Input prices and downstream overlapping ownership*

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Abstract

A key issue in antitrust economics is how overlapping ownership affects firm behavior and consumer welfare. This paper studies how horizontal ownership among downstream firms affects the input prices set by an upstream supplier. Using a general consumer demand function, I provide precise conditions on demand primitives for when downstream ownership raises, reduces, or has no effect on input prices. The key factor is shown to be how demand curvature varies with total output. When horizontal ownership lowers input prices, the price reduction is in turn passed on to consumers, which mitigates the standard anti-competitive effect in the output market.

Keywords: Overlapping ownership, Input markets, Vertical relations, Demand curvature, Anti-competitive effects.

JEL codes: D43, L13, L41.

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1 Introduction

In many markets, we observe that firms have partial ownership stakes in each other (cross-ownership) or that institutional investors hold shares in several competitors (common ownership) (Backus et al., 2021; Heim et al., 2022). These horizontal practices are often collectively referred to as overlapping ownership arrangements (OOAs) (López and Vives, 2019). The ability of such OOAs to reduce product market competition is well established theoretically (e.g., Reynolds and Snapp, 1986) and has also recently been documented empirically (Azar et al., 2018). The basic intuition for why OOAs can soften competition is that they incentivize firms to internalize how their strategic actions affect rivals’ profits. In recent years, this theory of harm has received increased interest from many antitrust agencies and policy makers (e.g., OECD, 2017; US Federal Trade Commission, 2018; European Commission, 2020). Moreover, leading antitrust experts have argued for stricter regulations of OOAs, such as a cap on the level of ownership allowed in a given market (Elhauge, 2015; Posner et al., 2016).

In the academic literature, the effects of OOAs have traditionally been studied using the classic oligopoly models (e.g., the Cournot and Bertrand models). When analyzing OOAs in such frameworks, an implicit assumption is that the level of overlapping ownership does not affect the firms’ input costs. This assumption may be reasonable if the firms produce inputs themselves or source them in the world market. In practice, however, inputs are often bought from powerful upstream firms, i.e., there are vertical relationships between the jointly held firms and their suppliers. In that case, the above-mentioned independence assumption is less compelling, since input costs will be a function of the input prices set by the upstream firms, and since there is empirical evidence suggesting that input prices may depend on the nature of downstream ownership (e.g., McGuckin and Nguyen, 2001; Bhattacharyya and Nain, 2011). Given this, new questions about OOAs emerge.

In particular, will a change in the level of downstream overlapping ownership elicit a strategic response from suppliers in the form of a change in input prices? If so, will input prices increase or decrease? How would such an upstream pricing
effect interfere with the standard view of OOAs? Can the supplier’s pricing response offset the anti-competitive effect of OOAs in the product market?

This paper adds to our understanding of OOAs by addressing these questions. To be precise, I study a canonical model of a vertical market structure, in which an upstream monopolist supplies two downstream firms who in turn produce final goods and compete in the downstream market. In this setting, I analyze how horizontal OOAs among the downstream firms affect the equilibrium input prices set by the supplier. A key feature of the model is that I consider a general consumer demand function and derive results in terms of primitives of this function. That approach distinguishes my paper from other recent contributions that have looked at similar questions while restricting attention to specific demand forms (see Section 2 for a detailed discussion).

The key finding of the paper is that the effect of downstream OOAs on input prices depends on the curvature (i.e., the elasticity of the slope) of the consumer demand function – and, more specifically, on how this curvature varies with total output. For example, I find that a higher level of overlapping ownership among the downstream firms leads to a higher input price if demand curvature is increasing in output. Conversely, downstream OOAs lead to lower input prices if demand curvature is decreasing in output. Finally, downstream OOAs have zero effect on input prices if and only if demand curvature is constant with respect to output. In short, the output-derivative of demand curvature is a sufficient statistic for the effect of downstream OOAs on input prices.

I also examine how downstream OOAs affect total output and consumer surplus. Here, I find that a higher level of overlapping ownership always reduces output and consumer welfare. While this result is in line with the traditional view OOAs, it is notable from an enforcement viewpoint that the supplier’s pricing response is important for the magnitude of consumer harm. In particular, it is interesting to note that when demand curvature is decreasing, the input price reduction is in turn passed on to consumers, which mitigates the direct anti-competitive effect that the OOA-expansion has in the downstream market.

The intuition for the input price effect is as follows. An increase in overlapping
ownership induces a downward pressure on total output. Such an output reduction affects the pass-through rate of input prices to final-goods quantities, through the curvature of demand. For example, if the curvature is increasing, lower output reduces curvature, which reduces the pass-through rate. As a result, the market-wide derived demand curve becomes less elastic and, in equilibrium, the supplier responds by raising the input price. Conversely, if the curvature is decreasing, lower output raises the pass-through rate and derived demand elasticity, forcing the supplier to reduce the equilibrium input price.

The remainder of the paper is organized as follows. I start in Section 2 by placing the paper within the related literature. Section 3 then lays out the model. The equilibrium of the model is derived in Section 4. In Section 5, I obtain my main results by studying how the equilibrium input prices and output levels depend on the level of downstream ownership. Section 6 concludes the paper. Some formal material is relegated to an Appendix.

## 2 Related literature

This paper is related to a nascent theoretical literature that examines the effects of overlapping ownership among downstream firms in vertically related markets.\(^1\) This literature shares some similarities with the more established literature on retail mergers (e.g., Lommerud et al., 2005; Inderst and Shaffer, 2007; Gaudin, 2018; Ghosh et al., 2022). However, a fundamental difference is the following: After a horizontal merger, the acquiring downstream firm obtains full control over the merging partner’s decisions. By contrast, after a small increase in the level of overlapping ownership, downstream firms continue to make their decisions independently and non-cooperatively. (See Section 5.1 and Section 6 for some further discussion of the relationship to the merger literature.)

\(^1\)There are some differences within this literature in terms of whether authors focus on partial cross-ownership (also called cross-holding or minority ownership), common ownership (also called horizontal shareholding), or both. While cross-ownership and common ownership are not identical in practice, they typically give rise to similar incentive effects in theoretical models. In this paper, I follow López and Vives (2019) in considering both practices under the umbrella term OOAs (see Section 3 for more details).
Hu et al. (2020) study a model with two downstream firms and one supplier who also can invest in cost-reducing R&D. They show that when the level of downstream overlapping ownership increases and the downstream firms reduce their outputs, the supplier responds by reducing its investment. Thus, a higher level of overlapping ownership leads to higher input prices.

Li and Shuai (2022) consider a model with asymmetric ownership, e.g., one downstream firm holding a share in its rival while the rival does not hold such a share (see also Hu et al., 2022). They show that the upstream supplier offers a lower input price to the downstream firm who holds the ownership share. This effect occurs because this buyer produces a lower quantity of the final good and therefore has a more elastic demand for the input.

Shuai et al. (2022) also study a model with asymmetric ownership, but where there also is an “outside” downstream firm who is not part of the OOA and potentially multiple suppliers. They find, similar to Li and Shuai (2022), that suppliers respond to downstream ownership by reducing the input price of the acquiring firm. However, their analysis also shows that the input price effects for the acquired firm and the outsider depends delicately on the upstream market structure.

Chen et al. (forthcoming) examine a model with $n \geq 2$ downstream firms and one supplier. In their model, the OOAs are not purely horizontal but also involve vertical ownership between the downstream firms and the supplier. They find that a higher ownership level reduces the elasticity of market-wide derived demand, thus allowing the supplier to raise its input prices.

The key difference between my paper and the above papers is the specification of consumer demand. In particular, whereas I consider a general demand function, the above papers restrict attention to linear and constant elasticity demands. This restriction is critical because these demand forms have constant curvatures. Thus,

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2While there has been relatively little attention on horizontal shareholding in vertical markets, there exists a substantial literature on purely vertical ownership between upstream and downstream firms, see, e.g., Levy et al. (2018).

3Hu et al. (2020) analyze both linear demand and a parametric constant elasticity form (see Section 3 for formal details). Li and Shuai (2022) and Chen et al. (forthcoming) focus on linear demand in their main analysis and consider general demand in appendices. However, there they do not solve for the equilibrium input prices or output levels. Shuai et al. (2022) use general demand to analyze the downstream firms’ incentives to engage in partial ownership with exogenous input costs. However, when analyzing endogenous input prices, they too focus on linear demand.

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the above models do not pick up the relationship between downstream ownership and input prices that works through variations in demand curvature. In this paper, I provide a clean and general characterization of this relationship and show that it can be positive or negative depending on the demand function. By contrast, the above-cited papers identify relationships that are specific to the demand systems they use, and whose directions (i.e., whether ownership raises or reduces input prices) depends on other features of the models (R&D, asymmetries, vertical ownership).

The closest paper in the traditional literature on OOAs is probably López and Vives (2019), who study a model in which oligopolistic firms set quantities and also invest in cost-reducing R&D. They show that a higher level of ownership can increase investment and reduce costs when certain conditions on demand curvature and R&D spillovers are met. My paper complements López and Vives (2019) by illustrating that, even in the absence of R&D investment, OOAs can lead to lower costs through the pricing decision of an upstream supplier.

On a methodological level, this paper is related to the influential work of Weyl and Fabinger (2013) and Mrázová and Neary (2017), who emphasized the importance of demand primitives such as pass-through rates and curvature for questions in IO (and many other sub-fields). Interestingly, the key factor in my analysis – the derivative of demand curvature with respect to output – was also identified by Chen and Schwartz (2015) as the key determinant for a very different issue, namely the welfare effects of differential pricing.

3 Model

I consider a setting with three firms: an upstream supplier, $U$, and two downstream firms, 1 and 2. The supplier produces an input at constant marginal cost $c \geq 0$. The downstream firms transform the input into a homogeneous final good using a one-to-one technology. Without loss of generality, I normalize the production costs of the downstream firms to zero.

The timing of events is as follows.
• \( t = 1 \): The supplier sets its per-unit input prices, \( w_1 \) and \( w_2 \).\(^4\) These price offers are publicly observable among the downstream firms.

• \( t = 2 \): The downstream firms purchase inputs and produce and sell final goods. Downstream competition takes place in a Cournot fashion, with the firms choosing quantities \( q_1 \) and \( q_2 \) simultaneously and non-cooperatively. Finally, consumers make their purchases and profits are realized.

The solution concept will be subgame perfect Nash equilibrium.

An important parameter of the model is \( \lambda \in [0, 1) \), which captures the degree of overlapping ownership among the downstream firms.\(^5\) In general, overlapping ownership induces (the manager of) each downstream firm to internalize with weight \( \lambda > 0 \) the rival’s profit in its own objective function (see (2) below). Moreover, as shown by López and Vives (2019, Section III), \( \lambda \) can represent both cross-ownership and common ownership, depending on the micro-foundation. Under cross-ownership, \( \lambda \) is an increasing function of the direct (symmetric) ownership stakes that the firms have in each other. Under common ownership, external investors hold shares in the competing firms, and \( \lambda \) increases with the number of investors and the size of their ownership positions.

The inverse demand function in the final-goods market is given by \( P(Q) \), where \( Q := q_1 + q_2 \) is total output. I assume that \( P(Q) \) is thrice continuously differentiable, and that \( P'(Q) < 0 \) for all \( Q \geq 0 \) such that \( P(Q) > 0 \). Furthermore, the curvature of inverse demand is defined as

\[
\sigma(Q) := -\frac{P''(Q)Q}{P'(Q)}.
\]

\(^4\) The assumption that \( U \) offers simple per-unit prices, which is also made in the related literature, can be defended on both theoretical and empirical grounds. In theory, more complex pricing schemes can be costly to implement (e.g., due to moral hazard problems), and may also be less efficient if trade occurs infrequently and volumes are subject to uncertainty (see Dobson and Waterson, 2007). In real-life input markets, we often observe the use of linear pricing, e.g., between TV channels and distributors (Crawford and Yurukoglu, 2012) and between hospitals and suppliers of medical equipment (Grennan, 2013).

\(^5\) It is perhaps most realistic to think of \( \lambda \) as belonging to the lower end of this spectrum (say, \( \lambda < 1/2 \)). For one, relatively small ownership stakes are what we typically observe in practice. Moreover, very high values of \( \lambda \) can be seen as logically inconsistent with the assumption that downstream firms choose quantities independently. Note that while I derive my main results by looking at small changes in \( \lambda \), the results do not hinge on the level of \( \lambda \) per se (i.e., whether ownership marginally increases starting from, say, 5% or 12%).
Note that $\sigma(Q)$ has the same sign as $P''(Q)$; thus, $\sigma(Q)$ can also be interpreted as a measure of demand convexity. In general, demand is concave if $\sigma(Q) < 0$ and convex if $\sigma(Q) > 0$. For linear demand, $\sigma(Q) = 0$.

Importantly, I allow $\sigma(Q)$ to vary with $Q$; the nature of this relationship will be key in the analysis. This is in contrast to the related literature discussed in Section 2 that focuses on demand forms with constant curvatures. This property is immediate for linear demand. Moreover, Hu et al. (2020) consider the parametric form $P(Q) = a - bQ^{(1-z)}/(1 - z)$ with $a, b \geq 0$ and $z < 1$, which has a curvature of $\sigma = -\left(b Q^{-(1+z)}/(1 - bQ^{-z})\right) Q$ for any $Q$.

The profit of the supplier is $\pi_U = (w_1 - c) q_1 + (w_2 - c) q_2$, and the individual profit of downstream firm $i \in \{1, 2\}$ is $\pi_i = [P(Q) - w_i] q_i$. Thus, for $k \neq i$ and a given $\lambda$, the objective functions of the downstream firms can be written as

$$\phi_i(q_i, q_k) = [P(Q) - w_i] q_i + \lambda [P(Q) - w_k] q_k.$$  \hspace{1cm} (2)

I make the following assumption.

**Assumption 1.** $2P'(Q) + P''(Q) Q < 0$ for all $Q > 0$.

Assumption 1 says that market-wide marginal revenue is decreasing in total output. Note that this condition is equivalent to $\sigma(Q) < 2$, i.e., inverse demand must not be too convex. The assumption is a standard one in Cournot-style models (see, e.g., Anderson and Renault, 2003). In our setting, Assumption 1 ensures (i) that the downstream firms’ second-order conditions are satisfied, and (ii) that there exists a unique and stable (symmetric) equilibrium in the subgame at $t = 2$ for given input prices (see Appendix A.1 for details). The supplier’s second-order condition is ensured only under an additional condition, to which I return in Section 4.2.

## 4 Equilibrium

In this section, I derive the symmetric subgame perfect Nash equilibrium of the above model. This analysis proceeds by backward induction.
4.1 Downstream competition

At $t = 2$, the first-order condition of downstream firm $i$ is $\partial \phi_i / \partial q_i = 0$, which can be written as

$$P(Q) + P'(Q) q_i + \lambda P'(Q) q_k = w_i. \quad (3)$$

The third term at the left-hand side of (3) reflects the standard anti-competitive effect of OOAs: Firm $i$ takes into account with weight $\lambda$ that marginally raising $q_i$ lowers the price on firm $k$’s units. In the following, I focus on a different question: How does overlapping ownership affect the input price at the right-hand side of (3), when this is chosen by an upstream supplier at a preceding stage of the game?

In the symmetric equilibrium, we have $q_i = q_k = q = \frac{1}{2}Q$. Lemma 1 below uses this to restate the equilibrium condition (3) in a more compact way and introduce some key variables that will feature throughout the analysis.

**Lemma 1.** In the downstream market, the symmetric equilibrium is characterized by

$$\mu(Q) = w_i, \quad (4)$$

where

$$\mu(Q) := P(Q) + \alpha P'(Q) Q \quad (5)$$

is the marginal revenue of firm $i$, and

$$\alpha := \frac{1 + \lambda}{2} \quad (6)$$

denotes the “Modified Herfindahl-Hirschman Index” (MHHI).\(^6\)

According to (4), each downstream firm equates its marginal revenue and marginal cost in equilibrium. Moreover, as can be seen from (5) and (6), a higher value of $\lambda$ increases the MHHI and thereby reduces the marginal revenue of firm $i$ (for a given $Q$), in line with the standard intuition.

\(^6\)The MHHI was introduced by Bresnahan and Salop (1986) and has featured heavily both in the empirical literature and in the recent policy debate on OOAs. In essence, for a given number of firms, the MHHI captures the “additional” reduction in competitive incentives caused by overlapping ownership.
4.2 Upstream pricing

At $t = 1$, the supplier’s profit can be written as

$$
\pi_U(w_i, w_k) = (w_i - c) q_i(w_i, w_k) + (w_k - c) q_k(w_k, w_i),
$$

(7)

where $q_i(w_i, w_k)$ and $q_k(w_k, w_i)$ are the downstream firms’ equilibrium quantities as functions of input prices. These quantity functions are implicitly defined by the first-order conditions at $t = 2$. The supplier chooses its input prices in order to maximize (7). The first-order condition, $\partial \pi_U / \partial w_i = 0$, is

$$
q_i(w_i, w_k) + (w_i - c) \frac{\partial q_i}{\partial w_i} + (w_k - c) \frac{\partial q_k}{\partial w_i} = 0.
$$

(8)

Before proceeding with the analysis, I now pin down the supplier’s second-order condition (i.e., $\partial^2 \pi_U / \partial w_i^2 + |\partial^2 \pi_U / \partial w_k w_i| < 0$). The following assumption uses the definition of marginal revenue in (5). In particular, it can be shown (see Appendix A.1) that, in the symmetric equilibrium, the supplier’s second-order condition holds under the following assumption.

**Assumption 2.** $2 \frac{\partial \mu}{\partial Q} + Q \frac{\partial^2 \mu}{\partial Q^2} < 0$ for all $Q$.

Note that Assumption 2 can equivalently be expressed as $2 - \tau(Q) > 0$, where

$$
\tau(Q) := -\frac{Q \frac{\partial^2 \mu}{\partial Q^2}}{\frac{\partial \mu}{\partial Q}}
$$

(9)

denotes the curvature of the derived demand function faced by the supplier.

To solve for the symmetric equilibrium input price, we need the pass-through rates of input prices to final-goods quantities for each individual downstream firm and the industry as a whole. These pass-through rates are found by evaluating the downstream first-order conditions at the equilibrium quantities $q_i(w_i, w_k)$ and $q_k(w_k, w_i)$, differentiating with respect to $w_i$ and $w_k$, and then solving the resulting system of equations (see Appendix A.2 for the formal proof).

**Lemma 2.** In the symmetric equilibrium, the variety-specific pass-through rates are
given by
\[
\frac{\partial q_i}{\partial w_i} = \frac{2 - \alpha \sigma (Q)}{4 (1 - \alpha)} \left( \frac{\partial \mu}{\partial Q} \right)^{-1},
\]
and
\[
\frac{\partial q_k}{\partial w_i} = -\alpha \frac{2 - \sigma (Q)}{4 (1 - \alpha)} \left( \frac{\partial \mu}{\partial Q} \right)^{-1},
\]
and the total pass-through rate is given by
\[
\frac{\partial Q}{\partial w_i} = \frac{\partial q_i}{\partial w_i} + \frac{\partial q_k}{\partial w_i} = \frac{1}{2} \left( \frac{\partial \mu}{\partial Q} \right)^{-1}
\]
To my knowledge, these pass-through rates have not previously been derived in the literature on OOAs. Yet, their interpretation is straightforward. We can start from (5) by noting that
\[
\frac{\partial \mu}{\partial Q} = (1 + \alpha) P' (Q) + \alpha P'' (Q) Q.
\]
By rearranging this expression, we see that
\[
\frac{\partial \mu}{\partial Q} < 0 \iff \sigma (Q) < \frac{1 + \alpha}{\alpha}.
\]
Thus, since \( \sigma (Q) < 2 \) and \( \frac{1 + \alpha}{\alpha} > 2 \) (as \( \alpha < 1 \)), we have \( \partial \mu / \partial Q < 0 \), and therefore \( \partial q_i / \partial w_i < 0 \) and \( \partial q_k / \partial w_i > 0 \). These signs are intuitive: If the input price of one firm increases, that firm produces less. In response, the rival firm produces more, since quantities are strategic substitutes.\(^7\) Furthermore, the fact that \( \partial Q / \partial w_i \) is negative reflects that the output-reducing effect of \( w_i \) on \( q_i \) dominates the output-expanding effect of \( w_i \) on \( q_k \). Thus, any change in the input price induces a change in the opposite direction in total output.

Note also that \( \partial Q / \partial w_i \) is the main determinant of the elasticity of the market-wide derived demand curve, which can be written as \( \epsilon = - [w/Q (w)] (\partial Q / \partial w_i) \). For example, we see from (12) that a steeper downstream marginal revenue curve gives a lower pass-through rate (i.e., lower in absolute terms, meaning that cost changes are

\(^7\)To show that quantities are strategic substitutes, we can apply the implicit function theorem to (3). This yields \( \partial q_i / \partial q_k = -\alpha \frac{2 - \sigma (Q)}{[2 - \alpha \sigma (Q)]} < 0 \), where the inequality follows from Assumption 1.
passed on to a *lesser* degree) and thus a less elastic derived demand curve, *ceteris paribus*. Conversely, if the marginal revenue curve is flatter, the pass-through rate is higher and derived demand is more sensitive to input price changes.

We can now return to the supplier’s first-order condition and derive the optimal input price. Focusing again on the symmetric equilibrium in which \( w_i = w_k = w \) and \( q_i(w, w) = q \), (8) can be written as

\[
\frac{1}{2} Q + (w - c) \frac{\partial Q}{\partial w_i} = 0.
\]

By using (12), we then obtain the following:

**Lemma 3.** The symmetric equilibrium input price is given by

\[ w = c - Q \frac{\partial \mu}{\partial Q}, \]

where the last term is evaluated at the equilibrium point.

The pricing rule in (13) is also intuitive. First, the supplier always charges a mark-up above its marginal cost (i.e., \( \partial \mu / \partial Q < 0 \Rightarrow w > c \)). Second, the size of the optimal mark-up reflects the elasticity of derived demand. For example, as explained above, a steeper marginal revenue curve gives a less elastic derived demand curve, which allows the supplier to set a higher input price. Conversely, if \( \partial \mu / \partial Q \) is closer to zero, derived demand is more elastic, and the supplier’s market power is more limited.

To summarize, (4) and (13) from Lemma 1 and Lemma 3, respectively, are the equilibrium conditions of the model. In the next section, I use these equations to address the key questions about the effects of downstream overlapping ownership.

## 5 Main results

I now take a comparative statics approach in studying the effect of a small increase in the ownership level on the equilibrium values. Specifically, I study the effect on input prices in Section 5.1 and effects on total output and consumer surplus in
Section 5.2. Note that equilibrium total output is given by \( Q = 2q(w, w) \), where \( w \) is set according to (13).

The first step is to totally differentiate the equilibrium conditions with respect to \( \lambda \). First, by differentiating (4) and using (5) and (6), we obtain

\[
\frac{\partial \mu}{\partial Q} \frac{dQ}{d\lambda} + \frac{d\mu}{d\lambda} = \frac{dw}{d\lambda} \iff \frac{dQ}{d\lambda} = \frac{1}{\frac{dw}{d\lambda} - \frac{P'(Q)Q}{2}}.
\]

(14)

Second, differentiating (13) yields

\[
\frac{dw}{d\lambda} = -\left\{ \frac{dQ}{d\lambda} \frac{\partial \mu}{\partial Q} + Q \left[ \frac{\partial^2 \mu}{\partial Q^2} \frac{dQ}{d\lambda} + \frac{d\left( \frac{\partial u}{\partial Q} \right)}{d\lambda} \right] \right\},
\]

or equivalently

\[
\frac{dw}{d\lambda} = - \left( \frac{\partial \mu}{\partial Q} + \frac{\partial^2 \mu}{\partial Q^2} Q \frac{dQ}{d\lambda} \right) \frac{dQ}{d\lambda} - Q \left[ P'(Q) + P''(Q)Q \right].
\]

(15)

The system given by (14) and (15) can then be solved simultaneously in order to determine \( \frac{dw}{d\lambda} \) and \( \frac{dQ}{d\lambda} \).

5.1 Input prices

Starting with the effect on the input price, we find that

\[
\frac{dw}{d\lambda} = \frac{-\frac{1}{2}Q^2}{2\frac{\partial \mu}{\partial Q} + Q \frac{\partial^2 \mu}{\partial Q^2}} \left[ P''(Q) \frac{\partial \mu}{\partial Q} - P'(Q) \frac{\partial^2 \mu}{\partial Q^2} \right].
\]

(16)

By using the definitions of \( \frac{\partial \mu}{\partial Q} \) and \( \tau(Q) \), and the fact that

\[
\frac{\partial^2 \mu}{\partial Q^2} = (1 + 2\alpha) P''(Q) + \alpha P'''(Q) Q,
\]

we can rewrite (16) as

\[
\frac{dw}{d\lambda} = \frac{-\frac{1}{2}Q^2}{\frac{\partial u}{\partial Q} [2 - \tau(Q)]} \left\{ Q \left[ P''(Q) \right]^2 - P'(Q) \left[ P''(Q) + P'''(Q) Q \right] \right\}.
\]

(17)
In addition, we have from (1) that
\[
\frac{\partial \sigma(Q)}{\partial Q} = -\left\{ \frac{[P''(Q)Q + P''(Q)]P'(Q) - Q[P''(Q)]^2}{[P'(Q)]^2} \right\}.
\] (18)

Thus, by combining (17) and (18), we obtain
\[
\frac{dw}{d\lambda} = \left( -\frac{\partial^2}{\partial Q^2} \left[ QP'(Q) \right] \right) \frac{\partial \sigma(Q)}{\partial Q}.
\] (19)

The term in brackets is positive due to the second-order conditions. Specifically, Assumption 1 implies \( \partial \mu/\partial Q < 0 \) and Assumption 2 implies \( 2 - \tau(Q) > 0 \). Thus, \( dw/d\lambda \) and \( \partial \sigma(Q)/\partial Q \) have the same sign. This yields the following result.

**Proposition 1.** An increase in the degree of downstream overlapping ownership leads to ...

- a higher input price if the curvature of consumer demand is increasing in total output, i.e., \( \partial \sigma(Q)/\partial Q > 0 \),
- a lower input price if the curvature of consumer demand is decreasing in total output, i.e., \( \partial \sigma(Q)/\partial Q < 0 \),
- no change in the input price if the curvature of consumer demand is constant with respect to total output, i.e., \( \partial \sigma(Q)/\partial Q = 0 \).

Proposition 1 is the main result of the paper. It shows that the derivative of the curvature of consumer demand with respect to total output is a sufficient statistic for the effect of downstream overlapping ownership on input prices. Moreover, it shows that the effect may be positive, negative, or zero, depending on whether the curvature is increasing, decreasing, or constant.\(^8\)

To see the intuition behind Proposition 1, it is useful to first recall the basic relationship between demand curvature and pass-through: A demand function with a higher curvature – i.e., one that is more convex – yields a higher (in absolute

\(^8\)Note that, as \( \partial \sigma(Q)/\partial Q \) may itself vary with output, the conditions in Proposition 1 are local conditions that apply at the equilibrium point.
terms) pass-through rate of input prices to final-goods output. This means that if the curvature (convexity) is increasing, a higher total output leads to a higher curvature and higher total pass-through rate, ceteris paribus. Conversely, if the curvature is decreasing, a higher total output leads to a lower pass-through rate.

With this relationship in mind, consider now the effect of a small increase in the degree of overlapping ownership. First, as will be shown in Section 5.2, more overlapping ownership always leads to lower total output (see (20) below). If the curvature is increasing, this output reduction then causes a reduction in the curvature and the pass-through rate, as explained above. Finally, as explained in Section 4.2, the lower pass-through rate gives a less elastic derived demand curve, which leads the supplier to set a higher input price. Thus, \( \frac{dw}{d\lambda} > 0 \). The converse argument applies in the case of decreasing curvature. Here, when the ownership level increases, the corresponding reduction in output leads to a higher pass-through rate, a more elastic derived demand curve, and a lower input price, ceteris paribus. Thirdly, in the case of constant curvature, the above mechanism does not come into play, and the increase in downstream ownership has no effect on the input price in this fundamental setting.

As discussed in Section 3, constant curvature is a property of some popular demand forms, e.g., linear and constant elasticity demands (see also Bulow and Pfleiderer, 1983). More recently, Fabinger and Weyl (2016) have characterized the curvature properties of various demand functions according to their underlying distributions of consumer valuations. They show that many commonly used demand forms (e.g., logit demand) have curvatures that are increasing in price or, equivalently, decreasing in quantity. For such specifications, downstream ownership reduces input prices in my model. A simple example that encompasses all three cases is \( P(Q) = (1 - Q)^x \) with \( x > 0 \), for which \( \sigma(Q) = -Q(1-x)/(1-Q) \) and

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9Intuitively, if demand is generated from an underlying distribution of consumer valuations, a more convex demand function corresponds to a greater heterogeneity, i.e., many consumers with high valuations and many consumers with low valuations. In this case, if the input price increases, the downstream firms may find it profitable to abandon the low-valuation consumers and sell only to high-valuation consumers, which leads to a large reduction in total quantity. See, e.g., Weyl and Fabinger (2013) and Miller et al. (2017).

10In their model, Hu et al. (2020) also briefly considered the case without upstream R&D and found – in line with my neutrality result – a null effect of OOAs on input prices. However, they did not relate this result to demand curvature.
\[ \frac{\partial \sigma(Q)}{\partial Q} = \frac{x - 1}{(1 - Q)^2}. \] Thus, this demand function has constant curvature if \( x = 1 \), decreasing curvature if \( x < 1 \), and increasing curvature if \( x > 1 \).

Moreover, the conditions on demand curvature given in Proposition 1 have two alternative yet equivalent interpretations. First, the curvature of demand being decreasing (respectively, increasing, constant) in output is equivalent to the pass-through rate of input prices onto the market price, i.e., \( \partial P/\partial w_i \), being increasing (resp., decreasing, constant) in \( w_i \). To see this, note that (12) can be used to obtain

\[ \frac{\partial P}{\partial w_i} = \frac{1}{2 \left[ 1 + \alpha - \alpha \sigma(Q) \right]^2} \frac{\partial \sigma(Q)}{\partial Q}. \]

Second, the curvature of demand being decreasing (respectively, increasing, constant) in output is also equivalent to the curvature of consumer demand being greater than (resp., smaller than, equal to) the curvature of derived demand.\footnote{These close connections between the derivative of demand curvature, the slope of the pass-through rate, and the comparison of curvature in the upstream and downstream markets have previously been noted by Gaudin (2018) and Ghosh et al. (2022). These authors examine the impact of retail mergers on input prices in models where the number of downstream firms is treated as a continuous variable.}

To see this, note that (18) and the definitions of \( \sigma(Q), \tau(Q), \frac{\partial \mu}{\partial Q}, \) and \( \frac{\partial^2 \mu}{\partial Q^2} \) can be used to show that

\[ \sigma(Q) - \tau(Q) = \frac{-\alpha Q P'(Q)}{\frac{\partial \sigma(Q)}{\partial Q}} \frac{\partial \sigma(Q)}{\partial Q}. \]

### 5.2 Output and consumer surplus

The preceding analysis shows that, in this model, overlapping ownership affects output in two ways: Directly, in the downstream market, and indirectly, through the input price. The direct effect is always negative: For a given input price, overlapping ownership reduces downstream competition and thereby total output (see (3)). The sign of the indirect effect, by contrast, depends on the curvature of demand as shown in Proposition 1, and its magnitude also depends on the pass-through rate of input prices to final-goods quantities. The net effect, which combines the direct
and indirect effects, is found by solving the equation system given by (14) and (15) for $\frac{dQ}{d\lambda}$:

$$\frac{dQ}{d\lambda} = \frac{-P'(Q)Q^2 - \sigma(Q)}{2\frac{\partial \nu}{\partial Q} 2 - \tau(Q)}.$$  \hfill (20)

We see that, under the second-order conditions in Assumption 1 and Assumption 2, the right hand side of (20) is always negative. Thus:

**Proposition 2.** An increase in the degree of downstream overlapping ownership unambiguously reduces total output.

Following the above intuition, Proposition 2 reflects that the sum of the direct effect and the indirect effect is negative. This is immediate when the curvature of demand is constant or increasing, as then the indirect effect is zero or negative (i.e., contributing to further reducing output) as well. In the case of decreasing curvature, the indirect effect is positive and the net effect is *a priori* ambiguous. Yet, as is evident from (20), the indirect effect cannot be strong enough to dominate the direct effect without coming in conflict with the upstream and downstream second-order conditions.

An implication of Proposition 2 is that, in this model, overlapping ownership reduces consumer surplus. (With homogeneous final goods, consumer surplus falls when output falls and the market price rises.) Intuitively, even in the case where a higher level of overlapping ownership gives a lower input price, the subsequent boost in quantity (i.e., the pass-through) is not large enough to fully compensate consumers for the softening of downstream competition. However, from an enforcement viewpoint, it is important to understand not just whether OOAs have anti-competitive effects but also how significant such an effect is likely to be, and what its main drivers are. In this respect, my model connects the magnitude of such anti-competitive effects to primitives of consumer demand – through the pricing strategy of an input supplier. In particular, the supplier’s strategic response *exacerbates* the anti-competitive effect if demand curvature is increasing (when the input price rises) and *mitigates* the anti-competitive effect if demand curvature is decreasing (when the input price falls). Finally, only in the special case of constant curvature does the direct effect and the total effect coincide.
6 Conclusion

In this paper, I studied the effect of overlapping ownership among downstream firms on the input prices set by an upstream supplier using a general specification of consumer demand. I found that, when the level of downstream ownership increases, the supplier’s pricing response depends on the relationship between demand curvature and total output. Specifically, the supplier responds by setting a higher input price if the curvature is increasing in output. Conversely, if demand curvature is decreasing, the supplier optimally reduces the input price.

When antitrust authorities evaluate a downstream horizontal merger, they may consider whether the transaction can lead to greater buyer power and lower input prices that offset the reduction in product market competition. Whether an effect of this type can occur also for minority acquisitions and other overlapping ownership practices is an interesting but unsettled question. My results suggest that, while such a mitigating effect can occur for some important demand forms, the effect can theoretically also be zero or even go in the opposite direction. More research is therefore needed to further clarify the relationship between overlapping ownership, input costs, and consumer welfare.

Appendix

A.1 Second-order conditions

Downstream level. Starting from the first-order condition in (3), we find

\[ \frac{\partial^2 \phi_i}{\partial q_i^2} = 2P'(Q) + P''(Q) (q_i + \lambda q_k). \]  (A1)

In the symmetric equilibrium where \( q_i = q_k = q \), we have \( q_i + \lambda q_k = (1 + \lambda) q = \alpha Q \). Thus,

\[ \frac{\partial^2 \phi_i}{\partial q_i^2} < 0 \iff 2P'(Q) + \alpha P''(Q) Q < 0 \iff \sigma(Q) < \frac{2}{\alpha}. \]  (A2)

\[12\text{See, e.g., the 2010 US Horizontal Merger Guidelines, Section 8.}\]
which is ensured by Assumption 1 (i.e., $\sigma(Q) < 2$), because $\alpha < 1$. Furthermore, we can sum (3) over $i = \{1, 2\}$ and obtain

$$2P(Q) + 2\alpha P'(Q) Q. \quad (A3)$$

The symmetric equilibrium is unique and stable if (A3) is decreasing in $Q$, i.e., if

$$2P'(Q) + 2\alpha [P''(Q)Q + P'(Q)] < 0 \iff \sigma(Q) < \frac{1+\alpha}{\alpha}, \quad (A4)$$

which, as $\alpha < 1$, is also ensured by Assumption 1.

**Upstream level.** Starting from (8), and focusing on the symmetric equilibrium in which $w_i = w_k = w$, we have

$$\frac{\partial^2 \pi_U}{\partial w_i^2} = 2 \frac{\partial q_i}{\partial w_i} + (w - c) \frac{\partial \left( \frac{\partial Q}{\partial w_i} \right)}{\partial w_i}, \quad (A5)$$

$$\frac{\partial^2 \pi_U}{\partial w_k w_i} = 2 \frac{\partial q_k}{\partial w_i} + (w - c) \frac{\partial \left( \frac{\partial Q}{\partial w_k} \right)}{\partial w_i}. \quad (A6)$$

Note that, in the symmetric equilibrium, i) $2 (\partial q_i/\partial w_i) + 2 (\partial q_k/\partial w_i) = (\partial \mu/\partial Q)^{-1}$ by (12), and ii) $\partial \left( \partial Q/\partial w_i \right)/\partial w_i = \partial \left( \partial Q/\partial w_k \right)/\partial w_i = - (\partial^2 \mu/\partial Q^2) / 4 (\partial \mu/\partial Q)^3$ by (12) and the definitions of $\partial \mu/\partial Q$ and $\partial^2 \mu/\partial Q^2$. By using i), ii), and (13), the second-order condition $\partial^2 \pi_U/\partial w_i^2 + |\partial^2 \pi_U/\partial w_i | < 0$ can be written as

$$\frac{1}{\partial \mu/\partial Q} + 2 \left[ -Q \frac{\partial \mu}{\partial Q} \left( -\frac{\partial^2 \mu}{\partial Q^2} \right)^3 \right] < 0 \iff 2 \frac{\partial \mu}{\partial Q} + Q \frac{\partial^2 \mu}{\partial Q^2} < 0, \quad (A7)$$

which corresponds to Assumption 2.

**A.2 Proofs**

**Proof of Lemma 2.** Let $Q_w = q_i(w_i, w_k) + q_k(w_k, w_i)$. The first-order condition in (3), evaluated at the equilibrium quantities, can then be written as

$$P(Q_w) + P'(Q_w) q_i(w_i, w_k) + \lambda P'(Q_w) q_k(w_k, w_i) = w_i. \quad (A8)$$
First, differentiating (A8) with respect to $w_i$ and collecting terms gives

$$\frac{\partial q_i}{\partial w_i} \{2P'(Q_w) + P''(Q_w) [q_i(w_i,w_k) + \lambda q_k(w_k,w_i)]\} + \frac{\partial q_k}{\partial w_i} \{(1 + \lambda) P'(Q_w) + P''(Q_w) [q_i(w_i,w_k) + \lambda q_k(w_k,w_i)]\} = 1.$$ 

(A9)

Second, differentiating (A8) with respect to $w_k$ and collecting terms gives

$$\frac{\partial q_i}{\partial w_k} \{2P'(Q_w) + P''(Q_w) [q_i(w_i,w_k) + \lambda q_k(w_k,w_i)]\} + \frac{\partial q_k}{\partial w_k} \{(1 + \lambda) P'(Q_w) + P''(Q_w) [q_i(w_i,w_k) + \lambda q_k(w_k,w_i)]\} = 0.$$ 

(A10)

For $i \neq k \in \{1, 2\}$, equations (A9) and (A10) comprise a system of four equations.

By solving this system, we find that

$$\frac{\partial q_i}{\partial w_i} = \frac{2P'(Q_w) + P''(Q_w) [q_k(w_k,w_i) + \lambda q_i(w_i,w_k)]}{P'(Q_w)(1 - \lambda)} \frac{(3 + \lambda) P'(Q_w) + (1 + \lambda) P''(Q_w) Q_w}{(3 + \lambda) P'(Q_w) + (1 + \lambda) P''(Q_w) Q_w},$$

(A11)

$$\frac{\partial q_k}{\partial w_i} = -\frac{(1 + \lambda) P'(Q_w) + P''(Q_w) [q_k(w_k,w_i) + \lambda q_i(w_i,w_k)]}{P'(Q_w)(1 - \lambda)} \frac{(3 + \lambda) P'(Q_w) + (1 + \lambda) P''(Q_w) Q_w}{(3 + \lambda) P'(Q_w) + (1 + \lambda) P''(Q_w) Q_w}.$$ 

(A12)

By symmetry, $\partial q_i/\partial w_i = \partial q_k/\partial w_k$ and $\partial q_i/\partial w_k = \partial q_k/\partial w_i$. Consider now the symmetric equilibrium, where $w_i = w_k = w$, $q_i(w,w) = q_k(w,w) = q$, and $Q_w = 2q = Q$. Note that $[(3 + \lambda) P'(Q) + (1 + \lambda) P''(Q) Q] = 2 (\partial \mu/\partial Q)$. By substituting in the latter equation, as well as $(1 + \lambda) = 2\alpha$, $(1 - \lambda) = 2 (1 - \alpha)$, and $P''(Q) Q = -P'(Q) \sigma(Q)$ from (1), (A11) and (A12) can be rewritten as (10) and (11), respectively.

\[\Box\]

References


