Zigzags on The Road to The Top: Matching with Leadership Spillovers in College Football*

- Preliminary and Incomplete -

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Abstract

We study the market for college football coaches and analyze the career paths of all the coaches that have become head coaches at the highest level (FBS) from 2000 to 2016. Mobility arise along two dimensions: the quality of the college and the position in the coaching hierarchy. Our main result document that coaches do not move monotonically up a productivity-ladder. 36.6% of the shifts between teams are moves we classify as zigzag moves. A zig-zag move is up in coaching hierarchy but down in Elo-ranking, or down in coaching hierarchy, but up in Elo ranking. We propose a matching model where assistant coaches learn from head coaches and show that these off-diagonal moves are equilibrium phenomenon. These zig-zag movements predict how quickly a coach can reach the top of the hierarchy in the FBS and the performance of the teams after hiring a new coach. Last, we document comparative statics in the movements around the median of the type distribution supporting median matching in our data.

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1 Introduction

This paper studies the market for college football coaches at the highest level (Division I Football Bowl Subdivision). In particular, we analyze the career paths for all coaches that have become a head coach from 2000 to 2016, from their first job as an assistant coach and all the way to the top of the coaching hierarchy.

As Lucas Jr (1978) and Rosen (1982) have noted, firms compete for individuals by offering not only a salary but also a set of responsibilities, a position within the firm's hierarchy. The assignment of individuals to positions in the hierarchy is especially important when a firm's organization is a determinant of success and when individuals are not perfect substitutes at different levels of a hierarchy (as in Kremer and Maskin, 1996; Garicano, 2000).

In the simplest assignment models à la Becker (1973) firms are differentiated by a quality, an exogenous type (could be reputation, infrastructure available for production, scope of products), individuals are characterized by an exogenous type (human capital, skills) that both affect the marginal productivity of the individual and shape the way firms and individuals match. Complementarities in production then implies that individuals with higher types are assigned to higher type firms.

When firms are hierarchical and individuals are not substitute at different levels of a hierarchy, the productivity of an individual depends on her position in this hierarchy, as well as on the types of the other members of the hierarchy and the type of the firm. With complementarities between individuals types and firm types, it is no longer true that higher individual types are assigned to higher type firms – this will be apparent from the static version of our model. However, *if we condition on the position of individuals in the hierarchy*, higher individual types are indeed assigned to higher type firms. This simple observation illustrates why a failure of assortative matching in the assignment of individual types to firm types is not an indication of a lack of complementarity in production but rather illustrates an important omitted variable in the analysis, the organizational structure of firms and the positions of individuals in their organigrammes.

But this observation has also consequences for the career path of individuals. Our focus in this paper is on the dynamic consequences of hierarchical production when individuals learn from their leaders. Learning will modify the distribution of individuals' types and will lead to a new assignment of individuals to firms and positions in the hierarchies. Some of these reassignments may look like standard demotions, when an individual joins a firm with a lower type and also goes down in the hierarchy ('down-down' or DD) or a promotion when an individual joins a firm with a higher type and also up in the hierarchy ('up-up' or UU). But some individuals who were high in a hierarchy may move to a better firm at a lower level ('down-up' or DU); other individuals who were among the middle levels of a hierarchy may elect to move to a less reputable firm but be higher up in the hierarchy ('up-down' or UD). Therefore, an individual moving to a lower productivity firm is not necessarily a sign of demotion or failure to perform. Indeed, because of complementarities, these moves can be accompanied by an increase in earnings.

To help frame the analysis, we propose a stylized two-period model, with two levels in the hierarchy – Head Coach (HC) and Assistant Coach (AC) – and colleges differentiated by quality. There are complementarities in the abilities of HC and AC but also in the quality of the colleges. When an AC works alongside a HC in a college, his ability to be an HC increases. We assume that the HC's ability to be a AC is not affected.

We show that the equilibrium has the following characteristics:

(a) There is assortative matching between colleges, HC and AC. By making an assumption on the support of abilities, matching is "median" in the sense that the top ability individuals is the HC in the best college with a AC if median ability. Assortative matching and measure consistency then determines the other matches.

(b) (Zigzags) There are equal proportions of UD and DU moves. This is a mechanical consequence of having a fixed number of head-coach positions at colleges: for a given population, if one individual is added to the pool of head-coaches, other individuals who were head-coaches must move down.

(c) (Permanence) Under median matching, individuals below the median are more likely to do UD moves, the higher their initial ability. Individuals above the median are more likely to do DU moves the lower is their initial ability.

In our empirical part, we collect the career of individuals and their positions in the hierarchy of the college that employ them. This market is ideally suited for our analysis of mobility because colleges have similar hierarchical organizations, and positions in the hierarchy are fixed. There are four main levels in the coaching hierarchy. The head coach is at the top. The second level consists of offensive and defensive coordinators. The third level consists of smaller unit coaches for the quarterback, linebackers, wide-receivers etc. And at the bottom are the assistants to these smaller units. We measure the quality of colleges by their ELO ranking, and provide here an alternative to the subjective measures of quality of colleges that are currently available.

At any time we can compute the number of times an individual has moved in a certain fashion: for instance, a one count of UD means that the individuals moved to a college with a lower Elo ranking than his initial college but to occupy a higher position on the hierarchical ladder. We count a move as being UD only if the difference in Elo rankings is larger than a bound.

Zigzags in the career paths of university coaches are prevalent, and there are equal measures of 'up-down' and 'down-up' moves (around 36.6% of the moves have this feature), consistent with the theoretical insight (b). Other empirical findings are summarized in Table 1.

The empirical results on permanence are consistent with our theoretical result (c): HCs

Move	Time to first HC position	Permanence as HC of college	Performance as HC
UD	0	0	good
DU	bad	bad	bad
DD	0	bad	0
UU	0	0	0
count	bad	0	0

Table 1: Empirical effects ('0' stands for non-significant, 'bad' for moves having a negative impact: 'good' for moves that have a positive impact; 'count' is the number of times an individual moved to a new college)

close to the median ability are more likely to do DU moves, hence are less permanent than HC who have higher ability.

The theory is silent on the effect of past moves on performance. The empirical results suggest that hiring as HC individuals who did a lot of UD moves has a positive effect while hiring those who did a lot of DU moves has a negative effect. These results are broadly consistent with our stylized model, and support the view that individuals learn from bosses.

We have organized the paper as follows. The first section sets up a theoretical model with learning from bosses in hierarchies. Next, we describe the economic environment and the data. Section 4 is the descriptive statistics and our 5 stylized facts. Section 5 contains our main result on the zigzag movement of coaches, and Section 6 inspect predictions from the movements to outcomes for the coach and team. Section 7 concludes.

1.1 Related Literature

We contribute to the literature in organizational economics on production in teams. Garicano and Hubbard (2003) use Census data on law firm positions and earning to discriminate among different production functions. Their evidence points to imperfect substitution of skills among positions in the organization, as in Kremer and Maskin (1996)'s model. Legros and Newman (2002) apply their general characterization of assortative matching to the Kremer and Maskin (1996) model and show that the matching outcomes are assortative in the sense that there is an order on "ordered pairs". They also provide conditions for the matching pattern to be "median." Chade and Eeckhout (2020) analyze a static model where teams compete. Anderson and Smith (2010) introduce the possibility of changing one's type as a function of the match. Types are binary and the transition probabilities depend on the two partners' types. Their main result is that the equilibrium matches do not satisfy positive assortativeness even if the production function exhibits complementarities. In their world, types are substitute and individuals either move down or up during the transition. Fudenberg and Rayo (2019) study master-apprentice relationships and the apprentice learning from the master (but not the reverse). The master tradeoffs the benefits of cheap labor, more productivity if the apprentice learns faster, but higher probability of separation if the apprentice learns everything the master has to teach.

We also contribute to the labor market search literature where workers sort based on unobserved characteristics and follow productivity ladders where workers move from less productive to more productive and higher-paying firms over the business cycle.

Haltiwanger et al. (2018) study the reallocation of workers across firms in the Longitudinal Employer-Household Dynamics program at the US Census. They find that job-to-job transitions move young workers from less productive to more productive firms. Moreover, they find that these job moves primarily reallocates less educated workers up the job ladder. More educated workers are less likely to match with low-productivity firms to begin with but are also less likely to separate from them.

Abowd et al. (1999) study the different wage compositions from observable and unobservable characteristics by the firm and the workers. Personal and firm fixed effects are important determinants of wages, but individual fixed effects account for most of the inter-industry wage dispersion. Furthermore, they find that firms that hire high-wage workers are more productive per worker.

Shimer and Smith (2000) studies conditions for when matching is assortative in a search model. In our model coach assistants and head coaches match in a frictionless environment.

Eeckhout and Kircher (2011) study sorting of workers across jobs in a search environment and show that wage data alone is impossible to identify whether assortative is positive or negative because the firm type is non-monotonic in wages. Wages are low at bad firms because the match value is low. Wages are also low at very good firms because the firm has to be compensated for matching with an agent that destroys the opportunity to match with a better worker. Hence, the wage schedule is non-monotonic around the optimal match of the worker and firm, and not monotonically increasing in the firm type.

Hagedorn et al. (2017) develop a new algorithm to rank workers and firms to circumvent the assumption that wages are monotone in the firms productivity. First, they rank workers within a firm based on their wage. Next, they use the coworkers' wages at the current firm as well as the coworkers at past firms to globally rank workers. This method relies heavily on coworker's movement to give relative rankings of workers.

We use an Elo measure to assign a quality to the teams instead of the usual productivity measures and let the placement within the hierarchy of coaches determine the ranking of the worker within the team to capture two dimensions of a job.

2 Learning from Bosses: A Model

There are two periods and each period, individuals are free to move among colleges. In the first period, individuals are characterized by a coaching talent a, where a has distribution $F(a)$ on a compact interval $[0, \overline{a}]$. Colleges have types x (wealth, reputation, quality of the infrastructure), uniformly distributed on $[0, 1]$, and need to hire a team consisting of a head-coach (HC) and an assistant coach (AC). There are fewer colleges than pairs of individuals that is there exists $\underline{a} \in (0, \overline{a})$ such that $F([\underline{a}, \overline{a}]) = 2$.

As in Kremer and Maskin (1996), the quality of a team with a HC of quality a and an AC of quality b is equal to $a^\theta b^{1-\theta}$, where $\theta \in (1/2, 1)$ indexes the relative importance of a HC for the success of the team. The output of the relationship within a college of type u is is equal to $c \cdot y(a, b)$. We assume perfect information among participants about talents and types of colleges.

Clearly, if the team consists of a, b with $a > b$, it is optimal to assign the individual of talent a to the HC position and the other individual to the AC position. We will use below the convention that teams are ordered with the first component being the talent of the HC, that is a team is a pair (a, b) where $a \geq b$. We write the quality of a team consisting of an HC of talent a and an AC of talent $b \leq a$

$$y(a, b) = a^\theta b^{1-\theta}.$$

Ignoring for the moment the match with colleges, Legros and Newman (2002) show that the match satisfies a positive assortative property: if there are two equilibrium teams (a, b) and $(\hat{a}, \hat{b})$ then $a > \hat{a} \Rightarrow b > \hat{b}$. This property extends to matches with colleges as should be intuitive if we decompose the assignment problem into two sub-problems. colleges compete for pairs (a, b) and because xy is supermodular, better colleges attract teams that generate a higher quality y. In the competition among individuals to join teams with high output, we have therefore assortative result. A formal proof is in the Appendix.

Proposition 1. *In a static environment, the equilibrium matching satisfies PAM. Hence, for two colleges of qualities $x > \hat{x}$, if the equilibrium matches are (x, a, b) and $(\hat{x}, \hat{a}, \hat{b})$, then $(a, b) \geq (\hat{a}, \hat{b})$.*

If there is no change in the distribution F, both the compositions of teams and the match with colleges are the same in the second period. However, changes in the distribution of talent is likely in this environment because coaching talent can be learned. There are many ways to model this learning from others. Individuals may get better at what they do in a given position, but they can also learn from their peers, ACs can learn from their HCs and HCs can also learn from managing and supervising ACs.

To simplify, we will consider a situation where it is only a subordinate AC that can learn from his HC, as in master-apprentice relationships.[1] We assume that the benefit of

[1]See Fudenberg and Rayo, 2019 for a model where the master can control the level of learning of his

learning is a linear function of the difference of talents of HC and AC, hence that HC's talent is more likely to "rub" on an AC the higher the talent of the HC is.[2]

Therefore, if individuals i, j of abilities (a, b), with $a > b$ match today, their abilities tomorrow will be respectively $a^* = a$ (HC has not learned from his AC) and $b^* = b + \lambda(a - b)$ (AC has learned from his boss). As long as $\lambda \in (0, 1)$, an AC cannot leapfrog his current HC in terms of ability, but he can leapfrog other current HCs. Because the distribution of abilities in the second period is a first-order stochastic shift of the initial distribution, a new matching pattern will be induced in the second period. PAM allows complex matching patterns (see Kremer and Maskin, 1996), but as Legros and Newman (2002) show simpler patterns exist when the support of types is not too large. Precisely, if the support of individuals types $[\underline{a}, \overline{a}]$ satisfies

$$\underline{a} \geq \left(\frac{1 - \theta}{\theta} \right)^{\frac{1}{\theta}} \overline{a} \tag{1}$$

there is median matching among individuals in the sense that matches 'cross' the median in a positive assortative way: if F is the distribution on $[\underline{a}, \overline{a}]$, and a_m is the median, the matches are (a, b) where $a \geq a_m$ and b is such that $F(a) - F(b) = \frac{1}{2}$.

Corollary 1. *Suppose that (1) holds. Let $\underline{a}$ be such that there is a measure 2 of individuals in $[\underline{a}, \overline{a}]$ and let a_m be the mean of the conditional distribution when the support is restricted to $[\underline{a}, \overline{a}]$. Then, individuals in $[\underline{a}, \overline{a}]$ are employed by colleges, all individuals in $[a_m, \overline{a}]$ become head coaches and match in a positive assortative way with colleges, and individuals in $[\underline{a}, a_m)]$ become assistant coaches and match with head coaches in an assortative way.*

As Anderson and Smith (2010) have noted when there is learning the socially efficient match may differ from the match in the static model. This result is related to the forward looking behavior of individual.[3] Because our objective here is to show that zigzags can arise as part of the equilibrium of an assignment model, we make the extreme assumption that individuals have a high discount factor and behave myopically.

Proposition 1 then implies that there is PAM in each period. This suggests how the transition from one period to the next evolves when there is median matching. For a first period distribution F, if individuals $1, 2$ with abilities $a_1 > a_2$ are matched, by median matching a_1 is above the median and a_2 is below the median. the next period ability of 1 is a_1 but the next period ability of 2 is $a_2 + \lambda(a_1 - a_2)$. Hence 2 will 'leapfrog' in terms of ability all individuals who are HCs in the first period with initial type less that $a_2 + \lambda(a_1 - a_2)$, and some will become HC in the second period. If this happens to a positive measure of individuals, it must be the case than an equivalent measure of individuals shift from a HC position to an AC position, which is our main result in Theorem 1 below.

apprentice.

[2]But one could also argue that it takes someone talented to understand someone talented or that bad bosses may provide examples of what should not be done as a HC.

[3]In their model, abilities are unknown and the (stochastic) outcome of joint production yields updating about the quality of each partner. In our model, abilities are known but increase deterministically.

That zigzags happen is now clear: an AC who is just below the median ability in the first period will have ability greater than all the HC in the first period who have abilities in the interval $(a_m, a_m + \lambda(\overline{a} - a_m))$, and the new median ability in the second period is greater than a_m, implying that this individual AC will become a HC in a college of lower quality while first period HCs whose ability is bounded by the two median abilities become ACs in a higher quality college.

Example 1. *There are four individuals* $\{i_1, i_2, i_3, i_4\}$ *with talents* $t(i_k) = k$, $\theta = 0.8$, *and* $\lambda = 0.6$. *There are two colleges* $coll_0, coll_1$ *with qualities* $x_1 > x_0$. *The reader can verify that the condition for median matching* (1) *holds. The equilibrium matches in the first period are therefore* $(coll_1, i_4, i_2), (coll_0, i_3, i_1)$.

After the first period, the abilities of individuals become $t^*(i_4) = 4$, $t^*(i_3) = 3$, $t^*(i_2) = 2 + \lambda 2$ *and* $t^*(i_1) = 1 + \lambda 2$. *As* $\lambda > 1/2$, *the new order of types is* $t^*(i_4) > t^*(i_2) > t^*(i_3) > t^*(i_1)$. *Hence, equilibrium matches are* $(coll_1, i_4, i_3), (coll_0, i_2, i_1)$.

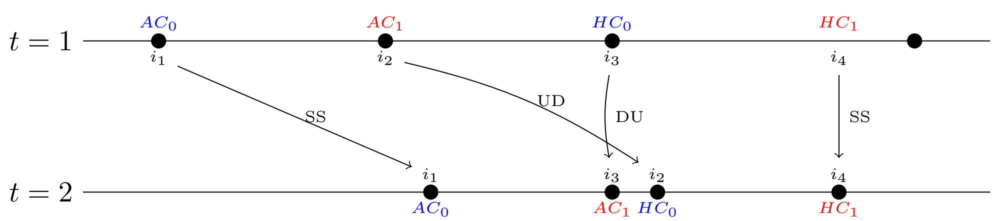

Figure 1: Moves in the two-period model.
The subscripts 0, 1 refer to the college
(e.g. HC_0 indicates the HC of college $coll_0$. "S" stands for 'Stay)

Therefore the individuals of initial talents 1 *and* 4 *stay in the same colleges and at the same levels of the hierarchies. But the individual* i_2 *moves 'down-up': she moves from an AC position in college* x_1 *to a HC position in college of inferior quality. Finally the individual* i_3 *moves 'up-down' from a HC position in college* $coll_0$ *to an AC position in college* $coll_1$. *This is a general property of equilibria in this model. Note that individual* i_2 *"leapfrogs" individual* i_3 *in the hierarchical position but not in the quality of the college he is eventually associated with.*[4]

Example 2 (More than two periods.)**.** *Let us revisit Example 1 with more than two periods, keeping our assumption that individuals have high discounting (hence will behave myopically), and assume that learning does not depreciate. As we have seen, individual* i_2 *becomes a HC in the second-period and has ability (assuming* $\lambda = 0.6$*)* $2 + \lambda 2 = 3.2 > 3$, *while individual* i_3 *becomes an AC under the supervision of* i_4. *Hence, in the third period,* i_3 *will have ability* $3 + \lambda 1 = 3.6$ *and leapfrogs* i_2. *Hence,* i_2 *becomes an AC under the supervision of* i_3. *This is illustrated in the figure below.*

[4]As we show in the next example, if there is a third period, individual i_3 will leapfrog individual i_2. It should be clear that after T periods, most individuals will catch up with individual i_4, which is pretty counterfactual. This would not be the case if individuals have an exogenous probability of leaving the market and be replaced by individuals of their same initial type.

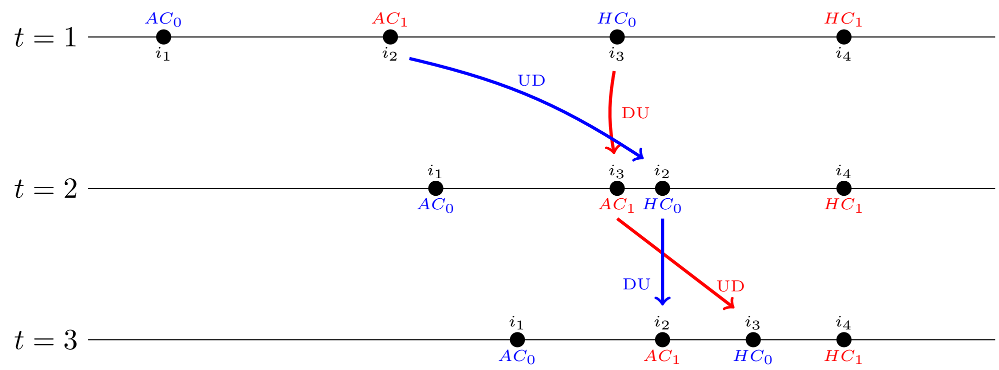

Figure 2: Moves in a three-period model

In the example, the UD move by i_2 can happen only if λ is greater than 1/2: this insures that the $t = 2$ ability of i_2 is greater than 3 hence that he becomes more attractive as a HC than i_3, who is therefore forced to do a *DU* move. When the distribution of talent has a continuous support, zigzags must happen for any positive value of λ, and the proportions of UD and DU moves are the same. The argument is a simple generalization of that in the example.

For a distribution with continuous support, PAM in the first period implies that there are *critical types* that could be both HC or AC (the set of critical types has zero measure). In the median matching case of Corollary 1 there is only one critical type, the median a_m. For the sake of intuition, consider such a case. Then all individuals with types in $(a_m, \overline{a}]$ are HC and all individuals with types in $[\underline{a}, a_m)$ are AC. An AC with type $b < a_m$ in the first period will have ability $b + \lambda(\mu(b) - b)$ in the second period if hired as a HC, where μ is the equilibrium map matching ACs to HCs. By PAM, for any value of λ, there exists $\underline{b} < a_m$ such that $\underline{b} + \lambda(\mu(\underline{b}) - \underline{b}) = a_m$. Therefore, next period the median a_m^* of the new distribution belongs to the interior of the interval $(a_m, a_m + \lambda(\overline{a} - a_m))$. Because the condition for median matching (1) continues to hold, there is again median matching, and there are individuals who were ACs in the first period who become HC and make a UD move and individuals who were HCs in the first period with ability in (a_m, a_m^*) who make a DU move. By measure consistency, these two sets of individuals have the same measure.[5] This proves the following theorem and two predictions (zigzags and equal measure of UD and DU moves) that provide a theoretical rationale for some of our main empirical findings.

Theorem 1. *Consider a distribution of talent with a continuous support. For high discounting, zigzags happen and the measure of 'up-down' moves is equal to the measure of 'down-up' moves.*

Our discussion of the median matching case yields another prediction on the probability

[5] If condition 1 is not satisfied and median matching fails, there is more than one critical type, but the previous argument applies. For instance, considering the critical type $\mu^{-1}(\overline{a})$ (which coincides with a_m in the median matching case), there are types slightly lower than this critical type who necessarily move UD; this is true for all critical types, and the equality of the measures follows measure consistency.

that individuals will go up or down in the hierarchy.

Corollary 2. *Suppose that condition 1 holds and that there is median matching. Then (i) the higher ability of an AC, the more likely the individual will become HC when changing college and (ii) the lower the ability of a HC, the higher the more likely the individual will become AC when changing college.*

This corollary is not true when there is not median matching. In this case, $\overline{a}$ matches with $b_1 > a_m$ (a 'critical type' who can be either AC or HC) who then matches with a critical type b_2, etc. With learning, ACs with ability slightly to the left of b_1 and b_2 will become HCs next period, but there will be individuals in (b_2, b_1) who stay ACs. Therefore, churning (going from AC to HC) is not monotonic in the abilities of ACs. Within the confines of this model, observing churning monotonic in the ability of ACs validates median matching.

3 Data

3.1 Environment and Data Collection

Our analysis focus on head coaches of Football Bowl Subdivision (FBS) teams from 2000 to 2016. The last season we use is the 2016-2017 season. As of the end of 2016-2017 season, there were 130 teams in the FBS, and 324 head coaches has worked for those teams from 2000 to 2016. We obtained career tenure data on all head coaches from their respective personal sites on Wikipedia, where information on coaching periods, positions and teams is available. The data is dated back to 1954, where the oldest coach in our data set, Bobby Bowden, first started his coaching career at Howard college as an assistant. In total, there are 668 teams in the career tenure data set, which include high school, college and professional teams.

For the Elo ranking, we collected data on game schedules and results from Cfbdatawarehouse.com. This website provides a comprehensive database on college football games from the 1890 to 2016. We chose 1930 as the starting year to construct the Elo ranking because the 1930s was often considered as the milestone period when college football started to become popular nationwide. Because we are only interested in college teams' Elo ranking, we divided all teams in our data set into 3 groups: high school, college and professional teams, and created Elo ranking for the college group only. Thus, we dropped all games in the data set played by high school, junior varsity or community college teams. We also dropped games which was played by a team that played less than 30 games (equivalent to about 3 football seasons) in the data set[6]. In total, the data set includes 213,627 games played by 975 college teams from 1930 to 2016. For each game, information is available on

[6] Elo rating is only reliable if a team plays more than a certain number of games.

the location of the game to determine which team is the home team, and the score margin of the game.

3.2 Elo

3.2.1 Overview

In this part, we will briefly describe the Elo ranking system and its modifications to match our studied context. The Elo system uses the following set of equations to determine the best prediction of games:

$$\lambda^i = \frac{1}{1 + 10^{(e_0^j - e_0^i)/d}} \text{ and } \lambda^j = 1 - \lambda^i \tag{2}$$

where λ^i and λ^j are the score predictions for team i and j respectively; e_0^i and e_0^j are team i's and team j's Elo ratings before the game. The actual score for the game is given by:

$$\alpha^i = \begin{cases} 1 & \text{if team i wins} \\ 0.5 & \text{if draws} \\ 0 & \text{if loses} \end{cases}$$

The Elo ratings are updated after the match and the new rating for team i is:

$$e_1^i = e_0^i + k(\alpha^i - \lambda^i) \tag{3}$$

and new rating for team j is calculated the same way. The rating is updated after every match so that teams carry the new Elo ratings to the next match. Next, we assign each new team an initial Elo rating of 1500 Elo points. Note that the Elo rating system only becomes a reliable predicting system after a sufficient number of games played. Thus, with schedule dated back to 1930, we have sufficiently enough past games to get reliable ratings for teams coached by the head coaches in our data set, starting from 1954.

There are two important modifications to the system above. First, we modify the system to account for home court advantage by adding θ to the home team's Elo rating before the game[7]. Thus, from equation 2, $e_0^j - e_0^i$ becomes $e_0^j + \theta - e_0^i$ if team j has home court advantage, and $e_0^j - e_0^i - \theta$ if team i has home court advantage. Second, we further improve the system to account for difference in point margin m by setting

$$k = k_0 \cdot ln\Big\{max(m, 1) + 1\Big\} \cdot \frac{2.2}{\beta^{0.001} + 2.2} \tag{4}$$

While k_0 is a fixed parameter, β is the Elo rating difference between winning team and losing team. Equation 4 improves the system by emphasizing on reward teams that

[7]Home field advantage has been proven to be a significant factor contributing to a home team's performance. See Courneya and Carron (1992) for a review.

win with a larger margin than expected more. However, the natural logarithm function of the margin m makes sure that the function is increasing but at a lower rate as margin becomes higher since it is not a big difference between a team winning by 50 or 60 points. β is added to the equation to avoid auto-correlation problem from the fact that higher Elo teams tend to win more often at a higher margin, while an upset from a lower Elo rating team tends to be at a very narrow margin. Thus, we want to reward upsets by underdogs more and diminish large margin win for higher Elo rating teams.

3.2.2 Calibration

Before running the simulation to get Elo rating for each team in each season, we need to identify four important parameters in the system. The first is d in equation (2), which determines the appropriate scale of the ratings. In the benchmark model, we use $d = 400$ following Elo (1978).

We then identify the other three parameters: k, k_0 and θ when both modifications are included by evaluating the predicting power of the ranking system. We use the log loss function (denoted as L^l) to assess how well the system predicts game outcomes. L^l emphasizes on penalizing wrong and confident prediction which is close to 0%. The goal is to minimize the expected value of the loss function to achieve the best predicted Elo ranking system. Since we focus on the ranking of teams from 1954, for the benchmark ranking, we use the minimum expected values of the loss function from 1954 to 2016 to identify the best values of parameters k_0 and θ.

For the benchmark model, the value of $k_0 = 30$ suggests that the updating parameter after each game is quite large compared to other values of k_0 in the literature (Hvattum and Arntzen, 2010; International Chess Federation, 2020). However, in our context, the number of games a team play each season is smaller than in other contexts such as chess or soccer; therefore, it is reasonable to have a large updating value so that if there is any significant change in a team's strength, its Elo rating can catch up quickly with the real strength of the team[8].

In Figure 3, we plot the Elo ratings as of 2016 season of all college teams in the data set to check our benchmark ranking's validity. Widely known as the best division in college football, Division 1A (FBS), is clearly also the best division according to our Elo ranking. The differences between the second-best division, Division 1AA, and other lower quality divisions are not too significant; however, the ranking of all divisions from best to worst from our ranking is consistent with the ranking in the real world, as shown in Figure 3.

[8] We use the other models with different configurations to perform robustness check later on in the paper.

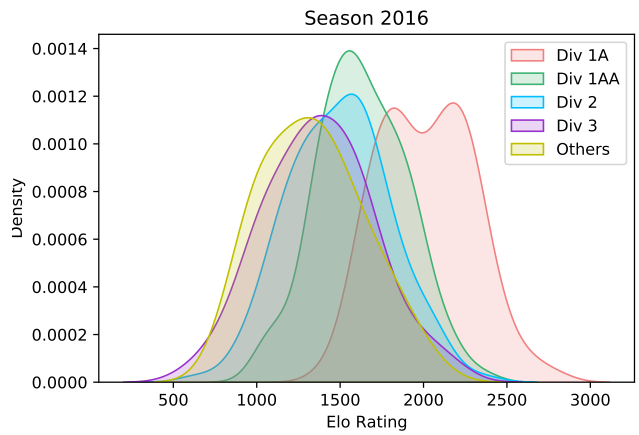

Figure 3: Distribution of teams across different divisions ranking as of 2016 season, from best to worst: Division 1A (FBS), Division 1AA, Division 2, Division 3 and others non-categorized teams.

4 Descriptive Statistics and Stylized Facts

Table 2: Descriptive statistics

Variable	Obs	Mean	Std.dev	Min	Max
Elo rank	9,691	2041.97	795.61	200	4000
Elo rank NCAA only	8,203	2023.57	301.74	819.97	2813.89
Position code	9,691	2.11	1.03	1	4
Tenure	9,691	16.77	10.60	1	56

4.1 Descriptive Statistics

Our sample is all head coaches at the Division 1 FBS in the years 2000 to 2016. Our sample consists of 324 coaches with careers from 648 teams spanning the years 1954 to 2016. In total, we have 9.691 coach-year observations.

The Elo-rating: All the college teams are ranked starting from the first season we have data from, 1930. However, we are interested in coaches that were head coaches in FBS between 2000 to 2016. Hence, the first coach-college pair in our sample is Bobby Bowden at Howard college in 1954 as a graduate assistant. The professional teams in the NFL are all given a rating of 4000 to be higher than the NCAA colleges, and all the high-school teams are given a rating of 200 to be lower than the NCAA colleges.

The mean Elo-rank across all seasons is 2.023,5. The lowest ranked college football team is Swarthmore (PA) with an Elo-rating of 819,94 in 1975 and the highest ranked college football team is college of Alabama in 2016 with an Elo-rating of 2813.89. college of Alabama won the College Football Playoff in 2016. The mean Elo ratings across all seasons is 2023.5. The lowest ranked university football team is Swarthmore (PA) with an

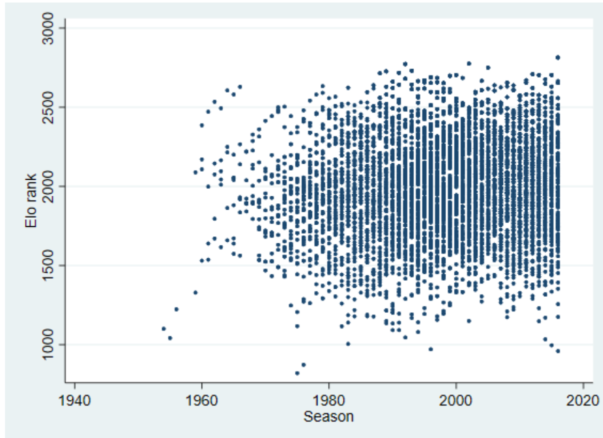

Figure 4: Scatter plot of the Elo ratings of the associated coach-college pairs from the 1954 to 2016 season for the NCAA colleges.

Elo rating of 819.94 in 1975 and the highest ranked university football team is University of Alabama in 2016 with an Elo rating of 2813.89. University of Alabama won the College Football Playoff in 2016.

The coaching hierarchy of an NCAA college football team consists of 9 unit leaders. At the top is the Head Coach (HC). The second level consists of the Offensive Coordinator (OC), Defensive Coordinator (DC) and Assistant Head Coach. The positions on level two are battalion leaders. The third level consists of unit leaders for the the quarterbacks (QB), offensive line, defensive line, receivers, and back positions. Last, there is a last level which constitutes offensive and defensive assistants, as well as graduate assistants. The coaching hierarchy is ranked from 4 at the bottom to 1 at the top.

4.2 Stylized Facts

In this section, we present stylized facts about the movements of coaches.

Fact 1. Most matches happens at the beginning of a coaches career
We inspect how many years the coach has been active when there is a new match tenure of a coach when there is a new match between a coach and a college, or a new position for the coach in the college.

There are 2.972 new matches in our data set. We inspect the cumulative years a coach has been active for all the different matches. There are new matching when the coach moves to a new college, or to a new position in the same college. The median number of years of a coach for a match in our data is 14.31 seasons with a standard deviation of 10.11. 50 percent of our matches has happened before 13 seasons and 90 percent of our matches happened before 29 seasons.

Fact 2. The jumps in Elo ranking and coaching hierarchy are larger in the beginning of the tenure compared to later in the tenure.
The left panel in Figure 6 shows the percentage change in Elo when changing to a new

Table 3: Descriptive statistics

	Obs	Mean	Std. Dev	50	90
Tenure	2,972	14.31	10.11	13	29

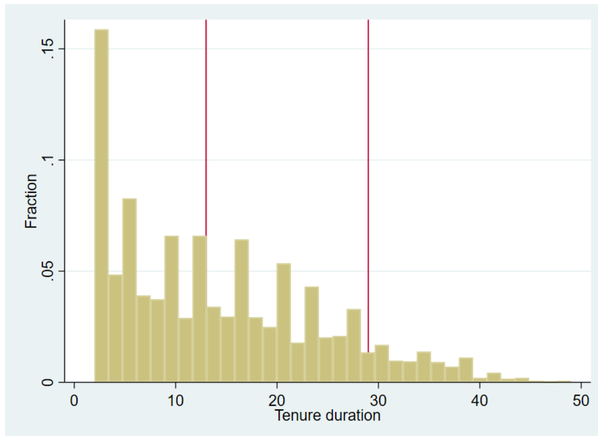

Figure 5: Histogram of tenure with there is a new match.

team and the tenure of the coach. The largest changes are a 112% increase and a 51.6% decrease. The mean change is -0.18%. A regression of percentage change on tenure shows a negative and significant relationship. A one standard deviation increase in tenure leads to a 1.61 percentage points reduction in the change in Elo.[9] Hence, as the coach becomes more seasoned the jumps in Elo ranking becomes smaller.

The right panel in Figure 6 shows the changes in the coaching hierarchy and the tenure of the coach. A regression of the changes in the hierarchy on tenure shows a positive and significant relationship. Hence, as the coach becomes more seasoned the shifts in the coaching hierarchy becomes smaller. A regression of position levels on tenure gives a negative and significant relationship. A one standard deviation increase in tenure leads to a 0.55 reduction in the coaching level. Hence, as the coaches becomes more seasoned they rise up in the hierarchy.[10]

Fact 3. The mean employment spell increases with tenure.
We inspect the employment spell for all new matches in our data. The mean employment spell is 2.95 seasons with a standard deviation of 2.36. The median employment spell is 2 seasons and the 90^{th} percentile is 6 seasons. There is a positive relationship between duration and tenure. A one standard deviation increase in tenure gives a 0.09 seasons longer spell.

Next, we contrast the employment spell before and after the first time the coach became a head coach in the FBS. For the matches before the first head coach position in the FBS the mean employment spell is 3.02 seasons with a standard deviation of 2.40. The median

[9] Coefficient on tenure is -0.16 with a standard error of 0.039

[10] Note: the top position is level 1 and the bottom position is 4.

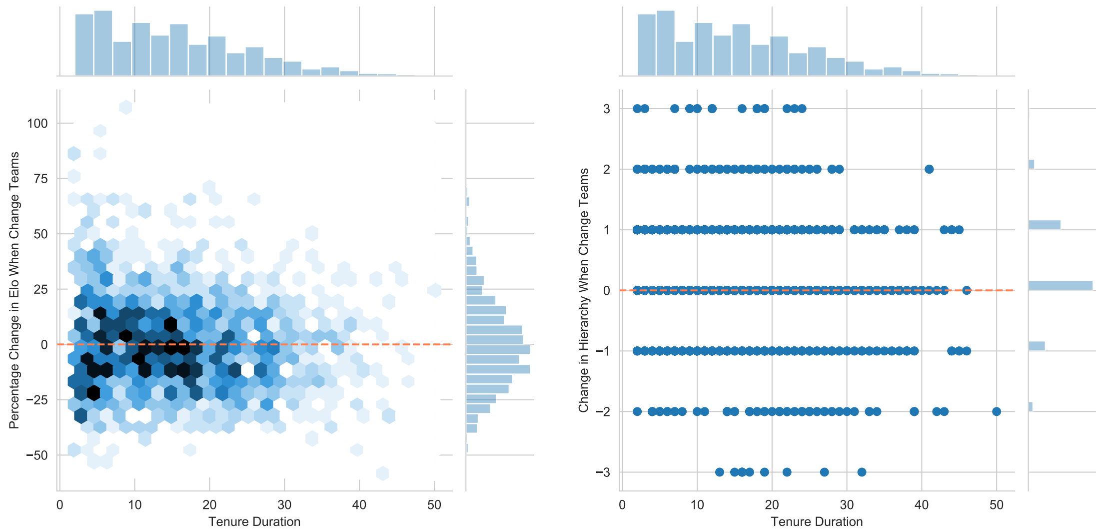

Figure 6: Separation distribution over tenure.

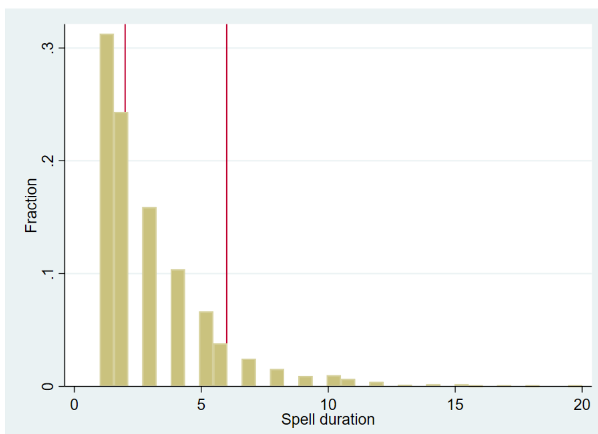

Figure 7: Histogram of the spell duration

employment is 2 seasons and the 90^{th} percentile is 6 seasons. The longest spell before becoming a head coach in the FBS is 20 seasons. For the matches after the first head coach position in the FBS the mean employment spell is 3.37 seasons with a standard deviation of 2.87. The median employment spell is 3 seasons and the 90^{th} percentile is 7 seasons. The longest spell after becoming a head coach in the FBS is 18 seasons.

Fact 4. It takes the median coach 14.2 seasons to become a head coach. In the left panel in Figure 8 we see that the median coach took 14.2 seasons to become a head coach at either high school, college or the NFL. The shortest spell as an assistant coach is 1 seasons, and the longest is 32 seasons. In the right panel we see that the media coach took 19.9 seasons to become a head coach in the FBS for the first time.

Fact 5. Most coach-college movements are to best teams in the Elo ranking. We partition the Elo rankings for the colleges each year into quartiles and place each

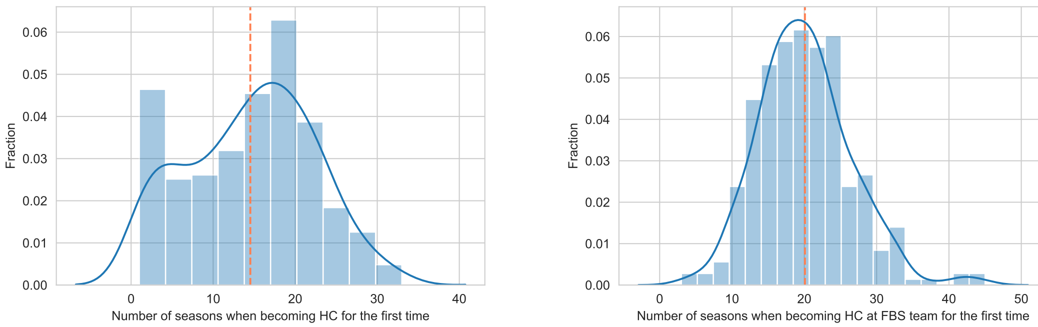

Figure 8: Number of seasons when becoming HC (Red dashed lines indicate the means)

college within a quartile. We assign the high schools and NFL outside of the quartiles. Then we inspect the fraction of moves from one quartile to the next. Table 4 is the transition probabilities between the different quartiles in the Elo ranking.

We see that the elements in the $Q4t+1$ column are the largest for all the college teams. This indicates that a coach within the NCAA are more likely to go to the Q4 level than any other level. For the coaches that move from a high school, they are most likely to go to a different high school or Q4. For the coaches that move from an NFL team, they are most likely to go to a different NFL team or Q4.

	Destination bin					
Origin bin	$\text{HS}t+1$	$\text{Q}1t+1$	$\text{Q}2t+1$	$\text{Q}3t+1$	$\text{Q}4t+1$	$\text{NFL}t+1$
HS_t	40.9	4.7	6.0	11.1	33.6	1.7
Q1_t	7.1	10.7	7.1	32.1	35.7	3.6
Q2_t	10.3	4.1	13.4	21.6	47.4	3.1
Q3_t	6.0	3.3	6.0	24.3	52.3	7.5
Q4_t	2.0	1.2	3.8	18.3	64.7	9.4
NFL_t	1.5	0.7	1.8	8.5	41.9	45.2

Table 4: Transition probability between the different quartiles of the Elo ranking

5 Results

In this section we present our main results. We first present non-parametric results for the movements in our two dimension: the coaching hierarchy and the Elo ranking. Second, we present parametric regressions for the movements in the two dimensions.

5.1 Main Result: Down in Elo ranking and Up in Coaching Hierarchy

We start with the new coach-college matches and isolate the coaching positions they have arrived at: HC, DC/OC, LB/QB and Level 4. Next, we combine the arrival position and the previous coaching position to say whether they have moved up or down in the coaching hierarchy. Next, we classify the number of moves that went up or down in the Elo ranking. A coach went to a worse team in period t if the Elo ranking of the the new team j was lower in the previous period $t-1$ than the original team i in previous period $t-1$. Last, we compute the likelihood making a particular move in the two dimensions across the positions.

In Table 5 we see that at the aggregate level, a coach moves up and down in the Elo ranking with equal probability. However, when we break up the moves to the different positions in the coaching hierarchy, we see that 64.7% of coaches moved down in the Elo ranking to become the head coach. For the LQ/QB coaches and the Level 4 coaches move up in the Elo ranking. The proportions are not different for the DC/OC positions.[11]

Table 5: Job-to-job movements across positions, frequency and percentage

			to become ...							
	Aggregate		Head coach		DC/OC		LB/QB		Lvl. 4	
coach went ...	Freq.	Pct.	Freq.	Pct.	Freq.	Pct.	Freq.	Pct.	Freq.	Pct.
down in Elo-rank	1,348	45.4	501	64.7	403	46.9	333	34.1	111	30.7
up in Elo-rank	1,318	44.3	167	21.6	378	44.0	566	57.9	207	57.2
no change	306	10.3	106	13.7	78	9.1	78	8.0	44	12.2
Sum	2,972		774		859		977		362	

In Table 6 we inspect each category in the hierarchy to see the coach positions they came from. The majority of coaches that came from a lower coaching position than the head coach moved down in the Elo ranking to become a head coach. Second, when we inspect the DC/OC positions, we see that if the move constitutes a step down in the coaching hierarchy (from HC), then the majority of moves went up in the Elo ranking. For the coaches that stayed in the same position in the coaching hierarchy, the fraction is similar to up and down. Last, for the coaches that moved up in the coaching hierarchy (from LB/QB and Lvl. 4) the majority moved down in the Elo ranking. Third, when we inspect the LB/QB we see that if coach moved down or stayed in the same position, then they moved up in the Elo ranking. While if the coach moved up in the coaching position from level 4, then they move down in the Elo ranking. Last, at Level 4, the majority of coaches that moved down in hierarchy moved up in the Elo ranking. And the coaches that stayed in the same hierarchy the majority moved up in the Elo ranking.

We draw two conclusions from this table. First, coaches move down in Elo ranking to rise in the coaching hierarchy. Second, if a coach moves down in the coaching hierarchy, they are more likely to up in the Elo ranking.

Next, we classify the moves from the previous table along moves in the two dimensions: movement in the coaching hierarchy and movement in the Elo ranking, and compute the overall fractions. The most common move, 23.1%, across all positions is up in the coaching hierarchy and down in the Elo ranking. We classify this movement as a zigzag movement.

[11] See appendix for tables for the different proportion tests.

Table 6: Movement to coaching position from past position, frequency

coach went ...	to become Head coach	from the position of ... HC	DC/OC	LB/QB	Lvl. 4
down in Elo-rank	501	164	224	79	34
up in Elo-rank	167	100	55	7	5
no change	106	57	25	2	22
coach went ...	to become DC/OC	HC	DC/OC	LB/QB	Lvl. 4
down in Elo-rank	403	30	129	201	43
up in Elo-rank	378	78	142	141	17
no change	78	15	24	36	3
coach went ...	to become LB/QB	HC	DC/OC	LB/QB	Lvl. 4
down in Elo-rank	333	13	45	169	106
up in Elo-rank	566	90	138	272	66
no change	78	-	14	50	14
coach went ...	to become Lvl. 4	HC	DC/OC	LB/QB	Lvl. 4
down in Elo-rank	111	5	3	9	94
up in Elo-rank	207	40	24	32	111
no change	44	1	1	3	39

For the head coach position, 337/774 = .44, and for the DC/OC 244/859 = .28 constitue a zigzag movements. The other zigzag movement is down in coaching hierarchy and up in Elo ranking. This move constitutes 13.5% of all moves. For the LB/QB the movement down in hierarchy and up in Elo ranking constitutes 228/977 = .23 and for the level 4 coaches 96/362 = .27. The second most common move overall is stay at the same level in the coaching hierarchy and move up in the Elo ranking.

Table 7: Number of movements in two dimensions

Movement Hierarchy	Elo	to become HC	DC/OC	LB/QB	Lvl.4	All	Perct.
Up	Up	67	158	66	N.A	291	9.8%
Down	Down	N.A.	30	58	17	105	3.5%
Up	Down	337	244	106	N.A	687	23.1%
Down	Up	N.A	78	228	96	402	13.5%
Stay	Down	164	129	169	94	556	18.7%
Stay	Up	100	142	272	111	625	21.0%
Stay	No change	57	24	50	39	170	5.7%
Up	No change	49	39	14	N.A	102	3.4%
Down	No change	NA	15	14	5	34	1.1%
	Sum	774	859	977	362	2,972	100%

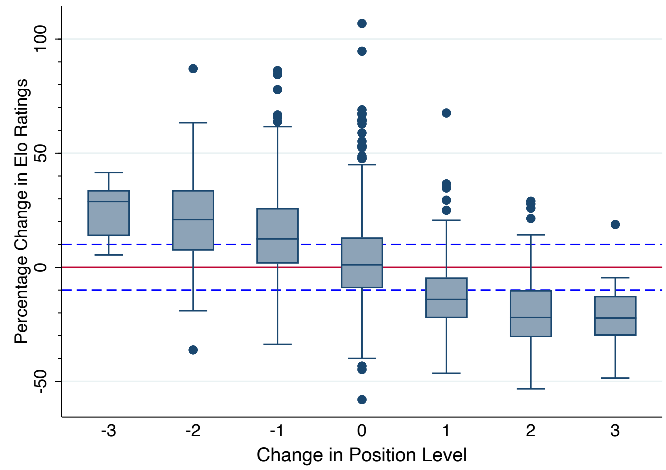

Figure 9: Relationship between the hierarchical ladder and Elo ranking ladder

5.2 Main Result: Negative Relation Between The Two Ladders

We estimate the relationship between the hierarchical ladder and Elo ranking ladder using the following regression:

$$Percentage\_Elo\_Change_{i,t} = \beta_1 \; Position\_Change_{i,t} + Controls \\ + Team\_FE + Coach\_FE + Decade\_FE + \epsilon_{i,t}$$

where i is the index for coach and t is the index for season. $Percentage\_Elo\_Change_{i,t}$ is defined as the percentage change in Elo ratings between the Elo rating of the departure team at $t-1$ and the Elo rating of the destination team at $t-1$ as the coach i changes his/her team. We assume that all movements happen in the time period after the end of the previous season and before the beginning of a new season[12]. $Position\_Change_{i,t}$ is the difference between coach i hierarchical position at $t-1$ and at t. For example, if a coach moves from the Offensive Coordinator position, which is Category 2, to the Head Coach position, which is Category 1, the difference is 1, representing that the coach moves 1 step up the hierarchical ladder. We also include two control variables: tenure duration and the new hierarchical position at the new team to control for experience and levels of responsibility.

First, we graph the relationship between the percentage change in Elo rating and position change in Figures 9 and 10. In both cases conditional on new position categories or not, we see a clear pattern that the relationship is negative, which is consistent with our findings in the previous section. Moreover, the Figures also show that the bigger the margin of jump in Elo rating is, the bigger the margin of jump in hierarchical position would be. This result continues to hold as shown in Table 8 in which we present 3 regres-

[12]Often from April to August.

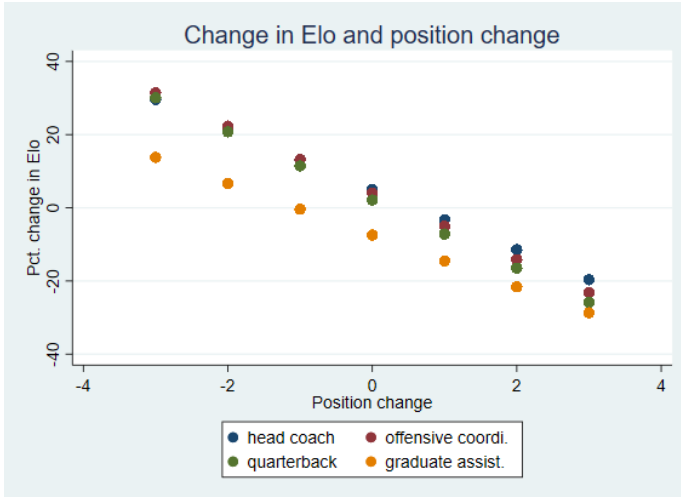

Figure 10: Relationship between the hierarchical ladder and Elo ranking ladder by new position

sion models controlling for different sets of fixed effects. In all 3 models, the coefficients for $Position\_Change_{i,t}$ are negative and statistically significant at 5%. Furthermore, the sizes of the coefficients are quite big (more than 8.8%), which means that if a coach move down 1 step on the hierarchical ladder, he/she is likely to go up the Elo ranking ladder to a team that is about 8.8% better in terms of Elo ratings[13].

6 Coach Movements and Outcomes

Are our mobility measures related to meaningful outcomes for the coach or the team? We are clear that the mobility patterns are not exogeneous but chosen by the coach and/or the team. Hence, we do not establish any causal relation between the movements and outcomes. However, if the mobility data were just noise, there should be no systematic relationship between the mobility patterns and coach and team outcomes. We therefore use our models to try to predict different outcomes for the coach and team.

6.1 Movements and Years Before First Head Coach Position in The FBS

The learning effect of team-to-team movements on the tenure duration of a coach to become a Head Coach for the first time at an FBS team is shown in Table 9. Our rationale to pick this particular point in a coach's career is that one can consider this point as the first point in a coach's career that he/she reaches a certain level of success. This can be equivalent to the first time that a person becomes the CEO of a top ranked firm (in terms of size, popularity or market share). The coefficient of UD is negative and non-significant, which is similar to UU and US (Up-Stay), shows that there is no different from going to a smaller

[13]This is a significant jump in quality of teams and win rate.

Table 8: Team-to-team Coach Movement

	(1) △ Elo Percentage	(2) △ Elo Percentage	(3) △ Elo Percentage
Position Change	-10.55***	-11.18***	-8.855***
	(0.638)	(0.785)	(0.849)
Destination Position	2.635***	1.869+	1.585+
	(0.690)	(0.987)	(0.953)
Tenure Duration	-0.224***	-0.169	-0.270
	(0.0625)	(0.197)	(0.190)
Constant	-0.768	65.38***	-1.257
	(2.331)	(11.23)	(7.758)
Team FE	No	No	Yes
Coach FE	No	Yes	Yes
Decade FE	No	Yes	Yes
Observations	1662	1662	1662
Standard Error	robust	robust	cluster
R-Squared	0.297	0.373	0.431

Only including observations when the coach changes to a new team.

$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. Standard errors in parentheses

team but up in hierarchical position to become a Head Coach to the traditional UU on both ladders and US on only one ladder.

On the other hand, moving DU has a negative effect on how long it takes to become a Head Coach at an FBS team for the first time. This result is intuitive because moving DU means that the coach has not had the chance to learn from his/her bosses at a top team before moving. Thus, it will take more time for that coach to get to a certain level of success because he/she will have to go through the learning process first. However, the size of DU coefficient is smaller than the traditional "demotion" DD and DS (Down-Stay), which suggests that the learning effect from better Elo rating teams is bigger than from moving to lower Elo rating teams.

Table 9 also shows the coefficients for variables that are proxies for a coach's first job Elo rating and the total number of team-to-team movements. As expected, the more a coach changes teams, the longer it would take him/her to reach success, and the better the starting job is, the faster he/she would reach success.

Table 9: Effects of Movements on Tenure Duration When Becoming Head Coach at FBS team for the first time

	Tenure Duration	
Numbers of Jumps on Hierarchy-Ranking:		
Up-Up	-0.803	(-1.11)
Up-Stay	-0.0549	(-0.08)
Up-Down†	-0.533	(-1.24)
Down-Up†	1.587**	(2.86)
Down-Down	2.678**	(2.76)
Down-Stay	2.100*	(2.32)
Stay-Up	0.0290	(0.08)
Stay-Stay	0.979*	(2.17)
Stay-Down	0.844	(1.78)
Pre-move Coach Elo	-0.000388	(-0.58)
Team Change Count	0.934**	(3.02)
First Job Elo	-0.00102*	(-2.16)
First Job Position	0.543	(1.14)
Pre-move Team Elo	0.00288*	(2.55)
Constant	-0.213	(-0.08)
Decade FE	Yes	
Observations	296	
Standard Error	robust	
R-Squared	0.444	

† represents zigzag movements.

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. t statistics in parentheses

Table 10: Effects of Movement Counts on Performance (Elo Gains)

	(1)		(2)		(3)		(4)		(5)		(6)	
	1 Year		2 Years		3 Years		1 Year		2 Years		3 Years	
Numbers of Jumps on Hierarchy-Ranking:												
Up-Up	-6.286	(-0.45)	32.52	(1.09)	27.36	(0.68)	-12.61	(-0.63)	17.86	(0.46)	1.088	(0.02)
Up-Stay	0.552	(0.04)	8.323	(0.42)	-14.26	(-0.49)	-3.349	(-0.26)	-7.221	(-0.35)	-31.04	(-1.01)
Up-Down†	-3.119	(-0.37)	35.16*	(2.02)	31.30	(1.18)	-14.02	(-1.40)	3.137	(0.16)	9.465	(0.31)
Down-Up†	12.36	(1.44)	25.94	(1.54)	2.948	(0.12)	10.13	(0.97)	1.560	(0.08)	-3.034	(-0.10)
Down-Down	35.95	(1.84)	58.43	(1.61)	129.7*	(2.30)	27.17	(1.12)	24.01	(0.58)	108.4	(1.56)
Down-Stay	4.505	(0.36)	33.00	(1.38)	44.80	(1.24)	1.502	(0.12)	20.06	(0.85)	27.65	(0.80)
Stay-Up	3.287	(0.40)	22.84	(1.60)	-5.790	(-0.24)	-0.768	(-0.09)	14.87	(0.95)	-19.77	(-0.74)
Stay-Stay	8.194	(1.00)	27.93	(1.88)	17.67	(0.70)	4.281	(0.50)	14.74	(0.95)	0.0607	(0.00)
Stay-Down	10.68	(1.25)	33.24*	(2.05)	55.27*	(2.23)	9.337	(0.97)	26.45	(1.61)	37.55	(1.46)
Average $\triangle$ Elo of Jumps:												
Up-Up							0.0653	(1.43)	0.0128	(0.24)	-0.195**	(-3.16)
Up-Down†							-0.00145	(-0.11)	-0.0532*	(-2.23)	-0.0302	(-0.88)
Down-Up†							0.0159	(1.15)	0.00498	(0.23)	0.0106	(0.34)
Down-Down							0.0913*	(2.05)	0.352	(1.67)	0.0634	(0.82)
Stay-Up							-0.00631	(-0.46)	-0.0124	(-0.48)	-0.0282	(-0.69)
Stay-Down							0.0194	(0.92)	0.0638	(1.74)	0.0929*	(2.00)
Interaction of Jumps and $\triangle$ Elo:												
Up-Up							-0.0412	(-0.93)	-0.0270	(-0.38)	0.144	(1.61)
Up-Down†							0.0197	(1.44)	0.0993***	(3.90)	0.0935*	(2.57)
Down-Up†							-0.00944	(-0.69)	-0.0108	(-0.44)	-0.0724*	(-2.14)
Down-Down							-0.0712	(-1.42)	-0.235	(-1.01)	0	(.)
Stay-Up							0.00613	(0.38)	-0.0342	(-1.44)	-0.0557	(-1.32)
Stay-Down							-0.00696	(-0.37)	0.00110	(0.03)	0.0155	(0.31)
Pre-move Coach Elo	0.00161	(0.32)	0.00554	(0.64)	0.0324*	(2.38)	0.000165	(0.03)	0.0303*	(2.34)	0.0847***	(4.32)
Tenure Duration	-1.131	(-0.98)	-0.587	(-0.28)	-4.794	(-1.68)	-1.341	(-1.15)	-1.377	(-0.66)	-6.169*	(-2.10)
Team Change Count	-2.282	(-0.32)	-23.41	(-1.78)	-6.128	(-0.31)	0.686	(0.09)	-12.37	(-0.89)	9.602	(0.46)
Coach FE	Yes		Yes		Yes		Yes		Yes		Yes	
Decade FE	Yes		Yes		Yes		Yes		Yes		Yes	
Observations	2229		1728		1236		2229		1728		1236	
Standard Error	cluster		cluster		cluster		cluster		cluster		cluster	

† represents zigzag movements.

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. t statistics in parentheses

6.2 Movements and Elo Gains

Elo gains are computed as the changes in Elo from one period to the next. A positive Elo gain constitutes an improvement in the performance by the team. The Elo gains take into account the difficulty of the opponent. A win against a stronger opponent will matter more than the win against a weak opponent.[14]

We show the regression results for the effects of different types of movements on Elo gains in performance of the destination team after a coach moves to a new team in Table 10. In models 1-3, we regress the Elo gains in performance in the first 1-3 years with a new team on the count numbers of different types of movements including different sets of fixed effects. In models 4-6, we add the average Elo margins of the movements and the interaction terms between the count variables and the margin variables.

While using the count variables only show no significant results for the zigzag movements, when both the margin and the interaction variables are included, the more one moves UD and the bigger the margin of jump is, the better the performance of the new team is after 2 and 3 years of working. This result is consistent with the prediction from the theoretical model that a coach that moves UD learns from his/her bosses at a better team to become a higher quality coach, and then replaces a boss at a lower rated team would perform better.

6.3 Persistence of Placement

We have demonstrated that the movements can predict the time it takes to become a head coach in the FBS for the first time. It is natural then to ask whether the time to become a head coach, the tenure, can say something about the persistent of the level. Is it such that a coach who is quick to rise to the top flairs out, or is there permanence in the placement?

We first investigate the likelihood of staying at the top level as a head coach in the next placement after the first placement as a head coach in the FBS. We use different levels to account for the top level. First, we use head coach in the FBS. Second, we use being a coach in the NFL at any of the coaching levels. Last, we use being a coach at the top two levels of the coaching hierarchy in the NFL.

In Table 11 and Figure 11, we see a negative relation between the tenure and the likelihood of becoming a head coach in the FBS in the next coach-college match. The shorter the tenure, the more likely is a coach to be a head coach in the FBS in the next match. There is a positive relation between the tenure and working in the NFL. The longer the tenure, the more likely is a coach to be working in the NFL in the next match. There is no relationship when we only focus on the top two coaching levels in the NFL.

Next, we investigate the likelihood of staying at the top level not just in the first match after becoming a head coach in the FBS, but also in the second and third match.

[14] See Section 3.2 for a description of the Elo rating system.

Table 11: Probit prediction of placement as a function of tenure

	(1) Head coach in FBS	(2) Coach in NFL	(3) HC/OC/DC in NFL
Tenure	-0.046**	0.025*	0.022
	(0.011)	(0.013)	(0.014)
Constant	1.379**	-1.78**	-2.103**
	(0.260)	(0.319)	(0.386)
Obs	299	299	299
Pseudo R2	0.051	0.017	0.014

Standard errors in parentheses. * $p < 0.05$, ** $p < 0.01$

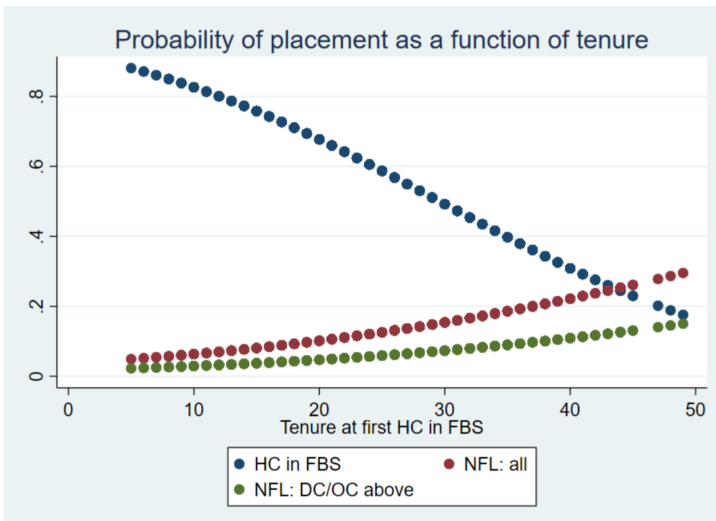

Figure 11: Predicted probabilities of different positions after the first head coach position in the FBS

Table 12 and Figure 12 show a negative relationship between the probability of being a head coach in an FBS team in the first, second or third match and the tenure of the coach at the first job as an FBS head coach. Hence. The longer the tenure before the first job as a head coach in the FBS, the less likely to have a success in the preceeding placement.

6.4 Median Matching

In this subsection we only focus on the movements within the NCAA teams because we are using the Elo-rating level in the predictions. There are 820 assistant coaches observations that went up in the hierarchy (and either up, down or no change in the elo). The other assistant coach movements constitute 1,082 observations. There are 136 head coaching observations that move down in the coaching hierarchy and other 255 coaching observation did not move down.

According to the median matching in our model, Corollary 2, the higher an assistant coach is in the Elo ranking, the more likely are they to move up in the coaching hierarchy.

Table 12: Probit prediction of placement as a function of tenure

	(1) First match	(2) Second match	(3) Third match
Tenure	-0.046*	-0.049**	-0.054**
	(0.011)	(0.013)	(0.011)
Constant	1.379**	1.333**	1.579**
	(0.260)	(0.261)	(0.270)
Obs	299	299	299
Pseudo R2	0.051	0.059	0.071

Standard errors in parentheses. * $p < 0.05$, ** $p < 0.01$

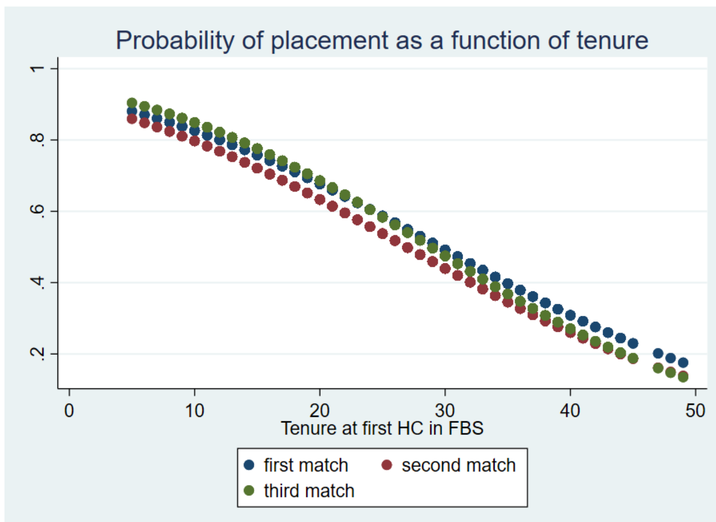

Figure 12: Predicted probabilities of being a head coach in the FBS in the first, second and third match after the first head coacing job in the FBS.

Second, the lower a head coach is in the Elo ranking, the more likely are they to move down in the coaching hierarchy. We test these prediction in in Table 13. Model 1 predicts the probability that a head coach moves in hierarchy as a function of the Elo ranking in the previous period. Model 2 predicts the probability that an assistant coach moves up in the coaching hierarchy as a function of the Elo ranking in the previous period. Figure 13 shows the corresponding predicted probabilities. The predicted probabilities match the prediction from the corollary.

Table 13: Probit predictions of movement patterns on elo ranking

	(1) Pr. moving down	(2) Pr. moving up
Past Elo ranking	-0.002**	0.0012**
	(0.0003)	(0.000)
Constant	3.41**	-2.65**
	(0.521)	(0.214)
Obs	391	1,902
Pseudo R2	0.129	0.061

Standard errors in parentheses. * $p < 0.05$, ** $p < 0.01$

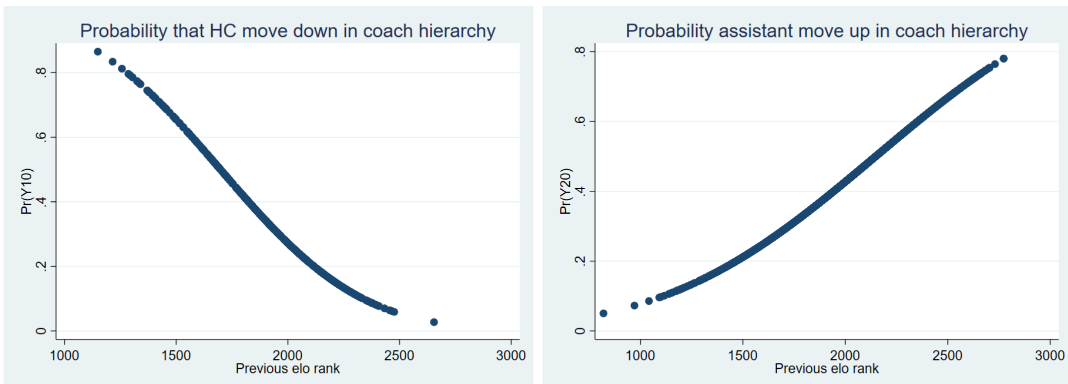

Figure 13: Left: The graph shows the predicted probability that a HC moves does in the coaching hierarchy as a function of the Elo ranking. Right: The graph shows the predicted probability that an assistant coach moves up in the coaching hierarchy as a function of the Elo ranking.

7 Conclusion

Using data on the careers for all head coaches at Divison I college football from 2000 to 2016 we construct new statistics on the tenure development to understand how coaches move and develop. Mobility arise along two position: the quality of the college and the position in the coaching hierarchy. We document a negative relationship between movements in the coaching hierarchy and in the Elo-ranking. A coach moves up in the coaching hierarchy and down in the Elo-ranking, or down in the coaching hierarchy and up in the Elo-ranking. We document that 36.6% of all moves are zig-zag moves: up in coaching hierarchy and down in Elo-ranking, or down in coaching hierarchy and up in Elo-ranking. Using data on the careers for all head coaches at Division 1 university football from 2000 to 2016 we construct new statistics on the tenure development to understand how coaches move and develop. Mobility arise along two position: the quality of the university and the position in the coaching hierarchy. We document a negative relationship between movements in the coaching hierarchy and in the Elo ranking. A coach moves up in the coaching hierarchy and down in the Elo ranking, or down in the coaching hierarchy and up in the Elo ranking. We document that 36.6% of all moves are zigzag moves: up in coaching hierarchy and down in Elo ranking, or down in coaching hierarchy and up in Elo ranking.

The movement patterns can predict how quickly a coach becomes a head coach in the FBS, and the improvements in the performance of the team as measured by the Elo gains. Last, we document a comparative statics supporting median match. A head coach is more likely to move down in the hierarchy when close to the median of the type distribution. In addition, an assistant coach is more likely to move up in the hierarchy when close to to the median of the type distribution.

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A Appendix

A.1 Proof of Results Missing From the Text

Proof of Proposition 1

Equilibrium matches satisfy PAM if for any equilibrium pairs (a, b) and $(\hat{a}, \hat{b})$ we have $a > \hat{a} \Rightarrow b > \hat{b}$ with the convention that in a pair (a, b), $a \geq b$.

Legros and Newman (2002) have established that there is PAM only if the production function satisfies the following Weak Increasing Difference Condition: for any $a > b \geq c > d$,

- either $y(a, c) - y(a, d) > y(b, c) - y(b, d)$
- or $y(a, b) - y(a, d) > y(b, c) - y(c, d)$

for then negative assortative matches $(a, d), (b, c)$ yield less surplus than positive assortative matches $(a, b), (c, d)$ or $(a, c), (b, d)$.

Consider now colleges of types $x > \hat{x}$. The two conditions for WID imply that (both sides of the inequalities are positive since $y(\cdot, \cdot)$ is increasing in each argument)

- either $x \cdot (y(a, c) - y(a, d)) > \hat{x} \cdot (y(b, c) - y(b, d))$
- or $x \cdot (y(a, b) - y(a, d)) > \hat{x} \cdot (y(b, c) - y(c, d))$

implying that matches $(x, a, d), (\hat{x}, b, c)$ are dominated (surplus wise) by matches $(x, a, b), (\hat{x}, c, d)$ or matches $(x, a, c), (\hat{x}, b, d)$, proving the result.

Proof of Corollary 1

The only thing to prove is that only individuals with type greater than $\underline{a}$ are employed, or more precisely that there is set of zero measure of individuals with types less than $\underline{a}$ who are employed. If this is not the case, there is necessarily a set of positive measure of individuals with type greater than $\underline{a}$ who are unemployed as coaches, but then a college that employs such an individual must be better off than employing an individual with type less than $\underline{a}$, a contradiction.