 2015. ICT capital includes information technologies (IT) and communication technologies (CT). IP capital includes software, organizational capital, and R&D capital. All data are from the 2021 release of the EU KLEMS database by the Luiss Lab of European Economics at Luiss University in Rome, Italy accessible at this link (last accessed: 29.08.2022). This database includes information on investment in capital stocks across both tangible and intangible assets. It is often used to study productivity and output, employment, labor income dynamics.

C Technical Appendix

I consider an infinitely lived firm that discounts its profits at the rate of return r and produces its output Y with the following technology:

$$Y_t = \left[\varpi_{LIK}^{\frac{1}{\varepsilon_1}} LIK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}} + \varpi_{TK}^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_{TK} t} TK_t\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right]^{\frac{\varepsilon_1}{\varepsilon_1-1}}, \tag{11}$$

where

$$LIK_t = \left[\varpi_{L}^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_L t} L_t\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}} + \varpi_{IK}^{\frac{1}{\varepsilon_2}} IK_t^{\frac{\varepsilon_2-1}{\varepsilon_2}}\right]^{\frac{\varepsilon_2}{\varepsilon_2-1}},$$

$$IK_t = \left[\varpi_{ICT}^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_{ICT} t} K_{ICT,t}\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}} + \varpi_{IP}^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_{IP} t} K_{IP,t}\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}}\right]^{\frac{\varepsilon_3}{\varepsilon_3-1}},$$

and ϖ-s are share parameters, ε-s are elasticity of susbstition parameters, γ-s are technological progress parameters, L is labor, K_{ICT} is ICT capital, K_{IP} is intellectual property capital, and TK is traditional capital.

The firm decides how much to invest in K_{ICT}, K_{IP} and TK taking the prices of investments p_{ICT}, p_{IP}, and p_{TK}, and r as given and solves the following problem:

$$\max_{\left\{L_t, I_{ICT,t}, I_{IP,t}, I_{TK,t}\right\}_{t=0}^{+\infty}} \sum_{t=0}^{+\infty} \left(\frac{1}{1+r_t}\right)^t \left(Y_t - w_t L_t - p_{ICT,t} I_{ICT,t} - p_{IP,t} I_{IP,t} - p_{TK,t} I_{TK,t}\right) \tag{12}$$

$s.t.$

$$I_{ICT,t} = K_{ICT,t+1} - (1-\delta_{ICT}) K_{ICT,t},$$

$$I_{IP,t} = K_{IP,t+1} - (1-\delta_{IP}) K_{IP,t},$$

$$I_{TK,t} = TK_{t+1} - (1-\delta_{TK}) TK_t,$$

where δ-s are depreciation rate parameters.

The first order conditions that follow from this problem are given by

$$\frac{r_{ICT,t}K_{ICT,t}}{Y_t} = \frac{\varpi_{LIK}^{\frac{1}{\varepsilon_1}} LIK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}}}{\varpi_{LIK}^{\frac{1}{\varepsilon_1}} LIK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}} + \varpi_{TK}^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_{TK}t} TK_t\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}}} \times \frac{\varpi_{IK}^{\frac{1}{\varepsilon_2}} IK_t^{\frac{\varepsilon_2-1}{\varepsilon_2}}}{\varpi_{L}^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_L t} L_t\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}} + \varpi_{IK}^{\frac{1}{\varepsilon_2}} IK_t^{\frac{\varepsilon_2-1}{\varepsilon_2}}} \times \frac{\varpi_{ICT}^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_{ICT}t} K_{ICT,t}\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}}}{\varpi_{ICT}^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_{ICT}t} K_{ICT,t}\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}} + \varpi_{IP}^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_{IP}t} K_{IP,t}\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}}}, \tag{13}$$

$$\frac{r_{IP,t}K_{IP,t}}{Y_t} = \frac{\varpi_{LIK}^{\frac{1}{\varepsilon_1}} LIK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}}}{\varpi_{LIK}^{\frac{1}{\varepsilon_1}} LIK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}} + \varpi_{TK}^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_{TK}t} TK_t\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}}} \times \frac{\varpi_{IK}^{\frac{1}{\varepsilon_2}} IK_t^{\frac{\varepsilon_2-1}{\varepsilon_2}}}{\varpi_{L}^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_L t} L_t\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}} + \varpi_{IK}^{\frac{1}{\varepsilon_2}} IK_t^{\frac{\varepsilon_2-1}{\varepsilon_2}}} \times \frac{\varpi_{IP}^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_{IP}t} K_{IP,t}\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}}}{\varpi_{ICT}^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_{ICT}t} K_{ICT,t}\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}} + \varpi_{IP}^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_{IP}t} K_{IP,t}\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}}}, \tag{14}$$

$$\frac{w_t L_t}{Y_t} = \frac{\varpi_{LIK}^{\frac{1}{\varepsilon_1}} LIK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}}}{\varpi_{LIK}^{\frac{1}{\varepsilon_1}} LIK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}} + \varpi_{TK}^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_{TK}t} TK_t\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}}} \times \frac{\varpi_{L}^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_L t} L_t\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}}}{\varpi_{L}^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_L t} L_t\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}} + \varpi_{IK}^{\frac{1}{\varepsilon_2}} IK_t^{\frac{\varepsilon_2-1}{\varepsilon_2}}}, \tag{15}$$

$$\frac{r_{TK,t}TK_t}{Y_t} = \frac{\varpi_{TK}^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_{TK}t} TK_t\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}}}{\varpi_{LIK}^{\frac{1}{\varepsilon_1}} LIK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}} + \varpi_{TK}^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_{TK}t} TK_t\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}}}, \tag{16}$$

where

$$r_{ICT,t} = \left[(1+r_t)\, p_{ICT,t-1} - (1-\delta_{ICT})\, p_{ICT,t}\right], \tag{17}$$

$$r_{IP,t} = \left[(1+r_t)\, p_{IP,t-1} - (1-\delta_{IP})\, p_{IP,t}\right], \tag{18}$$

$$r_{TK,t} = \left[(1+r_t)\, p_{TK,t-1} - (1-\delta_{TK})\, p_{TK,t}\right]. \tag{19}$$

I normalize the system of equations (11)-(16) using sample averages of the variables following León-Ledesma et al. (2010). I use geometric averages as in Herrendorf et al. (2015).

I denote by S_{IK_t,LIK_t} the share of compensation of ICT and IP capital out of the compensation of labor and ICT and IP capital and use S_{LIK_t,Y_t} to denote the share of labor, IP capital, and ICT capital compensation in value added:

$$
\begin{aligned}
S_{IK_t,LIK_t} &= \varpi_{IK}^{\frac{1}{\varepsilon_2}} \left(\frac{IK_t}{LIK_t} \right)^{\frac{\varepsilon_2 - 1}{\varepsilon_2}}, \\
S_{LIK_t,Y_t} &= \varpi_{LIK}^{\frac{1}{\varepsilon_1}} \left(\frac{LIK_t}{Y_t} \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}.
\end{aligned}
$$

I also use α_1, α_2, and α_3 to denote the following expressions:

$$
\alpha_1 = \varpi_{LIK}^{\frac{1}{\varepsilon_1}} \overline{\left(\frac{LIK_t}{Y_t} \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}}, \tag{20}
$$

$$
\alpha_2 = \varpi_{L}^{\frac{1}{\varepsilon_2}} \overline{\left(\frac{e^{\gamma_L t} L_t}{LIK_t} \right)^{\frac{\varepsilon_2 - 1}{\varepsilon_2}}}, \tag{21}
$$

$$
\alpha_3 = \varpi_{ICT}^{\frac{1}{\varepsilon_3}} \overline{\left(\frac{e^{\gamma_{ICT} t} K_{ICT,t}}{IK_t} \right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}}}, \tag{22}
$$

where $\bar{z}$ is the gemoteric average of a variable z.

I use α_1, α_2, and α_3 and write the logrithm of the normalized equation for output in the following way:

$$
\ln \frac{Y_t}{\bar{Y}} = \frac{\varepsilon_1}{\varepsilon_1 - 1} \ln \left[\alpha_1 \left(\frac{LIK_t}{\overline{LIK_t}} \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + (1 - \alpha_1) \left(e^{\gamma_{TK} \hat{t}} \frac{TK_t}{\overline{TK_t}} \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} \right], \tag{23}
$$

where

$$
\begin{aligned}
\frac{LIK_t}{\overline{LIK_t}} &= \left[\alpha_2 \left(e^{\gamma_L \hat{t}} \frac{L_t}{\overline{L_t}} \right)^{\frac{\varepsilon_2 - 1}{\varepsilon_2}} + (1 - \alpha_2) \left(\frac{IK_t}{\overline{IK_t}} \right)^{\frac{\varepsilon_2 - 1}{\varepsilon_2}} \right]^{\frac{\varepsilon_2}{\varepsilon_2 - 1}}, \\
\frac{IK_t}{\overline{IK_t}} &= \left[\alpha_3 \left(e^{\gamma_{ICT} \hat{t}} \frac{K_{ICT,t}}{\overline{K_{ICT,t}}} \right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}} + (1 - \alpha_3) \left(e^{\gamma_{IP} \hat{t}} \frac{K_{IP,t}}{\overline{K_{IPP,t}}} \right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}} \right]^{\frac{\varepsilon_3}{\varepsilon_3 - 1}},
\end{aligned}
$$

and $\hat{t}$ is demeaned trend,

$$\hat{t} = t - \frac{1}{T}\sum t. \tag{24}$$

I write the normalized equations corresponding to the first order conditions in the following way:

$$\begin{aligned}\ln r_{ICT,t} - \ln \overline{r_{ICT,t}} = {} & \left(1 - \frac{\varepsilon_3 - 1}{\varepsilon_3}\frac{\varepsilon_1}{\varepsilon_1 - 1}\right)\ln\left(\frac{S_{LIK_t,Y_t}}{\overline{S_{LIK_t,Y_t}}}\right) + \\ & \left(1 - \frac{\varepsilon_3 - 1}{\varepsilon_1}\frac{\varepsilon_2}{\varepsilon_2 - 1}\right)\ln\left(\frac{S_{IK_t,LIK_t}}{\overline{S_{IK_t,LIK_t}}}\right) + \\ & \frac{\varepsilon_3 - 1}{\varepsilon_3}\gamma_{ICT}\hat{t} - \frac{1}{\varepsilon_3}\ln\left(\frac{K_{ICT,t}/\overline{K_{ICT,t}}}{Y_t/\overline{Y_t}}\right),\end{aligned} \tag{25}$$

$$\begin{aligned}\ln r_{IP,t} - \ln \overline{r_{IP,t}} = {} & \left(1 - \frac{\varepsilon_3 - 1}{\varepsilon_3}\frac{\varepsilon_1}{\varepsilon_1 - 1}\right)\ln\left(\frac{S_{LIK_t,Y_t}}{\overline{S_{LIK_t,Y_t}}}\right) + \\ & \left(1 - \frac{\varepsilon_3 - 1}{\varepsilon_3}\frac{\varepsilon_2}{\varepsilon_2 - 1}\right)\ln\left(\frac{S_{IK_t,LIK_t}}{\overline{S_{IK_t,LIK_t}}}\right) + \\ & \frac{\varepsilon_3 - 1}{\varepsilon_3}\gamma_{IP}\hat{t} - \frac{1}{\varepsilon_3}\ln\left(\frac{K_{IP,t}/\overline{K_{IP,t}}}{Y_t/\overline{Y_t}}\right),\end{aligned} \tag{26}$$

$$\begin{aligned}\ln w_t - \ln \overline{w_t} = {} & \left(1 - \frac{\varepsilon_2 - 1}{\varepsilon_2}\frac{\varepsilon_1}{\varepsilon_1 - 1}\right)\ln\left(\frac{S_{LIK_t,Y_t}}{\overline{S_{LIK_t,Y_t}}}\right) + \\ & \frac{\varepsilon_2 - 1}{\varepsilon_2}\gamma_L\hat{t} - \frac{1}{\varepsilon_2}\ln\left(\frac{L_t/\overline{L_t}}{Y_t/\overline{Y_t}}\right),\end{aligned} \tag{27}$$

$$\ln r_{TK,t} - \ln \overline{r_{TK,t}} = \frac{\varepsilon_1 - 1}{\varepsilon_1}\gamma_{TK}\hat{t} - \frac{1}{\varepsilon_1}\ln\left(\frac{TK_t/\overline{TK_t}}{Y_t/\overline{Y_t}}\right). \tag{28}$$

I use equations (23)-(28) in the empirical estimations. The values of r_t, S_{IK_t,LIK_t} and S_{LIK_t,Y_t} are needed in the empirical exercise. I use the following expressions to determine the values of r_t:

$$\begin{aligned}Y_t - w_t L_t = {} & \left[(1 + r_t)\, p_{ICT,t-1} - (1 - \delta_{ICT})\, p_{ICT,t}\right] K_{ICT,t} + \\ & \left[(1 + r_t)\, p_{IP,t-1} - (1 - \delta_{IP})\, p_{IP,t}\right] K_{IP,t} + \\ & \left[(1 + r_t)\, p_{TK,t-1} - (1 - \delta_{TK})\, p_{TK,t}\right] TK_t,\end{aligned}$$

and

$$1 + r_t = \frac{Y_t - w_t L_t}{p_{ICT,t-1} K_{ICT,t} + p_{IP,t-1} K_{IP,t} + p_{TK,t-1} TK_t} + \frac{(1 - \delta_{ICT})\, p_{ICT,t} K_{ICT,t} + (1 - \delta_{IP})\, p_{IP,t} K_{IP,t} + (1 - \delta_{TK})\, p_{TK,t} K_{TK,t}}{p_{ICT,t-1} K_{ICT,t} + p_{IP,t-1} K_{IP,t} + p_{TK,t-1} TK_t}. \tag{29}$$

The values of S_{IK_t,LIK_t} and S_{LIK_t,Y_t} are given by

$$S_{IK_t,LIK_t} = \frac{r_{ICT,t} K_{ICT,t} + r_{IP,t} K_{IP,t}}{w_t L_t + r_{ICT,t} K_{ICT,t} + r_{IP,t} K_{IP,t}},$$
$$S_{LIK_t,Y_t} = \frac{w_t L_t + r_{ICT,t} K_{ICT,t} + r_{IP,t} K_{IP,t}}{Y_t},$$

where $r_{ICT,t}$ and $r_{IP,t}$ are given by equations (17) and (18).

In turn, I predict labor income share using the following expression:

$$\frac{w_t L_t}{Y_t} = \frac{\alpha_1 \left(\frac{LIK_t}{\overline{LIK_t}}\right)^{\frac{\varepsilon_3 - 1}{\varepsilon_1}}}{\alpha_1 \left(\frac{LIK_t}{\overline{LIK_t}}\right)^{\frac{\varepsilon_3 - 1}{\varepsilon_1}} + (1 - \alpha_1) \left(e^{\gamma_{TK} \hat{t}} \frac{TK_t}{\overline{TK_t}}\right)^{\frac{\varepsilon_3 - 1}{\varepsilon_1}}} \times \frac{\alpha_2 \left(e^{\gamma_L \hat{t}} \frac{L_t}{\overline{L_t}}\right)^{\frac{\varepsilon_2 - 1}{\varepsilon_2}}}{\alpha_2 \left(e^{\gamma_L \hat{t}} \frac{L_t}{\overline{L_t}}\right)^{\frac{\varepsilon_2 - 1}{\varepsilon_2}} + (1 - \alpha_2) \left(\frac{IK_t}{\overline{IK_t}}\right)^{\frac{\varepsilon_2 - 1}{\varepsilon_2}}}.$$

It can be derived from equation (27).

C.A One Type of Capital

I consider an infinitely lived firm that has the following production technology

$$Y_t = \left[\varpi_L^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_L t} L_t\right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + \varpi_K^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_K t} K_t\right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}\right]^{\frac{\varepsilon_1}{\varepsilon_1 - 1}}. \tag{30}$$

where K is total capital stock.

The firm discounts its profits at the rate of return r and solves the following problem:

$$\max_{\{L_t,I_t\}_{t=0}^{+\infty}} \sum_{t=0}^{+\infty} \left(\frac{1}{1+r_t}\right)^t (Y_t - w_t L_t - p_t I_t)$$

$$s.t.$$

$$I_t = K_{t+1} - (1-\delta) K_t.$$

The first order conditions that follow from this problem are given by

$$\frac{w_t L_t}{Y_t} = \frac{\varpi_L^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_L t} L_t\right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}}{\varpi_L^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_L t} L_t\right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + \varpi_K^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_K t} K_t\right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}}$$

$$\frac{r_t K_t}{Y_t} = \frac{\varpi_K^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_K t} K_t\right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}}{\varpi_L^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_L t} L_t\right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + \varpi_K^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_K t} K_t\right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}}$$

I use $\alpha_{1,R1}$ to denote

$$\alpha_{1,R1} = \varpi_L^{\frac{1}{\varepsilon_1}} \overline{\left(\frac{A_{L,t} L_t}{Y_t}\right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}} = \overline{\frac{w_t L_t}{Y_t}},$$

and the estimations use the following system of normalized equations:

$$\ln \frac{Y_t}{\bar{Y}} = \frac{\varepsilon_1}{\varepsilon_1 - 1} \ln \left[\alpha_{1,R1} \left(e^{\gamma_L \hat{t}} \frac{L_t}{\overline{L_t}} \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + (1 - \alpha_{1,R1}) \left(e^{\gamma_K \hat{t}} \frac{K_t}{\overline{K_t}} \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} \right], \tag{31}$$

$$\ln w_t - \ln \overline{w_t} = \frac{\varepsilon_1 - 1}{\varepsilon_1} \gamma_L \hat{t} - \frac{1}{\varepsilon_1} \ln \left(\frac{L_t / \overline{L_t}}{Y_t / \overline{Y_t}} \right), \tag{32}$$

$$\ln r_t - \ln \overline{r_t} = \frac{\varepsilon_1 - 1}{\varepsilon_1} \gamma_K \hat{t} - \frac{1}{\varepsilon_1} \ln \left(\frac{K_t / \overline{K_t}}{Y_t / \overline{Y_t}} \right). \tag{33}$$

The values of r are determined using $r_t = (Y_t - w_t L_t) / K_t$.

C.B Single Elasticity of Substitution

The firm solves the problem (12) where production function is now given by

$$Y_t = \Bigg[\varpi_L^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_L t} L_t \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + \varpi_{ICT}^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_{ICT} t} K_{ICT,t} \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} \tag{34}$$
$$\varpi_{IP}^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_{IP} t} K_{IP,t} \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + \varpi_{TK}^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_{TK} t} TK_t \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} \Bigg]^{\frac{\varepsilon_1}{\varepsilon_1 - 1}} .$$

The first order conditions that follow from this problem are given by

$$\frac{r_{ICT,t} K_{ICT,t}}{Y_t} = \varpi_{ICT}^{\frac{1}{\varepsilon_1}} \left(\frac{e^{\gamma_{ICT} t} K_{ICT,t}}{Y_t} \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}},$$
$$\frac{r_{IP,t} K_{IP,t}}{Y_t} = \varpi_{IP}^{\frac{1}{\varepsilon_1}} \left(\frac{e^{\gamma_{IP} t} K_{IP,t}}{Y_t} \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}},$$
$$\frac{w_t L_t}{Y_t} = \varpi_L^{\frac{1}{\varepsilon_1}} \left(\frac{e^{\gamma_L t} L_t}{Y_t} \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}},$$
$$\frac{r_{TK,t} TK_t}{Y_t} = \varpi_{TK}^{\frac{1}{\varepsilon_1}} \left(\frac{e^{\gamma_{TK} t} TK_t}{Y_t} \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}},$$

where $r_{ICT,t}$, $r_{IP,t}$, and $r_{TK,t}$ are given by equations (17)-(19) and r_t is given by equation (29).

I use $\alpha_{1,R2}$, $\alpha_{2,R2}$, and $\alpha_{3,R2}$ to denote the following expressions:

$$\alpha_{1,R2} = \varpi_{ICT}^{\frac{1}{\varepsilon_1}} \overline{\left(\frac{e^{\gamma_{ICT} t} K_{ICT,t}}{Y_t} \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}} = \overline{r_{ICT,t} \frac{K_{ICT,t}}{Y_t}},$$
$$\alpha_{2,R2} = \varpi_{IP}^{\frac{1}{\varepsilon_1}} \overline{\left(\frac{e^{\gamma_{IP} t} K_{IP,t}}{Y_t} \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}} = \overline{r_{IP,t} \frac{K_{IP,t}}{Y_t}},$$
$$\alpha_{3,R2} = \varpi_{TK}^{\frac{1}{\varepsilon_1}} \overline{\left(\frac{e^{\gamma_{TK} t} TK_t}{Y_t} \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}} = \overline{r_{TK,t} \frac{TK_t}{Y_t}}.$$

Finally, I estimate the following system of normalized equations:

$$\ln \frac{Y_t}{\bar{Y}} = \frac{\varepsilon_1}{\varepsilon_1 - 1} \ln \Bigg[(1 - \alpha_{1,R2} - \alpha_{2,R2} - \alpha_{3,R2}) \left(e^{\gamma_L \hat{t}} \frac{L_t}{\overline{\overline{L_t}}} \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + \alpha_{1,R2} \left(e^{\gamma_{ICT} \hat{t}} \frac{K_{ICT,t}}{\overline{\overline{K_{ICT,t}}}} \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + \alpha_{2,R2} \left(e^{\gamma_{IP} \hat{t}} \frac{K_{IP,t}}{\overline{\overline{K_{IP,t}}}} \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + \alpha_{3,R2} \left(e^{\gamma_{TK} \hat{t}} \frac{TK_t}{\overline{\overline{TK_t}}} \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} \Bigg], \tag{35}$$

$$\ln r_{ICT,t} - \ln \overline{r_{ICT,t}} = \frac{\varepsilon_1 - 1}{\varepsilon_1} \gamma_{ICT} \hat{t} - \frac{1}{\varepsilon_1} \ln \left(\frac{K_{ICT,t} / \overline{K_{ICT,t}}}{Y_t / \overline{Y_t}} \right), \tag{36}$$

$$\ln r_{IP,t} - \ln \overline{r_{IP,t}} = \frac{\varepsilon_1 - 1}{\varepsilon_1} \gamma_{IP} \hat{t} - \frac{1}{\varepsilon_1} \ln \left(\frac{K_{IP,t} / \overline{K_{IP,t}}}{Y_t / \overline{Y_t}} \right), \tag{37}$$

$$\ln w_t - \ln \overline{w_t} = \frac{\varepsilon_1 - 1}{\varepsilon_1} \gamma_L \hat{t} - \frac{1}{\varepsilon_1} \ln \left(\frac{L_t / \overline{L_t}}{Y_t / \overline{Y_t}} \right), \tag{38}$$

and

$$\ln r_{TK,t} - \ln \overline{r_{TK,t}} = \frac{\varepsilon_1 - 1}{\varepsilon_1} \gamma_{TK} \hat{t} - \frac{1}{\varepsilon_1} \ln \left(\frac{TK_t / \overline{TK_t}}{Y_t / \overline{Y_t}} \right). \tag{39}$$

C.C Separate Nests for Labor and Traditional Capital and for ICT and IP

The firm solves the problem (12) where production function is now given by

$$Y_t = \left(\varpi_{LTK}^{\frac{1}{\varepsilon_1}} LTK_t^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + \varpi_{IK}^{\frac{1}{\varepsilon_1}} IK_t^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} \right)^{\frac{\varepsilon_1}{\varepsilon_1 - 1}}, \tag{40}$$

where

$$LTK_t = \left[\varpi_L^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_L t} L_t \right)^{\frac{\varepsilon_2 - 1}{\varepsilon_2}} + \varpi_{TK}^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_{TK} t} TK_t \right)^{\frac{\varepsilon_2 - 1}{\varepsilon_2}} \right]^{\frac{\varepsilon_2}{\varepsilon_2 - 1}},$$

$$IK_t = \left[\varpi_{ICT}^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_{ICT} t} K_{ICT,t} \right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}} + \varpi_{IP}^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_{IP} t} K_{IP,t} \right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}} \right]^{\frac{\varepsilon_3}{\varepsilon_3 - 1}}.$$

The first order conditions that follow from this problem are given by

$$\frac{r_{ICT,t}K_{ICT,t}}{Y_t} = \frac{\varpi_{IK}^{\frac{1}{\varepsilon_1}} IK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}}}{\varpi_{LTK}^{\frac{1}{\varepsilon_1}} LTK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}} + \varpi_{IK}^{\frac{1}{\varepsilon_1}} IK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}}} \times \frac{\varpi_{ICT}^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_{ICT}t} K_{ICT,t}\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}}}{\varpi_{ICT}^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_{ICT}t} K_{ICT,t}\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}} + \varpi_{IP}^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_{IP}t} K_{IP,t}\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}}}, \tag{41}$$

$$\frac{r_{IP,t}K_{IP,t}}{Y_t} = \frac{\varpi_{IK}^{\frac{1}{\varepsilon_1}} IK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}}}{\varpi_{LTK}^{\frac{1}{\varepsilon_1}} LTK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}} + \varpi_{IK}^{\frac{1}{\varepsilon_1}} IK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}}} \times \frac{\varpi_{IP}^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_{IP}t} K_{IP,t}\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}}}{\varpi_{ICT}^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_{ICT}t} K_{ICT,t}\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}} + \varpi_{IP}^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_{IP}t} K_{IP,t}\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}}}, \tag{42}$$

$$\frac{w_t L_t}{Y_t} = \frac{\varpi_{LTK}^{\frac{1}{\varepsilon_1}} LTK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}}}{\varpi_{LTK}^{\frac{1}{\varepsilon_1}} LTK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}} + \varpi_{IK}^{\frac{1}{\varepsilon_1}} IK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}}} \times \frac{\varpi_{L}^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_{L}t} L_t\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}}}{\varpi_{L}^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_{L}t} L_t\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}} + \varpi_{TK}^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_{TK}t} TK_t\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}}}, \tag{43}$$

and

$$\frac{r_{TK,t}TK_t}{Y_t} = \frac{\varpi_{LTK}^{\frac{1}{\varepsilon_1}} LTK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}}}{\varpi_{LTK}^{\frac{1}{\varepsilon_1}} LTK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}} + \varpi_{IK}^{\frac{1}{\varepsilon_1}} IK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}}} \times \frac{\varpi_{TK}^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_{TK}t} TK_t\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}}}{\varpi_{L}^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_{L}t} L_t\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}} + \varpi_{TK}^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_{TK}t} TK_t\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}}}. \tag{44}$$

I denote by S_{LTK_t,Y_t} and S_{IK_t,Y_t} the shares of compensation of labor and traditional capital and ICT and IP capital in value added:

$$S_{LTK_t,Y_t} = \varpi_{LTK}^{\frac{1}{\varepsilon_1}} \left(\frac{LTK_t}{Y_t}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}},$$
$$S_{IK_t,Y_t} = \varpi_{IK}^{\frac{1}{\varepsilon_2}} \left(\frac{IK_t}{Y_t}\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}}.$$

I also use $\alpha_{1,R3}$, $\alpha_{2,R3}$, and $\alpha_{3,R3}$ to denote the following expressions:

$$\alpha_{1,R3} = \varpi_{LTK}^{\frac{1}{\varepsilon_1}} \overline{\left(\frac{LTK_t}{Y_t}\right)}^{\frac{\varepsilon_1-1}{\varepsilon_1}},$$
$$\alpha_{2,R3} = \varpi_{L}^{\frac{1}{\varepsilon_2}} \overline{\left(\frac{e^{\gamma_L t} L_t}{LTK_t}\right)}^{\frac{\varepsilon_2-1}{\varepsilon_2}},$$
$$\alpha_{3,R3} = \varpi_{ICT}^{\frac{1}{\varepsilon_3}} \overline{\left(\frac{e^{\gamma_{ICT} t} K_{ICT,t}}{IK_t}\right)}^{\frac{\varepsilon_3-1}{\varepsilon_3}}.$$

I use $\alpha_{1,R3}$, $\alpha_{2,R3}$, and $\alpha_{3,R3}$ and write the logrithm of the normalized equation for output in the following way:

$$\ln \frac{Y_t}{\overline{\overline{Y_t}}} = \frac{\varepsilon_1}{\varepsilon_1 - 1} \ln \left[\alpha_{1,R3} \left(\frac{LTK_t}{\overline{\overline{LTK_t}}} \right)^{\frac{\varepsilon_1-1}{\varepsilon_1}} + (1 - \alpha_{1,R3}) \left(\frac{IK_t}{\overline{\overline{IK_t}}} \right)^{\frac{\varepsilon_1-1}{\varepsilon_1}} \right], \tag{45}$$

where

$$\frac{LTK_t}{\overline{\overline{LTK_t}}} = \left[\alpha_{2,R3} \left(e^{\gamma_L \hat{t}} \frac{L_t}{\overline{\overline{L_t}}} \right)^{\frac{\varepsilon_2-1}{\varepsilon_2}} + (1 - \alpha_{2,R3}) \left(\frac{TK_t}{\overline{\overline{TK_t}}} \right)^{\frac{\varepsilon_2-1}{\varepsilon_2}} \right]^{\frac{\varepsilon_2}{\varepsilon_2-1}},$$
$$\frac{IK_t}{\overline{\overline{IK_t}}} = \left[\alpha_{3,R3} \left(e^{\gamma_{ICT} \hat{t}} \frac{K_{ICT,t}}{\overline{\overline{K_{ICT,t}}}} \right)^{\frac{\varepsilon_3-1}{\varepsilon_3}} + (1 - \alpha_{3,R3}) \left(e^{\gamma_{IP} \hat{t}} \frac{K_{IP,t}}{\overline{\overline{K_{IPP,t}}}} \right)^{\frac{\varepsilon_3-1}{\varepsilon_3}} \right]^{\frac{\varepsilon_3}{\varepsilon_3-1}},$$

and $\hat{t}$ is demeaned trend.

I write the normalized equations corresponding to the first order conditions in the following way:

$$\begin{aligned} \ln r_{ICT,t} - \ln \overline{r_{ICT,t}} = & \left(1 - \frac{\varepsilon_3 - 1}{\varepsilon_3} \frac{\varepsilon_1}{\varepsilon_1 - 1}\right) \ln \left(\frac{S_{IK_t,Y_t}}{\overline{\overline{S_{IK_t,Y_t}}}} \right) \\ & + \frac{\varepsilon_3 - 1}{\varepsilon_3} \gamma_{ICT} \hat{t} - \frac{1}{\varepsilon_3} \ln \left(\frac{K_{ICT,t}/\overline{K_{ICT,t}}}{Y_t/\overline{Y_t}} \right), \end{aligned} \tag{46}$$

$$\begin{aligned} \ln r_{IP,t} - \ln \overline{r_{IP,t}} = & \left(1 - \frac{\varepsilon_3 - 1}{\varepsilon_3} \frac{\varepsilon_1}{\varepsilon_1 - 1}\right) \ln \left(\frac{S_{IK_t,Y_t}}{\overline{\overline{S_{IK_t,Y_t}}}} \right) \\ & + \frac{\varepsilon_3 - 1}{\varepsilon_3} \gamma_{IP} \hat{t} - \frac{1}{\varepsilon_3} \ln \left(\frac{K_{IP,t}/\overline{K_{IP,t}}}{Y_t/\overline{Y_t}} \right), \end{aligned} \tag{47}$$

$$\ln w_t - \ln \overline{w_t} = \left(1 - \frac{\varepsilon_2 - 1}{\varepsilon_2}\frac{\varepsilon_1}{\varepsilon_1 - 1}\right) \ln\left(\frac{S_{LTK_t,Y_t}}{\overline{S_{LTK_t,Y_t}}}\right) + \frac{\varepsilon_2 - 1}{\varepsilon_2}\gamma_L \hat{t} - \frac{1}{\varepsilon_2}\ln\left(\frac{L_t/\overline{L_t}}{Y_t/\overline{Y_t}}\right), \tag{48}$$

and

$$\ln r_{TK,t} - \ln \overline{r_{TK,t}} = \left(1 - \frac{\varepsilon_2 - 1}{\varepsilon_2}\frac{\varepsilon_1}{\varepsilon_1 - 1}\right) \ln\left(\frac{S_{LTK_t,Y_t}}{\overline{S_{LTK_t,Y_t}}}\right) + \frac{\varepsilon_2 - 1}{\varepsilon_2}\gamma_{TK} \hat{t} - \frac{1}{\varepsilon_2}\ln\left(\frac{TK_t/\overline{TK_t}}{Y_t/\overline{Y_t}}\right). \tag{49}$$

C.D One Nest for Capital Types

The firm solves the problem (12) where production function is now given by

$$Y_t = \left[\varpi_L^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_L t} L_t\right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + \varpi_{TKIK}^{\frac{1}{\varepsilon_1}} IKTK_t^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}\right]^{\frac{\varepsilon_1}{\varepsilon_1 - 1}}, \tag{50}$$

where

$$IKTK_t = \left[\varpi_{IK}^{\frac{1}{\varepsilon_2}} IK_t^{\frac{\varepsilon_2 - 1}{\varepsilon_2}} + \varpi_{TK}^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_{TK} t} TK_t\right)^{\frac{\varepsilon_2 - 1}{\varepsilon_2}}\right]^{\frac{\varepsilon_2}{\varepsilon_2 - 1}},$$

and

$$IK_t = \left[\varpi_{ICT}^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_{ICT} t} K_{ICT,t}\right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}} + \varpi_{IP}^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_{IP} t} K_{IP,t}\right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}}\right]^{\frac{\varepsilon_3}{\varepsilon_3 - 1}}.$$

The first order conditions that follow from this problem are given by

$$\frac{r_{ICT,t} K_{ICT,t}}{Y_t} = \frac{\varpi_{TKIK}^{\frac{1}{\varepsilon_1}} IKTK_t^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}}{\varpi_L^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_L t} L_t\right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + \varpi_{TKIK}^{\frac{1}{\varepsilon_1}} IKTK_t^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}} \times \frac{\varpi_{IK}^{\frac{1}{\varepsilon_2}} IK_t^{\frac{\varepsilon_2 - 1}{\varepsilon_2}}}{\varpi_{IK}^{\frac{1}{\varepsilon_2}} IK_t^{\frac{\varepsilon_2 - 1}{\varepsilon_2}} + \varpi_{TK}^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_{TK} t} TK_t\right)^{\frac{\varepsilon_2 - 1}{\varepsilon_2}}} \times \frac{\varpi_{ICT}^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_{ICT} t} K_{ICT,t}\right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}}}{\varpi_{ICT}^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_{ICT} t} K_{ICT,t}\right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}} + \varpi_{IP}^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_{IP} t} K_{IP,t}\right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}}}, \tag{51}$$

$$\frac{r_{IP,t}K_{IP,t}}{Y_t} = \frac{\varpi_{TKIK}^{\frac{1}{\varepsilon_1}} IKTK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}}}{\varpi_L^{\frac{1}{\varepsilon_1}}\left(e^{\gamma_L t}L_t\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}} + \varpi_{TKIK}^{\frac{1}{\varepsilon_1}} IKTK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}}} \times \tag{52}$$

$$\frac{\varpi_{IK}^{\frac{1}{\varepsilon_2}} IK_t^{\frac{\varepsilon_2-1}{\varepsilon_2}}}{\varpi_{IK}^{\frac{1}{\varepsilon_2}} IK_t^{\frac{\varepsilon_2-1}{\varepsilon_2}} + \varpi_{TK}^{\frac{1}{\varepsilon_2}}\left(e^{\gamma_{TK} t}TK_t\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}}} \times$$

$$\frac{\varpi_{IP}^{\frac{1}{\varepsilon_3}}\left(e^{\gamma_{IP} t}K_{IP,t}\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}}}{\varpi_{ICT}^{\frac{1}{\varepsilon_3}}\left(e^{\gamma_{ICT} t}K_{ICT,t}\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}} + \varpi_{IP}^{\frac{1}{\varepsilon_3}}\left(e^{\gamma_{IP} t}K_{IP,t}\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}}},$$

$$\frac{w_t L_t}{Y_t} = \frac{\varpi_L^{\frac{1}{\varepsilon_1}}\left(e^{\gamma_L t}L_t\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}}}{\varpi_L^{\frac{1}{\varepsilon_1}}\left(e^{\gamma_L t}L_t\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}} + \varpi_{TKIK}^{\frac{1}{\varepsilon_1}} IKTK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}}}, \tag{53}$$

and

$$\frac{r_{TK,t}TK_t}{Y_t} = \frac{\varpi_{TKIK}^{\frac{1}{\varepsilon_1}} IKTK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}}}{\varpi_L^{\frac{1}{\varepsilon_1}}\left(e^{\gamma_L t}L_t\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}} + \varpi_{TKIK}^{\frac{1}{\varepsilon_1}} IKTK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}}} \times \tag{54}$$

$$\frac{\varpi_{TK}^{\frac{1}{\varepsilon_2}}\left(e^{\gamma_{TK} t}TK_t\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}}}{\varpi_{IK}^{\frac{1}{\varepsilon_2}} IK_t^{\frac{\varepsilon_2-1}{\varepsilon_2}} + \varpi_{TK}^{\frac{1}{\varepsilon_2}}\left(e^{\gamma_{TK} t}TK_t\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}}}.$$

I denote by S_{IKTK_t,Y_t} and $S_{IK_t,IKTK_t}$ the following expressions:

$$S_{IKTK_t,Y_t} = \varpi_{TKIK}^{\frac{1}{\varepsilon_1}}\left(\frac{IKTK_t}{Y_t}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}},$$

$$S_{IK_t,IKTK_t} = \varpi_{IK}^{\frac{1}{\varepsilon_2}}\left(\frac{IK_t}{IKTK_t}\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}}.$$

I also use $\alpha_{1,R4}$, $\alpha_{2,R4}$, and $\alpha_{3,R4}$ to denote the following expressions:

$$\alpha_{1,R4} = \varpi_L^{\frac{1}{\varepsilon_1}}\overline{\left(\frac{e^{\gamma_L t}L_t}{Y_t}\right)}^{\frac{\varepsilon_1-1}{\varepsilon_1}},$$

$$\alpha_{2,R4} = \varpi_{IK}^{\frac{1}{\varepsilon_2}}\overline{\left(\frac{IK_t}{IKTK_t}\right)}^{\frac{\varepsilon_2-1}{\varepsilon_2}},$$

$$\alpha_{3,R4} = \varpi_{ICT}^{\frac{1}{\varepsilon_3}}\overline{\left(\frac{e^{\gamma_{ICT} t}K_{ICT,t}}{IK_t}\right)}^{\frac{\varepsilon_3-1}{\varepsilon_3}}.$$

I use $\alpha_{1,R4}$, $\alpha_{2,R4}$, and $\alpha_{3,R4}$ and write the logarithm of the normalized equation for output in the following way:

$$\ln \frac{Y_t}{\overline{Y_t}} = \frac{\varepsilon_1}{\varepsilon_1 - 1} \ln \left[\alpha_{1,R4} \left(e^{\gamma_L \hat{t}} \frac{L_t}{\overline{L_t}} \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + (1 - \alpha_{1,R4}) \left(\frac{IKTK_t}{\overline{IKTK_t}} \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} \right], \tag{55}$$

where

$$\frac{IKTK_t}{\overline{IKTK_t}} = \left[\alpha_{2,R4} \left(\frac{IK_t}{\overline{IK_t}} \right)^{\frac{\varepsilon_2 - 1}{\varepsilon_2}} + (1 - \alpha_{2,R4}) \left(e^{\gamma_{TK} \hat{t}} \frac{TK_t}{\overline{TK_t}} \right)^{\frac{\varepsilon_2 - 1}{\varepsilon_2}} \right]^{\frac{\varepsilon_2}{\varepsilon_2 - 1}},$$

$$\frac{IK_t}{\overline{IK_t}} = \left[\alpha_{3,R4} \left(e^{\gamma_{ICT} \hat{t}} \frac{K_{ICT,t}}{\overline{K_{ICT,t}}} \right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}} + (1 - \alpha_{3,R4}) \left(e^{\gamma_{IP} \hat{t}} \frac{K_{IP,t}}{\overline{K_{IP,t}}} \right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}} \right]^{\frac{\varepsilon_3}{\varepsilon_3 - 1}}.$$

I write the normalized equations corresponding to the first order conditions in the following way:

$$\begin{aligned} \ln r_{ICT,t} - \ln \overline{r_{ICT,t}} = & \left(1 - \frac{\varepsilon_1}{\varepsilon_1 - 1} \frac{\varepsilon_3 - 1}{\varepsilon_3} \right) \ln \left(\frac{S_{IKTK_t,Y_t}}{\overline{S_{IKTK_t,Y_t}}} \right) + \\ & \left(1 - \frac{\varepsilon_2}{\varepsilon_2 - 1} \frac{\varepsilon_3 - 1}{\varepsilon_3} \right) \ln \left(\frac{S_{IK_t,IKTK_t}}{\overline{S_{IK_t,IKTK_t}}} \right) + \\ & \frac{\varepsilon_3 - 1}{\varepsilon_3} \gamma_{ICT} \hat{t} - \frac{1}{\varepsilon_3} \ln \left(\frac{K_{ICT,t}/\overline{K_{ICT,t}}}{Y_t/\overline{Y_t}} \right), \end{aligned} \tag{56}$$

$$\begin{aligned} \ln r_{IP,t} - \ln \overline{r_{IP,t}} = & \left(1 - \frac{\varepsilon_1}{\varepsilon_1 - 1} \frac{\varepsilon_3 - 1}{\varepsilon_3} \right) \ln \left(\frac{S_{IKTK_t,Y_t}}{\overline{S_{IKTK_t,Y_t}}} \right) + \\ & \left(1 - \frac{\varepsilon_2}{\varepsilon_2 - 1} \frac{\varepsilon_3 - 1}{\varepsilon_3} \right) \ln \left(\frac{S_{IK_t,IKTK_t}}{\overline{S_{IK_t,IKTK_t}}} \right) + \\ & \frac{\varepsilon_3 - 1}{\varepsilon_3} \gamma_{IP} \hat{t} - \frac{1}{\varepsilon_3} \ln \left(\frac{K_{IP,t}/\overline{K_{IP,t}}}{Y_t/\overline{Y_t}} \right), \end{aligned} \tag{57}$$

$$\ln w_t - \ln \overline{w_t} = \frac{\varepsilon_1 - 1}{\varepsilon_1} \gamma_L \hat{t} - \frac{1}{\varepsilon_1} \ln \left(\frac{L_t/\overline{L_t}}{Y_t/\overline{Y_t}} \right), \tag{58}$$

and

$$\begin{aligned} \ln r_{TK,t} - \ln \overline{r_{TK,t}} = & \left(1 - \frac{\varepsilon_1}{\varepsilon_1 - 1} \frac{\varepsilon_3 - 1}{\varepsilon_3} \right) \ln \left(\frac{S_{IKTK_t,Y_t}}{\overline{S_{IKTK_t,Y_t}}} \right) + \\ & \frac{\varepsilon_2 - 1}{\varepsilon_2} \gamma_{TK} \hat{t} - \frac{1}{\varepsilon_2} \ln \left(\frac{TK_t/\overline{TK_t}}{Y_t/\overline{Y_t}} \right). \end{aligned} \tag{59}$$