Correlated Persuasion and Information Leakage

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Very preliminary, please do not circulate

Abstract

To investigate the implications of information externality in information disclosure, we analyze a persuasion game with correlation. Each sender is endowed with a proposal with uncertain quality. She persuades her matched receiver to adopt the proposal by designing a signal (structure) and revealing its signal realization. The receiver adopts the proposal if and only if her posterior belief about the proposal is sufficiently favorable. There is no direct payoff dependence between the two sender-receiver pairs. However, the signal revealed by a sender is observable not only to her matched receiver, but also to the receiver of the other pair. At first blush, the strength of the correlation captures the severity of the information leakage, and consequently, while the receiver always benefits from a stronger correlation, the opposite is true for the senders. We show, however, that information leakage affects the outcome through two opposite forces. The higher the correlation, the harder it becomes for a sender to offset negative information regarding the other sender. However, the higher the correlation, conditional on the other sender's information, is at least mildly positive, the easier it is for a sender to persuade with only mildly positive information. While the former force dominates the latter when the correlation is weak, the latter dominates as the correlation becomes stronger. Therefore, the overall effect is non-monotone in the degree of correlation.

1 Introduction

Disclosure is a setting in which information externality can naturally arise. Entrepreneurs pursuing similar projects in the same industry design market studies or surveys with the objective of appealing to distinct groups of potential investors. Pharmaceutical companies with newly developed drugs adopting similar physiological mechanisms design their respective clinical trials and tests with the objective of gaining approval from the FDA.

In these instances, while senders do not directly compete with each other, the correlation of the state variables of different sender-receiver pairs implies that a receiver can learn payoff-relevant

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information from the disclosure of other senders, and this spillover establishes an informational linkage across different persuasion activities.

While the importance of information externality has been recognized and investigated in applications such as asset trading (Asriyan et al. (2017)), search (Caplin and Leahy (1998); Au (2019)), bank runs (Chen (1999)), and exploration for natural resources (Hendricks and Kovenock (1989)), the the implications of information externality/leakage in information disclosure are underexplored. This paper fills this gap in the literature.

To this end, we develop a model of correlated persuasion involving two sender-receiver pairs. Each sender is endowed with a proposal with uncertain quality. She persuades her matched receiver to adopt the proposal by designing a signal (structure) and revealing its signal realization. The receiver adopts the proposal if and only if her posterior belief about the proposal is sufficiently favorable. There is no direct payoff dependence between the two sender-receiver pairs. The linkage of the pairs is only through information: the qualities of the two senders are drawn from a joint distribution with a positive correlation. Moreover, the signal revealed by a sender is observable not only to her matched receiver, but also to the receiver of the other pair. In our model, the positive correlation captures the severity of the information leakage.

Our main findings are as follows. At first blush, the strength of the correlation captures the severity of the information leakage, and consequently, while the receiver always benefits from a stronger correlation, the opposite is true for the senders. We show, however, that information leakage affects the outcome through two opposite forces. The higher the correlation, the harder it becomes for a sender to offset negative information regarding the other sender. However, the higher the correlation, conditional on the other sender's information, is at least mildly positive, the easier it is for a sender to persuade with only mildly positive information. While the former force dominates the latter when the correlation is weak, the latter dominates as the correlation becomes stronger.

The intuition is driven primarily by the information spillover arising in correlated persuasion setting. Specifically, a sender can benefit from the fellow sender's good signal realization, but can also suffer a loss due to the fellow sender's bad signal realization. At low correlation, it is not likely that a sender's own good signal realization will come with a good signal realization by the fellow sender, so the aforementioned benefit is weak. As a result, each sender responds by a more revealing signal to counteract the information content of the fellow sender's possible bad signal realization. In contrast, at a high correlation, it is getting more likely that a sender's own good signal will come with a good signal realization by the fellow sender, making it easier for the sender to exploit its benefit. As a result, each sender responds by a less revealing signal, knowing that a mildly positive signal realization (when combined with the likely positive signal realization from the fellow sender) is able to persuade her own receiver to take the desired action. Less revealing signals would emerge in equilibrium.

2 Model

There are two senders, labeled as $i = 1, 2$. Each sender is endowed with a proposal with binary quality $U_i \in \{u_l, u_h\}$ with $u_h > u_l$. Assume the qualities are correlated with the joint distribution tabulated as follows.

	$U_2 = u_l$	$U_2 = u_h$
$U_1 = u_l$	$(1-\mu)^2 + \rho$	$\mu(1-\mu) - \rho$
$U_1 = u_h$	$\mu(1-\mu) - \rho$	$\mu^2 + \rho$

where $\mu < 1/2$ and $\rho \in [0, \mu(1-\mu)]$. Here, U_1 and U_2 share the same marginal distribution with $\Pr(U_i = u_h) = \mu$. The parameter ρ characterizes the degree of correlation between the two proposals: a higher value of ρ indicates stronger (positive) correlation between the proposal qualities.

There are two receivers. Receiver i is paired with sender i and decides whether to adopt the proposal by sender i. Being an expected-payoff maximizer, receiver i chooses adoption if and only if the belief that sender i's proposal has high quality, i.e., $U_i = u_h$, is sufficiently high. To lessen the burden of notations without losing the ability to illustrate the main insight, we suppose that receiver i adopts sender i's proposal if and only if the posterior belief that $U_i = u_h$ is no less than $1/2$. The objective of a sender is simply to maximize the probability that the paired receiver adopts his or her proposal.

The game is as follows. Without any prior knowledge of U_i, sender i chooses a disclosure mechanism/signal structure which consists of a message space M_i and a conditional distribution $\Phi_i : \{u_l, u_h\} \times M_i \to [0, 1]$. The two senders choose their disclosure mechanisms simultaneously. After observing the disclosure mechanisms and realized messages of both senders, each receiver decides whether to adopt the proposal of the paired sender.

The information disclosure mechanism on U_i induces a distribution of marginal distributions over U_i. We will refer to the marginal distribution over U_i conditional on a message realization $m_i \in M_i$, as sender i's signal realization, and denote it generically by $p_i = \Pr(U_i = u_h | m_i)$. By Kamenica and Gentzkow (2011), it is without loss of generality to focus on the game of information disclosure played between the senders, in which the set of pure strategies of sender i consists of all Bayes-plausible (marginal) distributions over signal realizations.(1) In this reduced game, given a pair of signal realizations (p_1, p_2), the receiver adopts sender i's proposal if $\Pr(U_i = u_h | p_1, p_2) \geq 1/2$. Given the symmetric nature of the game, we will focus on symmetric equilibria in what follows.

(1) A distribution G_i over signal realization is Bayes-plausible if and only if $\int p_i dG_i(p_i) = \mu$.

3 Preliminary Observation

3.1 Threshold signals:

As derived in Au and Kawai (2021), conditional on that a pair of realized signal being (p_1, p_2), we have

$$\Pr(U_1 = u_h | p_1, p_2) = \frac{p_1}{1-p_1}\frac{1-\mu}{\mu}\left(\frac{p_1}{1-p_1}\frac{1-\mu}{\mu} + \frac{((1-\mu)^2+\rho)\frac{1-p_2}{p_2}\frac{\mu}{1-\mu} + (\mu(1-\mu)-\rho)}{(\mu(1-\mu)-\rho)\frac{1-p_2}{p_2}\frac{\mu}{1-\mu} + (\mu^2+\rho)}\right)^{-1}.$$

Therefore, given sender 1's signal, receiver 1 will adopt sender 1's proposal if and only if[(2)]

$$p_2 \geq \tau(p_1) = \mu - \frac{\mu^2(1-\mu)^2(2p_1-1)}{\rho\left(p_1(1-\mu)+\mu(1-p_1)\right)}. \tag{1}$$

We discuss a few properties that τ satisfies. Firstly, τ is decreasing and convex. To see this, observe that when sender 1's signal marginal improves from p_1 to $p_1 + \Delta p_1$, the lower bound of the signal of sender 2 required to persuade receiver 1 goes down from $\tau(p_1)$ to $\tau(p_1 + \Delta p_1) < \tau(p_1)$. The corresponding change in $\Delta p_2(p_1) \equiv \tau(p_1 + \Delta p_1) - \tau(p_1)$ depends on the value of sender 1's signal. If p_1 is sufficiently small, i.e., sender 1's signal is bad, then a small improvement in sender 1's signal significantly reduces the amount of good information contain in sender 2's signal, i.e., $|\Delta p_2(p_1)|$ is large. In contrast, if sender 1's signal is already good so that p_1 is large, then then a small improvement sender 1's signal does not reduce the amount of good information contained in sender 2's signal much, i.e., $|\Delta p_2(p_1)|$ is small. That is, τ is convex.

When sender 1's signal is extremely bad, i.e., $p_1 = 0$, then regardless of the signal of sender 2, receiver 1 never adopts sender 1's proposal, i.e., $\tau(0) > 1$. In contrast, if sender 1's signal is sufficiently good, then regardless of the signal of sender 2, receiver 1 always adopts sender 1's proposal, i.e., $\tau(1) < 0$.

Additionally, as the correlation ρ goes up, for a given sender 1's signal p_1, whether the lowest signal of sender 2 required goes up or down depends on whether sender 1's signal p_1 is good or bad. First, consider the case where $p_1 > 1/2$, i.e., sender 1's signal is good. In this case, a negative impact by a bad signal by sender 2 is increasing in the correlation ρ. Consequently, for receiver 1 to be persuaded, sender 2's cannot be sufficiently bad. That is $\tau(p_1)$ is increasing in ρ when $p_1 > 1/2$. Conversely, when sender 1's signal is bad so that $p_1 < 1/2$, as the correlation ρ goes up, good signal by sender 1 has a larger positive impact in persuading receiver 1. Consequently, $\tau(p_1)$ is decreasing in ρ when $p_1 < 1/2$. This is why τ rotates counter-clock-wise at $(1/2, \mu)$.[(3)]

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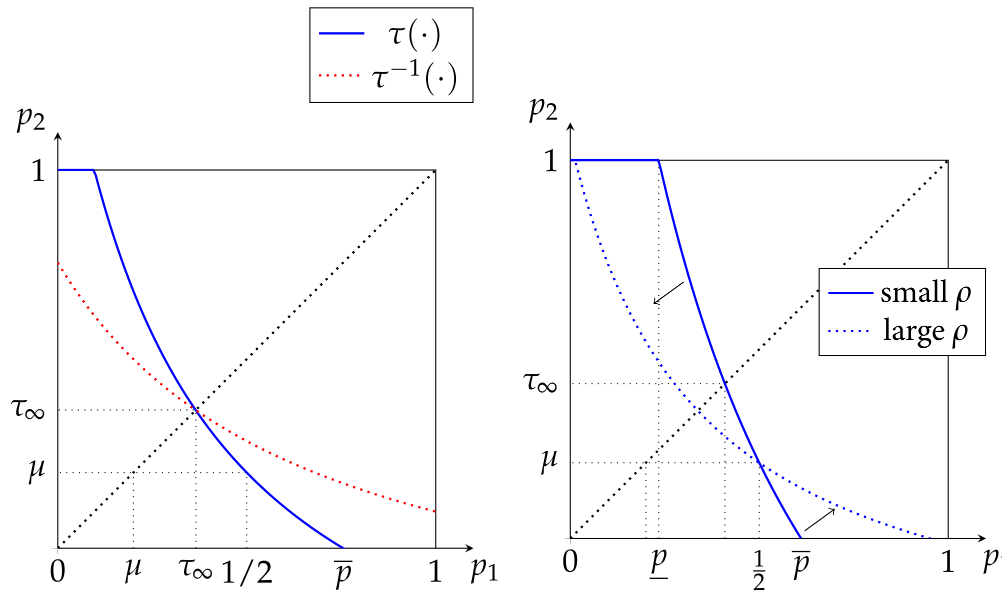
(2) That is, $\tau^{-1}(p) = \frac{\mu(\mu(1-\mu)^2-\rho(p-\mu))}{2\mu^2(1-\mu)^2+\rho(1-2\mu)(p-\mu)}$.

(3) Observe that if sender 2's signal does not contain any information, i.e., $p_2 = \mu$, sender 1 can persuade receiver 1 if and only if his signal is at least $1/2$, and this is why $\tau(1/2) = \mu$.

Lastly, the unique fixed point τ_∞ of τ is

$$\tau_\infty \equiv \frac{\mu\left[\mu\left(\rho+(1-\mu)^2\right)-(1-\mu)\sqrt{\left(\rho+\mu^2\right)\left(\rho+(1-\mu)^2\right)}\right]}{\rho\,(2\mu-1)}, \tag{2}$$

and $\tau\,(p) \geq \tau^{-1}\,(p)$ if and only if $\tau \leq \tau_\infty$. The fixed point is decreasing in ρ and $\lim_{\rho\to 0}\tau_\infty = 1/2$.



(a) $\tau(\cdot)$ and $\tau^{-1}(\cdot)$

(b) $\tau(\cdot)$ for small and for large ρ

Figure 1: $\tau(p_1)$ is the minimum p_2 required for receiver 1 to adopt sender 1's proposal. $\tau(\cdot)$ is decreasing and convex, and rotates counter-clockwise as ρ increases. The unique fixed point of τ_∞ of τ is decreasing in ρ, and $\tau(p) > \tau^{-1}(p)$ if and only if $p < \tau_\infty$.

Lemma 1 *The threshold belief $\tau\,(\cdot)$ satisfies the following properties: (i) τ is strictly decreasing and convex, (ii) $\underline{p} \equiv \tau^{-1}\,(1) \in (0, 1/2)$ and $\overline{p} \equiv \tau^{-1}\,(0) \in (1/2, 1)$ for all $\rho \in (0, \mu\,(1-\mu))$, (iii) $\tau\,(\cdot)$ is decreasing in ρ if $p_1 < 1/2$ and is increasing in ρ if $p_1 > 1/2$, (iv) $\underline{p}$ is decreasing in ρ and $\overline{p}$ is increasing in ρ for all $\rho \in (0, \mu\,(1-\mu))$, (v) $\tau_\infty \in (0, 1/2]$ and is decreasing in $\rho \in [0, \mu\,(1-\mu))$, (iv) $\tau^{-1}\,(p_1) < \tau\,(p_1)$ if $p_1 < \tau_\infty$ and $\tau^{-1}\,(p_1) > \tau\,(p_1)$ if $p_1 > \tau_\infty$.*

Proof. (i) follows from

$$\frac{d\tau\,(p_1)}{dp_1} = -\frac{\mu^2\,(1-\mu)^2}{\rho\,(p_1\,(1-\mu)+\mu\,(1-p_1))^2} < 0 \text{ and} \tag{3}$$

$$\frac{d^2\tau\,(p_1)}{dp_1^2} = \frac{2\mu^2\,(1-\mu)^2\,(1-2\mu)}{\rho\,(p_1\,(1-\mu)+\mu\,(1-p_1))^3} > 0.$$

(ii) follows from $\tau(0) = \mu\left(1 + \frac{(1-\mu)^2}{\rho}\right) > 1$, $\tau(1) = \mu\left(1 - \frac{\mu(1-\mu)}{\rho}\right) < 0$. (iii) holds as the sign of $d(\tau(p_1))/d\rho$ is $2p_1 - 1$ by (1). (iv) and (v) are both immediate from (iii). To see (iv), observe that

$$\tau^{-1}(p) \equiv \frac{\mu(\mu(1-\mu)^2 - \rho(p-\mu))}{2\mu^2(1-\mu)^2 + \rho(1-2\mu)(p-\mu)}.$$

Then, it is straightforward to verify that

$$\left|\frac{d\tau^{-1}(p)}{dp}\right| < \left|\frac{d\tau(p)}{dp}\right|.$$

■

3.2 Sender's Payoff

Suppose that sender 2 employs a strategy that induces signal p_2 with probability $\sigma(p_2)$.

Lemma 2 *Conditional on sender 1's realized signal being p_1, the probability that sender 2's realized signal being p_2 is*

$$\Pr(p_2|p_1) = \sigma(p_2)\left(1 + \rho\frac{(p_1-\mu)(p_2-\mu)}{\mu^2(1-\mu)^2}\right). \tag{4}$$

Consequently, when sener 2 uses strategy σ, the ex-ante probability of sender 1 succeeding in persuading receiver 1 is

$$\Pi(p_1|\sigma) = \sum_{p_2 \geq \max\{\tau(p_1),0\}} \Pr(p_2|p_1).$$

Proof. Let α and β the prob of sending a signal p_2 conditional on U_2 being u_h and u_l, respectively. Then

$$\alpha\mu + \beta(1-\mu) = \sigma(p_2) \text{ and } p_2 = \frac{\alpha\mu}{\sigma(p_2)}.$$

Consequently,

$$\alpha = \Pr(p_2|U_2 = u_h) = \frac{p_2\sigma(p_2)}{\mu}$$

and

$$\frac{\Pr(p_2|U_2 = u_l)}{\Pr(p_2|U_2 = u_h)} = \frac{\beta}{\alpha} = \frac{\frac{\sigma(p_2)-\alpha\mu}{1-\mu}}{\alpha} = \frac{\mu}{1-\mu} \times \frac{\sigma(p_2) - p_2\sigma(p_2)}{p_2\sigma(p_2)} = \frac{\mu}{1-\mu}\frac{1-p_2}{p_2}.$$

Therefore,

$$\begin{aligned}
&\Pr\left(p_2|p_1\right)\\
&=\Pr\left(p_2|U_2=u_h\right)\times\underbrace{\left(\mu+\frac{\rho\left(p_1-\mu\right)}{\mu\left(1-\mu\right)}\right)}_{\Pr(U_2=u_h|p_1)}+\Pr\left(p_2|U_2=u_l\right)\times\underbrace{\left(1-\left(\mu+\frac{\rho\left(p_1-\mu\right)}{\mu\left(1-\mu\right)}\right)\right)}_{\Pr(U_2=u_l|p_1)}\\
&=\Pr\left(p_2|U_2=u_h\right)\left[\left(\mu+\frac{\rho\left(p_1-\mu\right)}{\mu\left(1-\mu\right)}\right)+\frac{\Pr\left(p_2|U_2=u_l\right)}{\Pr\left(p_2|U_2=u_h\right)}\left(1-\left(\mu+\frac{\rho\left(p_1-\mu\right)}{\mu\left(1-\mu\right)}\right)\right)\right]\\
&=\frac{p_2\sigma\left(p_2\right)}{\mu}\left[\left(\mu+\frac{\rho\left(p_1-\mu\right)}{\mu\left(1-\mu\right)}\right)+\frac{\mu}{1-\mu}\frac{1-p_2}{p_2}\left(1-\left(\mu+\frac{\rho\left(p_1-\mu\right)}{\mu\left(1-\mu\right)}\right)\right)\right]\\
&=\sigma\left(p_2\right)\left(1+\rho\frac{\left(p_1-\mu\right)\left(p_2-\mu\right)}{\mu^2\left(1-\mu\right)^2}\right).
\end{aligned}$$

■

Lemma 3 *$\Pi\left(0|\sigma\right)=0$ and $\Pi\left(p|\sigma\right)=1$ for all $p\geq\overline{p}$.*

Proof. Since $p_1=0$ implies $\tau\left(p_1\right)>1$. Hence $\sum_{p_2\geq\tau(p_1)}\Pr\left(p_2|p_1\right)=0$. If $p_1\geq\overline{p}$, then $\tau\left(p_1\right)\leq 0$. Thus,

$$\sum_{p_2\geq\tau(p_1)}\Pr\left(p_2|p_1\right)=1+\rho\frac{\left(\overline{p}-\mu\right)}{\mu^2\left(1-\mu\right)^2}\times\underbrace{\left(\mathbb{E}_\sigma\left[p_2\right]-\mu\right)}_{=0}=1.$$

■

4 Lower and Upper Bounds of Symmetric Equilibrium Payoff

We now derive the bounds of a symmetric equilibrium payoff. We start with a lower bound. Observe that for an arbitrary strategy σ, the payoff of inducing signal $p_1\geq\overline{p}(=\tau^{-1}(0))$ is always one by **??** . Therefore, the sender can always secure the payoff of

$$\underline{\Pi}\left(\rho\right)\equiv\frac{\mu}{\overline{p}}=\mu\left(1+\frac{\left(1-\mu\right)}{\mu}\frac{\left(\mu\left(1-\mu\right)-\rho\right)}{\rho+\left(1-\mu\right)^2}\right),$$

by inducing signal 0 with probability $1-\mu/\overline{p}$ and $\overline{p}$ with probability $\mu/\overline{p}$. Additionally, since $\overline{p}$ is increasing in ρ by **??** , $\underline{\Pi}\left(\rho\right)$ is decreasing in ρ.

Observe that for any symmetric equilibrium σ, the highest signal that a sender induces, i.e., $\overline{\sigma}\equiv\sup\operatorname{supp}\left\{\sigma\right\}$, is bounded from above by $\overline{p}$ by **??** . Also by construction of τ_∞, $\overline{\sigma}\geq\tau_\infty$. We now note that for any symmetric equilibrium σ, 0 is in the support.

Lemma 4 *If σ is a symmetric equilibrium, then (i) σ has an atom at 0; and (ii) $\overline{\sigma}\equiv\sup\operatorname{supp}\left\{\sigma\right\}\in\left\{\tau_\infty,\overline{p}\right\}$.*

Proof. Let $\overline{\sigma} \equiv \sup \operatorname{supp}\{\sigma\}$, $\underline{\sigma} \equiv \inf \operatorname{supp}\{\sigma\}$, and and $\widetilde{\tau}_1 \equiv \tau^{-1}(\overline{\sigma})$. Since $\overline{\sigma} \leq \overline{p}$, for any $p_1 \in (0, \widetilde{\tau}_1)$, we have $\tau(p_1) > \overline{\sigma}$, as illustrated in Figure 2. That is, for any $p_1 \in (0, \widetilde{\tau}_1)$, $\Pi(p_1|\sigma) = 0$, and hence $(0, \widetilde{\tau}_1) \cap \operatorname{supp}\{\sigma\} = \emptyset$.

To show (i), suppose that $\underline{\sigma} > 0$ by contradiction. Then, $\underline{\sigma} \geq \widetilde{\tau}_1$, and hence $\Pi(p_1|\sigma) = 1$ on $(\widetilde{\tau}_2, 1]$, where $\widetilde{\tau}_2 \equiv \tau^{-1}(\widetilde{\tau}_1)$. Therefore, $(\widetilde{\tau}_2, 1] \cap \operatorname{supp}\{\sigma\} \neq \emptyset$. However, since $\tau(\widetilde{\tau}_1) > \tau^{-1}(\widetilde{\tau}_1)$ by **??** , and $\tau(\widetilde{\tau}_1) = \overline{\sigma}$ and $\widetilde{\tau}_2 = \tau^{-1}(\widetilde{\tau}_1)$, we have $\overline{\sigma} > \widetilde{\tau}_2$, which is a contradiction. This establishes that $\underline{\sigma} = 0$ and σ has an atom at $\underline{\sigma}$.

Next, we establish (ii). To that end, suppose that $\overline{\sigma} < \overline{p}$. Then the argument above implies that $\Pi(p|\sigma)$ is linear on $(\widetilde{\tau}_2, \overline{p})$, and exhibits an upward jump at $\overline{p}$. Therefore, $\overline{\sigma} \in \operatorname{supp}\{\sigma\}$ only if $\widetilde{\tau}_1 = \widetilde{\tau}_2$ so that $\widetilde{\tau}_1 = \widetilde{\tau}_2 = \tau_\infty$. ■

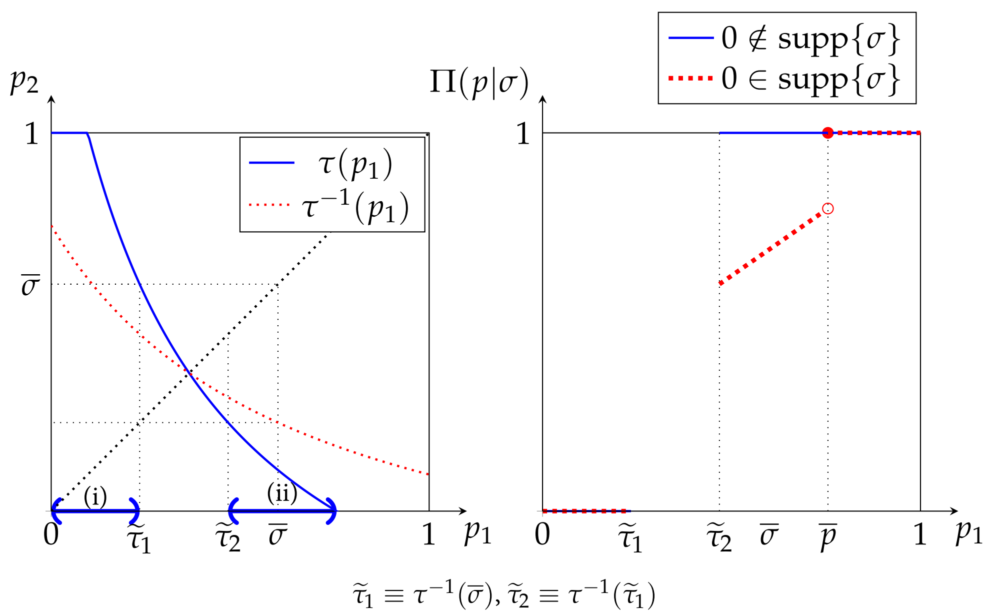


Figure 2: (Proof of **??**) Given $\overline{\sigma}$, any p in the interval (i) is not in the support of σ. Additionally, if $\underline{\sigma} > 0$, then any p in the interval (ii) is not in the support of σ.

Definition 1 *We say a symmetric equilibrium σ is a* ***coordinated equilibrium*** *if $\overline{\sigma} = \tau_\infty$, and* ***uncoordinated equilibrium*** *if $\overline{\sigma} = \overline{p}$.*

In light of **??** , a symmetric equilibrium σ is either coordinated or uncoordinated. Should a coordinated equilibrium exist, then $\operatorname{supp}\{\sigma\} = \{0, \tau_\infty\}$, i.e., the sender induces τ_∞ with probability $\sigma(\tau_\infty) \equiv \mu/\tau_\infty$ and 0 with probability $1 - \mu/\tau_\infty$; and the equilibrium payoff is

$$\begin{aligned} \overline{\Pi}(\rho) &\equiv \frac{\mu}{\tau_\infty} \times \sigma(\tau_\infty) \times \Pr(\tau_\infty|\tau_\infty) \\ &= \left(\frac{\mu}{\tau_\infty}\right)^2 \left(1 + \rho \frac{(\tau_\infty - \mu)^2}{\mu^2 (1-\mu)^2}\right). \end{aligned} \tag{5}$$

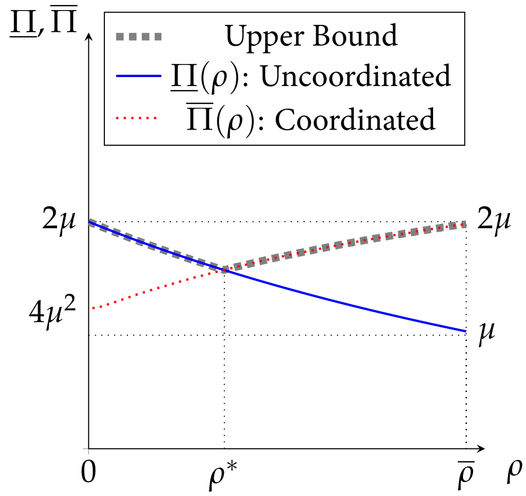


Figure 3: (**??**) If $\rho \leq \rho^*$, then guaranteed payoff $\underline{\Pi}(\rho)$ is the equilibrium payoff. If $\rho > \rho^*$ then the equilibrium payoff is either $\underline{\Pi}(\rho)$ (from an uncoordinated equilibrium) or $\overline{\Pi}(\rho)$ (from a coordinated equilibrium).

In contrast, should an uncoordinated equilibrium exist, then the equilibrium payoff is $\underline{\Pi}(\rho)$. Since $\underline{\Pi}(\rho)$ defines a lower bound of the equilibrium payoff, $\overline{\Pi}(\rho) \geq \underline{\Pi}(\rho)$ is a necessary condition for the existence of coordinated equilibrium.

Lemma 5 *$\overline{\Pi}(\rho)$ is increasing in ρ, and there exists $\rho^* \in (0, \mu(1-\mu))$ such that $\overline{\Pi}(\rho) > \underline{\Pi}(\rho)$ if and only if $\rho > \rho^*$. That is, a coordinated equilibrium exists only if $\rho > \rho^*$.*

Proof. $\overline{\Pi}(\rho)$ is increasing in ρ as

$$\overline{\Pi}'(\rho) = \frac{d\tau_\infty}{d\rho} \times 2 \times \left(\frac{\rho(\tau_\infty - \mu) - \mu(1-\mu)^2}{\tau_\infty^3 (1-\mu)^2} \right) + \frac{(\tau_\infty - \mu)^2}{(\tau_\infty(1-\mu))^2} > 0$$

where the last inequality follows from $\frac{d\tau_\infty}{d\rho} < 0$ by **??** , and $\rho(\tau_\infty - \mu) < \mu(1-\mu)^2$.

Recall that $\underline{\Pi}(\rho)$ is decreasing in ρ, as $\overline{p}$ is increasing in ρ. Additionally, $\underline{\Pi}(0) = 2\mu > \overline{\Pi}(0) = 4\mu^2$ ■

Therefore, for a given ρ, if there exists a symmetric equilibrium and $\rho \leq \rho^*$, the seller's equilibrium payoff must be equal to $\underline{\Pi}(\rho)$. In contrast, if there exists a symmetric equilibrium and $\rho > \rho^*$, then there may be multiple equilibrium payoffs, but it is bounded from below by $\underline{\Pi}(\rho)$ and from above by $\overline{\Pi}(\rho)$.

So the finding so far can be summarized as follows.

Theorem 1 *Suppose that σ^* is a symmetric equilibrium, and $\Pi^*(\rho)$ be the corresponding equilibrium payoff. If $\rho \leq \rho^*$, then $\Pi^*(\rho) = \underline{\Pi}(\rho)$. If $\rho > \rho^*$, then $\Pi^*(\rho) \in \{\underline{\Pi}(\rho), \overline{\Pi}(\rho)\}$, where $\underline{\Pi}(\rho)$ and $\overline{\Pi}(\rho)$ are the payoffs in the uncoordianted and coordinated equilibrium, respectively.*

5 Existence of Equilibrium

In this section, we show that a coodinated equilibria exists if and only if $\rho \geq \rho^*$, and an uncoordinated equilibria always exists. To simplify the exposition, for $\tau_0 \equiv 0$, we sequentially define $\tau_{n+1} \equiv \tau^{-1}(\tau_n)$, and $T \equiv \{\tau_n\}_{n=0}^{\infty}$. As illustrated in Figure 4, (i) $\tau_{2n+1} > \tau_\infty$ and is decreasing in n, (ii) $\tau_{2n} < \tau_\infty$ and is increasing in n, and (iii) $\tau_\infty = \lim_{n\to\infty} \tau_n$.

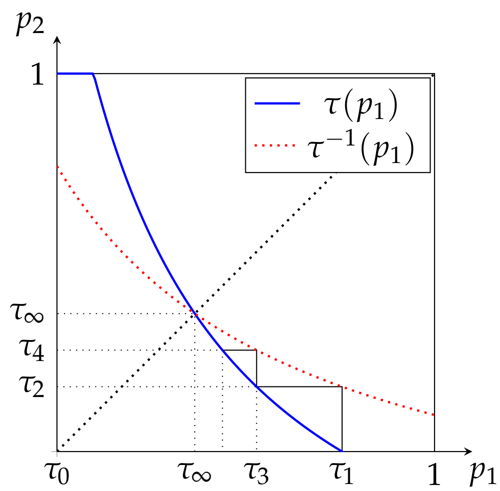


Figure 4: (Construction of τ_n) τ_{2n+1} is decreasing in n, and τ_{2n} is increasing in n.

As illustrated in **??** , observe that if sender 2's strategy σ_2 does not induce any signal in (p_l, p_h), then sender 1's payoff $\Pi(\cdot|\sigma_2)$ is linear on $\left(\tau^{-1}(p_h), \tau^{-1}(p_l)\right)$. Additionally, if $\sigma_2(p_l) > 0$, then $\Pi(\cdot|\sigma_2)$ exhibits an upward jump at $\tau^{-1}(p_l)$ and hence sender 1's best response does not induce any signal in $\left(\tau^{-1}(p_h), \tau^{-1}(p_l)\right)$; and induces $\tau^{-1}(p_h)$ only if $\sigma_2(p_h) > 0$.

Lemma 6 *Suppose that sender 2's strategy σ_2 only induces signals in T with positive probabilities, i.e., $\mathrm{supp}\{\sigma_2\} \subset T$. Then, sender 1's best-response σ_1 only induces τ_0 or $\tau^{-1}(s)$ for some $s \in \mathrm{supp}\{\sigma_2\}$ with positive probabilities, i.e.,*

$$\mathrm{supp}\{\sigma_2\} \subset \bigcup_{s\in\mathrm{supp}\{\sigma_2\}} \tau^{-1}(s) \cup \{\tau_0\}.$$

Recall that if σ is a symmetric equilibrium, then $\sigma(0) > 0$ and $(0, \tau_2) \cap \mathrm{supp}\{\sigma\} = \emptyset$. Therefore, the following lemma immediately follows

Lemma 7 *If σ is a symmetric coordinated equilibrium, then $\mathrm{supp}\{\sigma\} = \{\tau_0, \tau_\infty\}$. If σ is a symmetric uncoordinated equilibrium, then $\mathrm{supp}\{\sigma\} = T$, or there exists $\overline{n}$ such that $\mathrm{supp}\{\sigma\} = \{\tau_n\}_{n=0}^{\overline{n}}$ or $\mathrm{supp}\{\sigma\} = \{\tau_n\}_{n=0}^{\overline{n}} \cup \{\tau_\infty\}$.*

Lemma 8 *A coordinated equilibirum exists if and only if $\rho \geq \rho^*$.*

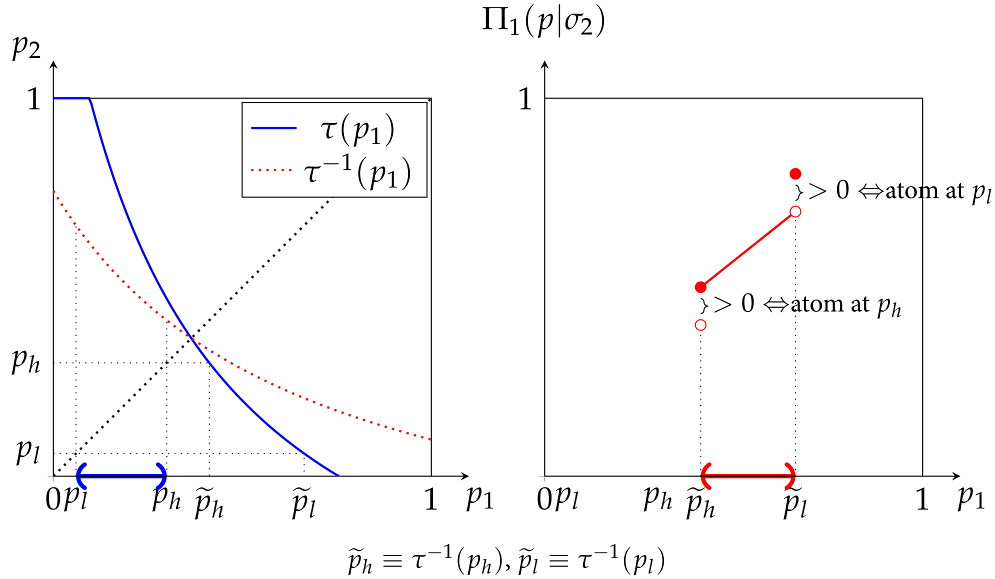


Figure 5: (**??**) If σ_2 does not contain any signal in the blue interval (p_l, p_h), then, $\Pi_1(\cdot|\sigma_2)$ is linear on the red interval $(\tau^{-1}(p_h)), \tau^{-1}(p_l))$. $\Pi_1(\cdot|\sigma_2)$ exhibits an jump at $\widetilde{p}_l \equiv \tau^{-1}(p_l)$ if and only if σ_2 has an atom at p_l.

Proof. We have already established that $\rho \geq \rho^*$ is necessary. To see the sufficiency, let σ be a strategy such that $\text{supp}\{\sigma\} = \{0, \tau_\infty\}$. Then, $\Pi(p_1|\sigma) = 0$ on $p_1 \in [0, \tau_\infty)$, and is linear on $[\tau_\infty, \tau_1)$. Additionally, by the definition of ρ^*, $\rho \geq \rho^*$ implies $\Pi(\tau_\infty|\sigma)/\tau_\infty \geq \Pi(\tau_1|\sigma)/\tau_1$. Thus the result follows. ■

Lemma 9 *If* $\sigma < \sigma^*$, *then the unique symmetric equilibrium* σ *is an uncoordinated equilibrium such that* $\text{supp}\{\sigma\} = \{0, \tau_1\}$.

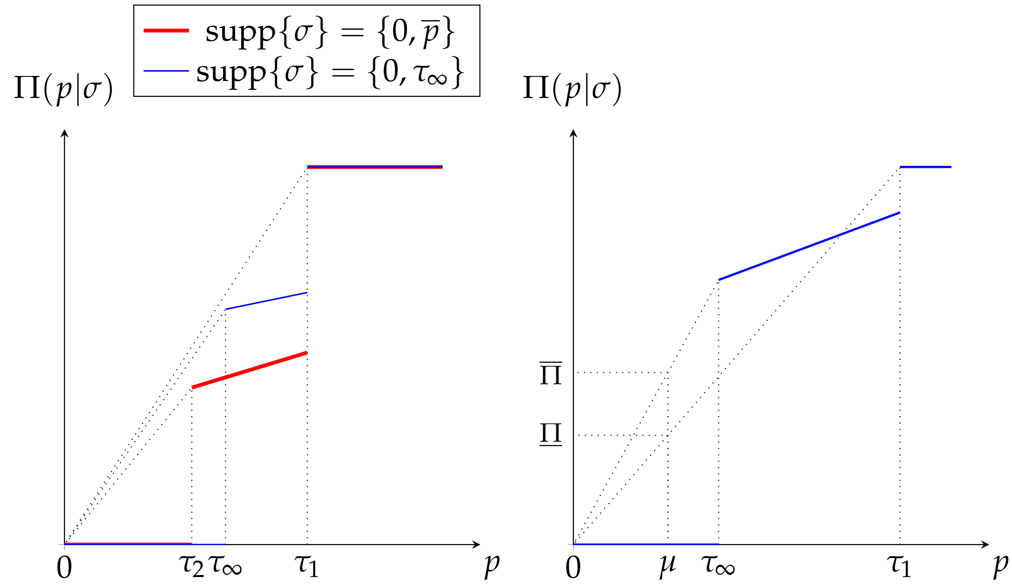
Proof. We first show that a symmetric equilibrium σ such that $\text{supp}\{\sigma\} = \{0, \tau_1\}$ exists first. To this end, let σ_p be a strategy such that $\text{supp}\,\sigma_p = \{0, p\}$. Observe that $\Pi(\cdot|\sigma_p) = 0$ on $[0, \tau^{-1}(p))$, linear on $[\tau^{-1}(p), p)$ and $\Pi(p_1|\sigma) = 1$ on $[p, 1]$. Therefore,

$$s(p) \equiv \frac{\Pi(\tau^{-1}(p)|\sigma_p)}{\tau^{-1}(p)} = \frac{\frac{\mu}{p} \times \left(1 + \rho\frac{(\tau^{-1}(p)-\mu)(p-\mu)}{\mu^2(1-\mu)^2}\right)}{\tau^{-1}(p)} = \frac{2\left(\mu^2(1-\mu) + \rho(p-\mu)\right)}{p\mu(1-\mu)}.$$

Observe that $s(p)$ is decreasing in p as

$$s'(p) = -2\frac{\mu(1-\mu) - \rho}{p^2(1-\mu)} < 0.$$

Additionally, we have already established that $s(\tau_\infty) < 1/\tau_1$ when $\rho < \rho^*$, and hence $s(\tau_1) < 1/\tau_1$. That is, σ_{τ_1} is indeed an equilibrium.



(a) $\rho < \rho^*$: No coordinated equilibrium but uncoordinated with a binary support exists

(b) $\rho > \rho^*$: coordinated equilibrium

Figure 6: **??** and **??** : Existence of a symmetric equilibrium.

Next, to establish the uniquness, suppose that there exists a symmetric equilibrium $\sigma \neq \sigma_{\tau_1}$. We have already established that $\sigma \neq \tau_\infty$. We also know that $\operatorname{supp}\sigma \leq \tau_1$. Therefore, we must have $(0, \tau_2] \cap \operatorname{supp}\sigma = \emptyset$. To see this, first support $\tau_1 \notin \operatorname{supp}\sigma$. Then, $\Pi(\tau_2|\sigma) = 0$. If $\tau_1 \in \operatorname{supp}\sigma$, then we have already established that $\Pi(\tau_2|\sigma)/\tau_2 < s(p_1) < 1/\tau_1$.

Let $q_1 \equiv \inf\operatorname{supp}\sigma \setminus \{0\}$. Then by the argument above, $q_1 \in (\tau_2, \tau_1)$. Additionally, since $\Pi(p_1|\sigma) = 0$ for all $p_1 < q_1$, $\Pi(\cdot|\sigma)$ must exhibit a jump at q_1, and hence σ must have an atom at $q_2 \equiv \tau(q_1)$. Additionally, by the definition of q_1, we have $q_2 \geq q_1$.

However, as $[0, q_1) \cap \operatorname{supp}\sigma = \emptyset$, $\Pi(p_1|\sigma)$ must be linear on $[\tau^{-1}(q_1), \tau_1)$; and as σ has an atom at 0, $\Pi(p_1|\sigma)$ exhibits an upward jump at $p_1 = \overline{p} = \tau^{-1}(0)$. Therefore, $\left(\tau^{-1}(q_1), \tau_1\right) \cap \operatorname{supp}\sigma = \emptyset$. Hence $\tau(q_1) < \tau^{-1}(q_1)$ must hold. That is, $q_1 > \tau_\infty$.

Then it is straightforward to show that

$$\frac{\Pi(q_1|\sigma)}{q_1} \leq \frac{\Pi\left(q_1|\sigma_{q_1}\right)}{q_1} < \frac{\Pi\left(q_\infty|\sigma_{\tau_\infty}\right)}{\tau_\infty}.$$

That is, $q_1 \notin \operatorname{supp}\sigma$, and hence we have a contradiction. ■

By applying the standard argument, we can show that an uncoordianted equilibrium always exists.

Lemma 10 *For all $\rho \in [0, \overline{\rho})$, an uncoordinated equilibrium exists.*

To summarize the results so far, we obtain the following results.

Theorem 2 *(i) For all* $\rho \in [0, \overline{\rho})$*, an uncoordinated equilibrium exists; (ii) For all* $\rho \in [0, \rho^*)$*, a unique symmetric equilibrium is the uncoordinated equilibrium with support* 0 *and* τ_1*; (iii) For all* $\rho \in [\rho^*, \overline{\rho})$*, a coordinated equilibrium exists.*

6 Conclusion

We have shown that the information leakage problem in general hurts the senders, but the effect is non-monotone in the degree of correlation. When the degree of correlation is low, a small increase in the degree of correlation incentivizes the senders to disclose more information. In contrast, when the degree of correltion is high, a small increase in the degree of correlation incentivizes the senders to disclose less information.

These findings imply that when the degree of correlation is determined by the sender's endogenous proposal design choices, they adopt distinct designs if the baseline correlation is low, but similar designs if the baseline correlation is high. Therefore, the information linkage can engender strategic complements between senders in their disclosure strategy. As a result, there is a strategic incentive for senders to over-invest in committing to less informative revelation, such as an improvement in their proposal's expected quality.

Another way to interpret our results is that the effect of transparency (i.e., the observability of a sender's signal by the unmatched receiver) on sender's disclosure strategy is non-monotone. Interestingly, in some parameter region, the extra information sources brought about by an improvement in transparency can hurt the receivers, as the senders endogenously reduce their information revelation in response.

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