Information and Pricing in Search Markets (Preliminary and Incomplete)

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Abstract

This paper studies a competition model with search frictions that combines a general information design problem with endogenous pricing. It includes a mix of savvy consumers with zero search costs and inexperienced consumers with strictly positive search costs. The presence of inexperienced consumers reduces equilibrium information provision, but this provision is non-monotonic in the search cost. Very low and high search costs result in full disclosure while intermediate search costs result in strategic information withholding tailored toward capturing the inexperienced consumers. The flexible information provision has a spillover effect on the equilibrium price and may allow firms to sustain a higher market price relative to the full information benchmark.

JEL Classifications: D4, D21, D83, L13

Keywords: information design, price competition, consumer search

1 Introduction

Market competition is often multifaceted, with prices being just one dimension on which firms compete. Another increasingly important dimension is targeted information provision that takes various forms, such as advertisement, product reviews, samples, generous return policies, or trial periods. Online retailers may provide detailed information about their products that includes technical specifications, images, videos, customer reviews, and safety records. Software developers may offer free or reduced trials of their products. Such business practices promote more informed purchases by consumers, but from the perspective of the firms, they are carefully designed marketing strategies to gain a competitive advantage over rivals.

This paper offers a general theoretical framework for studying firm competition in such a multifaceted environment. It utilizes a novel approach to Bayesian persuasion proposed by Dworczak and Martini (2019) to build a tractable search model in which firms compete both in information disclosure and pricing.

The model features two horizontally differentiated firms and a mix of inexperienced and savvy consumers who search for the best product fit. Consumers are privately informed about their search cost, with inexperienced consumers having a positive search cost and savvy consumers having no

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search cost. This framework is suitable for exploring the impact of both the search cost and the consumers' market sophistication, which is measured by the fraction of savvy consumers. Firms are ex-ante symmetric, with each consumer's valuation being independently and uniformly distributed. Consumers choose their search order and intensity, while the firms decide on their pricing and information disclosure strategies.

Following the Bayesian persuasion literature, the model imposes no structural restrictions on the firms' information disclosure. Given risk-neutral consumers, the firm's disclosure strategy involves selecting a posterior distribution of the consumer's expected value that satisfies Bayes plausibility, meaning that it is a mean-preserving contraction of the continuous prior distribution. In general, greater disclosure leads to higher product differentiation, with full disclosure (where the posterior is equal to the prior) and no disclosure (a mass point at the mean value) representing the maximum and minimum product differentiation, respectively. Between these two extremes, the firm is free to choose any Bayes-plausible distribution.

We characterize a symmetric perfect Bayesian equilibrium of the search game, in which the firms choose their information disclosure and pricing strategy upon being visited, and the consumers search optimally given their rational beliefs about the firms' strategies. The equilibrium characterization consists of two steps. First, we characterize a symmetric equilibrium of the information disclosure game for an arbitrary and symmetric fixed price. Given this characterization, we study the equilibrium of the pricing game.

The analysis reveals that the presence of inexperienced consumers has an overall adverse effect on information disclosure. Since inexperienced consumers are more easily persuaded to discontinue their search, the firms strategically tailor their information to maximize the likelihood that the inexperienced consumer's valuation of the product is higher than the expected return from continuing the search. To accomplish this in a Bayes-plausible way, the firms truncate their disclosure- they fully reveal very low-value realizations that encourage further search, and partially disclose higher realizations to induce immediate purchase by inexperienced consumers. Intuitively, low values trigger further search and call for maximum product differentiation, while higher values foreclose competition for inexperienced consumers and reduce firms' pressure to differentiate.

As the search cost or the fraction of inexperienced consumers increases, it becomes easier to capture consumers. This suggests that information provision is decreasing in the search cost, but we uncover an intriguing non-monotonicity. The relationship is indeed negative for low search costs when the firms' strategic information withholding significantly impacts the inexperienced consumers' search likelihood. However, for sufficiently high search costs, inexperienced consumers stop searching and purchase from the first firm they visit. In response, the firms shift their focus from capturing inexperienced consumers to attracting savvy consumers who always search both firms. This increases the pressure to differentiate their products by providing more information. As a result, information provision increases with the search cost.

Turning to the pricing decision, our analysis demonstrates how the endogeneity of the information disclosure impacts the pricing game. Unlike existing models of search that treat information as fixed, the present equilibrium derivation requires analyzing "double" deviations, in which a unilateral price deviation is accompanied by a corresponding deviation in the information revelation strategy. We characterize the symmetric pure strategy equilibrium price that prevents such "double" deviations. Similar to the equilibrium information disclosure, this candidate price also exhibits non-monotonicity in the search cost. For small search costs, firm competition is intense, and lower information provision is partially compensated by a lower price. As the search cost increases further, competition is eventually mitigated, allowing firms to increase their prices. Interestingly, for sufficiently high search costs, the equilibrium price exceeds the one obtained in the full information benchmark commonly studied in the existing literature.

Related Literature: This paper contributes to the growing literature on strategic information provision by firms. While the earlier papers on this topic focus on various types of advertising and disclosure in monopoly settings (Lewis and Sappington, 1994; Che, 1996; Ottaviani and Prat, 2001; Anderson and Renault, 2006; Johnson and Myatt, 2006), the more recent works examine competitive information disclosure (Ivanov, 2013; Board and Lu, 2018; Hwang et al., 2019; Au and Kawai, 2020; Au and Whitmeyer, 2021; Armstrong and Zhou, 2022; Dogan and Hu, 2022; Zhou, 2022). We contribute to these recent works in multiple dimensions. First, a major novelty of the model lies in the combination of a general information design problem and endogenous pricing. To the best of our knowledge, this paper is the first to tackle this challenging problem in the context of consumer search. Second, by allowing for both inexperienced and savvy consumers, our setting combines features of models with no search frictions (e.g., Ivanov, 2013; Hwang et al., 2019; Au and Kawai, 2020; Armstrong and Zhou, 2022) and models with costly search (e.g., Board and Lu, 2018; Au and Whitmeyer, 2021; Dogan and Hu, 2022; Zhou, 2022). This approach bridges the gap between these two strands of the literature to gain insight into the impact of consumers' market sophistication on information provision, pricing, and welfare.

Among the frictionless models, Armstrong and Zhou (2022) consider a general information design problem but focus on characterizing the firm and consumer-optimal information structures set by a third party, such as a platform. Among the papers that consider competitive information disclosure, Ivanov (2013) studies a class of rotation-ordered distributions, while Au and Kawai (2020) and Hwang et al. (2019) take a more general approach by imposing no structural restriction on the information disclosure. Similar to our setting, Hwang et al. (2019) endogenize both information and pricing but abstract from search frictions, which are central in our model. In that respect, this proposal generalizes their model to show how search frictions impact information disclosure, with the special case of zero search cost (or no inexperienced consumers) coinciding with their characterization.

Among the literature on consumer search, Zhou (2022) studies how improved information in terms of exogenous increasing convex order affects search intensity, price competition, and consumer welfare. Dogan and Hu (2022) study a general information design problem in a competitive environment but focus on the consumer-optimal information structure. Our focus on equilibrium information disclosure relates more closely to Board and Lu (2018) and Au and Whitmeyer (2021). Similar to our setting, Board and Lu (2018) consider a search model in which firms choose their information provision before being visited, but assume that sellers offer homogenous products. They characterize the conditions under which full disclosure and "monopoly level" of information are supported in equilibrium. We take a more comprehensive approach to information design by considering flexible partial disclosure strategies as well. In this respect, our environment is closer to Au and Whitmeyer (2021) who also allow for partial disclosure but assume that firms commit to their disclosure strategy prior to the consumers' search decision. This commitment adds an attraction motive to information disclosure that is absent in the current setting. Instead, the main focus of this paper is on the interaction between the information and pricing decisions that is absent in the existing search literature.

2 Setup

Consider a horizontally differentiated market with two competing firms and a unit mass of consumers. All parties are risk-neutral, and it is common knowledge that a consumer's valuation for each product is an iid draw from $U(0, 1)$. However, neither the firm nor the consumers know the realization of these values. Consumers must engage in search to gather information about their valuation for each firm's product. They are heterogeneous and privately informed about their search costs, with a fraction $\alpha \in [0, 1]$ being inexperienced consumers with a high search cost $s > 0$, and a fraction $1 - \alpha$ being savvy consumers with a zero search cost. The two extremes of $\alpha = 0$ and $\alpha = 1$ capture costless information acquisition as in Hwang et al. (2019), and symmetric and strictly positive search cost as in Au and Whitmeyer (2021), respectively.

Similar to Au and Whitmeyer (2021), the model departs from the standard search literature by allowing the two firms to engage in strategic information disclosure. Formally, each firm designs a signal space S_i and a corresponding joint distribution function H_i over $[0, 1] \times S_i$. Due to the risk-neutrality, the consumer's purchasing decision depends only on the posterior expected value $E[v_i|s_i]$. Thus, without loss of generality, we can formulate the firms' information design problem as choosing a posterior distribution G_i of the expected value subject to the standard Bayes plausibility constraint.

The timing of the game is as follows. First, each consumer chooses the order of approaching the two firms. Upon being visited, each firm chooses its information disclosure strategy G_i and price p_i. The consumer observes p_i and draws a posterior realization v_i from G_i. Then, she decides whether to purchase the product from firm i at p_i or continue the search by approaching the competing firm. If both firms are visited, the consumer purchases the product with the highest net expected value $E[v_i|s_i] - p_i$.

The analysis focuses on characterizing a symmetric Perfect Bayesian equilibrium, in which the firms charge the same price and use the same information disclosure strategy. Section 3 analyzes the information disclosure game for an arbitrary symmetric price p. Section 4 endogenizes the pricing decision by considering unliateral "double deviations" from a symmetric price and the optimal information disclosure characterized in Section 3.

3 Equilibrium information disclosure

Suppose that the firms' prices are fixed at p, and they compete only by choosing their information disclosure strategy G_i. For simplicity, let v_i denote the realized expected value drawn from G_i. To derive the firm's payoff as a function of v_i, we first consider the consumers' optimal search.

3.1 Consumers' search decisions

The savvy consumers are better off visiting both sellers and purchasing the product with the highest posterior expected valuation.[1] In contrast, the inexperienced consumers have an optimal search order and stopping rule characterized by Weitzman (1979). In particular, given firm j's posterior distribution $G_j(v)$ with support $[\underline{v}, \overline{v}]$, the reservation value r_j of visiting firm j solves:

$$\int_{r_j}^{\overline{v}} (v - r_j) dG_j(v) = s. \tag{1}$$

Weitzman shows that it is optimal to approach sellers in descending order of their reservation value and discontinue the search whenever the maximum value uncovered so far is (weakly) higher than the reservation value of the subsequent product in the queue. Note that the endogeneity of $G_j(v)$ implies that the reservation value r_j is an equilibrium object. Moreover, in a symmetric equilibrium with identical information disclosure strategies by the two firms, the reservation values for the two products are identical, and thus, consumers are indifferent in the search order. Given the consumers' optimal search decision, we next turn to the firms' information design problem.

3.2 Firms' information disclosure

To derive the firm's optimal posterior distribution, we first need to describe the firm's expected payoff as a function of the firm's posterior expected valuation. This payoff depends both on the competitor's information disclosure strategy and the inexperienced consumers' reservation values for the two products, as described by the lemma below.

Lemma 1 *Given an atomless information disclosure strategy by the competitor, $G_j(v)$, and the firms' reservation values r_i, r_j described by eq. (1), firm i's expected payoff is*

$$u_i(v_i | G_j, r_i, r_j) = \begin{cases} G_j(v_i) & \text{if } v_i < \min\{r_i, r_j\} \\ \tilde{\alpha}_i + (1 - \tilde{\alpha}_i) \min\{G_j(v_i), 1\} & \text{if } v_i \geq \min\{r_i, r_j\}, \end{cases} \tag{2}$$

where $\tilde{\alpha}_i$ is the firm's belief of facing an inexperienced consumer. Letting γ_i denote the inexperienced consumer's likelihood of visiting firm i first when indifferent, this belief is formed using Bayes rule as follows:

[1] Even a small amount of information disclosure would make the preference for visiting both sellers strict. While there always exists no disclosure-no search equilibrium (see Footnote 4), the analysis focuses on the more interesting equilibria that entail some disclosure and search.

$$\tilde{\alpha}_i = \begin{cases} \alpha & \textit{if } r_j < r_i \\ \frac{\alpha(\gamma_i+(1-\gamma_i)G_j(r_i))}{\alpha(\gamma_i+(1-\gamma_i)G_j(r_i))+(1-\alpha)} & \textit{if } r_j = r_i \\ \frac{\alpha G_j(r_i)}{\alpha G_j(r_i)+(1-\alpha)} & \textit{if } r_j > r_i. \end{cases} \tag{3}$$

To understand the payoff given by (2), note that if firm i reveals $v_i < \min\{r_i, r_j\}$, it will realize a sale only if $v_j < v_i$ independent of the search order- the savvy consumers compare all valuations, but the inexperienced consumers compare both only if i is visited first or $v_j < r_i$. In either case, a sale happens with probability $G_j(v_i)$.[2] For $v_i \geq \min\{r_i, r_j\}$, firm i sells to the savvy consumers with probability $G_j(v_i)$ as these consumers always compare both valuations. In contrast, firm i captures the inexperienced consumers for sure since these consumers either have already visited j (if $r_j \geq r_i$) and discovered $v_j < r_i$ or have chosen to visit i first (if $r_i \geq r_j$) and would discontinue the search since $v_i > r_j$.

The posterior belief of facing an inexperienced consumer $\tilde{\alpha}_i$, given by eq. (3), reflects the consumers' optimal search decisions. Given the firms' ex-ante symmetry, we are particularly interested in the symmetric case (i.e., $r_i = r_j = r$), in which the two firms employ identical information disclosure strategies, and as a result, consumers are indifferent in the search order. From eqs. (2) and (3), it is evident that the expected payoff of the firm is increasing in the probability of being a first mover (i.e., γ_i) since this increases $\tilde{\alpha}_i$. Therefore, symmetric payoffs that provide the same information disclosure incentives require that the consumers are randomizing equally in the order of approach (i.e., $\gamma_i = \frac{1}{2}$). In what follows, we characterize such an equilibrium.

3.2.1 Symmetric equilibrium dislosure

Given the prior distribution $F(v) = v$, a symmetric equilibrium of the persuasion game is a mean-preserving contraction (MPC, henceforth) of $F(v)$ that satisfies the following condition:

$$G^*(v) = \underset{G_i \in MPC(F)}{argmax} \int_0^1 u_i(v|G^*, r^*, r^*) dG_i(v), \tag{4}$$

where r^* satisfies eq. (1). To derive such an equilibrium, we employ the price theoretic approach of Dworczak and Martini (2019). Theorem 1 of their paper outlines sufficient conditions for the optimality of $G^*(v)$. It involves constructing an auxiliary function $\phi(v)$ that is convex and weakly greater than $u_i(v)$. Dworczak and Martini show that if 1) $supp(G) \subset \{v \in [0,1] : u(v) = \phi(v)\}$; 2) $\int_0^1 \phi(v) dG(v) = \int_0^1 \phi(v) dF(v)$; and 3) $G(v)$ is a MPC of $F(v)$, then $G(v)$ is an optimal disclosure strategy.

Given the above sufficient conditions, it is easy to see that if all consumers are savvy, i.e., $\alpha = 0$, full disclosure is an equilibrium of this game, as already established by Hwang et al. (2019). Given full disclosure by the competitor, the firm's expected payoff $u_i(v|F) = v$ is linear in the posterior realizations and thus $\phi(v) = v$ and $G(v) = F(v)$ satisfy the sufficient conditions for optimality.

[2] We restrict attention to posterior distributions that have no mass points. In Section 3.2.1 we characterize such a symmetric equilibrium. Moreover, it is easily argued that there is no symmetric equilibrium distribution that contains an atom as long as there is a positive probability of both firms being visited. Such distribution would give rise to strict deviation incentives to redistribute mass to higher valuations.

Intuitively, given the linearity of the payoff function, the firm's expected payoff is simply the expected value of the posterior mean, which is the same for any MPC of the prior. Therefore, the firm cannot do any better than full disclosure.[3][4]

The presence of inexperienced consumers (i.e., $\alpha > 0$) creates an upward jump in the firm's payoff function given by eq. (2). As long as the posterior valuation exceeds the reserve value r, the firm is able to fully capture the inexperienced consumers. Thus, full disclosure is no longer an equilibrium as the firms have strict incentives to conceal low valuations in order to increase their chance of capturing these consumers. We characterize a symmetric equilibrium in two steps. First, we treat the reservation value r as fixed and derive a symmetric equilibrium disclosure strategy. Then, we use this equilibrium characterization to derive the equilibrium reservation value. The following proposition describes a symmetric equilibrium disclosure for a fixed reservation value r.

Proposition 1 *Given $\alpha > 0$ and $r \in [0,1]$, there exists a symmetric equilibrium in which consumers randomize symmetrically in the order of search, and the firms employ the following information disclosure strategy:*

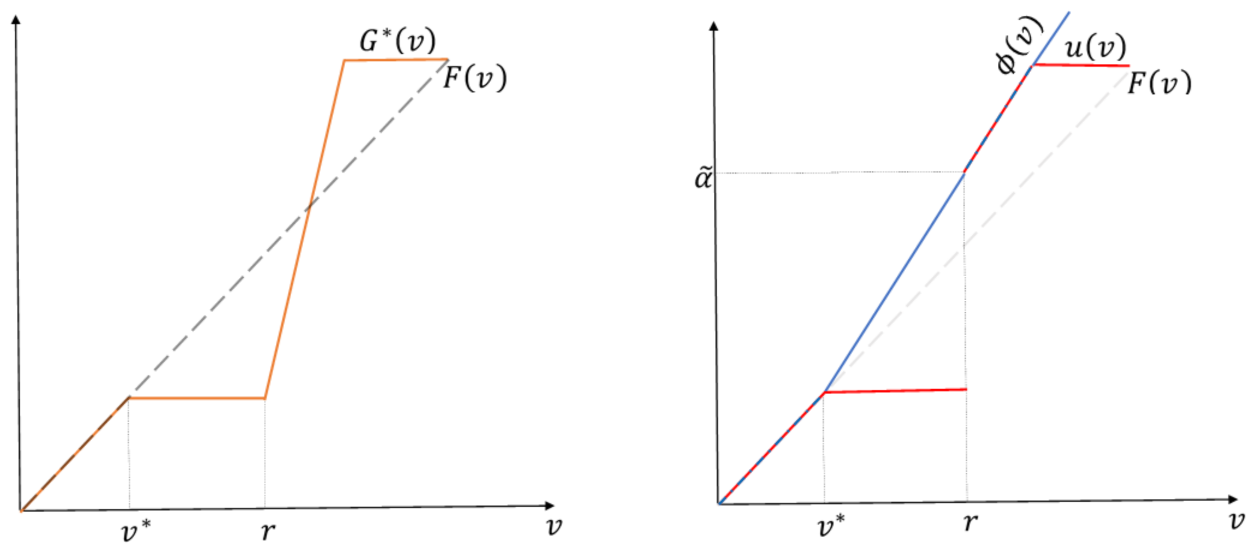
$$G^*(v) = \begin{cases} \min\{v, v^*\} & \text{if } v < r \\ \min\{v^* + \beta^*(v-r), 1\} & \text{if } v \geq r, \end{cases} \tag{5}$$

where $v^* = \max\{\frac{\sqrt{1-\alpha^2+r^2\alpha^2}-(1-r\alpha)}{\alpha}, 0\}$ and $\beta^* = \min\{\frac{1-v^*}{1+v^*-2r}, \frac{1}{1-2r}\}$. Savvy consumers search both sellers. The inexperienced consumers search both if the first posterior realization $v < r$ and buy from the first visited seller otherwise. For $r > \frac{\alpha}{2}$, $v^* > 0$, and the inexperienced consumers search with positive probability. For $r \leq \frac{\alpha}{2}$, $v^* = 0$, and the inexperienced consumers purchase from the first visited seller with prob. 1.

As the intuition suggests, the symmetric equilibrium described by Proposition 1 involves strategic information withholding in favor of realizations above the reserve value r. More interestingly, the posterior distribution is atomless at r. This is because a mass point at r generates strict incentives to shift weight to higher posterior values in order to discretely increase the savvy consumers' demand. Instead, the firms redistribute weight uniformly above r. Figure 1a) illustrates the equilibrium $G^*(v)$ for a sufficiently high reserve value (i.e., $r > \frac{\alpha}{2}$). The corresponding equilibrium payoff and the auxiliary function supporting this equilibrium are depicted in the right panel b). In equilibrium, the firms sacrifice high-value realizations, i.e., $v \in (r + \frac{1-v^*}{\beta^*}, 1]$ in order to increase the posterior expected values corresponding to low realizations $v \in (v^*, r)$. Since high-value realizations make the firm more competitive in the savvy market segment, there is a limit to the firm's

[3] There is a subtle difference between Hwang et al.'s setting and the current setting with $\alpha = 0$ in that the savvy consumers in our model may choose not to search. This gives rise to an additional no disclosure equilibrium with $G^*(v) = \mathbf{1}_v\{v \geq \frac{1}{2}\}$ that is absent in Hwang et al. (2019). In this equilibrium, the consumers do not search despite their zero search cost since they do not anticipate obtaining valuable information. Such trivial equilibrium exists for any α. The current characterization is focused on the more interesting equilibria that involve search by the savvy consumers and (partial) information disclosure by the firms.

[4] Hwang et. al.(2020) show that whenever the expected payoff is strictly convex, the firm has strict incentives to fully disclose as the payoff increase from high posterior realizations more than offsets the payoff reduction from low realizations.



(a) Equilibrium posterior distribution $G^*(v)$ (b) Payoff $u(v)$ and auxiliary function $\phi(v)$

Figure 1: Equilibrium disclosure for $r > \frac{\alpha}{2}$

willingness to make such a trade-off. As a result, the firm gives up on concealing values below v^* and instead fully discloses in this region. Such low realizations result in search by all consumers, while high realizations induce inexperienced consumers to discontinue their search.

As the reserve value decreases, it becomes easier to capture inexperienced consumers since it requires less information withholding to generate posterior beliefs above r. Thus, for r below $\frac{\alpha}{2}$, $v^* = 0$ and the firms disclose only values above r, ensuring that the first visited firm captures all inexperienced consumers. For $r = 0$, capturing inexperienced consumers becomes costless, and thus the firms revert back to full disclosure.

Note that the equilibrium information disclosure is non-monotonic in the reserve value. For $r = 1$, all consumers as savvy, and the firms' competition drives them to full disclosure. For $r = 0$, the inexperienced consumers never search, and the sole competition for the savvy consumers also results in full disclosure. Intermediate values of r result in partial dislosure due to the firms' strategic incentives to capture inexperienced consumers. This picture, however, is incomplete since the reserve price itself is an equilibrium object. Thus, to fully understand the impact of the inexperienced consumers on the market, we next turn to deriving the equilibrium reserve value r^*. To obtain r^*, we apply $G^*(v)$ to eq. (1). The following proposition characterizes the corresponding equilibrium reserve value.

Proposition 2 *Let $s \leq \frac{1}{2}$.*[5] *Then, the unique reservation value r^* that corresponds to the symmetric information equilibrium $G^*(v)$ is*

[5]For $s > \frac{1}{2}$, the value from search always falls short of the cost. This is because $\max\{v - r, 0\}$ is convex, and thus, by Jensen's inequality $\int_r^1 (v - r)dG(v) \leq \int_r^1 (v - r)dF(v)$ for any $G \in MPC(F)$. Therefore, the highest search value corresponds to full disclosure and $\int_r^1 (v - r)dF(v) \leq \frac{1}{2}$.

$$r^* = \begin{cases} \frac{1-\alpha^2+s\alpha^2-\sqrt{2s(1-\alpha^2)+s^2\alpha^2}}{1-\alpha^2} & \text{for } s \leq \frac{1-\alpha}{2} \\ \frac{1}{2} - s & \text{for } s \geq \frac{1-\alpha}{2}, \end{cases} \tag{6}$$

which is continuous and decreasing in s and α. For $s < \frac{1-\alpha}{2}$, there is a positive probability of search by inexperienced consumers. For $s \geq \frac{1-\alpha}{2}$, the inexperienced consumers purchase from the first visited firm.

Proposition 2 reveals intuitive comparative statics of the equilibrium reserve value r^*. Higher search cost s, and a higher share of inexperienced consumers α reduce the value of search. The search cost has a direct effect on the value from search as described by eq. (1), while α has an indirect effect via the firms' disclosure strategy. Intuitively, higher α mitigates firm competition, causing the firms to withhold more information. In fact, if all consumers are inexperienced (i.e., $\alpha = 1$), eq. (6) reveals that $r^* = \frac{1}{2} - s$ and there is no active search in equilibrium. Instead, as also pointed out by Au and Whitmeyer (2021), the consumers purchase from the first visited firm.

3.2.2 Information provision and consumer welfare

Propositions 1 and 2 show how search costs affect information provision by the competing firms. They reveal that the presence of inexperienced consumers also affects the information available to savvy consumers in the market. In this section, we study how information provision and consumer surplus change with the search cost.

Following Ganuza and Penalva (2010), we say that $G^*(v|s)$ is more informative than $G^*(v|s')$ if the former is a MPC of the latter.[6] The corresponding payoff of the zero search cost consumers is simply $CS_S = E_{G^*}[\max\{v_i, v_j\}]$ while the high search cost consumers realize a payoff of

$$CS_I = Pr(\max\{v_i, v_j\} < r)E[\max\{v_i, v_j\}|\max\{v_i, v_j\} < r] + Pr(\max\{v_i, v_j\} \geq r)r.$$

The following Proposition highlights the impact of the search cost s on the informativeness of $G^*(v)$ and the payoff of both types of consumers.

Proposition 3 *The search cost s has a non-monotonic impact on the informativeness of $G^*(v|s)$ with $G^*(v|0) = G^*(v|\frac{1}{2}) = v$. In particular, higher s results in a less (more) informative $G^*(v)$ for $s < \frac{1-\alpha}{2}$ ($s > \frac{1-\alpha}{2}$). As a result,*

(a) CS_S is decreasing in s for $s < \frac{1-\alpha}{2}$ and increasing in s for $s > \frac{1-\alpha}{2}$.

(b) CS_I is decreasing in s for all s.

The proposition reveals a non-monotonic impact of the search cost on the informativeness of the equilibrium disclosure. For low search costs ($s < \frac{1-\alpha}{2}$), information provision decreases with the

[6] This definition coincides with Ganuza and Penalva's integral precision criterion constrained to distributions with the same finite mean. It also closely related to Blackwell informativeness as described by Brooks et al. (2022).

search cost due to a reduction in the reservation value of inexperienced consumers, making it easier for firms to capture these consumers through information garbling. However, once s exceeds $\frac{1-\alpha}{2}$, all inexperienced consumers are captured by the first visited firm. Then, firms focus on competing for savvy consumers by increasing informativeness in order to enhance product differentiation. The savvy consumers' payoff is non-monotonic in the search cost because they always benefit from more information, which is hampered by the presence of uninformed consumers in the market. In contrast, the inexperienced consumers' payoff always decreases in the search cost because the negative effect of a higher search cost outweighs the positive effect of greater information provision.

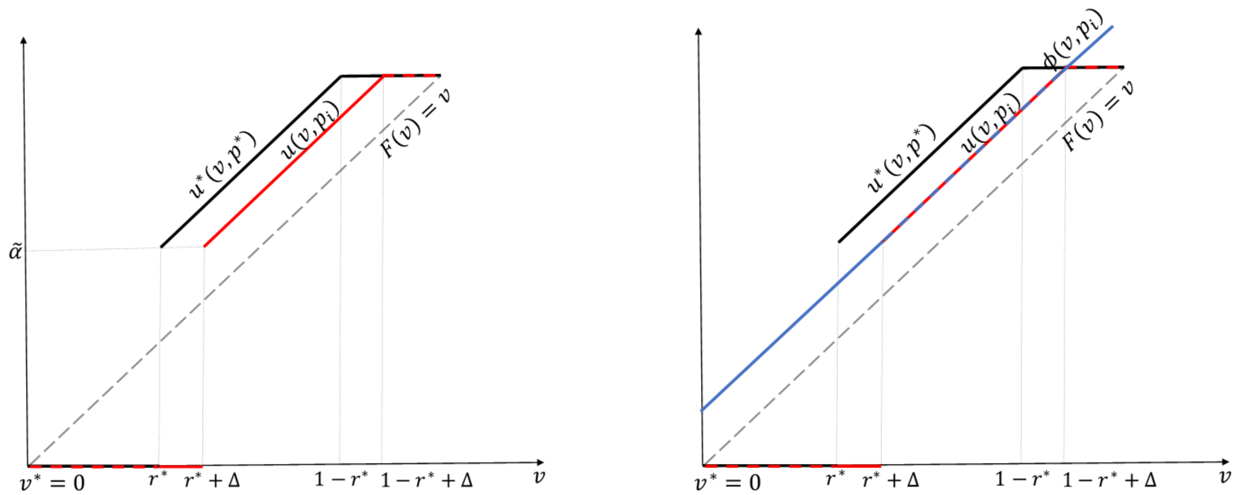
4 Endogenous pricing

So far, we have assumed that the firms are price takers and have control only over their information disclosure strategy. In this section, we allow the firms to choose their prices in addition to their information disclosure.

The consumers' search and purchasing decisions generalize seamlessly from the symmetric price model. In particular, given disclosure strategies G_i and prices p_i, the consumers' reserve value for i's product is given by $r_i - p_i$. Given symmetric strategies (G, p), each firm's reserve value is $r - p$, and the consumer's sequencing and purchasing decisions coincide with the base model. The firm's payoff for a posterior realization v is simply $\tilde{u}_i(v, p) = pu_i(v|G, r, r)$. Therefore, for any symmetric price p, the equilibrium disclosure G^* described in the previous section constitutes an equilibrium of this extended game. Turning to the equilibrium price, p^* must discourage unilateral deviations to a different price p_i. However, unlike a standard search model with fixed information disclosure, we need to rule out not only simple price deviations but also "double" deviations, in which a price deviation $\Delta = p_i - p^*$ is accompanied by a deviation in the information revelation strategy. In particular, given a strategy (p^*, G^*) by the competing firm, firm i's deviation demand is:

$$u_i(v, \Delta) = \begin{cases} \max\{0, v - \Delta\} & \text{if } v \leq v^* + \Delta \\ v^* & \text{if } v \in (v^* + \Delta, r^* + \Delta) \\ \tilde{\alpha}^* + (1 - \tilde{\alpha}^*)[v^* + \beta^*(v - r^* - \Delta)] & \text{if } v \in [r^* + \Delta, \min\{1 + v^* - r + \Delta, 1\}] \\ 1 & \text{if } v \geq \min\{1 + v^* - r + \Delta, 1\}, \end{cases} \tag{7}$$

where v^* corresponds to the threshold given by Proposition 1 and r^*- the reservation value given by Proposition 2. A price deviation Δ results in a shift in the expected demand $u_i(v, \Delta)$, with price increases resulting in a downward shift and price decreases- in an upward shift. Figure 2 illustrates the impact of a small price increase (i.e., $\Delta > 0$) for $v^* = 0$ (i.e., $s \geq \frac{1-\alpha}{2}$). The higher price makes the consumers less willing to purchase from the deviating firm for any value realization. This shifts the demand down from the black to the red function, as depicted in Figure 2a). Figure 2b) includes the auxiliary function $\phi(v, \Delta)$ corresponding to the optimal disclosure strategy under p_i. Clearly, the optimal disclosure region shifts to the right with $supp(G_i) \subset [r^* + \Delta, 1 - r^* + \Delta]$. By Proposition 2, $r^* = \frac{1}{2} - s < \frac{1}{2}$ for $s > \frac{1-\alpha}{2}$. Therefore, for small price deviations, $v = \frac{1}{2}$ belongs to the optimal disclosure region, implying that no disclosure, corresponding to putting all the mass on the prior



(a) Demand impact of $p_i > p^*$ (i.e., $\Delta > 0$) (b) Optimal disclosure region $[r^* + \Delta, 1 - r^* + \Delta]$

Figure 2: Impact of a price increase for $v^* = 0$

mean $\frac{1}{2}$, is an optimal strategy for the deviating firm. This results in an expected deviation payoff of:

$$\tilde{u}_i(\frac{1}{2}, p^*, \Delta) = (p^* + \Delta)\left(\tilde{\alpha}^* + (1-\tilde{\alpha}^*)\frac{\frac{1}{2} - r^* - \Delta}{1 - 2r^*}\right). \quad (8)$$

No profitable price deviation $\Delta > 0$ exists if $\tilde{u}(\frac{1}{2}, p^*, \Delta)$ is decreasing in Δ for $\Delta > 0$. Given the above expression, this requires that $p^* \leq \frac{(1+\tilde{\alpha}^*)(1-2r^*)}{2(1-\tilde{\alpha}^*)}$. Analogous analysis for a small downward price deviation reveals that $p^* \geq \frac{(1+\tilde{\alpha}^*)(1-2r^*)}{2(1-\tilde{\alpha}^*)}$. Thus, $p^* = \frac{(1+\tilde{\alpha}^*)(1-2r^*)}{2(1-\tilde{\alpha}^*)}$ is the only possible pure-strategy equilibrium price for $s > \frac{1-\alpha}{2}$. The following proposition describes the pure-strategy equilibrium price p^* for the entire relevant region of the search cost.

Proposition 4 *Let $s \in [0, \frac{1}{2}]$. Then, given $G^*(v)$ and r^*, the unique symmetric pure-strategy equilibrium price is given by:*

$$p^*(s) = \begin{cases} \frac{(1+\tilde{\alpha}^*(1-v^*)^2)}{2(1+\tilde{\alpha}^*(1-v^*))} & \text{if } s < \frac{1-\alpha}{2}, \\ \frac{(1+\tilde{\alpha}^*)(1-2r^*)}{2(1-\tilde{\alpha}^*)} & \text{if } s \geq \frac{1-\alpha}{2}, \end{cases} \quad (9)$$

where v^* and r^* are characterized by Proposition 1 and 2, respectively, and $\tilde{\alpha}^* = \frac{\alpha(1+v^*)}{\alpha(1+v^*)+2(1-\alpha)}$. Moreover, p^* is continous in s and

- *for $s \leq \frac{1-\alpha}{2}$, p^* is U-shaped in s with $p^*(0) = p^*(\frac{1-\alpha}{2}) = \frac{1}{2}$.*
- *for $s > \frac{1-\alpha}{2}$ is strictly increasing in s with $p^*(\frac{1}{2}) = \frac{1}{1-\alpha}$.*

To gain some preliminary insight, Figure 3 plots p^* for $\alpha = \frac{1}{2}$ as a function of the search cost. The figure also features the benchmark equilibrium price p^{full_info} representing exogenous full disclosure. While p^{full_info} is always increasing in the search cost, p^* features non-monotonicity due

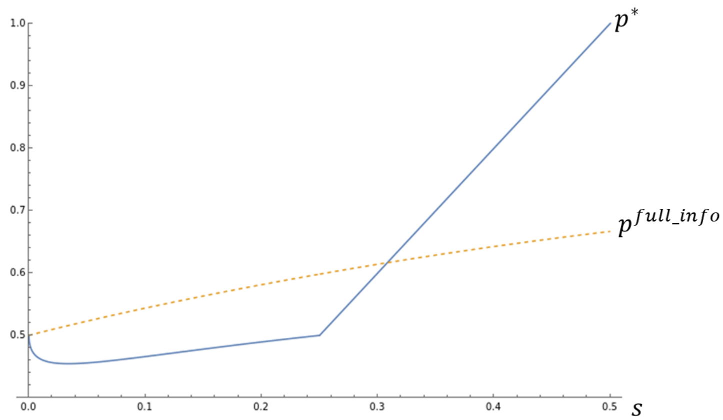


Figure 3: Pure-strategy symmetric equilibrium price

to the strategic information disclosure. For very low values of the search cost, the equilibrium price is decreasing in the search cost since the lower information provision in this region (see Proposition 3) and the high search propensity by the consumers lead to significant firm competition. A further increase in the search cost mitigates competition since it decreases the likelihood of search by inexperienced consumers. Thus, eventually, the price starts to increase with the search cost. At $s = \frac{1}{4}$, inexperienced consumers under endogenous pricing stop searching and purchase from the first visited firm, explaining the kink in p^*.

Comparing p^* and p^{full_info} reveals an interesting relationship between information disclosure and pricing. For $s = 0$, the two prices coincide since the firms fully disclose in equilibrium and all consumers engage in search. For low but positive values of the search cost, the firms offer a lower equilibrium price compared to the full information benchmark to offset the lower information provision in this region. The opposite is true for large search costs, where the equilibrium price exceeds the full information benchmark. Having captured the inexperienced consumers via their strategic information disclosure allows the firms to sustain a higher equilibrium price. Interestingly, even though the firms fully disclose their value at $s = \frac{1}{2}$, p^* exceeds p^{full_info}. This is likely driven by the fact that price deviations are more profitable when the firms are able to strategically adjust their information disclosure. Therefore, preventing such deviations requires a higher equilibrium price.

5 Appendix

Proof of Lemma 1.

Fix an equilibrium belief $G_i(v)$, $G_j(v)$, and the corresponding r_i and r_j satisfying eq. (1). Suppose that when indifferent, consumers break indifference in favor of purchasing from the last visited firm. The firm's payoff is simply the expected sale probability given a posterior realization v_i. The savvy consumers always compare the two expected values and purchase from i with probability $G_j(v_i)$. The sale probability to the inexperienced consumers depends on the relative values of r_i and r_j. There are 3 cases to consider:

- $r_i > r_j$: then firm i is visited first. Since all consumers visit at least one firm, the probability that the consumer is inexperienced conditional on visiting i is $\tilde{\alpha}_i = \alpha$. If $v_i < r_j$, then all consumers visit firm j and compare both valuations, resulting in an expected sale probability of $G_j(v_i)$. If $v_i \geq r_j$, the inexperienced consumers discontinue their search and purchase from i, resulting in an expected sale probability of $\alpha + (1-\alpha)G_j(v_i)$.

- $r_i < r_j$: then firm i is visited second. While the savvy consumers always visit i, the inexperienced ones visit i with probability $G_j(r_i)$, i.e., only if $v_j < r_i$. Thus, by Bayes' rule, the probability that the consumer is inexperienced conditional on visiting i is $\tilde{\alpha}_i = \frac{\alpha G_j(r_i)}{\alpha G_j(r_i)+(1-\alpha)}$. If $v_i < r_i$, i realizes a sale only if $v_j < v_i < r_i$, which would induce all consumers to visit. Thus, the expected probability of sale is $G_j(v_i)$. If $v_i \geq r_i$, the firm sells for sure to the inexperienced consumers since they must have observed $v_j < r_i$ in order to visit i. Therefore, the expected probability of sale is $\tilde{\alpha}_i + (1-\tilde{\alpha}_i)G_j(v_i)$.

- $r_i = r_j = r$: then the consumer is indifferent in the order of search. Letting γ_i denote the probability that the consumer approaches i first, the probability that the consumer is inexperienced conditional on visiting i is $\tilde{\alpha}_i = \frac{\alpha(\gamma_i+(1-\gamma_i)G_j(r))}{\alpha(\gamma_i+(1-\gamma_i)G_j(r))+(1-\alpha)}$. For $v_i < r$, i realizes a sale only if $v_j < v_i$ irrespective of the search order. The savvy consumers compare both values, while the inexperienced consumers compare both only if i is visited first or $v_j < r$. In either case, a sale happens only if $v_j < v_i$, corresponding to an expected payoff of $G_j(v_i)$. For $v_i \geq r$, firm i always sells to the inexperienced consumers since they either had already visited j and discovered $v_j < r$ or will search no further once they observe $v_i \geq r$. Therefore, the expected probability of sale is $\tilde{\alpha}_i + (1-\tilde{\alpha}_i)G_j(v_i)$.

Combining all three regions, we obtain the expected payoff given by eq. (2) and the corresponding posterior belief of facing an inexperienced consumer given by eq. (3). ■

Proof of Proposition 1. The proof follows by construction. In particular, we derive a cdf $G^*(v)$ that solves eq. (4) for an arbitrary r by ensuring that it satisfies the following optimality conditions outlined by Dworczak and Martini (2019).

Theorem 1 ***(Dworczak and Martini)***

Suppose that F is a distribution function with $supp(F) = [0,1]$. Consider the following programming problem:

$$\max_{G} \int_0^1 u(x)dG(x)$$

subject to the constraint that G is an MPC of F. A MPC G is a solution to the problem if there exists a ***convex*** *function $\phi : [0,1] \to \mathbb{R}$ such that (i) $\phi(v) \geq u(v)$ for all $v \in [0,1]$, (ii) supp$(G) \subset \{v \in [0,1] : u(v) = \phi(v)\}$, and (iii) $\int_0^1 \phi(v)dG(v) = \int_0^1 \phi(v)dF(v)$.*

Let $v' \in [0,r]$, and suppose that the equilibrium posterior distribution $G(v)$ takes the following form:

$$G(v) = \begin{cases} v & \text{for } v \leq v' \\ v' & \text{for } v \in (v', r) \\ v' + \beta(v-r) & \text{for } v \in [r, r + \frac{1-v'}{\beta}] \\ 1 & \text{for } v \geq r + \frac{1-v'}{\beta} = v''. \end{cases} \tag{10}$$

Let

$$W(z) = \int_0^z [F(v) - G(v)]dv.$$

Then, $G(v)$ is a MPC of $F(v) = v$ iff $W(0) = W(1) = 0$ and $W(z) \geq 0$ for all $z \in (0,1)$. Substituting for $G(v)$ from eq. (10) obtains:

$$W(z) = \begin{cases} 0 & \text{for } z \in [0, v'] \\ \frac{(z-v')^2}{2} & \text{for } z \in (v', r] \\ \frac{(z-v')^2 - \beta(z-r)^2}{2} & \text{for } z \in (r, v''] \\ \frac{(v''-v')^2 - \beta(v''-r)^2 + (z-v'')(z+v''-2)}{2} & \text{for } z \in (v'', 1] \end{cases} \tag{11}$$

$W(z)$ is a piece-wise continuous function with $W(0) = 0$ for $z \in [0, v']$. Moreover, $W(z)$ is non-negative for $z \in [0, r]$, strictly concave for $z \in (r, v'']$, and strictly decreasing in z for $z > v''$. This implies that 1) there is at most one $\tilde{z} > v'$ such that $W(\tilde{z}) = 0$ and 2) $W(z) > 0$ for all $z < \tilde{z}$. Therefore, $W(1) = 0$ is necessary and sufficient for $G(v)$ to be MPC of $F(v)$. Simplifying $W(1)$, we obtain:

$$W(1) = \frac{(1-v')(1-v'-\beta(1+v'-2r))}{2\beta}$$

From the above expression, $W(1) = 0$ if and only if:

$$\beta = \frac{1-v'}{1+v'-2r} = \tilde{\beta}. \tag{12}$$

Threfore, $\beta = \tilde{\beta}$ is necessary and sufficient for $G(v)$ given by eq. (10) to be MPC of $F(c)$. Let $\tilde{G}(v)$ denote the resulting distribution. The corresponding payoff $u(v)$ is

$$u(v) = \begin{cases} v & \text{if } v \leq v' \\ v' & \text{if } v \in (v', r) \\ \tilde{\alpha} + (1-\tilde{\alpha})(v' + \tilde{\beta}(v-r)) & \text{if } v \in [r, 1-r+v'] \\ 1 & \text{for } v \geq 1-r+v' = v''. \end{cases} \tag{13}$$

We next derive the unique v' such that $\tilde{G}(v)$ satisfies the sufficient conditions given by Theorem 1. By construction, $supp(\tilde{G}) = [0, v'] \cup [r, v'']$ and condition *(ii)* requires that the auxiliary function $\phi(v) = u(v)$ in this region. Moreover, $\phi(v)$ must be affine for $v \in [v', 1]$ since $\tilde{G}(v)$ is an MPC of $F(v)$ in this region and strict convexity of $\phi(v)$ implies $\int_{v'}^{1} \phi(v)dG(v) < \int_{v'}^{1} \phi(v)dF(v)$, contradicting condition (iii). Therefore, $\phi(v)$ must take the following form:

$$\phi(v) = \begin{cases} v & \text{if } v < v' \\ \tilde{\alpha} + (1-\tilde{\alpha})(v' + \tilde{\beta}(v-r)) & \text{if } v \geq v'. \end{cases} \tag{14}$$

The above $\phi(v)$ is uniquely defined to satisfy Properties *(ii)-(iii)* of Theorem 1. It remains to ensure that $\phi(v)$ is convex and $\phi(v) \geq u(v)$ for all $v \in [0, 1]$. We consider two possibilities for v', namely $v' > 0$ and $v' = 0$.

For $v' > 0$, $\phi(v) \geq u(v)$ and the convexity of $\phi(v)$ requires

$$\tilde{\beta} = \frac{\tilde{\alpha}(1-v')}{(1-\tilde{\alpha})(r-v')}, \tag{15}$$

and

$$\tilde{\beta} \geq \frac{1}{1-\tilde{\alpha}}. \tag{16}$$

Combining eqs. (12) and (15), we obtain:

$$r - v' = \tilde{\alpha}(1-r). \tag{17}$$

Therefore, by eq. (15), $\tilde{\beta} = \frac{1-v'}{(1-\tilde{\alpha})(1-r)} \geq \frac{1}{1-\tilde{\alpha}}$ since $r \geq v'$, establishing (16). Thus, $\phi(v)$ satisfies the convexity requirement. By eq. (3), $\tilde{\alpha} = \frac{\alpha(1+v')}{\alpha(1+v')+2(1-\alpha)}$ since $\tilde{G}(r) = \tilde{G}(v') = v'$. Substituting for $\tilde{\alpha}$ in eq. (17), we obtain:

$$v' = r - \frac{\alpha(1+v')}{\alpha(1+v') + 2(1-\alpha)}(1-r) \tag{18}$$

Notice that the left-hand side is increasing in v' while the right-hand side is decreasing in v'. The unique solution to eq. (18) is $v^* = \frac{\sqrt{1-\alpha^2+r^2\alpha^2}-(1-r\alpha)}{\alpha}$ with corresponding $\beta^* = \frac{1-v^*}{1+v^*-2r}$ that satisfies eqs. (12) and (15). Note that $v^* > 0$ iff $r > \frac{\alpha}{2}$.

For $r \leq \frac{\alpha}{2}$, the only candidate solution is $v' = 0$, corresponding to an affine $\phi(v)$. To ensure that $\phi(v) \geq u(v) \geq 0$, by eq. (14), $\tilde{\beta} \leq \frac{\tilde{\alpha}}{(1-\tilde{\alpha})r}$. By eq. (12), $\tilde{\beta} = \frac{1}{1-2r}$ and by eq. (3), $\tilde{\alpha} = \frac{\alpha}{2-\alpha}$. Therefore, $\beta \leq \frac{\tilde{\alpha}}{(1-\tilde{\alpha})r} \implies r \leq \frac{\alpha}{2}$. This confirms that $v^* = 0$ and $\beta^* = \frac{1}{1-2r}$ is the unique solution for $r \leq \frac{\alpha}{2}$.

If follows that $G(v)$, $u(v)$, and $\phi(v)$ given by eqs. (10), (13), and (14), respectively, satisfy Theorem 1 if and only if $v' = \max\{\frac{\sqrt{1-\alpha^2+r^2\alpha^2}-(1-r\alpha)}{\alpha}, 0\} = v^*$ and $\tilde{\beta} = \min\{\frac{1-v^*}{1+v^*-2r}, \frac{1}{1-2r}\} = \beta^*$. This completes the proof. ■

Proof of Proposition 2. We consider the two regions of r given by Proposition 1.

- $r > \frac{\alpha}{2}$. Then, $v''(r^*, \beta^*) = r^* + \frac{1-v^*}{\beta^*}$ and by eqs. (1) and (10), r^* solves

$$\int_{r^*}^{v''(r^*,\beta^*)} (v - r^*)\beta^* dv = \frac{\beta^*(v''(r^*,\beta^*) - r^*)^2}{2} = \frac{(1-v^*)^2}{2\beta^*} = s$$

  Substituting for v^* and β^* from Proposition 1 and solving for r^* obtains:

$$r^* = \frac{1 - \alpha^2 + s\alpha^2 - \sqrt{2s(1-\alpha^2) + s^2\alpha^2}}{1-\alpha^2}$$

  Since the equilibrium information disclosure requires $r^* > \frac{\alpha}{2} \Longrightarrow s < \frac{1-\alpha}{2}$, the above expression is the equilibrium reserve value for $s < \frac{1-\alpha}{2}$.

- $r \in [0, \frac{\alpha}{2}]$. Then, $v^* = 0$ and $v''(r^*, \beta^*) = 1 - r^*$. Therefore, by eqs. (1) and (10), r^* solves

$$\int_{r}^{1-r^*} (v - r^*)\frac{1}{1-2r^*} dv = \frac{1}{2} - r^* = s \Longrightarrow r^* = \frac{1}{2} - s$$

  Since the equilibrium information disclosure requires that $0 \le \frac{1}{2} - s \le \frac{\alpha}{2} \Longrightarrow s \in [\frac{1-\alpha}{2}, \frac{1}{2}]$, the above expression is the equilibrium reserve value for $s \in [\frac{1-\alpha}{2}, \frac{1}{2}]$.

■

Proof of Proposition 3.

Let $0 < s_1 < s_2 < \frac{1}{2}$. First, we show that $G^*(v|s_2)$ is a mean-preserving contraction of $G^*(v|s_1)$ if $s_2 \le \frac{1-\alpha}{2}$ and a mean-preserving spread if $s_1 \ge \frac{1-\alpha}{2}$. Let

$$W^*(z|s) = \int_0^z [F(v) - G^*(v|s)]dv, \tag{19}$$

and

$$H(z) = \int_0^z [G^*(v|s_2) - G^*(v|s_1)]dv. \tag{20}$$

By Bayes' plausibility, $W(0|s) = W(1|s)$ and $W(z|s) \ge 0$ for all z and s. Since $H(z) = W^*(z|s_1) - W^*(z|s_2)$, $H(0) = H(1) = 0$. We want to show that $H(z) \le 0$ for $s_2 \le \frac{1-\alpha}{2}$ and $H(z) \ge 0$ for $s_1 \ge \frac{1-\alpha}{2}$.

Note that for $s \le \frac{1-\alpha}{2}$, $v^*(s)$ and $r^*(s)$, and $\beta^*(s)$ are strictly decreasing in s. Moreover, the upper bound of the distribution $G^*(v)$, $v''(s) = 1-(r^* - v^*)$ is strictly decreasing in s. This implies that $G^*(v|s_2) \le G^*(v|s_1)$ for $v \le r^*(s_2)$ and $1 = G^*(v|s_2) > G^*(v|s_1)$ for $v \in [v''(s_2), v''(s_1))$.

Clearly, $H(z) \leq 0$ for $z \leq r^*(s_2)$. Suppose that there exists $\hat{z} \in (r^*(s_2), 1)$ such that $H(\hat{z}) > 0$. By eq. (20), this requires $G^*(z|s_2) > G^*(z|s_1)$ for some $\bar{z} \in (r^*(s_2), \hat{z})$. Note that $G^*(r^*(s_2)|s_2) = v^*(s_2) < \min\{r^*(s_2), v^*(s_2)\} = G^*(r^*(s_2)|s_1)$ and $G^*(v''(s_2)|s_2) = 1 > G^*(v''(s_2)|s_1)$. Moreover, $G^*(z|s_2)$ is affine for $z \in (r^*(s_2), v''(s_2))$ while $G^*(z|s_1)$ is convex. This implies that $G^*(z|s_2)$ and $G^*(z|s_1)$ intersect exactly once in $[r^*(s_2), v''(s_2)]$. Thus, $G^*(z|s_2) > G^*(z|s_1)$ for all $z \geq \hat{z} > \bar{z}$. This, in turn, implies that $H(z) > 0$ for all $z \in [\hat{z}, 1]$, contradicting $H(1) = 0$. Therefore, $H(z) \leq 0$ for all z, establishing that $G^*(v|s_2)$ is a mean-preserving contraction of $G^*(v|s_1)$ if $s_2 \leq \frac{1-\alpha}{2}$.

Analogously, for $s > \frac{1-\alpha}{2}$, $v^* = 0$, $r^*(s)$ and $\beta^*(s)$ are decreasing in s, while $v''(s)$ is increasing in s. This implies that $G^*(v|s_2) \geq G^*(v|s_1)$ for $v \leq r^*(s_1)$ and $1 = G^*(v|s_1) > G^*(v|s_2)$ for $v \in [v''(s_1), v''(s_2))$.Thus, $H(z) \geq 0$ for $z \leq r^*(s_1)$. Moreover, the linearity of $G^*(z|s_2)$ and the convexity of $G^*(z|s_1)$ for $z \in (r^*(s_2), v''(s_1)]$ implies that $G^*(z|s_2)$ and $G^*(z|s_1)$ intersect exactly ones. Therefore, $H(\tilde{z}) < 0$ for some $\tilde{z} \in (r^*(s_1), v''(s_1))$ implies that $H(z) < 0$ for all $z \in (\tilde{z}, 1]$, contradicting $H(1) = 0$. Therefore, $H(z) \geq 0$ for all z, establishing that $G^*(v|s_2)$ is a mean-preserving spread of $G^*(v|s_1)$ if $s_2 \leq \frac{1-\alpha}{2}$.

To prove (a), let $q(v_i|s) = \int_0^1 \max\{v_i, v_j\} dG(v_j|s)$ and $CS_S(s) = E_{G^*}[\max\{v_i, v_j\}] = \int_0^1 q(v_i|s) dG^*(v_i|s)$. Note that $q'(v_i|s) = G^*(v_i)$ and $q''(v_i|s) = g^*(v_i) \geq 0$.

Let $G(v|s_1)$ be a MPC of $G(v|s_2)$. To prove part (a), it suffices to show that $CS_S(s_1) < CS_S(s_2)$. The convexity of $\max\{v_i, v_j\}$ implies that $q(v_i|s_1) < q(v_i|s_2)$ and the convexity of $q(v_i|s)$ implies that $\int_0^1 q(v_i|s_1) dG^*(v_i|s_1) < \int_0^1 q(v_i|s_1) dG^*(v_i|s_2)$. Combining the two inequalities, we obtain:

$$CS_S(s_1) = \int_0^1 q(v_i|s_1) dG^*(v_i|s_1) < \int_0^1 q(v_i|s_1) dG^*(v_i|s_2) < \int_0^1 q(v_i|s_2) dG^*(v_i|s_2) = CS_S(s_2).$$

This establishes (a) since higher s generates a MPC for $s < \frac{1-\alpha}{2}$ and a MPS for $s > \frac{1-\alpha}{2}$.

For part (b), substituting for $G^*(v)$ from Proposition 1 in the expression for CS_I obtains:

$$CS_I = \int_0^{v^*} 2y^2 dy - (1 - (v^*)^2) r^* = \frac{2}{3}(v^*)^3 + r^* - v^* r^*. \tag{21}$$

Since r^* is decreasing in s, it suffices to show that the above expression is increasing in r^*. Taking a total derivative of eq. (21) with respect to r^* obtains:

$$\frac{dCS_I}{dr^*} = 1 - (v^*)^2 - 2v^*(r^* - v^*)\frac{\partial v^*}{\partial r^*} \tag{22}$$

Recall that v^* satisfies eq. (18). Solving for r^* from the equation, we obtain $r^* = \frac{2v^* + \alpha + \alpha(v^*)^2}{2(1+\alpha v^*)}$ and $r^* - v^* = \frac{\alpha(1-(v^*)^2)}{2(1+\alpha v^*)}$. Moreover, implicitly differentiating eq. (18) yields $\frac{\partial v^*}{\partial r^*} = \frac{2(1+\alpha v^*)^2}{(1+\alpha v^*)^2 + 1 - \alpha^2}$. Substituting for $r^* - v^*$ and $\frac{\partial v^*}{\partial r^*}$ in eq. (22) and simplifing obtains:

$$\frac{dCS_I}{dr^*} = (1 - (v^*)^2) \frac{1 - (\alpha v^*)^2 + 1 - \alpha^2}{(1 + \alpha v^*)^2 + 1 - \alpha^2} > 0.$$

This proves that $\frac{dCS_I}{ds^*} = \frac{dCS_I}{dr^*} \frac{dr^*}{ds} < 0$, establishing part (b).

■

Proof of Proposition 4.

Let $G_\Delta(v)$ denote the optimal disclosure given a price deviation $\Delta = p_i - p^*$. Then, the corresponding deviation demand is $E_{G_\Delta}[u_i(v,\Delta)]$ where $u_i(v,\Delta)$ is given by eq. (7). The corresponding deviation payoff is $E_{G_\Delta}[\tilde{u}_i(v,\Delta)] = (p^* + \Delta)E_{G_\Delta}[u_i(v,\Delta)]$. Let $\Delta^* = \underset{\Delta}{argmax}\ E_{G_\Delta}[\tilde{u}_i(v,\Delta)]$. No deviation from p^* requires that $0 \in \Delta^*$. To find such p^*, we first derive the optimal disclosure strategy $G_\Delta(v)$.

Consider the following convex auxiliary function $\phi(v,\Delta)$ that satisfies $\phi(v,\Delta) \geq u_i(v,\Delta)$:

$$\phi(v,\Delta) = \begin{cases} \max\{0, v_i - \Delta\} & \text{for } v < \hat{v} + \Delta, \\ \tilde{\alpha}^* + (1-\tilde{\alpha}^*)[v^* + \beta^*(v - r^* - \Delta)] & \text{for } v \geq \hat{v} + \Delta, \end{cases} \tag{23}$$

where $\hat{v}$ is set to ensure that continuity of $\phi(v,\Delta)$, i.e.,

$$\hat{v} = \begin{cases} r^* - \frac{\tilde{\alpha}^*}{(1-\tilde{\alpha}^*)\beta^*} & \text{if } v^* = 0, \\ v^* & \text{if } v^* > 0. \end{cases} \tag{24}$$

Given $\phi(v,\Delta)$, we can derive the optimal information structure and the corresponding optimal deviation payoff. We consider the case of $v^* = 0$ (i.e., $s < \frac{1-\alpha}{2}$) and $v^* > 0$ (i.e., $s \geq \frac{1-\alpha}{2}$) separately.

- $v^* = 0$ (i.e., $s \geq \frac{1-\alpha}{2}$)

  In this case, $\hat{v} \leq 0$ since $r^* \leq \frac{\tilde{\alpha}^*}{(1-\tilde{\alpha}^*)\beta^*}$ (see proof of Proposition 1). The optimal information structure depends on Δ as follows:

  - $\Delta \leq \min\{-\hat{v}, \frac{1}{2} - r^*\}$

    By eq. (23), $\phi(v,\Delta)$ is affine since $\hat{v}+\Delta \leq 0$. In this case, $G_\Delta(v|v^*=0) = \mathbf{1}(v \geq \frac{1}{2})$, i.e., a mass point at $\frac{1}{2}$, is an optimal disclosure strategy. To see this, note that $\Delta \leq \frac{1}{2} - r^*$ ensures that $\frac{1}{2} \geq r^* + \Delta$. Moreover, if $\Delta \geq r^* - \frac{1}{2}$, $\frac{1}{2} \in [r^*+\Delta, 1-r^*+v^*+\Delta]$ and by eqs. (7) and (23), $\phi(\frac{1}{2}, p_i) = u(\frac{1}{2}, p_i)$. Thus, $G_\Delta(v|v^*=0) = \mathbf{1}(v \geq \frac{1}{2})$ and an auxiliary function $\phi(v,\Delta)$ satisfies Theorem 1. For $\Delta \leq r^* - \frac{1}{2} < 0$, $\frac{1}{2} > 1 - r^* + \Delta$. Then, $G_\Delta(v|v^*=0) = \mathbf{1}(v \geq \frac{1}{2})$ and an auxiliary function $\hat{\phi}(v,\Delta) = 1$ satisfy Theorem 1.

  - $\min\{-\hat{v}, \frac{1}{2} - r^*\} < \Delta \leq \max\{1 - 2r^* + \hat{v}, \frac{1}{2} - r^*\}$

    If $-\hat{v} > \frac{1}{2} - r^*$, then $\frac{1}{2} - r^* > 1 - 2r^* + \hat{v}$ and this region for Δ is empty. If $-\hat{v} < \frac{1}{2} - r^*$, then $\max\{1-2r^*+\hat{v}, \frac{1}{2}-r^*\} = 1 - 2r^* + \hat{v}$. Then, for $\Delta > \min\{-\hat{v}, \frac{1}{2} - r^*\}$, $\phi(v,\Delta)$ is piece-wise linear and $\phi(v,\Delta) = u_i(v,\Delta)$ for $v \in [0, \hat{v}+\Delta] \cup [r^*+\Delta, 1-r^*+\Delta]$. Note that $E[v|v > \hat{v}+\Delta] = \frac{1+\hat{v}+\Delta}{2} \in [r^*+\Delta, 1-r^*+\Delta)$ since $2r^* - 1 + \hat{v} < -\hat{v} < \Delta \leq 1 - 2r^* + \hat{v}$. Then, the following MPC of $F(v)$ satisfies Theorem 1 and is an optimal disclosure strategy:

$$G_\Delta(v|v^*=0) = \begin{cases} \min\{v, \hat{v}+\Delta\} & \text{if } v < \frac{1+\hat{v}+\Delta}{2} \\ 1 & \text{if } v \geq \frac{1+\hat{v}+\Delta}{2}. \end{cases} \tag{25}$$

  - $\Delta > \max\{1 - 2r^* + \hat{v}, \frac{1}{2} - r^*\}$

Then, $E[v|v > \max\{\hat{v}+\Delta,0\}] = \frac{1+\max\{\hat{v}+\Delta,0\}}{2} < r^* + \Delta$. Thus, putting a mass point on $\frac{1+\max\{\hat{v}+\Delta,0\}}{2}$ is no longer optimal. Let's define

$$\tilde{v} = \min\{2[r^* + \Delta - \frac{1}{2}], 1\}, \tag{26}$$

which solves $E[v|v \geq \tilde{v}] = \frac{1+\tilde{v}}{2} = \min\{r^* + \Delta, 1\}$. Clearly, $\tilde{v} > \max\{\hat{v} + \Delta, 0\}$. Then, let's define the following auxiliary function:

$$\tilde{\phi}(v,\Delta) = \begin{cases} 0 & \text{for } v \leq \tilde{v} \\ \frac{\tilde{\alpha}^*}{1-r^*-\Delta}(v - \tilde{v}) & \text{for } v > \tilde{v}. \end{cases} \tag{27}$$

Notice that $\tilde{\phi}(v,\Delta) = u_i(v,\Delta)$ for $v \in [0,\tilde{v}] \cup (r^* + \Delta)$. Moreover, by eq. (24), $\frac{\tilde{\alpha}^*}{1-r^*-\Delta} = \frac{r^*-\hat{v}}{1-r^*-\Delta}(1-\tilde{\alpha}^*)\beta^*$ and by eqs. (7) and (27), $\tilde{\phi}(v,\Delta) \geq u_i(v,\Delta)$ for all v if and only if $\frac{r^*-\hat{v}}{1-r^*-\Delta} \geq 1 \implies \Delta \geq 1 - 2r^* + \hat{v}$. Therefore, for $\Delta \geq \max\{1 - 2r^* + \hat{v}, \frac{1}{2} - r^*\}$, $\tilde{\phi}(v,\Delta) \geq u_i(v,\Delta)$ and the following MPC of $F(v)$ satisfies Theorem 1:

$$G_\Delta(v|v^* = 0) = \begin{cases} \min\{v,\tilde{v}\} & \text{if } v < \min\{r^* + \Delta, 1\} \\ 1 & \text{if } v \geq \min\{r^* + \Delta, 1\}. \end{cases} \tag{28}$$

Given the characterized $G_\Delta(v|v^* = 0)$ and the payoff $u_i(v,\Delta)$ from (7), the expected deviation demand is:

$$E_{G_\Delta}[u_i(v,\Delta)] = \begin{cases} \tilde{\alpha}^* + (1-\tilde{\alpha}^*)\frac{\frac{1}{2}-r^*-\Delta}{1-2r^*} & \text{for } \Delta \leq \min\{-\hat{v}, \frac{1}{2} - r^*\} \\ \frac{1-\tilde{\alpha}^*}{1-2r^*}\frac{(1-\hat{v}-\Delta)^2}{2} & \text{for } \Delta \in \left(\min\{-\hat{v}, \frac{1}{2} - r^*\}, \max\{\frac{1}{2} - r^*, 1 - 2r^* + \hat{v}\}\right) \\ 2\tilde{\alpha}^*(\max\{1 - r^* - \Delta, 0\}) & \text{for } \Delta \geq \max\{\frac{1}{2} - r^*, 1 - 2r^* + \hat{v}\} \end{cases} \tag{29}$$

The above expression is continuous in Δ. Note that $\min\{-\hat{v}, \frac{1}{2}-r^*\} > 0$ and $E_{G_\Delta}[\tilde{u}_i(v,\Delta)|\Delta \leq \min\{-\hat{v}, \frac{1}{2}-r^*\}]$ is strictly concave in Δ with a unique maximizer $\hat{\Delta} = \frac{1-2r^*}{2(1-\tilde{\alpha}^*)}\left[\frac{1+\tilde{\alpha}^*}{2} - \frac{1-\tilde{\alpha}}{1-2r^*}p^*\right]$. Thus, a necessary condition for $\Delta^* = 0$ is $\hat{\Delta} = 0 \implies p = \frac{(1+\tilde{\alpha}^*)(1-2r^*)}{2(1-\tilde{\alpha}^*)} = p^*$.

It remains to establish that p^* also prevents larger upward deviations of $\Delta > \min\{-\hat{v}, \frac{1}{2} - r^*\}$. The intermediate region in eq. (29) exists if and only if $-\hat{v} < \frac{1}{2} - r^*$. In this case,

$$\begin{aligned} \frac{\partial E_{G_\Delta}[\tilde{u}_i(v,p_i)|\Delta \in (-\hat{v}, 1 - 2r^* + \hat{v})]}{\partial \Delta} &= \frac{1-\tilde{\alpha}^*}{1-2r^*}(1-\hat{v}-\Delta)(1-\hat{v}-2p^*-3\Delta) \overset{sign}{=} \\ &= (1-\hat{v}-2p^*-3\Delta) \leq \\ &\leq (1-2p^*+2\hat{v}) = \\ &= -4\frac{\tilde{\alpha}^* - (1+\tilde{\alpha}^*)r^*}{1-\tilde{\alpha}^*} \leq 0 \text{ since } r^* \leq \frac{\tilde{\alpha}^*}{1+\tilde{\alpha}^*} = \frac{\alpha}{2}. \end{aligned}$$

This establishes no profitable deviation to $\Delta \in (\min\{-\hat{v}, \frac{1}{2} - r^*\}, \max\{\frac{1}{2} - r^*, 1 - 2r^* + \hat{v}\}]$. Finally,

$$\frac{\partial E_{G_\Delta}[\tilde{u}_i(v,p_i)|\Delta \geq \max\{\frac{1}{2}-r^*, 1-2r^*+\hat{v}\})]}{\partial \Delta} = 2\tilde{\alpha}^*(-p^*-2\Delta+1-r^*) \leq$$
$$\leq 2\tilde{\alpha}(r^*-p^*) = 2\tilde{\alpha}\frac{4r^*-(1+\tilde{\alpha}^*)}{2(1-\tilde{\alpha}^*)} \leq$$
$$\leq -\tilde{\alpha}^*\frac{(1-\tilde{\alpha}^*)}{(1+\tilde{\alpha}^*)} < 0 \text{ since } r^* \leq \frac{\tilde{\alpha}^*}{1+\tilde{\alpha}^*}$$

Therefore, $p^* = \frac{(1+\tilde{\alpha}^*)(1-2r^*)}{2(1-\tilde{\alpha}^*)}$ is necessary and sufficient for $\in \underset{\Delta}{argmax}\ E_{G_\Delta}[\tilde{u}_i(v,\Delta)]$ and is the unique summetric equilibrium price for $v^*=0$.

- $v^* > 0$ (i.e., $s < \frac{1-\alpha}{2}$)

  In this case, $\hat{v} = v^* > 0$ and $1-2r^*+v^* > 0$. The optimal information structure depends on Δ as follows:

  - $\Delta \leq \max\{-v^*, r^*-\frac{1}{2}-v^*\}$

    In this case, $G_\Delta(v|v^*>0) = \mathbf{1}(v \geq \frac{1}{2})$ is an optimal disclosure strategy. To see this note, that $\Delta \leq \max\{-v^*, r^*-\frac{1}{2}-v^*\} < \frac{1}{2}-r^*$ since $1-2r^*+v^*>0$. For $r^* > \frac{1}{2}$, $\Delta \leq r^*-\frac{1}{2}-v^* \implies \frac{1}{2} \geq 1-r^*+v^*+\Delta$. Then, $G_\Delta(v|v^*>0) = \mathbf{1}(v \geq \frac{1}{2})$ and an auxiliary function $\hat{\phi}(v,\Delta) = 1$ satisfy Theorem 1. If $\Delta \in (r^*-\frac{1}{2}-v^*, -v^*] \neq \varnothing$, $\phi(v,\Delta)$ is affine and $\phi(\frac{1}{2},p_i) = u_i(\frac{1}{2},p_i)$ since $\frac{1}{2} < 1-r^*+v^*+\Delta$. Then, $\phi(v,\Delta)$ and $G_\Delta(v|v^*>0) = \mathbf{1}(v \geq \frac{1}{2})$ satisfy Theorem 1.

  - $\max\{-v^*, r^*-\frac{1}{2}-v^*\} < \Delta \leq 1-2r^*+v^*$

    Then, $\phi(v,\Delta)$ is piece-wise linear and $\phi(v,\Delta) = u_i(v,\Delta)$ for $v \in [0, v^*+\Delta] \cup [r^*+\Delta, 1-r^*+v^*+\Delta]$. Note that $E[v|v>v^*+\Delta] = \frac{1+v^*+\Delta}{2} \in [r^*+\Delta, 1-r^*+v^*+\Delta)$. Then, the following MPC of $F(v)$ satisfies Theorem 1 and is an optimal disclosure strategy:

$$G_\Delta(v|v^*=0) = \begin{cases} \min\{v, v^*+\Delta\} & \text{if } v < \frac{1+v^*+\Delta}{2} \\ 1 & \text{if } v \geq \frac{1+v^*+\Delta}{2}. \end{cases} \tag{30}$$

  - $\Delta > 1-2r^*+v^*$

    Then, $E[v|v>v^*+\Delta] < r^*+\Delta$ and, thus, putting a mass point at $\frac{1+v^*+\Delta}{2}$ is no longer optimal. Let's define the following auxiliary function:

$$\bar{\phi}(v,\Delta) = \begin{cases} \max\{v-\Delta, 0\} & \text{for } v \leq \tilde{v} \\ \tilde{v}-\Delta+\gamma(v-\tilde{v}) & \text{for } v > \tilde{v}, \end{cases} \tag{31}$$

    where $\tilde{v} > v^*+\Delta$ is defined by eq. (26) and $\gamma = \frac{\tilde{\alpha}^*+(1-\tilde{\alpha}^*)v^*-\tilde{v}+\Delta}{1-r^*-\Delta}$. By construction, $\bar{\phi}(v,\Delta)$ is continuous and concides with $u_i(v,\Delta)$ for $v \in [0, v^*+\Delta] \cup (r^*+\Delta)$. Note that $\tilde{v} > v^*+\Delta$ implies that $\phi(\tilde{v},p_i) > \bar{\phi}(\tilde{v},p_i) = \tilde{v}-\Delta$. Then, since both $\phi(v,\Delta)$ and $\bar{\phi}(v,\Delta)$ are affine for $v \geq \tilde{v}$ and $\phi(r^*+\Delta,p_i) = \bar{\phi}(r^*+\Delta,p_i) = u_i(v,\Delta)$, it follows that $\bar{\phi}(v,\Delta)$ is increasing faster than $\phi(v,\Delta)$ for $v \geq \tilde{v}$, i.e., $\gamma > \beta^*(1-\alpha^*)$. This, in turn, implies that $\phi(v,\Delta) \geq u_i(v,\Delta)$ for all v. Then, $\bar{\phi}(v,\Delta)$ and the following MPC of $F(v)$ satisfy Theorem 1:

$$G_\Delta(v|v^*>0) = \begin{cases} \min\{v,\Delta\} & \text{for } v < \frac{\Delta+\tilde{v}}{2} \\ \tilde{v} & \text{for } v \in [\frac{\Delta+\tilde{v}}{2}, \min\{r^*+\Delta,1\}) \\ 1 & \text{for } v \geq \min\{r^*+\Delta,1\}. \end{cases} \tag{32}$$

Given the characterized $G_\Delta(v|v^*>0)$ and the payoff $u_i(v,\Delta)$ from (7), the expected deviation demand is:[7]

$$E_{G_\Delta}[u_i(v,\Delta)] = \begin{cases} 1 & \text{for } \Delta \leq r^* - \frac{1}{2} - v^* \\ v^* + (1+\tilde{\alpha}^*)\left(\frac{1}{2} - v^* - \Delta\right) & \text{for } \Delta \in (r^* - \frac{1}{2} - v^*, -v^*] \\ \frac{(1-\Delta)^2 + \tilde{\alpha}^*(1-v^*-\Delta)^2}{2} & \text{for } \Delta \in \left(\max\{-v^*, r^* - \frac{1}{2} - v^*\}, 1 - 2r^* + v^*\right) \\ \frac{(2r^*+\Delta-1)^2}{2} + & \text{for } \Delta \in (1 - 2r^* + v^*, 1 - r^*] \\ +2(1-r^*-\Delta)\left(v^* + (1+\tilde{\alpha}^*)(r^*-v^*)\right) & \\ \frac{(\max\{1-\Delta,0\})^2}{2} & \text{for } \Delta > 1 - r^*. \end{cases} \tag{33}$$

The above expression is continuous in Δ and $0 \in \left(\max\{-v^*, r^* - \frac{1}{2} - v^*\}, 1 - 2r^* + v^*\right)$. Differentiating $E_{G_\Delta}[\tilde{u}_i(v,\Delta)|\Delta \in (\max\{-v^*, r^* - \frac{1}{2} - v^*\}, 1 - 2r^* + v^*)]$ obtains the following expression:

$$\frac{(1+\tilde{\alpha}^*)(1-\Delta) + \tilde{\alpha}^* v^*}{2} \left[\frac{(1-\Delta)^2 + \tilde{\alpha}^*(1-v^*-\Delta)^2}{2[(1+\tilde{\alpha}^*)(1-\Delta) + \tilde{\alpha}^* v^*]} - 2(p^*+\Delta) \right]$$

The first term is non-negative since $\Delta < 1$. It is also straighforward to verify that the first term in the brackets is decreasing in Δ for $\Delta < 1 - 2r^* + v^* \implies 1 - \Delta > 2r^* - v^* > v^*$. Therefore, there is at most one critial point $\hat{\Delta}$ in this region that maximizes the expected deviation payoff. Since no profitable deviation requires $\hat{\Delta} = 0$, it follows that $p^* = \frac{(1+\tilde{\alpha}^*(1-v^*)^2)}{2(1+\tilde{\alpha}^*(1-v^*))}$ is a necessary condition for preventing a profitable price deviation. It remains to establish that p^* prevents large price deviations by showing that $E_{G_\Delta}[\tilde{u}_i(v,p_i)]$ is strictly increasing in Δ for $\Delta < 0$ and strictly decreasing in Δ for $\Delta > 0$.

From the discussion above, this clearly holds for $\Delta \in (\max\{-v^*, r^* - \frac{1}{2} - v^*\}, 1 - 2r^* + v^*)]$. For $\Delta \leq r^* - \frac{1}{2} - v^* < 0$, $E[\tilde{u}_i(v,\Delta)] = (p^* + \Delta)$ is strictly increasing in Δ. For $\Delta \in (r^* - \frac{1}{2}, -v^*, -v^*] < 0$,

$$\begin{aligned} \frac{\partial E_{G_\Delta}[\tilde{u}_i(v,p_i)|\Delta \in (r^* - \frac{1}{2}, -v^*, -v^*]}{\partial \Delta} &= v^* + (1+\tilde{\alpha}^*)\left(\frac{1}{2} - v^* - 2\Delta - p^*\right) \geq \\ &\geq (2+\tilde{\alpha}^*)v^* + (1+\tilde{\alpha}^*)\left(\frac{1}{2} - p^*\right) \geq 0 \text{ since } p^* \leq \frac{1}{2} \end{aligned}$$

[7] The expected demand takes into account that for $v^* > 0$, $\beta^* = \frac{1+\tilde{\alpha}^*}{1-\tilde{\alpha}^*}$ (by eqs. (15) and (17)) and $\tilde{\alpha}^* + (1 - \tilde{\alpha}^*)[v^* + \beta^*(v^* - r^*)] = v^*$, which follows by the continuity of $\phi(v)$ given by eq. (14).

To establish the comparative statics with respect to s, note that $r^*(0) = 1$ and $r^*(\frac{1-\alpha}{2}) = \frac{\alpha}{2}$, which by Proposition 1 leads to $v^*(0) = 1$ and $v^*(\frac{1-\alpha}{2}) = 0$. Thus, by eq. (9) it follows immediately that $p^*(0) = p^*(\frac{1-\alpha}{2}) = \frac{1}{2}$. To show that p^* is $U-shaped$ in s for $s < \frac{1-\alpha}{2}$, note that $\frac{dp^*}{ds} = \frac{\partial p^*}{\partial v^*}\frac{dv^*}{ds}$, where by Proposition 1 and 2, $\frac{dv^*}{ds} = \frac{\partial v^*}{\partial r^*}\frac{\partial r^*}{\partial s} < 0$. Therefore, $\frac{dp^*}{ds} \overset{sign}{=} -\frac{\partial p^*}{\partial v^*}$. Substituting for $\tilde{\alpha}^*$ in p^* obtains:

$$p^* = \frac{2 - \alpha(v^*)^2(1-v^*)}{2[2+\alpha v^*(1-v^*)]}.$$

Differentiating p^* with respect to v^* obtains:

$$\frac{\partial p^*}{\partial v^*} = \frac{2[3(v^*)^2 - 1] - \alpha(v^*)^2(1-v^*)^2}{2[[2+\alpha v^*(1-v^*)]^2}$$

Clearly, $\frac{\partial p^*(v^*=1)}{\partial v^*} > 0$ and $\frac{\partial p^*(v^*=0)}{\partial v^*} < 0$, implying that $\frac{dp^*(0)}{ds} < 0$ and $\frac{dp^*(\frac{1-\alpha}{2})}{ds} > 0$. Moreover, $\frac{\partial^2 p^*}{\partial^2 v^*} = \frac{-2\alpha[3v^*(2-\alpha)+\alpha(1+3\alpha(v^*)^2+(v^*)^3)]}{[2+\alpha v^*(1-v^*)]^2} < 0$, implying that $p^*(s)$ is convex in s. Since $p^*(0) = p^*(\frac{1-\alpha}{2}) = \frac{1}{2}$, this establishes that $p^*(s)$ is U-shaped in s.

For $s \geq \frac{1-\alpha}{2}$, substituting for r^* given by Proposition 2 obtains $p^*(s) = \frac{s}{(1-\alpha)}$, which is strictly increasing in s with $p^*(\frac{1}{2}) = \frac{1}{1-\alpha}$.

■

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