Venture Capital Scarcity, Start-up Quality, and Returns Dispersion

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Abstract

Prior to asking venture investors for capital, entrepreneurs privately invest in the quality of their startups. We show that a negative venture capital supply shock not only discourages the ex ante investments due to complementarity between the quality and capital, but also distorts the alignment between the ex ante quality investments and ex post financial investments. When the entrepreneurs compete for scarce capital, their ex ante investments and equity offers have irreversible (all-pay) cost components, and the result is an all-pay auction with strategic uncertainty over the quality and equity offers. The all-pay auction distortions are robust to various contracting instruments (equity offers, convertible debt, and capped equity offers) and bargaining protocols over the equity shares (entrepreneurs' offers, Nash bargaining). With the observability of the opponent's ex ante investments, the allocation of scarce capital is even less predictable due to multiple equilibria and the sequential nature of strategic uncertainty. In the context of venture capital, we predict that a negative supply shock largely worsens allocation efficiency and increases the dispersion of venture capital returns.

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Keywords: venture capital, contracts, entrepreneurship, startup quality

1 Introduction

Venture capital markets are characterized by substantial search and matching frictions arising from heterogeneities of entrepreneurs and venture capital firms. The frictions exacerbate the effects of the short term supply and demand shocks. In particular, with shortages of venture capital, the venture capitalists can finance only some projects that have been identified, but search costs make it unprofitable for the entrepreneurs to look for alternative sources of financing. With limited outside options, a negative shock to the supply of venture capital has potential to significantly affect the allocation of capital, bargaining terms, profits, and efficiency.

There is extant literature on the effects of the supply of capital on the bargaining terms (among others, Hong, Serfes, and Thiele, 2020; Ewens, Gorbenko, and Korteweg, 2022; Gornall and Strebulaev, 2023) and the incontractible value-adding activities of the entrepreneurs (Kaplan and Strömberg, 2004; Inderst and Müller, 2004). To provide a novel perspective, we investigate the effects of capital scarcity on two, previously less studied, phenomena. First, we look into how the negative supply shock affects the level and variance of incontractible private investments of the entrepreneurs made prior to bargaining. The ex ante investments include setting up a motivated entrepreneurial team of early joiners, gaining leadership experience, building startup-specific human capital, designing a coherent information system to optimize project implementation, and conducting early customer research that allows re-engineering of the product. This type of organization capital is known to be critical for the startup performance (Choi, Haltiwanger, and Kim, 2023). Second, we study how capital scarcity intensifies competition for capital (and increases returns of venture capital) within the existing matches of startups and venture investors, where the key distinction is between the marginal and inframarginal capital.

While scarcity of capital discourages the provision of complementary inputs and thus destroys synergies, this standard effect is not the only effect we observe. With a negative capital supply shock, allocating scarce capital within existing matches turns into an auction mechanism, with bids for capital having irreversible (all-pay) cost components. As a consequence, the resulting all-pay auction generates strategic uncertainty that impacts efficiency and profits due to misalignment between the ex ante quality investments and the ex post financial investments. Thus, the shock affects not only the level of capital returns but also the capital returns dispersion.

To fix ideas, consider two entrepreneurial firms and an investor. Each entrepreneur needs financing for a project. The total supply of venture capital is limited because of exogenous matching frictions, and the investor has a fixed outside option (e.g., a government bond). Prior to negotiating with the investor, the entrepreneurs privately invest in the project quality, which subsequently increases the return to capital. The projects are scalable, with constant returns to capital up to a project capacity.

In the competition for scarce capital, the entrepreneurs are endowed with two instruments: an *equity offer* and *startup quality.* Both tools are costly, and the costs are independent on the amount of received capital. First, the effort spent to improve the startup quality is sunk at the negotiation stage. Second, offering a higher equity share to attract more capital increases not only the cost of the attracted (marginal) capital but also the cost of the capital that would be pledged even for a lower offer (inframarginal capital). Given that the costs associated with both competitive instruments are irreversible, the entrepreneurs' competition for the capital is an *all-pay* auction. The all-pay auction generates an ex post highly asymmetric capital allocation in which the winning entrepreneur obtains the maximal amount of capital (i.e., exploits full capacity of her project), and the losing entrepreneur is left with the remaining capital (i.e., the capacity of her project is not fully exploited). Ex ante, both entrepreneurs face the tradeoff between higher probabilities of financing versus more favorable deal terms (c.f., Casamatta and Haritchabalet, 2014); in a unique equilibrium, they inevitably impose strategic uncertainty on each other.

Our main result is robust to various extensions. In the main setting, the entrepreneurs cannot observe each other's project quality and offer the equity to the investor (an unrestricted simultaneous competition). We allow for alternative instruments (debt or convertible debt), and restrictions on the instrument (equity competition with a cap on the equity share, and debt competition with a fixed interest rate). In the extensions, the strategic uncertainties are replicated but with different structures. In addition, when the bilateral surplus is divided by Nash bargaining, the entrepreneurs are engaged in an all-pay auction, with the only difference that the auction is exclusively through the startup quality.

We also look into the effects of having the quality observable not only to the investor, but also to the other market participants. Interestingly, when the entrepreneurs observe the opponent's quality characteristics, extra strategic uncertainty occurs in the equilibrium. First, multiple equilibria exist; an asymmetric profile now becomes one of several equilibrium profiles. Second, in the symmetric equilibrium, the entrepreneurs randomize in two steps, and the sequential randomization makes the outcome even less predictable than the single-step randomization. Precisely, the equilibrium distributions of the quality-enhancing investments are identical in both regimes. Still, the distributions of the offers involve a much larger variance when the opponent's project quality is observable.

This paper's primary contribution is that endogenous strategic uncertainty occurs endogenously in the competition over scarce venture capital, and the uncertainty distorts the levels of the startup quality and increases dispersion in returns to capital. We bring an all-pay auction perspective to entrepreneurial finance, and also contribute to the search and matching literature in which multiple deals are considered at the same time. We specifically suggest that the dynamics with multiple simultaneous deals is richer due to differences in payoffs of winners (fully funded projects) and losers (partially funded or unfunded projects) and also due to the uncertainty over the payoffs resulting from endogenous strategic uncertainty.

The paper is organized as follows: After the literature review in Section 2 and the setup in Section 3, Section 4 begins with unconstrained simultaneous competition in both project quality and equity shares. In Section 5, we solve the effects of a cap on the equity share (e.g., due to agency frictions that restrict appropriability of cash flows). Section 6 documents the effect of observability of human capital investments, i.e., sequencing of competition into two steps. To show robustness, Section 7 shows the difference to a situation where bilateral bargaining is Nash bargaining. As another robustness check, Section 8 analyzes the competition when the financial contracts are restricted to a fixed interest rate (i.e., the investor provides a loan with a fixed interest rate). For each regime, we document frictions that exist in the absence of capital scarcity and use the outcome as a benchmark for the presence of capital scarcity. Section 9 provides an overview and summarizes empirical implications, and Section 10 concludes.

2 Literature

Venture capital markets are characterized by strong deviations from perfect markets. The classic concern of the literature on venture capital markets is optimal contracting in the presence of moral hazard of entrepreneurs (Gornall and Strebulaev, 2023). The starting point of the literature is to analyze how contracts improve incentives of the entrepreneurs and information sharing (Bolton and Dewatripont, 2004; Kaplan and Strömberg, 2004; Schmidt, 2003). In this literature, the investors are typically competitive and add no value beyond providing the capital.

Due to experience with organization and management of startups, monitoring ability, networks and industry-specific knowledge, venture investors add value beyond provision of capital. Their ability to add value is heterogeneous, as documented by persistent differences in their returns (Hochberg et al., 2014). When incontractible private efforts of venture capital and entrepreneurs are exerted in the project implementation stage, a double moral hazard problem arises and requires a delicate design of incentives of both sides (c.f., Bhattacharyya and Lafontaine, 1995; Repullo and Suarez, 2004; Hellman, 2006).

Additionally, the contracts reflect that the investors cannot be easily substituted and that the investors are heterogeneous. This creates a search and matching problem where two classes of agents meet and the contracts are endogenous. In such problems, the contracts reflect not only concerns about incentives of the entrepreneurs, but also investors' distributive concerns. This is solved in a class of search-and-matching models. Inderst and Müller (2004) develop a two-sided one-to-one exogenous matching model with endogenous contracts. There is a double-side moral hazard; the activities are exerted by both sides in the project implementation stage. The activities are incontractible and depend on the allocation of equity shares. The entrepreneur and investor bargain in the ex ante stage, and can transfer surplus only through equity shares. These transfers are therefore potentially destructive to surplus.

Bargaining powers (weights in Nash bargaining) are fixed, but the implied equity shares are endogenous. The difference is that the shares are entitlements to cash flows, whereas the bargaining divides deal utilities (cash flows, private effort costs and the capital cost).

Hong, Serfes, and Thiele (2020) construct another one-to-one matching model with double moral-hazard and test predictions of the model on the VC data from the U.S. from 1991 to 2010. The key prediction is that the entry of new venture investors has a differential effect: the effect on the success rate is positive for the low-value startups but negative for the high-value startups. With more competition, the venture capital receives lower equity shares. This shifts incentives from the investor to the entrepreneur. This shift improves the balance of incentives in the low-value startups but worsens the balance in the high-value startups. Using three measures of venture capital market competition, they find support for the differential effect of a more competitive supply of venture capital

Ewens, Gorbenko, and Korteweg (2022) build a tractable dynamic search-and-matching model with endogenous contracts to estimate the impact of venture capital contract terms on startup outcomes. The qualities of investors and entrepreneurs are exogenous, match-independent and complementary for the firm value. The investors give offers to the entrepreneurs and both sides can resume search with the hope of encountering a better partner. Like in the double-sided moral hazard problems, there is a interior optimal investor's equity share; this property is set in a reduced form without specific contractual microfoundations. The model estimates the optimal investor's share at average 15%, where the model-fitted average share is at 40%.

The literature investigates also the quality of matching. Hong et al. (2020) find evidence for positive assortative matching in the U.S. data and Fu et al. (2019) provide evidence for the Chinese venture capital. Sannino (2023) introduces adverse selection into matching environment to analyze the link between fund size and positive sorting of entrepreneurs across the high and low value-added segments of the market. Ewens et al. (2022) stress that positive assortative matching doesn't hold with endogenous contracts that impact value. In addition to the link between matching process and bargaining, Silveira and Wright (2016) investigate the stochastic length of the venture cycle, investments during implementation, and fund size.

3 Setup

3.1 Start-ups

A venture investor meets two start-ups, each owned by a wealth-constrained (female) entrepreneur. Each startup $i \in \{1, 2\}$ with the amount of the financial capital $k_i \in \mathbb{R}_+$ and the start-up quality $\pi_i \in \mathbb{R}_+$ generates cash flows with the expected value $V(k_i, \pi_i)$. Up to the project capacity k_H, returns to capital are constant and increasing in the startup quality,

$$V(k_i, \pi_i) := \pi_i \cdot \min\{k_i, k_H\}.$$

The outside option for the capital is normalized at 1. If the capacity is not exceeded, the start-up earns profits $V(k_i, \pi) - k_i = (\pi - 1)k_i$, the gross return from any unit of capital is π, and the net return is $\pi - 1$. The startups are ex ante symmetric in a sense that the expected values of cash flows conditional on the identical levels of quality and capital are identical. Since all agents are risk-neutral, it is straightforward to alternatively cast the cash flows in terms of the exit success rate.

3.2 Timeline and venture investor

At date 1, each entrepreneur exerts private incontractible effort into the quality of her startup. The cost of quality, $C(\pi_i)$, is symmetric, increasing and convex, and we normalize $C(1) = 0$.[1] At date 2, each entrepreneur offers an equity share $x_i \in [0, 1]$ to the venture investor. At date 3, the (male) investor allocates $(k_1, k_2) \in \mathbb{R}^2_+$ such that $k_1 + k_2 \leqslant K$, where $K \geqslant k_H$ is the investor's amount of capital. The investor's capital is either abundant ($K \geqslant 2k_H$) or scarce ($K < 2k_H$). The case of abundant capital serves as a benchmark. Scarce capital is rationed; still, if all capital is allocated, even the less successful entrepreneur receives at least the *uncontested* amount $k_L := K - k_H \geqslant 0$, and the entrepreneurs are engaged in competition over the *capital prize* $k_H - k_L$.

The investor's gross return from the financial capital employed in the startup i is $r_i := x_i\pi_i$; his net return per unit of capital in company i (*investor's return*) is $r_i - 1$.[2] The profit-maximizing investor is sequential rational. When allocating the capital, he chooses an allocation that maximizes his total profits, $(r_1 - 1)k_1 + (r_2 - 1)k_2$. (i) If each entrepreneur offers a sufficiently attractive gross return $r_i \geqslant 1$, the capital is rationed; the investor allocates k_H to the startup i and k_L to the startup j when $r_i > r_j$; for identical gross returns, the investor flips a coin. (ii) If not, the capital is not rationed; any entrepreneur that offers an insufficient return $r_i < 1$ receives zero capital, and any entrepreneur that offers $r_i \geqslant 1$ receives k_H.

Each equity offer is *unconditional* on the level of the provided capital. This assumption eliminates differences between contracting instruments. Precisely, an entrepreneur sets a pair (π_i, x_i) in equity competition, which is equivalent to setting a pair $(\pi_i, r_i) = (\pi_i, \pi_i x_i)$ in debt competition. Similarly, convertible preferred equity (convertible bond) is equivalent. Suppose the entrepreneur sets (π, r, x) where $r - 1$ is the interest rate on bond and x is the amount of shares into which is bond converted. When $r > \pi x$, the bond is not converted and the investor receives net return $r - 1$. Otherwise, the bond is converted and the investor receives net return $\pi x - 1$. To sum up, the convertible preferred equity is here equivalent to an equity offer of $x = \max\{x, \frac{r}{\pi}\}$.

[1] With this normalization, the entrepreneur's participation constraint is always met; the entrepreneur is willing to exert positive effort for any amount of capital and any positive cash flow (equity) share.

[2] This holds for $k_i \leqslant k_H$. However, allocating excessive capital is not rationalizable as it is decreasing investor's profits $r_i \cdot \min\{k_i, k_H\} - k_i$.

3.3 Surplus

The (ex ante) surplus of the startup, i.e., the transferrable value generated by the startup i after deducing both financial and non-financial capital costs, is

$$S(\pi_i, k_i) := V(\pi_i, k_i) - k_i - C(\pi_i).$$

The surplus-maximizing level of quality conditional on a given level of financial capital, k, is increasing in k, which reflects the complementarity between the quality and capital,

$$\Pi(k) := \arg\max_{\pi} S(\pi, k) = C_{\pi}^{\prime -1}(k).$$

When the low and high amounts of capital are expected, the conditional surplus-maximizing levels of quality are $\pi_L := \Pi(k_L)$ and $\pi_H := \Pi(k_H)$. The *maximal surplus* for a given level of capital k is

$$\bar{S}(k) := S(\Pi(k), k).$$

By envelope theorem, the maximal surplus is *increasing* in k (as long as $\pi > 1$) and therefore the unconditional surplus-maximizing level of capital is at the capacity constraint, and similarly π_H is the unconditional surplus-maximizing level of quality. Additionally, we assume that the surplus is sufficiently elastic in capital,[3]

$$\bar{S}'(k)k > \bar{S}(k) - \bar{S}(k_L), \quad k \in [k_L, k_H].$$

3.4 Startup quality

The startup quality represents two phenomena. From the *production perspective*, the startup quality reflects the level of human capital, i.e., the entrepreneur's expertise, skills, and understanding of the industry relevant for the project. Human capital increases the return by improving competence in strategic planning, market analysis, financial analysis, operations, and performance assessment. The skills and expertise are *not* project-specific; they do not directly reveal fundamental information about profitability of the proposed business project, but rather increase profitability by building competence of the entrepreneur and the founder's team. We can therefore speak about production from two types of capital (human and financial) that exhibit complementarity. There are two asymmetries. Non-financial capital has an increasing marginal cost, whereas the financial capital has a constant marginal cost. In addition, the financial capital is capacity-constrained.

From the *information perspective*, the startup quality reflects the quality of the informational framework for the operation of business. The quality increases with adopting a consistent and appropriate accounting system, with collecting relevant financial information, and with conducting pilot tests, studies and early customer research. With a higher quality,

[3] The assumption writes $\bar{S}(k)(1-\epsilon) < \bar{S}(k_L)$, where ϵ is the elasticity of the surplus with respect to k. With $\bar{S}(k_L) = 0$ as in our examples that follow, the condition simplifies to $\epsilon > 1$.

the startup is able to implement the project more efficiently, where implementation includes choosing an appropriate business model, re-engineering the product, and scaling variable inputs.

We adopt a reduced-form version of the company information system. Its precision is precision (inverse of variance) of a signal of market fundamentals. Consider a market fundamental $\theta_i \in \mathbb{R}$. The fundamental is normally distributed with precision τ_0,

$$\theta_i = \mu_0 + \varepsilon_i, \quad \varepsilon_i \sim \mathcal{N}(0, \tfrac{1}{\tau_0}).$$

The startup implements an information system that generates a noisy signal $s_i \in \mathbb{R}$ of the fundamental, where the noise is normally distributed with precision τ_i.

$$s_i = \theta_i + \delta_i, \quad \delta_i \sim \mathcal{N}(0, \tfrac{1}{\tau_i}).$$

The error terms ε_i and δ_i are independent. Upon observing the signal realization, the entrepreneur forms a posterior belief regarding the fundamental,

$$\theta_i \mid s_i \sim \mathcal{N}\left(\frac{\tau_0\mu_0 + \tau_i s_i}{\tau_0 + \tau_i}, \frac{1}{\tau_0 + \tau_i}\right).$$

The entrepreneur sets a company policy $\omega_i \in \mathbb{R}$. This policy affects the startup's ex post quality; the better the policy fits the fundamental, the higher is the quality. Without loss of generality, the ex post quality depends only on the fit between the policy and the fundamental and not on the fundamental itself:

$$Q(\theta_i, \omega_i) = \overline{\pi} - (\omega_i - \theta_i)^2.$$

The quality-maximizing entrepreneur sets the policy $\omega_i = \mathbb{E}(\theta_i \mid s_i) = \frac{\tau_0\mu_0 + \tau_i s_i}{\tau_0 + \tau_i}$. For any signal realization s_i, the entrepreneur thus generates the distribution of the ex post quality with the expected value (i.e., the interim quality)

$$\mathbb{E}\left(Q\left(\theta_i, \frac{\tau_0\mu_0 + \tau_i s_i}{\tau_0 + \tau_i}\right) \mid s_i\right) = \overline{\pi} - \frac{1}{\tau_0 + \tau_i}.$$

Since the interim quality is identical for any s_i, the ex ante expected quality is

$$\pi_i = \overline{\pi} - \frac{1}{\tau_0 + \tau_i}.$$

The entrepreneur's cost of building a company information system with precision τ_i is $C_\tau(\tau_i)$, where we normalize $C_\tau(1 - \tau_0) = 0$. The corresponding cost function of the (ex ante expected) quality is

$$C(\pi_i) := C_\tau\left(\frac{1}{\overline{\pi} - \pi_i} - \tau_0\right).$$

Since the precision cost function $C_\tau(\cdot)$ is increasing and convex and also the argument $\frac{1}{\overline{\pi} - \pi_i} - \tau_0$ is increasing and convex in π_i, Notice that the quality cost function $C(\cdot)$ is also increasing and convex in the quality. To obtain $C(1) = 0$, we normalize the precision cost function, $C_\tau\left(\frac{1}{\bar{\pi}-1} - \tau_0\right) = 0$.

This reduced-form specification disregards the potential role of limited liability for largely adverse events resulting in large mismatches between the company policy and the fundamental. At these events, the ex post quality is extremely low. For simplicity, we suppose unlimited liability, i.e., the resulting losses are shared by both the investor and the entrepreneur. This assumption is innocuous if the initial precision τ_0 is sufficiently high so that the large company failures are very rare; eliminating the rare events of redistributing liability for rare events would only slightly affect the optimal entrepreneur's and investor's decisions.

3.5 Discussion of assumptions

We will discuss the key phenomena and our modeling choices.

- Early investments into the startup. We think of activities that are exclusively or optimally made prior to the large-scale operation of the business; postponing the activities is associated with a 'lost momentum', breakdown risks or any other major risks. This is an important difference to the (single- or double-side) private efforts in the project implementation stage. A special case are investments to ensure consistency of information and decisions in time, e.g., adopting an appropriate company information system. The vast literature documents that key decisions for the survival and growth of startups are made at the early stages. The role of the founders in setting the initial vision and shaping the growth and performance of their ventures is known to be critical (Kaplan, Sensoy, and Stromberg, 2009; Becker and Hvide, 2022), and also the role of human capital of the other employees (early joiners) has been found key for the startups (Choi et al., 2023).

- Increasing returns to scale. The function of cash flows can be interpreted as a production function of two inputs, financial and human capital. The two inputs are complementary as the marginal product is increasing in the level of the other input. In addition to that, the marginal product is constant in own input; thus, returns to scale are increasing. The scale is limited by a limited capacity to absorb financial capital; this imposes a sharp discontinuity in the returns and indirectly a bound on the willingness to exploit the increasing returns to scale. In Appendix C.2, we demonstrate that complementarity (and increasing returns to scale) are not key for the existence strategic uncertainty; it exists also when the two inputs are perfect substitutes (and the returns to scale are constant), i.e., when the return to financial capital is independent on the startup quality amount of the other input.

- Scalable project with constant returns. For the levels $k_i \in [k_L, k_H]$, the project is scalable and the return to capital is invariant up to the capacity constraint. The capacity limit is then serves as an extreme version of the sharply decreasing returns. In practice, the success rate may be non-linear in the amount of capital. As a consequence, the investor is motivated to be less discriminating. But this is equivalent to having an imperfectly discriminating all-pay auction in which the strategic uncertainty is also present; more on that follows below.

- Perfectly discriminating venture investor: We assume that the investor is able to observe the project quality without noise and is sequentially rational (cannot commit to an ex post suboptimal capital allocation rule). However, a small noise doesn't affect the results. With a sufficient degree of discrimination, it is well established that contests motivate the competitors to strategically randomize, and thus a high degree of strategic uncertainty remains in the equilibrium. Ewerhart (2017) proves that, if the own-bid elasticities of player's odds of winning are larger enough and the corresponding all-pay auction has a unique equilibrium, then any equilibrium of the probabilistic contest is both payoff-equivalent and revenue-equivalent to the corresponding all-pay auction. The discriminatory/screening ability of venture investors is well documented (Ueda, 2004). Howell (2020) shows that venture investors are able to strongly predict venture success, and their judgments effectively serve as certificates.

- Ex ante symmetric and independent startups. Aside from the competition for capital, there are no additional (revenue, cost, or information) links or externalities between the entrepreneurs. Consequently, cash flows (and returns) are independent on the capital level in the other project. Ex ante symmetry allows us to avoid the issue of forced abstention of the weaker contestants who cannot by deviation generate a sufficiently large prize (recall our all-pay auctions is with endogenous prizes). Also, with ex ante symmetry, the ex post distortions are then much more straightforward to describe.

- Equity contracts. While the quality is incontractible, the financial capital is contractible. Like in the standard security design literature (Azarmsa and Cong, 2020), the security payoffs are contingent only on cash flows, and like in most of the literature on venture capital (Inderst and Müller, 2004; Hong et al., 2020), contracting for capital is through equity shares that entitle to shares of cash flows (gross returns), not to shares of company profits (net returns). With both players risk-neutral, we abstract from risk-sharing features of the contracts. We abstract from participation which effectively makes the contract a debt-equity mix (Ewens et al., 2022). Our model sticks to the tradition of venture capital contracting under symmetric information (Inderst and Müeller, 2004; Hellman and Thiele, 2015; Silviera and Wright, 2016; Hong et al., 2020; Ewens et al., 2022). For the design of securities under the entrepreneur's asymmetric information, see Inderst and Vladimirov (2019).

- Auction. In contrast to literature in which the security is exogenous or the investors post offers, here the entrepreneurs post offers of cash flow shares (bids). The bargaining power rests with the entrepreneurs and the investor only passively responds to the competing offers. The main idea is that we want to see unhindered competition driven purely by actions of the competitors (bidders); the investor is passive and thus cannot exploit the fact that parallel negotiations take place. In particular, the investor cannot extract surplus by giving discriminating offers to the two entrepreneurs. In Section 7, we alternatively let the investor bargain with the entrepreneurs after the early investments were made. Given transferability of the (now interim) surplus, Nash bargaining with the fixed bargaining powers gives a fixed division of the interim surplus. Like in Inderst and Müller (2004), the cash flows shares are endogenous, but here (given the absence of the project implementation) the difference between the interim surplus and the cash flow shares involves only the capital cost.

- k-unconditional equity offer. Each equity offer is unconditional on the level of the provided capital; i.e., the offer is valid both in the case of the win (receiving k_H) and in the case of the loss (receiving k_L). The investor receives an identical net return on each unit of capital. This constraint is critical for the existence of an irreversible cost of a higher bid; an increase in the return for marginal capital must be accompanied by an increased in the return from the inframarginal amount of capital. Nevertheless, in Appendix C.1 we show that the main result is robust also the ability to post a rich menu of k-specific contracts as long as the equilibrium is in weakly undominated strategies.

- Multi-lateral matching: Compared to classic one-to-one search and matching models of venture capital markets (both in discrete and continuous time), the investor in the case of capital scarcity meets multiple entrepreneurs. The value of the outside option in this static model is exogenous; an interpretation of the venture capital scarcity is that it represents a temporary negative supply shock that must be accommodated with the existing matches.

We abstract from the project implementation, and thus disregard the effects of the allocation of control and ownership on the investor's moral hazard. Our core phenomenon of interest is competition over the capital, and therefore the exclusively role of the cash flow shares is to provide more attractive terms of trade and not to fix incentives on the investor's side. The effects on the project implementation efforts are orthogonal to our analysis (Inderst and Müller, 2004; Hong et al., 2020).

3.6 Key mechanism: All-pay auction

Our setting is a sealed-bid all-pay auction with complete (but not perfect) information. Two ex ante symmetric bidders bid over a capital prize $k_H - k_L$, where the value of the prize is endogenous to the bidders through their investment into the quality. Given the endogeneity, the bidder can increase the bid (the investor's return) either directly or indirectly. The *direct* way is to increase the equity offer without increasing the valuation of the prize. The *indirect* way is to increase the valuation of the prize without increasing the equity offer. The all-pay auction arises because both ways to increase the bid involve an irreversible cost. A higher quality involves an irreversible investment. A higher equity offer involves an irreversible cost through the inframarginal capital; a higher equity offer redistributes not only the revenue from the marginal capital $k_H - k_L$ but also the revenue from the inframarginal capital k_L.

Without irreversible costs (i.e., zero inframarginal capital and zero quality investments), the bidders are engaged in a Bertrand-like competition and their competition leaves the surplus with the investor. This is equivalent to the monopoly investor posting a price on the capital prize (McAfee and McMillan, 1987). In this auction with known common value of the object, the winner's rent disappears in the equilibrium, and the ex post payoff flattens at the winning bid. This implies that an equilibrium exists where the capital is allocated to a single bidder.

In contract, with irreversible costs, there is ex post payoff discontinuity at the winning bid irrespective of the winner's rent. The ex post payoff discontinuity doesn't disappear because the bid is irreversible. The ex ante payoff discontinuity disappears only when the winning bid is stochastic. The nature of the mixed-strategy is then to flatten the ex ante expected payoff by combining a continuous (deterministic) cost with a continuous (stochastic) benefit. This however introduces strategic uncertainty, and since the uncertainty involves a productive quality investment, it affects both efficiency and profits.

4 Equity competition

In the main setting, entrepreneurs invest into their startups, and give the equity offers to the investor without restrictions but also without having observed the opponent's investment. The entrepreneurs bid jointly through their unobservable human capital investment π and the offer x. The investor's allocation rule is characterized by comparing the investor's net returns on capital, $r - 1 = \pi x - 1$, or equivalently investor's gross returns $r = \pi x$.

4.1 Benchmark: No capital scarcity

In the absence of scarcity, the entrepreneur extracts the interim surplus through a take-or-leave-it equity offer given to the investor and, as a consequence, also extracts the ex ante surplus. Precisely, the entrepreneur sets $\pi_i = \pi_H$ and offers $x_i = \frac{1}{\pi_H}$ such that the investor's net return is zero. Given her ability to extract ex ante surplus, there is no efficiency loss.[4]

[4] With bargaining power of the investor, there is efficiency loss due to the hold-up problem which introduces a wedge between interim and ex ante surplus. This is another argument why the entrepreneurs in the main setting post offers to the investors; the benchmark is than distortionless.

4.2 Bids

The investor allocates the capital prize in an auction where the bid is represented by a gross return r, and thus the entrepreneurs compete in a dimension of gross return. (The bids can be also defined as net returns.) Let $P(r)$ denote the symmetric equilibrium cumulative distribution function of the (gross) returns; $P(r_i)$ represents the win probability (success rate) of an entrepreneur i who offers a return r_i. For a given r and a corresponding probability (success rate) $P(r)$, the entrepreneur expects the capital level $k = (1 - P(r))k_L + P(r)k_H$. Since the entrepreneur's cost of capital is now fixed, the entrepreneur's objective is to maximize the surplus by setting the quality $\Pi(k)$.[5] This characterizes the r-conditional bid as

$$(\hat{\pi}(r), \hat{x}(r)) = \left(\Pi(k_L + P(r)(k_H - k_L)), \frac{r}{\hat{\pi}(r)}\right).$$

Intuitively, when the entrepreneur plays a mixed strategy, she is not randomizing π and x independently (i.e., choosing from a joint distribution where the two bid dimensions are independent), but chooses from a joint distribution where the two are perfectly correlated. The entrepreneur effectively chooses from a set of pairs of (π, x) that can be parameterized by r. Note also that the entrepreneur's expected payoff when he pays a gross return r and expects the capital $k = k_L + P(r)(k_H - k_L)$ is

$$W(r, k) = [\Pi(k) - r]k - C(\Pi(k)) = \bar{S}(k) - (r - 1)k.$$

4.3 The equilibrium

In the equilibrium, the expected level of capital k is increasing in r, and thus we can describe equilibrium strategy not only as a distribution of investor's gross returns but also as a distribution of the expected capital levels k. Formally, let $R(k)$ be the equilibrium (gross) investor's return that corresponds to the expected capital k when the entrepreneur's strategy is $P(r)$.[6] With this notation, we can interpret the equalizer property of the equilibrium mixed-strategy in two equivalent ways, either as $W(r, k)$ constant in r, where $k = k_L + P(r)(k_H - k_L)$, or as $W(r, k) = \bar{S}(k) - [R(k) - 1]k$ constant in k, where $R(k) = P^{-1}\left(\frac{k-k_L}{k_H-k_L}\right)$.

The latter (k-based) interpretation of the equalizer property helps us to understand the shape of the equilibrium distribution. Namely, to generate the entrepreneur's indifference over the capital levels (equivalently, over the returns), the equilibrium distribution always *extracts surplus* that is generated by provision of additional capital. In other words, using W^* for the equilibrium payoff, we require that in the equilibrium, the return is

$$R(k) = \frac{\bar{S}(k) - W^*}{k} + 1.$$

[5] The problem is to maximize $(1-x)\pi[(1-P(r))k_L+P(r)k_H]-C(\pi) = (\pi-r)[(1-P(r))k_L+P(r)k_H]-C(\pi)$ subject to $\pi x = r$. Here, notice that risk-neutrality and the fact that the cash flows are linear in both quality and capital imply that only the first moment of the uncertain value of capital matters.

[6] The value is given by $k_L + P(R(k))(k_H - k_L) = k$, i.e., $R(k) = P^{-1}\left(\frac{k-k_L}{k_H-k_L}\right)$.

Proposition 1 describes the mixed-strategy equilibrium in which the entrepreneurs randomize over pairs (π, x). A bid that is expected to be successful with probability $\sigma \in [0,1]$ and thus deliver the capital $k = k_L + \sigma(k_H - k_L)$ is characterized by $\Pi(k)$ and $R(k)$. Notice that for a mixed-strategy equilibrium to exist on an interval, $P(r)$ must be increasing on the interval, and consequently also $R(k) = P^{-1}\left(\frac{k-k_L}{k_H-k_L}\right)$ must be *increasing* in k. To satisfy this property, $\bar{S}'(k)k - \bar{S}(k) + \bar{S}(k_L) > 0$. This is guaranteed when the surplus is sufficiently elastic.

Proposition 1 (Main setting). *In unconstrained venture capital competition with unobserved opponent's quality, the quality of each startup follows a distribution $F(\pi)$ with support $\pi \in [\pi_L, \pi_H]$, where*

$$F(\pi) = \frac{\Pi^{-1}(\pi) - k_L}{k_H - k_L} = \frac{C'(\pi) - k_L}{k_H - k_L}.$$

The ex ante expected payoff of each entrepreneur is $W^* = \bar{S}(k_L)$.

To interpret, each return gives a *success rate* which is the percentile in the distribution of returns. The success rate also gives the expected amount of venture capital. In our production function of two complementary (multiplying) inputs, the marginal return to quality is given by the expected amount of venture capital. In the unconstrained optimal allocation, the marginal cost equals the marginal return, which implies that the expected amount of capital equals the marginal cost, $k = C'(\pi)$, and therefore the shape of the success rate (and consequently the shape of the distribution of qualities) directly follows the shape of the marginal cost of qualities.

4.4 Equilibrium properties

Entrepreneurs' offers. Each entrepreneur's equilibrium strategy can be characterized as a collection of functions of the expected level of capital, $(R(k), \Pi(k), X(k))$. $\Pi(k)$ is increasing, and by elasticity of surplus, also $R(k)$ is increasing. The latter means that when the entrepreneur wants to obtain more capital, she must also provide a higher return to the investor in the equilibrium. Whether more capital means a higher or lower equity share is uncertain; we observe that it primarily depends on the ratio between costs of private investments over cash flows:

$$X(k) = \frac{R(k)}{\Pi(k)} = 1 - \frac{C(\Pi(k)) + W^*}{\Pi(k)k}$$

Investor's returns. By inverting $r = R(k)$, we have $k = R^{-1}(r)$, and consequently the gross returns offered to the investors are distributed by

$$P(r) = P(R(k)) = \frac{k - k_L}{k_H - k_L} = \frac{R^{-1}(r) - k_L}{k_H - k_L}.$$

While any deviation from a uniform distribution of qualities is driven by non-linearity in $\Pi(k)$, any deviation from a uniform distribution of returns is driven by non-linearity in $R(k)$.

An interesting property of the all-pay auction is that the investor's net return from the capital is (almost always) positive even for the capital provided to the weaker (lower-quality) entrepreneur. The investor earns *zero* net return from the minimal capital provided to the entrepreneur only if the entrepreneur expects the minimal capital (and maximizes the surplus given the minimal capital). But, as the entrepreneurs randomize their bids, the investor gains a positive net return whenever the gross return from the relatively lower amount of capital is above the minimal gross return, $\min\{\pi_1, \pi_2\} > \pi_L$, which happens almost always.

Investor's cash flows and profits. The investor's cash flow from the startup can be expressed as a function of k, $R(k)k$. Since k is uniformly distributed with density $\frac{1}{k_H - k_L}$, the expected investor's cash flow from a startup is

$$\int_{k_L}^{k_H} \frac{R(k)k}{k_H - k_L} dk = \frac{1}{k_H - k_L} \int_{k_L}^{k_H} \Pi(k)k - C(\Pi(k))dk - W.$$

The investor's expected profit only subtracts the expected capital cost $\frac{K}{2}$. More precisely, the investor's profits are

$$\frac{1}{k_H - k_L} \int_{k_L}^{k_H} \bar{S}(k)\ dk - \bar{S}(k_L).$$

Startup cash flows, profits and surplus. To calculate the expected cash flows in a startup, see that the density of quality is $F'(\pi) = \frac{C''(\pi)}{k_H - k_L}$, and the conditional probability of winning the extra capital $k_H - k_L$ is $F(\pi) = \frac{C'(\pi) - k_L}{k_H - k_L}$. The expected cash flows are

$$k_L \int_{\pi_L}^{\pi_H} \frac{\pi C''(\pi)}{k_H - k_L} d\pi + (k_H - k_L) \int_{\pi_L}^{\pi_H} \frac{C'(\pi) - k_L}{k_H - k_L} \frac{\pi C''(\pi)}{k_H - k_L} d\pi = \int_{\pi_L}^{\pi_H} \frac{\pi C'(\pi) C''(\pi)}{k_H - k_L} d\pi.$$

Alternatively,

$$\frac{1}{k_H - k_L} \int_{k_L}^{k_H} \Pi(k)k\ dk.$$

The expected profits only subtract the expected capital cost $\frac{K}{2}$. The expected surplus is

$$\frac{1}{k_H - k_L} \int_{k_L}^{k_H} \bar{S}(k)\ dk.$$

4.5 Comparative statics

Scarcity. When capital is more scarce (i.e., k_L decreases), the strategic uncertainty increases. First, $\pi_L = \Pi(k_L)$ decreases, and the support of the quality (bids) expands. (Also the support of the investor's gross returns expands; while $R(k_L)$ is constant, $R(k_H)$ increases.) The probability mass shrinks without any effect on its shape. This clearly follows from the fact that k_L enters only denominator in $F(\pi_i) = 1 - \frac{k_H - C'(\pi_i)}{k_H - k_L}$. Consequently, the startup quality distributions are first-order stochastically ordered in k_L. With more scarce capital, the entrepreneurs are worse off. The effect on the investor is ambiguous, but at least for the low levels of scarcity, the investor is better off.[7]

4.6 Asymmetry

An asymmetric pure-strategy equilibrium doesn't exist. The argument is as follows: Suppose the entrepreneurs expect $(k_1, k_2) = (k_L, k_H)$. The loser sets $r_1 = 1$, where $(\pi_1, x_1) = (\pi_L, \frac{1}{\pi_L})$. Anticipating no competition, the winner also sets $r_2 = 1$, where $(\pi_2, x_2) = (\pi_H, \frac{1}{\pi_H})$ (or a slightly higher equity share). But then $U_1 = \bar{S}(k_L) < \bar{S}(k_H) = U_2$. This extra value motivates the loser to deviate to increase r_1 by a small increase in π_1 and beat the winner. A small increase in π_1 has a negligible effect on the private cost but step-wise increases cash flows which implies a step-wise increase in the entrepreneur's payoff.

5 Equity share cap

Suppose that offering the investor a high equity (cash flow) share is not credible; consequently, a cap on the investor's equity share exists, $x \leqslant X$. The cap on the appropriability of the cash flows by the investor reflects phenomena that are outside to our model. For instance, if the entrepreneur's share is too low, the moral hazard at the implementation stage becomes prohibitively severe, and setting the equity share at a certain moderate level is a constrained second-best contract. The moral hazard friction may also be in the form of diversion of cash flows to the private use of the manager; the incentive to divert may be sharply increasing when the manager's share is too low.

We will only study sufficiently low equity caps. A sufficiently low equity cap is *binding always*, i.e., in the equilibrium binds the entrepreneur for any return r that the entrepreneur decides to offer to the investor. With the equity share cap binding always, the entrepreneurs only compete through quality. This setting thus takes to the extreme an idea that in certain environments, price competition is much less relevant for the allocation of capital than competition in the entrepreneurial quality. Formally, for any offered $r \geqslant 1$ and for the symmetric equilibrium distribution $P^x(r)$, the investor receives the (expected) amount of capital $k \in [k_L, k_H]$, where $k = k_L + P_r^x(r)(k_H - k_L)$.[8] The allocation of capital is reflected by a function $K^x(r)$, and its inverse is $R^x(k)$. For any k that the entrepreneur expects, we require that the cap is binding in the equilibrium. Formally, the unconstrained optimum of the entrepreneur, $(\pi, x) = (\Pi(k), X^x(k))$ where $X^x(k) = R^x(k)/\Pi(k)$, is not available. That is, for

[7] When $k_L \to k_H$, the investor's profits disappear. For $k_L < k_H$, the effect of k_L on the investor's profits is

$$\frac{1}{(k_H - k_L)^2} \int_{k_L}^{k_H} \bar{S}(k)\ dk - \frac{1}{k_H - k_L} \bar{S}(k_L) - \bar{S}'(k_L).$$

[8] To distinguish the existence of the equity share cap, we will be using the upper index x.

any $k \in [k_L, k_H]$, the equity cap is sufficiently low:

$$X < \min_k X^x(k) = \min_k \frac{R^x(k)}{\Pi(k)}.$$

5.1 Benchmark: No capital scarcity

We will obtain two lessons from this benchmark. First, the equity share cap motivates the entrepreneurs to increase their quality investments, i.e., the equity cap *increases profitability* of the project. This lesson draws from the (capital-unconstrained) benchmarks with and without the equity cap. Second, imposing the equity share cap manifests differently in capital-constrained and capital-unconstrained allocations; while the effect on capital-unconstrained allocations (in this Section 5.1) is only to increase profitability, the effect on capital-constrained allocations (in Section 5.2) is more involved, albeit also featuring a shift towards higher levels of quality.

Without scarcity, the entrepreneur expects k_H with probability one. The entrepreneur receives $S(\pi, k_H) - k_H(\pi x - 1)$, and sets π to optimize her objective subject to the equity constraint, $x \leqslant X$, and investor's participation constraint, $\pi k_H \geqslant 1$. The equity constraint is relevant when a pair $(\pi, x) = (\pi_H, \frac{1}{\pi_H})$ is not available, i.e., when $X < \frac{1}{\pi_H}$.

Consider the entrepreneur's optimal's private investment conditional on a fixed equity share, $x = X < \frac{1}{\pi_H}$. There are two effects that are at play. First, due to lower appropriability of marginal returns, the entrepreneur prefers to make lower effort. Second, due to a constrained (less effective) transfer of profits to the investor, the entrepreneur needs to increase the returns to meet the investor's participation constraint. The second effect is decisive, and the optimal quality is a corner solution that primarily tries to deliver the minimal return to the investor.

To describe the first effect, the unconstrained optimal private investment is $C'^{-1}((1 - X)k_H) < C'^{-1}(k_H) = \pi_H$. This reflects that with a fixed equity share x, the entrepreneur that aims to compensate the investor is no longer a surplus-maximizer. The fixed equity share works like a tax on the early investments and the *tax distorts its amount* and consequently involves an efficiency loss (as well as lower profits). Put in the words of production, the equity share is a tax on financial capital gains that decreases demand for the complementary input.

However, for a fixed equity share $x = X < \frac{1}{\pi_H}$, the investor's participation condition is $\pi \geqslant \frac{1}{X} > \pi_H$. The investor's participation constraint cannot be disregarded now because capping the equity share is now *distorting efficiency of a transfer* to the investor. This second effect pushes the quality in the opposite director; by losing inability to efficiently compensate the investor through a sufficiently large equity share, the entrepreneur is left with the only remaining way, that is, to increase the cash flows that are shared, i.e., to increase the early quality investments.

Overall, for a fixed equity share, it is constrained optimal to set $\pi^x = \frac{1}{X} > \pi_H$. In other words, *imposing the equity share cap is increasing profitability in the project.* The increase has zero effect on the investor as she leaves with her outside option. This increase has a negative effect on the entrepreneur; while cash flows of the entrepreneur's are larger (due to both a larger share and large size of total cash flows), this positive effect is dominated by an increase in the private cost of quality.[9] The benchmark for the analysis of additional inefficiencies generated by capital competition is therefore different from the previous benchmark; it contains extra private investments stemming from the difference between a distortive and non-distortive way to provide a transfer to the investor.

5.2 The equilibrium

Anticipating that the equity share cap binds always, we proceed (like in the main problem) by reducing the problem to optimization along as single dimension, i.e., along the dimension of private investments π, keeping $x = X$ fixed. Proposition 2 derives $F^x(\pi)$ by applying the equalizer condition.

Proposition 2 (Equity share cap)**.** *With a binding cap on the investors' equity share, $x \leqslant X$, the quality of each startup follows a distribution $F^x(\pi)$ with support $\pi \in [\pi^x_L, \pi^x_H]$, where*

$$F^x(\pi) = \frac{S(\frac{1}{X}, k_L) - S(\pi, k_L) + (\pi X - 1)k_L}{S(\pi, k_H) - S(\pi, k_L) - (\pi X - 1)(k_H - k_L)}.$$

The ex ante expected payoff of each entrepreneur is $W^x = S(\frac{1}{X}, k_L)$.

The equity share cap forces the entrepreneurs to invest *more* in the quality. The minimal bid clearly increases, $\pi^x_L = \frac{1}{X} > \pi_L$. The proof also shows that also the maximal bid increases, $\pi^x_H > \pi_H$. Importantly, the entrepreneur earns a *lower* value than in the unconstrained case, $W < \bar{S}(k_L)$. Formally, since π_L is surplus-maximizing for k_L and the investors' net return is zero for k_L both in the absence and in the presence of the equity cap, we have $W^x = S(\frac{1}{X}, k_L) < \bar{S}(k_L)$. For instance, in the quadratic case, $W = \frac{4(1-X)X-1}{2X^2} < 0 = \bar{S}(k_L)$.

5.3 Comparative statics

Scarcity. Scarcity (a lower k_L) doesn't affect the minimal bid, but increases the maximal bid. Like in the unconstrained equity competition, the support expands.[10]

Equity cap. The more stringent equity cap, the larger are both minimal and maximal bids. This is an identical effect like we observe in the comparison of the capital-abundant benchmarks, i.e., benchmarks of the constrained and unconstrained equity competitions.

[9] Whether this increase in profitability is increasing social efficiency or not depends on whether the entrepreneur's outside option of private investments is correctly priced; if so, the equity share cap is harmful. But if the entrepreneur's outside option is overpriced like in the case when the outside option is socially harmful, then shifting the entrepreneur's effort towards this company is increasing social efficiency.

[10] The negative effect of k_L on the maximal bid is obtained from the effect of a higher k_L on the Eq. (1) in the proof of Proposition 2. A higher k_L increases W^x, which decreases the LHS. Since the LHS is decreasing in π, it means that the maximal bid decreases.

6 Observable quality

When the project quality is observable at date 2 (i.e., the entrepreneurs observe qualifications, experience, and history of the entrepreneurs, star team members in their teams, and when media coverage of early attempts is wide), the human capital investments and financial contracting are separated into two different steps. The sequence of the two stages is a sequence of two all-pay auctions. The sequence begins with entrepreneurs making upfront human capital investments. Their human capital levels are then observed in the market, and based on this knowledge, the entrepreneurs in the next step bid for capital. The second-stage all-pay auction is a (possibly asymmetric) auction for the capital prize, whereas the first-stage all-pay auction is a (symmetric) auction for the winner's rents in the second-stage all-pay auction.

The benchmark is like in the main setting; given abundant capital, each entrepreneur at date 2 can disregard the opponent's demand as well as the opponent's quality, and optimally offers $x_i = \frac{1}{\pi_i}$ such that $r_i = 1$. Anticipating $x_i = \frac{1}{\pi_i}$ and $k_i = k_H$, it is optimal for the entrepreneur to set $\pi_i = \pi_H$ at date 1. There is no efficiency loss.

6.1 Bids

By proceeding backwards, we will first solve the all-pay auctions (subgames) at date 2. The firm with a more experienced entrepreneur (a relatively higher β and consequently a relatively higher cash flow π) is called a *stronger* firm, and the firm with a less experienced entrepreneur is a *weaker* firm. Notice that human capital costs are sunk at this date.

We observe an all-pay auction where both entrepreneurs expect a guaranteed capital level k_L if they offer at least zero net return, and bid for additional capital $k_H - k_L$ by offering the investor positive net returns. The requirement of the net return to be non-negative, $r_i - 1 = x_i\pi_i - 1 \geqslant 0$, implies that the minimal bid to obtain the guaranteed capital level is

$$\underline{x}_i := \frac{1}{\pi_i}.$$

In a word, this all-pay auction is characterized by a minimal bid defined by the investor's participation condition. In addition, the entrepreneur's expected payoff at her minimal bid, $\underline{x}_i$, defines also her maximal rationalizable bid, $\overline{x}_i$. The link stems from the fact that the continuation payoff $(1 - \underline{x})k_L$ is guaranteed, which protects the entrepreneur from bidding excessively for k_H. Formally, in a mixed-strategy equilibrium, each entrepreneur i expects the guaranteed continuation payoff when offering the zero net return to the investor, and is thus willing to bid up to $\overline{x}_i$ which satisfies

$$(1 - \underline{x}_i)k_L = (1 - \overline{x}_i)k_H.$$

In other words, each entrepreneur i is willing to bid on $x \in [\underline{x}_i, \overline{x}_i]$, where

$$\overline{x}_i := \frac{k_H - k_L}{k_H} + \frac{k_L}{\pi_i k_H} = \left\{1 + (\pi_i - 1)\frac{k_H - k_L}{k_H}\right\} \underline{x}_i > \underline{x}_i.$$

To capture the possible asymmetry in expertise, let i be the (weakly) stronger entrepreneur, $\pi_i \geqslant \pi_j$, and j be the (weakly) weaker competitor. When submitting a minimal bid, both entrepreneurs offer an identical minimal net return, $\underline{x}_i\pi_i - 1 = \underline{x}_j\pi_j - 1 = 0$. In other words, both offer zero to the investor. But the stronger entrepreneur is willing to offer a higher maximal net return,

$$\overline{x}_i\pi_i - 1 = \pi_i\frac{k_H - k_L}{k_H} + \frac{k_L}{k_H} - 1 > \pi_j\frac{k_H - k_L}{k_H} + \frac{k_L}{k_H} - 1 = \overline{x}_j\pi_j - 1.$$

In the mixed-strategy equilibrium of the all-pay auction, however, it is well known that the players offer net returns from the largest *common* interval of net returns. This interval is defined by the feasible net returns of the weaker player. In other words, the stronger player's maximal bid is at most $\overline{x}_j\pi_j - 1$, which means that her top bid is lower than the maximal bid she is willing to submit, $\overline{x}_j\frac{\pi_j}{\pi_i} < \overline{x}_i$.

In an asymmetric all-pay auction of two competitors, we know that the stronger player typically bids more aggressively, while the weaker player abstains with a positive probability and bids less aggressively (Baye, Kovenock, and Vries, 1996; Che and Gale, 2003; Siegel, 2009). This is also true in our asymmetric auction with the minimal bid requirements (and with the maximal bids defined by the guaranteed payoffs at the minimal bids). Proposition 3 summarizes.

Proposition 3 (Equity competition with observed qualities)**.** *Let i be a (weakly) stronger entrepreneur, $\pi_i \geqslant \pi_j$, and $j \neq i$ be her competitor; let $\rho := \frac{\pi_j}{\pi_i} \in (0,1]$ be the handicap of the weaker entrepreneur. In the competition with observed qualities, the equity offers of the stronger entrepreneur follow a distribution*

$$G_i(x) = \frac{k_L}{k_H - k_L} \cdot \frac{x - \underline{x}_i}{\rho - x}$$

on an interval $x_i \in [\underline{x}_i, \rho\overline{x}_j]$. The offers of the weaker entrepreneur follow a distribution

$$G_j(x) = \frac{1-\rho}{1-\rho x} + \frac{k_L}{k_H - k_L} \cdot \frac{\rho x - \underline{x}_i}{1 - \rho x}$$

on an interval, $x_j \in [\underline{x}_j, \overline{x}_j]$. The continuation payoff of the weaker entrepreneur is

$$V_j(\pi_i, \pi_j) := (1 - \underline{x}_j)\pi_j k_L = (\pi_j - 1)k_L.$$

The stronger entrepreneur's continuation payoff is

$$V_i(\pi_i, \pi_j) := (1 - \underline{x}_i)\,\pi_i k_L + (1-\rho)\,\pi_i(k_H - k_L) = (\pi_i - 1)\,k_L + (\pi_i - \pi_j)\,(k_H - k_L).$$

While the weaker entrepreneur expects to receive $(\pi_j - 1)k_L$, the stronger entrepreneur in addition to her guaranteed continuation payoff $(\pi_i - 1)k_L$ receives *extra profits* $(\pi_i - \pi_j)(k_H - k_L)$. This extra payoff can be seen as an (endogenous) *prize* which an entrepreneur wins in an all-pay auction with irreversible human capital investments if her investment is relatively larger. The prize is not exogenous to the bidders; it is given by the fixed capital prize $k_H - k_L$ multiplied by the difference is gross returns $\pi_i - \pi_j$.[11]

Notice that for $k_L \to 0$, the mixed-strategy equilibrium converges to a *pure-strategy* equilibrium, where the weaker entrepreneur threatens with $\overline{x}_j = 1$. The stronger entrepreneur wins the prize, and the weaker entrepreneur keeps his threat active. The key difference is that with zero inframarginal capital, the threat is *costless*; with $k_L > 0$, this asymmetric equilibrium breaks down.

6.2 Startup quality

We now proceed to solving the all-pay auction at date 1. In this auction, the winning entrepreneur i receives extra profits $(\pi_i - \pi_j)(k_H - k_L)$ on top of her 'guaranteed' payoff, while the losing entrepreneur j doesn't receive the prize. We may interpret this auction as the auction for the benefits associated with the *relative strength* in the subsequent auction for capital. In addition to the prize, however, bids have also productive uses; first, each entrepreneur receives a guaranteed payoff $(\pi - 1)k_L$ at an increasing and convex cost $C(\pi)$ and second, a higher bid increases the value of the prize.

Next, derive the optimal levels of quality if an entrepreneur expects to be a winner (respectively, a loser) with probability one. Thereby, we will identify the extreme (minimal and maximal) values of cash flows, to be denoted π_L and π_H. Specifically for $\pi_j \leqslant \pi_i$, observe that the weaker entrepreneur prefers (conditionally on being a loser at π_j)

$$\pi_j = \arg\max V_j(\pi_i, \pi_j) - C(\pi_j) = \arg\max S(\pi_j, k_L) = \pi_L.$$

The stronger entrepreneur prefers (conditionally on being a winner at π_i)

$$\pi_i = \arg\max V_i(\pi_i, \pi_j) - C(\pi_i) = \arg\max S(\pi_i, k_L) - (\pi_j - 1)(k_H - k_L) = \pi_H.$$

In contrast to the auction for capital, this all-pay auction is symmetric, and we therefore identify a symmetric equilibrium, even if asymmetric equilibria may exist (more on that below).

[11] In addition, observe that $V_i(\pi_i, \pi_j) = V_j(\pi_i, \pi_j) + \pi_i k_H$; the difference between the winner's continuation payoff and the loser's continuation payoff is *exceeds* the prize (inframarginal capital exists, $k_H > k_H - k_L$); this difference consists of a *prize* plus a difference in the *guaranteed payoffs*. This difference is important if the relative payoffs, rather than absolute payoffs, matter at date $t = 1$. That is, if the entrepreneurs in the human capital investment stage care about relatively profits instead of absolute profits. As the relative prize is larger than the absolute prize, we would observe even more intense competition at date $t = 1$. (Notice that we keep the structure of APA in the bidding stage unchanged; the entrepreneurs would still care about absolute profits at date $t = 2$; if they cared about relative profits also at date $t = 2$, the competition would be more intensive at this later stage as well.)

Proposition 4 (Sequential all-pay auctions)**.** *In a symmetric equilibrium, the quality of each startup follows a distribution $F^\pi(\pi)$ with the support of $\pi \in [\pi_L, \pi_H]$, where*

$$F^\pi(\pi) = F(\pi).$$

The ex ante expected payoff of each entrepreneur is $W^\pi = W^*$.

In a symmetric equilibrium, we have replicated the equilibrium distribution of qualities from unrestrained equity competition. However, while the equity share conditional on a quality was deterministic, here the equity shares conditional on a quality are randomized.

6.3 Ex post inefficiency

While the distribution of the quality is identical for both observable and unobservable qualities, the allocation of capital is ex post efficient only in the main setting. In the main setting, only (gross) return $r = \pi x$ matters for the allocation, and π is positively correlated with r in the equilibrium. Therefore, a relatively higher return is equivalent to a relatively higher quality. In contrast, with observable quality, the entrepreneurs submit offers after making their (observable) quality investments. This allows them to compete for the capital prize even if their initial quality is lower; whenever the weaker entrepreneur wins the prize, the capital allocation is ex post inefficient.

6.4 Asymmetry

In contrast to the main setting, an asymmetric equilibrium exists always. Therefore, while the strategic uncertainty persists in a mixed equilibrium, the observability now opens the possibility that the entrepreneurs coordinate on an asymmetric equilibrium. Mismatches are therefore more an issue in rapidly-growing environments where the bargaining with investors precedes spreading of the information about the early investments; when startups earn reputation or when bargaining is delayed, mismatches are less likely.

An asymmetric pure-strategy equilibrium exists if each player (i) is not willing to change her status (winner or loser) and (ii) is not willing to change her investment, given her status. In other words, neither player is willing to deviate, where a deviation may or may not change the players' status. The latter conditions are clear; the winner wants to be as close as possible to π_H, and the loser wants to be as close as possible to π_L. This implies that the only asymmetric equilibrium can be (π_L, π_H); for any other asymmetric profile $\pi_1 < \pi_2$, at least one of the players can profitably deviate (π_1 to π_L and/or π_2 to π_H) without changing her status.

The former conditions are more complex. Take the pair (π_L, π_H). The winner's non-deviation condition is that a switch to π_L and losing is not profitable. Rearranging, the extra total returns from *maximal* capital must outweigh the extra cost,

$$(\pi_H - \pi_L)k_H \geqslant C(\pi_H) - C(\pi_L).$$

The loser's non-deviation condition is that a switch to any $\pi_D \geqslant \pi_H$ and winning is not profitable. Such a switch gives the loser the payoff $(\pi_H - 1)k_L + (\pi_D - \pi_H)k_H - C(\pi)$; this is maximized at $\pi_D = \pi_H$. Therefore, the loser prefers to switch at $\pi_D = \pi_W$ (or slightly above if tie is resolved unfavorably). Rearranging, the condition states that extra total returns from *minimal* capital must not outweigh the extra cost,

$$(\pi_H - \pi_L)k_L \leqslant C(\pi_H) - C(\pi_L).$$

We will now see that both conditions are satisfied. Let $C'(\pi)$ be the (increasing) marginal cost of the early investment. Using $C'(\pi_L) = k_L$ and $C'(\pi_H) = k_H$, the two conditions write as

$$\int_{\pi_L}^{\pi_H} k_L - C'(\pi)d\pi < 0 < \int_{\pi_L}^{\pi_H} k_H - C'(\pi)d\pi.$$

Since $k_L < C'(\pi) < k_L$ whenever $\pi_L < \pi < \pi_H$, the two conditions hold.

7 Nash bargaining

In this section, we suppose that the equity shares are set at date 2 by bilateral Nash bargaining between the entrepreneur and the investor. Precisely, we suppose that the investor at date 2 observes the private investments, allocates the capital to two parallel bargaining situations, and subsequently is engaged in bargaining over the profits generated by the allocated capital.[12] In the event of disagreement, the capital allocated to the startup is reallocated to an external use (e.g., government bond). In other words, reallocation of the capital to the other entrepreneur that has unused capacity is impossible.

Each bilateral bargaining is generalized Nash bargaining. The investor has bargaining power β in both bargaining situations; none of the entrepreneurs is a relatively weaker bargaining partner for the investor (and thus a more attractive partner). Given that the utility is transferrable in money, Nash bargaining solution is a division $(\beta, 1-\beta)$ of the bilateral interim surplus $(\pi_i - 1)k_i$ across the bargaining partners; in our setting, the surplus captures profits. The investor's net return from investment into startup i is $\beta(\pi_i - 1)$, and his gross return is $\beta\pi_i + 1 - \beta$. This corresponds to an endogenous equity share

$$x_i = \beta + \frac{1-\beta}{\pi_i} \in (\beta, 1].$$

7.1 Benchmark: No capital scarcity

In the main setting, the entrepreneur that expects capital k_H invests π_H. With Nash bargaining over the interim surpluses, two forces distort the ex-ante-surplus-maximizing investment.

[12] Inderst and Müller (2004) consider bargaining over the ex ante surplus S. This means that the private investments are contractible and not made prior to contracting; our interest is in early incontractible investments.

The first is cost-sharing due to the difference between the ex ante and interim surpluses. The second is the entrepreneur's appropriability of the interim surplus (profits). Precisely, the entrepreneur's ex ante objective is $(1-\beta)$-share of the interim surplus $S(\pi_i, k_H) + C(\pi_i)$ minus her full private costs, which is $(1-\beta)S(\pi_i, k_H) - \beta C(\pi_i)$. The optimal private investment is $\pi^\beta := \Pi((1-\beta)k_H)$. Only for zero investor's bargaining share, the level is not distorted, $\pi^\beta = \pi_H$.

7.2 The equilibrium

At date 2, prior to bilateral bargaining, the investor knows that bargaining settles at net her returns to capital $(\beta(\pi_1 - 1), \beta(\pi_2 - 1))$, and consequently fully funds the company with a higher quality and leaves the excess capital for the less attractive company.

Proposition 5 (Nash bargaining)**.** *In venture capital competition where the startup profits are divided by bilateral bargaining with the investor's bargaining power β, the quality of each startup follows a distribution $F^\beta(\pi)$ with support $\pi \in [\pi_L^\beta, \pi_H^\beta]$, where*

$$F^\beta(\pi) = \frac{(1-\beta)(\pi - \pi_L^\beta)k_L + C(\pi) - C(\pi_L^\beta)}{(1-\beta)(\pi - 1)(k_H - k_L)}.$$

The ex ante expected payoff of each entrepreneur is $W^\beta = (1-\beta)S(\pi_L^\beta, k_L) - \beta C(\pi_L^\beta)$.

Again, the equilibrium features strategic uncertainty. The minimal quality is $\pi_L^\beta = \Pi((1-\beta)k_L)$. The maximal bid is implicitly characterized by $W^\beta(\pi_H^\beta) = (1-\beta)S(\pi_H^\beta, k_H) - \beta C(\pi_H^\beta) = W$. The maximal quality exceeds the quality when the maximal capital is expected with certainty, $\pi_H^\beta > \pi^\beta$.[13]

7.3 Comparative statics

Scarcity. When the capital is less scarce, the support of the bids shrinks from both sides. The minimal quality increases, and by envelope theorem, also the entrepreneur's payoff increases. This implies that the maximal quality deviates less from the maximizer at π^β (it decreases). At the limit, the quality converges to the benchmark quality, π^β, where the strategic uncertainty distortion disappears and the only distortion is related to lower appropriability of the interim surplus.

Bargaining powers. With a higher investor's bargaining power, the appropriability distortion is pronounced. It is manifested in the lower minimal quality π_L^β and in the lower quality in the benchmark π^β.

[13] Let $W^\beta(\pi, k) := (1-\beta)S(\pi, k) - \beta C(\pi)$. We now π^β is a unique maximizer of $W^\beta(\pi, k_H)$ and π_L^β is a unique maximizer of $W^\beta(\pi, k_L)$. By envelope theorem, $W^\beta(\pi^\beta, k_H) > W^\beta(\pi_L^\beta, k_L)$. Therefore, an entrepreneur that is expecting k_H is willing to overinvest up to the level where she is indifferent over receiving k_L. This is in the decreasing part of the $W^\beta(\pi, k_H)$ function, where $\pi > \pi^\beta$.

8 Debt competition with a fixed interest rate

In this special robustness check, suppose the investor provides a *loan* with a *fixed* net interest rate $r-1$. For instance, suppose a non-profit (social impact) investor provides loans to the entrepreneurs and charges zero (or minimal) profits. The (social impact) investor cares about the quality. Consequently, like in the standard models with rationing, the capital is allocated ex post optimally (the entrepreneur with a higher net return and thus a higher willingness to pay for capital receives k_H). To avoid extra distortions of demand for financial capital, we assume that the fixed interest rate is not excessively high, $r \leqslant \pi_L$.

8.1 Benchmark: No capital scarcity

We start with a benchmark (no capital scarcity). Like the equity cap competition, the debt competition with the fixed interest rate is a pure quality competition. In contrast to the competition under the equity cap, here the price aspect of the competition is suppressed completely. Since a *fixed* payment rk_H is expected, the entrepreneur's decision is not distorted; she sets π_H and receives $\bar{S}(k_H) - (r-1)k_H$.

8.2 The equilibrium

The expected payoff of the entrepreneur i that receives funding k is $k(\pi_i - r) - C(\pi_i) = S(\pi_i, k) - (r-1)k$. Two observations arise: First, for a given k_i, the optimal quality is not distorted, $\pi_i = \Pi(k_i)$, since the entrepreneur *fully* appropriates an increase in cash flows due an increase in human capital. Second, even if the entrepreneur can *not fully* appropriate an increase in surplus due to an increase in financial capital (due to an extra capital cost $(r-1)k$), this doesn't distort their demand for financial capital. As long as $\pi > r$, the entrepreneurs demand full capacity k_H.

For any symmetric distribution of human capital investments, $F^r(\pi)$, the expected payoff of an entrepreneur i is

$$W^r(\pi_i) := [1 - F^r(\pi_i)]S(\pi_i, k_L) + F^r(\pi_i)[S(\pi_i, k_H) - (r-1)(k_H - k_L)] - (r-1)k_L.$$

Proposition 6 derives $F^r(\pi)$ by applying the equalizer condition; i.e., $W^r(\pi)$ is constant in π over the interval that defines the support of the distribution.

Proposition 6 (Fixed interest rate)**.** *In the competition for capital provided as loans with a fixed interest rate, $r - 1 \geqslant 0$, the quality of each startup follows a distribution $F^r(\pi)$ with support $\pi \in [\pi_L, \pi_H^r]$, where*

$$F^r(\pi) = \frac{\bar{S}(k_L) - S(\pi, k_L)}{S(\pi, k_H) - S(\pi, k_L) - (r-1)(k_H - k_L)} = \frac{\bar{S}(k_L) - S(\pi, k_L)}{(\pi - r)(k_H - k_L)}.$$

The ex ante expected payoff of each entrepreneur is $W^r = \bar{S}(k_L) - (r-1)k_L$.

The equilibrium bids are on the interval $[\pi_L, \pi_H^r]$, where the maximal quality is implicitly defined by $S(\pi_H^r, k_H) - (r-1)k_H = \bar{S}(k_L) - (r-1)k_L$. Intuitively, this level is the maximal return (i.e., maximal investment into human capital) that the loser is willing to set to steal the extra capital; at this level, all additional profits generated by extra capital are destroyed by setting an excessive large amount of human capital. Since $\pi_H^r > \pi_H$,[14] the maximal bid is *excessive* in a sense that winning the extra capital with a bid $\pi > \pi_H$ implies that the entrepreneur would benefit from the possibility to revise the bid downward.

8.3 Comparative statics

Scarcity. Again, like in the other settings, the existence of capital scarcity implies that the quality is dispersed. Now the quality π_H spreads around π_H.

Interest rate. With a higher interest rate, bids are more *compressed* (overbidding is reduced) as the maximal investment decreases towards π_H. The ex ante expected equilibrium profits of the entrepreneurs decrease, and also the expected gross return (i.e., the expected quality) decreases.[15]

8.4 Asymmetry

An asymmetric equilibrium in pure strategies, i.e., specialization (π_L, π_H), is not an equilibrium: The startup that is expected to receive a sufficient amount of capital (winner) invests π_H, but the startup that is expected to receive an insufficient amount of capital (loser) always prefers to invest above π_H to 'steal' the extra capital.

9 Overview

In the overview, we will briefly summarize and illustrate differences between the settings. We will focus mainly on profitability (due to startup quality) and equity shares. These differences give lessons on the effects of contracting changes. We will also provide a robust understanding of the effects of capital scarcity, conditional on various contracting distortions.

[14] Recall $S(\pi, k_H) - (r-1)k_H$ is increasing for $\pi \in [\pi_L, \pi_H]$ and decreasing for $\pi > \pi_H$ (as long as $\pi > r$, which holds here). Since $\bar{S}(k_H) - (r-1)k_H > \bar{S}(k_L) - (r-1)k_L$, we have that $S(\pi, k_H) - (r-1)k_H > \bar{S}(k_L) - (r-1)k_L$, and therefore the zero-extra-profit condition in the text is satisfied in the decreasing part of function $S(\pi, k_H) - (r-1)k_H$, i.e., for $\pi_H^r > \pi_H$.

[15] A higher r decreases denominator in F^r without affecting the numerator. Therefore, a F^r distribution with a lower r first-order stochastically dominates a F^r distribution with a higher r, as long as $r \leqslant \pi_L$. A higher required interest rate generates a loss when bidding π_L. The all-pay auction then involves randomization of the participation action. We are leaving it out here because this setting is only a benchmark for other settings where capital price is set endogenously and thus satisfies the participation constraint endogenously always.

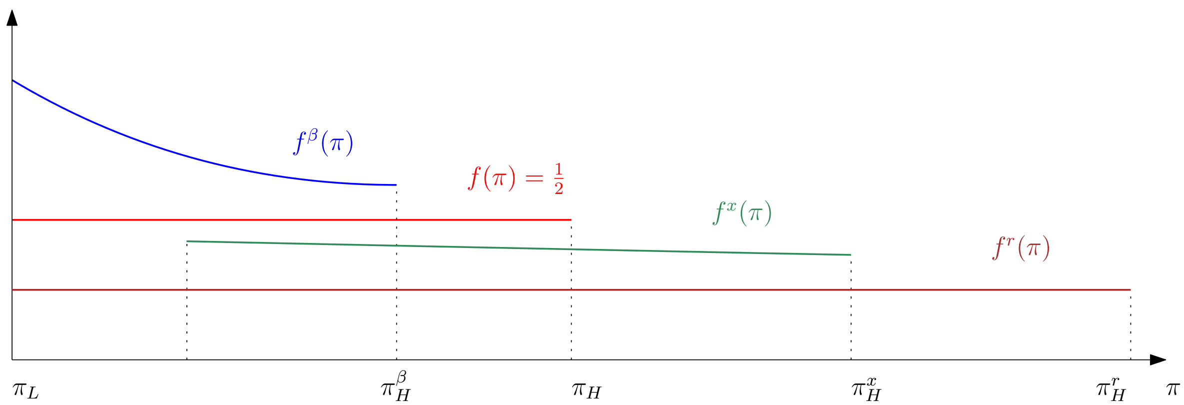

Figure 1: Startup qualities (the quadratic cost)

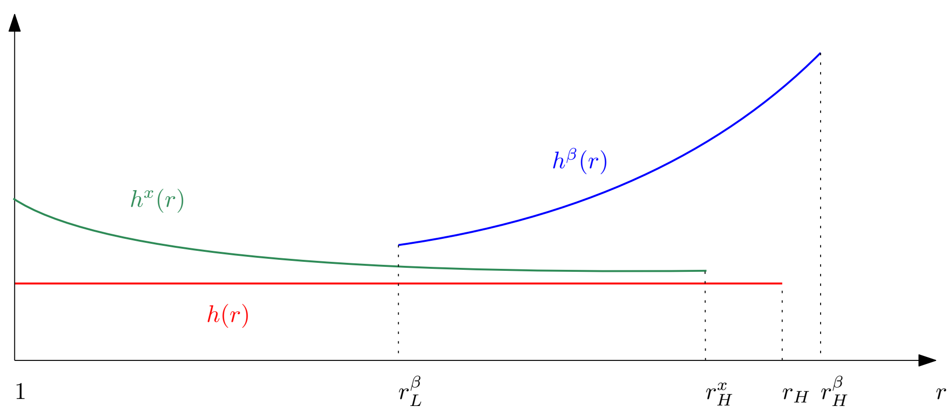

Figure 2: Gross returns offered to the investor (the quadratic cost)

9.1 Illustration: Quadratic cost

To begin with, we will illustrate the dispersion of the startup qualities (profits) and returns. Fig. 1 shows the equilibrium densities of startup quality for the quadratic cost with $(k_L, k_H) = (2, 4)$. For equity competition, the density is uniform, $f(x) = \frac{1}{2}$, where $(\pi_L, \pi_H) = (2, 4)$. The density $f^x(\pi) = \frac{2}{5} + \frac{8}{45\pi^2}$ is for a binding cap $X = \frac{3}{8}$, where $(\pi_L^x, \pi_H^x) = (\frac{8}{3}, \frac{15+\sqrt{209}}{6}) \doteq (\frac{8}{3}, \frac{87}{18})$. The density $f^r(\pi) = \frac{1}{4}$ is for a fixed interest rate $r - 1 = 1$, where $(\pi_L^r, \pi_H^r) = (2, 6)$. The density $f^{\beta}(\pi) = \frac{1}{2} + \frac{1}{2(\pi-1)^2}$ is for symmetric bargaining powers, $\beta = \frac{1}{2}$, where $(\pi_L^{\beta}, \pi_H^{\beta}) = (2, 2 + \sqrt{2})$.

In addition, Fig. 2 shows the equilibrium densities of returns offered to the venture investor for the quadratic cost and the levels of capital given above. For equity competition, the density is uniform, $h(r) = 1$, where $(r_L, r_H) = (1, 2)$. The density $h^x(r) = \frac{16}{15} + \frac{1}{15r^2}$ is for a binding cap $X = \frac{3}{8}$, where $(r_L^x, r_H^x) = (1, \frac{15+\sqrt{209}}{16}) \doteq (1, \frac{87}{48})$. For a fixed interest rate, the distribution is of course degenerate at the required level of the interest rate. The density $h^{\beta}(r) = r^2 - \frac{3}{4}$ is for symmetric bargaining powers, where $(r_L^{\beta}, r_H^{\beta}) = (\frac{3}{2}, \frac{3+\sqrt{2}}{2})$.

9.2 Properties of all-pay auctions

9.2.1 Ex post inefficiency

In a frictionless economy, the startup quality is set optimally to the startup capital. That is, since the ex post capital allocation is (k_L, k_H), the ex post optimal startup qualities are (π_L, π_H). The ex post qualities are, however, generically different due to strategic uncertainty over the amount of capital. In the main setting, the stronger entrepreneur's quality is (almost always) insufficient, and the weaker entrepreneur's quality is (almost always) excessive. These distortions are then affected by additional contracting distortions. With a binding equity cap, underbidding is diminished but overbidding is pronounced. With investor's bargaining share or investor's fixed return, overbidding is diminished but underbidding is pronounced.

When the qualities are unobserved, we observe positive (efficient) assortative matching between startup quality and capital. Intuitively, the quality is an instrument that is increasing the investor's return to the capital, and thus makes the capital allocation more attractive. In the context of venture capital, positive assortative matching between venture capital investors and entrepreneurs is typically attributed to the existence of complementarities (Ewens, Gorbenko and Korteweg, 2022). In Appendix C.2, we demonstrate that for perfect substitution, the quality is not increasing the return to the capital, and is in fact assortative matching is negative; in the equilibrium, the larger projects (with more capital) have a lower quality. This is because the entrepreneurs in the competition for capital increase the returns to capital only through giving more equity, but a higher equity offer implies a more pronounced moral hazard problem. The entrepreneurs are consequently discouraged to invest into the quality; when seeking to increase the scale of the company, they sacrifice the quality part of the production.

Additionally, with observed qualities, the stronger entrepreneur not necessarily receives more quality; while her equilibrium probability of winning the capital prize is larger, it is no longer certain. As the sequential equity competition with observed qualities is a sequence of two all-pay auctions, both entrepreneurs randomize in the second auction irrespective of their qualities, and the weaker entrepreneur wins the prize with a positive probability.

9.2.2 Startup profitability

In the absence of scarcity, observability and contracting through a fixed interest rate have no effect on the startup quality (profits). Imposing the equity cap is increasing profits; the lower is the cap, the higher are the profits. Investors' fixed bargaining power is decreasing the profits. In the presence of scarcity, observability again has no effect (except for equilibrium multiplicity). A fixed interest rate is now decreasing profits. Imposing the equity cap is again increasing profits. Imposing Nash bargaining is again decreasing the profits. Therefore, the robust (scarcity-irrelevant) effects of contracting frictions on profits are for the equity cap (positive), observability (neutral) and investor's bargaining power (negative). The fixed interest rate regimes has a (negative) effect that is conditional on scarcity.

The deviations from the main setting can be also characterized in the following way: $F(\pi)$ is the success rate in the auction for the extra capital, and thus the expected level of the capital is $k = k_L + F(\pi)(k_H - k_L)$. In the main setting, the entrepreneur is not constrained in setting a combination of quality/profitability and investor's return (π, r) since the return can be adjusted by the equity share. Consequently, the entrepreneur is motivated to set the quality that is maximizing the (ex ante) surplus given the expected level of the capital. The quality is distorted ex post only due to strategic uncertainty (the difference between the expected and realized level of the capital). With additional contracting distortions, the entrepreneur is no longer motivated to maximize the surplus. With an equity share cap, the entrepreneur cannot pay the investor's return through sufficient equity share, and thus overinvests. With limited (fixed) bargaining power, the entrepreneur is not maximizing the surplus as her weights on the profits and costs are distorted. With a fixed interest-rate payment, the entrepreneur cannot compete through the investor's interest rate but this forces her to compete only through quality; it is impossible to changing the quality without affecting the expected level of capital, and consequently the quality is not surplus-maximizing quality for a given expected level of capital.

9.2.3 Equity shares

The literature agrees that with capital scarcity, the equity share of the investors increases; with multiple investors, this reflects decreased competitiveness of the market (see, Ewens, Gorbenko and Korteweg, 2022). The effect of capital scarcity goes through several channels. In a two-sided exogenous matching model, Inderst and Müeller (2004) find that under good market conditions (capital abundance), the equity share of the investors are low, and with capital scarcity, the equity shares of the investors are high. The key channel is the role of the outside options; with scarcity, the investor's outside option is better and the entrepreneur's outside option is worse. This shifts contract frontier such that in Nash bargaining outcome, deal utilities are shifted from the entrepreneur to the investor.

Using a matching model with double-sided moral hazard, Hong, Serfes, and Thiele (2020) also predict that the venture capital receives lower equity shares with the entry of new venture investors (a more competitive supply side). This shifts incentives from the investor to the entrepreneur and thus affect how a double moral-hazard in the project implementation state is affected. Giving ex ante heterogeneity of startups, they predict a heterogenous effect: The shift improves the balance of incentives in the low-value startups but worsens the balance in the high-value startups.

Our paper primarily identifies links between capital scarcity and the equity share that go through the strategic uncertainty, and additionally looks into the channels due to additional contracting frictions. The standard channel is that with capital scarcity, valuations are lower due to less exploited complementarities; to compensate the investor, the equity shares must be higher. In Ewens, Gorbenko and Korteweg (2022), the startup quality involves both an exogenous component (entrepreneur's endowed type) and an endogenous component (entrepreneur's effort), and both components decrease the investor's cash flow share; as scarcity decreases quality, the investor's share increases. In our setting, this is most cleanly manifested when the investor's interest rate is fixed. With scarcity, the expected quality drops and to preserve the investor's return, the investor's share of cash flows must increase.

There are other channels as well. In Nash bargaining, the wedge between endogenous cash flow shares and exogenous bargaining shares over deal utilities is through the fixed capital cost. This wedge is asymmetric as it affects the investor more. With scarcity, the role of the (exogenous) capital cost in the total surplus is larger, and therefore the investor's equity share increases. In contrast, with capital abundance, the role of the capital cost is diluted and the equity shares converge down to the fixed bargaining shares.

In addition, in our main setting, while the investor's return is positively correlated with the startup quality (as both $\Pi(k)$ and $R(k)$ are increasing in k), a higher return is not necessarily through both a higher quality and a higher investor's share. For instance, as Appendix A shows, in a class of exponential cost functions, we observe neutrality; the endogenous investor's equity share is constant in the startup quality. Additionally, contracting frictions affect the link; with an equity cap, the correlation disappears by definition. Paying a fixed interest rate or bargaining with fixed bargaining powers restores the negative correlation that is observed in the other literature.

9.2.4 Venture capital returns dispersion

Our key prediction is that with capital scarcity, the interval of the returns offered to the venture investor is larger, and thus the dispersion of returns increases. This is observed for any contracting friction except for the special case when the investor's returns is forced to be constant. The dispersion of returns is associated with the dispersion in startup qualities (early investments). This is similar to Hong et al. (2020), where the gap between the low-type and high-type entrepreneurs' success rates increases with a less competitive supply (capital scarcity). In contrast to our setting, the wider gap is given by a differential effect on the heterogenous entrepreneurs, and reflects the balance of incentives in the double-hazard problem that is absent here.

9.2.5 Entrepreneurs

Entrepreneurs are adversely affected by the equity cap (due to excessive quality), by losing bargaining power (due to insufficient quality), and by paying a fixed interest rate above the market rate (due to paying a higher investor's rent). The types of losses of the entrepreneurs are thus friction-specific. Entrepreneurs are additionally affected by observability as the observability increases uncertainty over the equilibrium type.

9.3 Capital scarcity

Capital scarcity is present in four ways: (i) A change between abundance and scarcity (at $K = 2k_H$), (ii) increasing scarcity of the inframarginal capital (at $K \in (k_H, 2k_H)$), (iii) a change between positive and zero inframarginal capital (at $K = k_H$), and (iv) increasing scarcity of the marginal capital (at $K < k_H$). The effects in (i) have been covered in details in the benchmarks. In short, with this switch, strategic uncertainty arises in any setting. The effects in (ii) follow from comparative statics provided for each setting; the key is that with higher scarcity, there is also more uncertainty, which distorts the economy, and specifically harms the entrepreneurs.

At $K \leq k_H$, the inframarginal capital no longer exists. This implies that competing through the equity shares no longer has an irreversible cost component through the inframarginal amount of capital. But the investments into quality are still sunk costs, and this preserves the all-pay auction. The main effect is that the weaker entrepreneur is not funded which drives the bargaining power of the entrepreneurs in the equity-share competition to minimum; their (ex ante expected) equilibrium payoff is zero. The equilibrium strategy remains, only $k_L = 0$, k_H is replaced by K, and $\pi_L = \pi^\beta = 1$. (Recall $C(1) = 0$.) The only structural effect is on the equity cap competition. Now, since the investor doesn't allocate any capital to the loser, her net return on the allocated capital (zero capital) can be negative. In addition, for loans provided for a fixed interest rate, the condition that the interest payment doesn't distort the demand for capital, $r \leq \pi_L$, is met only for $r = 1$. When $r = 1$, we can also see that providing loans generates higher quality, $F^\beta(\pi) = \frac{C(\pi)}{(1-\beta)(\pi-1)k_H} < \frac{C(\pi)}{(\pi-1)k_H} = F^r(\pi)$. This is intuitive, because with $r = 1$, the entrepreneur has full bargaining power whereas in Nash bargaining, her power is only $1 - \beta$.

10 Conclusions

In venture capital markets, search and matching frictions imply that temporary imbalances in the supply and demand are often resolved within the existing matches. In this paper, we specifically analyze how a shock to the supply of venture capital affects the entrepreneurs' early incontractible investments (efforts) in the quality of their startups and the subsequent contracting. To that end, we build a static, ex ante symmetric, and one-side moral-hazard model with endogenous quality, where multiple entrepreneurs with scalable but capacity-constrained startups post competitive offers to win scarce capital.

The key observation is that the competition for scarce venture capital has a structure of an all-pay auction (if the opponent's quality is not observed), or a sequence of two all-pay auctions (if the opponent's quality is not observed). In the equilibrium, the entrepreneurs face a tradeoff between a higher probability of financing on one side and better deals and less effort on the other side. The competition generates endogenous (strategic) ex ante uncertainty, which negatively impacts ex post efficiency; additionally, and with observed opponent's quality, the uncertainty even distorts efficient (positive assortative) matching between the startup quality and the level of capital. Our key prediction is that the adverse effects of supply shocks are magnified by endogenous strategic uncertainty, and the uncertainty manifests in the larger dispersion of the venture capital returns.

Our setting not only predicts endogenous dispersion of startup qualities and investors' returns but also demonstrates how the dispersion and startup profitability are affected by additional contracting features. When the price aspect of competition is suppressed (e.g., when large investor's returns or equity shares are not credible or involve extra prohibitive agency cost), we observe that the missing price competition is substituted by more intensive competition in the startup quality, and the profitability (exit success rate) increases. When the investor's bargaining power increases, the entrepreneurs are less willing to incur incontractible investments and the profitability decreases. Importantly, while additional contracting features change the structure of the auction, the strategic uncertainty remains.

Our model is parsimonious, but the model admits a broader interpretation. On the side of the venture investor, we analyze the allocation of fixed capital, but we can equally think of allocating the fixed investor's capacity to develop the startup. Second, in the main setting, the irreversible costs stem from the quality investment and inframarginal capital; but we show that the uncertainty remains even if one of the two components disappears. Third, while we employ complementarity between the startup quality and financial capital, the complementarity is not necessary for the strategic uncertainty to emerge. Even when the quality and finances generate independent cash flows, the core problem is that the entrepreneurs cannot offer equity shares only to the cash flows generated by the finance component. By giving a more attractive equity offer, the entrepreneurs thus pay an irreversible cost associated with the lower share of cash flows generated by the quality component. This irreversible cost component again generates the all-pay auction in which the irreversible costs (and also the success rates) are randomized.

In summary, this paper gives predictions on the contracts, project profits (success rates), capital allocations, and qualities of entrepreneurs in an environment with scarce capital. We are able to characterize testable links between the industry characteristics, entrepreneur's endogenous qualities, profitability (or success rates) of projects, and contract terms. We can also describe channels through which different contracting frictions affect competition for venture capital and give predictions on the expected levels and dispersions of the characteristics of startups and contracts.

References

[1] Azarmsa, E., Cong, L. W. (2020). Persuasion in relationship finance. *Journal of Financial Economics*, 138(3), 818–837.

[2] Baye, M. R., Kovenock, D., De Vries, C. G. (1996). The all-pay auction with complete information. *Economic Theory*, 8(2), 291–305.

[3] Becker, S. O., Hvide, H. K. (2022). Entrepreneur death and startup performance. *Review of Finance*, 26(1), 163–185.

[4] Bhattacharyya, S., Lafontaine, F. (1995). Double-sided moral hazard and the nature of share contracts. *RAND Journal of Economics*, 761–781.

[5] Bolton, P., Dewatripont, M. (2004). *Contract theory.* The MIT Press, Cambridge, MA .

[6] Casamatta, C., Haritchabalet, C. (2014). Dealing with venture capitalists: shopping around or exclusive negotiation. *Review of Finance*, 18(5), 1743–1773.

[7] Che, Y. K., Gale, I. (2003). Optimal design of research contests. *American Economic Review*, 93(3), 646–671.

[8] Choi, J., Goldschlag, N., Haltiwanger, J., Kim, J. D. (2023). Early joiners and startup performance. *Review of Economics and Statistics*, forthcoming.

[9] Da Rin, M., Hellmann, T., Puri, M. (2013). A survey of venture capital research. In: *Handbook of the Economics of Finance* (Vol. 2, pp. 573–648). Elsevier.

[10] De Bettignies, J. E., Brander, J. A. (2007). Financing entrepreneurship: Bank finance versus venture capital. *Journal of Business Venturing*, 22(6), 808–832.

[11] Ewens, M., Gorbenko, A., Korteweg, A. (2022). Venture capital contracts. *Journal of Financial Economics*, 143(1), 131–158.

[12] Ewerhart, C. (2017). Contests with small noise and the robustness of the all-pay auction. *Games and Economic Behavior*, 105, 195–211.

[13] Fu, H., Yang, J., An, Y. (2019). Made for each other: Perfect matching in venture capital markets. *Journal of Banking & Finance*, 100, 346–358.

[14] Gornall, W., Strebulaev, I. A. (2023). The Contracting and Valuation of Venture Capital-Backed Companies. In: Eckbo, B., Phillips, G., Sorensen, M. (eds.) *Handbook of the Economics of Corporate Finance*, 1 (1), 3–76.

[15] Hellmann, T. (2006). IPOs, acquisitions, and the use of convertible securities in venture capital. *Journal of Financial Economics*, 81(3), 649–679.

[16] Hellmann, T., Thiele, V. (2015). Friends or foes? The interrelationship between angel and venture capital markets. *Journal of Financial Economics*, 115(3), 639–653.

[17] Hochberg, Y. V., Ljungqvist, A., Vissing-Jørgensen, A. (2014). Informational holdup and performance persistence in venture capital. *Review of Financial Studies*, 27(1), 102–152.

[18] Hong, S., Serfes, K., Thiele, V. (2020). Competition in the venture capital market and the success of startup companies: Theory and evidence. *Journal of Economics & Management Strategy*, 29(4), 741–791.

[19] Howell, S. T. (2020). Reducing information frictions in venture capital: The role of new venture competitions. *Journal of Financial Economics*, 136(3), 676–694.

[20] Inderst, R., Müller, H. M. (2004). The effect of capital market characteristics on the value of start-up firms. *Journal of Financial Economics*, 72(2), 319–356.

[21] Inderst, R., Vladimirov, V. (2019). Growth firms and relationship finance: A capital structure perspective. *Management Science*, 65(11), 5411–5426.

[22] Kaplan, S.N., Strömberg, P. (2004). Characteristics, contracts, and actions: evidence from venture capital analyses. *Journal of Finance*, 59, 2177–2210.

[23] Kaplan, S.N., Sensoy, B. A., Strömberg , P. (2009). Should investors bet on the jockey or the horse? Evidence from the evolution of firms from early business plans to public companies. *Journal of Finance*, 64(1), 75–115.

[24] McAfee, R. P., McMillan, J. (1987). Auctions and bidding. *Journal of Economic Literature*, 25(2), 699–738.

[25] Repullo, R., Suarez, J. (2004). Venture capital finance: A security design approach. *Review of Finance*, 8(1), 75–108.

[26] Sannino, F. (2023). The Equilibrium Size and Value-Added of Venture Capital. *Journal of Finance*, forthcoming.

[27] Schmidt, K.M. (2003). Convertible securities and venture capital finance. *Journal of Finance*, 58, 1139–1166.

[28] Siegel, R. (2009). All-pay contests. *Econometrica*, 77(1), 71–92.

[29] Silveira, R., Wright, R. (2016). Venture capital: A model of search and bargaining. *Review of Economic Dynamics*, 19, 232–246.

[30] Ueda, M. (2004). Banks versus venture capital: Project evaluation, screening, and expropriation. *Journal of Finance*, 59(2), 601–621.

A Examples

A.1 Exponential cost

Consider an exponential quality cost function, $C = \frac{\pi^{1+\alpha}}{1+\alpha}$, where $\alpha > 0$. The optimal quality, $\Pi(k) = k^{\frac{1}{\alpha}}$, is convex if $\alpha \in (0,1)$, linear if $\alpha = 1$, and concave if $\alpha > 1$. To normalize $\bar{S}(k_L) = 0$, we must set $\pi_L = \frac{1+\alpha}{\alpha}$, which implies normalization to $k_L = \left(\frac{1+\alpha}{\alpha}\right)^{\alpha}$. Notice also the surplus is sufficiently elastic with any $\alpha > 0$; this holds even with $\bar{S}(k_L) > 0$.

A.1.1 Equity competition

The equilibrium quality distribution, $F(\pi) = \frac{\pi^{\alpha} - k_L}{k_H - k_L}$, is concave if $\alpha \in (0,1)$, linear if $\alpha = 1$, and convex if $\alpha > 1$. For any α, the entrepreneurs compete only in the quality dimension and voluntarily suppress competition in the equity-share dimension:

$$X(k) = \frac{R(k)}{\Pi(k)} = \frac{\bar{S}(k) - \bar{S}(k_L)}{\Pi(k)k} + \frac{1}{\Pi(k)} = \frac{\alpha}{1+\alpha}$$

The expected cash flows are

$$\int_{\pi_L}^{\pi_H} \frac{\alpha \pi^{2\alpha}}{k_H - k_L} d\pi = \frac{\alpha}{1+2\alpha} \frac{1}{k_H - k_L} \left(\pi_H^{1+2\alpha} - \pi_L^{1+2\alpha}\right).$$

For example, if $(k_L, k_H) = (2,4)$, we have $(\pi_L, \pi_H) = (2^{1/\alpha}, 2^{2/\alpha})$, and the expected cash flows are

$$\frac{\alpha 2^{\frac{1}{\alpha}+2}\left(\alpha 2^{\frac{1}{\alpha}+2} - 1\right)}{2(1+2\alpha)}.$$

A.2 Example: Quadratic cost

Consider a quadratic cost function of quality, $C = \frac{\pi^2}{2}$, with the optimal return $\Pi(k) = k$. This is a special case of the exponential cost function with $\alpha = 1$. Consider $(k_L, k_H) = (2,4)$, i.e., $(\pi_L, \pi_H) = (2,4)$. With this specification, $\bar{S}(k_L) = 0$. Notice that the condition on the surplus is satisfied for any $k > 0$, since $\bar{S}(k) = \frac{k^2}{2} - k < k^2 - k$.

A.2.1 Equity competition

We are interested in the equilibrium quality, interest rates and equity shares. (i) Using $\Pi^{-1}(\pi) = \pi$, the equilibrium returns are uniformly distributed, $F(\pi) = \frac{\pi - k_L}{k_H - k_L} = \frac{\pi}{2} - 1$. The returns are in the range $\pi \in [\pi_L, \pi_H] = [2,4]$. (ii) We obtain the equilibrium distribution of the investor's returns in two steps. First, we calculate $R(k)$ from the equalizer condition, $\bar{S}(k) - [R(k) - 1]k = 0$, where we used that in our case $\bar{S}(k_L) - [R(k_L) - 1]k_L = 0 - 0 = 0$. This gives $R(k) = \frac{k}{2}$. Therefore, the investor's returns are in the range $r \in [1,2]$. Second, entering $k = 2R(k)$ into $k = k_L + P(r)(k_H - k_L) = 2 + 2P(r)$, we obtain uniform distribution, $P(r) = r - 1$. (iii) Using $\hat{\pi}(r) = k = k_L + (r-1)(k_H - k_L) = 2 + 2\left(\frac{r}{4} - 1\right) = 2r$, the cash flow share conditional on r is $\hat{x}(r) = \frac{r}{\hat{\pi}(r)} = \frac{r}{k} = \frac{1}{2}$. In other words, the entrepreneurs compete only in the human-capital dimension and voluntarily suppress competition in the equity-share dimension and offer exactly the 'fair' share of 50%.

Notice that the outcome is identical to the outcome under the fixed equity share at $\frac{1}{2}$, including the distribution of project returns. In addition, project returns (i.e., human capital investments) are exactly like in late contracting (but late contracting then involves randomized shares, with randomization dependent on the human capital investments.) Also, see that the investor's expected cash flow is $4\frac{2}{3}$. This is close to the naive estimate $4\frac{1}{2}$ that disregards the correlation between the level of capital and the level of private investments. (The naive estimate is obtained by multiplying the mean private investment $\pi = 3$, the mean/constant equity share $\frac{1}{2}$ and the mean capital $k = 3$.) Similarly, the expected cash flow is $9\frac{1}{3}$, which is close 9 which is the naive estimate that disregards conditional probability, i.e., the multiple of the mean private investment $\pi = 3$ and the mean capital $k = 3$.

A.2.2 Equity competition with observable qualities

The distributions of the interest rates and the investor's cash flow shares are not easy to get, since we have derived only conditional distributions, i.e., conditional on pairs (π_1, π_2). For illustration: The median cash flow share offered by a stronger entrepreneur j, $\pi_j \geqslant \pi_i$, is given implicitly by $\frac{1}{2} = G_j(x)$. The median is $x = \frac{(2-\pi_j)k_L + \pi_j k_H}{\pi_i(k_H + k_L)} = \frac{2+\pi_j}{\pi_i}$. Another illustration: Suppose investments are identical, $\pi := \pi_1 = \pi_2$. Then, $G(x) = \frac{k_L}{k_H - k_L}\frac{\pi x - 1}{\pi(1-x)}$. The offers are largely concentrated at high offers, since the density is $g(x) = \frac{k_L}{k_H - k_L}\frac{\pi - 1}{\pi}\frac{1}{(1-x)^2}$. Notice that this shape is independent on the cost function.

A.2.3 Nash bargaining

Consider fair bargaining shares. The distribution is $F^{\beta}(\pi) = \frac{\pi(\pi-2)}{2(\pi-1)}$, and the density is $f^{\beta}(\pi) = \frac{1}{2} + \frac{1}{2(\pi-1)^2}$. The returns are distributed on $(\pi_L^{\beta}, \pi_H^{\beta}) = (2, 2+\sqrt{2})$. Since the bargaining share are fair, the investments are not distorted at the bottom. Since $2 + \sqrt{2} < 4$, the quality at the top is more compressed relative to the setting where the entrepreneurs make offers. Notice that they are less compressed relative to the bargaining setting where the capital is abundant, since $\pi^{\beta} = \Pi(\frac{1}{2}k_H) = 2 < 2 + \sqrt{2} = \pi_H^{\beta}$. Intuitively, the effect of bargaining is to decrease the quality due to lower appropriability, but the effect of scarcity is dispersion of quality; the latter effect dominates and the bid under scarcity that guarantees full capacity with certainty is larger than the bid under abundance.

A.2.4 Debt competition with a fixed interest rate

We are interested in the equilibrium qualities, interest rates and cash flow shares. (i) The equilibrium distribution of quality is $F^r(\pi) = \frac{(\pi-2)^2}{4(\pi-1)}$. The maximal quality is $\pi_H^r = \pi_H + 2\sqrt{6} > \pi_H$. The corresponding density of returns is $f^r(\pi) = \frac{\pi(\pi-2)}{4(\pi-1)^2}$. (ii) The interest rate is fixed. (iii) To obtain the equilibrium distribution of the investor's equity shares, notice that $r = x\pi$, and therefore $G(x) = 1 - F(\frac{r}{x}) = \frac{r^2-8rx+8x^2}{4x(x-r)}$. The shares are in the range $x \in [\frac{1}{\pi_H^r}, \frac{1}{\pi_L}]$.

B Proofs

Proof of Proposition 1

Proof. First, to obtain the equilibrium expected payoff of the entrepreneur, W^*, we evaluate the payoff at the minimal level of capital. When the entrepreneur expects zero success, k_L, she is effectively not competing over the capital and offers the investor the minimal acceptable return, $r = 1$. Consequently, $R(k_L) = 1$, and from the properties of $R(k)$, the equilibrium payoff is

$$W^* = W^k(k_L) = \bar{S}(k_L).$$

Second, we will characterize the equilibrium through the success rate $\sigma \in [0, 1]$. The expected amount of capital is $k = k_L + \sigma(k_H - k_L)$. Since $\Pi(k)$ is increasing in k, it is consequently also increasing in σ. Therefore, when the distribution of quality is denoted $F(\pi)$, we have $F(\pi) = \sigma$ when $\pi = \Pi(k) = \Pi(k_L + \sigma(k_H - k_L))$. Then,

$$F(\pi) = \sigma = \frac{k - k_L}{k_H - k_L} = \frac{\Pi^{-1}(\pi) - k_L}{k_H - k_L} = \frac{C'(\pi) - k_L}{k_H - k_L}.$$

□

Proof of Proposition 2

Proof. We begin with the $k = k_L$, which gives us the minimal amount of π. We prove that $R^x(k_L) = 1$. Suppose not and (π, X) satisfies $\pi X > 1$ or equivalently $\pi > \frac{1}{X}$. We know that for any r, the unconstrained optimum is π_L. Since the equity cap is sufficiently low, the equity cap binds, $X < X(k_L) = \frac{1}{\pi_L}$; equivalently $\pi_L < \frac{1}{X}$, and therefore $\pi > \frac{1}{X} > \pi_L$. Now, find the optimal π that maximizes the entrepreneur's objective for a fixed equity share X and for a fixed capital k_L, $\pi_L^X = C'^{-1}((1 - X)k_L) < \pi_L$. Since $\pi > \pi_L > \pi_L^X$, the entrepreneur is strictly better of by decreasing π. The entrepreneur cannot decrease only if the equity cap binds, $\pi = \frac{1}{X}$; at this level, $r = 1$ and thus $R^x(k_L) = 1$. Therefore, the minimal bid is $\pi_L^x = \frac{1}{X}$. The value for the entrepreneur is $W^x(\pi) = S(\pi_L^x, k_L) - k_L(\pi X - 1) = S(\pi_L^x, k_L)$ which gives us (by equalizer property) also the equilibrium expected value. Notice that $S(\pi_L^x, k_L) < S(\pi_L, k_L) = \bar{S}(k_L)$, since the level π_L^x is now excessive relative to the surplus-maximizing level and destroys part of the surplus.

Next we proceed to any $k > k_L$. The entrepreneur's payoff for any (π, k) is $(1 - x)\pi k - C(\pi) = S(\pi, k) - k(\pi X - 1)$. For a symmetric distribution of human capital investments and symmetric distribution of returns, $F^x(\pi)$, we obtain the entrepreneur's expected payoff as

$$W^x(\pi) = [1 - F^x(\pi)][S(\pi, k_L) - k_L(\pi X - 1)] + F^x(\pi)[S(\pi, k_H) - k_H(\pi X - 1)].$$

By equalizer condition, $W^x(\pi)$ must be constant in π, which gives us $F^x(\pi)$. We now analyze the maximal bid that is implicitly defined as a solution to

$$S(\pi_H^x, k_H) - W^x = k_H(\pi_H^x X - 1). \tag{1}$$

First, evaluate at $\pi_H^x = \pi_H$.

- Take a hypothetical (benchmark) case where $W^x = \bar{S}(k_L)$ and $X = X(k_H)$. Then, at $\pi_H^x = \pi_H$, the LHS equals the RHS, since the equation (1) then reflects our well-know condition from the main setting, $R(k_H) = \pi_H X(k_H) = \frac{\bar{S}(k_H) - \bar{S}(k_L)}{k_H} + 1$.
- Now evaluate how the differences between our case and the benchmark case affect the equation (1).
- LHS: Since $W^x < \bar{S}(k_L)$, the LHS is larger than in the benchmark case.
- RHS: Since the cap is sufficiently low (binding always), it is binding also the original distribution; i.e., $X < X(k_H)$. As a consequence, the RHS is lower than in the benchmark case.

Second, see that the LHS is decreasing in π (if $\pi \geqslant \pi_H$ which is true here) and the RHS is increasing in π. Combined together, the LHS equals the RHS when $\pi_H^x \geqslant \pi_H$, because the large LHS decreases and a small RHS increases and restore equality that was in the benchmark case with $\pi_H^x = \pi_H$, $W^x = \bar{S}(k_L)$ and $X = X(k_H)$. □

Proof of Proposition 3

Proof. We will look for distributions $G_i(x)$ and $G_j(x)$ that are the optimal distributions of bids that generate investor's net returns on a common interval $[0, \overline{x}_j \pi_j - 1]$. Following the literature on APAs, we will look for a profile of mixed strategies in which the stronger entrepreneur is never passive, $G_i(\underline{x}_i) = 0$, whereas the weaker entrepreneur is passive with a positive probability, $G_j(\underline{x}_j) > 0$. By the equalizer property of a mixed strategy, the expected payoff of each player can be expressed as the payoff at her minimal bid. For the weaker player, it is simply

$$V_j = (1 - \underline{x}_j)\pi_j k_L = (\pi_j - 1)k_L.$$

For the stronger player, this payoff implies either of two events: (i) a win with probability $G_j(\underline{x}_j)$ (the weaker player is passive), or (ii) a loss with probability $1 - G_j(\underline{x}_j)$ (the weaker player is active). In total, her expected payoff is

$$V_i = (1 - \underline{x}_i)\,\pi_i[k_L + G_j(\underline{x}_j)(k_H - k_L)] = (\pi_i - 1)[k_L + G_j(\underline{x}_j)(k_H - k_L)].$$

Next, we exploit the equalizer property to obtain the densities of the distributions of bids. For the weaker entrepreneur, the expected payoff conditional on her bid x_j is

$$v_j(x_j) := (1 - x_j)\pi_j \left[k_L + G_i\left(\frac{\pi_j}{\pi_i} x_j\right)(k_H - k_L)\right].$$

By the equalizer property, $v_j'(x_j) = 0$. We rearrange this derivative and use that the ex ante expected payoff is for each bid x_j on the support equal to the expected payoff conditional on the bid x_j, $v_j(x_j) = V_j$, and obtain the density for the stronger player,

$$g_i(x_i) = \frac{\pi_i}{(\pi_j - \pi_i x_i)^2} \cdot \frac{V_j}{k_H - k_L} = \frac{\pi_i(\pi_j - 1)}{(\pi_j - \pi_i x_i)^2} \frac{k_L}{k_H - k_L}.$$

By integrating, and using $\rho := \frac{\pi_j}{\pi_i}$, we have $G_i(x_i) = \frac{k_L}{k_H - k_L} \frac{\rho - \underline{x}_i}{\rho - x} - B$, where B is a constant. We derive the constant $B = \frac{k_L}{k_H - k_L}$ by knowing that the stronger entrepreneur is active, $G_i(\underline{x}_i) = 0$, which completes the derivation of $G_i(x_i)$.

By analogy, we obtain the density for the weaker player. We use that the for the stronger entrepreneur, the expected payoff conditional on her bid x_i is

$$v_i(x_i) := (1 - x_i)\pi_i \left[k_L + G_j\left(\tfrac{\pi_i}{\pi_j} x_i\right)(k_H - k_L)\right].$$

We use $v_i'(x_i) = 0$, exploit that $v_i(x_i) = V_i$, and obtain the density for the weaker player,

$$g_j(x_j) = \frac{\pi_j}{(\pi_j - \pi_i x_i)^2} \cdot \frac{V_i}{k_H - k_L} = \frac{\pi_j(\pi_i - 1)}{(\pi_i - \pi_j x_j)^2} \left[\frac{k_L}{k_H - k_L} + G_j(\underline{x}_j)\right].$$

By integrating, we have $G_j(x_j) = \frac{\pi_i - 1}{\pi_i - \pi_j x_j}\left[\frac{k_L}{k_H - k_L} + G_j(\underline{x}_j)\right] - B$. To derive $G_j(\underline{x}_j)$ and the constant B, we can first evaluate the distribution function at the minimal bid, $x_j = \underline{x}_j$, to obtain

$$G_j(\underline{x}_j) = \frac{k_L}{k_H - k_L} + G_j(\underline{x}_j) - B.$$

As a result, we obtain the constant as $B = \frac{k_L}{k_H - k_L}$. Inserting the constant into the distribution function, we get $G_j(\underline{x}_j) = \frac{\pi_i}{\pi_i - 1}(1 - \rho)$, and therefore

$$G_j(x_j) = \frac{\pi_i - 1}{\pi_i - \pi_j x_j}\left[\frac{k_L}{k_H - k_L} + \frac{\pi_i}{\pi_i - 1}(1 - \rho)\right] - \frac{k_L}{k_H - k_L} = \frac{1 - \rho}{1 - \rho x_j} + \frac{k_L}{k_H - k_L} \cdot \frac{\rho x_j - \underline{x}_i}{1 - \rho x_j}.$$

Finally, it is useful to express the stronger player's payoff as

$$V_i = (\pi_i - 1)[k_L + G_j(\underline{x}_j)(k_H - k_L)] = (\pi_i - 1)k_L + (\pi_i - \pi_j)(k_H - k_L) = \pi_i(k_H + k_L) + V_j.$$

□

Proof of Proposition 4

Proof. Consider an entrepreneur i who selects an admissible π_i in a late contracting regime. Let $W_L^{late}(\pi_i) := (\pi_i - 1)k_L - C(\pi)$ be her ex ante expected payoff when she is the loser in the subsequent APA ($\pi_i < \pi_j$) and $W_H^{late}(\pi_i) := W_L^{late}(\pi_i) + (\pi_i - \pi_j)(k_H - k_L)$ when she is the winner in the subsequent APA ($\pi_i > \pi_j$). Let $F(\pi)$ be the symmetric equilibrium distribution of cash flows. Given that the prize in the case of victory is increasing in π_i, we write her expected payoff as

$$W_i^{late}(\pi_i) = W_L^{late}(\pi_i) + (k_H - k_L)\int_{\pi_L}^{\pi_i}(\pi_i - \pi_j)df(\pi_j).$$

We use that the second term in $W_i^{late}(\pi_i)$ is $(k_H - k_L)\left(\pi_i[F(\pi_i) - F(\pi_L)] - \int_{\pi_L}^{\pi_i}\pi_j f(\pi_j)d\pi_j\right)$ and its derivative is $(k_H - k_L)(\pi_i f(\pi_i) + F(\pi_i) - F(\pi_L) - \pi_i f(\pi_i)) = (k_H - k_L)F(\pi_i)$. By the equalizer property, the expected payoff is constant in π_i, and therefore

$$\frac{\partial W_L^{late}(\pi_i)}{\partial \pi_i} + (k_H - k_L)F(\pi_i) = k_L - C'(\pi) + (k_H - k_L)F(\pi_i) = 0.$$

It clearly follows that the lowest bid (i.e., the bid π satisfying $F(\pi) = 0$) is π_L, since $k_L - C'(\pi_L) = 0$ and the highest bid (i.e., the bid π satisfying $F(\pi) = 1$) is π_H, given that $k_H - C'(\pi_H) = 0$. □

Proof of Proposition 5

Proof. For a symmetric distribution of qualities, $F^\beta(\pi)$, the entrepreneur's expected payoff when setting a quality π is

$$W^\beta(\pi) := [1 - F^\beta(\pi)](1-\beta)[S(\pi, k_L) + C(\pi)] + F^\beta(\pi)(1-\beta)[S(\pi, k_H) + C(\pi)] - C(\pi).$$

At the minimal bid, $\pi_L^\beta = \Pi((1-\beta)k_L)$, we have $W^\beta = W^\beta(\pi_L^\beta) = (1-\beta)S(\pi_L^\beta, k_L) - \beta C(\pi_L^\beta)$. Using equalizer property,

$$F^\beta(\pi) = \frac{W^\beta - (1-\beta)S(\pi, k_L) + \beta C(\pi)}{(1-\beta)[S(\pi, k_H) - S(\pi, k_L)]} = \frac{(\pi - \pi_L^\beta)k_L + \frac{1}{1-\beta}\left[C(\pi) - C(\pi_L^\beta)\right]}{(\pi - 1)(k_H - k_L)}.$$

□

Proof of Proposition 6

Proof. We first prove that an equilibrium in pure strategies doesn't exist.

- In a symmetric profile in pure strategies, each entrepreneur i expects $k_i = \frac{K}{2}$ and conditional on this amount of capital prefers $\pi_i = \Pi(\frac{K}{2})$. But as $\bar{S}(k) - (r-1)k$ is increasing in k (as long as the interest rate is not excessive, $r < \pi_L$), the entrepreneur prefers to deviate to $\pi_i = \pi_H$ and receive k_H.

- In an asymmetric profile in pure strategies, the entrepreneur with a higher return expects k_H and conditional on this amount of capital prefers $\pi_H = \Pi(k_H)$. The entrepreneur with a lower return expects k_L and conditional on this amount of capital prefers $\pi_L = \Pi(k_L)$. However, this entrepreneur prefers to deviate to $\pi_i = \pi_H + \epsilon$ (where $\epsilon > 0$ is sufficiently small) and receive k_H, because $S(\pi_H + \epsilon, k_H) - (r-1)k_H = \bar{S}(k_H) - (r-1)k_H - \Delta > \bar{S}(k_L) - (r-1)k_L$, because $\Delta > 0$ is sufficiently small due to continuity of $S(\pi, k)$ in both arguments.

In a mixed strategy equilibrium with support on an interval-type of support, both entrepreneurs need that the equalizingproperty holds for any strategy from the interval. (If the support is not on an interval, there are pure strategies within the interval that are not in the best response, and therefore the equalizing property doesn't apply; it changes into the property that the expected payoff is strictly lower.) Both entrepreneurs solve an identical problem which yields an identical F^r function with an identical support.

We use that the minimal bid is π_L and for $F^r(\pi_L) = 0$, and thus $W^r = \bar{S}(k_L) - (r-1)k_L$. By equalizer condition, $W^r(\pi)$ is constant in a mixed-strategy equilibrium. Using $W^r(\pi) = \bar{S}(k_L) - (r-1)k_L$ for $\pi > \pi_L$, we obtain $F^r(\pi)$. □

C Additional analyses

C.1 Equity competition with k-specific contracts

Allowing the entrepreneurs to offer a *menu of k-specific contracts* doesn't affect this result if the equilibrium is in weakly undominated strategies. We prove that an asymmetric equilibrium doesn't exist. Expecting the asymmetric allocation, the weaker entrepreneur (loser) again sets π_L and offers a share $\frac{1}{\pi_L}$ for the low level of capital $k_1 = k_L$: $x_1(k_L) = \frac{1}{\pi_L}$.

Now, there are two ways how the richness of menus may help the winner solidify her winning position (and protect the winner from loser's challenge). First, the winner may discourage the investor by making reallocation of low level of capital to the winner less attractive; it means to decrease the investor's return when the investor allocates only low level of capital. However, since the investor earns zero, there is no room to discourage the investor.

Second, the winner may effectively exploit that the richness gives the loser an opportunity to pose a threat to the winner. The threat is through a more attractive offer at $k_1 = k_H$, i.e., she may offer a menu with a high $x_1(k_H)$. If the loser's equilibrium menu involves the threat, then the winner is *disciplined* by the threat. This extra discipline may help the winner to protect her rents; the winner will be forced to a higher return to the investor which subsequently means that the loser's deviation will be discouraged.

Does this channel work, however? For any loser's counterproposal to be attractive for the investor, we must have that $x_1(k_H) \geqslant \frac{\pi_2}{\pi_L}\, x_2(k_H)$.[16] Second, the loser is better off with a threat; evaluating at the best (costless) loser's counteroffer $x_1(k_H) = \frac{\pi_2}{\pi_L}\, x_2(k_H)$, the loser is better off with a threat if

$$\left(1 - \frac{1}{\pi_L}\right) k_L < \left(1 - \frac{r_2(k_H)}{\pi_L}\right) k_H.$$

In other words, we require that the winner avoids a credible threat by setting a return:

$$r_2(k_H) := \pi_2 x_2(k_H) \geqslant \frac{\pi_L(k_H - k_L) + k_L}{k_H}.$$

[16]For example, if the winner sets $(\pi_2, x_2) = (\pi_H, \frac{1}{\pi_H})$, the threat is simply $x_1(k_H) > \frac{1}{\pi_L}$.

This is a necessary condition if we seek an equilibrium in weakly undominated strategies. In such an equilibrium, any counteroffer must be a credible threat; the loser must not be worse off by 'accidentally' winning (even if she expects to lose). Since the winner effectively sets the investor's return $r_2(k_H)$ (the condition doesn't restrict how the return is achieved), it is optimal to not distort the private investments and satisfy this constraint with $\pi_2 = \pi_H$. As a result, the winner's share

$$x_2(k_H) = \frac{\pi_L(k_H - k_L) + k_L}{\pi_H k_H}$$

Now, it is not difficult to evaluate that the winner is better off with this menu instead of deviating to a menu in which she wins only a low level of capital (i.e., by providing the zero return). The point is that the loser is disadvantaged; with a lower private investment π_H, her counteroffer must promise a significantly higher equity share than the winner promises, and consequently makes the threat less attractive. The winner is protected by the fact that the contracts cannot modify (i.e., increase) the early private investments:

$$\left(1 - \frac{1}{\pi_L}\right) k_L < \left(1 - \frac{r_2(k_H)}{\pi_L}\right) k_H.$$

However, this allocation is not an equilibrium allocation. In this allocation, the winner still earns rents that the loser wants to capture. In the equilibrium, we must take into account also deviation in the early investments. Specifically, when the loser beats the winner's allocation $(\pi_H, x_2(k_H))$ by imitating it (only with a slightly more attractive equity offer), the loser is better off as the previous equation shows. Therefore, the asymmetric allocation is not an equilibrium. The key problem is again that the loser can fully imitate the winner and consequently can seize the positive rents that are unavoidable. With richer menus of contracts, the rents are lower, but still exist, and this makes the asymmetric allocation unstable.

C.2 Equity competition with perfect substitution

Like in the main setting, each startup has constant returns to both inputs; in addition, each return is independent on the other input, and consequently cash flows feature constant returns to scale (up to the capacity limit),

$$V(k_i, \pi) = \pi + \rho \min\{k_i, k_H\}.$$

To create profits by employing the capital, we let the $\rho > 1$ be the constant return to capital. When the investor allocates the capital, she compares returns to capital $r = x\rho$, because earnings from the quality $x\pi$ are independent on the level of capital. The equity competition is competition in returns in which the quality is set only residually, namely to maximize $(1-x)\pi - C(\pi) = (1 - \frac{r}{\rho}) - C(\pi)$. Precisely, for a return r, the entrepreneur sets

$$(\hat{\pi}(r), \hat{x}(r)) = \left(\Pi(1 - \tfrac{r}{\rho}), \tfrac{r}{\rho}\right).$$

Notice that the quality is decreasing in the offered return and the equity is increasing in the offered return. The maximal quality is therefore $\overline{\pi} = \Pi(1 - \frac{r}{\rho})$. The minimal offered return is $\underline{r} = 1$, which includes the minimal equity offer $\underline{x} = \frac{1}{\rho}$. At the minimal offer, the probability of winning the capital prize is zero, $P(1) = 0$, and the entrepreneur's payoff is $(1-\frac{1}{\rho})\overline{\pi} - C(\overline{\pi}) + (\rho-1)k_L$. It is useful to denotes the entrepreneur's value from the production of quality as $Q(r) := (1 - \frac{r}{\rho})\Pi(1 - \frac{r}{\rho}) - C(\Pi(1 - \frac{r}{\rho}))$; this value is decreasing in r.

By equalizer property, the entrepreneur's payoff for any offered return r is identical to her payoff for the minimal return 1,

$$Q(1) + (\rho - 1)k_L = Q(r) + (\rho - r)[k_L + P(r)(k_H - k_L)].$$

The equilibrium distribution of the offered returns is

$$P(r) = \frac{Q(1) - Q(r) + (r - 1)k_L}{(\rho - r)(k_H - k_L)}.$$

See that $P(r)$ is increasing because the numerator is increasing and the denominator is decreasing in r. The maximal return offered to the investor is implicitly characterized by $Q(1) - Q(r) + rk_H = (1 - \rho)k_L + \rho k_H$.

To summarize: Even if the startup quality doesn't increase attractiveness of the bid, there is no full separation between the production through quality and the production from capital. The problem is that the equity offer affects both productions, and thus a higher bid for capital involves an *irreversible* cost in the production of capital (i.e., a drop in the entrepreneur's value $Q(r)$). As a result, the competition for capital is an all-pay auction.

C.3 Restricted equity vs. debt competition

What are the differences between equity and debt in this economy? This setting abstracts from the screening role of the investor and from the investor's value added to the project implementation. Without these features, it is expected that debt financing is more efficient as it provides stronger incentives to the entrepreneurs (de Bettignies and Brander, 2007; Da Rin, Hellmann, and Puri, 2013). However, when the entrepreneurs post unrestricted offers (debt, convertible debt or equity), Section 4 shows that the instrument is irrelevant in our economy.

With restrictions to a fixed interest rate and fixed equity share, the differences exist. First, compare the maximal bids. When r is fixed, the maximal bid is defined by $S(\overline{\pi}^r, k_H) - \bar{S}(k_L) = (r - 1)(k_H - k_L)$, while for a fixed x it is defined by $S(\overline{\pi}^x, k_H) - \bar{S}(k_L) = (\pi x - 1)k_H = (\frac{\pi}{\pi_L} - 1)k_H$. See that the RHS represents the *premium* that the winner demands to be willing to keep her winning position (to get more capital). There are two effects. (i) First, given an identical investor's return $r = \pi x$, the premium is now larger and thus the bids are lower. (ii) Second, it is useful to take the maximal bid in the previous regime and evaluate the endogenous interest rate in this regime, which is the interest rate $\frac{\overline{\pi}^r}{\pi_L} - 1$. If the endogenous interest rate is lower than the fixed interest rate $\frac{\overline{\pi}^r}{\pi_L} - 1 < r$, then the winner's premium drops, and the entrepreneurs can afford to overbid more. The second effect is opposite to the first effect. However, if $\frac{\overline{\pi}^r}{\pi_L} - 1 > r$, the second effect is consistent with the first effect; the premium is larger and the bids are lower. In other words, the entrepreneurs overbid (destroy winner's surplus) less.

Specifically, if the fixed interest rate is zero, $r = 1$, it is not difficult to derive that $F^r(\pi) \leqslant F^x(\pi)$. That is, *fixing the interest rate instead of the equity share incentivizes the private effort of the entrepreneurs better*. The difference is clear: With a fixed zero interest rate, the entrepreneurs know that the cost of capital for each unit of capital is zero; with a fixed equity share, the cost of capital is positive unless the human capital investment is minimal, $\pi = \pi_L$.