Deregulation: Why we should sometimes welcome even low-quality firms*

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Abstract

The quality of newly invented goods or services can only be assessed after they have been in use for a significant period. This applies to both high-quality and low-quality products—typically, the former survive, whereas the latter become obsolete in the market, resulting in resource wastage. We investigate this situation by developing a general equilibrium model. In a competitive market equilibrium, when the entry cost is relatively high compared to the research and development costs, only high-quality firms enter the market, ensuring efficiency. However, when the entry cost is low, low-quality firms enter the market, resulting in inefficiency. In this case, although entry regulations may exclude low-quality firms, they also reduce market competition and negatively impact economic welfare.

Keywords: General Equilibrium, Innovation, Regulation

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1 Introduction

The cryptocurrency boom, which has dominated the news in recent years, is only the latest in a long line of similar historical economic booms. From the railroad mania of the 1800s to the flood of green companies sponsored by the American Recovery and Reinvestment Act (ARRA) of 2009, these booms demonstrate how easily new businesses can suddenly emerge and attract countless customers and investors.

The intense competition that follows a boom can be a boon for a country in the long run. For example, the widespread construction of railroads during the boom of the 1800s significantly boosted the economy of Britain. Some of the largest companies today, such as Amazon and Google, emerged during the dotcom era in the early 2000s.

However, booms are notorious for the losses that they leave in their wake. The same boom that produced Amazon and Google also resulted in a loss of $5 trillion or approximately 75 % of the market. Due to poor management and quality problems, approximately 8 % of the ARRA-funded companies collapsed within years of being funded, most notably Solyndra and A123, wasting billions of dollars in taxpayer money.

Thus, booms can affect the economy both positively and negatively, complicating their effects on economic welfare. Fly-by-night operators who exploit these booms can be controlled by regulations. However such regulations can also limit creative destruction.

Therefore, in this study, we construct a general equilibrium model and examine the aggregate effect of regulations on economic welfare. We focus on the impact of fly-by-night operators, who inflict critical economic harm but are rarely considered in general equilibrium models.[1]

The proposed model has four features. First, entrants can decide whether to invest in product quality. Even those that do not invest in product quality can survive in the market for a while because product quality is not immediately apparent to consumers. Atkeson

[1] To the best of our knowledge, Atkeson et al. (2014) is the only study to consider the effect of fly-by-night operators in the general equilibrium model.

et al. (2014) considered a similar situation in which buyers do not know the quality of entrants' products and must rely on their reputation to evaluate them. Thus, low-quality firms can survive as long as their products are not perceived to be of low quality. Instead of considering the reputation-building process, we simplify it to the disclosure of firm quality after one period; however, the incentive for low-quality firms to enter the market and earn short-term profits remains.

Second, intermediate goods firms are monopolistically competitive, as per the expanding product variety model of Grossman and Helpman (1991); thus, the effect of competitiveness is considered.

Third, labor is used in the production of goods, firm entry, and research and development (R&D), as well as the official costs of entry.[2] Therefore, a change in labor demand in one sector affects employment in another.

Fourth, in the extended version of our model, we assume that low-quality firms not only waste economic resources but also cause negative externalities on economic welfare.

Regulation has several effects on economic performance in the steady state when these features are considered. First, regulation uses labor in the economy, although this labor does not directly produce anything, which negatively affects economic welfare because it diverts labor away from other sectors. Second, it reduces the proportion of low-quality entrants who waste the economy's labor, thereby increasing welfare. This effect is supported by empirical evidence. For example, Darnihamedani et al. (2018) demonstrated that the tax burden on entrepreneurs encourages innovative entrepreneurship. Other studies investigated the effect of deregulation on the quality of entrants and concluded that deregulation reduces entrant quality (e.g., Branstetter et al., 2014; Schulz et al., 2016; Rostam-Axchar, 2014).[3] Third, regulation reduces the total number of entrants, and therefore, incumbents, in the long

[2]Djankov et al. (2002), "with official costs of entry [being] extremely high in most countries."

[3]Branstetter et al. (2014) examined data from Portugal and observed that the deregulation of entry requirements allowed "marginal firms," which were typically small and had low-quality operators, to enter the market. These firms tended to be forced out within two years, showing the limitations of deregulation. Schulz et al. (2016), using German data, and Rostam-Axchar (2014), using Mexican data, found similar results: deregulation of entry requirements allows untrained workers to enter the market.

run, which decreases competition and lowers total factor productivity (TFP). This result is consistent with the findings of Bruhn (2011), who shows that "simplifying" market entry procedures increases competition, resulting in improved economic performance. Several other studies (e.g., Bruhn 2011; Yakovlev and Zhuravskaya 2013) add credence to the hypothesis that deregulation increases competition and TFP.

The overall effect of regulation on economic welfare depends on the ratio of entry fixed costs (including regulation) to R&D costs. If the entry fixed cost is relatively high, then all entrants are already high-quality firms; therefore, regulation is unnecessary, as it wastes economic resources and reduces TFP through the competition effect. If the entry cost is relatively low, at least a few low-quality firms will enter. In this case, regulations can help improve the entrant quality by excluding low-quality firms. However, this effect is insufficient to significantly improve economic welfare. Regulations may improve economic welfare in the case of negative externalities caused by low-quality firms, such as pollution, health issues, or product safety problems, depending on the degree of negative externalities.

Our analysis contributes to Schumpeterian growth theory[4] as we provide a tractable model that considers the effect of fly-by-night operators as well as labor market equilibrium. Unlike Atkeson et al. (2014), who focused on the reputation-building process of intermediate goods firms in a general equilibrium model, we direct our attention to resource constraints by including labor as input for intermediate goods, firm entry, and R&D.

The remainder of this paper is organized as follows. Section 2 presents the model. Section 3 defines and analyzes steady-state equilibrium. Section 4 examines economic welfare and how deregulation impacts it, and Section 5 concludes the paper.

2 The model

In this section, we introduce the model and describe each agent's activity.

[4]See Aghion et al. (2014) for a survey.

2.1 Structure of the model

Time is discrete and indexed by $t = 0, 1, 2, ..., \infty$. The economy consists of four kinds of agents: representative households, final good firms, intermediate goods firms, and potential entrants. While final good firms are perfectly competitive, intermediate goods firms compete monopolistically.

The quality of an intermediate good cannot be observed until it is consumed. Therefore, final good firms have expectations for the quality of intermediate goods. Every period, new intermediate firms can enter the market by paying a fixed entry cost, which includes regulatory costs such as legal costs and non-regulatory costs. They can produce high-quality goods if they choose to pay the R&D costs. These firms are hereafter termed "H-firms." However, if they decide not to pay the R&D costs, they will produce low-quality goods. Such firms are hereafter termed "L-firms." Once the final goods are consumed, the quality of each intermediate good becomes known to the public. Consequently, final good firms discontinue buying from L-firms, eventually forcing them to exit the market. This results in only H-firms remaining in the market.

A representative household lives infinitely. In each period, individuals allocate their income between consumption and savings to maximize their lifetime utility.

For convenience, Table 1 provides the definitions of the notations frequently used in this study. The timing of the events within each period is summarized as follows:

Table 1: Notations

Notation	Definition
n_t	the size of the incumbent at the beginning of the period
n_t^e	the size of the entrant at the beginning of the period
m_t	the size of the active incumbent
m_t^e	the size of the active entrant
ϕ_t	the share of H-firms out of the entrants
V_t	the value of an incumbent at the beginning of the period
V_t^H	the value of an H-firm at the beginning of the period
V_t^L	the value of an L-firm at the beginning of the period
$1 - \omega$	the exogenous exit rate for all firms

- At the beginning of the period, new intermediate goods firms enter the market, with or without investing in R&D.
- An external macro shock occurs in the intermediate goods market, forcing $1-\omega$ of firms to exit, leaving ω of firms in the market as active firms.
- Final good firms buy intermediate goods from active intermediate goods firms. After the final goods are consumed, the quality of each intermediate good is revealed.
- L-firms exit the market.

In the following subsections, we define the activities of each economic agent.

2.2 Firms

Here, we define the activities of final good and intermediate goods firms.

2.2.1 Final good firms

The production of final goods in period t (i.e., Y_t) is defined as follows:

$$Y_t = \left[\int_i z_t(i)^\alpha di \right]^{\frac{1}{\alpha}}, \quad 0 < \alpha < 1,$$
$$where\ z_t(i) = q(i)x_t(i),$$

where $x_t(i)$ is the amount and $q(i)$ is the quality of the intermediate good i. $q(i)$ equals 1 if the quality is high and 0 if the quality is low.

Final good firms buy intermediate goods from two types of sellers: incumbents and entrants. While final good firms know that all incumbent intermediate goods firms are H-firms, they do not know the quality of entrants because they have not used the products of those entrants. They believe that ϕ_t of those entrants are H-firms, and that finding one such unit requires buying from $1/\phi_t$ unit of random entrants.

For clarity, we redefine $x_t(i)$ as the amount of intermediate goods sold by the incumbent i, $x_t^e(i)$ as that sold by the entrant i, $p_t(i)$ as the price of the intermediate goods sold by the incumbent i, and $p_t^e(i)$ as that sold by the entrant i. The profit of the final goods firm, expressed as Π_t, is

$$\begin{aligned}\Pi_t &= P_t\left[\int_i z_t(i)^\alpha di\right]^{\frac{1}{\alpha}} - \int_0^{m_t} p_t(i)x_t(i)di - \int_0^{m_t^e} p_t^e(i)x_t^e(i)di, \\ &= P_t\left[\int_0^{m_t} x_t(i)^\alpha di + \int_0^{m_t^e}\{q(i)x_t^e(i)\}^\alpha di\right]^{\frac{1}{\alpha}} - \int_0^{m_t} p_t(i)x_t(i)di - \int_0^{m_t^e} p_t^e(i)x_t^e(i)di, \quad (1)\end{aligned}$$

where m_t is the size of active incumbent, m_t^e is the size of active entrant, and P_t is the price of the final good. Considering that the products of an L-firm do not add any value to the final goods, that is, $q(i) = 0$, the contribution of entrants to output in equation (1), that is $\int_0^{m_t^e}\{q(i)x_t^e(i)\}^\alpha di$, is expressed as $\int_0^{\phi_t m_t^e} x_t^e(i)^\alpha di$. In addition, under the belief ϕ_t, final good firms can find one unit of H-entrants out of $1/\phi_t$ units of random entrants. Hence, to buy $x_t^e(i)$ units of goods from one unit of H-entrants, they must spend $\int_0^{1/\phi_t} p_t^e(j)x_t^e(j)dj$ on $1/\phi_t$ units of random entrants. Therefore, the profit is expressed as:

$$\Pi_t = P_t\left[\int_0^{m_t} x_t(i)^\alpha di + \int_0^{\phi_t m_t^e} x_t^e(i)^\alpha di\right]^{\frac{1}{\alpha}} - \int_0^{m_t} p_t(i)x_t(i)di - \int_0^{\phi_t m_t^e}\int_0^{1/\phi_t} p_t^e(j)x_t^e(j)djdi.$$

The respective first-order conditions concerning $x_t(i)$ and $x_t^e(i)$ are:

$$P_t Y_t^{1-\alpha} x_t(i)^{\alpha-1} = p_t(i), \quad (2)$$

$$P_t Y_t^{1-\alpha} x_t^e(i)^{\alpha-1} = \frac{p_t^e(i)}{\phi_t}. \quad (3)$$

Given the first-order conditions (2) and (3), the respective demand for $x_t(i)$ and $x_t^e(i)$ are as follows:

$$x_t(i) = \left(\frac{p_t(i)}{P_t}\right)^{\frac{1}{\alpha-1}} Y_t, \tag{4}$$

$$x_t^e(i) = \left(\frac{p_t^e(i)}{\phi_t P_t}\right)^{\frac{1}{\alpha-1}} Y_t. \tag{5}$$

By combining the first-order conditions (2) and (3) with the zero profit condition for the final goods firm, P_t is expressed as follows:

$$P_t = \left[\int_0^{m_t} p(i)^{\frac{\alpha}{\alpha-1}} di + \int_0^{m_t^e} \hat{\phi}_t p_t^e(i)^{\frac{\alpha}{\alpha-1}} di\right]^{\frac{\alpha-1}{\alpha}}, \tag{6}$$

where $\hat{\phi}_t \equiv \phi_t^{\frac{1}{1-\alpha}}$. Equation (6) implies that the price of the final goods decreases with the share of H-entrants.

2.2.2 Intermediate goods firms

Intermediate goods firms compete monopolistically. We assume that producing a unit of intermediate goods costs ψ units of labor regardless of product quality. Each firm determines the amount of production to maximize its profit based on the demand functions (4) or (5). Therefore, the respective profits of incumbents π_t and entrants π_t^e are:

$$\pi_t(i) = (p_t(i) - \psi w_t) x_t(i),$$

$$\pi_t^e(i) = (p_t^e(i) - \psi w_t) x_t^e(i),$$

where w_t denotes the wage rate. By solving the profit-maximization problem for each intermediate firm, the prices of goods produced by incumbents and entrants are:

$$p_t(i) = p_t^e(i) = \frac{\psi w_t}{\alpha} \equiv p_t. \tag{7}$$

As evident from equation (7), all active firms set the same price whether they are entrants or incumbents, that is, $p_t(i) = p_t^e(i)$. This, together with equations (4) and (5), suggests that production among incumbents is the same, as is production among entrants. Therefore, instead of using i, we define x_t as the production of incumbents and x_t^e as that of entrants. Despite them setting the same price, equations (4), (5), and (7) also suggest that final good firms utilize goods produced by incumbents more than those by entrants, that is, $x_t > x_t^e$. This is because the cost of discovering one unit of H-firms in the entrant market is $1/\phi_t$ times higher than that in the incumbent market.

As final good firms cannot know the quality of the entrants, all the entrants can sell their products and earn the same profits, regardless of their quality. The respective resulting profits for the incumbents and entrants are:

$$\pi_t = (1-\alpha)\left(\frac{\psi w_t}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} P_t^{\frac{1}{1-\alpha}} Y_t, \tag{8}$$

$$\pi_t^e = \hat{\phi}_t(1-\alpha)\left(\frac{\psi w_t}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} P_t^{\frac{1}{1-\alpha}} Y_t = \hat{\phi}_t \pi_t. \tag{9}$$

This indicates that the profits of entrants are smaller than those of incumbents.

Although the profits of H- and L-entrants are the same, L-entrants cannot survive because their qualities are revealed once their products are used, compelling them to exit the market. Therefore, the respective values of the incumbents, V_t, H-entrants, V_t^H, and L-entrants, V_t^L, are:

$$V_t = \omega\left(\pi_t + \frac{V_{t+1}}{1+r_t}\right), \tag{10}$$

$$V_t^H = \omega\left(\hat{\phi}_t \pi_t + \frac{V_{t+1}}{1+r_t}\right), \tag{11}$$

$$V_t^L = \omega \hat{\phi}_t \pi_t. \tag{12}$$

2.2.3 Entry

When entering the market, entrants can choose to invest in R&D, becoming H-firms, or proceed without investing in R&D, becoming L-firms that are eventually forced out of the market after profiting once. We assume that K units of labor are needed to succeed in R&D. In addition, to enter the market and produce intermediate goods, entrants must pay F units of labor as an entry fixed cost, which includes regulatory costs such as legal costs and non-regulatory costs. The entrants issue equities to finance these costs.

The respective free-entry conditions for H- and L-entrants are:

$$\underbrace{\omega\left(\hat{\phi}_t\pi_t + \frac{V_{t+1}}{1+r_t}\right)}_{V_t^H} \leq (K+F)w_t, \tag{13}$$

$$\underbrace{\omega\hat{\phi}_t\pi_t}_{V_t^L} \leq Fw_t, \tag{14}$$

where, in each condition above, the left-hand side represents the expected benefit after entry and the right-hand side represents the cost of entering the market. In either condition, the equality is satisfied when entries occur.

2.3 Households

Representative households are infinitely-lived. The utility of households in period t is the sum of all discounted utilities from future consumption, expressed as:

$$U_t = \sum_{\tau=t}^{\infty} \beta^{\tau-t} \ln c_\tau, \tag{15}$$

where c_τ is the consumption in period τ and β is the time preference rate. The intertemporal budget constraint is:

$$w_t + m_t\pi_t + m_t^e\pi_t^e = E_t + n_t^e V_t^e. \tag{16}$$

The left-hand side of equation (16) represents the income of the representative agent: w_t is labor income, π_t is dividend income from active incumbents, and π_t^e is dividend income from active entrants. The right-hand side denotes expenditures: E_t is the consumption expenditure on final goods, which is equal to $P_t c_t$, and $n_t^e V_t^e$ is the expenditure on entrants' equities, where $V_t^e = \phi_t V_t^H + (1-\phi_t)V_t^L$, and n_t^e is the size of the entrant at the beginning of the period before the external macro shock. Households have all assets in the form of stocks of intermediate firms, and we define these amounts at the beginning of the period t, that is, $n_t V_t$, as a_t. Therefore, the intertemporal budget constraint is expressed as follows[5]:

$$w_t = E_t + \frac{a_{t+1}}{1+r_t} - a_t. \tag{17}$$

Given the budget constraint (17), a household chooses the expenditure on the final goods E_t and the amount of the asset in the next period a_{t+1} for $t = 0, 1, \cdots, \infty$ to maximize utility (15). The Euler equation is derived from the first-order condition with respect to a_{t+1} as follows:

$$\frac{E_{t+1}}{E_t} = \beta(1+r_t). \tag{18}$$

We normalize the expenditure on the consumption good in every period to one, that is, $E_t = 1$. Subsequently, the Euler equation (18) implies the following:

$$\frac{1}{1+r_t} = \beta. \tag{19}$$

[5]By substituting equations (11) and (12) into $V_t^e = \phi_t V_t^H + (1-\phi_t)V_t^L$, and using $n_t^e = \frac{m_t^e}{\omega}$, the aggregate value of entrants is expressed as $n_t^e V_t^e = m_t^e \pi_t^e + \frac{\phi_t m_t^e V_{t+1}}{1+r_t}$. On the other hand, from equation (10), the dividend from active incumbent firms is expressed as $m_t \pi_t = \omega n_t \left(\frac{V_t}{\omega} - \frac{V_{t+1}}{1+r_t} \right) = a_t - \frac{m_t V_{t+1}}{1+r_t}$, because $n_t V_t = a_t$ and $\omega n_t = m_t$. Moreover, the size of incumbents at the beginning of period $t+1$ (i.e., n_{t+1}) is the size of active H-firms at the end of period t; therefore, $n_{t+1} = m_t + \phi_t m_t^e$. By substituting the derived equations for $n_t^e V_t^e$, $m_t \pi_t$, and n_{t+1} into equation (16), we obtain equation (17).

3 Market equilibrium

In this section, we define market equilibrium and efficient allocation and demonstrate that the allocation in market equilibrium is not always efficient.

Definition 1. *The market equilibrium is provided by the trajectory* $\{c_t, Y_t, m_t^e, m_t, n_t^e, n_t, \phi_t, w_t, r_t, P_t, V_t, V_t^H, V_t^L\}$ *such that, in each period, the following conditions hold:*

1. *Households choose consumption and savings to maximize their lifetime utility, taking* r_t*,* w_t*, and* P_t *as given.*

2. *Final good firms maximize profit, taking* $P_t, w_t, p_t(i)$*, and* $p_t^e(i)$ *as given. Thus, the demands for intermediate goods produced by incumbents and entrants are expressed as in equations (4) and (5), respectively.*

3. *The final good producers' belief* ϕ_t *is consistent with the share of H-entrants out of all entrants.*

4. *Free-entry conditions (13) and (14) are satisfied.*

5. *The final good market, intermediate goods market, labor market, and asset market clear.*

Here, we characterize the market equilibrium.

Corollary 1. *There is no market equilibrium in which only L-firms enter.*

Proof. Suppose that only L-firms enter the market in equilibrium, that is, $\phi_t = 0$ and $n_t^e > 0$. In this situation, final goods firm do not buy products from entrants because they know that the entrants' products only incur costs without adding any value to the final goods. Therefore, the profit of entrants becomes zero, and the free-entry condition (14) is satisfied with inequality, implying that L-firms do not enter the market. This result contradicts the assumption that $n_t^e > 0$. □

This corollary shows that if entries occur, a certain number of those entrants are H-entrants. We first examine the production side of the economy. As the prices of all intermediate goods are identical, from equations (6) and (7), the final good price is:

$$P_t = \frac{\psi w_t}{\alpha} \left[m_t + \hat{\phi}_t m_t^e \right]^{\frac{\alpha-1}{\alpha}}. \tag{20}$$

The final good market clearing condition is:

$$Y_t = \frac{1}{P_t}, \tag{21}$$

where the right-hand side $\frac{1}{P_t} = c_t$ is the consumer demand. By substituting equations (7), (21), and (20) into equations (4) and (5), we obtain the production of the incumbents and entrants as follows:

$$x_t = \frac{\alpha}{\psi w_t (m_t + \hat{\phi}_t m_t^e)}, \tag{22}$$

$$x_t^e = \frac{\hat{\phi}_t \alpha}{\psi w_t (m_t + \hat{\phi}_t m_t^e)} = \hat{\phi}_t x_t. \tag{23}$$

Moreover, by substituting equations (7), (22), and (23) into equations (8) and (9), the profits of the incumbents and entrants can be expressed as:

$$\pi_t = \frac{1-\alpha}{m_t + \hat{\phi}_t m_t^e}, \tag{24}$$

$$\pi_t^e = \frac{\hat{\phi}_t (1-\alpha)}{m_t + \hat{\phi}_t m_t^e} = \hat{\phi}_t \pi_t. \tag{25}$$

Next, the labor market clearing condition is given by:

$$F n_t^e + K \phi_t n_t^e + \psi (m_t x_t + m_t^e x_t^e) = 1, \tag{26}$$

where the first two terms on the left-hand side represent the labor demanded by the entrants to enter the market and the third term represents the labor demanded to produce intermediate goods.

Finally, we investigate the dynamics of the intermediate goods firms. Given that only H-firms remain in the market after production, the size of incumbents at the beginning of period $t+1$, that is, n_{t+1}, is:

$$n_{t+1} = m_t + \phi_t m_t^e. \tag{27}$$

In addition, an exogenous shock impacts all firms, implying that $1-\omega$ of firms are forced to exit the market regardless of their quality; the sizes of the active incumbent and entrant are ω of those at the beginning of the period, that is:

$$m_t = \omega n_t, \;\; m_t^e = \omega n_t^e. \tag{28}$$

Therefore, the transition in the size of the active incumbent m_t is:

$$m_{t+1} = \omega(m_t + \phi_t m_t^e). \tag{29}$$

The following proposition summarizes the relationship between the entry rate (i.e., $n_t^e/(n_t + n_t^e)$) and the entrant relative size (i.e., $m_t^e x_t^e/(m_t x_t + m_t^e x_t^e)$) in the equilibrium.

Proposition 1. *Given ϕ_t, the relationship between the entry rate and the entrant relative size is as follows:*

1. *If $0 < \phi_t < 1$, the entry rate is larger than the entrant's relative size, that is, $\frac{n_t^e}{n_t+n_t^e} > \frac{m_t^e x_t^e}{m_t x_t + m_t^e x_t^e}$.*

2. *If $\phi_t = 1$, the entry rate is equal to the entrant relative size, that is, $\frac{n_t^e}{n_t+n_t^e} = \frac{m_t^e x_t^e}{m_t x_t + m_t^e x_t^e}$.*

Proof. As $n_t = \frac{m_t}{\omega}$ and $n_t^e = \frac{m_t^e}{\omega}$ from equation (28), the entry rate is $\frac{n_t^e}{n_t+n_t^e} = \frac{m_t^e}{m_t+m_t^e}$. By contrast, by using equations (22) and (23), the entrant relative size is $\frac{m_t^e x_t^e}{m_t x_t + m_t^e x_t^e} = \frac{\hat{\phi}_t m_t^e}{m_t + \hat{\phi}_t m_t^e}$.

Therefore,

$$\frac{n_t^e}{n_t + n_t^e} - \frac{m_t^e x_t^e}{m_t x_t + m_t^e x_t^e} = \frac{(1-\hat{\phi}_t) m_t m_t^e}{(m_t + m_t^e)(m_t + \hat{\phi}_t m_t^e)},$$

which is zero when $\phi_t = 1$ and positive when $\phi_t < 1$. □

When $0 < \phi_t < 1$, the result in Proposition 1 is consistent with the observations (U.S. 1963–82) by Dunne et al. (1998). Here, we focus on two types of market equilibria: one in which only H-firms enter, that is, $\phi_t = 1$, and the other in which both H- and L-firms enter, that is, $0 < \phi_t < 1$.

3.1 Market equilibrium with H- and L-firms ($0 < \phi_t < 1$)

First, we focus on the market equilibrium in which both H- and L-firms enter, that is, $0 < \phi_t < 1$ and $n_t^e > 0$. In this case, free-entry conditions (13) and (14) are satisfied with equality. By substituting equation (25) into the free-entry condition for L-firms (14), we obtain:

$$\frac{\omega(1-\alpha)\hat{\phi}_t}{m_t + \hat{\phi}_t m_t^e} = F w_t. \tag{30}$$

On the contrary, the free-entry condition for H-entrants (13) and L-entrants (14) together with equation (19) imply the following:

$$\omega \beta V_{t+1} = K w_t. \tag{31}$$

Substituting equations (14), (19), and (31) into equation (10), the value of the incumbent is:

$$V_t = \left(\frac{F}{\hat{\phi}_t} + K \right) w_t. \tag{32}$$

By substituting equations (22), (23), and (28) into equation (26), the labor market equilibrium condition is expressed as follows:

$$(F + \phi_t K)\frac{m_t^e}{\omega} + \frac{\alpha}{w_t} = 1. \tag{33}$$

Therefore, the market equilibrium when $0 < \phi_t < 1$ is characterized by equations (29)–(33).

3.2 Market equilibrium with only H-firms ($\phi_t = 1$)

Next, we characterize the equilibrium in which only H-firms enter, that is, $\phi_t = 1$ and $n_t^e > 0$. As L-firms do not enter the market, the inequality holds in the free-entry condition (14). By substituting $\phi_t = 1$ and equation (25) into this, the free-entry condition for L-firms is:

$$\frac{\omega(1-\alpha)}{m_t + m_t^e} < F w_t. \tag{34}$$

By contrast, the free-entry condition for H-firms is satisfied with equality. As $\phi_t = \hat{\phi}_t = 1$, the values of H-entrants and incumbents are identical, that is, $V_t^H = V_t$. Therefore, equations (10), (11), and (13) imply the following:

$$V_t = (K + F)w_t. \tag{35}$$

By substituting $\phi_t = 1$ and equation (24) into equation (10), the value of the incumbents is:

$$V_t = \omega\left(\frac{1-\alpha}{m_t + m_t^e} + \beta V_{t+1}\right). \tag{36}$$

By substituting equations (22), (23), (28), and $\phi_t = 1$ into equation (26), the labor market equilibrium becomes:

$$(F + K)n_t^e + \frac{\alpha}{w_t} = 1. \tag{37}$$

Finally, the transition in the size of active incumbent m_t is:

$$m_{t+1} = \omega(m_t + m_t^e). \tag{38}$$

Therefore, the market equilibrium when $\phi_t = 1$ is characterized by equations (35)–(38) and free-entry condition (34).

To compare the market equilibrium with efficient allocation, we define efficient allocation as follows:

Definition 2. *Efficient allocation occurs with perfect information, where the quality of each firm is immediately observable.*

Under perfect information, no one buys the products produced by L-firms. Hence, entrants have no incentives to enter the market as L-firms. In this case, $\phi_t = 1$, and the next lemma characterizes its allocation.[6]

Lemma 1. *Efficient allocation is characterized by equations (35)–(38).*

Although the equations characterizing efficient allocation are similar to those characterizing the market equilibrium with only H-firms, the efficient allocation differs because it does not require the free-entry condition for L-firms.

4 Steady-state analysis

In this section, we focus on the steady state, where all variables are constant regardless of time, and evaluate economic welfare in the steady state. As an external macro shock forces some firms to exit the market randomly, entries always occur in the steady state, that is, $n^e > 0$. Hereafter, the steady-state variables are indicated by omitting the time subscript.

[6] The efficient allocation coincides with the allocation of the discrete version of Grossman and Helpman (1991) Ch. 3.

4.1 Resource allocation in the steady state

The steady-state properties are summarized in Proposition 2.

Proposition 2. *(i) When* $0 < \frac{F}{K} < \frac{1-\omega\beta}{\omega\beta}$*, a unique steady state exists in the market equilibrium, where both the H- and L-firms enter the market, that is,* $0 < \phi < 1$*. In the steady state, the following equations hold:*

$$\phi = \left(\frac{\omega\beta}{1-\omega\beta}\frac{F}{K}\right)^{1-\alpha}, \tag{39}$$

$$m = \frac{\omega^2(1-\alpha)}{(1-\alpha)(1-\omega)\left(\frac{F}{\phi}+K\right)+\alpha\omega F\left(\left(\frac{1}{\phi}\right)^{\frac{1}{1-\alpha}}+\frac{1-\omega}{\omega}\frac{1}{\phi}\right)}, \tag{40}$$

$$w = \frac{\omega(1-\alpha)}{F\left(\left(\frac{1}{\phi}\right)^{\frac{1}{1-\alpha}}+\frac{1-\omega}{\omega}\frac{1}{\phi}\right)m}, \tag{41}$$

$$P = \frac{\omega\psi(1-\alpha)\phi^{\frac{1}{1-\alpha}}}{\alpha F m^{\frac{1}{\alpha}}\left(1+\frac{1-\omega}{\omega}\phi^{\frac{\alpha}{1-\alpha}}\right)^{\frac{1}{\alpha}}}. \tag{42}$$

(ii) When $\frac{1-\omega\beta}{\omega\beta} < \frac{F}{K}$*, a unique steady state exists in the market equilibrium, where only H-firms enter the market, that is,* $\phi = 1$*. Here, the steady-state variables are as follows:*

$$m = \frac{\omega^2(1-\alpha)}{(K+F)\left\{1-\omega(1-\alpha+\alpha\beta)\right\}}, \tag{43}$$

$$w = \frac{\omega^2(1-\alpha)}{(1-\omega\beta)(K+F)m}, \tag{44}$$

$$P = \frac{\omega^{\frac{1+\alpha}{\alpha}}\psi(1-\alpha)}{\alpha(1-\omega\beta)(F+K)m^{\frac{1}{\alpha}}}. \tag{45}$$

(iii) In an efficient allocation, regardless of $\frac{F}{K}$*, a unique steady state exists in which only H-firms enter the market. Each variable in the steady state is the same as in Case (ii).*

Proof. See Appendix. □

Proposition 2 suggests that the steady-state property is affected by the relative cost of entry F compared to the R&D cost K. The relationship between the share of H-entrants among all entrants (ϕ) and the relative cost of entry compared to the R&D cost (F/K) is summarized in Figure 1. When F/K is relatively small, that is, $F/K < \frac{1-\omega\beta}{\omega\beta}$, ϕ is increasing in F/K. This is because an increase in F decreases the entry of both L- and H-firms via the free-entry conditions, which reduces competition among active firms. As H-firms survive longer than L-firms, an increase in profit from weakened competition raises the value of H-firms more than that of L-firms. Therefore, the entry of H-firms decreases less than that of L-firms, as indicated by the increase in ϕ. By contrast, an increase in K negatively affects only H-entrants through the free-entry condition, leading to a decrease in ϕ.

Moreover, because an increase in either ω or β increases the value of incumbents, it raises the incentives for H-entrants to enter the market. Therefore, as indicated by the broken line in Figure 1, an increase in ω or β increases ϕ for a given F/K.

As F can be partly controlled by governments by imposing legal procedures, an increase in F can be interpreted as an increase in regulations. In summary, an increase in regulations improves the average quality of entrants when L-entrants exist. As L-firms waste economic resources without contributing to production, an increase in regulations improves the average productivity of intermediate goods firms. Simultaneously, this discourages the entry of H-firms, leading to a decrease in incumbents, which weakens competition. Furthermore, regulations require labor that is not used for productive activities, tightening the labor market and decreasing its supply for productive activities. Therefore, the aggregate effect of an increase in regulations on economic welfare remains unclear. In the following subsection, we discuss this effect in detail.

4.2 Welfare

Welfare in the steady state is given by:

$$U = -\frac{1}{1-\beta} \ln P,$$

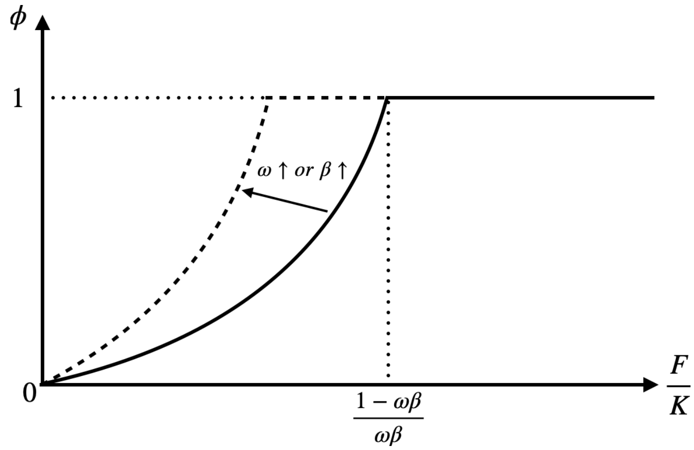

Figure 1: Relationship between ϕ and F/K

where P is defined in equation (42) when $0 < \frac{F}{K} < \frac{1-\omega\beta}{\omega\beta}$ and in equation (45) when $\frac{1-\omega\beta}{\omega\beta} < \frac{F}{K}$.

Proposition 3. *In the steady state, for a given innovation cost K, the final good price P increases with the entry cost F. Therefore, utility in the steady state is decreasing in the entry cost.*

Proof. See Appendix. □

Proposition 3 suggests that an increase in regulations harms economic welfare, although it can improve the average productivity of intermediate goods firms by excluding L-firms. When only H-firms enter, that is, $F/K > (1 - \omega\beta)/\omega\beta$, an increase in regulations only deprives labor of productive sectors, and the quality of entrants does not improve any further. In this case, the variety of goods decreases and the intermediate goods market becomes less competitive, leading to a decrease in economic welfare. By contrast, when both H- and L-firms enter, that is, $F/K < (1 - \omega\beta)/\omega\beta$, an increase in regulations still deprives labor of productive sectors, although now it improve the quality of entrants, which positively affects economic welfare. However, as entrants produce less than incumbents, the positive effect of the decrease in the share of L-firms among entrants is insufficient to compensate for the negative effects of the decrease in the variety of goods and competition. On the opposite end, Proposition 3 suggests that deregulation improves economic welfare even though it encourages the entry of L-firms.

4.3 Effect of negative externality

As suggested in the previous subsection, even when we consider the existence of low-quality firms that waste resources, regulations cannot be rationalized from the perspective of economic welfare. However, in reality, low-quality firms not only waste economic resources but also cause negative externalities such as pollution, health issues, and product safety concerns. In this subsection, we consider the impact of this type of negative externality on economic welfare. The utility function is defined as:

$$U_t = \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[\ln c_\tau - v(b_\tau) \right], \tag{46}$$

where b_τ is the amount of intermediate goods produced by L-firms in period τ, that is, $b_\tau = \omega(1-\phi_\tau)m_\tau^e x_\tau^e$, and $v(b_\tau)$ is the cost of the negative externality on economic welfare. We assume $v' > 0,\ v'' > 0$.

As resource allocation in competitive equilibrium is the same as that in Section 4.1 and welfare from consumption is the same as that in Section 4.2, we focus on the effect of an increase in regulations on the size of the negative externality in the steady state, that is, b. When $\frac{1-\omega\beta}{\omega\beta} \leq \frac{F}{K}$, no negative externality exists because L-firms never enter the market, and hence, $b = 0$. When $\frac{1-\omega\beta}{\omega\beta} > \frac{F}{K}$, as $1-\phi$ of L-firms enter the market, by applying the steady state allocation in Section 4.1 to $b = \omega(1-\phi)m^e x^e$, the size of the negative externality is given by:

$$b = \frac{\alpha\omega(1-\omega)}{\psi} \frac{(1-\phi)}{1-\omega+\left\{(1-\alpha)(1-\omega)+\frac{\alpha(1-\omega\beta)}{\beta}\right\}\frac{\omega\beta}{1-\omega\beta}\phi^{\frac{-\alpha}{1-\alpha}}}, \tag{47}$$

where ϕ is defined by equation (39). As ϕ is monotonically increasing in F, we analyze the

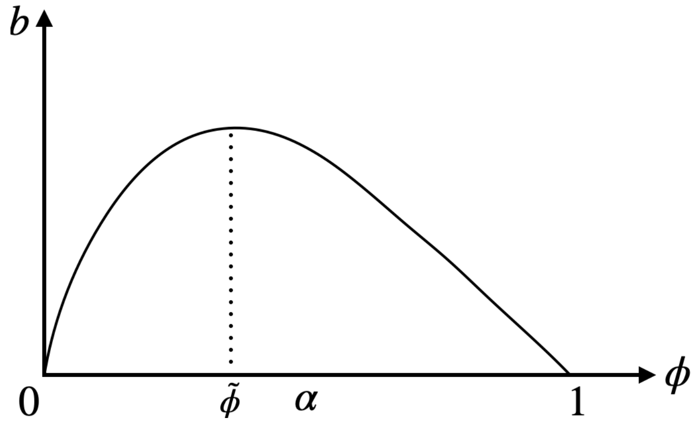

Figure 2: Relationship between b and ϕ

effect of ϕ on b. By differentiating equation (47) with respect to ϕ, we obtain:

$$\frac{\partial b}{\partial \phi} = \frac{\alpha\omega(1-\omega)}{\psi} \frac{\frac{A}{1-\alpha}\frac{\alpha-\phi}{\phi^{\frac{1}{1-\alpha}}} - (1-\omega)}{\left\{1-\omega+A\phi^{\frac{-\alpha}{1-\alpha}}\right\}^2}, \tag{48}$$

where $A = \left\{(1-\alpha)(1-\omega) + \frac{\alpha(1-\omega\beta)}{\beta}\right\}\frac{\omega\beta}{1-\omega\beta}$. The relationship between b and ϕ is shown in Figure 2.[7]

An increase in F affects b through three channels: first, it reduces the share of L-entrants among entrants; second, the number of entrants decreases and along with the first, these conditions result in a decrease in b; and third, each entrant's production increases as buyers expect a rise in the quality of their products, which increases b. When F is small, the third effect dominates the first and second effects, implying that an increase in regulations increases the negative externality. Thus, an increase in regulations has a negative impact on economic welfare. When F is large, the first and second effects outweigh the third, meaning that an increase in regulations reduces the negative externality. This improves economic welfare.

Although regulations negatively affect welfare from consumption, as analyzed in Section

[7]The denominator of equation (48) is positive and the numerator is decreasing in ϕ. Therefore, a unique ϕ that satisfies $\frac{\partial b}{\partial \phi} = 0$ exists. Let $\tilde{\phi}$ denote such ϕ. As $\frac{\partial b}{\partial \phi}\big|_{\phi=\alpha} = \frac{-\alpha(1-\omega)^2}{\psi\left\{1-\omega+A\phi^{\frac{-\alpha}{1-\alpha}}\right\}^2} < 0$, $\tilde{\phi}$ is smaller than α, that is, $\tilde{\phi} < \alpha$.

4.3, the impact of negative externality on welfare depends on the entry cost, as analyzed here. Therefore, the overall effect of regulations on economic welfare is ambiguous. However, when F is sufficiently small, regulations are detrimental to economic welfare because they negatively affect both consumption and externality. By contrast, when F is sufficiently large, the negative effect of regulations on consumption may be outweighed by the positive effect on the externality. In this case, regulation is helpful, which is consistent with the findings of Atkeson et al. (2015).

5 Conclusion

By developing a general equilibrium model, we examined the situation in which the quality of firms are not observable until their products are consumed, allowing low-quality firms to enter the market. In a competitive market equilibrium, when the entry cost is relatively high compared to the R&D costs, only high-quality firms enter the market, ensuring efficiency. However, when the entry cost is relatively low, low-quality firms enter the market, resulting in inefficiency. In this case, while entry regulations may exclude low-quality firms, they also reduce market competition and adversely impact economic welfare. Therefore, deregulation can be effective to improve economic welfare despite it encouraging the existence of low-quality firms.

A Appendix

A.1 Proof of Proposition 2

(i) Steady-state when $0 < \frac{F}{K} < \frac{1-\omega\beta}{\omega\beta}$

We derive the steady-state variables when $0 < \frac{F}{K} < \frac{1-\omega\beta}{\omega\beta}$. Suppose that $\phi < 1$. From equations (30)–(33) and (29), variables (m, m^e, V, w, ϕ) in the steady state are characterized by the following equations:

$$\frac{\omega(1-\alpha)\hat{\phi}}{m+\hat{\phi}m^e} = Fw, \tag{A.1}$$

$$\omega\beta V = Kw, \tag{A.2}$$

$$V = \left(\frac{F}{\hat{\phi}} + K\right) w, \tag{A.3}$$

$$(F+\phi K)\frac{m^e}{\omega} + \frac{\alpha}{w} = 1, \tag{A.4}$$

$$m = \omega(m+\phi m^e). \tag{A.5}$$

The division of (A.2) by (A.3) implies the following:

$$\phi = \left(\frac{\omega\beta}{1-\omega\beta}\frac{F}{K}\right)^{1-\alpha}. \tag{A.6}$$

When $0 < \frac{F}{K} < \frac{1-\omega\beta}{\omega\beta}$, ϕ in equation (A.6) is smaller than 1, which is consistent with the assumption that $\phi < 1$. Equation (A.5) leads to the following:

$$m^e = \frac{1-\omega}{\phi\omega} m. \tag{A.7}$$

By substituting equations (A.7) and (A.5) into equation (A.1), w is expressed as follows:

$$w = \frac{\omega(1-\alpha)}{F\left(\left(\frac{1}{\phi}\right)^{\frac{1}{1-\alpha}} + \frac{1-\omega}{\omega}\frac{1}{\phi}\right) m}. \tag{A.8}$$

Therefore, by substituting equations (A.7) and (A.8) into equation (A.4), m is given as:

$$m = \frac{\omega^2(1-\alpha)}{(1-\alpha)(1-\omega)\left(\frac{F}{\phi}+K\right) + \alpha\omega F\left(\left(\frac{1}{\phi}\right)^{\frac{1}{1-\alpha}} + \frac{1-\omega}{\omega}\frac{1}{\phi}\right)}, \tag{A.9}$$

where ϕ is determined in equation (A.6). We derive the price of the final goods P by substituting equations (A.7), (A.8), and (A.9) into equation (20).

(ii) Steady-state when $\frac{1-\omega\beta}{\omega\beta} < \frac{F}{K}$

Next, we present the steady-state variables when $\frac{1-\omega\beta}{\omega\beta} < \frac{F}{K}$. Assume that $\phi = 1$. From equations (34)–(38), the variables (m, m^e, V, w) in the steady state are characterized by the following equations:

$$\frac{\omega(1-\alpha)}{m+m^e} < Fw, \tag{A.10}$$

$$V = (K+F)w, \tag{A.11}$$

$$V = \omega\left(\frac{1-\alpha}{m+m^e} + \beta V\right), \tag{A.12}$$

$$(F+K)\frac{m^e}{\omega} + \frac{\alpha}{w} = 1, \tag{A.13}$$

$$m = \omega(m+m^e). \tag{A.14}$$

By solving equation (A.14) with respect to m^e, we obtain:

$$m^e = \frac{1-\omega}{\omega}m. \tag{A.15}$$

Substituting equations (A.11) and (A.15) into equation (A.12) yields:

$$w = \frac{\omega^2(1-\alpha)}{(1-\omega\beta)(K+F)m}. \tag{A.16}$$

By substituting equations (A.15) and (A.16) into equation (A.13), we obtain:

$$m = \frac{\omega^2(1-\alpha)}{(K+F)\left\{1-\omega(1-\alpha+\alpha\beta)\right\}}. \tag{A.17}$$

Therefore, by using equations (A.15) and (A.16), we obtain:

$$\frac{\omega(1-\alpha)}{m+m^e} - Fw = \frac{\omega^2(1-\alpha)}{m} - \frac{\omega^2(1-\alpha)F}{(1-\omega\beta)(F+K)m},$$
$$= \frac{\omega^3\beta(1-\alpha)K}{(1-\omega\beta)(F+K)m}\left(\frac{1-\omega\beta}{\omega\beta} - \frac{F}{K}\right), \tag{A.18}$$

where the left-hand side of equation (A.18) is negative when $\frac{1-\omega\beta}{\omega\beta} < \frac{F}{K}$. Therefore, the free-entry condition for L-firms (A.10) holds, which means that only H-firms enter when $\frac{1-\omega\beta}{\omega\beta} < \frac{F}{K}$. Therefore, the assumption that $\phi = 1$ is consistent with the result. We derive the price of the final goods P by substituting equations (A.15), (A.16), and (A.17) into (20).

A.2 Proof of Proposition 3

As welfare U is decreasing in the final good price P, we examine the relationship between P and F.

(i) Welfare when $0 < \frac{F}{K} < \frac{1-\omega\beta}{\omega\beta}$

By substituting equations (39) and (40) into equation (42), we can express the final good price as follows:

$$P = \frac{\psi\beta\omega^2(1-\alpha)}{\alpha(1-\omega\beta)K}[g(F)]^{\frac{1}{\alpha}}, \tag{A.19}$$

where $g(F)$ is defined as:

$$g(F) = \frac{(1-\omega)\left(\frac{F}{\phi}+K\right)}{\omega^2\left(1+\frac{1-\omega}{\omega}\frac{\hat{\phi}}{\phi}\right)} + \frac{\alpha\omega(1-\omega\beta)}{\omega\beta}K. \tag{A.20}$$

As $sign\left(\frac{dP}{dF}\right) = sign(g'(F))$, we analyze the properties of $g(F)$. Differentiating equation (A.20) yields the following:

$$g'(F) = \frac{1-\omega}{\omega^2} \frac{\phi - F\frac{d\phi}{dF}}{\left(\phi + \frac{(1-\omega)\beta F}{(1-\omega\beta)K}\right)^2}. \tag{A.21}$$

As $\frac{d\phi}{dF} = (1-\alpha)\left(\frac{\omega\beta}{1-\omega\beta}\frac{F}{K}\right)^{1-\alpha}\frac{1}{F}$, equation (A.21) can be expressed as:

$$g'(F) = \frac{\alpha\phi(1-\omega)(1-\beta)}{\omega^2(1-\omega\beta)\left(\phi + \frac{(1-\omega)\beta F}{(1-\omega\beta)K}\right)^2} > 0. \tag{A.22}$$

Therefore, P is increasing in F, which means welfare is decreasing in regulations.

(ii) Welfare when $\frac{F}{K} > \frac{1-\omega\beta}{\omega\beta}$

By substituting equation (43) into equation (45), P can be expressed as follows:

$$P = \frac{\psi\{1-\omega(1-\alpha+\alpha\beta)\}^{\frac{1}{\alpha}}}{\alpha(1-\omega\beta)(1-\alpha)^{\frac{1-\alpha}{\alpha}}\omega^{\frac{1-\alpha}{\alpha}}}(F+K)^{\frac{1-\alpha}{\alpha}}. \tag{A.23}$$

Equation (A.23) suggests that P is increasing in F, which implies that welfare is decreasing in F.

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