# Contracts that Reward Innovation: Delegated Experimentation with an Informed Principal

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**Abstract**

We examine the nature of contracts that optimally reward innovations in a risky environment, where the innovator is privately informed about the quality of her innovation and must engage an agent to develop it. We model the innovator as a principal who has private but imperfect information about the quality of her project: the project might be worth exploring or not, but even a project of high quality may fail. We characterize the best equilibrium for the high type principal, which is either a separating equilibrium or a pooling one. Due to the interaction between the signaling incentives of the principal and dynamic moral hazard of the agent, the best equilibrium induces an inefficiently early termination of the high quality project. The high type principal is forced to share the surplus – with the agent in the separating equilibrium, or the low type principal in the pooling equilibrium. A mediator, who offers a menu of contracts and keeps the agent uncertain about which contract will be implemented, can increase the payoff of the high type principal to approximate her full information surplus.

**Keywords:** Contracts, Innovation, Experimentation, Informed Principal, Dynamic Moral Hazard, Signaling Games, Mechanism Design

**JEL Codes:** D82, D83, D86

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# 1 Introduction

How should we design contracts in order to reward innovation, when poor ideas can masquerade as worthwhile ones? Consider an innovator who has a new idea, and has private but imperfect information about the quality of the idea. Specifically, the idea may be worth exploring, but even high quality ideas may result in failure, just as ideas that are of low quality ex ante may sometimes be successful. The innovator needs to engage an agent to explore the idea, and thus the agent's moral hazard must also be confronted. Since the agent becomes more pessimistic over time about the probability of success in the absence of a breakthrough, moral hazard is dynamic. Moreover, the innovator also needs to convince the agent that the idea's quality is high and it is worth exploring. Our question is: what are the contracts that provide maximal rewards for high quality innovations? If high quality innovations are not rewarded properly, innovators would not invest in better ideas in the first place. The question is particularly important in the knowledge economy, where economic growth is driven by innovations. Furthermore, all the ingredients listed are important considerations in knowledge industries: innovators have private information, but even the best ones are not omniscient, and sometimes come up with unworkable ideas; the innovators themselves may not be the best ones to undertake project development, and may have to delegate the task to specialized bodies; only these specialized bodies know how intensively they are working on the innovator's project.

We study a model where a privately informed principal engages an agent to work on a project. A senior professor may need to hire a research assistant to conduct lab experiments; a tech firm may need to recruit user participants for usability tests of its new product in the early experimental phase; a multi-level marketing company, such as Amway and Herbalife, relies on salespeople to pay visits to potential customers. In the gig economy, similar situations are prevalent when a firm enters a new industry but has informational advantage over independent contractors or gig workers who need to learn the quality of the product/technology or the market conditions; for instance, drivers work for a new delivery service. The pharmaceutical industry is another example. Scientists in biotech firms have strong insights into the fundamental mechanism of diseases, but other entities, such as Contract Research (or Manufacturing) Organizations, are specialized in other research support services.[1] When they cooperate on a drug development project, the biotech firm not only needs to incentivize those organizations to exert efforts towards exploring the project's viability, but also to earn their trust and convince them of the project's quality. The problem is particularly severe when high quality projects are relatively scarce, and cannot be distinguished from low quality projects.[2] Moreover, without recognizing the complex situation, inexperienced workers may also be vulnerable for employment scams; for example, when they work as salespeople for insurance firms with hard-to-sell inferior products, but accept the arrangement to pay "training fees" first and then get compensated with "contingent commissions."

[1]The services may include biopharmaceutical and formulation development, commercialization, stability studies, preclinical and clinical research, clinical trials management, and pharmacovigilance.

[2]This is very typical in research industries. Stevens and Burley (1997) estimate that there is only one commercial success in 3,000 raw ideas of innovation across most industries. Klees and Joines (1996) report that only one compound of drug development is approved for marketing among every 5,000 to 10,000 compounds that enter preclinical testing.

This paper has two parts. In the first part, we combine a signaling game with an exponential bandit model to study the dynamic agency problem. An innovator or a principal (she) is endowed with a project. Neither she nor the hired agent (he) knows whether the project is *viable*, i.e., whether it can succeed and generate profits. However, the principal is privately aware of the project's *quality*, i.e., how long the project, if it is viable, will take in expectation to succeed.[3] Only the high quality project is worth exploring. The principal can commit to a long-term contract to incentivize the agent to exert private efforts. The contract also serves as a signal of her project's quality, or her type. When the agent works on the project, he gradually learns about both the viability and the quality of the project. Thus, the principal has two tasks at once: signaling her type and providing incentives for the agent to work. To isolate the conflict between signaling and providing incentives, we assume both players are risk neutral and have unlimited liability.

Our focus is on rewarding innovation. Thus, we characterize the best equilibrium contract for the high type principal. The high type principal can never obtain her full information surplus, i.e., the payoff she would obtain if she were known to be a high type. Depending on the proportion of high quality projects in the population, the best equilibrium for her is either a separating equilibrium or a pooling equilibrium.

When high quality projects are scarce, a separating equilibrium gives the best outcome to the high type principal. With unlimited liability, the high type principal could sell her project to the agent, leaving him to be the residual claimant that solves the moral hazard problem. Indeed, selling the project would be optimal if the quality of the project were publicly known. That is, the agent makes an upfront payment that equals the expected surplus of the project, in exchange for the entire profit when the project succeeds. However, when only the principal knows her type, such a contract does not do a good job separating the two types. Both types value the upfront payment the same, and the low type principal has incentives to exaggerate the quality of her project to sell it for a good price. To separate from the low type principal and convince the agent of the project's quality, the high type principal will not charge the upfront payment but rather pay the agent some base wage that is independent of the outcome, and at the same time share the profits with him as bonus payments conditional on a successful project development. The base wage helps the high type principal separate from the low type. The bonus payments are increasing over time to encourage an increasingly pessimistic agent to continue working. Thus, the high type principal shares a portion of the total surplus with the agent. Moreover, to separate from the low type in the least costly way, the high type principal will terminate the project inefficiently early. This is not only because the dynamic moral hazard problem, but also because an early termination discourages the low type principal to mimic the high type since the low type, knowing her chance of a success is low, is less concerned about sharing profits and would prefer longer project development. Thus, both the inefficient termination and sharing the surplus with the agent reduce the payoff of the high type principal.

On the other hand, when high quality projects are not rare, it is better for the high type principal to pool with the low type to avoid signaling costs. In that case, the best equilibrium

[3] More specifically, the quality of a project is modeled as the success probability in one period when the project is viable and when the agent exerts effort.

for the high type principal is a pooling equilibrium, where both types charge the agent a positive sign-up fee and then share the profits with him once the project succeeds. Thus, instead of leaving rents to the agent, the high type principal "shares" rents with the low type principal. Moreover, the equilibrium contract still features an inefficiently early termination, compared to a project whose quality is ex ante unknown.

The above results show that the high quality project is operated inefficiently and the high type principal cannot obtain her full information surplus. It provides a need for mediators to facilitate the agency process. Online platforms, such as Science Exchange, Scientist and Upwork, are such mediators. Science Exchange and Scientist help researchers outsource their research to other scientific institutions around the world. Upwork is a global freelancing platform that connects independent professionals. Those platforms facilitate matching between relevant parties, smooth contract implementation, and provide guidance for partnerships. Law firms specialized in contracts may also play a similar role as mediators.

In the second part of the paper, we analyze the mechanism design problem faced by a mediator who seeks to maximize the reward for superior innovations, i.e., high quality projects. The designer offers a menu consisting of two contracts, one for each type of principal. This menu must satisfy incentive compatibility for the principal, which induces truthful revelation by each type (it must also satisfy individual rationality). The agent observes the menu, and infers that each of the contracts will be implemented with a probability corresponding to his prior, and decides whether to accept or reject the menu. However, the agent is not told which element is being implemented until it is essential for him to know. In other words, the agent is confronted with an opaque contract, and remains uncertain about his exact rewards for some time. This relaxes the agent's individual rationality and incentive compatibility constraints; they only need to be held on average. In addition, the opaqueness of the contingent transfers allows the two types of principal to bet on a success, which provides an additional device for the high type principal to separate herself. In this way, the mediator designs a mechanism that improves the payoff of the high type principal. Moreover, the inefficiency costs are minimal, since the mediator can recommend the low type project running for only one period. In the optimal mechanism for the high type principal with pure recommendations, the high type principal obtains approximately all the surplus from her innovation when the period length is small. Thus, the contract with the mediator allows the innovator of a superior project to appropriate almost her entire contribution to social surplus. Furthermore, an innovator who comes up with an inferior project is left with no surplus, thus the menu simultaneously minimizes the rewards for wasteful innovations.

Such a mechanism resembles a widely used incentive contract in Chinese venture capital market, called the Valuation Adjustment Mechanism (VAM) or the "Bet-on Agreement." A VAM specifies certain future performance conditions, which, upon reached, grant different parties certain rights to adjust the originally agreed valuation and investment plan. Lin (2020) argues that the VAM is designed to protect investors in a market with severe information asymmetry and immature legal environment. The optimal mechanism to reward high quality projects in our setting also features a crucial condition that determines the future project development and how profits are shared. That condition is the principal's private information about the project's quality and will be revealed only during the project development.

The rest of the paper is organized as follows. Section 2 discusses the related literature. Section 3 introduces the model. Section 4 discusses the efficient solution and other benchmarks. Section 5 characterizes the best equilibrium for the high type principal in the signaling game. Section 6 considers the third-party mechanism design problem. Section 7 concludes. The Appendix provides proofs.

# 2 Related Literature

This paper examines the incentives for experimentation when the principal is informed. First, it relates to a growing literature on incentives for experimentation. These papers, as well as the current one, build on a two-armed bandit model of learning, as in Keller, Rady, and Cripps (2005), and focus on how to provide incentives for agents to experiment through contingent contracts. Most papers, such as Bergemann and Hege (1998, 2005), and Hörner and Samuelson (2013), consider how to incentivize one party to experiment, who is subject to the moral hazard problem. They typically consider a repeated interaction between the principal and the agent, assume limited liability, and demonstrate an inefficiency result due to the agency costs. Guo (2016) studies a dynamic relationship in which a principal delegates experimentation to a biased agent who has private information about the prior belief that the state is good. Thus, the incentive problem comes from the hidden information of an informed but biased agent. Halac, Kartik, and Liu (2016) is a closely related paper. They examine an agency problem subjected to both moral hazard and adverse selection in the context of experimentation. They assume that the principal can commit to a long-term contract, and no limited liability. The major difference is that, instead of considering the private information on the side of the agent, our paper examines the case where the private information is on the side of the principal. In the screening problem of Halac, Kartik, and Liu (2016), the high type agent has an incentive to pretend to be the low type. In our signaling problem, the low type principal has an incentive to pretend to be the high type. Therefore, Halac, Kartik, and Liu (2016) show that the optimal contract has no distortion for the high type agent, but requires the low type agent to terminate the project inefficiently early.[4] By contrast, we show that it is the high type project that is terminated inefficiently early in the best equilibrium for the high type principal. Moreover, in that equilibrium, either there is no distortion for the low type project, or it is distorted towards over-experimentation, depending on the prior belief about the low type principal. Thus, the economic forces underlying the analyses are very different in the two papers. Furthermore, we also allow for a mediator and show that the approximate efficiency can be achieved, a result that has no counterpart in Halac, Kartik, and Liu (2016).

Second, our paper relates to a relatively small literature on the informed principal problem with moral hazard. Myerson (1983) first considers the informed principal problem from an axiomatic point of view. Maskin and Tirole (1990, 1992) develop a noncooperative game framework to analyze the informed principal problem with no moral hazard. According to their categorization, our model is the "common value" informed principal problem, since the

[4]It is a typical result in screening problems that distortions occur at the bottom, but not at the top.

agent cares directly about the type of principal. Beaudry (1994) is an early paper concerning an informed principal problem with moral hazard. He characterizes a separating equilibrium where the principal leaves rents to the agent. More recent papers, such as Silvers (2012), Wagner, Mylovanov, and Tröger (2015), Bedard (2016), and Karle, Schumacher, and Staat (2016) also consider the informed principal problem in the presence of moral hazard. In all these papers, the moral hazard problem is static. Few papers examine this problem in a dynamic setting.[5] One exception is Kaya (2010). She studies a similar informed principal problem with moral hazard when the principal and the agent interact repeatedly, but assumes the principal and the agent start with symmetric uncertainty about the productivity information. In each period, the principal has a choice to acquire that information without costs, but would rather delay acquiring it in order to save the costs for signaling. However, in our paper, the agent learns both the type of the principal and the viability of the project during experimentation, which makes the incentive problem very different.[6]

Last, our paper is also related to the information design problem with moral hazard. Jehiel (2015) examines whether a principal with private signals prefers to commit to a non-transparent information disclosure policy to overcome the agent's moral hazard. He finds that full transparency is generically suboptimal under some mild conditions. Our result in the mechanism design part echoes his result – keeping the agent in the dark improves the payoff of the high type principal. However, we focus on the signaling problem of the principal after her private information is realized, while Jehiel (2015) assumes the principal can commit to an information disclosure policy before the realization of the private information. Ely and Szydlowski (2020) study a model where a principal is privately informed about the duration of the required efforts for completing a project. The principal's objective is to induce the agent to work as much as possible, thus they find that the optimal information disclosure policy is "moving the goalposts": at the outset, the principal tries to make the agent optimistic that the task is easy in order to induce him to start working, but persuades him that the task is hard when the difficult goal is within reach. In their setting, there is no signaling consideration, and the principal's problem is to keep the agent from quitting.

# 3 The Model

Time is discrete and the horizon is infinite, with a small but positive period length $\Delta > 0$. The per period discount factor is $\delta = e^{-\rho\Delta}$, where $\rho \geq 0$. To get rid of the integer problem and have a tractable solution, we will look at the case as $\Delta \to 0$ for some results.[7]

[5]Fryer and Holden (2012) consider a two period informed principal problem with moral hazard. However, they make a behavioral assumption that the agent only learns from a noisy signal, but not from the contract proposed by the principal.

[6]Kaya (2010) uses "money burning," i.e., a pure production-irrelevant cost, as a signaling device for the principal after acquiring information. This money burning signaling device, together with the assumption that the agent has limited liability, are essential to her results. See Kaya (2010) Footnote 15.

[7]Those results approximate the circumstance when $\Delta$ is small. We do not use a continuous time model to avoid technical difficulties, since stochastic integration with respect to general transfer scheme may not be well-defined. We also use the following notational conventions. $N$ and $n$ are used for time periods when we talk about the model with a fixed $\Delta > 0$; $T$ and $t$ are used for time when we examine the model in the limit $\Delta \to 0$. For a variable $x_n$ that has time subscript $n$, let $x_t$ be the limit of $x_n$ as $\Delta \to 0$ and $n\Delta \to t$

There are two risk neutral players, a principal $P$ and an agent $A$. The principal (she) hires the agent (he) to complete a project with uncertain viability. The agent, after accepting the principal's offer, chooses whether to exert efforts in every period before the game ends. The agent has a flow cost $c\Delta > 0$ when he exerts efforts in any period. The efforts are not observable or verifiable by the principal.

The principal is privately informed about the *quality* of the project, $\theta \in \Theta = \{H, L\}$. $H$ represents a high quality project and $L$ represents a low quality project. The quality is persistent and is chosen by nature at the beginning. The private quality can also be regarded as the *type* of the project/principal. The agent does not know the principal's type, but has a prior $\beta_0 \in (0, 1)$ on the $H$ type, which is common knowledge.

A project of either type may be in one of the two *states*: a good state $G$, or a bad state $B$. The state is also persistent and chosen by nature at the beginning. However, neither party knows the state of the project. They have a common prior $q_0 \in (0, 1)$ on the $G$ state, and the distribution of states is independent of the distribution of types.

The following figures summarize the information structure when nature moves. $P$ knows the types, but not the states. $A$ knows neither of them.

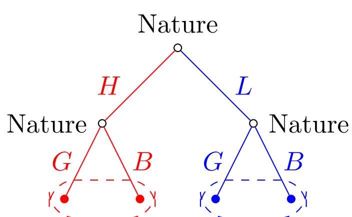


Figure 1: Principal's Information

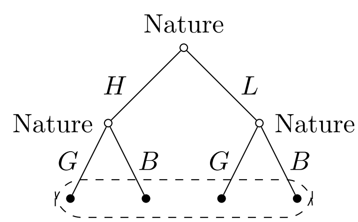


Figure 2: Agent's Information

In the $B$ state, a project, independent of its type and the agent's efforts, never generates any profits. In the $G$ state, when the agent exerts efforts in one period, a $\theta \in \Theta$ type project can generate a lump-sum profit $h > 0$ to the principal in that period with a probability $\lambda^\theta \Delta$, where $\lambda^H > \lambda^L > 0$. It is a *success* of the project in a period when the profit arrives, otherwise we call it a *failure*. A success of the project can be publicly observed and verified,[8] and will end the game. Hence, the difference between the $H$ and the $L$ type projects is the arrival rate of success conditional on the project is in the $G$ state and the agent exerts efforts. In addition, we will maintain the following assumption:

**Assumption.** $q_0 \lambda^H h > c \geq q_0 \lambda^L h$.

Thus, given the cost and the benefit of the project, and the prior belief about the state, only

(when it exists).

[8]The public observability and verifiability of a success is an inconsequential assumption in the model. If a success is only observed by the agent, the agent can choose to misreport a failure to the principal when a success actually arrives. However, the agent has no incentive to hide a success from the principal in any equilibrium contract in this paper. When a contract gives exact incentives for the agent to work if a success is contractible, it also gives enough incentives for the agent to truthfully report the success if it is not contractible. See Section 5.2.

the $H$ type project is worth experimenting on. If it were commonly known that the project's type was $L$, it would exit the market.

The timing of the game is the following. The principal first learns her private type, $H$ or $L$, and then proposes a take-it-or-leave-it long-term contract to the agent. The principal can fully commit to her contract, which specifies the following:

- A termination date $N \in \mathbb{N}_0$ of the project conditional on no success.[9]
- A lump-sum payment $W \in \mathbb{R}$ from the principal to the agent at time zero.
- A contingent payment plan, two vectors $\boldsymbol{b} \in \mathbb{R}^N$ and $\boldsymbol{p} \in \mathbb{R}^N$, both from the principal to the agent. $\boldsymbol{b}$ is the bonus vector that determines the payment when the project succeeds in any period; $\boldsymbol{p}$ is the penalty vector that determines the payment when the project fails in any period. In other words, if the project succeeds in the $n$-th period, where $1 \leq n \leq N$, the payments to the agent are $p_k$ in the $k$-th period for $1 \leq k < n$, and $b_n$ in the $n$-th period.

All payments can be either positive or negative. Thus, a *contract* is a quadruple $C = \{N, W, \boldsymbol{b}, \boldsymbol{p}\}$.[10] We call a contract a *bonus contract* if $\boldsymbol{p} = \mathbf{0}$, and a *penalty contract* if $\boldsymbol{b} = \mathbf{0}$. If $N = 0$, the only payment is $W$. A contract is called a *null contract* if $N = W = 0$.

After the principal proposes a contract, the agent chooses whether to accept it. If he rejects it, both parties obtain their reservation payoff zero and the game ends. If he accepts it, the contract is implemented and the agent decides whether to exert efforts in each period, until the project succeeds or the termination date is reached.

This is a signaling game as the informed player (the principal) moves first. We will consider Perfect Bayesian Equilibria (PBE).

A contract specifies $2N + 1$ transfers from the principal to the agent, however, some are redundant. We can restrict attention to a smaller set of contracts, i.e., the set of bonus contracts or the set of penalty contracts, without loss of generality.

The idea is similar to Halac, Kartik, and Liu (2016). Given a termination date $N$ of a contract, there are $N + 1$ possible outcomes, i.e., a success arrives after $n$-th experiment for $n \in \{1, 2, \cdots, N\}$ and it never arrives. Both parties are risk neutral and only care about the discounted transfers on any realized outcome. Thus, $N + 1$ transfers (contingent on the $N + 1$ outcomes) are sufficient to characterize any payment plan. We say two contracts with the same termination date are *payoff equivalent* if for any action plan of the agent, any type of the principal, and any belief of the agent, both contracts deliver the same expected payoffs to the principal and to the agent. It is easy to see that, for any contract, there exists a bonus (or penalty) contract that is payoff equivalent to it,[11] because, for any realized outcome, bonuses (or penalties) together with the lump sum payment $W$ can deliver any discounted transfers that a general contract could.

[9] The principal, whatever type she is, never wants to experiment forever. Therefore, without loss, we do not consider the option to run the project forever.

[10] Note that all terms depend on $\Delta$, but we omit that to save notation.

[11] See Proposition 1 of Halac, Kartik, and Liu (2016).

Different from Halac, Kartik, and Liu (2016), one more thing needs to be taken care of in our signaling game. The agent may form different beliefs on payoff-equivalent contracts. But the principal and the agent would feel indifferent to payoff-equivalent contracts as long as the agent forms the same belief, which would induce the same action plan of the agent and deliver the same expected payoffs to both parties. Hence, we shall have no reason to think that the agent would regard different payoff-equivalent contracts as different signals. Formally, we assume that the agent forms the same belief about the type of the principal for any payoff-equivalent contracts. This is a restriction on the off-path beliefs.

From now on, without loss of generality, we can focus on bonus contracts or penalty contracts. In this paper, we will restrict attention to bonus contracts, or *contracts* for simplicity. Thus, a contract is a triple $C = \{N, W, \boldsymbol{b}\}$.

# 4 Benchmarks

## 4.1 Efficient Solution

We first consider the efficient solution without the agency problem, where a social planner seeks to maximize the total surplus, given he knows both the private type of the principal and the hidden actions of the agent but not the state of the project.

The social planner solves an optimal stopping problem. The optimal strategy is to stop the project whenever the posterior belief about the project's state being $G$ falls below some cutoff belief. This strategy is also equivalent to specifying how long to experiment.[12]

Let $V^\theta(N)$ be the expected discounted value of the $\theta \in \Theta$ project when the social planner experiments $N$ times. It is given by

$$V^\theta(N) = \sum_{n=1}^{N} \delta^n f_{n-1}^\theta(q_0) \left( q_n^\theta \lambda^\theta h - c \right) \Delta, \tag{1}$$

where $f_m^\theta(q) = q(1 - \lambda^\theta \Delta)^m + 1 - q$ is the probability that a $\theta$ project, with a prior $q$ being in the $G$ state, fails $m$ times. $\left( q_n^\theta \lambda^\theta h - c \right) \Delta$ is the expected payoff for the $n$-th experiment conditional on a success not having arrived yet. Here $q_n^\theta$ is the posterior belief about the state being $G$ for a $\theta$ project before the start of the $n$-th experiment,[13]

$$q_n^\theta = \frac{q_0(1 - \lambda^\theta \Delta)^{n-1}}{q_0(1 - \lambda^\theta \Delta)^{n-1} + 1 - q_0}.$$

Thus, the optimal policy for the social planner, with a $\theta$ project, is to conduct the project as long as

$$q_n^\theta \lambda^\theta h \geq c.$$

[12] More precisely, a strategy specifies how long to experiment without a success. Since a success ends the game, we omit repeating "without a success."

[13] After a project fails $n - 1$ previous experiments.

Given the assumption $q_0\lambda^L h \le c$, the social planner will abort the $L$ type project immediately; the $L$ type project is a lemon. Thus, the *efficient termination date for the $L$ type project* is zero. On the other hand, given the assumption $q_0\lambda^H h > c$, the *efficient termination date for the $H$ type project*, denoted by $N_*^H$, is strictly positive. To get rid of the integer problem of $N_*^H$, it will be helpful to examine the limit of the experimenting time:

$$T_*^H := \lim_{\Delta\to 0} N_*^H \Delta = \frac{1}{\lambda^H}\log\frac{l_0}{l^H},$$

where $l_0 = \frac{q_0}{1-q_0}$ is the likelihood ratio of the prior belief that the state of the project is $G$, and $l^H = \frac{c}{\lambda^H h - c}$ is the likelihood ratio of the efficient cutoff posterior belief that the state of the $H$ project is $G$. We also denote $V_0^\theta(T) := \lim_{\Delta\to 0, N\Delta\to T} V^\theta(N)$.

The efficient solution is obtained assuming away both the private information of the principal and the hidden actions of the agent. Next, we consider benchmarks where the incentive problem is one-sided.

## 4.2 One-sided Incentive Problem

Our model has both private information on the side of the principal and hidden actions on the side of the agent. Both are crucial, because the incentive problem would be trivial without either of them.

**No Private Information** – Let us first consider the case where the project's private type is public information, while the agent's actions are still private. Although the principal still needs to incentivize the agent to experiment on the commonly unknown state of the project, the incentive problem is trivial. The principal can sell the project to the agent as the agent has no financial constraint. After the agent becomes the residual claimant of the project, he will implement the efficient solution. Thus, the $H$ type extracts all the surplus by selling her project with a price at its expected value, and the $L$ type exits the market.

**Observable Efforts** – Another simple case is where the agent's efforts are publicly observable and verifiable. Thus, the principal can contract directly on the efforts. Even though the principal has private information on the type of the project, the incentive problem becomes trivial. There exists a separating equilibrium in which both projects are implemented efficiently and the $H$ type principal extracts all the surplus of her project. In the equilibrium, the $H$ type proposes an "honest" contract that pays the agent for his efforts each period until her efficient termination date if and only if he exerts efforts, while the $L$ type exits the market. The agent optimally accepts the "honest" contract and exerts efforts, and believes contracts other than the "honest" contract are offered by the $L$ type.

In the above two scenarios, both projects are implemented efficiently and all the surplus is retained by the principal. We call such an outcome the *full information benchmark* (*FIB*). In the presence of both private information on the side of the principal and hidden actions on the side of the agent, the principal needs to signal her type and incentivize the agent to work at the same time. We will show that the FIB can never be achieved.

# 5 Equilibrium Characterization

In this section, we first show there is no equilibrium that achieves the FIB. Multiple equilibria exist as a PBE does not restrict the off-equilibrium beliefs. Because a $L$ type project is a lemon, we examine how much a $H$ type project can be rewarded in the equilibrium. That is, we study the best equilibrium for the $H$ type. We characterize the best equilibrium for the $H$ type principal, which is either a separating equilibrium or a pooling equilibrium, depending on the prior belief about the $H$ type principal.

## 5.1 The Impossibility of Achieving the FIB

If an equilibrium implements the FIB, it must be a separating equilibrium: the $L$ type aborts the project, while the $H$ type incentivizes the agent to work until the efficient termination date and extracts all the surplus. Moreover, the agent believes the principal's type is $H$ if the $H$ type contract is proposed, but the $L$ type cannot benefit from proposing the $H$ type contract.

We first characterize all contracts that satisfy the agent's binding individual rationality (IR) constraint, and the agent's incentive compatible (IC) constraints until the efficient termination date $N_*^H$, given the agent believes the principal's type is $H$. Then we find the worst one for the $L$ type among that set of contracts, and show that the $L$ type can still obtain a strictly positive payoff from the worst contract. Thus, any contract that implements the FIB must violate the IC constraint for the $L$ type. Hence, the FIB cannot be achieved in any equilibrium.

When the agent believes that the principal's type is $\theta \in \Theta$, a contract $C = \{N, W, \boldsymbol{b}\}$ satisfies the IC constraints for the agent to work from period one to $N$, if for all $1 \le n \le N$,

$$\sum_{s=n}^{N} \delta^{s-n} f_{s-n}^{\theta}(q_n^{\theta}) \left(q_s^{\theta}\lambda^{\theta} b_s - c\right) \Delta \ge \sum_{s=n+1}^{N} \delta^{s-n} f_{s-n-1}^{\theta}(q_n^{\theta}) \left(q_{s-1}^{\theta}\lambda^{\theta} b_s - c\right) \Delta. \qquad IC_A^{\theta}(N)$$

The above defines $N$ inequality constraints. The left-hand side (LHS) is the agent's expected discounted payoff when he experiments from period $n$ until the termination date $N$, after failing $n-1$ previous experiments. It has the same structure as the social planner's expected discounted value in expression (1), except the agent's value of a success in period $s$ is $b_s$, instead of $h$. The right-hand side (RHS) is the agent's expected discounted payoff when he shirks in period $n$ but experiments from then on until the termination date $N$, after failing $n-1$ previous experiments. Thus, $IC_A^{\theta}(N)$ prevents a one-time profitable deviation (shirking) of the agent in all histories when he never shirks before. If the agent deviates and shirks in some past period, it is still optimal for him to work thereafter, since he is more optimistic than he would be had he worked.

The moral hazard problem is dynamic. In the $n$-th period, the expected payoff of working from the current period is $\left(q_n^{\theta}\lambda^{\theta} b_n - c\right)\Delta$, while shirking gives a current payoff zero. The continuation values of working and shirking also differ due to two effects. First, the *learning effect*: (unexpected) shirking makes the agent more optimistic about the project than the principal. The principal (not knowing the unexpected shirking of the agent) believes that the probability of the state being $G$ is $q^\theta_{n+1} < q^\theta_n$, while the agent (correctly) believes that it is still $q^\theta_n$. Second, the *end-of-game effect*: the game has $f^\theta_1(q^\theta_n) < 1$ probability to continue if the agent works, while it continues for sure if the agent shirks. Those two effects increase the continuation value of shirking compared to working. Thus, the bonus must compensate the agent for both his current period cost and his continuation value loss.

When the agent believes that the principal's type is $\theta \in \Theta$, a contract $C = \{N, W, \boldsymbol{b}\}$ satisfies the IR constraint for the agent, if

$$W^\theta_A = W + \sum_{n=1}^{N} \delta^n f^\theta_{n-1}(q_0) \left( q^\theta_n \lambda^\theta b_n - c \right) \Delta \geq 0. \qquad IR^\theta_A(N)$$

Let $S(N)$ denote the set of contracts that satisfy both $IC^H_A(N)$ and a binding $IR^H_A(N)$. Hence, $S(N^H_*)$ is the set of contracts that implement the FIB for the $H$ type given that the agent believes her type is $H$.

The worst contract $C^{wt} = \{N^H_*, W^{wt}, \boldsymbol{b^{wt}}\}$ for the $L$ type in the above set of contracts solves the following Program I:

$$\min_{\boldsymbol{b}, W} \Pi^L = -W + \sum_{n=1}^{N^H_*} \delta^n f^L_{n-1}(q_0) q^L_n \lambda^L \left( h - b_n \right) \Delta$$
$$\text{s.t. } IC^H_A(N^H_*) \text{ and a binding } IR^H_A(N^H_*).$$

The $L$ type's expected payoff is determined by the payment transferred to the agent at time zero and the kept share of profits once the project succeeds. Here $q^L_n \lambda^L (h - b_n)\Delta$ is her expected payoff for the $n$-th experiment conditional on a success not having arrived.

**Lemma 1.** *The worst contract for the L type in the set $S(N^H_*)$ of contracts is the contract in which the agent's IC constraints bind in every period $n = 1, 2, \ldots, N^H_*$.*

This result is important as it shows how to incentivize the agent to work and at the same time make the $L$ type's mimicry least profitable. More importantly, as it will be shown later, it greatly simplifies our search for the best equilibrium for the $H$ type as the similar result holds whenever the $H$ type has a signaling concern.[14]

We now provide intuition for why all the IC constraints must bind. The set of contracts that implement the FIB for the $H$ type is large, because the principal has the discretion to give more high-powered incentives than required, so that the agent's IC constraints are slack, and then extract the surplus so conferred via a larger sign-up fee, i.e., a lower value of $W$. However, such a contract is more attractive for the $L$ type. We now explain why.

[14] See Lemma 5 in the Appendix.

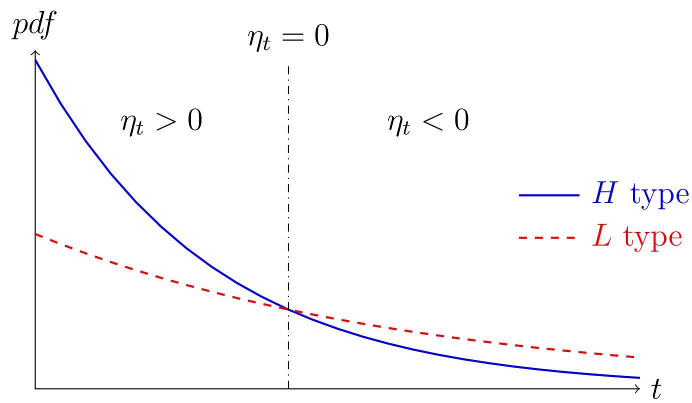


Figure 3: The probability of success of the two type

Figure 3 shows the probability density functions of the success time. Let

$$\eta_n := f^H_{n-1}(q_0)q^H_n\lambda^H - f^L_{n-1}(q_0)q^L_n\lambda^L.$$

Thus, $\eta_n\Delta$ is the difference of success probabilities for the $n$-th experiment between the $H$ and $L$ types. Although the $H$ type is more likely to succeed than the $L$ type conditional on the state being $G$, the posterior beliefs about the state being $G$ conditional on no success decrease faster for the $H$ type than for the $L$ type – the $H$ type learns faster. Thus, the two curves cross once. The $H$ type is more likely to succeed in the earlier periods ($\eta_n > 0$), but the $L$ type is more likely to succeed in the later periods ($\eta_n < 0$).

Suppose the agent believes the principal's type is $H$, but her actual type is $L$. Thus, the agent overvalues the bonus payments in the earlier periods ($\eta_n > 0$) as the success probability is lower than what he believed, and is therefore willing to pay a higher sign-up fee than he would pay had he known the principal's type is actually $L$. Consequently, to minimize the $L$ type's incentive to mimic the $H$ type, bonus payments must be minimized in any such period where the probability of success is greater for the $H$ type than that for the $L$ type. Thus, the IC constraints must bind there.

Consider now any period where the probability of success is greater for the $L$ type than that for the $H$ type. By the preceding argument, it would seem that the bonus payment in such a period should be increased to reduce the $L$ type's incentive to mimic the $H$ type. But this is not true. Consider the period right after the crossing point. Raising the bonus in that period makes the agent's continuation value of shirking, in all previous periods, higher. To incentivize the agent to work, the principal has to raise bonus payments in all previous periods proportionally. It actually makes the $L$ type better off, because the distribution of success for the $L$ type first-order stochastic dominates that for the $H$ type, i.e., $\phi_n := \sum_{s=1}^{n} \eta_s\Delta > 0$ for any $n \geq 1$. To minimize the $L$ type's mimicry incentive, the bonus payment must also be minimized in the period right after the crossing point. The same logic continues. We show that the bonus payments in any period must be minimized in the worst contract for the $L$ type. Thus, all IC constraints must bind.

Even in the worst contract for the $L$ type, the bonus scheme is an increasing sharing plan that leaves the $L$ type a positive expected payoff during experimentation. Moreover, the agent is also willing to pay a positive sign-up fee, if he believes the principal's type is $H$. Thus, the $L$ type obtains a strictly positive payoff from that worst contract, and has incentives to mimic the $H$ type. Hence, we conclude:

**Proposition 1.** *There is no equilibrium that implements the FIB.*

## 5.2 Separating Equilibrium

The FIB cannot be implemented in any equilibrium, since any contract that achieves the FIB for the $H$ type must violate the IC constraint for the $L$ type. How can the $H$ type principal separate herself from the $L$ type?

The simplest way is to increase the upfront payment $W$. The $L$ type has a lower success probability than the $H$ type. Thus, an upfront payment $W$ that makes the $H$ type profitable could be unprofitable for the $L$ type. Consider the contract $C^{wt}$ we found. Let $\Pi^L(C^{wt})$ be the payoff of the $L$ type from $C^{wt}$, when the agent believes her type is $H$. Now we introduce a new contract, $C^1 = \{N_*^H, W^1, \boldsymbol{b^{wt}}\}$, with the same termination date and bonus payments, but a higher upfront payment, where $W^1 = W^{wt} + \Pi^L(C^{wt})$. Hence, the $L$ type would obtain a zero payoff from $C^1$, even when the agent believes her type is $H$. But the $H$ type can obtain a strictly positive payoff from $C^1$, when the agent believes her type is $H$. This is simply because the total surplus is larger for the $H$ type project, compared to the $L$ type.

Thus, $C^1$ can separate the $H$ type from the $L$ type. Furthermore, the $H$ type and the $L$ type proposing $C^1$ and the null contract respectively, can be supported in an equilibrium. In fact, for any other contract, we can find some belief that prevents a profitable deviation by either type. We have the following result:

**Lemma 2.** *For any contract, if the agent believes the principal's type is $L$, the principal, whatever type she actually is, cannot obtain a strictly positive payoff.*

Thus, a pessimistic belief that assigns probability one about the principal's type being $L$ for any off-equilibrium path contract can support the above equilibrium.

The contract $C^1$ simply uses the upfront payment to separate the two types. The $H$ type could design a more sophisticated contract to separate herself and improve her payoff. Now we consider the *best separating equilibrium for the $H$ type*, or *BSEH*, i.e., the equilibrium that gives the $H$ type the highest payoff among all separating equilibria.

Given the structure of a contract, the $H$ type principal could use three potential devices to separate herself. First, she could increase the upfront payment $W$ as we have seen. Second, she could reduce the experimentation by terminating it earlier. Since the $L$ type learns slower than the $H$ type, the value of an additional experiment decreases more slowly for the former than for the latter. Hence, an experiment could be more valuable to the $L$ type than to the $H$ type in the later periods. Thus, shutting down the project earlier makes mimicry less attractive. Third, she could delay the project, by having the agent not work in some periods before terminating the project permanently. Those devices are all costly to the $H$ type; the question is how to use them in the least costly way.

Although it is not clear at first whether delaying the project temporarily permits separation, we show this is not the case. Delaying the project essentially increases the discount factor between two consecutive experiments.[15] Each discount factor affects discounted payoffs linearly, if all other terms in the payoff functions are independent of the discount factor. To make the agent follow the induced action plan, bonus payments have to be correlated with future discount factors. However, allowing for the possibility to delay the project does not change the fact that the least costly separation and pooling both require the agent's IC constraints bind. It, in turn, makes the current bonus payment linearly correlate with any future discount factor. Thus, discounted payoffs are linear in each discount factor between two consecutive experiments, given the agent's IC constraints binding. Therefore, from the $H$ type's point of view, it is optimal to either never delay the project, or delay it forever (i.e., terminating the project). Hence, the $H$ type project will never be temporarily delayed in the best equilibrium for the $H$ type. From now on, we will not consider delaying the project temporarily in the main body of the paper. More details can be found in the Appendix.

In the BSEH, the $L$ type must obtain a zero payoff; let her propose the null contract in the equilibrium. We also assume that the agent assigns probability one about the principal's type being $L$ for any off-equilibrium path contract. Lemma 2 ensures both types of principal will not deviate to those contracts. We shall not worry about the $H$ type's IR constraint and her incentives to choose the $L$ type's equilibrium contract in the BSEH. The last constraint for the principal is the $L$ type's incentives to deviate to the $H$ type's equilibrium contract. Given the $H$ type's contract $C = \{N, W, \boldsymbol{b}\}$, the $L$ type's IC constraint is

$$\Pi^L = -W + \sum_{n=1}^{N} \delta^n f_{n-1}^L(q_0) q_n^L \lambda^L (h - b_n) \Delta \leq 0. \qquad IC_L^H(N)$$

The agent's IR and IC constraints are the same as before. Hence, the $H$'s equilibrium contract, $C^{sep} = \{N^{sep}, W^{sep}, \boldsymbol{b^{sep}}\}$, in the BSEH solves the following Program II:

$$\max_{N,W,\boldsymbol{b}} \Pi^H = -W + \sum_{n=1}^{N} \delta^n f_{n-1}^H(q_0) q_n^H \lambda^H (h - b_n)\Delta$$
$$\text{s.t. } IC_A^H(N),\ IR_A^H(N), \text{ and } IC_L^H(N).$$

To solve this program, we drop the constraint $IR_A^H(N)$ and leave it to be verified. In the relaxed program, $IC_L^H(N)$ must bind, which determines the upfront transfer $W$, otherwise the $H$ type can decrease $W$ to obtain a higher payoff. For any given $N$, the agent's IC constraints in $IC_A^H(N)$ also bind in every period $1 \leq n \leq N$ due to the same reason as in Lemma 1, which determines the bonus scheme $\boldsymbol{b}$. The last thing is to determine the termination date $N$. By taking the limit $\Delta \to 0$, we have the following result:

**Proposition 2.** *In the limit $\Delta \to 0$, there is a unique equilibrium contract for the $H$ type $C^{sep} = \{T^{sep}, W^{sep}, \boldsymbol{b^{sep}}\}$ in the BSEH. It has the following features:*

- *Under experimentation: $0 < T^{sep} < \frac{1}{2} T_*^H$, where $T^{sep}$ satisfies*

$$\lambda^H e^{-\lambda^H T^{sep}} - \lambda^L e^{-\lambda^L T^{sep}} = \left(\lambda^H - \lambda^L\right) e^{-\lambda^H T_*^H} e^{(\lambda^H - \lambda^L) T^{sep}};$$

[15] The discount factor between the $(n-1)$-th and $n$-th experiments is $\delta_n = \delta^k$, where $k \in \mathbb{N} \cup \{\infty\}$ is the number of delaying periods. $k = 1$ means no delay, and $k = \infty$ means terminating the project.

- *A positive base wage:* $W^{sep} > 0$*;*
- *A (weakly) increasing bonus plan:* $b_t^{sep}$ *is weakly increasing in* $t$ *and* $b_t^{sep} \in (0, h)$.

**Rent Sharing** – In such an equilibrium, the $H$ type principal shares some surplus with the agent. She obtains less than $1 - \lambda^L/\lambda^H$ of the total surplus and the agent obtains more than $\lambda^L/\lambda^H$ of that. The surplus is shared through both a positive base wage $W^{sep}$ before experimentation, and an increasing bonus scheme in the period when the success arrives. Although the principal has all the bargaining power and the agent has no financial constraint, she leaves some rents to the agent in order to signal her type is $H$. This phenomenon is first shown in Beaudry (1994). The $L$ type has no rents and exits the market.

**Role of Learning** – An increasing bonus scheme means the principal gives a larger reward to the agent if the success comes later. This is due to the decline of the posterior beliefs about the viability of the project. A failed experiment drives the agent's belief about the viability of the project down, and greater incentives are needed in later periods.

The inefficiently early termination is the second source of the signaling costs. The payment $W$ must equal the $L$ type's expected share of the profits if the $L$ type proposes the $H$ type's contract. Thus, the $H$ type's payoff can be seen as the difference between her expected share of the profits and that of the $L$ type. Extending the experiment has two effects.

Consider payoffs only in the last period. The marginal net benefit for extending the experiment is the difference between the $H$ type's expected share of her profits and that of the $L$ type in that period:

$$q_0 e^{-\rho T} \left( \lambda^H e^{-\lambda^H T} - \lambda^L e^{-\lambda^L T} \right) (h - b_T), \tag{2}$$

where $\lambda^\theta e^{-\lambda^\theta T}$ is the probability of success in the last period $T$ for the $\theta \in \Theta$ type project in the $G$ state, and $(h - b_T)$ is the share retained by the principal.

Consider payoffs in all other periods. Due to the dynamic moral hazard problem, the marginal net cost for extending the experiment is the difference between the increased accumulated bonuses that are given up by the $H$ type and that by the $L$ type:

$$q_0 e^{-\rho T} \left( e^{-\lambda^L T} - e^{-\lambda^H T} \right) \left( \lambda^H b_T - c \right), \tag{3}$$

where $\left( e^{-\lambda^L T} - e^{-\lambda^H T} \right)$ is the probability difference of failures until the last period $T$ between the $H$ and $L$ type in the $G$ state, and $\left( \lambda^H b_T - c \right)$ represents the increased bonuses.[16]

Equalizing the above two effects gives the equilibrium terminating date $T^{sep}$. The inefficiency comes from the combination of the private information and hidden actions on both sides. Moreover, it is aggravated because of the dynamic moral hazard problem due to *learning*. If the state of the project is known, extending the experiment does not increase the agent's share of the profits in previous periods; the principal does not need to give the agent compensation more than one-time incentive costs. Hence, the effect represented by expression (3) is zero, if there is no learning towards the state of the project. Thus, the inefficiency is solely

[16] The marginal increase of a bonus payment at time $t$ by extending the experiment is $\left( \lambda^H b_T - c \right)$ with a proper discounting, i.e., $\frac{\mathrm{d}}{\mathrm{d}T} b_t = e^{-\rho(T-t)} \left( \lambda^H b_T - c \right)$.

determined by expression (2), and it diminishes (i.e., the terminating date goes to be infinity) as $\lambda^L$ goes to zero. However, in our model, the state of the project is unknown; thus, prolonging experimentation increases the bonus payments in all periods. Even when $\lambda^L$ goes to zero, expression (3) is strictly positive. In the limit case ($\lambda^L = 0$), the equilibrium termination date $T^{sep}$ is determined by

$$q_0 e^{-\rho T} e^{-\lambda^H T} \lambda^H \left(h - b_T\right) = q_0 e^{-\rho T} \left(1 - e^{-\lambda^H T}\right) \left(\lambda^H b_T - c\right),$$

where $\lambda^H(h - b_T)$ and $(\lambda^H b_T - c)$ are the expected payoffs for the principal and for the agent in the $G$ state, respectively. Therefore, for $T = T^{sep}$

$$e^{-\lambda^H T} = \frac{\lambda^H b_T - c}{\lambda^H h - c} = \frac{l^H}{l_T} = e^{-\lambda^H \left(T_*^H - T\right)},$$

where $l^H = \frac{c}{\lambda^H h - c}$ and $l_T = \frac{c}{\lambda^H b_T - c}$ are the likelihood ratio of the posterior beliefs that the state of the $H$ project is $G$ at the efficient termination time $T_*^H$ and at the termination time $T$, respectively. Hence, we have $T^{sep} = T_*^H/2$ when $\lambda^L = 0$.

We can show that the equilibrium termination date $T^{sep}$ is decreasing in $\lambda^L$, therefore the $H$ type project is only operated less than a half of the efficient time.

**Limited Liability** – Consider a different setting where the principal is publicly known to be the $H$ type, but the agent is protected by limited liability, i.e., $W$ and $\boldsymbol{b}$ must be positive. It is easy to show that the optimal contract in that case is the $H$ type's equilibrium contract in the BSEH of our original model with $\lambda^L = 0$, which implies $W^{sep} = 0$. Hence, our model provides an alternative explanation why the agent often has limited liability, i.e., the principal cannot charge anything from the agent or sell the project to him. A standard argument is that the agent may have some financial constraints. However, in our model where the agent does not have any financial constraints, the principal is still willing not to charge anything from him in order to separate herself from an inferior project. Thus, without financial constraints, we still may observe the "limited liability" phenomenon in equilibrium.

**Unobservable Successes** – We assume a success is publicly observable and verifiable. One may think it is sometimes very costly for the principal to verify a success. If that cost is prohibitively high, contracts can only depend on the agent's report on a success. Thus, the principal may face an additional problem: the agent may delay reporting a success to obtain a higher payoff if the increase of bonus payments over time offsets the discounting cost.

However, the structure of the bonus payments in the equilibrium contract ensures that does not happen. Actually, the agent has no incentive to hide a success from the principal, as long as he is given the exact incentive to experiment when a success can be costlessly verified, as in our model. In other words, given the binding IC constraints for the agent, the agent will report a success truthfully when it arrives. Formally, Lemma 6 in the Appendix shows that binding IC constraints for the agent implies, for any $n$ before the termination date,

$$b_n \geq \delta b_{n+1}.$$

When $\delta < 1$, the above inequality is strict. Since all the remaining results in this paper feature no temporary suspension and binding IC constraints for the agent, the agent has no incentive to delay reporting a success even when it cannot be observed by the principal.

The BSEH shows the maximal reward the $H$ type can obtain in a separating equilibrium. A robust feature of the BSEH is that it is the unique separating equilibrium that survives the intuitive criterion of Cho and Kreps (1987). In fact, any equilibrium that gives a lower payoff to the $H$ type than the BSEH fails the intuitive criterion.

## 5.3 Pooling Equilibrium

In this section we examine pooling equilibria. A pooling equilibrium is an equilibrium in which both types of principal propose the same equilibrium contract, and the agent holds the prior belief $\beta_0$ about the $H$ type after the equilibrium contract is proposed.

We first consider the efficient solution when the type of the project is unknown at the ex ante stage. We call it the *mixed* project. When the prior belief is $\beta_0$, the value of the mixed project with a termination date $N$ is

$$V(N) := \beta_0 V^H(N) + (1-\beta_0)V^L(N).$$

Let $N_*$ be the *efficient termination date for the mixed project*, which maximizes $V(N)$. Let $T_*$ be the limit of $N_*\Delta$ when $\Delta \to 0$. When the prior $\beta_0$ is too pessimistic, i.e., $q_0\lambda_0 h \le c$, where $\lambda_0 := \beta_0\lambda^H + (1-\beta_0)\lambda^L$, the efficient termination date is zero. Any pooling equilibrium is trivial and features no experiments or transfers. Otherwise, when the prior $\beta_0$ is not too small, i.e., $q_0\lambda_0 h > c$, the efficient termination date for the mixed project is in between zero and the efficient termination date for the $H$ type. We will only consider the latter case.

One simple pooling equilibrium is that both types of principal sell the mixed project to the agent, and extract all expected surplus of the mixed project. Lemma 2 ensures no one will deviate to any off-equilibrium path contract, if the agent believes the deviator is the $L$ type. However, the $H$ type can obtain a higher payoff in other pooling equilibria. Again, we consider the *best pooling equilibrium for the H type*, or *BPEH*.

When both types of principal pool together, the agent updates his beliefs on both the type and the state of the project during experimentation. Furthermore, even the type and the state are independent at the outset, the posterior beliefs after some failures are correlated. Let $q_n$ be the posterior belief about the project's state being $G$, after $n-1$ failures, and $\beta_n$ be the posterior belief about the project's type being $H$ conditional on the state being $G$, after $n-1$ failures. The posterior belief about the project's type being $H$ conditional on the state being $B$ is the prior $\beta_0$, no matter how many failures the project has. Hence,

$$q_n = \frac{q_0\left[\beta_0\left(1-\lambda^H\Delta\right)^{n-1} + (1-\beta_0)\left(1-\lambda^L\Delta\right)^{n-1}\right]}{q_0\left[\beta_0\left(1-\lambda^H\Delta\right)^{n-1} + (1-\beta_0)\left(1-\lambda^L\Delta\right)^{n-1}\right] + 1 - q_0},$$

and

$$\beta_n = \frac{\beta_0\left(1-\lambda^H\Delta\right)^{n-1}}{\beta_0\left(1-\lambda^H\Delta\right)^{n-1} + (1-\beta_0)\left(1-\lambda^L\Delta\right)^{n-1}}.$$

Let $\lambda_n := \beta_n \lambda^H + (1-\beta_n)\lambda^L$ be the expected arrival rate of success conditional on the state being $G$, after $n-1$ failures. It is easy to see that $q_n$, $\beta_n$ and $\lambda_n$ are strictly decreasing over time. That is, both the posterior belief about the state being $G$ and the posterior belief about the type being $H$ conditional on the state being $G$ decline after a failure.

The learning process makes the agent's IC constraints more complex. But they can be easily transformed from the constraints in the separating equilibrium by using correct beliefs. Given $C = \{N, W, \boldsymbol{b}\}$, the agent's IC constraints become, for $1 \leq n \leq N$

$$\sum_{s=n}^{N} \delta^{s-n} f_{s-n}(q_n, \beta_n)\left(q_s \lambda_s b_s - c\right)\Delta \geq \sum_{s=n+1}^{N} \delta^{s-n} f_{s-n-1}(q_n, \beta_n)\left(q_{s-1}\lambda_{s-1} b_s - c\right)\Delta, \quad IC_A(N)$$

where $f_m(q,\beta) = q\left[\beta(1-\lambda^H\Delta)^m + (1-\beta)(1-\lambda^L\Delta)^m\right] + 1 - q$ is the probability of failing $m$ times for a project starting with a probability $q$ being in state $G$, and a probability $\beta$ being type $H$ conditional on being in state $G$.

The agent's IR constraint becomes

$$W_A = W + \sum_{n=1}^{N} \delta^n f_{n-1}(q_0, \beta_0)(q_n \lambda_n b_n - c) \geq 0. \qquad IR_A(N)$$

In the BPEH, the principal's IC constraints are trivially satisfied on the equilibrium path. Lemma 2 ensures no type has an incentive to deviate to any other contracts once we assume the agent believes the principal who proposes other contracts is the $L$ type. We shall not worry about the IR constraint for the $H$ type. The IR constraint for the $L$ type is

$$\Pi^L = -W + \sum_{n=1}^{N} \delta^n f_{n-1}^L(q_0) q_n^L \lambda^L \left(h - b_n\right)\Delta \geq 0. \qquad IR_L(N)$$

Thus, the equilibrium contract $C^{pl} = \{N^{pl}, W^{pl}, \boldsymbol{b^{pl}}\}$ in the BPEH solves the following Program III:

$$\max_{N,W,\boldsymbol{b}} \Pi^H = -W + \sum_{n=1}^{N} \delta^n f_{n-1}^H(q_0) q_n^H \lambda^H (h - b_n)\Delta$$
$$\text{s.t. } IC_A(N),\ IR_A(N), \text{ and } IR_L(N).$$

We solve the relaxed program without $IR_L(N)$ as it can be verified that it is slack. $IR_A(N)$ must bind in the relaxed program, otherwise decreasing $W$ gives both types a higher payoff. This determines the time-zero transfer $W$. Given any $N$, we can show that the agent's IC constraints $IC_A(N)$ must bind for all $n$, which determines the bonus scheme $\boldsymbol{b}$.[17] Last, we solve the program by finding the equilibrium termination time.

**Proposition 3.** *Assume $q_0\lambda_0 h > c$. In the limit $\Delta \to 0$, there is a unique equilibrium contract for the $H$ type $C^{pl} = \{T^{pl}, W^{pl}, \boldsymbol{b^{pl}}\}$ in the BPEH. It has the following features:*

[17] The reason for binding IC constraints of the agent is similar to Lemma 1 and Proposition 2. Notice that the agent obtaining no rents implies that the payoff of the $H$ type principal equals the sum of the project's value and $(1-\beta_0)$ portion of the payoff difference between the $H$ and $L$ type. Hence, given a fixed amount of experimentation, the pooling contract in the BPEH must maximize the payoff difference.

- *Under experimentation:* $0 < T^{pl} < T_*$;
- *A sign-up fee:* $W^{pl} < 0$;
- *A (weakly) increasing bonus plan:* $b_t^{pl}$ *is weakly increasing in* $t$ *and* $b_t^{pl} \in (0, h)$.

*Moreover, the equilibrium payoff for the* $H$ *type in the BPEH is strictly increasing in the prior belief about the* $H$ *type* $\beta_0$ *and converges to* $V_0^H(T_*^H)$ *as* $\beta_0 \to 1$.

The above contract shares some common features with the $H$ type's equilibrium contract in the BSEH. It gives the agent exact enough incentives to work, i.e., the IC constraints for the agent always bind. The bonus payments are constant over time if there is no discounting (i.e., $\rho = 0$), and they are strictly increasing if $\rho > 0$. Furthermore, as in the BSEH, the increase of bonus payments cannot offset the cost for discounting. Hence, even when a success is not publicly observed, the agent has no incentive to delay reporting a success.

**Pooling Costs** – In the BSEH, the $H$ type gives a positive base wage to the agent. By contrast, in the BPEH, both types charge a sign-up fee from the agent. Thus, the agent obtains no rents but the $L$ type obtains positive rents from pooling. The agent does not have financial constraints and is willing to pay the sign-up fee to exchange for some share of profits once the project succeeds. The amount the $H$ type principal extracts from the agent cannot recover her FIB surplus. When the $L$ type pools with the $H$ type, the latter has to compensate more costs to the agent, but only partially extracts the promised bonuses back from the agent. Both the additional compensation costs and the non-recoverable bonuses are proportional to the population of the $L$ type, $1 - \beta_0$. Thus, the pooling costs diminish as the prior about the $H$ type goes to one.

**Learning Viability and Quality** – When both types pool together, the agent needs to learn both the viability and the quality of the project. A failure means the agent is more pessimistic about both the viability and the (conditional) quality[18] – learning creates dynamic moral hazard costs. However, learning the viability and the quality has different consequences. If what needs to be learnt is the viability ($0 < q_0 < 1$), it is very costly to extend the experiment, since the posterior beliefs of success eventually goes to zero. However, if what needs to be learnt is solely the quality ($q_0 = 1$ and $0 < \beta_0 < 1$), the principal can always incentivize the agent to work with bounded bonuses, i.e., $b_t \leq c/\lambda^L$ for any $t > 0$. Formally, let $T$ be the termination date, then the limit of the marginal effect of extending experiment for the last period bonus is

$$\lim_{T \to \infty} \frac{\mathrm{d}b_T}{\mathrm{d}T} = \begin{cases} \infty & \text{if } q \in (0,1), \\ 0 & \text{if } q = 1. \end{cases}$$

**Under Experimentation**[19] – It is straightforward that the equilibrium termination time

[18]The conditional quality means the project's quality conditional on it being viable. Note that the conditional quality is what matters for the agent's incentives, and the unconditional quality could be increasing with one more failure after some point.

[19]The contract in the BPEH features under experimentation only for the $H$ type principal, while the same contract requires over experimentation for the $L$ type principal.

is less than the efficient termination time for the $H$ type. In addition, it is also less than the efficient termination time for the mixed project. We now provide some intuition.

Because the agent has no rents in the equilibrium, the surplus of the mixed project is shared between the two types of principal. Thus, the equilibrium payoff of the $H$ type is equal to the sum of (1) the surplus of the mixed project, and (2) $(1-\beta_0)$ portion of the equilibrium payoff difference between the $H$ and $L$ type. We now show that the marginal effect of extending the mixed project at its efficient termination time $T_*$ is negative for the $H$ type. Thus, she benefits from terminating the project earlier than the efficient time $T_*$.

Consider the first part of the $H$ type's payoff – the surplus of the mixed project. By the definition of the efficient termination time, extending the mixed project has zero marginal effect on the total surplus. For the second part of the $H$ type's payoff, extending the project at $T_*$ does not generate any marginal benefit for the principal, since the required bonus (for incentivizing the agent to work) has to be equal to the profit of a success. However, due to the dynamic moral hazard problem and the fact that the $H$ type is more likely to succeed before the termination date than the $L$ type, the costs for sharing more bonuses with the agent in all periods incurred by the $H$ type is larger than that incurred by the $L$ type. Thus, extending the project does incur additional costs. Therefore, the marginal effect of extending the project at $T_*$ is negative and the $H$ type principal prefers to terminate the project earlier than the efficient time for the mixed project.

## 5.4 The Best Equilibrium for the High Type

We have examined both the BSEH and the BPEH. In the BSEH, the equilibrium payoff for the $H$ type is independent of the prior belief $\beta_0$ and is less than her FIB surplus. In the BPEH, the equilibrium payoff for the $H$ type is strictly increasing in $\beta_0$ when her payoff is strictly positive (i.e., when $q_0\lambda_0 h > c$), and converges to her FIB surplus as $\beta_0$ goes to one. Obviously, there exists a cutoff prior belief about the $H$ type, $\beta_c \in (0, 1)$, such that the BSEH gives the $H$ type a higher payoff than the BPEH does when $\beta_0 < \beta_c$, and the BPEH gives the $H$ type a higher payoff than the BSEH does when $\beta_0 > \beta_c$.

To find the best equilibrium for the $H$ type, we still need to study other equilibria, such as partial separating equilibria, where the principal may randomize over different contracts. We categorize all equilibria into two classes: (1) the set of equilibria that give the $L$ type a strictly positive payoff, and (2) the set of equilibria that give the $L$ type a zero payoff. We now show that (1) among all equilibria that give the $L$ type a strictly positive payoff, the BPEH gives the $H$ type the highest payoff, and (2) among all equilibria that give the $L$ type a zero payoff, the BSEH gives the $H$ type the highest payoff. Thus, we can conclude that the best equilibrium for the $H$ type is either BSEH or BPEH, depending on $\beta_0$.

First, consider the set of equilibria that give the $L$ type a strictly positive payoff. This means that no equilibrium path contract fully reveals the $L$ type. In other words, the $L$ never chooses any contract by herself alone in the equilibrium, she always pools with the $H$ type. Moreover, there must be at least one contract proposed by the $L$ type in the equilibrium such that the agent's belief about the principal being the $H$ type does not exceed the prior $\beta_0$. Together with Proposition 3, we can show that:

**Lemma 3.** *For any equilibrium such that the $L$ type obtains a strictly positive payoff, the $H$ type's payoff cannot exceed her payoff in the BPEH.*

Now consider the set of equilibria that give the $L$ type a zero payoff. Fix any such equilibrium $E$ and any equilibrium contract $C$ proposed by the $H$ type in $E$. Suppose the agent forms belief $\beta \in [0, 1]$ about $H$ type after $C$ is proposed. Then $C$ must satisfy the agent's IC and IR constraints, and the $L$ type cannot obtain a strictly positive payoff by proposing the same contract. We solve the best contract for the $H$ type subjected to the above constraints and show that the best outcome the $H$ type can obtain is increasing in $\beta$. In addition, we come back to our BSEH result when $\beta = 1$. Hence, we have the following result:

**Lemma 4.** *For any equilibrium such that the $L$ type obtains a zero payoff, the $H$ type's payoff cannot exceed her payoff in the BSEH.*

With the above two lemmas, we conclude:

**Proposition 4.** *There exists a cutoff belief $\beta_c \in (0, 1)$,*

- *when $\beta_0 < \beta_c$, the BSEH gives the $H$ type the highest payoff among all equilibria;*
- *when $\beta_0 > \beta_c$, the BPEH gives the $H$ type the highest payoff among all equilibria.*

Thus, in the best equilibrium for the $H$ type, the termination time of the $H$ type project is inefficiently early, and she has to share rents with either the agent (in the BSEH) or with the $L$ type (in the BPEH). The equilibrium contract for the $H$ type in the BSEH also maximizes the payoff difference between the $H$ and $L$ type when the agent believes the contract proposer is the $H$ type. Lemma 7 in the Appendix shows that such a payoff difference is increasing in the agent's belief about the principal being the $H$ type. Therefore, we conclude that the BSEH is the equilibrium that maximizes the payoff difference between the two types of principal. Hence, if we consider a pre-game where the principal can invest in the quality of a project before the signaling game, namely exerting investment efforts to increase the probability $\beta_0$ of having a $H$ type project, her investment decision and the prior $\beta_0$ are endogenously determined by the payoff difference between the two types in the equilibrium of the signaling game. Thus, the BSEH permits the highest prior, as long as the investment cost is increasing and convex in $\beta_0$.

# 6 Introducing a Mediator

The above results illustrate the conflict between signaling and providing incentives to the agent. We now consider the implication of allowing for a mediator who designs a mechanism (i.e., a menu of contracts).

As in the rest of the paper, we assume the goal of the mediator is to maximize the payoff of the $H$ type principal. This is a natural benchmark that facilitates the comparison across the signaling game and the mechanism design problem. The mediator needs to attract the $H$ type project, which generates the social surplus. Thus, it must provide her with a higher payoff than she can obtain in the signaling game without the mediator. Moreover, if there are multiple profit-oriented mediators who compete in a Bertrand fashion, competition will drive them to maximize the $H$ type's payoff in the equilibrium.

The basic role of the mediator is to communicate with both the principal and the agent, and disclose information at a proper time. By exploring the fact that the two types of principal have different beliefs about their projects, separation between them can be achieved immediately, but this information can be concealed from the agent, at least temporarily. In this way, the agent's IR and IC constraints only need to be held in expectation, although different principals offer different contracts.

Science Exchange, an online platform that connects scientific researchers with experimental service providers, is an example of a mediator. A researcher corresponds to the principal in our model, while a service provider is the agent. The platform has its own system to verify the qualification of service providers, so one shall not worry much about their abilities may be private. The major concern for service providers and for the platform is that the quality of ideas brought in by researchers is, by their nature, hard to evaluate. Moreover, moral hazard on the part of service providers is likely to be important, given the uncertainties associated with research processes. The role played by the platform is not only to reduce search and transaction costs, but also to design contracts, protect intellectual properties and confidentiality, and facilitate communications.[20] Thus, researchers' confidential information can be concealed from service providers at the outset and will be disclosed to them only when it is necessary.

We now formally examine the mediator's mechanism design problem. A mechanism, $\mathcal{M} = \{\mathcal{C}^H, \mathcal{C}^L\}$, is an extensive form game containing a menu of two contracts. A contract, $\mathcal{C}^\theta = \{N^\theta, W^\theta, \boldsymbol{b}^{\boldsymbol{\theta}}\}$, is a triple as before, where $\theta \in \Theta$, $N^\theta \in \mathbb{N}_0$, $W^\theta \in \mathbb{R}$, and $\boldsymbol{b}^{\boldsymbol{\theta}} \in \mathbb{R}^{N^\theta}$.

Given a mechanism $\mathcal{M} = \{\mathcal{C}^H, \mathcal{C}^L\}$, the agent and the principal simultaneously make their participation decisions, and the principal reports her type to the mediator. If at least one party rejects the mechanism, both of them obtain their reservation payoff zero. If both accept it and the principal reports $\theta$, the mediator implements the contract $\mathcal{C}^\theta$:

- In any period $n \leq N^\theta$ (before success), the mediator recommends that the agent work. If the project succeeds in that period, the principal transfers $b_n^\theta$ to the agent and the game ends; otherwise, there is no payment.
- In the period $n = N^\theta + 1$ (before success), the mediator recommends that the agent not work and the game ends.
- The principal transfers $W^\theta$, which is independent of the outcome and is measured in the time-zero discounted value, to the agent when the game ends.

The agent's action set is the same as before. He chooses whether to follow the mediator's recommendations (i.e., whether to exert effort), which is unobservable to both the principal and the mediator.

Before we go any further, we explain the space of mechanisms that we study. They are direct

[20]See `https://www.scienceexchange.com/trust` and other sites under the same domain for details.

mechanisms with two restrictions. First, recommendations for delaying a project temporarily are not considered, but this is without loss for the purpose of finding the *optimal mechanism for the $H$ type principal.* Second, random recommendations are not allowed, which is with loss of generality. However, we show that, as the period length shrinks, the $H$ type can obtain a payoff that converges to her FIB surplus via pure recommendations. We will discuss later how random recommendations improve the $H$ type's payoff for a fixed period length.

Now we consider constraints in this new environment. We say a mechanism $\mathcal{M} = \{\mathcal{C}^H, \mathcal{C}^L\}$ is *feasible*, if both types of principal and the agent are willing to participate, both types of principal are willing to report truthfully, and the agent is willing to follow recommendations. In other words, a feasible mechanism satisfies the following IR and IC constraints.

The type $\theta \in \Theta$ principal's IR constraint is

$$\Pi^\theta = -W^\theta + \sum_{n=1}^{N^\theta} \delta^n f^\theta_{n-1}(q_0) q^\theta_n \lambda^\theta \left(h - b^\theta_n\right) \Delta \geq 0. \qquad IR_\theta(\mathcal{M})$$

Let $\beta^H_0 = \beta_0$ and $\beta^L_0 = 1 - \beta_0$. The agent's IR constraint is

$$W_A = \sum_{\theta \in \Theta} \beta^\theta_0 \left[-W^\theta + \sum_{n=1}^{N^\theta} \delta^n f^\theta_{n-1}(q_0) \left(q^\theta_n \lambda^\theta b^\theta_n - c\right) \Delta\right] \geq 0. \qquad IR_A(\mathcal{M})$$

The type $\theta \in \Theta$ principal's IC constraint (for not misreporting $\theta' \in \Theta \backslash \{\theta\}$) is

$$-W^\theta + \sum_{n=1}^{N^\theta} \delta^n f^\theta_{n-1}(q_0) q^\theta_n \lambda^\theta \left(h - b^\theta_n\right) \Delta \geq -W^{\theta'} + \sum_{n=1}^{N^{\theta'}} \delta^n f^\theta_{n-1}(q_0) q^\theta_n \lambda^\theta \left(h - b^{\theta'}_n\right) \Delta. \quad IC^{\theta'}_\theta(\mathcal{M})$$

The above constraints are straightforward extensions from the signaling game. The only difference is that transfers now depend on the reported type.

The agent's IC constraints are more involved, since signaling can take place at any time. Suppose the $H$ type experiments longer than the $L$ type, i.e., $N^H \geq N^L$.[21] Because recommendations for both types are the same until $N^L$, the agent's beliefs about the principal's type and about the project's state evolve in the same way as in the pooling equilibrium. However, if $N^H > N^L$ and if the agent is recommended to work in the period $N^L + 1$, his posterior belief about the principal's type being $H$ jumps to one, but his posterior belief about the project's state of being $G$ jumps down, since a failure from the $H$ type is more informative about the state being $B$ than a failure from the $L$ type. Specifically, the agent's IC constraints are

$$\begin{cases} \nu_n \chi^H_n + (1 - \nu_n) \chi^L_n \geq 0, & \text{for } 1 \leq n \leq N^L\ , \\ \chi^H_n \geq 0, & \text{for } N^L + 1 \leq n \leq N^H \text{ if } N^H > N^L, \end{cases} \qquad IC_A(\mathcal{M})$$

where $\nu_n = q_n \beta_n + (1 - q_n)\beta_0$ is the posterior belief about the principal's type being $H$ after $n - 1$ failures, and $\chi^\theta_n$ is the payoff difference for the agent between always following

[21] When $N^H < N^L$, we can define the IC constraints for the agent in the same way, though it is irrelevant for the optimal mechanism for the $H$ type principal.

recommendations, and shirking in period $n$ but following all remaining recommendations thereafter, conditional on the principal's type being $\theta$. That is,

$$\chi_n^\theta = \sum_{s=n}^{N^\theta} \delta^{s-n} f_{s-n}^\theta(q_n^\theta) \left(q_s^\theta \lambda^\theta b_s^\theta - c\right) \Delta - \sum_{s=n+1}^{N^\theta} \delta^{s-n} f_{s-n-1}^\theta(q_n^\theta) \left(q_{s-1}^\theta \lambda^\theta b_s^\theta - c\right) \Delta.$$

Note that the equilibrium contracts in the signaling game remain feasible for the mediator. Thus, the mediator can replicate the equilibrium outcomes of the signaling game. We now show how the mediator can achieve a strictly higher payoff for the $H$ type, compared to both the BPEH and the BSEH, by relaxing only the IR constraint of the agent.

Consider feasible mechanisms in which the agent is recommended to not work at all if the principal reports $L$, i.e., $N^L = 0$. Thus, the agent learns the principal's type before he starts to work, but after he accepts the offer. The reason the mediator can achieve a strictly higher payoff for the $H$ type, compared to the BPEH, is (1) the total social surplus increases if the mediator recommends the agent not work for the $L$ type, and (2) it is cheaper to incentivize the agent to work without the $L$ type pooling during the experimentation. Comparing to the BSEH, the mediator can also achieve a strictly higher payoff for the $H$ type. Recall that the $H$ type leaves some rents to the agent in the BSEH. The mediator can lower the independent transfers $W$ in both contracts by the same amount, while keeping the agent's IR constraint satisfied in expectation before he learns the principal's type. This does not change the $L$ type's incentive, but increases both types' payoffs. Thus, separation is less costly when it occurs right after the acceptance of the offer. However, it is not hard to see that the $H$ type cannot obtain her FIB payoff by solely relaxing the agent's IR constraint, because instead the $L$ type must be left with some rents to induce separation.[22]

The mediator can further improve the $H$ type's payoff by also relaxing the IC constraints of the agent. This requires the mediator recommends the agent work on the $L$ type project for some time. Thus, separation takes place in the period when recommendations differ for different types of principal.

In our quasi-linear environment, the total surplus generated in a feasible mechanism is shared by both types of principal and the agent. Let $\Pi^\theta$ be the payoff obtained by the $\theta$ type of principal and $W_A$ be the payoff obtained by the agent. In a feasible mechanism, we have

$$\beta_0 V^H(N^H) + (1-\beta_0) V^L(N^L) = \beta_0 \Pi^H + (1-\beta_0)\Pi^L + W_A.$$

On the one hand, feasibility requires that both types of principal and the agent must obtain a non-negative payoff. On the other hand, the value generated by the $L$ type $V^L(N^L)$ is strictly decreasing in $N^L$ for $N^L \geq 1$, and the value generated by the $H$ type $V^H(N^H)$ is maximized at $N^H = N_*^H$. Thus, for any $N^L \geq 1$, we have

$$\Pi^H = V^H(N^H) + \frac{1-\beta_0}{\beta_0}\left(V^L(N^L) - \Pi^L\right) - \frac{1}{\beta_0} W_A \leq V^H(N_*^H) + \frac{1-\beta_0}{\beta_0} V^L(1). \qquad (4)$$

[22] The first part of the proof of Proposition 5 solves the optimal mechanism for the $H$ type when $N^L = 0$. In addition to leaving rents to the $L$ type, it also features a termination date of the $H$ type in between its equilibrium termination date in the BSEH and its efficient termination date. Thus, there is no feasible mechanism that implements the FIB.

This means in any feasible mechanism with $N^L \geq 1$, the $H$ type's payoff has an upper bound, which is strictly positive for a small $\Delta$.

We now show there exists a feasible mechanism with $N^L \geq 1$ that achieves the upper bound. Such a mechanism must (1) satisfy feasibility, (2) leave no rents to both the agent and the $L$ type, and (3) implement the $H$ type project efficiently ($N^H = N_*^H$) and the $L$ type project almost efficiently ($N^L = 1$). The last condition determines the efficiency of both projects, and therefore the time when recommendations differ for different reports of the principal. Together with the second condition, they determine the independent transfers:

$$W^L = \delta q_0 \lambda^L \left(h - b_1^L\right) \Delta, \tag{5}$$

$$W^H = -\left[V^H(N_*^H) + \frac{1-\beta_0}{\beta_0} V^L(1)\right] + \sum_{n=1}^{N_*^H} \delta^n f_{n-1}^H(q_0) q_n^H \lambda^H \left(h - b_n^H\right) \Delta. \tag{6}$$

Now we show how to use bonus payments to incentivize both the agent (to work) and the $L$ type principal (to report truthfully) at the same time.

The mediator reveals the principal's report, which is her type if she reports truthfully, in the second period by recommending differently. If the mediator recommends the agent work, he will learn the principal's type is $H$ and his IC constraints from the second period to $N_*^H$ are the same as in the separating equilibrium. However, in the first period, the agent's belief about the principal's type being $H$ is the prior $\beta_0$, and his IC constraint is simplified to

$$q_0 \left[\beta_0 \lambda^H b_1^H + (1-\beta_0)\lambda^L b_1^L\right] - c \geq \beta_0 q_0 \lambda^H \sum_{s=2}^{N_*^H} \delta^{s-1} \left(1 - \lambda^H \Delta\right)^{s-2} \left(\lambda^H b_s^H - c\right) \Delta. \tag{7}$$

The agent requires enough expected bonus payment to work in the first period. Thus, given the bonus payment from the $H$ type, the inequality (7) determines a lower bound for the bonus payment $b_1^L$ from the $L$ type.

The $L$ type's IC constraint is $-W^H + \sum_{n=1}^{N_*^H} \delta^n f_{n-1}^L(q_0) q_n^L \lambda^L \left(h - b_n^H\right) \Delta \leq 0$, i.e.,

$$V^H(N_*^H) + \frac{1-\beta_0}{\beta_0} V^L(1) - \sum_{n=1}^{N_*^H} \delta^n \eta_n \left(h - b_n^H\right) \Delta \leq 0. \tag{8}$$

Because the $H$ type is more likely to succeed in the first period than the $L$ type ($\eta_1 > 0$), the inequality (8) determines an upper bound for the bonus payment $b_1^H$ from the $H$ type.

Thus, distinct bonus payments for the first experiment can be used to incentivize both the $L$ type principal and the agent. We can make $b_1^H$ small to prevent the $L$ type from mimicking the $H$ type, but keep $b_1^L$ large so that the expected bonus payment is high enough for the agent to work. This virtually resolves the conflicts between signaling and providing incentives to the agent. When the period length is small, the inefficiency is negligible and the $H$ type principal can approximately obtain her FIB payoff. We have the following result:

**Proposition 5.** *There exists a $\bar{\Delta} > 0$, such that for any $\Delta \in (0, \bar{\Delta})$, there exists an optimal mechanism for the $H$ type, in which the $H$ type obtains a payoff $V^H(N_*^H) + \frac{1-\beta_0}{\beta_0} V^L(1)$. As $\Delta \to 0$, the $H$ type's payoff converges to her FIB surplus.*

The optimal mechanism features a trial period, i.e., the first period. During the trial period, the agent is always recommended to work, regardless of the principal's report. Hence, the agent does not know which project he is working on. The principal's private type will be revealed to the agent only after the trial period, i.e., he will be told to stop working for the $L$ project, but to keep working for the $H$ project until the efficient termination time. From the agent's point of view, the failure in the trial period will trigger the principal's option to terminate the project, but only the $L$ type will exercise her option. Such a design protects the agent from wasteful projects, and thus protects the $H$ type principal from skepticism. In addition, the payment structure during the trial period is designed to elicit the private information of the principal. Both types of principal believe that the chance to succeed during the trial period for the $H$ type is larger than that for the $L$ type. Thus, the mediator can use a mixture of base payments and bonus payments to explore such differential beliefs. The contract designed for the $H$ type contains a high base payment but a low bonus payment for the success during the trial period, while the contract designed for the $L$ type contains a low base payment but a high bonus payment. Thus, the two contracts are two bets on the event of the trial period success, which induce the principal to truthfully report her type due to the differential beliefs about the event. Therefore, the average bonus payment during the trial period – a high bonus from the $L$ type and a low bonus from the $H$ type – provides an incentive for the agent to work, while the structure of the payments provides an incentive for the principal to reveal her type. From the efficiency point of view, this mechanism also induces efficient experimentation for the $H$ type project. However, running the $L$ type project for the trial period is inefficient. But this trial period is short. It lasts only one period, and it is negligible when the period length is small. Hence, the $H$ type principal can obtain a payoff close to her FIB surplus for a small period length, while the $L$ type principal and the agent are left with no rents.[23]

If random recommendations are allowed, the mediator can further improve the $H$ type's payoff for a fixed $\Delta$. Instead of recommending the agent work in the first period for both projects, the mediator can recommend him work with a small probability $\mu \in (0, 1)$ for the $L$ type, while keep the recommendations for the $H$ type unchanged. In this way, it reduces the inefficiency loss as the $L$ type project is less likely to be implemented. It also makes the agent more willing to work upon receiving a recommendation of working as such a recommendation is more likely come from the $H$ type project. To induce separation, the mediator, of course, has to increase the size of the bets for the success in the first period between the two types. Note that the probability $\mu$ can be arbitrarily small, but it must be strictly positive. When $\mu$ diminishes, the $H$ type keeps virtually all the surplus of her innovation, while the $L$ type who has an inferior innovation is left with no rents. Therefore, for a fixed $\Delta$, although the FIB cannot be exactly implemented, it can be virtually implemented.

The reader may ask, can we achieve approximate efficiency without a mediator? Suppose we consider a game where the principal can propose an arbitrary menu of contracts, with the provision that the exact contract that is to be implemented will depend upon a private message sent by the principal. Furthermore, the menu provides a disclosure date, at which

[23] The agent gains some rents from working for the $H$ type, and loses some rents from working for the $L$ type, but breaks even on average.

it will be revealed to the agent which contract has been chosen. Is there a PBE where both types of principal propose the above mechanism? This is an appropriate generalization, to our context, of the question that has been examined in the three-stage mechanism proposing game of Maskin and Tirole (1992). The key problem here is ensuring that deviations, by either type of principal to a different menu of contracts, are unprofitable. In our context, the set of possible deviating contracts is extremely large and complex, and we have been unable to show that every such deviation is unprofitable. More specifically, we need to show that for any other menu, there exists a belief of the agent about the type of the principal that prevents a profitable deviation by either type. Our Lemma 2 shows that a pessimistic belief that assigns probability one about the principal's type being $L$ can prevent a profitable deviation in the original signaling game, but such a simple belief does not work in the menu proposing game.[24] An explicit construction of beliefs that prevents profitable deviations appears to be intractable. In this context, the paper by Wagner, Mylovanov, and Tröger (2015) is relevant. They examine moral hazard in a static setting and show that when the FIB is feasible, it is an equilibrium outcome. However, they do not study the case where the FIB is not feasible. The key feature of the present model is that the FIB is not feasible, although we can approximate it.[25] Consequently, their results and methods cannot be applied in our context.

# 7 Conclusion

We analyze a model where an informed principal engages an agent to explore the viability of her project. The principal has private information about the quality of her project. Thus, in addition to providing incentives to the agent to experiment on the project, the high type principal has to convince the agent of her project's quality. We examine the best outcome for the high type principal both in a signaling game where the principal commits to a transparent contract and in a third-party mechanism design problem.

In the signaling game, when the prior probability of a low quality project is large, the best equilibrium for the high type principal is a separating equilibrium. The high type principal separates himself from the low type by leaving rents to the agent, and also by terminating the project inefficiently early. When the prior probability of a low quality project is small, the best equilibrium for the high type is a pooling equilibrium, which also features an early termination of the high quality project.

In neither equilibrium of the signaling game is the innovator with a superior project able to capture her contribution to social surplus. This leads us to study the role of a mediator. The mediator offers a menu of two contracts, but does not disclose the type of the principal to the agent unless it is necessary to do so. We find that an opaque mechanism is able to approximately implement the optimal outcome, in terms of both inducing efficient experimentation and ensuring high rewards for the high type principal. It is necessary to induce experimentation by the low type only for a single period, but as the period length becomes

[24]We can construct a menu such that the $H$ type would find it profitable offering it to the agent if he believed that she was the $L$ type.

[25]Recall that the FIB requires an immediate termination of the $L$ types project and a zero payoff for her.

small, this is approximately efficient.

# Appendices

## A Temporary Delays of the Project

In this section, we formalize the idea of delaying the project temporarily. Lemma 5 will show that it is never optimal for the $H$ type to delay her project temporarily.

Given a contract $C = \{N, W, \boldsymbol{b}\}$ and the agent's belief $\beta_0$ about the principal being $H$, after accepting the contract, the agent solves a dynamic decision problem. Suppose the agent's optimal action plan is to exert efforts only for periods in $R \subseteq \{1, 2, \cdots, N\}$, then the contract must satisfy the agent's IC constraints for those periods, i.e., for all $1 \leq n \leq \#R$,[26]

$$\sum_{s=n}^{\#R} \Pi_{j=n+1}^{s} \delta_j f_{s-n}(q_n, \beta_n)(q_s \lambda_s b_{r_s} - c)\Delta \geq \sum_{s=n+1}^{\#R} \Pi_{j=n+1}^{s} \delta_j f_{s-n-1}(q_n, \beta_n)(q_{s-1} \lambda_{s-1} b_{r_s} - c)\Delta,$$

where $\delta_j = \delta^{r_j - r_{j-1}}$, $r_j$ is the $j$-th minimal item in $R$, and $r_0 = 0$.

$\#R$ is the actual number of experiments. Everything remains the same in the IC constraints, except for the discount factor of each experiment. Thus, we can represent contracts in a different way: the principal has to incentivize the agent to work for every experiment, but she is free to choose how long to suspend the project between two consecutive experiments, or equivalently, choose the discount factors between consecutive experiments.

From the perspective of the induced actions, we define the following direct contracts. A *direct contract* is a quadruple, $\mathscr{C} = \{N, W, \boldsymbol{b}, \boldsymbol{\delta}\}$, where $N \in \mathbb{N}_0$ is the number of experiments, $W$ is the time-zero transfer from the principal to the agent, $\boldsymbol{b} \in \mathbb{R}^N$ is the vector of bonus scheme for a success that *must* incentivize the agent to work for *all* $N$ experiments, and $\boldsymbol{\delta} \in [0, \delta]^N$ is the vector of discount factors between consecutive experiments, and for $1 \leq n \leq N$, $\delta_n = \delta^k$ for some $k \in \mathbb{N} \cup \{\infty\}$ so that $0 \leq \delta_n \leq \delta$. The principal is free to choose any discount factors and any number of experiments. But given the principal's choice, the bonus payments must satisfy the IC constraints for the agent for all $N$ experiments.

With a direct contract $\mathscr{C} = \{N, W, \boldsymbol{b}, \boldsymbol{\delta}\}$, nothing changes except for discount factors. We can easily rewrite the payoffs of all parties, and their IR and IC constraints by properly adjusting their discount factors. For example, given the agent's belief about the principal's

[26] We use $\#R$ to denote the size of $R$. Other notations are from the section 5.3. The constraints are the agent's IC constraints when the agent believes that the $H$ and $L$ type pool with the prior belief $\beta_0$. The degenerate case $\beta_0 = 0$ (or $\beta_0 = 1$) means the agent believes the principal is the $L$ (or $H$) type. The bonus scheme also needs to satisfy the IC constraints that the agent does not work in the periods $n \in \{1, 2, \cdots, N\} \backslash R$. We omit the requirement of $b_n$ for $n \in \{1, 2, \cdots, N\} \backslash R$, since they are payoff irrelevant in the equilibrium as long as they are low enough to discourage the agent to experiment.

type being $H$ is $\beta_0$, his IC constraints become, for $1 \leq n \leq N$,[27]

$$\sum_{s=n}^{N} \Pi_{j=n+1}^{s} \delta_j f_{s-n}(q_n, \beta_n)(q_s \lambda_s b_s - c)\Delta \geq \sum_{s=n+1}^{N} \Pi_{j=n+1}^{s} \delta_j f_{s-n-1}(q_n, \beta_n)(q_{s-1}\lambda_{s-1} b_s - c)\Delta.$$

Thus, we can use direct contracts to analyze the signaling game and the third-party mechanism design problem. However, Lemma 5 implies that delaying the $H$ type project temporarily never happens in the best equilibrium for the $H$ type. As we will see later, this is also true for the optimal mechanism for the $H$ type.

# B Three Useful Lemmas

We provide three useful lemmas for other proofs. Lemma 5 shows the IC constraints for the agent always bind whenever there are signaling concerns and a temporary delay of the project is never optimal for separation. Lemma 6 characterizes the recursive relations between two consecutive bonuses when the IC constraints for the agent bind. Lemma 7 explores the limit of the payoff difference between the two types of principal from the same contract when the period length shrinks, and shows that it is monotonic in the belief about the type of principal when the termination time is less than the efficient termination time for the $H$ type.

## B.1 Lemma 5

**Lemma 5.** *Fix any $N > 1$, let $\boldsymbol{b}^*, \boldsymbol{\delta}^*$ be a solution to the following program:*

$$\max_{\boldsymbol{b}, \{0 \leq \delta_n \leq \delta\}_{n=1}^{N}} \sum_{n=1}^{N} \Pi_{j=1}^{n} \delta_j \left[ f_{n-1}^{H}(q_0) q_n^H \lambda^H \Delta - f_{n-1}^{L}(q_0) q_n^L \lambda^L \Delta \right] (h - b_n)$$

$$s.t. \sum_{s=n}^{N} \Pi_{j=n+1}^{s} \delta_j f_{s-n}(q_n, \beta_n) \left( q_s \lambda_s b_s - c \right) \Delta \geq \sum_{s=n+1}^{N} \Pi_{j=n+1}^{s} \delta_j f_{s-n-1}(q_n, \beta_n) \left( q_{s-1} \lambda_{s-1} b_s - c \right) \Delta$$

*Let $m^* + 1 = \min\{n : \delta_n^* = 0\}$ or $m^* = N$ if $\{n : \delta_n^* = 0\} = \varnothing$. For any $n \leq m^*$, the above constraints must bind. Moreover, $\boldsymbol{b}^*$ and $\delta_n^* = \delta$ for $n \leq m^*$ also constitutes a solution.*

The constraints are the agent's IC constraints. The objective function is the payoff difference between the $H$ and $L$ type when they propose the same contract. To find the best equilibrium for the $H$ type, the time-zero payment $W$ is pinned down by either a binding IC constraint for the $L$ type as in the BSEH or a binding IR constraint for the agent as in the BPEH. In other words, the $H$ type has no reasons to leave rents to the agent other than prevent the $L$ type mimicking her. If the $L$ type obtains a zero payoff, the objective function equals the $H$ type's payoff. If the agent obtains a zero payoff, the objective function represents the part of the $H$ type's payoff that involves the bonus payments. The lemma shows that for a given termination date, the IC constraints for the agent must bind for all periods even when arbitrary delay of the project is allowed. Moreover, it shows that a temporary delay of the

[27] To include the first and the last terms into the compact form, we use the convention that $\Pi_{j=n+1}^{n} \delta_j = 1$ for any $n$ and $\delta_j$, and $\sum_{s=N+1}^{N} \omega_s = 0$ for any $N$ and $\omega_s$.

project is never optimal for the $H$ type. Furthermore, the result holds with respect to the following transformation of the objective function: $H \longmapsto \xi H + G(\delta_1, \dots, \delta_n)$, where $\xi > 0$ and $G(\delta_1, \dots, \delta_n)$ is linear in every $\delta_n$ and is independent of every $b_n$ for $1 \le n \le N$.

*Proof.* The result is trivial if $m^* = 1$. Consider $m^* > 1$. All terms after $m^*$ in the objective function are zero, we can rewrite the program so that the last term is in the $m^*$ period.

Note that $\eta_n := f^H_{n-1}(q_0)q^H_n\lambda^H - f^L_{n-1}(q_0)q^L_n\lambda^L$, and $\eta_n\Delta$ is the difference of the probabilities to succeed for the $n$-th experiment between the $H$ and $L$ type. It is easy to see that $\eta_n \ge 0$ if $n \le n^* := \lfloor \frac{\log\lambda^H - \log\lambda^L}{\log(1-\lambda^L\Delta) - \log(1-\lambda^H\Delta)} \rfloor + 1$, and $\eta_n < 0$ if $n > n^*$.

Therefore, for $1 \le n \le n^*$, it is optimal to set $b_n$ as small as possible. Since $b_n$ is bounded from below, the IC constraints for $1 \le n \le n^*$ must bind.

Given the binding constraints for $1 \le n \le n^*$, consider $n = n^* + 1$. Note that a change of $b_{n^*+1}$, call it $y_{n^*+1}$, will change all the previous bonuses according to the binding constraints from period one to period $n^*$.

We can show that the change of $b_n$ for $1 \le n \le n^*$, call it $y_n$, from $y_{n^*+1}$ is

$$y_n = \Pi_{j=n+1}^{n^*+1}\delta_j \left[\lambda^H - \left(\lambda^H - \lambda^L\right)\frac{\lambda^L(1-\beta_{n^*})}{\lambda_{n^*}}\right]\Delta y_{n^*+1}.$$

Here $\zeta_{n^*} := \lambda^H - (\lambda^H - \lambda^L)\frac{\lambda^L(1-\beta_{n^*})}{\lambda_{n^*}}$ is a positive constant. It equals $\lambda^H$ if $\beta_0 = 1$ and equals $\lambda^L$ if $\beta_0 = 0$. When $\beta_0 \in (0,1)$, it is in between $\lambda^L$ and $\lambda^H$ and converges to $\lambda^L$ as $n^* \to \infty$.

Thus, the change of the objective function from $y_{n^*+1}$ is

$$\begin{aligned} -\sum_{n=1}^{n^*+1}\Pi_{j=1}^{n}\delta_j\eta_n\Delta y_n &= -\sum_{n=1}^{n^*}\Pi_{j=1}^{n^*+1}\delta_j\eta_n\Delta\zeta_{n^*}\Delta y_{n^*+1} - \Pi_{j=1}^{n^*+1}\delta_j\eta_{n^*+1}\Delta y_{n^*+1}, \\ &= -\Pi_{j=1}^{n^*+1}\delta_j y_{n^*+1}\left(\zeta_{n^*}\Delta\sum_{n=1}^{n^*}\eta_n\Delta + \eta_{n^*+1}\Delta\right). \end{aligned} \tag{9}$$

Note that the distribution on success for $L$ type first-order stochastic dominates that for $H$ type; the $H$ project is more likely to succeed. Formally, for any $s \ge 1$

$$\phi_s := \sum_{n=1}^{s}\eta_n\Delta = -\left(f^H_s(q_0) - f^L_s(q_0)\right) = q_0\left[(1-\lambda^L\Delta)^s - (1-\lambda^H\Delta)^s\right] > 0.$$

Therefore, the terms in the parenthesis of equation (9) becomes

$$\zeta_{n^*}\Delta q_0\left[(1-\lambda^L\Delta)^{n^*} - (1-\lambda^H\Delta)^{n^*}\right] + q_0\left[(1-\lambda^H\Delta)^{n^*}\lambda^H\Delta - (1-\lambda^L\Delta)^{n^*}\lambda^L\Delta\right],$$

which is larger than $q_0(\lambda^H - \lambda^L)\Delta(1-\lambda^H\Delta)^{n^*} > 0$ since $\zeta_{n^*} \ge \lambda^L$.

Therefore, to maximize the objective function, $b_{n^*+1}$ has to be as small as possible. Thus, the IC constraints in period $n^* + 1$ also binds.

By induction, we obtain a necessary condition for the solution to the program: the IC constraints to incentivize agent to work must bind in each period. In addition, it is easy to see that those binding constraints imply that $b_n$ is a linear function of $\delta_{n+1}$, $\delta_{n+2}$, $\cdots$, $\delta_N$.

Thus, we can see that every discount factor enters the objective function linearly. Consider the discount factor in $n+1$ period, $\delta_{n+1}$. It will not affect any $b_s$ for $s \geq n+1$, so $\delta_{n+1}$ enters into the terms linearly beyond period $n+1$ as a discount factor. It will linearly affect every $b_s$ for $s \leq n$, but they are not discounted by $\delta_{n+1}$. Therefore, the discount factor $\delta_{n+1}$ enters objective function linearly.

Thus, there exists an optimal discount factor that is a corner solution, i.e., $\delta_n^* \in \{\delta, 0\}$ for any $1 \leq n \leq m^*$. Since $m^*+1 = \min\{n : \delta_n^* = 0\}$, we have $\delta_n^* = \delta$ for all $1 \leq n \leq m^*$. □

## B.2 Lemma 6

**Lemma 6.** *Given the binding IC constraints for the agent when he has prior $\beta_0$ about the $H$ type, i.e., for $1 \leq n \leq N$,*

$$\sum_{s=n}^{N} \delta^{s-n} f_{s-n}(q_n, \beta_n)(q_s\lambda_s b_s - c)\Delta = \sum_{s=n+1}^{N} \delta^{s-n} f_{s-n-1}(q_n, \beta_n)(q_{s-1}\lambda_{s-1} b_s - c)\Delta.$$

*Then $q_N\lambda_N b_N - c = 0$, and for $1 \leq n \leq N-1$,*

$$q_n\lambda_n b_n - c = \delta(q_n\lambda_n b_{n+1} - c).$$

*Proof.* Immediately, the binding IC constraint for the last period $N$ implies

$$q_N\lambda_N b_N - c = 0.$$

For the IC constraint in period $N-1$, it gives us

$$q_{N-1}\lambda_{N-1} b_{N-1} - c + \delta f_1(q_{N-1}, \beta_{N-1})(q_N\lambda_N b_N - c) = \delta(q_{N-1}\lambda_{N-1} b_N - c).$$

Since the second term on the LHS is zero, it implies

$$q_{N-1}\lambda_{N-1} b_{N-1} - c = \delta(q_{N-1}\lambda_{N-1} b_N - c).$$

Suppose for all $m+1 \leq n \leq N-1$, we have $q_n\lambda_n b_n - c = \delta(q_n\lambda_n b_{n+1} - c)$. Consider the IC constraint in the period $m$,

$$\sum_{s=m}^{N-1} \delta^{s-m} f_{s-m}(q_m, \beta_m)(q_s\lambda_s b_s - c)\Delta = \sum_{s=m+1}^{N} \delta^{s-m} f_{s-m-1}(q_m, \beta_m)(q_{s-1}\lambda_{s-1} b_s - c)\Delta.$$

Note that the LHS has no terms in the period $N$ since $q_N\lambda_N b_N - c = 0$. Relabeling the terms on the LHS gives us

$$\sum_{s=m+1}^{N} \delta^{s-m-1} f_{s-m-1}(q_m, \beta_m)(q_{s-1}\lambda_{s-1} b_{s-1} - c)\Delta = \sum_{s=m+1}^{N} \delta^{s-m} f_{s-m-1}(q_m, \beta_m)(q_{s-1}\lambda_{s-1} b_s - c)\Delta.$$

Since $q_n\lambda_n b_n - c = \delta(q_n\lambda_n b_{n+1} - c)$ for $m+1 \leq n \leq N-1$, it is clear that the recursive relation remains valid for $n = m$, which concludes this lemma. □

The following observations will be useful for other proofs:

1. By iteration, for $1 \leq n \leq N$

$$b_n = \delta^{N-n}\frac{c}{q_N\lambda_N} + (1-\delta)\sum_{s=n}^{N-1}\delta^{s-n}\frac{c}{q_s\lambda_s};$$

2. For $n < N$, $q_n\lambda_n b_n > c$;
3. For $n < N$, $\delta b_{n+1} \leq b_n \leq b_{n+1}$, and both inequalities are strict when $\delta < 1$.

## B.3 Lemma 7

With a slight abuse of notation, we denote $q_t$, $\lambda_t$, $\eta_t$, $\phi_t$, $f_t^\theta(q)$ and $f_t(q,\beta)$, where $\theta \in \Theta$, as the limit of $q_n$, $\lambda_n$, $\eta_n$, $\phi_n$, $f_n^\theta(q)$ and $f_n(q,\beta)$ when $\Delta \to 0$ and $n\Delta \to t$.

**Lemma 7.** *Let $T = N\Delta$. Define*

$$H^{\beta_0}(N) := \sum_{n=1}^{N}\delta^n\eta_n(h-b_n)\Delta,$$

*where $b_n = \delta^{N-n}\frac{c}{q_N\lambda_N} + (1-\delta)\sum_{s=n}^{N-1}\delta^{s-n}\frac{c}{q_s\lambda_s}$. Then*

$$\lim_{\Delta\to 0}H^{\beta_0}(N) = H_0^{\beta_0}(T) := e^{-\rho T}\left(h - \frac{c}{q_T\lambda_T}\right)\phi_T + \rho\int_0^T e^{-\rho t}\left(h - \frac{c}{q_t\lambda_t}\right)\phi_t\,\mathrm{d}t.$$

*In addition, $H_0^{\beta_0}(T)$ is strictly increasing in $\beta_0 \in [0,1]$ for any $T \in (0, T_*^H]$.*

*Proof.* Note that

$$\sum_{n=1}^{N}\delta^n\eta_n h\Delta = \sum_{n=1}^{N}\delta^n h(\phi_n - \phi_{n-1}) = \delta^N h\phi_N + (1-\delta)\sum_{n=1}^{N-1}\delta^n h\phi_n,$$

and by changing the order of summation we have

$$\begin{aligned}\sum_{n=1}^{N}\delta^n\eta_n b_n\Delta &= \delta^N\frac{c}{q_N\lambda_N}\phi_N + (1-\delta)\sum_{n=1}^{N}\sum_{s=n}^{N-1}\delta^s\eta_n\frac{c}{q_s\lambda_s}\Delta,\\ &= \delta^N\frac{c}{q_N\lambda_N}\phi_N + (1-\delta)\sum_{s=1}^{N-1}\delta^s\frac{c}{q_s\lambda_s}\phi_s.\end{aligned}$$

Therefore,

$$H^{\beta_0}(N) = \delta^N\left(h - \frac{c}{q_N\lambda_N}\right)\phi_N + \frac{(1-\delta)}{\Delta}\sum_{n=1}^{N-1}\delta^n\left(h - \frac{c}{q_n\lambda_n}\right)\phi_n\Delta.$$

Let $T = N\Delta$, we have

$$\lim_{\Delta\to 0} \delta^N \left(h - \frac{c}{q_N \lambda_N}\right) \phi_N = e^{-\rho T} \left(h - \frac{c}{q_T \lambda_T}\right) \phi_t \text{ and } \lim_{\Delta\to 0} \frac{1-\delta}{\Delta} = \rho.$$

In addition, we can show that for any $t \in [0, T]$,

$$\left|\delta^n (h - \frac{c}{q_n \lambda_n})\phi_n - e^{-\rho t}(h - \frac{c}{q_t \lambda_t})\phi_t\right| \underset{\Delta\to 0}{=} \mathcal{O}(\Delta),$$

where $t = n\Delta$. Thus,

$$\lim_{\Delta\to 0} \sum_{n=1}^{N-1} \left|\delta^n (h - \frac{c}{q_n \lambda_n})\phi_n - e^{-\rho t}(h - \frac{c}{q_t \lambda_t})\phi_t\right| \Delta = 0.$$

Thus, we have

$$\lim_{\Delta\to 0} \sum_{n=1}^{N-1} \delta^n \left(h - \frac{c}{q_n \lambda_n}\right) \phi_n \Delta = \lim_{\Delta\to 0} \sum_{n=1}^{N-1} e^{-\rho t} \left(h - \frac{c}{q_t \lambda_t}\right) \phi_t \Delta = \int_0^T e^{-\rho t} \left(h - \frac{c}{q_t \lambda_t}\right) \phi_t \, \mathrm{d}t,$$

where the last equality comes from Riemann integral. Therefore, $\lim_{\Delta\to 0} H^{\beta_0}(N) = H_0^{\beta_0}(T)$. Moreover, we have

$$\frac{\mathrm{d}}{\mathrm{d}\beta_0} H_0^{\beta_0}(T) = -\left(e^{-\rho T} \frac{\mathrm{d}\frac{c}{q_T \lambda_T}}{\mathrm{d}\beta_0} \phi_T + \rho \int_0^T e^{-\rho t} \frac{\mathrm{d}\frac{c}{q_t \lambda_t}}{\mathrm{d}\beta_0} \phi_t \, \mathrm{d}t\right).$$

Note that for all $0 \le t \le T_*^H$,

$$\frac{\mathrm{d}q_t \lambda_t}{\mathrm{d}\beta_0} = \frac{f_t^H(q_0) f_t^L(q_0)}{f_t^2(q_0, \beta_0)} \left(q_t^H \lambda^H - q_t^L \lambda^L\right) > 0,$$

since $q_t^H \lambda^H \ge \frac{c}{h} \ge q_t^L \lambda^L$ and at least one inequality is strict. Thus, for all $0 \le t \le T_*^H$, $\frac{\mathrm{d}}{\mathrm{d}\beta_0}\left(\frac{c}{q_t \lambda_t}\right) < 0$. Therefore, $\frac{\mathrm{d}}{\mathrm{d}\beta_0} H_0^{\beta_0}(T) > 0$ for $T \in (0, T_*^H]$ as $\phi_t > 0$ for $t > 0$. □

Actually, we can show that when $\Delta \to 0$, $n\Delta \to t$, and $N\Delta \to T$, $b_n$ converges to

$$b_t = e^{-\rho(T-t)} \frac{c}{q_T \lambda_T} + \rho \int_t^T e^{-\rho(\tau - t)} \frac{c}{q_\tau \lambda_\tau} \, \mathrm{d}\tau,$$

and $H_0^{\beta_0}(T) = \int_0^T e^{-\rho t} \eta_t (h - b_t) \, \mathrm{d}t$. The latter can be seen by differentiating the RHS,

$$\frac{\mathrm{d}}{\mathrm{d}T}\left[\int_0^T e^{-\rho t} \eta_t \left(h - b_t\right) \mathrm{d}t\right] = e^{-\rho T} \eta_T (h - b_T) - \int_0^T e^{-\rho t} \eta_t \frac{\mathrm{d}b_t}{\mathrm{d}T} \, \mathrm{d}t = e^{-\rho T} \frac{\mathrm{d}}{\mathrm{d}T}\left[\left(h - \frac{c}{q_T \lambda_T}\right) \phi_T\right],$$

and applying the fundamental theorem of calculus and integration by parts,

$$\begin{aligned}\int_0^T e^{-\rho t} \eta_t (h - b_t) \, \mathrm{d}t &= \int_0^T e^{-\rho t} \, \mathrm{d}\left(h - \frac{c}{q_t \lambda_t}\right) \phi_t, \\ &= e^{-\rho T} \left(h - \frac{c}{q_T \lambda_T}\right) \phi_T + \rho \int_0^T e^{-\rho t} \left(h - \frac{c}{q_t \lambda_t}\right) \phi_t \, \mathrm{d}t.\end{aligned}$$

Hence, $H_0^{\beta_0}(T) = \int_0^T e^{-\rho t} \eta_t (h - b_t) \, \mathrm{d}t$. Moreover, we have

$$\frac{\mathrm{d}}{\mathrm{d}T} H_0^{\beta_0}(T) = e^{-\rho T} \left[\eta_T h + \phi_T c - c q_0 \frac{(\lambda^H - \lambda^L) \lambda_0 e^{-(\lambda^L + \lambda^H)T}}{q_T \lambda_T (\beta_0 \lambda^H e^{-\lambda^H T} + (1 - \beta_0) \lambda^L e^{-\lambda^L T})}\right].$$

# C Proof of Lemma 1 and Proposition 1

*Proof.* The worst contract $C^{wt} = (N_*^H, W^{wt}, \boldsymbol{b^{wt}})$ for the $L$ type solves Program I. Since $IR_A^H(N_*^H)$ binds,

$$-W = \sum_{n=1}^{N_*^H} \delta^n f_{n-1}^H(q_0) \left( q_n^H \lambda^H b_n - c \right) \Delta,$$

and the objective function in the Program I becomes

$$-\sum_{n=1}^{N_*^H} \delta^n \eta_n (h - b_n)\Delta + V^H(N_*^H).$$

According to Lemma 5, the solution to Program I must have binding constraints in $IC_A^H(N_*^H)$ for any $1 \le n \le N_*^H$, which gives us Lemma 1.

Furthermore, from Lemma 6, $q_{N_*^H}^H \lambda^H b_{N_*^H} - c = 0$, and for $1 \le n < N_*^H$,

$$q_n^H \lambda^H b_n - c = \delta \left( q_n^H \lambda^H b_{n+1} - c \right).$$

Clearly, $0 < b_n \le b_{N_*^H} \le h$, and $q_n^H \lambda^H b_n - c > 0$ for all $n < N_*^H$. It means that the $L$ type can obtain positive payment from the agent before experimentation, i.e., $-W^{wt} \ge 0$, and positive share of profits during experimentation, i.e., $\sum_{n=1}^{N_*^H} \delta^n f_{n-1}^L(q_0) q_n^L \lambda^L (h - b_n)\Delta \ge 0$. Additionally, at least one of the two payoffs is strictly positive. If $-W^{wt} = 0$, then $N_*^H = 1$. Thus, $b_{N_*^H} = c/q_0\lambda^H < h$ and the share of profits during experimentation is strictly positive.

Therefore, by choosing the worst contract for the $L$ type, the $L$ type can obtain a strictly positive payoff, which violates her IC constraint. This gives us Proposition 1. □

# D Proof of Lemma 2

*Proof.* The statement is obviously true for the $L$ type since the maximal expected surplus is zero for the $L$ project. When the agent believes that the principal's type is $L$, he either rejects the contract that gives him a negative payoff, or accepts the contract that gives the $L$ type a negative payoff. In either case, the $L$ type cannot obtain a strictly positive payoff.

Now we consider the $H$ type. We will solve the best contract for the $H$ type if the agent believes that she is a $L$ type but is still willing to accept the contract. The best contract solves the following program:

$$\max_{N,W,\boldsymbol{b}} \Pi^H = -W + \sum_{n=1}^{N} \delta^n f_{n-1}^H(q_0) q_n^H \lambda^H (h - b_n)\Delta$$

$$\text{s.t.} \; \sum_{s=n}^{N} \delta^{s-n} f_{s-n}^L(q_n^L) \left( q_s^L \lambda^L b_s - c \right) \Delta \ge \sum_{s=n+1}^{N} \delta^{s-n} f_{s-n-1}^L(q_n^L) \left( q_{s-1}^L \lambda^L b_s - c \right) \Delta,$$

$$W + \sum_{n=1}^{N} \delta^n f_{n-1}^L(q_0) \left( q_n^L \lambda^L b_n - c \right) \Delta \ge 0.$$

The IR constraint for agent must bind, so the objective function becomes

$$\Pi^H = \sum_{n=1}^{N} \delta^n \eta_n (h - b_n)\Delta + V^L(N).$$

According to Lemma 5 and 6, the IC constraints must bind, and

$$b_n = \delta^{N-n} \frac{c}{q_N^L \lambda^L} + (1-\delta) \sum_{s=n}^{N-1} \delta^{s-n} \frac{c}{q_s^L \lambda^L}.$$

We now show that reducing the number of experiments by one makes the $H$ type better off. Thus, the termination date of the best contract for the $H$ type is zero and her maximal profit is zero. Let $b_n^N$ be the bonus in period $n$, and $\Pi^H(N)$ be the profit of the $H$ type given the termination date is $N$. We can show that for $N \geq 1$

$$b_n^{N+1} - b_n^N = \delta^{N+1-n} \left( \frac{c}{q_{N+1}^L \lambda^L} - \frac{c}{q_N^L \lambda^L} \right).$$

Moreover, because $\frac{c}{q_{N+1}^L \lambda^L} - \frac{c}{q_N^L \lambda^L} \geq 0$, $\phi_N \geq 0$, and $\frac{c}{q_{N+1}^L \lambda^L} \geq \frac{c}{q_0 \lambda^L} \geq h$, we have

$$\Pi^H(N) - \Pi^H(N+1) = \delta^{N+1} \left[ \left( \frac{c}{q_{N+1}^L \lambda^L} - \frac{c}{q_N^L \lambda^L} \right) \phi_N + f_N^H(q_0) q_{N+1}^H \lambda^H \left( \frac{c}{q_{N+1}^L \lambda^L} - h \right) \Delta \right] \geq 0.$$

□

# E Proof of Proposition 2

*Proof.* The $H$ type's equilibrium contract $C^{sep} = \{N^{sep}, W^{sep}, \boldsymbol{b^{sep}}\}$ in the BSEH solves Program II. We now solve the relaxed program without the agent's IR constraint.

First, $IC_L^H(N)$ must bind in the relaxed program, otherwise we can decrease $W$ without violating any other constraint and obtain a higher payoff. The binding $IC_L^H(N)$ determines the time-zero payment $W$. Thus, the objective function becomes

$$\sum_{n=1}^{N} \delta^n \eta_n (h - b_n)\Delta.$$

From Lemma 5, all IC constraints must bind. Furthermore, Lemma 6 gives an explicit characterization of the bonus scheme. In addition, Lemma 7 gives the limit form of the above objective function. Let $\Pi_{sep}^H(T)$ be the limit of the $H$ type's payoff when her termination time $N\Delta$ goes to $T$ and $\Delta$ goes to 0. Thus, we have[28]

$$\Pi_{sep}^H(T) = V_0^H(T) - V_0^L(T) - c\frac{\lambda^H - \lambda^L}{\lambda^H} \left[ (1-q_0) \frac{1 - e^{-(\rho+\lambda^L-\lambda^H)T}}{\rho + \lambda^L - \lambda^H} + q_0 \frac{1 - e^{-(\rho+\lambda^L)T}}{\rho + \lambda^L} \right].$$

[28] We assume that $\rho + \lambda^L - \lambda^H \neq 0$ so that the denominator is not 0. However, when $\rho + \lambda^L - \lambda^H = 0$, we can obtain the same results. The proofs in Section I of the Appendix also apply, where the term $\rho + \lambda^L - \lambda^H$ is in the denominator.

Taking the derivative with respect to $T$, we have

$$\frac{\mathrm{d}}{\mathrm{d}T}\Pi^H_{sep}(T) = \frac{q_0}{\lambda^H}\left(\lambda^H h - c\right)e^{-(\rho+\lambda^L)T}\left[\lambda^H e^{(\lambda^L-\lambda^H)T} - \left(\lambda^H - \lambda^L\right)\frac{l^H}{l_0}e^{\lambda^H T} - \lambda^L\right].$$

Note that $\left[\lambda^H e^{(\lambda^L-\lambda^H)T} - \left(\lambda^H - \lambda^L\right)\frac{l^H}{l_0}e^{\lambda^H T} - \lambda^L\right]$ is strictly decreasing in $T$. At $T = 0$, it is $\frac{q_0}{\lambda^H}\left(\lambda^H h - c\right)\left(\lambda^H - \lambda^L\right)\left(1 - \frac{l^H}{l_0}\right) > 0$, and it goes to negative infinity as $T$ goes to infinity. Thus, there exists a $T^{sep}$ such that

$$\lambda^H e^{-\lambda^H T^{sep}} - \left(\lambda^H - \lambda^L\right)\frac{l^H}{l_0}e^{(\lambda^H-\lambda^L)T^{sep}} - \lambda^L e^{-\lambda^L T^{sep}} = 0.$$

$\Pi^H_{sep}(T)$ is strictly increasing in $T$ when $T < T^{sep}$ and is strictly decreasing in $T$ when $T > T^{sep}$. Thus, it attains its maximum at $T^{sep}$.

Moreover, because $\frac{l^H}{l_0} = e^{-\lambda^H T^H_*}$, we have

$$\lambda^H e^{-\lambda^H T^H_*} - \left(\lambda^H - \lambda^L\right)\frac{l^H}{l_0}e^{(\lambda^H-\lambda^L)T^H_*} - \lambda^L e^{-\lambda^L T^H_*} = \lambda^H\left(e^{-\lambda^H T^H_*} - e^{-\lambda^L T^H_*}\right) < 0.$$

Therefore, $T^{sep} < T^H_*$. There is always under experimentation. In addition, since $T^{sep} < T^H_*$ and $b^{sep}_t$ is increasing, we have $b^{sep}_t \leq b^{sep}_{T^{sep}} < h$. Furthermore, $W^{sep} > 0$ since all $b^{sep}_t < h$.

In addition, we can show that $T^{sep}$ is decreasing in $\lambda^L$. When $\lambda^L = 0$, $T^{sep} = T^H_*/2$ and $W^{sep} = 0$. Therefore, $T^{sep} < T^H_*/2$ when $\lambda^L > 0$.

Last, we check the agent's IR constraint. His payoff $V^H_0(T^{sep}) - \Pi^H_{sep}(T^{sep})$ equals

$$\begin{aligned}
& V^L_0(T^{sep}) + c\frac{\lambda^H - \lambda^L}{\lambda^H}\left[(1-q_0)\frac{1 - e^{-(\rho+\lambda^L-\lambda^H)T^{sep}}}{\rho + \lambda^L - \lambda^H} + q_0\frac{1 - e^{-(\rho+\lambda^L)T^{sep}}}{\rho + \lambda^L}\right] \\
= \quad & \frac{\lambda^L}{\lambda^H}V^H_0(T^{sep}) + c\frac{\lambda^H - \lambda^L}{\lambda^H}(1-q_0)\left[\frac{1 - e^{-(\rho+\lambda^L-\lambda^H)T^{sep}}}{\rho + \lambda^L - \lambda^H} - \frac{1 - e^{-\rho T^{sep}}}{\rho}\right] \\
& + \frac{\lambda^L}{\lambda^H}q_0\left(\lambda^H h - c\right)\left[\frac{1 - e^{-(\rho+\lambda^L)T^{sep}}}{\rho + \lambda^L} - \frac{1 - e^{-(\rho+\lambda^H)T^{sep}}}{\rho + \lambda^H}\right] > 0.
\end{aligned}$$

All terms are strictly positive. The first term is strictly positive since $0 < T^{sep} < T^H_*$. The other two terms are strictly positive since $\frac{1-e^{-xt}}{x}$ is strictly decreasing in $x$ when $t > 0$. Therefore, the solution we find is the $H$ type's equilibrium contract in the BSEH. In such an equilibrium, the agent obtains a payoff more than $\frac{\lambda^L}{\lambda^H}V^H_0(T^{sep})$, and the $H$ type principal obtains a payoff less than $\left(1 - \frac{\lambda^L}{\lambda^H}\right)V^H_0(T^{sep})$. □

# F Proof of Proposition 3

*Proof.* The equilibrium contract $C^{pl} = \{N^{pl}, W^{pl}, \boldsymbol{b^{pl}}\}$ in the BPEH solves Program III. We now solve the relaxed program without $IR_L(N)$. In the relaxed program, $IR_A(N)$ must bind, otherwise decreasing $W$ increases the objective function without violating any constraints.

The binding $IR_A(N)$ determines the time-zero payment $W$. Taking it into the objective function, the $H$ type's payoff becomes

$$(1-\beta_0)\sum_{n=1}^{N}\delta^n\eta_n(h-b_n)\Delta+V(N).$$

From Lemma 5, all IC constraints should bind. Furthermore, Lemma 6 gives an explicit characterization of the bonus scheme. In addition, Lemma 7 gives the limit form of the first part of the objective function. Let $\Pi^H_{pl}(T)$ denote the limit of the $H$ type's payoff when her termination time $N\Delta$ goes to $T$ and $\Delta$ goes to 0. Then, we have

$$\Pi^H_{pl}(T)=(1-\beta_0)H_0^{\beta_0}(T)+\beta_0V_0^H(T)+(1-\beta_0)V_0^L(T).$$

Therefore, taking the derivative with respect to $T$, $\frac{\mathrm{d}}{\mathrm{d}T}\Pi^H_{pl}(T)$ equals

$$e^{-\rho T}\left[-(1-\beta_0)q_0c\frac{\left(\lambda^H-\lambda^L\right)\lambda_0e^{-(\lambda^L+\lambda^H)T}}{q_T\lambda_T\left(\beta_0\lambda^He^{-\lambda^HT}+(1-\beta_0)\lambda^Le^{-\lambda^LT}\right)}+q_0\left(\lambda^Hh-c\right)e^{-\lambda^HT}-(1-q_0)c\right].$$

Thus, $\frac{\mathrm{d}}{\mathrm{d}T}\Pi^H_{pl}(T)>0$ is equivalent to

$$q_0\left(\lambda^Hh-c\right)>(1-\beta_0)q_0c\frac{\left(\lambda^H-\lambda^L\right)\lambda_0}{q_T\lambda_T\left(\beta_0\lambda^He^{(\lambda^L-\lambda^H)T}+(1-\beta_0)\lambda^L\right)}+(1-q_0)ce^{\lambda^HT}.$$

Note that the RHS of above inequality is strictly increasing in $T$ and goes to infinity as $T$ goes to infinity. In addition, at $T=0$, the RHS equals

$$\begin{aligned}(1-\beta_0)q_0\left(\lambda^H-\lambda^L\right)\frac{c}{q_0\lambda_0}+(1-q_0)c&<(1-\beta_0)q_0\left(\lambda^H-\lambda^L\right)h+(1-q_0)c,\\&=c-q_0\lambda_0h+q_0\left(\lambda^Hh-c\right)<q_0\left(\lambda^Hh-c\right),\end{aligned}$$

where both inequities come from the fact that $q_0\lambda_0h>c$. Hence, there exists a $T^{pl}>0$, such that $\frac{\mathrm{d}}{\mathrm{d}T}\Pi^H_{pl}(T)|_{T=T^{pl}}=0$. $\Pi^H_{pl}(T)$ is strictly increasing when $T<T^{pl}$ and is strictly decreasing when $T>T^{pl}$. $T^{pl}$ maximizes $\Pi^H_{pl}(T)$.

To show $T^{pl}<T_*$, we show that $\frac{\mathrm{d}}{\mathrm{d}T}\Pi^H_{pl}(T)|_{T=T_*}<0$. Since $q_{T_*}\lambda_{T_*}h=c$, we have

$$\beta_0\frac{l_0}{l^H}e^{-\lambda^HT_*}+(1-\beta_0)\frac{l_0}{l^L}e^{-\lambda^LT_*}=1.$$

Thus, $\frac{\mathrm{d}}{\mathrm{d}T}\Pi^H_{pl}(T)|_{T=T_*}$ is equal to

$$e^{-\rho T}(1-\beta_0)q_0\left(e^{-\lambda^HT_*}-e^{-\lambda^LT_*}\right)\frac{\beta_0\left(\lambda^H-\lambda^L\right)\left(\lambda^Hh-c\right)e^{-\lambda^HT_*}+\lambda^L\frac{c}{l_0}}{\beta_0\lambda^He^{-\lambda^HT_*}+(1-\beta_0)\lambda^Le^{-\lambda^LT_*}}<0.$$

Furthermore, $T^{pl}<T_*$ implies $b_{T^{pl}}<h$, because $q_{T^{pl}}\lambda_{T^{pl}}b_{T^{pl}}=q_{T_*}\lambda_{T_*}h=c$.

It is easy to see that $W^{pl} < 0$. Note that the agent's expected payoff is zero, i.e.,

$$W^{pl} + \int_0^{T^{pl}} e^{-\rho t} f_t(q_0, \beta_0)(q_t \lambda_t b_t - c)\, \mathrm{d}t = 0,$$

and $q_t \lambda_t b_t - c > 0$ for all $t < T^{pl}$. Obviously, $W^{pl} < 0$. This means the $L$ type must obtain a strictly positive payoff in the equilibrium, because her payoff before experimentation is strictly positive ($W^{pl} < 0$) and her payoff during the experimentation is also strictly positive as $b_t < h$ for all $t$. Thus, the $L$ type's IR constraint is satisfied and the contract we solved is indeed the best pooling equilibrium for the $H$ type.

At last, we now show that $\frac{\mathrm{d}}{\mathrm{d}\beta_0}\Pi^H_{pl}(T^{pl}) > 0$. Note that

$$\frac{\partial}{\partial \beta_0}\Pi^H_{pl}(T) = (1-\beta_0)\frac{\mathrm{d}}{\mathrm{d}\beta_0} H_0^{\beta_0}(T) + V_0^H(T) - V_0^L(T) - H_0^{\beta_0}(T).$$

From Lemma 7, for $0 < T \le T^H_*$, $(1-\beta_0)\frac{\mathrm{d}}{\mathrm{d}\beta_0} H_0^{\beta_0}(T) > 0$, and

$$\begin{aligned} V_0^H(T) - V_0^L(T) - H_0^{\beta_0}(T) &= \int_0^T e^{-\rho t}(\eta_t b_t + \phi_t c)\, \mathrm{d}t, \\ &= e^{-\rho T}\frac{c}{q_T \lambda_T}\phi_T + \rho \int_0^T e^{-\rho t}\frac{c}{q_t \lambda_t}\phi_t \, \mathrm{d}t + \int_0^T e^{-\rho t}\phi_t c \, \mathrm{d}t > 0. \end{aligned}$$

Therefore, $\frac{\partial}{\partial \beta_0}\Pi^H_{pl}(T) > 0$ for $T \in (0, T^H_*]$. The Envelope Theorem implies $\frac{\mathrm{d}}{\mathrm{d}\beta_0}\Pi^H_{pl}(T^{pl}) = \frac{\partial}{\partial \beta_0}\Pi^H_{pl}(T^{pl}) > 0$ since $0 < T^{pl} < T_* < T^H_*$. Thus, $T^{pl} \to T^H_*$ and $\Pi^H_{pl}(T^{pl}) \to V_0^H(T^H_*)$ as $\beta_0 \to 1$. □

# G Proof of Lemma 3

*Proof.* Fix any equilibrium $E$ that gives the $L$ type a strictly positive payoff. Let $CL$ and $CH$ be the sets of equilibrium contracts chosen by the $L$ and $H$ type, respectively. Then $CL \subseteq CH$. In addition, the $L$ (or $H$) type must obtain the same payoff by proposing any $C \in CL$ (or $C \in CH$).

For any contract $C \in CL$, let $x^C \in (0, 1]$ and $y^C \in (0, 1]$ be the probabilities that $C$ is chosen by the $L$ and $H$ type in the equilibrium, respectively.[29] Then $\sum_{C \in CL} y^C \le \sum_{C \in CL} x^C = 1$.

Thus, there are only two possible cases.

1. $CL = CH$, and $x^C = y^C$ for all $C \in CL = CH$;
2. There is at least one contract $C$, such that $x^C > y^C$.

In the first case, for any $C \in CL = CH$, when the principal proposes the contract $C$, the agent's belief about the principal's type is just the prior $\beta_0$. In addition, given this belief

[29]For technical simplicity, we assume that every contract over which one principal randomizes, she chooses that contract with a strictly positive probability.

and the contract $C$, we know that the $H$'s payoff in such an equilibrium cannot exceed the payoff she can obtain in the BPEH when the prior is $\beta_0$, by definition.

In the second case, for the contract $C$ such that $x^C > y^C$, the agent's belief about the principal' type $\beta$ is strictly below the prior $\beta_0$ by Bayesian rule. Given such belief and the contract $C$, the $H$ type's payoff cannot exceed the payoff she can obtain in the BPEH when the prior is $\beta$, by definition. From Proposition 3, the $H$ type's payoff in the BPEH is increasing in the prior. Thus, we can conclude that the $H$ type's payoff cannot exceed the payoff she can obtain in the BPEH when the prior is $\beta_0$. □

# H Proof of Lemma 4

*Proof.* For the equilibrium in which the $L$ type obtains zero payoff, the best contract for the $H$ type $\hat{C} = \{\hat{N}, \hat{W}, \hat{\boldsymbol{b}}\}$, given the agent's belief about the $H$ type is $\beta_0$, solves the following program:

$$\max_{N,W,\boldsymbol{b}} \Pi^H = -W + \sum_{n=1}^{N} \delta^n f_{n-1}^H(q_0) q_n^H \lambda^H (h - b_n)\Delta$$
$$\text{s.t. } IC_A(N) \text{ , } IR_A(N) \text{ , } IC_L^H(N).$$

As in the BPEH, when $\beta_0$ is small ($\frac{c}{q_0\lambda_0} \geq h$), the optimal termination time is zero and the payoff of the $H$ type is zero. Now we consider the non-trivial case when $\frac{c}{q_0\lambda_0} < h$.

We again drop the agent's IR constraint, solve the relaxed program, and then verify it later. However, for the purpose to prove this lemma, we do not have to. The reason is that the solution to the relaxed program cannot be smaller than that to the original program, and observe that the relaxed program is the same program solves the BSEH if $\beta_0 = 1$. All we need to show is the value function for this relaxed program is increasing in $\beta_0$.

Consider the relaxed program without $IR_A(N)$. Clearly, $IC_L^H(N)$ must bind. Solving for $W$, the objective function becomes

$$\sum_{n=1}^{N} \delta^n \eta_n (h - b_n)\Delta.$$

By Lemma 5 and 6, the agent's IC constraints $IC_A(N)$ must bind. Let $\hat{\Pi}^H(T)$ denote the $H$ type's payoff limit as $N\Delta$ goes to $T$ and $\Delta$ goes to zero. From Lemma 7,

$$\hat{\Pi}^H(T) = e^{-\rho T}\phi_T \left(h - \frac{c}{q_T\lambda_T}\right) + \rho \int_0^T e^{-\rho t}\phi_t \left(h - \frac{c}{q_t\lambda_t}\right) \mathrm{d}t.$$

For any $T > T^*$, since $q_T\lambda_T h < q_{T_*}\lambda_{T_*} h = c$, we have

$$\hat{\Pi}^H(T^*) - \hat{\Pi}^H(T) = -e^{-\rho T}\phi_T \left(h - \frac{c}{q_T\lambda_T}\right) - \rho \int_{T^*}^T e^{-\rho t}\phi_t \left(h - \frac{c}{q_t\lambda_t}\right) \mathrm{d}t > 0.$$

Hence, the principal never wants to extend the project beyond the efficient termination time of the mixed project – we can focus on solutions in the compact set $[0, T_*]$. The objective function is continuous, so there exists a solution $\hat{T} \in [0, T_*]$ to the relaxed program. We can further show that $\hat{T} \in (0, T_*)$ by showing $\frac{\mathrm{d}}{\mathrm{d}T}\hat{\Pi}^H(T)|_{T=0} > 0$ and $\frac{\mathrm{d}}{\mathrm{d}T}\hat{\Pi}^H(T)|_{T=T_*} < 0$.

Lemma 7 also shows $\frac{\partial}{\partial \beta_0}\hat{\Pi}^H(T) > 0$ for $0 < T \leq T_*^H$. Thus, the Envelope Theorem implies $\frac{\mathrm{d}}{\mathrm{d}\beta_0}\hat{\Pi}^H(\hat{T}) = \frac{\partial}{\partial \beta_0}\hat{\Pi}^H(\hat{T}) > 0$, because $0 < \hat{T} < T_* \leq T_*^H$.

Hence, the value function of the relaxed program is zero for $\frac{c}{q_0 \lambda_0} \leq h$, and is strictly increasing in $\beta_0$ for $\frac{c}{q_0 \lambda_0} > h$. Note that when $\beta_0 = 1$, it is the equilibrium payoff for the $H$ type in the BSEH. Therefore, all equilibria that give the $L$ type a zero payoff cannot give the $H$ type a higher payoff than the BSEH. □

# I Proof of Proposition 5

In this section, we solve the optimal mechanism for the $H$ type when $\Delta$ is small. First, we restrict our attention to feasible mechanisms when $N^L = 0$. Then, we focus on feasible mechanisms when $N^L \geq 1$. Last, we compare them when $\Delta$ is small.

*Proof.* (a) We now show that the $H$ type's payoff is strictly lower than her FIB payoff when $N^L = 0$. For this purpose, it is not necessary to characterize the optimal mechanism for the $H$ type when $N^L = 0$, but we nevertheless solve it to illustrate how much the mediator can help the $H$ type by solely relaxing the IR constraint of the agent.

The optimal mechanism for the $H$ type when $N^L = 0$, $\mathcal{M}^0 = \{\mathcal{C}^{H0}, \mathcal{C}^{L0}\}$, solves the following program:

$$\max_{\mathcal{M}} \Pi^H(\mathcal{M}) = -W^H + q_0 \sum_{n=1}^{N^H} \delta^n \left(1 - \lambda^H \Delta\right)^{n-1} \lambda^H \left(h - b_n^H\right) \Delta$$
$$\text{s.t. } IR_H(\mathcal{M}) \text{ , } IR_L(\mathcal{M}) \text{ , } IR_A(\mathcal{M}) \text{ , } IC_H^L(\mathcal{M}) \text{ , } IC_L^H(\mathcal{M}) \text{ , } IC_A(\mathcal{M}) \text{ , and } N^L = 0$$

The constraints $IR_H(\mathcal{M})$, $IR_L(\mathcal{M})$, and $IC_H^L(\mathcal{M})$ are all slack. We solve the relaxed program without them. In the relaxed program, $IC_L^H(\mathcal{M})$ and $IR_A(\mathcal{M})$ must bind, otherwise we can adjust $W^H$ and $W^L$ to increase the payoff of the $H$ type.

From the binding $IC_L^H(\mathcal{M})$ and $IR_A(\mathcal{M})$, we can obtain $W^H$ and $W^L$, which also pins down

$$\begin{aligned}
\Pi^H =& \beta_0 V^H(N^H) + (1 - \beta_0) \sum_{n=1}^{N^H} \delta^n \eta_n \left(h - b_n^H\right) \Delta, \\
\Pi^L =& \beta_0 V^H(N^H) - \beta_0 \sum_{n=1}^{N^H} \delta^n \eta_n \left(h - b_n^H\right) \Delta.
\end{aligned}$$

For any given $N^H$, according to Lemma 5 and 6, the agent's IC constraints in $IC_A(\mathcal{M})$ must also bind, and we can obtain $b_n^H$ for $1 \leq n \leq N^H$.

Let $\Pi_0^\theta(T)$ denote the limit of the $\theta$ type's payoff when $\Delta$ goes to zero and the termination time of the $H$ type $N^H \Delta$ goes to $T$, where the subscript 0 is reminiscent of the $L$ type's experimenting time. Thus, we have

$$\Pi_0^H(T) = V_0^H(T) - (1-\beta_0)V_0^L(T) - (1-\beta_0)c\frac{\lambda^H - \lambda^L}{\lambda^H}\left[(1-q_0)\frac{1-e^{-(\rho+\lambda^L-\lambda^H)T}}{\rho+\lambda^L-\lambda^H} + q_0\frac{1-e^{-(\rho+\lambda^L)T}}{\rho+\lambda^L}\right],$$

$$\Pi_0^L(T) = \beta_0 V_0^L(T) + \beta_0 c\frac{\lambda^H - \lambda^L}{\lambda^H}\left[(1-q_0)\frac{1-e^{-(\rho+\lambda^L-\lambda^H)T}}{\rho+\lambda^L-\lambda^H} + q_0\frac{1-e^{-(\rho+\lambda^L)T}}{\rho+\lambda^L}\right].$$

The last thing is to find the optimal termination time for the $H$ type. Taking the derivative with respect to $T$, we have

$$\frac{\mathrm{d}}{\mathrm{d}T}\Pi_0^H(T) = (1-\beta_0)\frac{q_0}{\lambda^H}\left(\lambda^H h - c\right)e^{-(\rho+\lambda^L)T}\alpha(T),$$

where $\alpha(T)$ is strictly decreasing in $T$ and is defined by

$$\alpha(T) := \frac{1}{1-\beta_0}\lambda^H e^{(\lambda^L-\lambda^H)T} - \frac{\beta_0}{1-\beta_0}\lambda^H\frac{l^H}{l_0}e^{\lambda^L T} - \left(\lambda^H - \lambda^L\right)\frac{l^H}{l_0}e^{\lambda^H T} - \lambda^L.$$

First, $\alpha(0) = \left(\frac{1}{1-\beta_0}\lambda^H - \lambda^L\right)\left(1 - \frac{l^H}{l_0}\right) > 0$. Moreover, $\alpha(T)$ converges to $-\lambda^L$ as $T \to \infty$. Thus, $\Pi_0^H(T)$ is quasi-concave in $T$, and $\alpha(T^{H0}) = 0$ determines the unique solution $T^{H0}$ to the relaxed program. We can also show that $\alpha(T^{sep}) > 0 > \alpha(T_*^H)$ so that $T^{sep} < T^{H0} < T_*^H$.

We now verify the constraints we have dropped from the original program. $IR_H(\mathcal{M})$ is automatically satisfied. Now we check $IR_L(\mathcal{M})$. Note that

$$\frac{\mathrm{d}}{\mathrm{d}T}\Pi_0^L(T) = \beta_0 q_0\left(\lambda^H h - c\right)e^{-\rho T}\left[\frac{\lambda^L}{\lambda^H}e^{-\lambda^L T}\left(1 - \frac{l^H}{l_0}e^{\lambda^H T}\right) + \frac{l^H}{l_0}\left(e^{(\lambda^H-\lambda^L)T} - 1\right)\right].$$

When $T < T_*^H$, $e^{\lambda^H T} < e^{\lambda^H T_*^H} = \frac{l_0}{l^H}$, and thus $1 - \frac{l^H}{l_0}e^{\lambda^H T} > 0$. We also have $e^{(\lambda^H-\lambda^L)T} - 1 > 0$ for $T > 0$. Thus, $\frac{\mathrm{d}}{\mathrm{d}T}\Pi_0^L(T) > 0$ for $T \in [0, T_*^H]$, i.e., $\Pi_0^L(T)$ is strictly increasing in $T$. Hence, $\Pi_0^L(T^{H0}) > \Pi_0^L(0) = 0$ and $IR_L(\mathcal{M})$ is slack.

Finally, we show $IC_H^L(\mathcal{M})$ is also slack. By deviating to $\mathcal{C}^{L0}$, the $H$ type obtains $\Pi_0^L(T^{H0})$. Because $b_t^H < h$, we have

$$\Pi_0^L(T^{H0}) - \Pi_0^H(T^{H0}) = -\int_0^{T^{H0}} e^{-\rho t}\eta_t\left(h - b_t^H\right)\mathrm{d}t < 0.$$

Therefore, the solution to the relaxed program is the optimal mechanism for the $H$ type when $N^L = 0$. Moreover, $\Pi_0^L(T^{H0}) > 0$ implies $\Pi_0^H(T^{H0}) < V_0^H(T^{H0}) < V_0^H(T_*^H)$. Hence, the $H$ type's payoff is strictly less than her FIB payoff in the limit $\Delta \to 0$.

(b) Now we consider the optimal mechanism for the $H$ type when $N^L \geq 1$. The inequality (4) shows, for a small $\Delta$, the $H$ type's payoff has an upper bound $V^H(N_*^H) + \frac{1-\beta_0}{\beta_0}V^L(1)$ when $N^L \geq 1$. We show the following mechanism $\mathcal{M}^1$ is feasible and achieves this upper bound.

**Recommendations** – The mediator recommends the agent work until $N_*^H$ if the principal reports $H$, but work once if the principal reports $L$.

**Bonus schemes** – Let $\{b_n^H\}_{2\leq n\leq N_*^H}$ be the bonus transfers that satisfy the agent's binding IC constraints given the agent believes the principal's type is $H$.[30] For $2 \leq n \leq N_*^H$,

$$b_n^H = \delta^{N_*^H - n}\frac{c}{q_{N_*^H}^H \lambda^H} + (1-\delta)\sum_{s=n}^{N_*^H - 1}\delta^{s-n}\frac{c}{q_s^H\lambda^H}.$$

Given $\{b_n^H\}_{2\leq n\leq N_*^H}$, the inequality (8) determines an upper bound for $b_1^H$. We choose some $b_1^H$ which is below the upper bound. Fix the chosen $\{b_n^H\}_{1\leq n\leq N_*^H}$, the inequality (7) determines a lower bound for $b_1^L$. We choose some $b_1^L$ which is above both the lower bound and $h$. Therefore, the above bonus scheme satisfies both the agent's and the $L$ type's IC constraints. Moreover, because $b_1^L \geq h$, if the $H$ type reports she is $L$, she will obtain a negative payoff:

$$-W^L + \delta q_0 \lambda^H \left(h - b_1^L\right)\Delta = \delta q_0 \left(\lambda^H - \lambda^L\right)\left(h - b_1^L\right)\Delta \leq 0.$$

Thus, the above bonus scheme also satisfies the IC constraint of the $H$ type.

**Independent transfers** – Given the above bonus schemes, we choose $W^L$ and $W^H$ that satisfy equation (5) and (6) to make the $L$ type and the agent obtain a zero payoff. Last, the $H$ type's IR constraint is automatically satisfied for a small $\Delta$.

Therefore, $\mathcal{M}^1$ is feasible. In addition, the $H$ type achieves her payoff upper bound $V^H(N_*^H) + \frac{1-\beta_0}{\beta_0}V^L(1)$ from $\mathcal{M}^1$, because all surplus is retained by the $H$ type. When $\Delta \to 0$, her payoff converges to $V_0^H(T_*^H)$, since $\lim_{\Delta\to 0} V^L(1) = 0$.

(c) We have shown that in the limit $\Delta \to 0$, the $H$ type's payoff in $\mathcal{M}^0$ is strictly below $V_0^H(T_*^H)$, while her payoff in $\mathcal{M}^1$ converges to $V_0^H(T_*^H)$. Therefore, when $\Delta$ is small enough, $\mathcal{M}^1$ is the optimal mechanism for the $H$ type. $\square$

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[30] Whether the IC constraints for the agent bind or not does not affect the result here. However, binding IC constraints are robust in the sense that they also provide the proper incentives for the agent to truthfully report a success when a success cannot be observed by the principal.

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