The Effects of Personal Data Management on Competition and Welfare*

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Abstract

This study examines how consumers' personal data management affects firms' competition in the data collection and data application markets and welfare outcomes. Consumers purchase products from differentiated firms in two markets. Firms compete to collect consumer data first to predict their preferences in the data application market, where each firm offers personalized prices to its targeted consumers and a uniform price to untargeted consumers. Before firms offer prices, their targeted consumers can erase data to become untargeted for a fixed cost. We show that consumers' privacy management mitigates price competition, reduces firms' profits, and harms consumer surplus and social welfare in the data application market; privacy management intensifies competition and improves consumer surplus in the data collection market. Across these two markets, profits and social welfare decline. The change in consumers' two-market surplus depends on their foresight regarding the outcomes in the data application market, with only forward-looking consumers having a higher surplus. We extend the model in several directions, including data-enabled product personalization, privacy costs, data portability, and data ownership, and discuss the implications for privacy laws.

Keywords: privacy management, data collection, data application, price discrimination, privacy laws

JEL code: D11, D43, D62, L13, L51

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1 Introduction

By collecting consumer data and using data analytics, firms can gain a competitive edge in developing data-utilizing markets such as those of healthcare, insurance, and streaming services (Goldfarb and Tucker, 2019, Farboodi et al., 2019, Hagiu and Wright, 2022). As a result, many firms devote significant resources to collecting and utilizing consumer data, often leveraging it across multiple industries.

Big tech firms' entry into the healthcare market is a typical example of data collection and data application. Google has become a central player in the healthcare industry since it made public plans to develop Google Health for health insurers and doctors in 2008 (Ozalp et al., 2022). In 2021, Google acquired Fitbit, which produces smartwatches that collect real-time health data from millions of customers, for $2.1 billion. Through the Apple Watch, which collects a wide range of biometric data, Apple can provide users with health support. Moreover, Amazon allows consumers to communicate with healthcare companies through its smart speaker, Alexa; the company has completed serial acquisitions of pharmacy and health firms in recent years and is now offering employees its own virtual healthcare (Farr, 2019).[1] In sum, these companies collect customer data through their devices and provide data-utilizing services to users.

Data-utilized services (e.g., digital healthcare services) not only improve the trade surpluses of such services but also lead to two major concerns: data privacy and personalized pricing.[2] Consumer demand for data privacy has sparked a global wave of legislation granting consumers enhanced control over their data. A typical example of such laws is the European Union's General Data Protection Regulation (GDPR). GDPR Article 17, the right to erasure ('right to be forgotten'), stipulates that consumers can order a firm to erase their data, which it must do

[1] Amazon purchased online pharmacy PillPack in 2018, acquired digital health startup Health Navigator in 2019, and proposed the acquisition of One Medical for approximately $3.9 billion in 2022. In 2022, Amazon proposed the acquisition of iRobot, the maker of the popular Roomba vacuum cleaner, which can enhance its data collection of customers' homes and private lives.

[2] For instance, consumers' privacy was a major concern in the Google-Fitbit and Amazon-iRobot acquisitions. Transferring patient data from Ascension to Google Cloud led to an ethical dispute, although Ascension obeyed the Health Insurance Portability and Accountability Act (HIPAA) (see Schneble et al. (2020) for details).

without undue delay, even if the firm had obtained consent from customers to process their personal data. Privacy laws in many other countries also clearly grant consumers the right to erase their data.[3] Although privacy laws secure the right of consumers to manage their personal data, the impact of this right in competitive environments remains unclear and under debate (Aridor et al., 2022; Ali et al., 2023).

Personalized pricing, where firms use data gathered from personal devices (e.g., smartphones and smartwatches) to offer customized prices to consumers, facilitates consumer surplus exploitation (Wagner and Eidenmüller, 2019).[4] Several studies, such as Shiller (2020), Dubé and Misra (2023), and Smith et al. (2022), have quantitatively confirmed the effectiveness of such exploitation.[5] We therefore explore the effects of consumers' personal data management to escape personalized pricing, which arises under privacy laws, on profits and welfare.

This study considers a duopoly model in which consumers purchase products in two independent markets: market B (e.g., wearable devices) is for data collection, and market A (e.g., healthcare and insurance) is for data application. Consumers' product preferences in the two markets are independent. The two firms compete in the two markets and first attempt to attract consumers in the data collection market with uniform pricing. The personal data collected uniquely by each firm uncover its customers' preferences for firms' products in the data application market. For example, firms in the healthcare or insurance market can use the biometric data collected from wristbands to predict a suitable healthcare and insurance plan for each consumer. Such a plan can be different from the firm's standard product. After competition in market B, each firm in market A offers personalized prices to its targeted customers, for

[3]In addition to data erasure, consumers can order the firm to transmit their data from the firm to another firm if technically feasible—so-called data portability (GDPR Article 20). The California Consumer Privacy Act (CCPA) and the Treasury Laws Amendment (Consumer Data Right) Bill 2019 in Australia also give consumers the right to data portability. We discuss the effects of data portability in one extension.

[4]See Esteves and Resende (2016) and Ezrachi and Stucke (2016) for real-world examples of personalized offers.

[5]Although the surplus exploitation concern does not match the standard result in the literature of personalized pricing in which personalized pricing is worse for firms than is uniform pricing in oligopoly markets (Thisse and Vives, 1988; Shaffer and Zhang, 1995; Zhang, 2011), some studies show that personalized pricing does not always negatively affect firms (e.g., Shaffer and Zhang, 2002; Choudhary et al., 2005; Matsumura and Matsushima, 2015; Esteves, 2022).

whom the firm has collected data, and a uniform price to untargeted consumers, for whom the firm does not have data.[6] However, before firms decide prices in market A, each firm's targeted customers can erase their data from the firm's database to become untargeted consumers at a fixed cost, an option referred to as *privacy management*.[7] Consumers who erase their data (i.e., opt-out consumers) can escape personalized prices and choose uniform prices offered by firms. Our benchmark is the scenario in which consumers cannot manage their privacy.

We find that consumers' privacy management mitigates price competition, reduces firms' profits, and negatively affects welfare in market A, the data application market. The explanation of competition mitigation starts with which types of consumers self-select to manage data. Consumers who strongly prefer one firm in market A choose to erase their data because they expect to be charged high personalized prices otherwise. Consequently, each firm's uniform price rises because the price is applied to those opt-out consumers who strongly prefer the firm. Such increases in uniform prices induce firms to set higher personalized prices for consumers who do not erase their data, i.e., opt-in consumers. In other words, opt-out consumers bring negative externalities to opt-in consumers by raising firms' prices.

We sequentially explain the impacts on profits, consumer surplus, and social welfare in market A. Firms' profits in this market decline because their uniform prices cannot efficiently extract the surplus from opt-out consumers with high reservation values. Opt-in consumers are worse off, and opt-out consumers are barely better off because they pay an inflated uniform price and privacy management costs, lowering consumer surplus in market A. The consumer-firm mismatch becomes larger, reducing social welfare, because higher uniform prices help each firm protect its targeted customers through personalized prices in market A. In addition, privacy management costs further negatively affect welfare. Interestingly, increasing the privacy management cost can benefit firms, consumers, and society simultaneously because fewer con-

[6]Generally, consumer data can not only help the firm provide personalized products that generate more trade surplus but also enable the firm to extract the surplus more efficiently with sophisticated pricing tools. We focus on firms' pricing tools in the main model and discuss personalized products in an extension.

[7]Consumers can exercise their right to object to their data being used in the data application market under GDPR Article 21, which is equivalent to data erasure in the model.

sumers would opt out, and the above equilibrium changes would thus become less impactful.

We show that consumers' privacy management intensifies competition in market B (i.e., enhances firms' incentives to collect data) compared to its absence. The reason for this is that a firm's profit in market A increases faster with its market share in market B under privacy management. Specifically, when a firm gains more consumers in market B, its uniform price in market A becomes higher because it has more consumers who strongly prefer the firm and opt out. This higher uniform price, in turn, diminishes the incentives of *marginal* consumers to erase their data because the gains from escaping personalized prices are lower. Thus, the firm can price discriminate against more consumers with a higher willingness to pay and efficiently extract their surplus in market A. Under privacy management, inflated personalized prices further facilitate surplus extraction. As price competition intensifies in market B, firms earn lower profits, and consumers benefit. In our model of symmetric firms, the two firms always split the market equally and generate a fixed consumer-firm match, which leaves the social welfare of market B unchanged with privacy management.

When combining these two markets, privacy management lowers the two-market profits and social welfare but enhances total consumer surplus only when consumers are forward-looking regarding the outcome in market A. Compared to myopic consumers, forward-looking consumers are more price sensitive in market B because the expected surplus that they will obtain from a firm in market A is positively related to the firm's market share in market B. Consumers' higher price sensitivity further accelerates competition in market B and results in larger consumer surplus gains.

We consider several factors as extensions, two of which we explain here: data portability (GDPR Article 20) and data ownership. When data portability is available, consumers with weak preferences for both firms make their data available to firms with data portability to trigger consumer-by-consumer Bertrand competition on personalized pricing, intensifying competition in market A but mitigating competition in market B because the value of collecting data plummets. When consumers own data property rights (i.e., opt out by default and opt in by choice), compared to the main model, the equilibrium outcome is better from the welfare viewpoint, provided that the compensation paid to consumers for data usage is appropriate.

Our study provides the following policy implications. Consumers' two-market surplus increases only when they have foresight regarding the data application market, and thus, firms should explain how to use collected data when they gain customers in the data collection market.[8] Both data erasure and data portability result in surplus redistribution, with consumers who manage privacy benefiting. Data erasure destroys surplus simultaneously due to increased consumer-firm mismatch, while data portability enhances social welfare because it enables consumers to be targeted and attracted by their preferred firm.

The remainder of this paper is organized as follows. After reviewing the literature in Section 2 and setting up the model in Section 3, we establish the benchmark equilibrium without privacy management and the equilibrium when privacy management is available in Section 4. Section 5 provides several extensions, such as data-enabled product personalization, data portability, privacy costs, and ownership of data property rights. Section 6 discusses the policy implications. Finally, Section 7 concludes the paper.

2 Literature review

Our study contributes to the literature on behavior-based price discrimination (BBPD) with personalized pricing, consumer data management, and the role of data in competition.

Our study relates to those studies in which firms collect data (e.g., purchasing history) and then apply them to pricing in a competitive environment. Caminal and Matutes (1990), Chen (1997), and Fudenberg and Tirole (2000) are earlier works that consider two-period models with third-degree price discrimination based on consumers' purchase history (so-called behavior-based price discrimination (BBPD)). Moreover, Choe et al. (2018), Choe et al. (2022), and Laussel

[8]In November 2022, Google reached a record $392M privacy settlement with 40 U.S. states over location data, requiring Google to provide consumers with more information and to be more transparent about its data collection and utilization (https://www.nytimes.com/2022/11/14/technology/google-privacy-settlement.html).

and Resende (2022) incorporate data-enabled personalized pricing into the BBPD framework.[9] Among those studies, our work is closely related to Chen et al. (2022), who consider two-stage duopoly models in which only one of the firms can apply consumer data collected in a market to another market to offer personalized prices.[10] However, they do not investigate how consumers' data management affects competition in the two markets, which is the focus of our study.

We contribute to the literature on consumer data management, particularly discussions on the interaction between consumer data management and firms' pricing strategy. Belleflamme and Vergote (2016) and Koh et al. (2017) show that consumers' data management choices function as signals of their willingness to pay and that the monopolist adjusts discriminatory prices against two consumer groups (those who manage data and those who do not) accordingly.[11] While we share this signaling effect, our study is embedded in a competitive environment where each firm's discriminatory prices face competition.[12] Therefore, whether data management consumers can cause pricing externalities for other consumers and the magnitude of such externalities can differ.

There are three recent papers on consumer data management. First, in a Hotelling linear city model, Ali et al. (2023) assume that duopoly firms collect personal data only from consumers' revelation of preferences and show that consumers can strategically disclose their preferences to amplify firms' competition and benefit.[13] The result is similar to our analysis of data portability. One of our contributions is that we consider the interaction between consumer data manage-

[9]Choe et al. (2018) show that personalized pricing intensifies competition in the first period and that the resulting outcomes are asymmetric. Choe et al. (2022) incorporate data-sharing agreements between competing firms into Choe et al. (2018). Laussel and Resende (2022) incorporate endogenous product differentiation and product personalization into Choe et al. (2018) and show a contrasting result to that of Choe et al. (2018).

[10]Herresthal et al. (2023) consider two-stage oligopoly models in which one of the firms can apply customer data collected in a market to an insurance market and show that the firm's data application benefits consumers.

[11]Using the BBPD framework, Acquisti and Varian (2005) and Conitzer et al. (2012) consider consumers' identity management to escape expected high prices in monopoly models.

[12]Montes et al. (2019) and Valletti and Wu (2020) consider consumer data management to escape personalized prices, with the former focusing on the data broker's optimal data sales strategy and the latter focusing on the endogenization of profiling technology quality.

[13]Ichihashi (2020) discusses the interaction between information disclosure by a consumer and product recommendation by a seller in buyer-seller models.

ment and firms' incentives to collect data and demonstrate that data management substantially enhances such incentives.

Anderson et al. (2022) consider a two-stage game with targeted discounting in stage two, assuming that consumers can escape target discounts *before* knowing their preferences for firms' products, meaning that consumers' denial does not signal their preferences, different from our game. They show that the refusal of personalized discounts can benefit firms and consumers by mitigating discount competition and lowering uniform prices.

Using the two-period BBPD framework with consumers' heterogeneous willingness to pay, Ke and Sudhir (2022) study how consumers' data management affects competition among ex ante homogeneous firms. Their findings, contrary to ours, show that consumers who strongly prefer a firm opt in because, in their model setting, these consumers can reap greater benefits from personalized products, but the firm cannot efficiently extract the surplus due to uncertainty about their valuations. In their model, consumer surplus increases under privacy management. We reach diverging welfare results due to various modeling differences, including market structure, product differentiation, and the firm's ability to expand and distribute the "pie" with the help of data.

The final section discusses recent research on the role of data and categorizes them into two categories: competitive edge from data accumulation (Farboodi et al., 2019; Hagiu and Wright, 2022; Prüfer and Schottmüller, 2021; Cordorelli and Padilla, 2022; Fainmesser et al., 2022; de Cornière and Taylor, 2023) and consumer data markets (Choi et al., 2019; Bergemann and Bonatti, 2019; Argenziano and Bonatti, 2021; Ichihashi, 2021; Acemoglu et al., 2022; Bergemann et al., 2022). However, the interaction between consumer data management and firms' competition is beyond the scope of these papers.

3 Model

Consider two markets, A and B. Market B is data rich, and firms can collect substantial consumer data in this market. Market A is lucrative, and firms can predict consumers' preferences in the product space of this market using their data from market B. Our model is best understood with the help of a concrete example. Thus, we refer to market B as the market for wearable devices, such as Fitbit wristbands and Apple watches, and market A as the market for healthcare and insurance. Needless to say, our model applies to other markets where data are collected in one market and applied in another market.

Two firms serve each market—A_1 and A_2 in market A and B_1 and B_2 in market B—where firms A_1 and B_1 are two subsidiaries of firm 1 and firms A_2 and B_2 are two subsidiaries of firm 2. The two markets have the same group of consumers, and their mass is normalized to one. In each market, a consumer demands one unit of product. We normalize the marginal cost of production to zero and treat prices as profit margins.

In market B, consumers are uniformly distributed on $[0, 1]$. Firms B_1 and B_2 are located at 0 and 1, respectively, and compete on uniform prices. Firm B_i's price is β_i ($i = 1, 2$). Consumer $y \in [0, 1]$ obtains utility $u = v_B - ty - \beta_1$ from firm B_1 and utility $u = v_B - t(1 - y) - \beta_2$ from firm B_2. We assume that the value of v_B is large such that the market is fully covered. If a consumer purchases from firm B_i, then the firm can collect consumer data through the consumer's usage of its product. We assume that consumers must consent to data collection in the market B to use products like wristbands and smartwatches properly.

In market A, firms A_1 and A_2 are located at 0 and 1, respectively. If a consumer is located at $y \in [0, 1]$ in market B, then her location x in market A is a random variable on $[0, 1]$ that follows a uniform distribution. In other words, consumers' product preferences in the two markets are independent of each other because the products serve different purposes (Matutes and Regibeau, 1988; Armstrong and Vickers, 2010). Consumers privately know their exact locations in market A, which can be interpreted as the healthcare or insurance plan that perfectly matches their needs.[14] Firm A_i obtains all of firm B_i's collected consumer data. This data transfer means that

[14]Healthcare or insurance plans are commonly believed to differ substantially (Gaynor and Vogt, 2000; Gaynor and Town, 2011; Biglaiser and Ma, 2003). For instance, healthcare can differ in terms of lists of approved physicians, diagnostic testing, real-time monitoring, and high-tech services, while insurance plans may vary in terms of their coverage, care (hospital) providers, reimbursement policies, and other characteristics of the plan provider. The Hotelling model is widely used to model differentiation in healthcare and insurance plans (Gal-or, 1997; Ellis, 1998, Biglaiser and

if a consumer has purchased from firm B_i, then firm A_i can uncover her exact realized location x in market A and target her with the help of consumer data. However, firm A_j ($j \neq i$) knows only that her location x is uniformly distributed on $[0, 1]$. Firm A_i charges personalized prices $p_i(x)$ to its targeted consumers and a uniform price α_i to untargeted consumers.

Before firm B_i transfers consumer data to firm A_i, knowing her preferences in market A, each consumer can choose whether to erase her data from firm B_i's datasets or object to firm B_i's transfer of her data to firm A_i, which is called *privacy management*. We assume that the cost of privacy management is $\varepsilon \geq 0$ for every consumer. This formulation is consistent with existing U.S. privacy laws that give the property rights over data to the entities that collect them, and consumers opt in by default and can manage their privacy (Economides and Lianos, 2021).

If a consumer does not erase her data, then she continues to be a targeted consumer of firm A_i. Firm A_i's targeted consumer has two choices: receiving utility $u = v_A - tx_i - p_i(x)$ from firm A_i or receiving utility $u = v_A - tx_j - \alpha_j$ from firm A_j, where $x_i = 1 - x_j$. If she erases her data, she cannot be targeted by either firm and thus has two choices: obtaining utility $u = v_A - tx_i - \alpha_i$ from A_i or obtaining utility $u = v_A - tx_j - \alpha_j$ from A_j. We maintain the assumption that the value of v_A is large such that the market is fully covered.

The whole game proceeds as follows. The two firms in market B simultaneously decide their uniform prices, and consumers make their purchase decisions in market B after observing the prices. After the purchase decisions in market B, consumers recognize their product preferences over A_1 and A_2 in market A, and firms collect customer data through the consumer usage of the products in market B. Then, consumers simultaneously decide whether to erase their data. In market A, the two firms simultaneously post publicly observable uniform prices. Thereafter, firm A_i offers private personalized prices to its targeted consumers. After observing all available offers, consumers make purchasing decisions in market A. The sequential timing of price offers in market A is standard in the literature on personalized pricing (Thisse and Vives, 1988; Shaffer and Zhang, 2002; Choe et al., 2018), and reflects the flexibility in choosing personalized prices

Ma, 2003; Olivella and Vera-Hernández, 2007; Chen et al., 2022).

and allows us to solve for the subgame perfect Nash equilibrium in pure strategies.

We consider two cases regarding consumer expectations of the outcome in market A when they make purchase decisions in market B. In the myopic case, consumers do not expect their subsequent choices in the adjacent market and base their decisions solely on utility in market B. In the forward-looking case, consumers correctly recognize their expected surpluses in market A and make decisions based on their total surpluses in markets A and B.

We make several simplifying assumptions in the main model and relax them in Section 5. First, consumer data in the main model are used only for price discrimination in market A (i.e., to divide the "pie") and not to enlarge the pie by creating data-enhanced products. Moreover, consumers do not incur any privacy costs, even if they do not manage their personal data. These assumptions are relaxed in Section 5.1. Second, in addition to data erasure, we consider data portability as an extra option for privacy management in Section 5.2. Third, market size is fixed in the main model, regardless of privacy policies. Privacy-sensitive consumers may leave the market if privacy management is absent and return when it is available. This case is discussed in Section 5.3. Finally, Section 5.4 examines an alternative privacy setting where consumers own data property rights so that they opt out by default and opt in by choice.

The main model assumes that the market structure is symmetric. In an asymmetric market structure in which firm A_2 and firm B_2 are independent and there is no data transfer between them, our key insights remain valid, as shown in Section 4 of the online appendix.

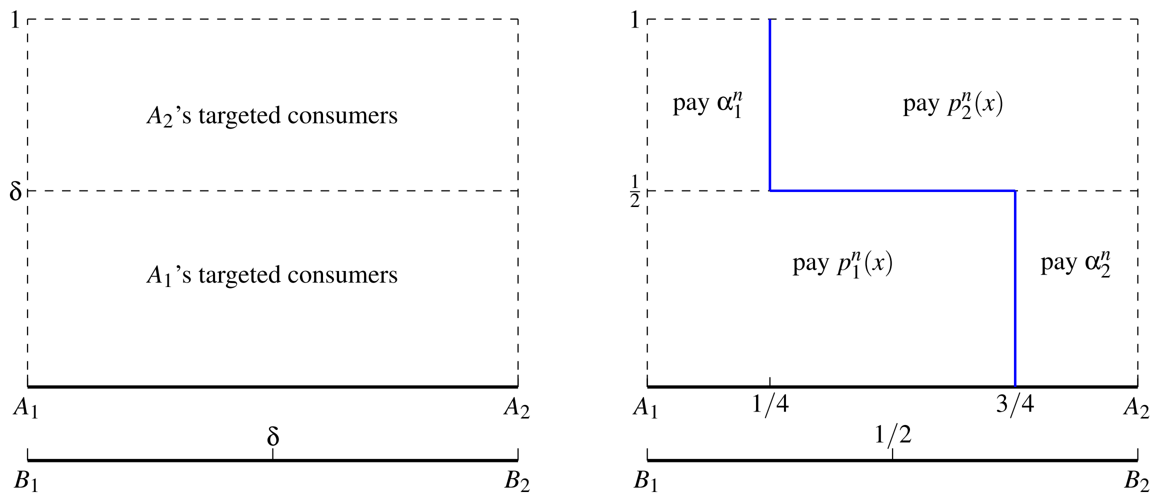
Four additional extensions are available in the online appendix: some consumers not purchasing in market B; consumers in market A following a nonuniform distribution; consumers in market B choosing whether to consent to data collection; and preferences of a part of consumers have perfect correlations across markets.[15]

[15]Our insights are robust when some consumers purchase nothing in market B and when consumers' preference x in market A follows another distribution, which includes the uniform distribution as a special case. Moreover, we discuss two other privacy management settings, ex ante and sequential privacy management, in which firms need consumers' consent to collect their data in market B. In ex ante privacy management, consumers cannot erase their data in market A and thus need to decide whether to agree to data collection and application before collection begins. Consumers in sequential privacy management decide whether to consent to data collection in market B and then decide whether to consent to data application in market A after data collection. We find that ex ante and sequential privacy management

4 Analysis

4.1 Benchmark: No privacy management

We start with the benchmark in which consumers cannot erase their data from firm B_i's datasets. In this case, if a consumer is firm B_i's targeted consumer, then she must be firm A_i's targeted consumer. Suppose that firm B_1 wins consumers on $[0, \delta]$ and that firm B_2 wins consumers on $[\delta, 1]$ in market B. As shown in Figure 1(a), at any point $x \in [0, 1]$ in market A, a segment, δ, of consumers is targeted by firm A_1, and the remaining $1 - \delta$ segment is targeted by firm A_2. In other words, at any point x in market A, firm A_i's targeted consumers are firm A_j's untargeted consumers.



(a) Targeted consumers without privacy management

(b) Equilibrium market outcome

Figure 1: Market structure and equilibrium without privacy management

Firm A_i sets personalized prices $p_i(x)$ for targeted consumers and a uniform price, α_i, for untargeted consumers. The equilibrium prices are the same as those in Thisse and Vives (1988):

$$\alpha_1^n = \alpha_2^n = t/2, \tag{1}$$

leads to lower consumer surplus compared to the setting where firms do not request consent to collect consumer data. When some consumers' locations are the same in both markets, they still erase data if they strongly prefer firm A_i. Therefore, our key insights remain robust.

$$p_1^n(x) = \begin{cases} 2t(3/4 - x) & \text{if } x \leq 3/4, \\ 0 & \text{if } x \geq 3/4, \end{cases} \qquad p_2^n(x) = \begin{cases} 0 & \text{if } x \leq 1/4, \\ 2t(x - 1/4) & \text{if } x \geq 1/4, \end{cases} \tag{2}$$

where superscript n indicates no privacy management. Firm A_1 wins consumers to the left of the blue lines in Figure 1(b). Concretely, firm A_1 wins its targeted consumers on $[0, 3/4]$ and wins the rival's targeted consumers on $[0, 1/4]$. Firm A_2 wins the remaining consumers. Therefore, the two firms' profits are $\pi_{A_1}^n = t(2 + 7\delta)/16$ and $\pi_{A_2}^n = t(9 - 7\delta)/16$, which increase with their market shares in market B.

We now focus on the equilibrium analysis of market B. First, we derive the ex ante expected surplus of firm B_1's consumers in market A, $E[CS_{B_1}]$, as follows:

$$E[CS_{B_1}] = \int_0^{3/4} (v_A - p_1^n(x) - tx)dx + \int_{3/4}^1 (v_A - \alpha_2^n - t(1-x))dx = v_A - t.$$

Similarly, we have that $E[CS_{B_2}] = v_A - t$. The indifferent consumer δ in market B is determined by

$$v_B - \beta_1 - t\delta + gE[CS_{B_1}] = v_B - \beta_2 - t(1 - \delta) + gE[CS_{B_2}]. \tag{3}$$

Parameter $g \in \{0, 1\}$ specifies the extent of consumers' *foresight* regarding the expected outcome in market A, in which $g = 0$ indicates that consumers are myopic, while $g = 1$ indicates that consumers are forward looking. The indifferent consumer is $\delta = (t + \beta_2 - \beta_1)/(2t)$, as in the standard Hotelling model. The profits of firms B_1 and B_2 are $\pi_{B_1}^n = \beta_1\delta$ and $\pi_{B_2}^n = \beta_2(1 - \delta)$, respectively. Firms 1 and 2 decide their uniform prices β_i to maximize two-market profits: $\Pi_1^n = \pi_{A_1}^n + \pi_{B_1}^n$ and $\Pi_2^n = \pi_{A_2}^n + \pi_{B_2}^n$, respectively. The equilibrium uniform prices in market B are $\beta_1^n = \beta_2^n = 9t/16$, implying that the indifferent consumer is $\delta^n = 1/2$. We have that $\pi_{B_1}^n = \pi_{B_2}^n = 9t/32$ and $\pi_{A_1}^n = \pi_{A_2}^n = 11t/32$. The equilibrium profits of firms 1 and 2 are $\Pi_1^n = \Pi_2^n = 5t/8$.

4.2 Equilibrium under privacy management

Consumers in this section can manage privacy by erasing their data from firm B_i's datasets. We start by characterizing consumers' data erasure strategies. Consumers make privacy management decisions based on anticipated prices in market A, i.e., α_1^a, α_2^a, $p_1^a(x)$, and $p_2^a(x)$, where superscript a indicates anticipation. Moreover, we focus on the case in which firms in market A use pure strategies in uniform pricing. Lemma 1 characterizes what kinds of consumers choose to erase their data.

Lemma 1. *Given consumers' price anticipations in market A, firm B_1's consumers erase data if and only if their locations in market A are smaller than $\tilde{x}_1$, and firm B_2's consumers erase data if and only if their locations in market A are larger than $\tilde{x}_2$, in which*

$$\tilde{x}_1 \equiv \frac{1}{2} + \frac{\alpha_2^a - \alpha_1^a}{2t} - \frac{\varepsilon}{2t} \text{ and } \tilde{x}_2 \equiv \frac{1}{2} + \frac{\alpha_2^a - \alpha_1^a}{2t} + \frac{\varepsilon}{2t}. \tag{4}$$

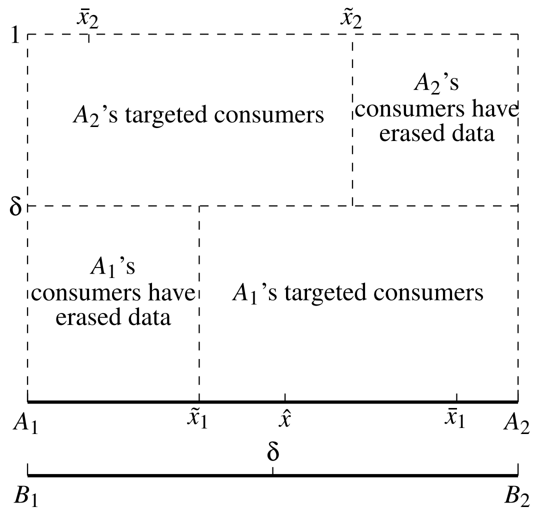
Lemma 1 states that consumers who match well with a firm in market A erase data. Figure 2(a) shows these opt-out consumers, who face very high personalized prices if they do not erase their data.[16] As an example, let us consider a consumer of firm B_1. If she erases her data, then she receives utility $v_A - tx - \alpha_1^a - \varepsilon$ or utility $v_A - t(1-x) - \alpha_2^a - \varepsilon$ in market A. Otherwise, she receives utility $v_A - tx - p_1^a(x)$ or utility $v_A - t(1-x) - \alpha_2^a$. When her location x is smaller than $\bar{x}_1 \equiv 1/2 + \alpha_2^a/(2t)$, in which $\bar{x}_1$ is indifferent between firm A_1's zero personalized price and firm A_2's uniform price, firm A_1 charges the optimal personalized price $p_1^a(x) = \alpha_2^a + t(1-2x)$, equalizing $v_A - tx - p_1^a(x)$ and $v_A - t(1-x) - \alpha_2^a$, and wins over the consumer. Therefore, consumer $x \in [0, \bar{x}_1]$ does not erase her data if and only if

$$v_A - tx - p_1^a(x) \geq \max\{v_A - tx - \alpha_1^a, v_A - t(1-x) - \alpha_2^a\} - \varepsilon,$$

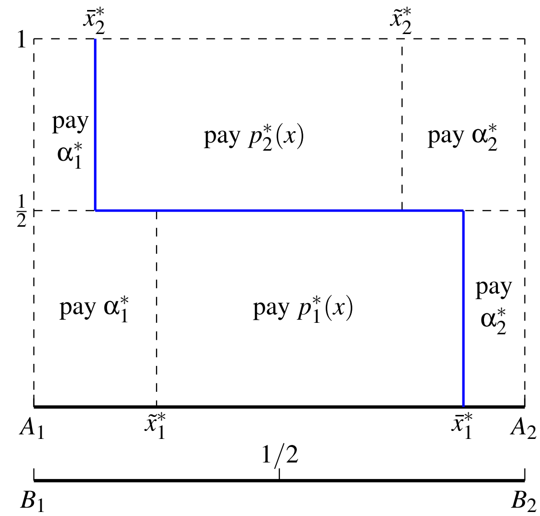
which is equivalent to $x > \tilde{x}_1$. When the consumer's location x is larger than $\bar{x}_1$ (i.e., she strongly prefers A_2's product), she purchases from firm A_2 under price α_2 and does not erase data because erasing data does not generate any benefit but brings costs ε.

As shown in Figure 2(a), after privacy management, firm A_i's untargeted consumers are those who have purchased from firm B_j and those who have purchased from firm B_i and have erased

[16]Several recent empirical studies on consumer privacy endorse Lemma 1. Chen and Gal (2021) use experiments to find that consumers have greater concerns over their privacy when they perceive that they do not receive fair value created from their personal information and are less willing to disclose their data. Through experiments, Lin (2022) finds that consumers' instrumental preference for privacy, which is endogenous to how the firm utilizes consumer data to generate targeting outcomes, significantly affects consumers' data sharing with the firm.



(a) Targeted consumers after privacy management



(b) Equilibrium market outcome

Figure 2: Market structure and equilibrium under privacy management

their data (i.e., opt-out consumers). Firm A_i's targeted consumers are firm B_i's consumers who choose to opt in.

We now analyze the equilibrium prices in market A after consumers engage in privacy management. Suppose that two firms' uniform prices are α_1 and α_2. Firm A_1's opt-out consumers purchase from firm A_1 if and only if $x < \hat{x} \equiv \frac{1}{2} + \frac{\alpha_2 - \alpha_1}{2t}$. Consumers' price anticipations should be correct in equilibrium due to rational expectations, implying that inequality $\tilde{x}_1 < \hat{x} < \tilde{x}_2$ should hold in equilibrium.

We formulate the objectives of firms A_1 and A_2 to derive their optimal uniform prices. Firm A_1 wins the rival's targeted consumers $x < \max\{\bar{x}_2, 0\}$ with uniform price α_1, in which $\bar{x}_2 \equiv 1/2 - \alpha_1/(2t)$. When $\alpha_1 \geq t$, firm A_1 forgoes poaching the rival's targeted consumers. Therefore, firm A_1's profit from its uniform price α_1 is

$$\begin{cases} \alpha_1 \left[(1-\delta) \left(\dfrac{1}{2} - \dfrac{\alpha_1}{2t} \right) + \delta \tilde{x}_1 \right] & \text{when } \alpha_1 \leq t \\ \alpha_1 \delta \tilde{x}_1 & \text{when } \alpha_1 \geq t \text{ and } \hat{x} \geq \tilde{x}_1. \end{cases}$$

Given the determined $\tilde{x}_1$, firm A_1's local optimal price for the second case of the above profit function is α_1 such that $\hat{x} = \tilde{x}_1$, which violates the definition of $\tilde{x}_1$ in (4) because $\tilde{x}_1 = \hat{x} - \varepsilon/(2t)$.

Therefore, only the first case can be sustainable in equilibrium. Similarly, firm A_2's profit from its uniform price α_2 is

$$\begin{cases} \alpha_2 \left[\delta\left(\frac{1}{2} - \frac{\alpha_2}{2t}\right) + (1-\delta)(1-\tilde{x}_2)\right] & \text{when } \alpha_2 \leq t \\ \alpha_2(1-\delta)(1-\tilde{x}_2) & \text{when } \alpha_2 \geq t \text{ and } \hat{x} \leq \tilde{x}_2. \end{cases}$$

Only the first case can be sustainable in equilibrium.

Given $\tilde{x}_1$ and $\tilde{x}_2$, the two firms' optimal uniform prices are $\alpha_1 = \frac{t}{2} + \frac{t\tilde{x}_1\delta}{1-\delta}$ and $\alpha_2 = \frac{t}{2} + \frac{t(1-\tilde{x}_2)(1-\delta)}{\delta}$. Since consumers have rational expectations (i.e., $\alpha_1^a = \alpha_1$ and $\alpha_2^a = \alpha_2$), based on (4), we have that

$$\alpha_1^* = \frac{t}{2} + \delta(t-\varepsilon), \quad \alpha_2^* = \frac{t}{2} + (1-\delta)(t-\varepsilon), \tag{5}$$

and

$$\tilde{x}_1^* = \frac{(1-\delta)(t-\varepsilon)}{t}, \quad \tilde{x}_2^* = 1 - \frac{\delta(t-\varepsilon)}{t}. \tag{6}$$

Given any $\delta \in [0,1]$, we need to make the equilibrium cutoffs (in Figure 2(b)) lie between zero and one, which requires $t/2 \leq \varepsilon \leq t$, and ensure that no firm has an incentive to deviate, which requires assumption 1.[17]

Assumption 1. *We assume that* $\sqrt{3}t/2 \leq \varepsilon \leq t$*.*

Under this assumption, the share of consumers erasing data is less than 7% on the equilibrium path, moderately aligning with real-world consumer behaviors on privacy management.[18] For instance, the GDPR decreases total recorded online searches by 10.7% (Aridor et al., 2022). When ε approaches its upper bound t, the number of opt-out consumers approaches zero, and the equilibrium under privacy management converges to that under no privacy management.

In comparison to uniform prices under no privacy management in (1), uniform prices under privacy management in (5) increase. The intuition behind this result is as follows. Firm A_i uses

[17]Section A.1 of the appendix shows how Assumption 1 is derived. When ε is small (i.e., $\varepsilon < \sqrt{3}t/2$), many consumers choose to erase data due to the low cost. As a result, firm A_i has the incentive to deviate from the equilibrium uniform price unilaterally under some δ. Specifically, when δ is large, firm A_1 has many opt-out consumers. It will profitably deviate to a uniform price α_1 higher than t to exploit its nearby opt-out consumers better and refrain from poaching the rival's targeted consumers. On the other hand, when δ is small, firm A_2 will profitably deviate to a uniform price α_2 higher than t to exploit its nearby opt-out consumers better.

[18]The equilibrium δ^* is equal to $1/2$, as we show later.

its uniform price to serve its own opt-out consumers, who strongly prefer the firm, and to poach the rival's targeted consumers. Its uniform price needs to compete with the rival's uniform price α_j for the first batch of consumers and compete with the rival's personalized prices $p_j(x)$ for the second batch. The rival's uniform price α_j is higher than the flexible personalized price $p_j(x)$ for marginal consumers, implying that opt-out consumers are less price elastic. Therefore, opt-out consumers' high willingness to pay and low price elasticity make firm A_i a "fat cat" in uniform price competition (Fudenberg and Tirole, 1984).[19] As shown in Figure 3, the uniform price charged by firm A_2 increases from α_2^n to α_2^*.

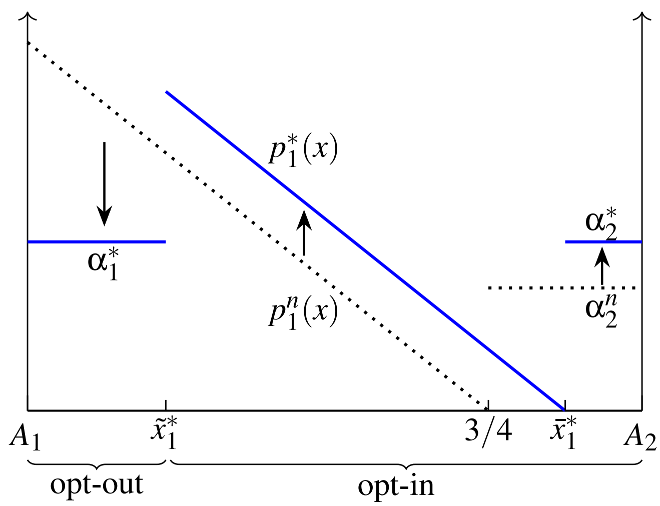


Figure 3: Comparison of equilibrium prices paid by A_1's consumers

Each firm's personalized price competes with the rival's uniform price, implying that $p_1^*(x) = \alpha_2^* + t(1-2x)$ and $p_2^*(x) = \alpha_1^* + t(2x-1)$. Therefore, firms' personalized prices also increase under privacy management. In Figure 3, firm A_1's personalized prices increase from $p_1^n(x)$ to $p_1^*(x)$ due to the inflated α_2^* under privacy management.

Proposition 1. *When consumers can manage their privacy, firms in the data application market charge higher uniform and personalized prices compared to the benchmark without privacy*

[19]When the market structure is asymmetric such that firm A_2 and firm B_2 are independent and there is no data transfer between them, firm A_1's uniform price increases under privacy management due to the same intuition. Firm A_2's uniform price also increases under privacy management because prices are strategic complements.

management. Moreover, a firm's price increases are larger

(i) when the privacy management cost ε declines or

(ii) when the firm's market share in the data collection market expands.

Compared to uniform prices under no privacy management, the terms $\delta(t-\varepsilon)$ and $(1-\delta)(t-\varepsilon)$ in (5) represent the increases in uniform prices due to opt-out consumers. A lower ε leads to larger uniform price increases because more consumers opt out, as in (6), and the firm becomes more concerned about opt-out consumers who strongly prefer the firm when deciding its uniform price.

Related to the changes in uniform prices, a larger δ leads to a higher α_1^* and a lower α_2^* because firm A_1 focuses more on its opt-out consumers, whose reservation values are high, and firm A_2 focuses more on its rival's opt-in consumers and charges a lower uniform price to poach these consumers. The changes in uniform prices lead to lower $p_1^*(x)$ and higher $p_2^*(x)$. The effect of δ on prices becomes stronger as the privacy management cost decreases because consumers are more likely to opt out.

Consumers' incentives to opt out are directly impacted by δ and ε. A larger δ leads to a higher α_1^* and a lower α_2^*, which implies lower personalized prices $p_1^*(x) = \alpha_2^* + t(1-2x)$. Therefore, firm A_1's consumers have weaker incentives to opt out because the benefits of escaping personalized prices decline. Formally, $\tilde{x}_1^*$ in (6) decreases with δ. Conversely, a higher δ leads to higher $p_2^*(x) = \alpha_1^* + t(2x-1)$ and a lower α_2^*, increasing the incentives of firm A_2's consumers to opt out. Therefore, $\tilde{x}_2^*$ in (6) decreases with δ. As before, a decrease in ε strengthens the effects of δ on $\tilde{x}_1^*$ and $\tilde{x}_2^*$.

Corollary 1. *As the value of δ increases,*

(i) α_1^* and $p_2^*(x)$ increase, and α_2^* and $p_1^*(x)$ decrease;

(ii) firm A_1 consumers' incentives to opt out weaken, and firm A_2 consumers' incentives to opt out strengthen (i.e., $\tilde{x}_1^*$ and $\tilde{x}_2^*$ decrease); and

(iii) the impacts of δ in (i) and (ii) become stronger as the value of ε decreases.

A firm cannot win its targeted consumers who strongly prefer the rival's product, even if its personalized price is zero, as shown in Figure 2(b). Specifically, firm A_1 loses its targeted consumers on $[\bar{x}_1^*, 1]$, and firm A_2 loses its targeted consumers on $[0, \bar{x}_2^*]$. In equilibrium, we obtain $\bar{x}_1^* = \frac{3}{4} + \frac{(1-\delta)(t-\varepsilon)}{2t}$ and $\bar{x}_2^* = \frac{1}{4} - \frac{\delta(t-\varepsilon)}{2t}$.

Opt-out consumers in market A bring negative externalities on other opt-in consumers through price and mismatch channels, although those opt-out consumers pay strictly lower uniform prices than personalized opt-in prices under no privacy management, as illustrated in Figure 3. Compared to no privacy management, a firm's opt-in consumers either pay higher personalized prices or pay the rival firm's higher uniform price under privacy management. Furthermore, higher uniform prices help firms protect their targeted consumers by personalized pricing, allowing firms to retain lukewarm consumers who would switch if privacy management were not feasible. For instance, as shown in Figure 3, firm A_1 keeps its targeted consumers on $[3/4, \bar{x}_1^*]$ only under privacy management.

Corollary 2. *Opt-out consumers in market A pay strictly lower prices than under no privacy management but cause negative externalities on opt-in consumers in market A by increasing their prices or making them incur higher mismatch costs.*

Finally, the equilibrium profits of firms A_1 and A_2 in market A are as follows:

$$\pi_{A_1} = \alpha_1^*[\delta\tilde{x}_1^* + (1-\delta)\bar{x}_2^*] + \delta \int_{\tilde{x}_1^*}^{\bar{x}_1^*} p_1^*(x)dx,$$

$$\pi_{A_2} = \alpha_2^*[(1-\delta)(1-\tilde{x}_2^*) + \delta(1-\bar{x}_1^*)] + (1-\delta) \int_{\bar{x}_2^*}^{\tilde{x}_2^*} p_2^*(x)dx.$$

Firm A_1's profit increases with δ, and firm A_2's profit increases with $1-\delta$. Their profits increase with privacy management cost ε under Assumption 1.

Lemma 2. *As ε decreases from its upper bound t, firm A_1's profit increases faster with $\delta \in [1/2, 1]$, and firm A_2's profit increases faster with $(1-\delta) \in [1/2, 1]$, implying that compared to no privacy management, privacy management makes a firm's profit in the data application market increase* *faster with its market share in the data collection market.*

Lemma 2 says that as ε declines from t, more consumers erase data in market A, resulting in firm A_i's profit increasing with its consumer base faster. The intuition behind Lemma 2 is as follows. Based on Corollary 1(iii), as the value of ε decreases, an increase in δ further enlarges α_1^*, which makes the surplus extraction with uniform price more efficient. At the same time, a larger δ further diminishes the range of A_1's opt-out consumers (i.e., $[0, \tilde{x}_1^*]$), and thus, A_1 can price discriminate against more consumers with a high willingness to pay with personalized prices. These two effects increase the profitability of A_1. However, as the value of ε decreases, an increase in δ diminishes the range in which A_1 serves A_2's targeted consumers (i.e., $\bar{x}_2^*$) more and decreases A_1's personalized prices $p_1^*(x)$ more. These two effects diminish the profitability of A_1. We can check that the former two positive effects dominate the latter effects, i.e., that $\partial^2 \pi_{A_1}/\partial\delta\partial\varepsilon < 0$ holds if $\delta \geq 1/2$ and ε is not small.

Now, we turn our attention to the equilibrium analysis of market B. First, we derive the ex ante expected surplus of firm B_i's consumer in market A, $E[CS_{B_i}]$, as follows:

$$\begin{aligned}
E[CS_{B_1}] &= \int_0^{\tilde{x}_1^*}(v_A - \alpha_1^* - tx - \varepsilon)dx + \int_{\tilde{x}_1^*}^{\bar{x}_1^*}(v_A - p_1^*(x) - tx)dx + \int_{\bar{x}_1^*}^{1}(v_A - \alpha_2^* - t(1-x))dx \\
&= v_A - (1+\delta-\delta^2)t + (1-\delta)(2\delta-1)\varepsilon + \frac{(1-\delta)^2\varepsilon^2}{t}, \\
E[CS_{B_2}] &= \int_0^{\bar{x}_2^*}(v_A - \alpha_1^* - tx)dx + \int_{\bar{x}_2^*}^{\tilde{x}_2^*}(v_A - p_2^*(x) - t(1-x))dx + \int_{\tilde{x}_2^*}^{1}(v_A - \alpha_2^* - t(1-x) - \varepsilon)dx \\
&= v_A - (1+\delta-\delta^2)t - \delta(2\delta-1)\varepsilon + \frac{\delta^2\varepsilon^2}{t}.
\end{aligned}$$

The difference between $E[CS_{B_1}]$ and $E[CS_{B_2}]$ is

$$E[CS_{B_1}] - E[CS_{B_2}] = \frac{(t-\varepsilon)\varepsilon(2\delta-1)}{t} > 0 \text{ if and only if } \delta > \frac{1}{2}. \tag{7}$$

The firm with a larger market share in market B provides a higher expected consumer surplus to its customers in market A.[20]

[20] It is straightforward to check that $E[CS_{B_1}]$ increases with firm B_1's market share δ and that $E[CS_{B_2}]$ increases with firm B_2's market share $1-\delta$ under Assumption 1. If firm B_1 earns a larger market share (i.e., a larger δ value), then firm A_1's uniform price α_1^* increases and the rival's α_2^* decreases, which leads to a lower $p_1^*(x)$. As a result, it is ex ante less likely for B_1's consumers to opt out and they are more likely to enjoy the lower $p_1^*(x)$ in market A, leading to a higher $E[CS_{B_1}]$.

The indifferent consumer δ in market B is determined the same way as in (3), which leads to $\delta = \frac{1}{2} + \frac{t(\beta_2-\beta_1)}{2(t^2-g(t-\varepsilon)\varepsilon)}$. Demand in market B is more price elastic if consumers are forward looking (i.e., $g = 1$). Firms' profits in market B are $\pi_{B_1} = \beta_1\delta$ and $\pi_{B_2} = \beta_2(1-\delta)$. Firms 1 and 2 decide their uniform prices β_i to maximize two-market profits: $\Pi_1 = \pi_{A_1} + \pi_{B_1}$ and $\Pi_2 = \pi_{A_2} + \pi_{B_2}$. The equilibrium uniform prices in market B are

$$\beta_1^* = \beta_2^* = \frac{(2t+\varepsilon)(4t-\varepsilon)}{16t} - \frac{g(t-\varepsilon)\varepsilon}{t}, \tag{8}$$

implying that $\delta^* = 1/2$. The equilibrium profits of firms B_1 and B_2 are $\pi^*_{B_1} = \pi^*_{B_2} = (2t+\varepsilon)(4t-\varepsilon)/(32t) - g(t-\varepsilon)\varepsilon/(2t)$.

The equilibrium outcomes in market A are determined by replacing δ with $1/2$. Marginal consumers who are indifferent between erasing and keeping their data are

$$\tilde{x}_1^* = \frac{1}{2} - \frac{\varepsilon}{2t}, \quad \tilde{x}_2^* = \frac{1}{2} + \frac{\varepsilon}{2t}. \tag{9}$$

We have that $\bar{x}_1^* = 1 - \varepsilon/(4t)$ and $\bar{x}_2^* = \varepsilon/(4t)$. The equilibrium prices in market A are

$$\alpha_1^* = \alpha_2^* = t - \varepsilon/2. \tag{10}$$

$$p_1^*(x) = \begin{cases} 2t(1-x) - \varepsilon/2 & \text{when } \tilde{x}_1^* < x < \bar{x}_1^*, \\ 0 & \text{when } \bar{x}_1^* \le x, \end{cases} \quad p_2^*(x) = \begin{cases} 0 & \text{when } x \le \bar{x}_2^*, \\ 2tx - \varepsilon/2 & \text{when } \bar{x}_2^* < x < \tilde{x}_2^*. \end{cases} \tag{11}$$

The equilibrium profits of firms A_1 and A_2 are $\pi^*_{A_1} = \pi^*_{A_2} = (12t^2 + 3\varepsilon^2 - 4t\varepsilon)/(32t)$. The equilibrium profits of firms 1 and 2 are $\Pi_1^* = \Pi_2^* = (10t^2 + \varepsilon^2 - t\varepsilon)/(16t) - g(t-\varepsilon)\varepsilon/(2t)$. Figure 2(b) shows the equilibrium outcomes in the two markets.

Compared with the benchmark without privacy management, equilibrium prices in market B decrease when privacy management is available. This is because firm A_i's profit grows faster with B_i's market share (Lemma 2) under privacy management, motivating firm B_i to compete more aggressively. Specifically, as a firm's market share in market B becomes larger, the firm's uniform price in market A becomes higher because it faces a larger number of opt-out consumers with a higher willingness to pay (Corollary 1(i)). This higher uniform price diminishes the incentives of marginal consumers to erase their data because the gains from escaping personalized prices are lower (Corollary 1(ii)). Thus, the firm can price discriminate against more consumers with a high willingness to pay and efficiently extract their surplus. The inflated personalized prices in market A under privacy management further facilitate surplus extraction. Therefore, firms in market B compete more aggressively to obtain a larger market share.

Proposition 2. *Compared to no privacy management, when consumers can manage their privacy, price competition in market B intensifies and becomes more severe as consumers' foresight regarding market A improves (i.e., $g = 1$).*

As consumers care more about their anticipated surplus in market A ($g = 1$), price competition in market B becomes more intense. The intuition is as follows. The firm with a larger market share in market B provides a higher expected consumer surplus in market A (see (7)). When consumers are forward looking, firm B_i with a larger market share is more likely to win consumers, leading to more fierce price competition in market B. In contrast, when consumers are myopic (i.e., $g = 0$) or do not manage their privacy (i.e., $\varepsilon = t$), the utility difference between $E[CS_{B_1}]$ and $E[CS_{B_2}]$ vanishes in consumers' purchase decisions in market B.

Corollary 3. *As privacy management cost ε increases, equilibrium prices under privacy management decrease in market A and increase in market B.*

As ε increases, fewer consumers choose to erase their data in the equilibrium, leading to intensified uniform price competition in market A and hence lower personalized prices. Based on Lemma 2, a higher privacy management cost leads to a lower marginal gain in market A from increasing customers in market B. Thus, a higher privacy management cost mitigates uniform price competition in market B. Since we can interpret no privacy management as the polar case in which no consumer erases data because of prohibitively high costs (i.e., $\varepsilon = t$), Corollary 3 immediately implies the price changes under privacy management in both markets (Proposition 1 and Proposition 2).

Proposition 3. *When consumers can manage their privacy, all firms earn lower profits than they would under no privacy management, and their profits increase with privacy management cost ε.*

Interestingly, all firms' profits under privacy management increase with privacy management cost ε, despite that it is a deadweight loss.[21] Three forces lead to this result. First, as ε increases, fewer consumers opt out, allowing firms to price discriminate against more consumers with a high willingness to pay. Second, firm A_i attracts more of the rival's targeted consumers with α_i^* and loses more of its targeted consumers. The gain outweighs the loss because the price α_i^* charged to poached consumers is higher than $p_i^*(x)$ for its lost distant targeted consumers. These two forces bring firms higher profits in market A. Third, based on Corollary 3, firms compete less aggressively and obtain higher profits in market B as ε increases.

4.3 Impacts of privacy management on welfare

We compare consumer surplus in the two markets under privacy management with that under no privacy management. The consumer surplus without privacy management in market A is $CS_A^n = v_A - t$, which declines to $CS_A^* = v_A - (5t^2 - \varepsilon^2)/(4t)$ under privacy management because of the weakened price competition (Proposition 1). In more detail, opt-in consumers become worse off (Corollary 2). The surplus of opt-out consumers remains the same as in the benchmark because some opt-out consumers are better off by avoiding high personalized prices, while others are worse off by paying inflated uniform prices and incurring privacy management costs.

The consumer surplus in market B without privacy management is $CS_B^n = v_B - 13t/16$, which increases to $CS_B^* = v_B - (12t^2 + 2t\varepsilon - \varepsilon^2)/(16t) + g(t-\varepsilon)\varepsilon/t$ under privacy management because the price competition intensifies (Proposition 2). Additionally, consumer surplus CS_B^* and total consumer surplus $CS_A^* + CS_B^*$ increase with the degree of consumer foresight g. Concretely, if consumers are myopic about the outcome in market A ($g = 0$), then $CS_A^* + CS_B^* < CS_A^n + CS_B^n$ holds, and if they are forward looking ($g = 1$), then $CS_A^* + CS_B^* > CS_A^n + CS_B^n$ holds. These two inequalities suggest that providing consumers with information about how their data will be

[21] This result helps explain why Facebook and Google deliberately increase consumers' costs to delete cookies (https://www.cnil.fr/en/cookies-cnil-fines-google-total-150-million-euros-and-facebook-60-million-euros-non-compliance). In January 2022, CNIL, the Data Protection Authority for France, fined Google and Facebook 150 million and 60 million euros, respectively, for "not providing an equivalent solution (button or other) enabling the Internet user to easily refuse the deposit of these cookies".

used benefits them by intensifying competition.[22]

Social welfare in market A without privacy management is $SW_A^n = v_A - 5t/16$ and declines to $SW_A^* = v_A - (8t^2 - 7\varepsilon^2 + 4t\varepsilon)/(16t)$ under privacy management because more consumers mismatch with the less preferred firm. The costs of privacy management, a deadweight loss, further diminish social welfare. Social welfare in market B is always equal to $v_B - t/4$, regardless of privacy management. Therefore, total social welfare in the two markets decreases under privacy management.

Proposition 4. *Compared to no privacy management, when consumers can manage their privacy, the total social welfare in the two markets declines; the total consumer surplus declines if consumers are myopic about the outcome in market A and increases if they are forward looking. Specifically,*

(i) opt-out consumers' surplus in market A does not change, and opt-in consumers in market A are worse off, leading to lower consumer surplus in market A, and consumer surplus in market B increases;

(ii) social welfare declines in market A and does not change in market B.

Propositions 3 and 4 conclude that compared to no privacy management, firms' profits, total consumer surplus, and total social welfare can all be negatively affected under privacy management if consumers are myopic about the outcome in market A. In this case, only some opt-out consumers are better off. The resulting changes in firms' competition in both markets harm other agents and society. In other words, privacy management leads to simultaneous surplus redistribution and destruction.

Corollary 4. *Under privacy management, as cost ε increases,*

(i) consumer surplus increases in market A and decreases in market B, and the total consumer

[22] An experimental study by Lin (2022) indicates that consumers are able to engage in strategic reasoning when making data-sharing decisions and that their beliefs are accurate to the first order only when the information environment is transparent.

surplus of the two markets increases (decreases) if consumers are myopic (forward looking), and

(ii) social welfare in market A increases, and the total social welfare of the two markets increases.

Corollary 4 states how privacy management cost ε affects welfare. A higher ε directly negatively affects welfare because it is a deadweight loss. In addition, it impacts welfare by affecting consumers' decision to opt out and subsequent market competition. In market B, a higher ε hurts consumer surplus by mitigating competition (Corollary 3). In market A, a higher ε reduces the number of opt-out consumers and intensifies price competition, enhancing consumer surplus; social welfare improves as well due to less consumer-firm mismatch.

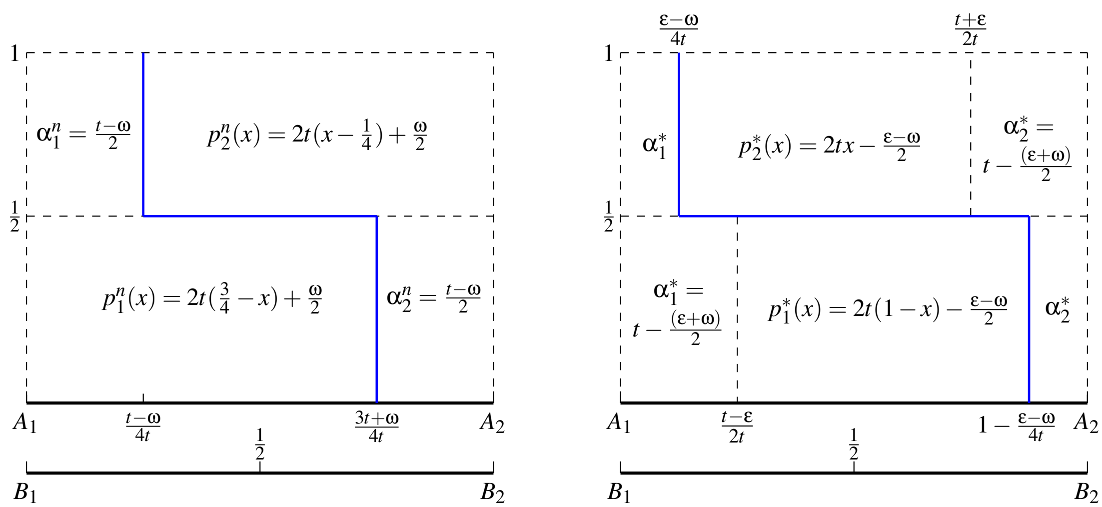
Consumers' foresight influences how ε affects consumer surpluses in the two markets. When consumers are forward looking, an increase in ε diminishes the difference between $E[CS_{B_1}]$ and $E[CS_{B_2}]$ (see (7)), which mitigates price competition in market B because obtaining market share yields a smaller advantage in attracting consumers. Therefore, when consumers are forward looking, they lose more surplus in market B as ε increases.

In fact, as ε approaches its upper bound t, the number of opt-out consumers converges to zero, and the welfare level under privacy management approaches that under no privacy management.

5 Extensions and discussions

5.1 Data-enabled product personalization and privacy costs

Now, we assume that firms can provide data-enabled personalized products and that consumers incur a privacy cost from their data being exploited. Concretely, if firm A_i's consumer does not erase her data, then firm A_i can utilize data analytics to offer her a personalized product that has a higher matching value for her than A_i's standard product. We parameterize this by $\Delta v > 0$ such that the consumer derives intrinsic value $v_A + \Delta v$ from the personalized product. However, the consumer incurs a privacy cost $c \geq 0$ from her data being exploited in market A. If the consumer erases data, then she cannot enjoy the personalized product but saves the privacy costs. We define $\omega = \Delta v - c$ as the net benefit of product personalization and adopt the following assumptions: $\max\{-3t, 5t - 2\varepsilon - 2v_A\} \leq \omega \leq 2\varepsilon - t$ and $\hat{t} \leq \varepsilon \leq t$.[23] We can establish equilibria when consumers cannot manage their privacy (the benchmark) and when they can. Figure 4 shows the equilibrium outcomes when consumers are myopic and forward looking.



(a) Equilibrium without data erasure

(b) Equilibrium with data erasure

Figure 4: Equilibrium under production personalization and privacy costs

Compared to the benchmark, firm A_i earns a lower profit under privacy management when benefit ω is large (i.e., $\omega > (t - 3\varepsilon)/6$). The intuition behind this finding is as follows. As ω rises, A_i's personalized prices without privacy management $p_i^n(x)$ increase and apply to more consumers because the rival's consumer poaching becomes more difficult, resulting in higher profits for the firm. In contrast, under privacy management, firm A_i's uniform price α_i^* decreases with ω. The firm's profit loss in inefficient surplus extraction from opt-out consumers rises with ω. As in the main model, firms in market B always earn lower profits under privacy management

[23]If ω is very large and consumers can retain part of the benefit under endogenous prices, then no consumer will erase her data. In this case, all of firm A_i's targeted consumers always purchase from the firm, implying the largest consumer-firm mismatch.

due to intensified competition.

Proposition 5. *Compared to no privacy management, when consumers can manage their privacy,*

(i) firms in market A earn lower profits if and only if the net benefit of product personalization is relatively large (i.e., $\omega > (t - 3\varepsilon)/6$), and firms in market B always earn lower profits, and

(ii) the welfare comparison results are identical to Proposition 4, except that opt-out consumers become strictly better off now.

In the equilibrium under privacy management, as ω increases, firm A_i has to lower its uniform price to poach the rival's targeted consumers, but its personalized prices increase and apply to more consumers. As a result, firm A_i's equilibrium profit increases with ω as long as it is not small (i.e., $\omega > 2t/3 - \varepsilon$). This relationship between ω and $\pi^*_{A_i}$ implies that a larger customer base generates higher profits in market A, thus accelerating competition in market B. Ultimately, a higher ω negatively affects firm i's total profits.

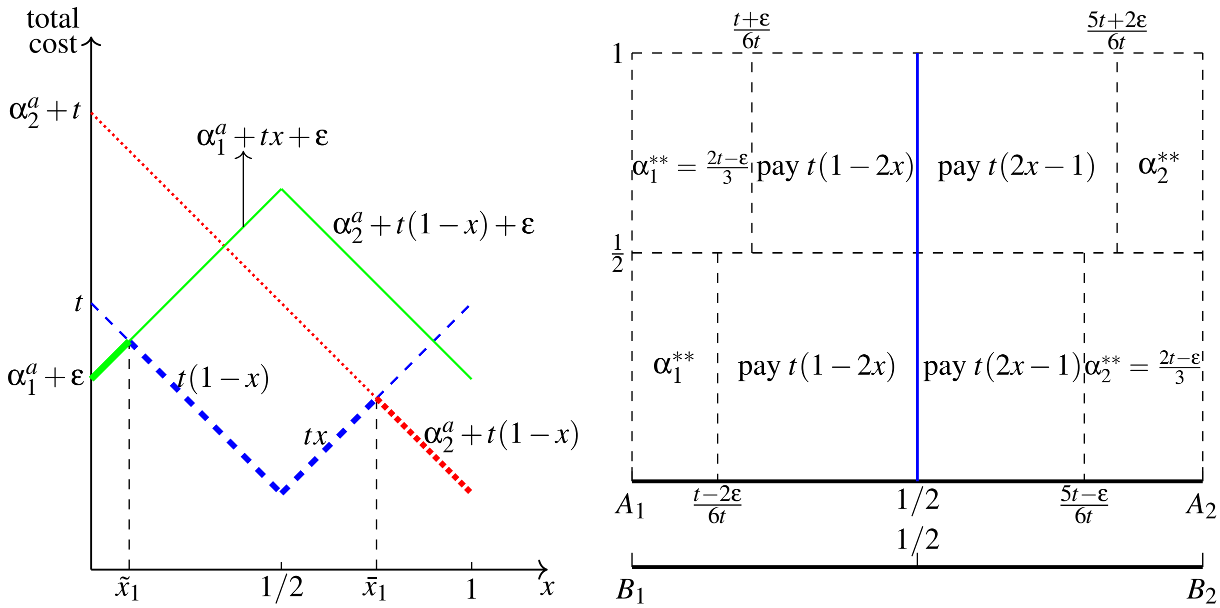
5.2 Data portability

We discuss the case where consumers can use data erasure and data portability in privacy management. Specifically, data portability means that firm B_i's targeted consumers can ask the firm to transfer their data to the rival firm at no cost.[24] Due to analytical difficulty, we assume that consumers are myopic about the outcome in market A. The benchmark equilibrium of no privacy management is the same as that in Section 4.1.

Firm A_1's consumer x chooses from four privacy management options to minimize her anticipated total cost (including price, transportation costs, and data erasure costs if applicable). First, if the consumer erases data and opts out, then her total cost is either $\alpha_1^a + tx + \varepsilon$ or $\alpha_2^a + t(1-x) + \varepsilon$, as shown by the solid green lines in Figure 5(a). Second, if consumer x chooses

[24] According to privacy laws, the responsibility of data portability falls mainly on the firm, and consumers do not incur high costs. For example, the guidelines in the GDPR state that "the overall system implementation costs should not be charged to the data subjects". The CCPA clearly requires no charge to consumers for data portability. The results do not change if we assume that data portability entails a small cost for consumers.

data portability and does not erase her data in A_1's datasets, then she is targeted by both firms. Her total cost is $t(1-x)$ when $x \leq 1/2$ and tx when $x \geq 1/2$ (dashed blue lines). Third, if consumer x chooses data portability and erases her data in A_1's datasets, then she is targeted only by firm A_2 and incurs total cost $\alpha_1^a + tx + \varepsilon$ (upward-sloping green line). Fourth, if she does not manage privacy, then her total cost is $\alpha_2^a + t(1-x)$ (dotted red line).



(a) Total costs

(b) Equilibrium outcome

Figure 5: Total costs and equilibrium outcome

In the equilibrium of market A, firm A_1's consumers on $[0, \tilde{x}_1]$ opt out by erasing data; consumers on $[\tilde{x}_1, \bar{x}_1]$ choose data portability and do not erase their data in firm A_1's datasets; and consumers on $[\bar{x}_1, 1]$ do not manage privacy and purchase from firm A_2. Firm A_2's consumers adopt a similar privacy management strategy. Figure 5(b) shows the equilibrium outcome.

Compared to no privacy management, equilibrium uniform prices in market A increase because of opt-out consumers. As a result, consumers who do not manage their privacy become worse off. Firm A_i's consumers who have chosen data portability benefit from fierce consumer-by-consumer price competition and are not affected by inflated uniform prices. Firms in market A earn lower profits than them under no privacy management. In addition to the inability to price discriminate against nearby opt-out consumers, fierce competition for consumers using data portability further reduces profits. Price competition in market B significantly mitigates as the value of targeted consumers in market A diminishes substantially. As a result, firms in market B obtain higher profits. In total, firms 1 and 2 earn higher profits under privacy management.

Consumer surplus in market A increases compared to that under no privacy management. However, consumer surplus in market B declines dramatically due to the mitigated price competition in this market, leading to a lower total consumer surplus. When data portability is available, consumers purchase from their preferred firm in both markets, implying that social welfare is maximized.

Proposition 6. *Suppose that consumers have access to data erasure and data portability in privacy management. Compared to no privacy management,*

(i) uniform prices increase in market A due to opt-out consumers, but personalized price competition intensifies;

(ii) firms in market A earn lower profits, and those in market B earn higher profits, resulting in higher profits for firms 1 and 2;

(iii) consumer surplus in market A increases, and that in market B and total consumer surplus decrease; and

(iv) social welfare in both markets achieves the maximum value.

5.3 Heterogeneous consumer privacy types and market failure

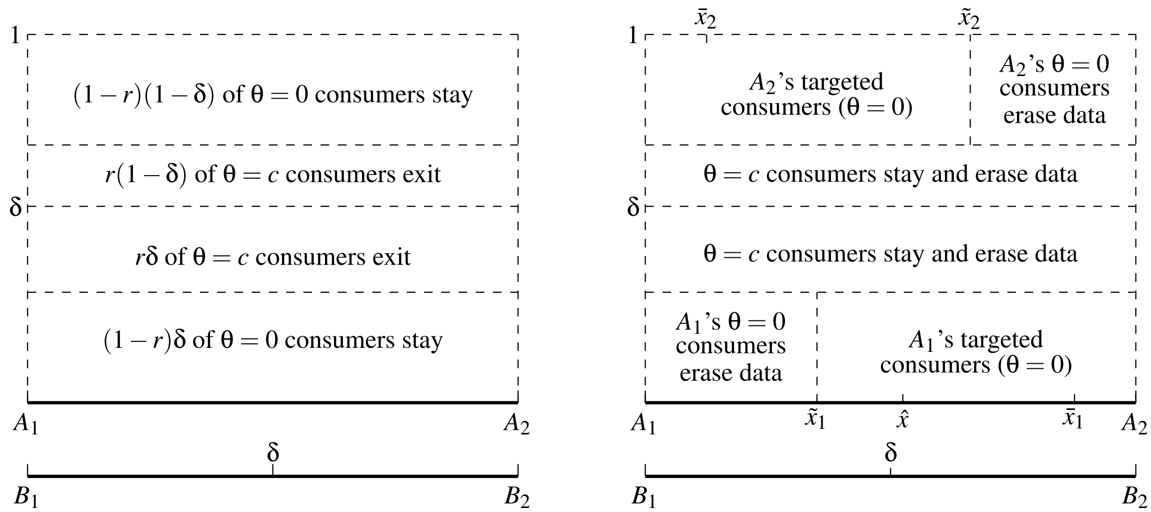
This section extends the main model by incorporating heterogeneous consumer privacy concerns. We focus on a scenario in which privacy-sensitive consumers may exit the market if they cannot manage their privacy, leading to market failure. Privacy management fixes such failure by attracting these consumers back to the market, thus producing a new type of "good". The return of these consumers, however, weakens price competition in the data application market and expands negative externalities on (privacy-insensitive) opt-in consumers, aggravating the "bad" and "ugly" aspects of privacy management.

To capture privacy types, we label a consumer in market A as (x, θ), where x is her location and $\theta \in \{0, c\}$ indicates her privacy cost, which is her private information. Privacy cost θ accrues in market A if her data are transferred and exploited in this market. With probability $r \in [0, 1]$, the consumer is a privacy-sensitive type with privacy cost $\theta = c$; with probability $1 - r$, the consumer is a privacy-insensitive type without privacy cost $\theta = 0$.

Before firm B_i transfers consumer data to firm A_i, anticipating the data transfer and privacy cost θ, its consumers can decide whether to exit market A. If a consumer exits market A, then her utility in this market becomes zero. Conversely, if she stays and does not erase her data, then firm A_i knows her (x, θ), and firm A_j ($j \neq i$) knows only the distribution of x and θ. Then, she is firm A_i's targeted consumer and has two choices: obtain utility $u = v_A - tx_i - p_i(x) - \theta$ from firm A_i or obtain utility $u = v_A - tx_j - \alpha_j - \theta$ from firm A_j. If she stays and erases her data, then the consumer does not incur any privacy costs and can escape targeting by either firm.

The game proceeds as in the main model except for the privacy management stage, in which all consumers simultaneously decide whether to exit the market and erase their data. To neatly illustrate our idea, we assume that privacy-sensitive consumers' privacy cost is very high: $c > v_A$. Moreover, when c is very high, the option of data portability is no longer relevant because consumers never choose it.

□ **Benchmark: No privacy management and market failure** Our first observation is that all privacy-sensitive consumers choose to exit market A because they receive negative utility from staying in the market due to the high privacy cost c. As shown in the left panel of Figure 6, firms A_1 and A_2 lose $r\delta$ and $r(1 - \delta)$ shares of privacy-sensitive consumers, causing "market failure." These firms have only privacy-insensitive consumers left in the market. The equilibrium analysis is the same as that in Section 4.1. However, equilibrium profits, consumer surplus, and social welfare all decline due to exiting consumers. The larger r and v_A are, the more significant the welfare losses.



(a) Consumers' distribution without privacy management (b) Consumers' distribution under privacy management

Figure 6: Distribution of consumers without and with privacy management

□ **Equilibrium under privacy management** Our analysis begins with the observation that when privacy management is available, all privacy-sensitive consumers stay in market A and erase their data at the cost of ε. Privacy-insensitive consumers' privacy management strategy is the same as that in Lemma 1. Figure 6(b) shows the distribution of opt-in consumers and those consumers who have erased their data. Compared to the main model (Figure 2(a)), both firms in market A now have larger numbers of untargeted consumers due to opt-out privacy-sensitive consumers.

Here, opt-out privacy-insensitive consumers mitigate price competition in market A and bring negative externalities to other consumers, consistent with Corollary 2. Moreover, equilibrium prices α_i^* and $p_i^*(x)$ increase with r, suggesting that having a larger portion of privacy-sensitive consumers who erase their data further mitigates price competition in market A. The competition to obtain privacy-sensitive consumers, which involves uniform versus uniform prices, is less severe than that for acquiring opt-in privacy-insensitive consumers, which requires uniform versus personalized prices. Thus, price competition becomes less aggressive as the number of privacy-sensitive consumers increases (i.e., a larger r value). Furthermore, $\bar{x}_1^*$ increases with r, and $\bar{x}_2^*$ decreases with r. These relations imply that a larger r generates more consumer-firm mismatch in market A. As a result, the return of privacy-sensitive consumers yields negative externalities for privacy-insensitive consumers by increasing prices and making them incur higher mismatch costs.

We find that compared to no privacy management, firm B_i always earns lower profits under privacy management because price competition in market B intensifies as the benefits of winning consumers increase in market A. Consistent with the main model, forward-looking consumers motivate firms to compete more aggressively in market B. Conversely, firms in market A earn higher profits under privacy management, except when r is extremely small, leading to firms 1 and 2 earning higher profits under privacy management, except when r is small.

Proposition 7. *Suppose that consumers have heterogeneous privacy sensitivity. Compared to no privacy management, when privacy management is available,*

(i) market failure due to privacy-sensitive consumers exiting the market is fixed,

(ii) the return of privacy-sensitive consumers and the opting out of privacy-insensitive consumers mitigate price competition in market A, and

(iii) price competition intensifies in market B.

We now discuss the welfare implications of Proposition 7. Privacy management enhances consumer surplus by attracting privacy-sensitive consumers back to market A. However, the return of these consumers further mitigates price competition and negatively affects consumer surplus in market A. Therefore, when the former effect dominates the latter, that is, when the share of privacy-sensitive consumers is larger than a threshold level, privacy management improves consumer surplus in market A. Proposition 7(iii) implies that privacy management increases consumer surplus in market B. Similarly, when the share of privacy-sensitive consumers is larger than a threshold level, privacy management improves social welfare in market A. However, the availability of privacy management does not change social welfare in market B.

5.4 Who should own data property rights?

We now discuss how consumers' personal data management affects market competition and welfare when they own data property rights. In this section, consumers opt out by default and opt in by choice, and firms can exploit a consumer's data only when she opts in. This setting mirrors the privacy model in GDPR, where firms have to obtain consumers' consent to collect and process their data. The game proceeds as in the main model, except that the two firms in market A commit to a benefit, $b \in [0, t]$, for opt-in consumers and deliver this benefit once they opt in. In addition, opt-in consumers obtain a net benefit, $\omega \in [0, 2b - t]$, from product personalization in the data application market (as in Section 5.1).[25] The equilibrium analysis follows a similar process as that in the main model.

We compare the equilibrium to the benchmark in Section 5.1 where firms own data and consumers can opt out with a fixed cost, ε. The comparative results depend on the value of b. The first case focuses on $b = \varepsilon$, which equalizes consumers' tradeoff between opting in and opting out, regardless of who owns the data. The second case focuses on the profit-maximizing b value for firms 1 and 2: $b = t$ when consumers are forward looking ($g = 1$), and $b = (t + \omega)/2$ when consumers are myopic ($g = 0$). Table 1 shows the comparative results.

In market A, firms earn a lower profit because they pay benefits to opt-in consumers; consumer surplus increases due to the benefits and saved privacy management cost ε. Firm B_i is less incentivized to collect data with an attractive price when consumers own the data property rights, resulting in higher profits and lower consumer surplus in market B. However, the price mitigation in market B is not so significant when consumers are forward looking ($g = 1$), making total consumer surplus increase, as in the $b = t$ row of Table 1.

[25] Benefit b serves the same role as ε in the equilibrium analysis; it can be a lump-sum payment, a discount, a tie-in gift, or any other benefit to induce consumers to opt in. The assumption of $\omega \leq 2b - t$ ensures pure strategy equilibrium in the price competition of market A.

Table 1: Profits and welfare change when consumers own the data

	$\pi^*_{A_i}$	$\pi^*_{B_i}$	Π^*_1 & Π^*_2	CS^*_A	CS^*_B	$CS^*_A + CS^*_B$	SW^*_A	SW^*_B
$b = \varepsilon$	↓	↑	↑	↑	↓	0	↑	0
$b = t$ $(g = 1)$	↓	↑	↑	↑	↓	↑	↑	0
$b = \frac{t+\omega}{2}$ $(g = 0)$	↓	↑	↑	↑	↓	↓ iff $\omega < 2\varepsilon - t$	↓ iff $\hat{\omega}_1 < \omega < \hat{\omega}_2$	0

Note: ↑ indicates an increase, ↓ indicates a decrease, 0 indicates no change, and iff indicates if and only if.

Proposition 8. *Suppose that consumers own the data property rights. Compared to the scenario where firms own the data,*

(i) firms in market A earn a lower profit, and firms in market B earn a higher profit due to mitigated competition, leading to higher profits for firms 1 and 2;

(ii) consumer surplus increases in market A and decreases in market B; and

(iii) giving data property rights to consumers leads to higher total consumer surplus and social welfare when these consumers are forward looking.

6 Policy implications

6.1 Rights to data erasure and data portability

We can extend the main model to show that firm B_i has no incentive to voluntarily offer consumers the choice of data erasure.[26] To understand this aspect, suppose that firm B_i offers data erasure. Firm A_i cannot charge high personalized prices to opt-out consumers who have a high willingness to pay and wins fewer of the rival's targeted consumers because of its inflated uniform price. This finding implies that privacy laws are necessary to ensure consumers' right to delete their personal data. However, such a right is likely to backfire on consumers economically

[26]Due to analytical difficulty, here, we assume that consumers are myopic about the outcome in market A.

and negatively affect society (Proposition 4).

In the following policy discussion on data portability as an additional option, we focus on the case in which Assumption 1 holds and adopt two benchmarks: one is no privacy management (Section 4.1) and the other is data erasure only (Section 4.2). Similar to Section 5.2, which requires $\varepsilon < t/2$, we can establish equilibria for forward-looking and myopic consumers when $t/2 \leq \varepsilon \leq t$ holds. The equilibrium outcome in market A is similar to that in Figure 5(b), but no consumer chooses to erase data (i.e., $\tilde{x}_1^{**} = 0$ and $\tilde{x}_2^{**} = 1$) because opting out with high cost ε and paying the inflated uniform price is no longer the best choice. Instead, a firm's consumers with a high willingness to pay choose data portability to be targeted by both firms in market A and pay low personalized prices. Compared to no privacy management and data erasure only, offering consumers the additional choice of data portability improves the profits of firms 1 and 2 and social welfare but reduces total consumer surplus, consistent with Proposition 6.

Proposition 9. *Compared to no privacy management, data erasure and data portability change competition in the data collection and application markets in opposite directions; total consumer surplus is always negatively affected, while total social welfare is enhanced whenever data portability is available.*

Data erasure weakens competition in the data application market and intensifies competition in the data collection market, and the opposite is true for data portability. Consumers' surplus loss from competition mitigation in one market outweighs their gains from intensified competition in the other market. Data erasure negatively affects social welfare due to a larger consumer-firm mismatch, while data portability improves social welfare because it enables consumers to be targeted and hence attracted by their preferred firm.

6.2 Banning cross-market data transfer and data walls

Suppose that regulators prohibit firm B_i from transferring consumer data to firm A_i. Alternatively, suppose that firms 1 and 2 commit to setting up an inside data wall to block the data flow from business B to business A.[27] As a result of the ban, consumers no longer need to actively manage their data; instead, they all opt out passively. Firms in market A cannot target any consumers, implying that the equilibrium prices are the same as those in the classical Hotelling model: $\alpha_1^* = \alpha_1^* = t$. Price competition in market B also becomes the Hotelling type: $\beta_1^* = \beta_2^* = t$.

Compared to Section 4.2 where consumers can only erase their data, all firms earn higher profits under the ban. In market A, the severe competition between uniform prices and personalized prices downgrades to competition between uniform prices. Specifically, uniform price $\alpha_i^* = t$ is higher than that in (10) and the personalized prices in (11) for most opt-in consumers. The ban also mitigates price competition in market B. As a result, consumer surplus declines in each market. Since all consumers purchase from their preferred firms in each market, social welfare achieves the maximum value under the ban.

The same results are obtained when we compare the equilibrium under the ban with that in Section 4.1, where consumers cannot manage their privacy, or with the scenario in which consumers have access to data erasure and data portability.

Proposition 10. *Suppose that firms* 1 *and* 2 *cannot transfer data across markets. Compared to the case with no such restriction, regardless of whether consumers can manage their privacy,*

(i) all firms earn higher profits due to weaker price competition in each market, and

(ii) while consumer surplus declines in each market, social welfare weakly improves in market A*.*

7 Conclusions

We consider a duopoly model in which consumers purchase products in two markets: one for data collection, and one for data application. The two firms compete in the two markets and first

[27] For example, in the Google-Fitbit merger, Google assured global Fitbit users that it would not use their health and wellness data for Google ads and that it would separate the Fitbit data from other Google ad data and store Fitbit data in a separate data silo (https://blog.google/products/devices-services/fitbit-acquisition/).

attempt to gain consumers in the data collection market. The data collected uniquely by each firm capture its customers' preferences for firms' products in the data application market. Following competition in the data collection market, each firm in the data application market offers personalized prices to its targeted customers and a uniform price to untargeted consumers. Before firms' pricing decisions, each firm's targeted customers can erase their data from the firm's database to become untargeted at a fixed cost. Consumers who erase their data can escape personalized prices and choose uniform prices offered by firms.

We find that consumers who strongly prefer one firm in the data application market choose to erase their data. These consumers opting out increases firms' uniform prices, which induces firms to set higher personalized prices. Nevertheless, firms earn lower profits in the data application market because they cannot extract enough surplus from opt-out consumers who have a high willingness to pay. Higher prices clearly negatively affect consumers. Moreover, due to higher uniform prices, each firm easily protects its targeted customers through personalized pricing, resulting in increased consumer-firm mismatches and decreased social welfare. Interestingly, consumer privacy management intensifies competition in the data collection market, although it diminishes the profitability of data applications. Combining these two markets, consumer privacy management leads to lower profits and social welfare; consumers benefit only when they are forward looking. The results are robust to various extensions.

Our study has direct implications for privacy laws. Data erasure and data portability can negatively affect consumers because the former weakens competition in the data application market, while the latter dampens competition in the data collection market. However, regarding social welfare, data portability is more likely to be beneficial for welfare than is data erasure because the former enables a consumer to be targeted and attracted by her preferred firm, while data erasure tends to worsen consumer-firm mismatch. Furthermore, banning firms' cross-market data transfer harms consumers because it mitigates competition in both markets. Giving consumers data property rights can be beneficial to consumers themselves and society as a whole if they are forward looking.

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A Appendix

A.1 Proof of equilibrium uniform prices in market A

In this section, we show how to determine the conditions under which firm A_1 and firm A_2 do not deviate from the subgame equilibrium $\alpha_1^* = \frac{t(1-\delta)+2t\delta\tilde{x}_1}{2(1-\delta)}$ and $\alpha_2^* = \frac{t\delta+2t(1-\delta)(1-\tilde{x}_2)}{2\delta}$. Firm A_1 has two possible deviations. The first one is increasing α_1 above t and does not poach the rival's targeted consumers. The second is reducing α_1 to attract firm A_2's consumers who have erased their data (i.e., consumers on $[\tilde{x}_2^*, 1]$).

□ **Firm A_1's deviation one:** $\alpha_1 \geq t$ Notice that $v_A - t - \alpha_2^* > 0$ always holds based on our Assumption 1, implying firm A_1 cannot become a local monopoly for its consumers on $[0, \tilde{x}_1^*]$. Given its deviation price α_1^d, the indifferent consumer is $\hat{x}^d = \frac{1}{2} + \frac{\alpha_2^* - \alpha_1^d}{2t}$. Firm A_1 maximizes its deviation profit from uniform price $\pi_{A_1}^d = \delta\alpha_1^d\hat{x}^d$ subject to $\hat{x}^d \leq \tilde{x}_1^*$. The optimal deviation price is $\alpha_1^d = \frac{t(1+2\delta)+2(1-\delta)\varepsilon}{2}$ and the deviation profit is $\pi_{A_1}^d = \frac{\delta(1-\delta)(t-\varepsilon)[(2\delta+1)t+2(1-\delta)\varepsilon]}{2t}$. Its equilibrium profit from the uniform price is $\pi_{A_1}^* = \frac{(1-\delta)(t+2\delta t-2\delta\varepsilon)^2}{8t}$. Then we have

$$\pi_{A_1}^* - \pi_{A_1}^d = \frac{(1-\delta)\left[-4(t-\varepsilon)^2\delta^2 - 8\varepsilon(t-\varepsilon)\delta + t^2\right]}{8t} > 0 \text{ if and only if } \delta < \min\left\{1, \frac{\sqrt{4\varepsilon^2+t^2}-2\varepsilon}{2(t-\varepsilon)}\right\}.$$

□ **Firm A_1's deviation two:** $\hat{x}^d \geq \tilde{x}_2^*$ To make $\hat{x}^d \geq \tilde{x}_2^*$ hold, firm A_1's deviation price $\alpha_1^d \leq \left(\delta + \frac{1}{2}\right)t - (\delta+1)\varepsilon$. Firm A_1's deviation profit from the uniform price is

$$\pi_{A_1}^d = \delta\alpha_1^d\tilde{x}_1^* + (1-\delta)\alpha_1^d(\bar{x}_2^d + \hat{x}^d - \tilde{x}_2^*) = \frac{-4(1-\delta)(\alpha_1^d)^2 + (1-\delta)[3t(1+2\delta) - (2+6\delta)\varepsilon]\alpha_1^d}{4t}.$$

Firm A_1 maximizes $\pi_{A_1}^d$ subject to $\alpha_1^d \leq \left(\delta + \frac{1}{2}\right)t - (\delta+1)\varepsilon$. Based on our Assumption 1, firm A_1's optimal deviation is $\alpha_1^d = \left(\delta + \frac{1}{2}\right)t - (\delta+1)\varepsilon$, implying $\hat{x}^d = \tilde{x}_2^*$. Such deviation is clearly unprofitable.

Firm A_2 has two possible deviations similar to firm A_1. We find firm A_2 has no incentive to deviate if and only if $\delta > \max\left\{0, \frac{2t-\sqrt{4\varepsilon^2+t^2}}{2(t-\varepsilon)}\right\}$. In summary, no firm has the incentive to deviate if and only if

$$\max\left\{0, \frac{2t-\sqrt{4\varepsilon^2+t^2}}{2(t-\varepsilon)}\right\} \leq \delta \leq \min\left\{1, \frac{\sqrt{4\varepsilon^2+t^2}-2\varepsilon}{2(t-\varepsilon)}\right\}.$$

Under Assumption 1, this condition is always satisfied for any $\delta \in [0,1]$.

A.2 Proof of Proposition 4

Consumer surplus in market A without privacy management is

$$\begin{aligned} CS_A^n =& \frac{1}{2}\left[\int_0^{3/4}(v_A - tx - p_1^n(x))dx + \int_{3/4}^1 (v_A - t(1-x) - \alpha_2^n)dx\right] \\ &+ \frac{1}{2}\left[\int_0^{1/4}(v_A - tx - \alpha_1^n)dx + \int_{1/4}^1 (v_A - t(1-x) - p_2^n(x))dx\right] \\ =& v_A - t. \end{aligned}$$

Consumer surplus in market A under privacy management is

$$\begin{aligned} CS_A^* =& \frac{1}{2}\left[\int_0^{\tilde{x}_1^*}(v_A - tx - \alpha_1^* - \varepsilon)dx + \int_{\tilde{x}_1^*}^{\bar{x}_1^*}(v_A - tx - p_1^*(x))dx + \int_{\bar{x}_1^*}^1 (v_A - t(1-x) - \alpha_2^*)dx\right] \\ &+ \frac{1}{2}\left[\int_0^{\bar{x}_2^*}(v_A - tx - \alpha_1^*)dx + \int_{\bar{x}_2^*}^{\tilde{x}_2^*}(v_A - t(1-x) - p_2^*(x))dx + \int_{\tilde{x}_2^*}^1 (v_A - t(1-x) - \alpha_2^* - \varepsilon)dx\right] \\ =& v_A - \frac{5t^2 - \varepsilon^2}{4t}. \end{aligned}$$

It is straightforward to check that $CS_A^* < CS_A^n$ holds.

Consumer surplus in market B without and with privacy management is

$$\begin{aligned} CS_B^n &= \int_0^{1/2}(v_B - tx - \beta_1^n)dx + \int_{1/2}^1 (v_B - t(1-x) - \beta_2^n)dx = v_B - \frac{13}{16}t, \\ CS_B^*(g=0) &= \int_0^{1/2}(v_B - tx - \beta_1^*)dx + \int_{1/2}^1 (v_B - t(1-x) - \beta_2^*)dx = v_B - \frac{12t^2 + 2t\varepsilon - \varepsilon^2}{16t}, \\ CS_B^*(g=1) &= \int_0^{1/2}(v_B - tx - \beta_1^*)dx + \int_{1/2}^1 (v_B - t(1-x) - \beta_2^*)dx = v_B - \frac{12t^2 - 14t\varepsilon + 15\varepsilon^2}{16t}. \end{aligned}$$

We can check that $CS_B^* > CS_B^n$ always holds. Moreover, $CS_A^* + CS_B^* < CS_A^n + CS_B^n$ holds when $g = 0$ and the reverse holds when $g = 1$.

Social welfare in market A without privacy management is $SW_A^n = v_A - \frac{5t}{16}$. Social welfare in market A under privacy management is $SW_A^* = v_A - \frac{8t^2 - 7\varepsilon^2 + 4t\varepsilon}{16t}$. We have $SW_A^* < SW_A^n$. Social welfare in market B always equals $v_B - \frac{t}{4}$.

A.3 Proof for Section 5.1

□ **Benchmark: no privacy management** The analysis is similar to Section 4.1. In market A, the equilibrium uniform prices are $\alpha_1^n = \alpha_2^n = (t-\omega)/2$ and the personalized prices are

$$p_1^n(x) = \begin{cases} 2t(3/4-x)+\omega/2 & \text{if } x \le 3/4+\omega/(4t) \\ 0 & \text{if } x \ge 3/4+\omega/(4t) \end{cases} \quad p_2^n(x) = \begin{cases} 0 & \text{if } x \le 1/4-\omega/(4t) \\ 2t(x-1/4)+\omega/2 & \text{if } x \ge 1/4-\omega/(4t). \end{cases}$$

In market B, the indifferent consumer is $\delta = \frac{1+\beta_2-\beta_1}{2}$, which is independent of g because $E[CS_{B_1}] = E[CS_{B_2}]$. Firm 1 and firm 2 decide their uniform prices to maximize total profits $\Pi_1 = \beta_1\delta + \pi_{A_1}^n$ and $\Pi_2 = \beta_2(1-\delta) + \pi_{A_2}^n$, respectively. The equilibrium uniform prices in market B are $\beta_1^n = \beta_2^n = (\omega^2 - 10t\omega + 9t^2)/(16t)$. The indifferent consumer in market B is $\delta^n = 1/2$. We have $\pi_{B_1}^n = \pi_{B_2}^n = (\omega^2-10t\omega+9t^2)/(32t)$ and $\pi_{A_1}^n = \pi_{A_2}^n = (3\omega^2+2t\omega+11t^2)/(32t)$. The equilibrium profits of firms 1 and 2 are $\Pi_1^n = \Pi_2^n = (\omega^2-2t\omega+5t^2)/(8t)$.

□ **Equilibrium under privacy management** The analysis is similar to Section 4.2. Given δ, the equilibrium uniform prices in market A is $\alpha_1^* = t(1/2+\delta)-\delta\varepsilon-\frac{\omega}{2}$ and $\alpha_2^* = t(3/2-\delta)-(1-\delta)\varepsilon-\frac{\omega}{2}$. The personalized prices are $p_1^*(x) = \alpha_2^* + t(1-2x)+\omega$ and $p_2^*(x) = \alpha_1^* - t(1-2x)+\omega$. The cutoffs are $\tilde{x}_1^* = (1-\delta)(t-\varepsilon)/t$, $\tilde{x}_2^* = 1-\delta(t-\varepsilon)/t$, $\bar{x}_1^* = ((5-2\delta)t+\omega-2(1-\delta)\varepsilon)/(4t)$, and $\bar{x}_2^* = ((1-2\delta)t+2\delta\varepsilon-\omega)/(4t)$. It is straightforward to check that $\tilde{x}_1^* \in (0,1)$ and $\tilde{x}_2^* \in (0,1)$ always hold. Under our assumption of $\omega > -3t$, $\bar{x}_1^* > 0$ and $\bar{x}_2^* < 1$ always hold. We have $\bar{x}_1^* < 1$ is equivalent to $\delta > \frac{\omega+t-2\varepsilon}{2(t-\varepsilon)}$ and $\bar{x}_2^* > 0$ is equivalent to $\delta < \frac{t-\omega}{2(t-\varepsilon)}$. When $\omega \le 2\varepsilon - t$, $\frac{t-2\varepsilon+\omega}{2(t-\varepsilon)} < \delta < \frac{t-\omega}{2(t-\varepsilon)}$ holds for any $\delta \in [0,1]$. We assume $\omega \le 2\varepsilon - t$ holds in the following analysis. Similar to the main model, firms in market A should not have incentive to deviate from α_1^* and α_2^*, which requires

$$\max\{\hat{\delta}_2(\varepsilon,\omega), \tilde{\delta}_2(\varepsilon,\omega), 0\} \le \delta \le \min\{\hat{\delta}_1(\varepsilon,\omega), \tilde{\delta}_1(\varepsilon,\omega), 1\}. \tag{12}$$

The derivation of (12) can be found in the online appendix. We find that (12) holds for any $\delta \in [0,1]$ when $\varepsilon \in [\hat{t}(\omega,t), t]$, in which $\hat{t}(\omega,t)$ increases with ω and ranges from $0.35t$ to t.

In market B, the equilibrium uniform prices are $\beta_1^* = \beta_2^* = ((2t+\varepsilon)(4t-\varepsilon)-\omega(10t-\omega))/(16t) - g(t-\varepsilon)\varepsilon/t$, implying the indifferent consumer is $\delta^* = 1/2$.[28] The profits of firms are $\pi_{A_1}^* = \pi_{A_2}^* = (12t^2 +$

[28]The ex-ante expected surpluf of firm B_i's consumer in market A is $E[CS_{B_i}] + \omega/2$, where $E[CS_{B_i}]$ is in Section 4.2.

$3\varepsilon^2-4t\varepsilon+\omega(6\varepsilon+3\omega-4t))/(32t)$ and $\pi^*_{B_1}=\pi^*_{B_2}=((2t+\varepsilon)(4t-\varepsilon)-\omega(10t-\omega))/(32t)-g(t-\varepsilon)\varepsilon/(2t)$. The equilibrium profits of firms 1 and 2 are $\Pi_1^*=\Pi_2^*=(10t^2+\varepsilon^2-t\varepsilon+\omega(3\varepsilon+2\omega-7t))/(16t)-g(t-\varepsilon)\varepsilon/(2t)$. The welfare comparison is similar to the main model and is omitted here.

A.4 Proof for data portability in Section 5.2 and Section 6.1

Consumers' total costs from different options are shown in Figure 7. The left panel shows the case of small ε, and the right panel shows the case of large ε.

□ **When ε is small** Based on the left panel of Figure 7, consumers on $[0,\tilde{x}_1]$ opt out by erasing data, consumers on $[\tilde{x}_1,\bar{x}_1]$ choose data portability, and consumers on $[\bar{x}_1,1]$ do not manage privacy and opt in. We have $\tilde{x}_1=\frac{1}{2}-\frac{\alpha_1^a+\varepsilon}{2t}$ and $\bar{x}_1=\frac{1}{2}+\frac{\alpha_2^a}{2t}$. Firm B_2's targeted consumers adopt a similar privacy management strategy, in which $\tilde{x}_2=\frac{1}{2}+\frac{\alpha_2^a+\varepsilon}{2t}$ and $\bar{x}_2=\frac{1}{2}-\frac{\alpha_1^a}{2t}$.

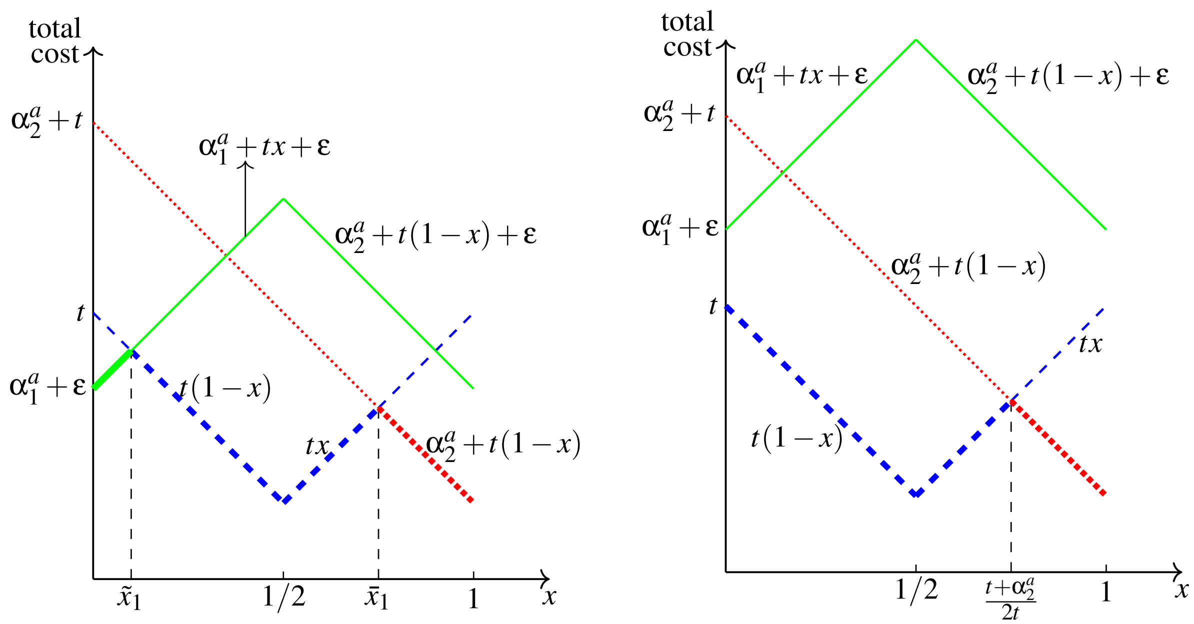

Figure 7: Total costs and consumers' privacy management strategy

Firm A_1's profit from its uniform price α_1 and firm A_2's profit from its uniform price α_2 are

$$\begin{cases}\alpha_1\left[(1-\delta)\bar{x}_2+\delta\tilde{x}_1\right] & \text{when } \alpha_1\le t-\varepsilon\\ \alpha_1(1-\delta)\bar{x}_2 & \text{when } t-\varepsilon\le\alpha_1\le t,\end{cases}\quad \begin{cases}\alpha_2\left[\delta(1-\bar{x}_1)+(1-\delta)(1-\tilde{x}_2)\right] & \text{when } \alpha_2\le t-\varepsilon\\ \alpha_2\delta(1-\bar{x}_1) & \text{when } t-\varepsilon\le\alpha_2\le t.\end{cases}$$

In the equilibrium of market A, we have $\alpha_1^{**}=\frac{t-\delta\varepsilon}{2-\delta}$, $\alpha_2^{**}=\frac{t-\varepsilon+\delta\varepsilon}{1+\delta}$, $\tilde{x}_1^{**}=\frac{(1-\delta)(t-2\varepsilon)}{2t(2-\delta)}$, $\tilde{x}_2^{**}=\frac{2t+t\delta+2\delta\varepsilon}{2t(1+\delta)}$, $\bar{x}_1^{**}=\frac{t(2+\delta)-(1-\delta)\varepsilon}{2t(1+\delta)}$, and $\bar{x}_2^{**}=\frac{t(1-\delta)+\delta\varepsilon}{2t(2-\delta)}$. The constraints $\tilde{x}_1^{**}<1$, $0<\tilde{x}_2^{**}$, $0<\bar{x}_1^{**}<1$, and

$0 < \bar{x}_2^{**} < 1$ always hold. The constraints $0 < \tilde{x}_1^{**}$ and $\tilde{x}_2^{**} < 1$ hold for any δ if and only if $0 < \varepsilon < \frac{t}{2}$. Firm A_1 and firm A_2 have no incentive to deviate if and only if $\max\{0, \delta_2(t,\varepsilon)\} \le \delta \le \min\{1, \delta_1(t,\varepsilon)\}$, which holds for any $\delta \in [0,1]$ when $0.37t \le \varepsilon < \frac{t}{2}$.[29] Two firms' equilibrium profits in market A are

$$\pi_{A_1}^{**} = \alpha_1^{**}[\delta\tilde{x}_1^{**} + (1-\delta)\bar{x}_2^{**}] + \delta \int_{\tilde{x}_1^{**}}^{\frac{1}{2}} t(1-2x)dx + (1-\delta)\int_{\bar{x}_2^{**}}^{\frac{1}{2}} t(1-2x)dx,$$
$$\pi_{A_2}^{**} = \alpha_2^{**}[(1-\delta)(1-\tilde{x}_2^{**}) + \delta(1-\bar{x}_1^{**})] + \delta \int_{\frac{1}{2}}^{\bar{x}_1^{**}} t(2x-1)dx + (1-\delta)\int_{\frac{1}{2}}^{\tilde{x}_2^{**}} t(2x-1)dx.$$

In market B, the indifferent consumer is $\delta = \frac{1}{2} + \frac{\beta_2 - \beta_1}{2t}$. Firm i decides β_i to maximize its total profit $\Pi_i = \pi_{B_i} + \pi_{A_i}$. The equilibrium uniform prices are $\beta_1^{**} = \beta_2^{**} = \frac{100t^2 + 8t\varepsilon - 11\varepsilon^2}{108t}$, implying $\delta^{**} = \frac{1}{2}$. The equilibrium outcomes in market A are determined by replacing δ with $\frac{1}{2}$. The uniform price α_i^{**} increases compared to no privacy management.

Under data erasure and data portability, consumer surplus in market A is $CS_A^{**} = v_A - \frac{25t^2 - 4t\varepsilon + 7\varepsilon^2}{36t}$, and consumer surplus in market B is

$$CS_B^{**} = \int_0^{1/2} (v_B - tx - \beta_1^{**})dx + \int_{1/2}^{1} (v_B - t(1-x) - \beta_2^{**})dx = v_B - \frac{127t^2 + 8t\varepsilon - 11\varepsilon^2}{108t}.$$

Compared to no privacy management, we have $CS_A^{**} > CS_A^n$, $CS_B^{**} < CS_B^n$, $CS_A^{**} + CS_B^{**} < CS_A^n + CS_B^n$, $SW_A^{**} > SW_A^n$, and $SW_B^{**} > SW_B^n$.

□ **When ε is relatively large** This case requires $\frac{t}{2} < \varepsilon < t$. The right panel of Figure 7 shows the total costs of consumers' different options. In this case, no consumer opts out by erasing data. Firms' optimal uniform prices are $\alpha_1^{**} = \alpha_2^{**} = \frac{t}{2}$, implying $\bar{x}_1^{**} = \frac{3}{4}$ and $\bar{x}_2^{**} = \frac{1}{4}$. It is straightforward to check that no firm has incentive to deviate from α_1^{**} and α_2^{**}. Two firms' equilibrium profits in market A are

$$\pi_{A_1}^{**} = \alpha_1^{**}(1-\delta)\bar{x}_2^{**} + \delta\int_0^{\frac{1}{2}} t(1-2x)dx + (1-\delta)\int_{\bar{x}_2^{**}}^{\frac{1}{2}} t(1-2x)dx = \frac{t}{16}(3+\delta),$$
$$\pi_{A_2}^{**} = \alpha_2^{**}\delta(1-\bar{x}_1^{**}) + \delta\int_{\frac{1}{2}}^{\bar{x}_1^{**}} t(2x-1)dx + (1-\delta)\int_{\frac{1}{2}}^{1} t(2x-1)dx = \frac{t}{16}(4-\delta).$$

Notice that $E[CS_{B_1}] = E[CS_{B_2}]$ always holds, implying the indifferent consumer in market B is independent of g. The analysis in market B is the same as in the case of small ε. The equilibrium uniform prices are $\beta_1^{**} = \beta_2^{**} = \frac{15t}{16}$, implying $\delta^{**} = \frac{1}{2}$.

[29]The detailed proof of this condition can be found in the online appendix.

When $\frac{t}{2} < \varepsilon < t$, we have two benchmarks to compare. In benchmark one (with superscript n), consumers cannot manage their privacy (Section 4.1). In the other benchmark (with superscript $*$), consumers can only use data erasure in privacy management (Section 4.2). We have the following ranking about firms' profits: $\pi^n_{A_i} > \pi^*_{A_i} > \pi^{**}_{A_i}$, $\pi^{**}_{B_i} > \pi^n_{B_i} > \pi^*_{B_i}$, and $\Pi^{**}_i > \Pi^n_i > \Pi^*_i$.

In market A, consumer surplus without privacy management is $CS^n_A = v_A - t$; consumer surplus under data erasure is $CS^*_A = v_A - \frac{5t^2-\varepsilon^2}{4t}$; consumer surplus under data erasure and data portability is

$$\begin{aligned}CS^{**}_A =& \frac{1}{2}\left[\int_0^{\frac{1}{2}}(v_A - tx - t(1-2x))dx + \int_{\frac{1}{2}}^{\bar{x}^{**}_1}(v_A - t(1-x) - t(2x-1))dx + \int_{\bar{x}^{**}_1}^{1}(v_A - t(1-x) - \alpha^{**}_2)dx\right] \\ &+ \frac{1}{2}\left[\int_0^{\bar{x}^{**}_2}(v_A - tx - \alpha^{**}_1)dx + \int_{\bar{x}^{**}_2}^{\frac{1}{2}}(v_A - tx - t(1-2x))dx + \int_{\frac{1}{2}}^{1}(v_A - t(1-x) - t(2x-1))dx\right] \\ =& v_A - \frac{11}{16}t.\end{aligned}$$

It is straightforward to check that $CS^*_A < CS^n_A < CS^{**}_A$ holds.

In market B, consumer surplus without privacy management is $CS^n_B = v_B - \frac{13}{16}t$; consumer surplus under data erasure is $CS^*_B = v_B - \frac{12t^2+2t\varepsilon-\varepsilon^2}{16t}$; consumer surplus under data erasure and data portability is

$$CS^{**}_B = \int_0^{1/2}(v_B - tx - \beta^{**}_1)dx + \int_{1/2}^{1}(v_B - t(1-x) - \beta^{**}_2)dx = v_B - \frac{19}{16}t.$$

We can check that $CS^*_B > CS^n_B > CS^{**}_B$ holds. Moreover, $CS^n_A + CS^n_B > CS^{**}_A + CS^{**}_B$ always holds, and $CS^*_A + CS^*_B > CS^{**}_A + CS^{**}_B$ holds under our assumption of $\varepsilon > \frac{\sqrt{3}}{2}t$.

In market A, social welfare without privacy management is $SW^n_A = v_A - \frac{5t}{16}$; social welfare under data erasure is $SW^*_A = v_A - \frac{8t^2-7\varepsilon^2+4t\varepsilon}{16t}$; social welfare under data erasure and data portability is $SW^{**}_A = v_A - \frac{t}{4}$. We have $SW^*_A < SW^n_A < SW^{**}_A$. Social welfare in market B always equals $v_B - \frac{t}{4}$.

A.5 Proof for Section 5.3

□ **Benchmark: no privacy management** The equilibrium prices in market A are the same as in Section 4.1. Given δ, two firms' profits are $\pi^n_{A_1} = t(1-r)(2+7\delta)/16$ and $\pi^n_{A_2} = t(1-r)(9-7\delta)/16$. The equilibrium uniform prices in market B are $\beta^n_1 = \beta^n_2 = t(9+7r)/16$, implying the indifferent consumer in market B is $\delta^n = 1/2$. We have $\pi^n_{B_1} = \pi^n_{B_2} = t(9+7r)/32$ and $\pi^n_{A_1} = \pi^n_{A_2} = 11t(1-r)/32$. The equilibrium profits of firms 1 and 2 are $\Pi^n_1 = \Pi^n_2 = t(5-r)/8$. It is straightforward to calculate that consumer surplus is $CS^n_A = (1-r)(v_A - t)$ and $CS^n_B = v_B - (13+7r)t/16$, and social welfare is $SW^n_A = (1-r)(16v_A - 5t)/16$ and $SW^n_B = v_B - t/4$.

□ **Equilibrium under privacy management** The profits of firms A_1 and A_2 from their uniform prices are

$$\begin{cases} \alpha_1\left[(1-r)(1-\delta)\bar{x}_2 + (1-r)\delta\tilde{x}_1 + r\hat{x}\right] & \text{when } \alpha_1 \le t, \\ \alpha_1\left[(1-r)\delta\tilde{x}_1 + r\hat{x}\right] & \text{when } \alpha_1 \ge t \text{ and } \hat{x} \ge \tilde{x}_1. \end{cases}$$

$$\begin{cases} \alpha_2\left[(1-r)\delta(1-\bar{x}_1) + (1-r)(1-\delta)(1-\tilde{x}_2) + r(1-\hat{x})\right] & \text{when } \alpha_2 \le t, \\ \alpha_2\left[(1-r)(1-\delta)(1-\tilde{x}_2) + r(1-\hat{x})\right] & \text{when } \alpha_2 \ge t \text{ and } \hat{x} \le \tilde{x}_2. \end{cases}$$

The definitions of $\tilde{x}_i$, $\bar{x}_i$, and $\hat{x}$ are the same as in Section 4.2. Still, in the two cases of A_i's profit function, only the first case can sustain in the equilibrium. Given δ, the equilibrium outcomes in market A are

$$\alpha_1^* = \frac{-\delta(r-1)(\varepsilon(r-2)+2t) + \varepsilon(r-1)r + 2rt + t}{r+2}, \quad \alpha_2^* = \frac{\varepsilon(r-1)(\delta(r-2)+2) + t(2\delta(r-1)+3)}{r+2},$$
$$\tilde{x}_1^* = \frac{\varepsilon\left(2\delta\left(r^2-3r+2\right) - r^2 + 2r - 4\right) + t(4\delta(r-1) - r + 4)}{2(r+2)t}, \quad \tilde{x}_2^* = \frac{2\delta(r-1)(\varepsilon(r-2)+2t) - (r-4)(\varepsilon r + t)}{2(r+2)t},$$
$$\bar{x}_1^* = \frac{\varepsilon(r-1)(\delta(r-2)+2) + t(2\delta(r-1)+r+5)}{2(r+2)t}, \quad \bar{x}_2^* = \frac{(r-1)(\delta\varepsilon(r-2) + 2\delta t - \varepsilon r - t)}{2(r+2)t},$$
$$\hat{x}^* = \frac{(2\delta-1)\varepsilon\left(r^2-3r+2\right) + t(4\delta(r-1) - r + 4)}{2(r+2)t}.$$

We find that $\bar{x}_i^* \in (0,1)$, $\hat{x}^* \in (0,1)$, $\tilde{x}_1^* < 1$, and $\tilde{x}_2^* > 0$ always hold when $\varepsilon > t/2$. The $\tilde{x}_1^* > 0$ holds when $t/2 < \varepsilon < 3t/4$ or $3t/4 < \varepsilon < t$ and $0 < \delta < \min\left\{1, \frac{\varepsilon r^2 - 2\varepsilon r + 4\varepsilon + rt - 4t}{2\varepsilon r^2 - 6\varepsilon r + 4\varepsilon + 4rt - 4t}\right\}$. The $\tilde{x}_2^* < 1$ holds when $t/2 < \varepsilon < 3t/4$ or $\max\left\{0, \frac{\varepsilon r^2 - 4\varepsilon r + 3rt}{2\varepsilon r^2 - 6\varepsilon r + 4\varepsilon + 4rt - 4t}\right\} < \delta < 1$. Two firms' equilibrium profits are

$$\pi^*_{A_1} = \alpha_1^*\left[(1-r)(1-\delta)\bar{x}_2^* + (1-r)\delta\tilde{x}_1^* + r\hat{x}^*\right] + (1-r)\delta\int_{\tilde{x}_1^*}^{\bar{x}_1^*} p_1^*(x)dx,$$
$$\pi^*_{A_2} = \alpha_2^*\left[(1-r)\delta(1-\bar{x}_1^*) + (1-r)(1-\delta)(1-\tilde{x}_2^*) + r(1-\hat{x}^*)\right] + (1-r)(1-\delta)\int_{\bar{x}_2^*}^{\tilde{x}_2^*} p_2^*(x)dx.$$

Now we focus on market B. Firm B_i's privacy-insensitive consumers' expected surplus in market A are

$$E[CS_{B_1}^{\theta=0}] = \int_0^{\tilde{x}_1^*} (v_A - \alpha_1^* - tx - \varepsilon)dx + \int_{\tilde{x}_1^*}^{\bar{x}_1^*} (v_A - p_1^*(x) - tx)dx + \int_{\bar{x}_1^*}^{1} (v_A - \alpha_2^* - t(1-x))dx,$$
$$E[CS_{B_2}^{\theta=0}] = \int_0^{\bar{x}_2^*} (v_A - \alpha_1^* - tx)dx + \int_{\bar{x}_2^*}^{\tilde{x}_2^*} (v_A - p_2^*(x) - t(1-x))dx + \int_{\tilde{x}_2^*}^{1} (v_A - \alpha_2^* - t(1-x) - \varepsilon)dx.$$

Firm B_i's privacy-sensitive consumers' expected surplus in market A are

$$E[CS_{B_1}^{\theta=c}] = E[CS_{B_2}^{\theta=c}] = \int_0^{\hat{x}^*} (v_A - \alpha_1^* - tx - \varepsilon)dx + \int_{\hat{x}^*}^{1} (v_A - \alpha_2^* - t(1-x) - \varepsilon)dx.$$

The indifferent consumer δ in market B is determined by

$$\begin{aligned}&(1-r)(v_B - \beta_1 - t\delta + gE[CS_{B_1}^{\theta=0}]) + r(v_B - \beta_1 - t\delta + gE[CS_{B_1}^{\theta=c}])\\ =&(1-r)(v_B - \beta_2 - t(1-\delta) + gE[CS_{B_2}^{\theta=0}]) + r(v_B - \beta_2 - t(1-\delta) + gE[CS_{B_2}^{\theta=c}]),\end{aligned}$$

which implies $\delta = \frac{(r+2)t(\beta_1-\beta_2-t)+\varepsilon^2 g(r-2)(r-1)^2+2\varepsilon g(r-1)^2 t}{2\varepsilon^2 g(r-2)(r-1)^2+4\varepsilon g(r-1)^2 t-2(r+2)t^2}$. Two firms' profit in market B are $\pi_{B_1} = \beta_1\delta$ and $\pi_{B_2} = \beta_2(1-\delta)$. Firm 1 and firm 2 decide their uniform prices to maximize the total profit $\Pi_1 = \pi_{B_1} + \pi_{A_1}$ and $\Pi_2 = \pi_{B_2} + \pi_{A_2}$.

The equilibrium uniform prices in market B are

$$\beta_1^* = \beta_2^* = \frac{\varepsilon^2(r-1)\left(-16g\left(r^2-3r+2\right)+r^3+9r+2\right)+4\varepsilon(r-1)t\left(-8g(r-1)+r^2+2r-1\right)+4\left(r^2+7r+4\right)t^2}{16(r+2)t}.$$

The indifferent consumer is $\delta^* = \frac{1}{2}$. The equilibrium profits of firm B_1 and firm B_2 are $\pi_{B_1}^* = \beta_1^*/2$ and $\pi_{B_2}^* = \beta_2^*/2$.

The equilibrium outcomes in market A are determined by replacing δ with $\frac{1}{2}$. We have $\tilde{x}_1^* = \frac{1}{2} - \frac{\varepsilon}{2t}$, $\tilde{x}_2^* = \frac{1}{2} + \frac{\varepsilon}{2t}$, $\bar{x}_1^* = 1 - \frac{(1-r)\varepsilon}{4t}$, and $\bar{x}_2^* = \frac{(1-r)\varepsilon}{4t}$. The equilibrium prices in market A are $\alpha_1^* = \alpha_2^* = t - \frac{1}{2}(1-r)\varepsilon$. The equilibrium personalized prices are

$$p_1^*(x) = \begin{cases} 2t(1-x) - \frac{1}{2}(1-r)\varepsilon & \text{when } \tilde{x}_1^* < x < \bar{x}_1^* \\ 0 & \text{when } x \geq \bar{x}_1^* \end{cases} \quad p_2^*(x) = \begin{cases} 0 & \text{when } x \leq \bar{x}_2^* \\ 2tx - \frac{1}{2}(1-r)\varepsilon & \text{when } \bar{x}_2^* < x < \tilde{x}_2^*. \end{cases}$$

The equilibrium profits of firm A_1 and firm A_2 are $\pi_{A_1}^* = \pi_{A_2}^* = \frac{\varepsilon^2\left(r^3-3r^2-r+3\right)+4\varepsilon\left(r^2-1\right)t+4(r+3)t^2}{32t}$.

The equilibrium profits of firm 1 and firm 2 are

$$\Pi_1^* = \Pi_2^* = \frac{\varepsilon^2(r-1)^2\left(-8g(r-2)+r^2+r+2\right)+2\varepsilon(r-1)t\left(-8g(r-1)+2r^2+5r+1\right)+4\left(r^2+6r+5\right)t^2}{16(r+2)t}.$$

Consumer surplus in market A is

$$
\begin{aligned}
CS_A^* =& \frac{1-r}{2}\left[\int_0^{\bar{x}_2^*}(v_A - tx - \alpha_1^*)dx + \int_{\bar{x}_2^*}^{\tilde{x}_2^*}(v_A - t(1-x) - p_2^*(x))dx + \int_{\tilde{x}_2^*}^{1}(v_A - t(1-x) - \alpha_2^* - \varepsilon)dx\right] \\
&+ \frac{1-r}{2}\left[\int_0^{\tilde{x}_1^*}(v_A - tx - \alpha_1^* - \varepsilon)dx + \int_{\tilde{x}_1^*}^{\bar{x}_1^*}(v_A - tx - p_1^*(x))dx + \int_{\bar{x}_1^*}^{1}(v_A - t(1-x) - \alpha_2^*)dx\right] \\
&+ r\left[\int_0^{1/2}(v_A - tx - \alpha_1^*)dx + \int_{1/2}^{1}(v_A - t(1-x) - \alpha_2^*)dx\right] \\
=& v_A + \frac{\varepsilon^2(1-r)}{4t} - r\varepsilon - \frac{5t}{4}.
\end{aligned}
$$

Consumer surplus in market B is

$$
\begin{aligned}
CS_B^* &= \int_0^{1/2}(v_B - tx - \beta_1^*)dx + \int_{1/2}^{1}(v_B - t(1-x) - \beta_2^*)dx \\
&= v_B - \frac{\varepsilon^2(r-1)\left(-16g\left(r^2-3r+2\right)+r^3+9r+2\right)}{16(r+2)t} - \frac{\varepsilon(r-1)\left(-8g(r-1)+r^2+2r-1\right)}{4(r+2)} - \frac{\left(r^2+8r+6\right)t}{4(r+2)}.
\end{aligned}
$$

Social welfare in market A is $SW_A^* = v_A + \frac{\varepsilon^2\left(r^3-3r^2-5r+7\right)+4\varepsilon t\left(r^2-4r-1\right)+4t^2(r-2)}{16t}$. Social welfare in market B is equal to $v_B - \frac{t}{4}$. The comparisons of firms' profits, consumer surplus, and social welfare are easy to derive and are omitted here.

A.6 Proofs of Section 5.4

The equilibrium analysis in market A is the same as in Section 5.1 after replacing the privacy management cost ε with the opt-in benefit b. In the equilibrium of market A, we have $\alpha_1^* = t(1/2+\delta) - \delta b - \frac{\omega}{2}$, $\alpha_2^* = t(3/2-\delta) - (1-\delta)b - \frac{\omega}{2}$, $\tilde{x}_1^* = \frac{(t-b)(1-\delta)}{t}$, $\tilde{x}_2^* = 1 - \frac{\delta(t-b)}{t}$, $\bar{x}_1^* = \frac{5t-2b-2(t-b)\delta+\omega}{4t}$, and $\bar{x}_2^* = \frac{t-2(t-b)\delta-\omega}{4t}$. Two firms' personalized prices are

$$
p_1^*(x) = \begin{cases} t(5/2-\delta) - (1-\delta)b + \frac{\omega}{2} - 2tx & \text{if } \tilde{x}_1^* < x < \bar{x}_1^* \\ 0 & \text{if } x \geq \bar{x}_1^* \end{cases} \qquad p_2^*(x) = \begin{cases} 0 & \text{if } x \leq \bar{x}_2^* \\ t(\delta-1/2) - \delta b + \frac{\omega}{2} + 2tx & \text{if } \bar{x}_2^* < x < \tilde{x}_2^*. \end{cases}
$$

Two firms' equilibrium profits in market A are

$$
\begin{aligned}
\pi_{A_1}^* =& \alpha_1^*[\delta\tilde{x}_1^* + (1-\delta)\bar{x}_2^*] + \delta\int_{\tilde{x}_1^*}^{\bar{x}_1^*} p_1^*(x)dx - b\delta(1-\tilde{x}_1^*), \\
\pi_{A_2}^* =& \alpha_2^*[(1-\delta)(1-\tilde{x}_2^*) + \delta(1-\bar{x}_1^*)] + (1-\delta)\int_{\bar{x}_2^*}^{\tilde{x}_2^*} p_2^*(x)dx - b(1-\delta)\tilde{x}_2^*.
\end{aligned}
$$

To make such an equilibrium valid, we need $0 < \tilde{x}_1^* < 1$, $0 < \tilde{x}_2^* < 1$, $0 < \bar{x}_1^* < 1$, $0 < \bar{x}_2^* < 1$. These constraints are equivalent to $\frac{t-2b+\omega}{2(t-b)} < \delta < \frac{t-\omega}{2(t-b)}$, which holds for any $\delta \in [0,1]$ when

$\omega \leq 2b - t$. In the following analysis, we assume $\omega \leq 2b - t$ holds. In addition, we need to make sure no firm has the incentive to deviate from α_1^* and α_2^*, which requires

$$\max\{\hat{\delta}_2(b,\omega), \tilde{\delta}_2(b,\omega), 0\} \leq \delta \leq \min\{\hat{\delta}_1(b,\omega), \tilde{\delta}_1(b,\omega), 1\}. \tag{13}$$

The expressions of $\hat{\delta}_i(b,\omega)$ and $\tilde{\delta}_i(b,\omega)$ are the same as $\hat{\delta}_i(\varepsilon,\omega)$ and $\tilde{\delta}_i(\varepsilon,\omega)$ in the proof of Section 5.1 after replacing ε with b. Condition (13) holds for any $\delta \in [0,1]$ if and only if $\hat{b}(\omega,t) \leq b \leq t$.

In market B, the expected surpluses of firm B_i's targeted consumers are

$$\begin{aligned}
E[CS_{B_1}] &= \int_0^{\tilde{x}_1^*} (v_A - \alpha_1^* - tx)dx + \int_{\tilde{x}_1^*}^{\bar{x}_1^*} (v_A - p_1^*(x) - tx + b)dx + \int_{\bar{x}_1^*}^{1} (v_A - \alpha_2^* - t(1-x) + b)dx \\
&= v_A - (1+\delta-\delta^2)t + (1-\delta)(2\delta-1)\varepsilon + \frac{(1-\delta)^2\varepsilon^2}{t}, \\
E[CS_{B_2}] &= \int_0^{\bar{x}_2^*} (v_A - \alpha_1^* - tx + b)dx + \int_{\bar{x}_2^*}^{\tilde{x}_2^*} (v_A - p_2^*(x) - t(1-x) + b)dx + \int_{\tilde{x}_2^*}^{1} (v_A - \alpha_2^* - t(1-x))dx \\
&= v_A - (1+\delta-\delta^2)t - \delta(2\delta-1)\varepsilon + \frac{\delta^2\varepsilon^2}{t}.
\end{aligned}$$

The indifferent consumer δ is determined by $v_B - t\delta - \beta_1 + gE[CS_{B_1}] = v_B - t(1-\delta) - \beta_2 + gE[CS_{B_2}]$, which leads to $\delta = \frac{2b^2g - bg(\omega+2t) + t(-2\beta_1+2\beta_2+g\omega+2t)}{4b^2g - 2bg(\omega+2t) + 2t(g\omega+2t)}$. Two firms' profits in market B are $\pi_{B_1} = \beta_1\delta$ and $\pi_{B_2} = \beta_2(1-\delta)$. Firm 1 and firm 2 decide their uniform prices to maximize total profit $\Pi_1 = \pi_{A_1}^* + \pi_{B_1}$ and $\Pi_2 = \pi_{A_2}^* + \pi_{B_2}$. The equilibrium uniform prices in market B are

$$\beta_1^* = \beta_2^* = \frac{16b^2g - b^2 - 8bg\omega - 16bgt + 18bt + 8g\omega t + \omega^2 - 10\omega t + 8t^2}{16t}.$$

The indifferent consumer in market B is $\delta^* = \frac{1}{2}$. We have $\pi_{B_1}^* = \pi_{B_2}^* = \beta_1^*/2$.

The equilibrium in market A is straightforward to get by replacing δ with $1/2$. The profits of firm A_1 and firm A_2 are $\pi_{A_1}^* = \pi_{A_2}^* = \frac{12t^2 - 5b^2 - 12tb + \omega(6b+3\omega-4t)}{32t}$. The equilibrium profits of firm 1 and firm 2 are $\Pi_1^* = \Pi_2^* = \frac{b^2(8g-3) + b(3-4g)\omega + b(3-8g)t + (4g-7)\omega t + 2\omega^2 + 10t^2}{16t}$.

Consumer surplus in market A is

$$\begin{aligned}
CS_A^* =& \frac{1}{2}\left[\int_0^{\tilde{x}_1^*} (v_A - tx - \alpha_1^*)dx + \int_{\tilde{x}_1^*}^{\bar{x}_1^*} (v_A + \omega - tx - p_1^*(x) + b)dx + \int_{\bar{x}_1^*}^{1} (v_A - t(1-x) - \alpha_2^* + b)dx\right] \\
&+ \frac{1}{2}\left[\int_0^{\bar{x}_2^*} (v_A - tx - \alpha_1^* + b)dx + \int_{\bar{x}_2^*}^{\tilde{x}_2^*} (v_A + \omega - t(1-x) - p_2^*(x) + b)dx + \int_{\tilde{x}_2^*}^{1} (v_A - t(1-x) - \alpha_2^*)dx\right] \\
=& v_A - \frac{5t^2 - b^2 - 4tb}{4t} + \frac{\omega}{2}.
\end{aligned}$$

Consumer surplus in market B is

$$CS_B^*(g=0) = \int_0^{1/2} (v_B - tx - \beta_1^*)dx + \int_{1/2}^1 (v_B - t(1-x) - \beta_2^*)dx = v_B - \frac{12t^2 + 18tb - b^2}{16t} - \frac{\omega^2 - 10t\omega}{16t},$$
$$CS_B^*(g=1) = \int_0^{1/2} (v_B - tx - \beta_1^*)dx + \int_{1/2}^1 (v_B - t(1-x) - \beta_2^*)dx = v_B - \frac{15b^2 + 2bt + 12t^2}{16t} - \frac{\omega^2 - \omega(8b+2t)}{16t}.$$

Under our assumption of $\omega \le 2b - t$ and $b < t$, $CS_B^*(g=1) > CS_B^*(g=0)$ holds. Social welfare in market A is $SW_A^* = v_A - \frac{8t^2-4tb+b^2}{16t} + \frac{\omega(3\omega+4t+6b)}{16t}$. Social welfare in market B always equals $v_B - \frac{t}{4}$.

When $b = \varepsilon$, the comparison to Section 5.1 is straightforward and is omitted here. When $g = 1$, we have $\Pi_1^* = \Pi_2^* = \frac{5b^2}{16t} - \frac{b(\omega+5t)}{16t} + \frac{2\omega^2-3\omega t+10t^2}{16t}$. Since $b \ge \hat{b}(\omega, t) \ge \hat{b}(\omega = 0, t) \approx 0.67t$ and $0.67t > \frac{\omega+5t}{10}$, the optimal $b^* = t$. The following comparisons are straightforward. When $g = 0$, we have $\Pi_1^* = \Pi_2^* = -\frac{3b^2}{16t} + \frac{b(3\omega+3t)}{16t} + \frac{2\omega^2-7\omega t+10t^2}{16t}$, implying the optimal $b^* = \frac{t+\omega}{2}$. In this case, social welfare in market A reduces if and only if

$$\frac{1}{11}\left[3(4\varepsilon - 3t) - 2\sqrt{113\varepsilon^2 + t^2 - 98t\varepsilon}\right] < \omega < \frac{1}{11}\left[3(4\varepsilon - 3t) + 2\sqrt{113\varepsilon^2 + t^2 - 98t\varepsilon}\right].$$

A.7 Proof for Section 6.1

We now prove firm B_i has no incentive to offer the choice of data erasure to consumers voluntarily. The game proceeds as in the main model, except that the two firms in market B simultaneously decide whether to offer the choice before competition in market A begins. Let us first characterize the market equilibrium when only firm B_1 offers privacy management to consumers. Firm A_2's optimal $\alpha_2^* = \frac{t}{2}$, leading to $\bar{x}_1^* = \frac{3}{4}$. Given $\tilde{x}_1$, firm A_1's optimal α_1 is $\alpha_1 = \frac{t(1-\delta)+2t\delta\tilde{x}_1}{2(1-\delta)}$. Since consumers have rational expectations, based on (4), we have $\alpha_1^* = \frac{t(2+\delta)-2\delta\varepsilon}{2(2-\delta)}$ and $\tilde{x}_1^* = \frac{(1-\delta)(t-\varepsilon)}{t(2-\delta)}$. Using α_1^*, firm A_1 could successfully poach A_2's targeted consumers on $[0, \bar{x}_2^*]$, in which $\bar{x}_2^* = \frac{2\delta\varepsilon-3\delta t+2t}{8t-4\delta t}$. Two firms' personalized prices are

$$p_1^*(x) = \begin{cases} \alpha_2^* + t(1-2x) & \text{when } \tilde{x}_1^* < x < 3/4 \\ 0 & \text{when } x \ge 3/4 \end{cases} \quad p_2^*(x) = \begin{cases} 0 & \text{when } x \le \bar{x}_2^* \\ \alpha_1^* + t(2x-1) & \text{when } \bar{x}_2^* < x \le 1. \end{cases}$$

Two firms' equilibrium profits in market A are

$$\pi_{A_1}^* = \alpha_1^*[\delta\tilde{x}_1^* + (1-\delta)\bar{x}_2^*] + \delta\int_{\tilde{x}_1^*}^{\bar{x}_1^*} p_1^*(x)dx, \quad \pi_{A_2}^* = \delta\alpha_2^*/4 + (1-\delta)\int_{\bar{x}_2^*}^1 p_2^*(x)dx.$$

To make such an equilibrium valid, we need $0 < \tilde{x}_1^* < 1$ and $\bar{x}_2^* > 0$, which always hold under our Assumption 1.

In market B, firm 1 and firm 2 decide their uniform prices to maximize total profits $\Pi_1 = \beta_1\delta + \pi^*_{A_1}$ and $\Pi_2 = \beta_2(1-\delta) + \pi^*_{A_2}$, respectively. We can solve the optimal β_1 and β_2 and determine all firms' profits by numerical methods. Firm 1 and firm 2 simultaneously determine whether to offer privacy management to consumers. The analysis of Nash equilibrium is straightforward and is omitted here.