Entry Regulations and Product Variety in Retail*

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Abstract

This paper evaluates the impact of entry regulations on stores' incentives to adjust product variety and long-run performance. We use rich Swedish data on stores, product categories, and local regulations to estimate a dynamic model of endogenous product offerings where stores utilize multiproduct service technology to generate sales. The median long-run benefit of adding one more product category is approximately 20% higher in restrictive than in liberal markets. A more liberal regulation spurs repositioning and increases product variety, driven by a new mechanism of cost reductions and productivity gains. Incumbents' long-run profits decrease from more intense competition but stores that adjust variety gain market share. A constant marginal cost of adding variety mainly raise long-run profits in rural markets with limited demand by 2 percent.

Keywords: Retail; entry regulations; competition dynamics; variety; productivity.
JEL Classification: L11, L13, L81.

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1 Introduction

There is substantial literature on the economic impacts of regulations but their impact on product variety remains an open question. The European Union, for instance, emphasize entry regulations as crucial for enhancing consumers' access to product variety (European Commission, 2018).[1] Firms offer multiple products that are important for consumer welfare and growth in the long run. Yet most work on product offerings consider the static costs and benefits of product adjustments rather than dynamic effects, and omit firms' internal operations related to economies of scale and scope. This paper analyzes how the tradeoff between costs and future benefits of adjusting product variety affect a firm's product repositioning incentives when regulations and the competitive environment change.

Entry regulations alter the intensity of competition and affect product offerings through at least two opposing forces. On the one hand, competition can decrease profit margins, making it more challenging to recover investments associated with more product offerings, incentivizing firms to hold back on variety. On the other hand, competition can incentivize firms to offer a wider range of products to attract customers. Theory alone cannot predict whether product variety increases or decreases, and thus empirical work is required to assess what force prevails. This paper delivers an empirical judgment on this theoretically ambiguous question in the context of retail markets.

The goal of this paper is to examine the impact of entry regulations on repositioning incentives in product variety and long-term store performance. We estimate a dynamic model where stores make endogenous decisions over product variety by trading off the short-run costs against the long-run benefits. Stores compete in a dynamic oligopoly where the stringency of entry regulations affects the costs of altering product offerings, future productivity, multiproduct sales (sales per product) and market share. The model is estimated using rich Swedish retail data on stores, product categories, and entry regulations across local markets for the retail sale of new

[1]Product offerings and the link to competition are also emphasized in, for example, EU and US merger guidelines (European Commission, 2004; US Department of Justice, 2010; Rose and Shapiro, 2022) and the Digital Market Act (European Commission, 2022). Although 85 percent of global retail is still offline (The Economist, 2021), the extent of variety in online markets and platforms is debated (e.g., Brynjolfsson et al., 2022). The economics literature on product variety and competition dates back to 1950s (e.g., Baumol and Ide, 1956).

goods in specialized stores between 2004–2009. This paper defines product variety as a retailer offering more distinct product categories.[2] We perform counterfactual experiments of a more liberal entry regulation, and explore the importance of the productivity- and cost channels for product variety and store performance.

Entry regulations and government subsidies are common in OECD countries, but their stringency and design differ.[3] Given that services represent 70 percent of GDP and employment and constitute a large share of household budgets, assessing the consequences of different regulatory designs is essential. According to the Swedish Plan and Building Act (PBL), all stores are subject to regulation, and each municipality has the power to make land-use decisions. Local authorities typically require each store seeking entry to complete a formal application. The application is approved or rejected after the potential consequences of entry on factors such as market shares and product variety have been evaluated. Rarely are all applications approved in Sweden. We follow the previous literature and use the number of approved PBL applications for both commercial and residential purposes divided by population density to measure regulatory stringency, which more closely approximate actual regulation enforcement and captures key aspects essential for retail such as market size and geographic density.

Incumbents choose product categories and inputs in each period based on their own and competitors' productivity and demand primitives and local market characteristics (Ericson and Pakes, 1995). Stores sell the products using a novel multiproduct technology that embeds their inputs and economies of scope. Stores face costs in adjusting their product offerings, which include reorganizing the store, managing inventory, modifying the store layout, etc. Entry regulations in local markets affect stores' product variety adjustment costs and future productivity, altering product and store sales, market share and the future benefits from repositioning variety. Stores respond differently to changes in local regulations, and a store's market share is

[2]The definition of variety lacks consensus in the literature. There are many definitions of variety depending on data across papers in macroeconomics, trade, industrial organization and marketing. Focusing on dynamic incentives and supply-side operations in terms of adjustment cost (e.g., advertising, distribution and other operational costs) and productivity justifies using product categories as our measure of variety.

[3]Countries such as the United States have more flexible zoning laws, while the United Kingdom and France explicitly regulate large entrants. See Rose (2014), Pozzi and Schivardi (2016), and European Commission (2018).

determined by its own product variety and that of rivals in local markets.

We estimate the store's period payoffs and policy controls to recover the value functions, i.e., long-run profits, in an approach similar to the two-step dynamic game estimation (e.g., Bajari et al., 2007; Pakes et al., 2007). In the first step, we estimate the store multiproduct service technology and market index function that form two connected systems of equations and use them to recompute short-run sales per product, total sales and market shares in a local market when store inputs and market structure change after the stringency of entry regulation is altered. In the second step, we identify the variety adjustment cost by matching the observed data with the model prediction. Unlike previous work, we solve the dynamic model for the store's two optimal continuous decisions (the number of product categories and inventory before sales) and reduce computational complexity using recent methods based on reinforcement learning and function approximations (Sutton and Barto, 2018).

This paper contributes to the literature by providing an empirically tractable approach to explore how changes in competition due to entry regulations influence the dynamic incentives behind endogenous product variety decisions among incumbents. We introduce novel channels whereby entry regulations change stores' dynamic incentives to offer product variety through variety adjustment cost and future productivity changes, which affect local competition. When adding a new product category, the store trades off the cost increase against efficient utilization of economies of scope to increase short-run sales and future discounted benefits from variety repositioning. Central is how regulations influence firms' dynamic incentives to offer variety through the lens of firms' internal operations. We extend work that the cost-side determines variety dating back to 1980s, and draw on recent arguments that supply-side features inside stores merit more attention than demand-side factors with respect to sustaining competition intensity in services (The Economist, 2021). Moreover, this paper extends the existing work on entry regulations by analyzing their impact on stores' product variety decisions and uncovering the mechanisms behind the observed effects, including the role of competitors. The dynamic framework fully endogenizes the store's product categories, inventory before sales, sales per product and market share using all products in stores and without requiring detailed price data. Because many sectors consist almost entirely of multiproduct firms and our approach does not require price data, it more easily scales to multiple industries and can be more broadly applicable when scanner data are not available.

To make our model of local market competition tractable, we follow the theoretical literature that approximates the Markov perfect equilibrium based on oblivious and moment-based equilibrium concepts (Weintraub et al., 2008; Benkard et al., 2015; Ifrach and Weintraub, 2017), as well as recent applications (e.g., Barwick et al., 2023; Gowrisankaran et al., 2022; Jeon, 2022). Stores consider rivals through the aggregate state, which is attractive when there are many rivals, as in our retail application. We discuss the identification of the model and provide Monte Carlo simulations for validation purposes.

The proposed multiproduct technology used by stores to generate sales is transparent regarding the aggregation across products and the rate of substitution between products and is consistent with stores' profit maximization behavior, as discussed in the early theoretical literature on production technology (Mundlak, 1964; Fuss and McFadden, 1978). The multiproduct technology uses sales for all stores in the industry and allows us to model the impact of competition on store revenue productivity and sales per product when the market structure changes, although we do not precisely model the impact on prices. The estimation of the multiproduct technology and a market index function are also used to recover store-level revenue productivity and demand shocks using a control function estimator at the product-category level, relying on input demand for labor and inventory (Doraszelski and Jaumandreu, 2013; Kumar and Zhang, 2019; Maican et al., 2023). Recovered demand shocks are associated with consumers' quality of the shopping experience and other demand factors that affect store sales and market share.

Our framework offers new insights into how competition and regulation affect firms' incentives to offer product variety under uncertainty, which can inform policy design and assessment. There is a general trend that firms frequently adjust their products and drastically increase the supply of product offerings over time. If firms can adjust product variety, policymakers need to consider repositioning in the assessment and design of regulation (Pakes, 2021). We capture the impact of regulation and industrial policies on firm behavior in industries with heterogeneous product offerings across firms, where firms appreciate the services attached to each product or group of products.

Data on product information connected to a census are scarce for services industries. Having access to such data, we use product categories to measure product variety at the store level. The facts emerging from our data and recent policy concerns about regulations in services guide the formal model (European Commission, 2018). Stores frequently adjust their product offerings, and there is substantial variation in product categories within and between stores over time. We find a positive association between competition arising from entry regulation liberalization and the number of product categories. Competition also alters the distribution of sales per product, where bottom-selling categories increase more than top-selling categories.

Reduced-form regressions robust to possible endogeneity of the regulatory measure show the first evidence of the two opposing channels in our model: a more liberal regulation has a positive association with labor productivity and inventory productivity and a negative association with store market share. However, to fully understand the forces behind how competition affects product offerings, to quantify the cost and long-run benefits of product adjustments, and to simulate counterfactuals under alternative regulations and policies require a dynamic structural model.

The results from the dynamic model show that entry regulations have a significant impact on the costs and benefits that determine the store's optimal product variety. The median adjustment cost of product categories is 17% higher in markets with restrictive rather than liberal regulation. The median long-run benefit of adding one more product category is approximately 20% higher in restrictive than in liberal markets. Stores in rural markets have the highest dispersion in the benefit. The median benefit of adding variety is 2% lower for stores in rural rather than urban markets, reflecting less variety for consumers in rural areas.

Counterfactual policy experiments show that more liberal regulation encourages product repositioning and leads to more product categories, driven by new mechanisms of productivity improvements and lower adjustment costs. Modest liberalization (one more PBL approval in all markets) induces the largest net entry of product categories in rural and restrictive markets. However, most entry and exit of categories occur in urban and liberal markets but net entry is lower. Long-run profits decrease among incumbents in all markets by 1-2% due to increased competition, whereas stores that adjust their product offerings gain market share.

Strong liberalization (double PBL approvals) results in more repositioning in all markets except rural with larger magnitudes than under modest liberalization. Importantly, strong liberalization implies that incumbents that already hold many categories adjust to a larger extent than under modest liberalization, reflecting importance of economies of scope. The counterfactual simulations of strong liberalization shows that the losses in long-run profits are up to ten times larger than under a modest increase in competitive pressure. Long-run profits decrease the most in rural markets due to limited demand. In contrast, long-run profits decrease the least in restrictive markets where incumbents that adjust variety obtain the largest gain in market share. There is thus particularly room for liberalization in markets with restrictive regulation. We empirically document new supply-side channels behind variety adjustments. Without the productivity channel, there is less variety and larger reductions in long-run benefits, e.g., reduction in restrictive markets (8%) is about double than in the other three market types. Similarly, lower adjustment costs (i.e., constant marginal cost of adding variety) imply more product categories and particularly benefit incumbents in rural markets with limited demand, where the long-run profits increase by 2%. These new mechanisms of productivity and adjustment costs driving dynamic incentives behind product repositioning that boosts variety are left out of the traditional analysis.

Relation to the literature. This paper adds to the general literature on product variety and competition (e.g., Baumol and Ide, 1956; Dixit and Stiglitz, 1977; Broda and Weinstein, 2006). That the cost-side determines the supply of varieties dates back to Panzar and Willig (1981), Baumol et al. (1982) and Bailey and Friedlaender (1982), suggesting that economies of scale and scope drive multiproduct firms. The role of competition for variety is often highlighted in a more general sense in empirical IO literature (e.g., Berry and Waldfogel, 2001; Sweeting, 2010; Watson, 2009; Hsieh and Rossi-Hansberg, 2019; Ellickson et al., 2020). This paper adds to a small but growing literature on endogenous product differentiation and competition. The recent literature uses structural methods based on discrete choice demand and scanner data to investigate the impact of mergers on a firm's product variety choices (Berry and Haile, 2022; Fan and Yang, 2022; Gandhi and Nevo, 2022; Garrido, 2022; Rossi, 2023) and cross-category pricing (Thomassen et al., 2017). Most of this literature rely on narrowly defined products, e.g., brands or flavours, rather than broader product categories driven by firms' internal operations related to economies of scale and scope.

The paper complements existing demand and entry literature on endogenous variety. We focuses less on static consumer demand and pricing decisions and more on dynamic aspects of the impact of regulations in local markets. Prices restrict quantity demanded, while purchasing costs related to traveling and waiting in the checkout line limit demand for product variety (Bronnenberg, 2015). Existing work on variety and entry regulations primarily focuses on profits and market structure but omits endogenous variety decisions based on firm productivity, demand shocks, cost and economies of scope and their dynamic implications (e.g., Suzuki, 2013; Fan and Xiao, 2015; Maican and Orth, 2018). This paper adds a rich modeling of the supply-side decisions (all payers and local markets) and the distributional effects of entry regulation across local market types. We estimate the distribution and the endogenous evolution of key store primitives (supply and demand) from novel multiproduct sales technology, allowing for different sources of uncertainty and rich store and market heterogeneity. Then, we use them in a dynamic model with complex state space to understand the channels through which competition affects variety and short versus long-run tradeoffs.

Recent work emphasizes that product offerings are dynamic decisions that policymakers need to account for in their assessment and design of regulation (Sweeting, 2013; Pakes, 2021). Our dynamic framework of endogenous product variety decisions relates to the literature on competition dynamics and firm responses to industry policies (e.g., Gowrisankaran and Town, 1997; Ryan, 2012; Collard-Wexler, 2013; Kalouptsidi, 2014; Fowlie et al., 2016; Igami and Uetake, 2020). In particular, our work relates to the theoretical literature on equilibrium concepts that approximates Markov perfect equilibrium.[4]

Our paper also relates to work on productivity in multiproduct firms (De Loecker et al., 2016; Orr, 2022; Dhyne et al., 2023) and in service sectors (Maican and Orth, 2021). The nature of services makes it difficult to measure physical quantities and prices and to aggregate across products, complicating the issue of defining technical productivity (Oi, 1992). Maican and Orth (2021) utilize a simplified version of the multiproduct function in a static framework omitting modeling regulation, endogenous variety, and competition dynamics to study the relationship between revenue productivity and variety using simulations from reduced-form policy functions.[5]

[4]Ackerberg et al. (2007) and Aguirregabiria et al. (2021) survey some recent empirical literature on dynamic games.

[5]Previous work, such as Syverson (2011), Maican and Orth (2015), and Backus (2020), has typically found positive effects of stronger competition on productivity due to external factors such as less restrictive regulation.

Section 2 presents the entry regulations and data. Section 3 presents the dynamic model. Section 4 discusses the empirical results, and Section 5 reports the counterfactual experiments. Section 6 concludes the paper. In several places, we refer to an online appendix.

2 Swedish Retail Trade and Entry Regulations

The goal of policymakers is to ensure that all individuals in society have access to a wide variety of products at low prices and in stores within a reasonable geographic distance. To reach this goal, most OECD countries empower local governments to make decisions regarding the entry of new stores. The Swedish Plan and Building Act (PBL) regulates the use of land, water and buildings. The regulation contains a comprehensive plan that covers and guides the use of the entire municipality and detailed development plans that cover only a fraction of the municipality. The detailed development plans divide municipalities into smaller areas for which limits on use and design (e.g., construction rights for real estate and use for workplaces, housing, schools, parks, etc.) are set. Entry by a new store requires that the PBL allows retail operations in the geographic area where the store wants to enter. A formal application needs to be sent to the municipal government, which is supposed to evaluate the consequences for prices, accessibility of store types and products for different consumer groups, traffic, broader environmental issues, etc. The local government can accept or reject an application. Because the Swedish regulation is typical of the regulations in many other countries, our application to Sweden is relevant and offers broad implications for other countries (Pozzi and Schivardi, 2016). Appendix D discusses the PBL in detail.

Regional development policies. Regional subsidies are alternative policy tools employed to encourage stores to provide a wide variety of products. In 2001, the Swedish government announced a new regional development policy designed to maintain a sustainable service level in all parts of Sweden (bill 2001/02:4 *A policy for growth and viability for the whole country).* One of the programs embedded in the policy was *Stores in the countryside.* The aim of the program was to improve stores in rural areas by implementing store performance actions, such as store refitting, improvements to the distribution of products and technical equipment, modernization of inventory and assignment of mentors to enhance communication between store managers and local authorities. In 2015, the Swedish government announced the *Rural Development Programme* (RDP). The RDP contains support and compensation for municipalities and is intended to facilitate living and operating businesses in rural areas by investing in local services and technologies (e.g., broadband). The RDP emphasizes retail stores, as they also provide numerous other utilities, such as postal services. The stringency of entry regulations is crucial for achieving the goals of the RDP because entry regulations include considerable details on investments in infrastructure and access to services.

Local markets. Sweden consists of 290 municipalities that make decisions regarding entry regulations and regional development policies. Following previous studies and considering that municipal governments decide on entry and regional programs, we take the municipality as a local market (Maican and Orth, 2015; Maican and Orth, 2018). We classify municipalities by market type. The first classification rests on the stringency of entry regulations. Markets with regulatory stringency below the median value are defined as restrictive; otherwise, they are defined as liberal. The second classification refers to rural or urban markets. Markets with fewer than 10,000 inhabitants are defined as rural; those with more inhabitants are defined as urban. Restrictive and liberal markets are defined based on the potential competitive pressure from entrants, whereas rural and urban markets are defined based on potential demand. Because entry regulations affect all markets and regional programs target rural markets, our classification by market type is crucial for evaluating policy changes.

Data. We focus on the three-digit industry *retail sale of new goods in specialized stores* (Swedish National Industry (SNI) code 524). This industry includes the following subsectors at the five-digit SNI level: clothing; furniture and lighting equipment; electrical household appliances and radio/television goods; hardware, paints and glass; books, newspapers and stationery; and other specialized stores. The empirical analysis accounts for heterogeneity across these subsectors.

We use three data sets provided by Statistics Sweden and the Swedish Mapping, Cadastral and Land Registration Authority (SMA). The first data set covers detailed annual information on all retail firms in Sweden (census) during the period from 2000 to 2010. The data contain financial statistics of input and output measures: sales, value added, number of employees, capital stock, inventories, cost of products bought, investment, etc. Inventories capture the value of products held in stock at the end of each year and are taken from book values (accounting data).

The cost of products bought measures a store's cost of buying products from wholesalers. The cost of products bought and inventories both rely on the input prices of goods (what stores pay to wholesalers). In other words, sales and value added are measured in output prices, whereas the cost of products bought and inventories are measured in input prices. Because of difficulties in measuring quantity units in retail arising from the nature and complexity of the product assortments, quantity measures of output and inventories are not available.

Our second data set includes information on a sample of approximately 1,100 stores per year and covers store-level data on all product categories and their yearly sales from 2004 to 2009. The product data are provided by Statistics Sweden, which ensures that the sample is representative and accurate. Unique identification codes allow us to perfectly match the product categories to the stores. The product categories have 6–8 digit codes assigned, which define categories such as clothes for women, men, and children.[6] The number of product categories is our measure of product variety in a store. Although one would ultimately like to access data on individual products, such detailed data are rarely available for the extensive coverage of firms linked to a census. Nevertheless, the vast majority of stores in our data offer several product categories, and this enables us to take a step toward capturing variety within industries along with input and output measures. The use of product categories also facilitates applications of our approach across multiple industries. The number of product categories captures the extensive margin of product variety in a store. Data on sales per product category capture the intensive margin of product lines (range) inside a category. Most importantly, the combination of the two data sets allows us to compute product market shares within a store and a store's market share in a geographic market (municipality).

The third data set contains the number of applications approved by local authorities for each municipality and year (SMA). This data set also includes applications to alter land-use plans and the total number of existing land-use plans. We follow previous work and define the stringency of regulation in the local market as the number of approved PBL applications for both commercial and residential purposes divided by population density (Bertrand and Kramarz, 2002; Suzuki, 2013; Turner et al., 2014; Maican and Orth, 2018).[7] Our variable does not

[6]The structure of the product data is similar to that of PRODCOM manufacturing data (Eurostat).

[7]Municipalities with a nonsocialist majority approve more PBL applications. The correlation between nonsocialist seats and the number of approved PBL applications in local markets is

measure market size or competition rather than regulation because rarely are all PBL applications approved in a local market and the decisions are binding (see also Appendix D).[8]

Descriptive statistics and stylized facts. Table 1 shows that there is an aggregate increase in sales, value added, average number of product categories, investments, and labor over time. From 2004 to 2009, sales, investments, and the number of employees increased by 64 percent, 81 percent and 53 percent, respectively. The average store has 4 product categories. The number of product categories varies between 1 and 17. Our regulation measure, the average number of approved PBL applications over population density, increased from 0.23 to 0.29 during our study period. That more approvals are associated with fiercer competition is confirmed by the negative correlation between product-category sales and regulation.

TABLE 1: Descriptive statistics

Year	Sales	Value added	Investment	No. of employees	Mean no. of product categories at store	Mean no. of PBL approvals by pop. density	Corr. product sales and no. of PBL approvals by pop. density
2004	80.454	17.518	1.286	31,424	3.101	0.228	-0.020
2005	97.144	22.358	1.531	39,468	4.514	0.263	-0.005
2006	103.116	23.448	1.796	38,640	4.151	0.253	-0.004
2007	147.852	30.497	2.466	47,104	4.399	0.289	-0.020
2008	130.613	26.427	2.528	49,130	4.185	0.285	-0.040
2009	131.826	27.123	2.335	47,940	4.223	0.234	-0.019

NOTE: Sales (excl. VAT), value added, inventories (includes products bought), and investment are in billions of 2000 SEK (1 USD= 7.3 SEK, 1 EUR= 9.3 SEK). Sales per product category are computed at store level.

Product repositioning is more frequent in retail than in manufacturing because retailers employ the same technology to sell a different set of product categories. We observe adjustments in product categories in 52 percent of store–year observations, a result also confirmed by the median number of years that a store adjusts product categories, which is approximately half of the total number of years in the sample. Nevertheless, the mean of cumulated yearly adjustments of the number of product categories is positive (i.e., product variety increases over time). The yearly adjustments in a store's number of product categories between $t-1$ and t vary considerably. The interquartile range of yearly changes in a store's number of product categories is 2. We also find substantial variation in the yearly changes across five-digit subsectors; i.e., the median of the five-digit interquartile range is 1, and the maximum is 3.

Figure 1 presents boxplots showing the distributions of store performance measures in the

0.6.

[8]While the approval rate is an alternative regulation measure, it requires observations of the number of approvals and rejections (not observed) and is limited because the decision to apply in a market is a function of the expected success rate.

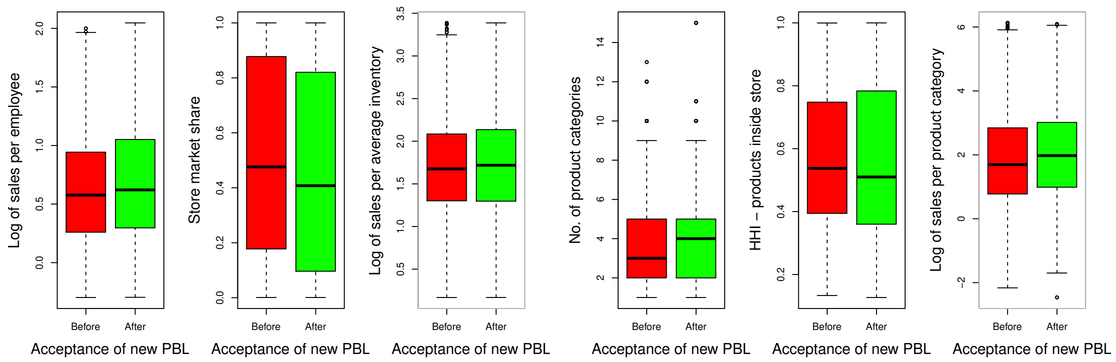

FIGURE 1: Store performance distributions in the year before and after approved PBL applications

year before and after the acceptance of new PBL applications. We measure store performance by labor productivity (log of sales per employee), inventory performance (log of sales per average inventory and log of cost of goods sold over average inventory), and market share. Median labor and inventory productivity are higher, whereas median market share is lower after the acceptance of new PBL applications. That competition arising from more liberal regulation has a positive association with productivity and a negative association with market share supports results from previous literature and provides strong evidence for the channels in our model. It also suggests that we have to control for entry regulation when developing more sophisticated measures of store performance such as total factor productivity.

Figure 1 also shows that the median store has more product categories and higher sales per product category after the acceptance of new PBL applications. Entry regulations inducing repositioning provide support for the structural model. Consumers benefit from more product variety, and incumbents benefit from higher sales per product category in markets with more liberal regulation. However, the underlying factors driving these patterns are not well understood in the absence of a structural model.

TABLE 2: Impact of accepted PBL applications on stores' product variety

	No. of products (1)	Log no. of products (2)	Product sales entropy (3)
New applications accepted	0.344	0.047	-0.069
	(0.150)	(0.020)	(0.029)
Year fixed effect	Yes	Yes	Yes
Subsector fixed effect	Yes	Yes	Yes
Adjusted R2	0.236	0.204	0.242

NOTE: The independent variable is a dummy variable that takes the value one if there are new applications accepted in a local market. Entropy measures store diversification in sales and is computed for each store j based on the market share of each product category i inside the store, i.e., $E_{jt} = \sum_i ms_{ijt} ln(ms_{ijt})$. Clustered standard errors at the market level are in parentheses.

The reduced-form regressions in Table 2 show that new PBL applications increase the number of product categories and decrease the entropy of product sales. Entropy measures sales diversification and is computed based on the market share of each product category i sold by store j, $E_{jt} = \sum_i ms_{ijt} ln(ms_{ijt})$ (Bernard et al., 2011). A store that focuses on top sales categories has high entropy. On average, stores in markets with new applications accepted have approximately 5 percent more product categories and 7 percent lower product-sales entropy. This suggests that regulatory changes are associated with adjustments in product variety.

Table 3 shows that one additional PBL approval per population density increases the number of product categories in stores by 4.7 percent and decreases stores' product-sales entropy by 5.2 percent. We use $AR(1)$ reduced-form regressions that include year, subsector, and local market fixed effects (i.e., $\Delta z_{jmt} = \alpha_z z_{jmt-1} + \alpha_r r_{mt-1} + f_s + f_t + f_m + u_{jmt}$), where z is the level and logarithm of the number of product categories and sales entropy. The average persistence in the number of product categories and sales entropy are approximately 60 and 63 percent, respectively.

TABLE 3: Impact of entry regulation on dynamics of stores' product variety

	Change in no. of products		Change in log no. of products		Change in sales entropy	
	(1)	(2)	(3)	(4)	(5)	(6)
No. of products in $t-1$	-0.396	-0.400				
	(0.053)	(0.014)				
Log of no. of products in $t-1$			-0.524	-0.526		
			(0.034)	(0.013)		
Product sales entropy in $t-1$					-0.372	-0.376
					(0.013)	(0.014)
Entry regulation in $t-1$	0.310	0.688	0.047	0.101	-0.052	-0.170
	(0.174)	(0.289)	(0.022)	(0.051)	(0.030)	(0.053)
Year fixed effect	Yes	Yes	Yes	Yes	Yes	Yes
Market fixed effect	Yes	Yes	Yes	Yes	Yes	Yes
Subsector fixed effect	Yes	Yes	Yes	Yes	Yes	Yes
Adjusted R^2	0.381		0.455		0.305	
F test (weak IV)		237.698		218.022		265.208
Sargan test (p value)		0.230		0.094		0.961

NOTE: The OLS estimator is used in columns (1), (3) and (5). An IV estimator is used to control for the endogeneity of entry regulation in columns (2), (4) and (6). The IV regressions use three instruments, i.e., the share of nonsocialist seats in the local market, the number of approved applications in the neighboring municipalities, and one internal instrument exogenous variable (e.g., income and income squared) (see Lewbel, 2012). Entropy measures store diversification in sales and is computed for each store j based on the market share of each product category i inside the store, i.e., $E_{jt} = \sum_i ms_{ijt} ln(ms_{ijt})$. Clustered standard errors at the market level are in parentheses.

Endogeneity of regulation. Our results are robust to the endogeneity of the entry regulation measure. The reduced form specifications are informative for the sign of the impact and less on the magnitude because of the complexity of the bias from endogeneity of entry regulations.

The included local market-year fixed effects control for unobserved factors that lead to at least two potential sources of endogeneity: market shock to demand or costs may induce both entry (i.e., more approved applications) and an increase in the number of product categories; a change in local preferences (demographics) affects both the elected local government, which affect the number of approved applications and product categories. On the other hand, store productivity and quality shocks are other sources of endogeneity (they are modeled in the structural model): more applications may be approved in markets with low-productivity stores that can only offer fewer product categories; more approved applications force incumbents to cut costs and improve their service quality by selling fewer product categories. Specifications (2), (4) and (6) in Table 3 control for the endogeneity of regulation using an instrumental variable (IV) approach. We use three instruments based on previous literature: the share of nonsocialist seats in the local government (Maican and Orth, 2015; Pozzi and Schivardi, 2016), the number of approved applications in neighboring municipalities, and one internal instrument based on variables (e.g., income and income squared) exogenous to stores (Lewbel, 2012). The first instrument relies on nonsocialist local governments being more positive toward entry. To be an effective instrument for entry regulation, the share of nonsocialist seats should not be related to local market demand. This instrument raises the following concerns. First, the outcomes of elections might be influenced by economic conditions. Political business cycles can affect our results only if there is a substantial ability to predict future demand shocks when politicians are elected. The second concern is that political preferences might capture local policies other than entry regulations. In Sweden, the PBL is rather exceptional because it enables local politicians to play a key role. Furthermore, the number of PBLs in other markets is an appropriate instrument if it reflects common trends, such as nationwide trends in regulatory strictness, or other shocks specific only to entry regulations. The exclusion restriction requires that regulatory strictness in other markets is not correlated with store-specific shocks (Maican and Orth, 2015). Although the proposed instruments are not perfect, we believe that they are the best instruments given previous work and the available data. The results of the Sargan test show that the overidentifying restrictions are valid; i.e., the test fails to reject the null hypothesis that the instruments are uncorrelated with the remaining shocks.[9] The statistically significant F-test shows that the instruments are not weakly correlated with the entry regulation measure (Staiger and Stock,

[9]The p-value is larger than 0.05 in all IV specifications.

1997).

In summary, the descriptive statistics and reduced-form specifications highlight that less restrictive entry regulation is associated with stores offering more products and having lower entropy of product sales, a lower market share, and higher labor productivity. However, the reduced-form specifications omit the fact that the choice of product variety is a dynamic and endogenous decision and that entry regulations can influence it through different channels. The reduced-form approach, moreover, leaves no room for strategic interactions. Answering our research question on how entry regulations affect incentives to offer variety and long-run performance requires a structural model of oligopolistic competition that enables calculations of counterfactual outcomes under alternative regulatory designs. The structural model fully endogenizes the store's choice of product variety by emphasizing the store's tradeoff between the short-run cost and future benefits from product repositioning due to changes in competition. Notably, the model recognizes channels through which entry regulations affect the intensive and extensive margins of product variety, market share and long-run performance.

3 A Dynamic Model of Multiproduct Service in Markets with Entry Regulations

We develop an infinite-horizon dynamic equilibrium model of the supply of product variety in markets with different entry regulation stringency. We consider a retail sector where all stores focus on a well-defined service activity (e.g., selling apparel or shoes). Stores operate in a set of local markets $\{1, \ldots, \overline{M}\}$ (i.e., municipalities) and use a novel multiproduct technology that incorporates economies of scale and scope and the role of all inputs to generate sales.[10] In each year t, there are NS_{mt} stores currently operating in market m. Each store j is characterized by its revenue productivity ω_{jt}, demand shocks μ_{jt}, log of capital stock k_{jt}, log of the number of employees l_{jt}, and log of the inventory level at the beginning of period t, n_{jt}. Each local market m is characterized by an entry regulation stringency measure r_{mt} and other observed characteristics grouped in $\mathbf{x}_{mt}$ (population, population density, average income).[11]

[10] Panzar and Willig (1981) and Bailey and Friedlaender (1982) highlight the key role of economies of scale and scope for the existence of multiple products at the firm/store level.

[11] We use capital letters for levels, lower-case letters for logs and entry regulation measure r_{mt} (ratio), and bold for a vector of variables.

Each period t proceeds as follows. First, policymakers in local market m determine the stringency of entry regulation r_{mt}. A store cannot influence local decisions on entry regulations. Entry regulations affect stores' decisions through their direct impact on cost structure (e.g., adjustment costs related to variety and inputs) and indirect impact on future productivity as well as product- and store-level sales and market share when competition changes. Second, stores compete annually in a dynamic oligopoly. At the beginning of year t, store j observes its own and rivals' states after receiving productivity and demand shocks and accounting for the stringency of local entry regulations. The incumbents decide whether to exit or continue to operate. If the store continues, it chooses the optimal levels of the number of product categories np_{jt}, inventory holdings before sales a_{jt} (i.e., log of the sum of the inventory level at the beginning of period t and the products bought during period t), and other inputs to generate sales and maximize profits. Third, conditional on local entry regulations and product category and input repositioning, stores compete in markets and earn profits from selling products and services.

Store j has an adjustment cost in product variety, $c_n(np_{jt}, a_{jt}, r_{mt})$, which is increasing in a_{jt} and is affected by regulation r_{mt} (Joskow and Rose, 1989; Maican and Orth, 2018). A more restrictive entry regulation increases stores' operating costs as a result of an increase in the costs due to, for example, a more expensive location or building costs, which affect stores' adjustment costs. Repositioning product categories implies adjustment costs to optimally integrate the new product category mix within the store, e.g., reorganizing the store, managing inventory, establishing long-term contracts with suppliers, advertising, logistics, training labor, and changing the store's layout and machinery equipment. Stores balance the adjustment costs of changing product variety against the expected future benefits. Stores can react to high demand shocks by increasing inventories without changing product categories (i.e., greater love for variety), which implies more inventory. Section 3.2 provides a detailed discussion of the link between entry regulations and cost $c_n(\cdot)$. Less restrictive entry regulations increase competitive pressure, forcing stores to improve their future productivity and invest in technology, creating incentives for stores to increase their product variety and store size. Technological advances in the store can benefit the existing number of product categories through faster product lines and a higher frequency of turnover (Holmes, 2001).

We model and solve the store's dynamic optimization problem, highlighting the dynamic role of entry regulations and adjustment costs for incumbents' endogenous product variety decisions, competition, and long-run profits. Store j maximizes the discounted expected value of future net cash flows using the Bellman equation:

$$V(\mathbf{s}_{jt}) = \max_{np_{jt}, a_{jt}} \left[\pi(\mathbf{s}_{jt}; np_{jt}, a_{jt}) - c_n(np_{jt}, a_{jt}, r_{mt}) + \zeta \mathbb{E}[V(\mathbf{s}_{jt+1}) | \mathcal{F}_{jt}] \right], \tag{1}$$

where the state vector $\mathbf{s}_{jt}$ includes store-specific state variables, market characteristics, and a list of state variables of other stores active in a local market, $\pi(\mathbf{s}_{jt})$ is the static profit function, and ζ is a store's discount factor.[12] The store information set $\mathcal{F}_{jt}$ includes only current and past information on productivity, demand shocks, product variety in the previous period, input prices, and local market characteristics (not future values).[13] Inventory holdings (a_{jt}) and the number of product categories (np_{jt}) have dynamic implications due to adjustment costs. Although we measure product variety by the number of product categories in a store, our framework can allow modeling of individual product-level data if available.

Equilibrium. Local and industrial policies affect stores' costs, inventory, investment in technology, labor and exit. We assume that these policies are unexpected and permanent once they are implemented. Ericson and Pakes (1995) discuss the existence of a Markov perfect equilibrium (MPE) in a modeling setting similar to ours. We assume that the market reflects a Markov equilibrium where each store includes aggregated local market characteristics in its state. Our approach approximates an MPE similarly to a moment-based Markov equilibrium (MME), but where every store recognizes that it can influence the aggregate state (Weintraub et al., 2008; Benkard et al., 2015; Ifrach and Weintraub, 2017). This simplification is essential for estimation because a curse of dimensionality problem would appear with many competitors, as in our data.

[12]The actual choice of the number of product categories np_{jt} can be written as $np_{jt} = np_{jt-1} + \Delta np_{jt}$, where Δnp_{jt} is the adjustment in the number of product categories. Therefore, the store's optimization problem given by the Bellman equation (1) is consistent with optimizing over Δnp_{jt} instead of np_{jt} because the previous number of product categories np_{jt-1} is part of the state space (it is part of inventory at the beginning of the year).

[13]Estimating stores' expectations and adaptation to changes in the local environment is challenging without high-frequency time-series data for a long period (see Doraszelski et al. (2018) for an application to electricity markets).

The equilibrium in the industry is stationary and includes the policies for the number of products $\tilde{np}_t(\mathbf{s}_{jt})$, inventory holdings before sales $\tilde{a}_t(\mathbf{s}_{jt})$ and the value function $V(\mathbf{s}_{jt})$ that are consistent with the stores' optimization problem (1). Conditional on the states, the stationarity of the equilibrium implies that the value functions are not indexed by time. The equilibrium implies that the states satisfy the Markov property before and after a change in a policy. To form expectations, stores use the optimal policies. We use the optimal policy functions that result to estimate productivity ω_{jt} and demand shocks μ_{jt}.

Modeling roadmap. Section 3.1 introduces a multiproduct technology and discuss its theoretical foundations. It is used to construct a product-category sales-generating function and to compute store revenues and static per-period profits. This multiproduct sales-generating function is similar to a standard production function but includes a demand shock and the usual productivity shock. Section 3.1.1 discusses identification and estimation of multiproduct sales function. The estimation is similar to Olley and Pakes (1996)[OP] but requires an additional control function to recover two store-specific variables that are unobservable to the researcher (revenue productivity and demand shocks). Estimation of the dynamic model is presented in Section 3.2.

3.1 Multiproduct Service Technology in Retail

Retailers offer multiple products and services to consumers. Offering multiple products creates difficulties in aggregating service output when there is not a single value function because the composite service output of a store depends on other things, including prices. In addition, the productivity of resources in a product or service is not independent of the level of services in other products.

ASSUMPTION 1: *The multiproduct service-generating function of a retailer can be written as an implicit function, which can be described by the transcendental function that generalizes the Cobb–Douglas function (Mundlak, 1964):*

$$F(\mathbf{Q}, \boldsymbol{\Psi}) = G(\mathbf{Q}) - H(\boldsymbol{\Psi}) = 0 \tag{2}$$

where $G(\mathbf{Q}) = Q_1^{\tilde{\alpha}_1} \times \cdots \times Q_{np}^{\tilde{\alpha}_{np}} exp(\tilde{\gamma}_1 Q_1 + \cdots + \tilde{\gamma}_{np} Q_{np})$; $H(\boldsymbol{\Psi}) = \Psi_1^{\tilde{\beta}_1} \times \cdots \times \Psi_d^{\tilde{\beta}_d} exp(\tilde{\omega})$; $\mathbf{Q}$ is the vector of service output (product categories in our case); Q_i is the i-th service output of the store (quantity of product category i), $i = \{1, \cdots, np\}$; Ψ_e is the e-th service input of the store (e.g., labor, capital, inventory), $e = \{1, \cdots, d\}$; and $\tilde{\omega}$ is the retailer's technical productivity (i.e., quantity-based total factor productivity).[14] The parameters $\tilde{\alpha}_1, \cdots, \tilde{\alpha}_{np}$ and $\tilde{\gamma}_1, \cdots, \tilde{\gamma}_{np}$ define the production frontier and affect product–product and product–input substitutions, playing a key role in profit maximization, and $\tilde{\beta}_1, \cdots, \tilde{\beta}_d$ affect product–input and input–input substitutions.

The assumption on the transformation function $G(\mathbf{Q}) - H(\mathbf{\Psi}) = 0$ is known as the separability property. It implies that retailers nearly always sell the product categories jointly. That is, the product categories cannot be sold separately using a sales technology for each product category (nonjoint sales). Second, it can be shown that a necessary and sufficient condition for separability is that the total cost function is multiplicatively separable (in quantity and input prices), which implies that the ratio of the two marginal costs is independent of input prices (Hall, 1973).[15] Under competitive equilibrium, this implies that product-category price ratios depend on the product-category mix. Third, a necessary and sufficient condition for nonjointness is that the total cost of selling all product categories is the sum of the cost of selling each product category separately. Nonjointness in sales technology is restrictive in retail because economies of scale and scope are not modeled explicitly (Panzar and Willig, 1981). It also implies that marginal cost ratios are independent of the product-category mix.

The theoretical results of the multiproduct service function related to profit maximization play a crucial role in the identification of sales technology. For example, productivity is typically defined as aggregate output over aggregate input; that is, the output and input coefficients $\tilde{\alpha}_i$ and $\tilde{\beta}_j$ affect the productivity measure. We show the general restrictions on the coefficients of transcendental multiproduct functions that are required to satisfy the static profit maximization conditions in settings with many inputs ($d \geq 2$) and outputs ($np \geq 2$) (see Mundlak (1964) for $d = 2$ and $np = 2$). Readers not interested in theoretical details can move directly to Section 3.1.1.

THEOREM 1: *Consider a general service-generating function* $F(\mathbf{Q}, \mathbf{\Psi}) = G(\mathbf{Q}) - H(\mathbf{\Psi}) =$

[14]Hicks (1946) discusses the general implicit production function. The exponential term in $G(\cdot)$ destroys the homogeneity of $H(\cdot)$ but allows inflexion points in the function (Halter et al., 1957).

[15]Hall (1973) proposes a multiproduct cost function specification where separability and nonjointness are introduced as parametric restrictions.

0, *where* $G(\mathbf{Q}) = Q_1^{\tilde{\alpha}_1} \times \cdots \times Q_{np}^{\tilde{\alpha}_{np}} exp(\tilde{\gamma}_1 Q_1 + \cdots + \tilde{\gamma}_{np} Q_{np})$; $H(\mathbf{\Psi}) = \Psi_1^{\tilde{\beta}_1} \times \cdots \times \Psi_d^{\tilde{\beta}_d} exp(\tilde{\omega})$. *If the parameters satisfy the following conditions (a)* $\tilde{\alpha}_i < 0$ *and* $\tilde{\gamma}_i > 0$ *for all* $i = \{1, \cdots, np\}$; *(b)* $\tilde{\beta}_e > 0$ *for all* $e = \{1, \cdots, d\}$, *then the conditions for profit maximization are satisfied.*

PROOF: The static profit maximization problem at the store level is $\max_{\mathbf{\Psi}} \Pi = \mathbf{P}'\mathbf{Q} - \mathbf{W}'\mathbf{\Psi}$ subject to $F(\mathbf{Q}, \mathbf{\Psi}) = 0$, where $\mathbf{P}$ and $\mathbf{W}$ are vectors of output and input prices, respectively. The main idea of the proof is that the sign of the determinant of the bordered Hessian matrix of the optimization problem should satisfy the second-order requirement for profit maximization. The proof and an additional discussion are provided in Appendix A.■

For certain values of $\tilde{\gamma}_i$, the service output is sold at the minimum cost, and the optimal inputs yield minimum revenues. However, we want to avoid these situations (saddle points) in the empirical applications.

PROPOSITION 1: *If the service function is simple Cobb–Douglas in outputs* ($\tilde{\gamma}_i = 0$ *for all* i) *and inputs and the first-order conditions are satisfied, then the optimal service quantity* $\mathbf{Q}^*$ *is sold at the minimum cost, and any inputs* $\mathbf{\Psi}^*$ *yield minimum revenues. The profit* $\pi(Q^*, \Psi^*)$ *at point* (Q^*, Ψ^*) *is a saddle point, i.e.,* $\pi(Q^*, \Psi) \leq \pi(Q^*, \Psi^*) \leq \pi(Q, \Psi^*)$.

PROOF: The proof uses the sign of the determinant of the Hessian matrix. For the full proof and an additional discussion, we refer readers to Appendix A (see also Mundlak, 1964).■

A direct consequence of Proposition 1 is that when the inputs $\mathbf{\Psi}$ produce minimum revenues and the first-order conditions are satisfied, then the profit can be maximized by a selection of product categories (a corner solution). This problem does not exist in the case of a single output. The conditions $\tilde{\alpha}_i < 0$ and $\tilde{\gamma}_i > 0$ for all i are not the only second-order conditions for profit maximization.[16] Another key aspect of multiproduct technology is that the sign of the $\tilde{\gamma}_i$ parameters determines the sign of the product category (factor) substitution (Appendix A). The marginal rate of substitution for $\tilde{\gamma}_i = 0$ implies that the product–product marginal rate of substitution is a convex function. This function is concave when $\tilde{\gamma}_i > 0$, which has key implications in empirical applications that allow for economies of scope.

Aggregation and the role of sales. To write the service-generating function at the product-category level, we normalize one parameter to one, i.e., the i-th output, which can be done by raising the service function to the power of $-\tilde{\alpha}_i$. The resulting parameters of product

[16]The result in Theorem 1 holds when some (not all) $\tilde{\alpha}_i$ are positive and the corresponding $\tilde{\gamma}_i$ can be set to zero, which reduces the number of parameters.

categories other than i have the opposite sign when $\tilde{\alpha}_i$ is negative. When quantity is unobserved, we want to set the weights $\tilde{\gamma}_i$ to obtain a meaningful interpretation of the aggregation across the store's product-category mix. Similarly to Mundlak (1964), we consider $\tilde{\gamma}_i = \tilde{\alpha}_y P_i$, where P_i is the price index of product category i (the price of a representative basket), which yields the product-category sales and reduces the number of parameters to be estimated. Thus, $\sum_{i=1}^{np} \tilde{\gamma}_i Q_i = \tilde{\alpha}_y \sum_{i=1}^{np} P_i Q_i = \tilde{\alpha}_y Y$, which is total store-level sales Y multiplied by $\tilde{\alpha}_y$, and it has a meaningful interpretation. The store's total sales thus play a key role in the relationship between inputs and product categories for the multiproduct service-generating function because they drive substitution between product categories. We use this result from transcendental production functions to write a product-category sales-generating function that accounts for sales of other products.

3.1.1 Empirical Framework: Multiproduct Sales-Generating Function

We model a multiproduct sales-generating function that accounts for changes in local entry regulations. Without loss of generality, we write the model at the product-category level using the simplest demand setting. If one accesses data on products inside a category, one can derive product-level sales while accounting for the nested structure.

ASSUMPTION 2: *All stores use the same service technology to sell their product categories, and this technology does not depend on the product.*

Based on transcendental technology (2), the multiproduct service-generating function of store j in logs is given by

$$\sum_{i=1}^{np_j} \tilde{\alpha}_i q_{ijt} + \tilde{\alpha}_y Y_{jt} = \tilde{\beta}_l l_{jt} + \tilde{\beta}_k k_{jt} + \tilde{\beta}_a a_{jt} + \tilde{\omega}_{jt} + \tilde{u}_{jt}^p, \tag{3}$$

where q_{ijt} is the log of the quantity of product category i sold by store j in period t, Y_{jt} is the total sales of store j in period t, l_{jt} is the log number of employees, k_{jt} is the log capital stock, a_{jt} is the log of the sum of the inventory level at the beginning of period t (n_{jt}) and the products bought during period t, and $\tilde{u}_{jt}^p$ are the remaining service output shocks. Assumption 2 allows us to reduce the number of parameters to be estimated. With sufficient data for all product categories across markets over a long period, assumption 2 can be relaxed to allow separate technologies for each product. Because each store is unique in our data, we omit the market index m and refer to store j in market m.

In a multiproduct setting, the sales technology possibilities require aggregation over the products. We use product equilibrium prices from a demand equation to obtain product sales and aggregate over products. A product category consists of physical products and store-specific services associated with each product. Two stores that sell product categories having the same label (e.g., kitchen furniture) do not sell exactly the same products in our model. Even if stores sell the same product brands in a category, it is unlikely that they offer the same purchase service to consumers for each product.

In our model, the total number of product categories across stores in a local market is the choice set of a consumer. For simplicity of exposition, we assume that consumers have constant elasticity of substitution (CES) preferences over differentiated product categories, and the researcher observes product information only for a sample of stores and total sales for all stores in local markets. The set of product categories from stores with the same service activity in a local market for which the researcher does not have product information defines the consumer's outside option.

The consumer's decision is how much of each product category to purchase from stores with product information available and from the outside option (i.e., from other stores in a local market that belong to the same five-digit subsector for which we do not have product information). The link between a CES demand system and a discrete choice demand system is used to write the consumer choice probability equation consistent with CES preferences[17]

$$q_{ijt} - q_{ot} = \mathbf{x}'_{ijt}\boldsymbol{\beta}_x + \sigma_a a_{jt} - \sigma p_{ijt} + \tilde{\mu}_{ijt}, \tag{4}$$

where p_{ijt} is the log of the price of product category i in store j; $\mathbf{x}_{ijt}$ represents the observed determinants of the intensive and extensive margins of the utility function when the consumers buy product category i from store j; σ is the elasticity of substitution; $\tilde{\mu}_{ijt}$ represents the unobserved product characteristics at the store level, for example, the quality of the shopping

[17] The demand system is similar to the logit discrete choice system based on unit demand, but the logarithm of price is used. A nested demand framework can be integrated, but the form of the sales-generating function will then include more terms.

experience attached to product i in store j; and q_{ot} is the outside option quantity.[18]

Before making decisions on inputs and variety, the stores observe only aggregate information on demand shocks at the store level (i.e., μ_{jt}). Stores observe the realizations of demand shocks μ_{ijt} for each product category only after they select product categories.[19] The presence of a_{jt} in a demand equation captures that consumers prefer stores with products in stock.

Multiplying the price p_{ijt} from (4) by the output weights (elasticities) $\tilde{\alpha}_i$, summing over the number of products, and using the result in (3), we obtain the store-level sales-generating function that is used to obtain sales for product i, y_{ijt}

$$y_{ijt} = -\alpha_y y_{-ijt} + \beta_l l_{jt} + \beta_k k_{jt} + \beta_a a_{jt} + \beta_q y_{ot} + \mathbf{x}'_{jt}\boldsymbol{\beta}_x + \omega_{jt} + \mu_{jt} + u^p_{ijt}, \tag{5}$$

where y_{-ijt} is the log of sales of product categories other than i, y_{ot} measures the sales of the outside option, $\mathbf{x}_{jt}$ sums all observed characteristics at the store and market levels, and u^p_{ijt} represents i.i.d. remaining shocks to sales that are mean-independent of all control variables and store inputs ($E[u^p_{ijt}|\mathcal{F}_{jt}] = 0$). We show the derivation of equation (5) in Appendix B.[20]

In the empirical implementation, sales y_{ot} measures the sales of product categories by stores that belong to the same five-digit subsector for which we do not have product information in the local market m.[21] We include only local market variables in $\mathbf{x}_{jt}$ (e.g., population, population density, income) and therefore use the notation $\mathbf{x}_{mt}$ instead of $\mathbf{x}_{jt}$ in what follows. The observed and unobserved product characteristics are aggregated to the store level with $\tilde{\alpha}_i$ as weights. The variable μ_{jt} is the weighted sum of all product demand shocks μ_{ijt} at the store level, summarizing the aggregate information on demand shocks observed by stores when they

[18]The outside option q_{ot} measures the aggregate quantity of product categories sold by stores that belong to the same five-digit subsector for which we do not have product information in a local market. The left-hand side of equation (4) is a simplification of the well-known difference in logarithms of market shares in quantities in logit models (Appendix B). σ is globally identified for the set of products with positive choice probabilities. The system satisfies the connected substitutes condition and is invertible.

[19]Eizenberg (2014) uses a similar assumption in an empirical discrete choice demand framework.

[20]Equation (5) is derived by rewriting the linear sum of product category sales $\sum_{i=1}^{np_{jt}} \left[\tilde{\alpha}_i y_{ijt} + \left(1 - \frac{1}{\sigma}\right) \tilde{\alpha}_y Y_{ijt}\right] \equiv \alpha_i y_{ijt} + \alpha_y y_{-ijt}$ and normalizing $\alpha_i = 1$.

[21]If the outside option is "do not buy," y_{ot} is total sales in market m (aggregate sales). Appendix B derives y_{ot} using equations (3) and (4).

make input decisions. Demand shocks related to product quality, location, checkout speed, the courteousness of store employees, parking, bagging services, cleanliness, etc., are part of μ_{jt}. Thus, demand shocks μ_{jt} include factors related to customer satisfaction and the quality of shopping in store j in period t.

The multiproduct sales-generating function (5) differs from a single-product function in that it controls for "competition" within the store, which is represented by the effect of the sales of other product categories on the sales of a product category in a store. The multiproduct service technology (3) allows for rich information on service product substitution and models the relationship between sales per product category and total sales. In our data, stores do not sell the same product categories, and therefore, we cannot identify all parameters $\tilde{\alpha}_i$ and $\tilde{\alpha}_y$ without a selection over products or panel data that cover a long time period. By using the sales of different products in equation (5), we reduce the number of parameters to be estimated and obtain information on economies of scope. Therefore, we estimate only the coefficient of sales of products other than product i in store j (α_y) and not all coefficients α_i, $i = \{1, \cdots, np_j\}$.

The economies of scope parameter α_y provides a critical understanding of stores' changes in total sales when they change the number of product categories while their resources remain the same or retain the same product categories and change the store's resources (see Maican and Orth (2021) for a detailed discussion and numerical simulations). The coefficient α_y plays a key role in both the persistence and level of productivity. The input coefficients in the multiproduct sales-generating function (5), i.e., β_l, β_k, β_a, β_q, are functions of the elasticity of substitution σ and are similar to the aggregate sales-generating function at the store level, which allows us to compare them with the estimates for a single-output technology.[22]

In service industries, it is difficult to define a clean measure of technical productivity due to the complexity of measuring output and economies of scale and scope (Oi, 1992). Estimating only one coefficient for the other product categories (α_y) when controlling for prices has a cost—we cannot obtain a clean measure of technical productivity $\tilde{\omega}_{jt}$ because the coefficients of labor, capital and inventories include demand shocks even if we control for the elasticity of

[22]The coefficients of the multiproduct sales technology are functions of σ, i.e., $\beta_q = 1/\sigma$, $\beta_l = \tilde{\beta}_l(1 - 1/\sigma)$, $\beta_k = \tilde{\beta}_k(1 - 1/\sigma)$ and $\beta_a = \overline{\beta}_a(1 - 1/\sigma)$. The parameters σ_a and $\tilde{\beta}_a$ are included in $\overline{\beta}_a$, and they cannot be separately identified. Thus, we cannot separately identify the effect of inventory on supply and demand; that is, we identify the net effect through β_a (see Section 3.1.2 and Appendix B).

substitution. Therefore, the variable $\omega_{jt} \equiv (1 - 1/\sigma)\tilde{\omega}_{jt}$ measures revenue (sales) productivity. We simply refer to ω_{jt} as store productivity in what follows. The productivity measure ω_{jt} might include sales shocks due to the approximations in (5), but all these sales shocks are different from demand shocks μ_{jt} that affect consumer preferences for product categories in a store. Nevertheless, the productivity shocks ω_{jt} can be separated from the store's demand shocks μ_{jt}, which are part of the demand and affect store market share.

A few aspects of the multiproduct sales-generating function should be noted. First, store productivity and demand shocks affect sales, and they are not observed by the researcher but are observed by stores when decisions are made. Second, the multiproduct setting in Section 3.1 requires a positive α_y for static profit maximization to hold. This condition also holds in a dynamic setting because a policy function (input choice) should be optimal in each period.[23] We now discuss the evolution of the main state variables.

ASSUMPTION 3: *Store productivity and demand shocks follow two first-order Markov processes: (i) an endogenous process $P_\omega(\omega_{jt}|\omega_{jt-1}, \mu_{jt-1}, r_{mt-1})$, where r_{mt-1} measures regulation in local market m in period $t-1$, and (ii) an exogenous process, $P_\mu(\mu_{jt}|\mu_{jt-1})$, and (iii) the distributions $P_\omega(\cdot)$ and $P_\mu(\cdot)$ are stochastically increasing in ω and μ and are known to stores. Local market characteristics evolve according to a process known by all stores. All stores perceive regulation policies to be permanent.*

Assumption 3 states that the demand shocks μ_{jt} are correlated over time according to a first-order Markov process

$$\mu_{jt} = h_t^\mu(\mu_{jt-1}; \gamma^\mu) + \eta_{jt}, \tag{6}$$

where $h_t^\mu(\cdot)$ is an approximation of the conditional expectation and η_{jt} are shocks that are mean-independent of all information known at $t-1$. Store productivity ω_{jt} follows an endogenous first-order Markov process where productivity, previous demand shocks, and entry regulation affect future productivity:

$$\omega_{jt} = h_t^\omega(\omega_{jt-1}, \mu_{jt-1}, r_{mt-1}; \gamma^\omega) + \xi_{jt}, \tag{7}$$

[23]The sign conditions on the first and second derivatives that are used to prove Theorem 1 and Proposition 1 remain the same in a dynamic setting.

where $h_t^{\omega}(\cdot)$ is an approximation of the conditional expectation and ξ_{jt} are shocks to productivity that are mean-independent of all information known at $t-1$. In the empirical implementation, the functions $h^{\omega}(\cdot)$ and $h^{\mu}(\cdot)$ in the Markov processes of ω_{jt} and μ_{jt} are polynomials, i.e., $\mu_{jt} = \gamma_0^{\mu} + \gamma_1^{\mu}\mu_{jt-1} + \gamma_2^{\mu}(\mu_{jt-1})^2 + \gamma_3^{\mu}(\mu_{jt-1})^3 + \eta_{jt}$ and $\omega_{jt} = \gamma_0^{\omega} + \gamma_1^{\omega}\omega_{jt-1} + \gamma_2^{\omega}(\omega_{jt-1})^2 + \gamma_3^{\omega}(\omega_{jt-1})^3 + \gamma_4^{\omega}\mu_{jt-1} + \gamma_5^{\omega}r_{mt-1} + \gamma_6^{\omega}\omega_{jt-1} \times \mu_{jt-1} + \gamma_7^{\omega}r_{mt-1} \times \omega_{jt-1} + \gamma_8^{\omega}r_{mt-1} \times \mu_{jt-1} + \xi_{jt}$. To survive fiercer competition after entry, incumbents improve productivity by learning practices from entrants (external learning). Stores can also use information about previous demand shocks, capturing why consumers choose a store to improve productivity. For example, rearranging products on shelves such that consumers have faster access improves a store's efficiency in allocating resources. Modeling the impact of less restrictive entry regulation on revenue productivity and repositioning in product categories and inventory choices allows for the possible net agglomeration effects (e.g., productivity and demand spillover) on store sales, even if we do not model the channels and spatial relationships (Duranton and Puga, 2020).

ASSUMPTION 4: *The inventory level in period $t+1$ evolves according to $N_{jt+1} = \tilde{N}_t(A_{jt}, Y_{jt})$, where A_{jt} is the adjusted inventory, i.e., inventory at the beginning of period N_{jt} adjusted by the products bought in period t. The function $\tilde{N}_t(\cdot)$ is increasing in A_{jt} and decreasing in Y_{jt}.*[24] *The capital stock accumulates according to $K_{jt+1} = (1-\delta_K)K_{jt} + I_{jt}$, where δ_K is the depreciation rate and I_{jt} is investment.*

Inventory affects stores' service output because high inventory is costly to keep in stock and low inventory reduces consumers' choices. Products bought from wholesalers are an input that together with inventory at the beginning of period t (A_{jt}) lead to inventory levels in the beginning of period $t+1$ after realization of sales in period t (N_{jt+1}). Stores with high μ_{jt} increase their products bought from wholesalers. However, this also leads to a drop in inventories at the beginning of the next year because of the unexpected increase in sales. In other words, there is a distinction between how μ_{jt} affects current inventories and products bought from the wholesaler and the realization of inventories at the end of the year.

We now turn to the assumptions on the policy functions (input demand functions) that are required to recover productivity ω_{jt} and demand shocks μ_{jt}. We assume that labor $l_{jt} = \tilde{l}_{jt}(\mathbf{s}_{jt})$,

[24] In physical units, inventory in $t+1$ evolves according to $N_{jt+1} = A_{jt} - Y_{jt}$.

which is part of profits $\pi(\cdot)$, is chosen to maximize short-run profits.[25]

ASSUMPTION 5: *The multivariate function* $(\tilde{l}_{jt}, \tilde{a}_{jt})$ *is a bijection onto* (ω_{jt}, μ_{jt}).

Strict monotonicity guarantees the inversion in the case of a single policy function and unobservable factor.[26] In our case with two unobservables, invertibility implies solving systems of nonlinear equations. A key condition for invertibility is that the determinant of the Jacobian is not zero. This condition is satisfied when productivity and demand shocks have different impacts on labor and inventory, and the relative impact is not the same $(\partial\tilde{l}/\partial\omega)/(\partial\tilde{l}/\partial\mu) \neq (\partial\tilde{a}/\partial\omega)/(\partial\tilde{a}/\partial\mu)$. This requirement is not restrictive and can be empirically tested with the estimated policy functions (Section 4). Appendix C discusses invertibility in detail.

Market share index function. Following the production function literature to control for unobservables, we use an output index function and an input process to recover the demand shocks μ_{jt} (Ackerberg et al., 2007). The aim of the index function is to identify μ_{jt} separately from ω_{jt} and not to infer, e.g., changes in price elasticities due to repositioning in product categories. Store demand shocks μ_{jt} are defined as a weighted sum of product category–specific demand shocks to store j from the demand system (4) and include information that affects consumers' store choice and the store's market share. Most importantly, the aggregation weights in μ_{jt} arise from the multiproduct service technology (2). Thus, the store's market share is an informative output for the index function, which is computed using product-category sales that are affected by demand shocks. We use inventory before sales, as it contains information about μ_{jt}, as input demand.

We consider the output of an index function with store and market characteristics $\boldsymbol{\delta}_{jt}$ (which can include $\mathbf{x}_{mt}$) and μ_{jt} as arguments

$$\tau_{jt} = \tau_t(\boldsymbol{\delta}_{jt}; \boldsymbol{\rho}) + \mu_{jt} + \nu_{jt}, \tag{8}$$

[25]If labor has dynamic implications, then l_{jt-1} is part of the state space, and the optimal labor $l_{jt} = \tilde{l}_t(\cdot)$ solves the Bellman equation.

[26]Note that the fact that labor is strictly increasing in ω_{jt} can be shown when using Cobb–Douglas technology (Doraszelski and Jaumandreu, 2013; Maican and Orth, 2017). This also holds in our case because the transcendental technology is a generalization of Cobb–Douglas technology. The fact that the inventory demand function before sales $\tilde{a}_t(\cdot)$ is increasing in μ_{jt} is valid in retail markets. a_t does not depend on product prices p_{ijt}. The invertibility condition is difficult to satisfy with end-of-year inventory n_t instead of a_t; e.g., we might have large inventory n_t because of ongoing high demand or low demand realization.

where the output index $\tau_{jt} = ln(ms_{jt}) - ln(ms_{0t})$ is the ratio of the store market share and the market share of the outside option; ms_{jt} is the market share of store j in local market m in period t computed at the five-digit industry sector level using sales; ms_{0t} is the outside option, i.e., the market share of other stores in market m computed at the five-digit industry sector level (we have the same outside option as in equation (5), but here we use a share-based measure); and ν_{jt} is an error term that is mean-independent of all controls. In the empirical implementation, we choose a simple linear specification for $\tau_t(\cdot)$, i.e., $\tau_t(\boldsymbol{\delta}_{jt};\boldsymbol{\rho}) = \rho_{np}np_{jt} + \rho_{inc,1}inc_{mt} + \rho_{inc,2}inc^2_{mt}$, where inc_{mt} is the logarithm of the average income in the local market.[27]

The index function is consistent with multiproduct sales. It aggregates stores' category sales from the multiproduct sales function in the output index τ_{jt} and is informative about store demand and consistent with aggregate demand in a local market (it includes μ_{jt}). A complication of using a store-level aggregate demand system, where consumers obtain utility from choosing a store, is the need for price data and to define a basket of products to calculate a price index consistent with the multiproduct service technology.[28] Indeed, our framework relies on all stores in five-digit service industries for which price data are scant. Although one could access price data, it is difficult to define an annual price index given that labor and capital are observed yearly.

We now explain the market share index function and its link to the multiproduct sales technology. First, services frequently rely on sales that depend on both demand and supply to measure output. In our model, sales depend on both the store's demand shocks μ_{jt} and productivity ω_{jt}, whereas a store's market share index function depends only on μ_{jt}. To guarantee consistency and identification, the demand shocks μ_{jt} enter additively and connect the market share index function (8) and the sales-generating function (5). Because the sales-generating function (5) controls for the capital stock k_{jt} and inventory a_{jt}, they are not part of μ_{jt}, and we do not need to control for them in the market share index function.[29] The number of product

[27]Because we have a rich state space in the control functions and two equations, the estimates of the market index specification remain robust after controlling for other omitted factors, such as the number of stores (part of the outside market share).

[28]As in a nested-logit model, we can derive the probability of choosing store j as a function of p_{ijt} and μ_{ijt} using the conditional choice probability. However, this is not helpful in the identification because p_{ijt} and μ_{ijt} are not observed.

[29]Even if we control for the capital stock k_{jt} and inventory a_{jt} in the market share index equation, we cannot separately identify their effects on demand and supply; i.e., we identify the

categories np_{jt} affects a_{jt}, which includes additional information such as the volume of each product, and products are aggregated based on monetary value.

Second, supply-side weights included in μ_{jt} and remaining shocks ν_{jt} restrict us from relying on nonparametric inversion from the discrete choice literature to recover μ_{jt}. Although the index function (8) is not a logit demand specification, being a function of np_{jt} and ν_{jt} but not the price, it is useful for understanding store local market demand. The market share index function uses the same output as a logit demand system consistent with CES assumptions. The reason is that market share captures information about local market demand and enables a simple expression for the logarithm of store sales and the outside option.[30]

Regulation and endogenous product sales and market shares. We obtain a joint system of equations from the multiproduct sales equation and the market share index function and find its solution using the nested fixed-point algorithm. Note that μ_{jt} has an additive effect on the log of market share and a nonlinear one on sales because multiproduct technology forms a nonlinear system of equations that gives product sales. We have two systems of equations: sales per product category at the store level (equation (5)) and store local market share (equation (8)). Using recovered demand shocks, we solve the joint systems of equations to obtain sales per product category and the outside option local market sales (total sales) following policy interventions that affect stores' primitives.[31]

We describe how the model can be used to compute changes in product-category sales and sales of stores in the outside option (y_{ot}) after policy changes. A numerical implementation of the model also helps improve the understanding of the integration of different parts of the model. For simplicity of exposition, we assume only one store ($j = 1$) for which we observe the number of product categories and have recovered productivity and demand shocks. The

net effect. Appendix B discusses the identification of β_a.

[30] The ratio of market shares of two stores depends on the number of product categories that they offer and demand shocks. Because store-specific demand shocks depend on the product-category mix, the market share ratio changes if one of the stores alters its product-category mix without changing the number of product categories. One way to avoid the IIA problem specific to logit models in equation (8) is to group product categories by a store characteristic and rewrite equations (4) and (8) in nested-logit form. This is beyond the scope of this paper.

[31] The market share index function is not useful in counterfactuals if the outside option sales are unaffected by changes in the local environment. In this case, the index function is used only in identification to recover demand shocks μ_{jt}.

multiproduct sales equation can be written as $y_{i1t} = -\alpha_y y_{-i1t} + (1/\sigma)y_{ot} + T_{1t} + \mu_{1t}$, where the term T_{1t} groups all store characteristics (labor, capital, inventory, productivity, and market characteristics) that are in equation (5) and $i = \{1, \cdots, np_1\}$ indexes the product categories of the store. The index equation can be written as $ln(\sum_{i=1}^{np_1} exp(y_{i1t})) - ln(y_{ot}) = \boldsymbol{\delta}_{1t}\boldsymbol{\rho} + \mu_{1t}$. We start with an initial value for y_{ot} denoted by $y_{ot}^{(0)}$. Then, we use the multiproduct equation to compute product-category sales $y_{i1t}^{(0)}$ using the fixed-point algorithm to solve the multiproduct sales system of equations (the number of equations is given by the number of categories). The computed product-category sales $y_{i1t}^{(0)}$ are used to obtain the next sales of the outside option $y_{ot}^{(1)}$, which are used to compute the next period's product-category sales $y_{i1t}^{(1)}$. We repeat this process until $\|y_{i1t}^{(n+1)} - y_{i1t}^{(n)}\| < tol$ and $\|y_{ot}^{(n+1)} - y_{ot}^{(n)}\| < tol$, where tol is a numerical tolerance level and n is the number of iterations. The same algorithm is applied if there are many stores in a market for which we observe their product categories.[32]

Discussion on regulation and competition effects. The design of the Swedish entry regulation targets the use of land, water and buildings, and its implementation covers all activities. Therefore, entry regulation is exogenous to stores in our setting that focuses on product variety repositioning by incumbents. More lenient entry regulation can affect the number of stores in our retail sectors and the number of firms in other industries that are not in our data. Changes in the local business environment affect competition and create new potential demand for retailers and opportunities for suppliers. To accentuate product variety decisions among incumbents and to keep the model tractable, we do not explicitly model store entry and exit. However, our framework models all stores and is rich in modeling local competition. First, we allow for oligopolistic competition. Changes in entry regulation affect incumbents' productivity and cost-enhancing product variety. Therefore, the optimal choice of variety and inventory before sales, which are functions of the rivals' productivity and demand shocks, are obtained by solving the store's dynamic programming problem. Second, having the store's optimal choices, we recompute the product- and store-level sales and total sales of all stores in a local market for which we do not have product variety information (i.e., outside option y_{ot}) by solving the nonlinear systems of equations given by the multiproduct sales and market share index equations. Thus, even if we do not access prices, we fully endogenize the store's variety and sales and total

[32]The Monte Carlo simulations show a fast convergence of the algorithm for a large number of products and stores.

sales in the outside option in a local market to account for the complex impact of changes in competition due to regulation.

Identification and estimation of multiproduct sales function. The identification and estimation of the sales-generating function and the Markov processes for ω_{jt} and μ_{jt}(i.e., β_l, β_k, β_a, α_y, σ, ρ_{np}, $\rho_{inc,1}$, $\rho_{inc,2}$, $\boldsymbol{\gamma}^{\omega}$, and $\boldsymbol{\gamma}^{\mu}$) are based on the well-established two-step methods in OP and following production function literature (Levinsohn and Petrin, 2003; Ackerberg et al., 2015; Gandhi et al., 2020). Identification comes from a system of equations (multiproduct sales and market share) and two unobservables (productivity and demand shocks), where one of the unobservables is part of only one equation. Two control functions based on the store's optimal policy functions are used to proxy for ω_{jt} and μ_{jt}.[33] Compared to OP, we have two unobservables to recover, and we show how the market share index function helps to recover demand shocks μ_{jt} separate from productivity ω_{jt} and ensures the identification of the model. By inverting these policy functions to solve for ω_{jt} and μ_{jt} (Assumption 5 must hold), we obtain $\omega_{jt} = f_t^1(l_{jt}, k_{jt}, n_{jt}, w_{jt}, a_{jt}, y_{ot}, \mathbf{x}_{mt}, r_{mt})$ and $\mu_{jt} = f_t^2(l_{jt}, k_{jt}, n_{jt}, w_{jt}, a_{jt}, y_{ot}, \mathbf{x}_{mt}, r_{mt})$, which are nonparametric functions of the observed variables in the state space and the controls (Appendix C). In the first step, we use the inversions of the policy functions to construct measures of productivity ω_{jt} and demand shocks μ_{jt} as functions of the structural parameters that do not include the remaining shocks u_{ijt}^p and ν_{jt}. Then, we rewrite the sales and market share equations (i.e., 5) and (8)) using parametric forms of productivity ω_{jt} and demand shocks μ_{jt} and Markov processes. The parameters of the multiproduct sales function and market share equation are identified using moment conditions on the remaining shocks in these equations, $\xi_{jt}+u_{ijt}^p$ and $\eta_{jt}+\nu_{jt}$. Online Appendix B.2 presents an extensive discussion of all identification details for the multiproduct sales function and market share equation (see also Maican and Orth (2021)).

The vector of parameters to be estimated is $\boldsymbol{\theta} = (\beta_l,\ \beta_k,\ \beta_a,\ \alpha_y,\ \sigma,\ \boldsymbol{\beta}_x,\ \rho_{np},\ \rho_{inc,1},\ \rho_{inc,2},\ \boldsymbol{\gamma}^{\omega}$, and $\boldsymbol{\gamma}^{\mu})$. Productivity ω_{jt} and μ_{jt} are functions of $\boldsymbol{\theta}$. We can identify the $\boldsymbol{\theta}$ coefficients using moment conditions based on $(\xi_{jt} + u_{ijt}^p)$ and $(\eta_{jt} + \nu_{jt})$ and the generalized method of moments (GMM) estimator. The identification uses the fact that the current shocks are conditionally independent of information in $t-1$, $\mathcal{F}_{jt-1}$. To identify $\boldsymbol{\theta}$, we use the moment conditions

[33] Ackerberg et al. (2007) (Section 2.4) and Matzkin (2008) discuss the core of identification of such systems of equations.

$E[\xi_{jt} + u^p_{ijt}|\ y_{-ijt-1},\ l_{jt-1},\ k_{jt-1},\ a_{jt-1},\ \mathbf{x}_{mt-1}] = 0$ and $E[\eta_{jt} + \nu_{jt}|np_{jt-1}, inc_{mt-1}, inc^2_{jt-1}] = 0$. The parameters β_l, β_k, and β_a are identified using l_{jt-1}, k_{jt-1}, and a_{jt-1} as instruments. Thus, we exploit the fact that the current remaining productivity and sales shocks are not correlated with previous inputs to form moment conditions. To identify the economies of scope parameter α_y, we use y_{-ijt-1} as an instrument, which requires that the previous output is not correlated with current remaining sales and productivity shocks. The Monte Carlo experiments discussed below show the robustness of the identification of the scope parameter α_y using previous output.[34] The fact that previous local market characteristics $\mathbf{x}_{mt-1}$ are not correlated with current remaining sales and productivity shocks allows us to identify $\boldsymbol{\beta}_x$.[35] To identify the coefficients of the market share equation, we use the fact that $(\eta_{jt} + \nu_{jt})$ are not correlated with the previous number of product categories and income. The Markov process parameters $\boldsymbol{\gamma}^\omega$ and $\boldsymbol{\gamma}^\mu$ are identified using the corresponding polynomial terms in equations (7) and (6) as instruments. The $\boldsymbol{\theta}$ parameters are estimated by minimizing the GMM objective function

$$\min_{\beta} Q_N = \left[\frac{1}{N}W'v(\boldsymbol{\theta})\right]' A \left[\frac{1}{N}W'v(\boldsymbol{\theta})\right], \tag{9}$$

where $v_{jt} = (u_{ijt} + \xi_{jt}, \nu_{jt} + \eta_{jt})'$, W is the matrix of instruments, and A is the weighting matrix defined as $A = \left[\frac{1}{N}W'v(\boldsymbol{\beta})v'(\boldsymbol{\beta})W\right]^{-1}$.[36]

Monte Carlo simulations. In Appendix B.1, we provide Monte Carlo simulations to show the identification of multioutput technology with two inputs, labor and capital, using the control function approach. We also discuss the bias of the labor and capital coefficients of a single-output technology when the true data-generating process (DGP) of total output is a multiproduct technology.

Alternative specifications. Appendix C discusses several extensions: an alternative estimator, nested demand specifications, a specification controlling for the endogeneity of regulation, and the relationship with other multiproduct estimators.

[34] Appendix C discusses a more computationally demanding estimator.

[35] $\mathbf{x}_{mt}$ are also valid instruments because market characteristics are exogenous.

[36] Standard errors are computed according to Ackerberg et al. (2012).

3.2 Estimation of the Dynamic Model

This section discusses the estimation of the dynamic model that is used to compute the optimal number of product categories and long-run profits after changes in entry regulations or other changes in the local business environment.

Regulation and adjustment costs of product variety. Entry regulations affect stores' operating costs. First, more restrictive regulation can increase stores' operating costs through higher fixed costs, e.g., expensive location or building costs (Joskow and Rose, 1989; Maican and Orth, 2018). Any product repositioning that requires reconstruction of the store (e.g., larger store space, installation of special equipment, access for trucks) and that might change the store's environmental aspects are subject to entry regulation. Integrating new product offerings in the store also involves labor training, inventory management, advertising, logistics, and the working out of contracts with suppliers. Second, in markets with fewer stores (due to restrictive regulation), the cost of logistics can increase, and product differentiation decreases. Consumers in these markets need to travel longer distances and can compensate for the longer travel time by spending less time in a store. This industry background highlights that controlling for entry regulation in the cost of managing variety is essential for our understanding of stores' decisions. Regulations impact the adjustment costs in variety (and inventory) through both demand and supply channels because regulation changes induce agglomeration effects, affecting the stores' cost (Duranton and Puga, 2020). However, our model remains consistent even if entry regulations do not affect the cost of managing variety. That is, the store's optimal inputs are still affected by entry regulations through the productivity channel. Most importantly, we can test whether the coefficients of the terms in $c_n(\cdot)$ that include entry regulations are statistically significantly different from zero in the empirical implementation.

We assume that stores have quadratic adjustment costs in product categories (M SEK): $c_n(np_{jt}, a_{jt}, r_{mt}; \boldsymbol{\varphi}) = \varphi_1 np_{jt} + \varphi_2 np_{jt}^2 + \varphi_3 exp(a_{jt})^2 + \varphi_4 exp(a_{jt}) np_{jt} + \varphi_5 np_{jt} r_{mt} + \varphi_6 exp(a_{jt})$ r_{mt}. The marginal effect of more liberal entry regulation (an increase in r_{mt}) on adjustment costs in variety depends on the store's number of product categories and size of inventory a_{jt}. A change in regulations affects the store's cost and productivity and, consequently, the number of product categories, sales, and market share. Thus, the store's value function is given by the following Bellman equation:

$$V(\mathbf{s}_{jt}) = \max_{np_{jt},a_{jt}} \{\pi(\mathbf{s}_{jt}; np_{jt}, a_{jt}) - c_n(\cdot;\boldsymbol{\varphi}) + \zeta\mathbb{E}[V(\mathbf{s}_{jt+1})|\mathcal{F}_{jt}]\},$$

where $\pi_{jt}(\mathbf{s}_{jt})$ measures the variable profits and $\boldsymbol{\varphi} = (\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5, \varphi_6)$ are the parameters to be estimated in the dynamic stage using value function approximation and simulation (Ryan, 2012; Sweeting, 2013).

State space. The state vector $\mathbf{s}_{jt}$ is large because it includes many store-specific :state variables (i.e., ω_{jt}, μ_{jt}, k_{jt}, n_{jt}, np_{jt-1}, w_{jt}) and local market variables (i.e., y_{ot}, $\mathbf{x}_{mt}$, r_{mt}) and a list of state variables of other stores active in a local market. Most local markets contain a large number of stores (i.e., high-dimensional rival states). We reduce the computational complexity of the dynamic model by assuming that stores do not keep track of the state variables of every rival store, which reduces the state space. Instead, stores use the sum of rivals' productivity and demand shocks in a local market in a subsector as sufficient statistics. Our approach relates to the theoretical literature that approximates MPE based on oblivious and moment-based equilibrium concepts. Recent empirical literature uses these approximation methods to reduce the computational complexity of MPE (e.g., Sweeting, 2013; Barwick et al., 2023; Gowrisankaran et al., 2022; Jeon, 2022). Thus, in the empirical implementation, the state space vector $\mathbf{s}_{jt}$ includes ω_{jt}, μ_{jt}, k_{jt}, n_{jt}, np_{jt-1}, w_{jt}, y_{ot}, $\mathbf{x}_{mt}$, r_{mt} and $\sum_{s\neq j}(\omega_{st} + \mu_{st})$.[37]

Value function approximation. We approximate the value function V using radial basis function networks (RBFN), i.e., $V(\mathbf{s}_{jt}) = \mathbf{bs}(\mathbf{s}_{jt})\boldsymbol{\kappa}$, where $\mathbf{bs}(\cdot)$ are the basis functions (Mai-Duy and Tran-Cong, 2003; Sutton and Barto, 2018).[38] This allows us to rewrite the Bellman equation as

$$V(\mathbf{s}_{jt};\boldsymbol{\kappa}) = \max_{np_{jt},a_{jt}} \{\pi(\mathbf{s}_{jt}; np_{jt}, a_{jt}) - c_n(\cdot;\boldsymbol{\varphi}) + \zeta\mathbb{E}[V(\mathbf{s}_{jt+1};\boldsymbol{\kappa})|\mathcal{F}_{jt}]\}.$$

[37]Our model estimates are robust when we use only the sum of rivals' productivity ($\sum_{s\neq j}\omega_{st}$) in the state space (we find only minor changes in the cost function coefficients). Previous versions of the paper presented single-agent estimates of the model. The results revealed a smaller competitive impact of lenient entry liberalization on long-run profits than in the dynamic game findings.

[38]RBFN are derived from the theory of function approximation and are commonly used in the AI and machine learning literature.

For each set of cost parameters $\boldsymbol{\varphi}$, we use the linearity property of the value function approximation to find the approximation parameters $\boldsymbol{\kappa}$ such that the Bellman equation holds. We use the RBFN approximation to find the optimal policies using the state variables in t and $t+1$. The multiproduct technology estimation gives the transitions for productivity and demand shocks. The inventory at the end of the period (i.e., the beginning of the next period) is estimated using a b-splines approximation in a_{jt} and y_{jt}.[39]

To compute total store-level sales using our model, we need to solve a system of nonlinear equations (for each store), which is given by the multiproduct technology. This system of equations has a unique solution and is solved using fixed-point iteration. The store's total sales are a function of the number of product categories. Net profits $\pi(\cdot)$ are computed as sales minus labor cost, i.e., $\pi(\cdot; np_{jt}, a_{jt}) = \sum_{i=1}^{np_{jt}} exp(y_{ijt}) - exp(w_{jt}) \times exp(l_{jt})$, where log sales y_{ijt} are computed by solving the system of equations given by the multiproduct technology (5) and market share index function (8) for each store at the optimal choices of np_{jt} and a_{jt}.

Estimation. Given an initial estimate of $\boldsymbol{\varphi}$ and approximation parameters $\boldsymbol{\kappa}$, we solve the first-order condition in the Bellman equation to find the optimal number of product categories np_{jt} and inventory a_{jt} in each state. Then, new value function approximation parameters $\boldsymbol{\kappa}$ are found by solving the Bellman equation. The cost parameters are estimated using the method of moments (Gourieroux and Monfort, 1996). The estimator matches the percentiles of the observed number of product categories (np_{jt}) and inventory (a_{jt}) distributions $\mathcal{P}_x$ ($x = [.05, .10, .15, \cdots, .95]$) with percentiles generated by the policy functions from the model (i.e., solving the system of first-order conditions). We denote the vector moments generated by the model as $\tilde{\mathcal{P}}(\boldsymbol{\varphi})$, which depend on the structural parameters, and $\mathcal{P}$ as the corresponding vector of data moments. The criterion function minimizes the distance between the moments $\tilde{\mathcal{P}}(\boldsymbol{\varphi})$ and $\mathcal{P}$

$$J(\boldsymbol{\varphi}) = [\mathcal{P} - \tilde{\mathcal{P}}(\boldsymbol{\varphi})]' \boldsymbol{W} [\mathcal{P} - \tilde{\mathcal{P}}(\boldsymbol{\varphi})], \tag{10}$$

where $\boldsymbol{W}$ is a weighting matrix.[40] The cost function coefficients are identified by matching the observed and predicted percentiles of the distribution of np_{jt} and a_{jt}. The standard errors are computed using subsampling.

[39] We assume that regulatory changes do not affect the structural form of this relationship. However, regulation affects the variables of this function, a_{jt} and y_{jt}.

[40] The identity matrix is used in the empirical setting.

4 Results

First, we discuss the multiproduct sales-generating function estimates and the role of entry regulations and the determinants of the number of product categories and product-category sales competition in a store. Then, we present the dynamic model estimates, long-run profits and benefits of adding products in different markets.

Sales-generating function estimates. Table 4 shows the estimates of the multiproduct sales-generating function in equation (5) by OLS and the nonparametric two-step estimators presented in Section 3.1.[41] The estimated coefficients of labor and inventories decrease from 0.784 (OLS) to 0.571 and from 1.037 (OLS) to 0.411, respectively, using the two-step estimator. The coefficient of capital increases; i.e., it is 0.061 (OLS) and 0.289 (the two-step estimator). The changes in the estimated coefficients are in line with the previous literature, which suggests upward bias in the coefficients of labor and inventories when the correlation between inputs and productivity is not controlled for.

TABLE 4: Estimation of multiproduct sales-generating function

	OLS		Two-step estimation	
	Estimate	Std.	Estimate	Std.
Log no. of employees	0.784	0.035	0.571	0.033
Log of capital	0.061	0.029	0.289	0.036
Log of inventory	1.037	0.021	0.411	0.054
Log of sales of other products	-0.896	0.009	-0.857	0.061
Log of sales outside option	-0.005	0.006	0.287	0.043
Log of population	0.014	0.022	0.176	0.032
Log of pop. density	0.018	0.016	0.697	0.032
Coef. of no. of products (ρ_{np})			0.213	0.096
Log of income	38.120	13.360	0.289	0.058
Log of income squared	-3.620	1.257	0.043	0.058
Elasticity of substitution			3.480	
Year fixed effect	Yes		Yes	
Subsector fixed effect	Yes		Yes	
R squared	0.558			
No. of obs.	16,759		16,759	

NOTE: The dependent variable is the log of sales of a product category at the store level. The OLS specification controls for the current impact of entry regulation. Two-step estimation refers to the estimation method presented in Section 3.1. Municipalities are defined as local markets. Reported standard errors (in parentheses) are computed using Ackerberg et al. (2012).

The estimates are consistent with the profit maximization behavior of multiproduct firms

[41] The two-step estimated coefficients are adjusted for the elasticity of substitution σ and the coefficient of other product categories $\tilde{\alpha}_y$ to allow for comparisons across specifications. The two-step estimator controls for the endogeneity of store input choices and entry regulation and allows us to separately identify two shocks.

because the sales of a product category decrease when the sales of other product categories increase (Mundlak, 1964).[42] On average, a one percent increase in sales of other products decreases the sales of a product category by 0.857 percent, suggesting relatively fierce competition for sales space in a store. The magnitude of the coefficient of the other product categories (α_y) is key for the productivity measure, as it influences the input coefficients. The estimated elasticity of demand for product substitution is 3.480, which is in line with previous literature.

Stores in large and densely populated markets sell more per product category. The number of product categories and income have a positive impact on stores' market share. That consumers benefit from more product categories is consistent with findings from previous literature (on love for variety). On average, a store with a 30 percent market share gains 5 percent market share by adding one more product category.

Entry regulations and store primitives. Table 5 shows the estimates of the processes for productivity ω_{jt} and demand shocks μ_{jt} (equations (49-a) and (50-a)). We reject the null hypothesis that the coefficients of demand shocks μ_{jt} in the productivity process equal zero (*p-value*=0.000). More liberal entry regulation has a positive impact on revenue productivity; i.e., one more approval increases productivity by 0.120 percent on average.[43] However, the impact of entry regulations on productivity is decreasing in productivity and demand shocks. This implies high heterogeneity in stores' future productivity due to changes in regulation, which affects long-run profits.

Demand shocks also have a positive impact on future revenue productivity, and the impact is increasing in productivity. A one percent increase in μ_{jt} raises productivity by 0.018 percent on average. We expect stores to learn from demand to improve future productivity in services where demand shocks affect inventory management, input and product variety choices that lead to productivity advances.

A key factor that drives the dynamics in productivity and demand shocks is persistence. The average persistence of the productivity process (0.869) is lower than the persistence of the store's demand shocks (0.943) (Table 5). The size of the persistence in productivity is similar to what has been found in the literature.[44]

[42]The conditions of Theorem 1 are satisfied because $-\alpha_y = -0.857$, i.e., $\alpha_y > 0$.

[43]The average is computed based on the observed population density, where the largest marginal effect is approximately 9 percent.

[44]See, e.g., Doraszelski and Jaumandreu (2013) and Maican and Orth (2017).

TABLE 5: Estimation of productivity and demand shock processes

Productivity (ω_t) process			Demand shocks (μ_t) process		
	Estimate	Std.		Estimate	Std.
Productivity (ω_{t-1})	0.846	0.013	Demand shock (μ_{t-1})	0.987	0.018
Productivity sq. (ω_{t-1}^2)	0.025	0.006	Demand shock sq. (μ_{t-1}^2)	-0.012	0.004
Productivity cubic (ω_{t-1}^3)	-0.002	0.001	Demand sh. cubic (μ_{t-1}^3)	-0.0006	0.0002
Demand shock (μ_{t-1})	0.025	0.004			
Prod.*Dem.. sh. ($\omega_{t-1} \times \mu_{t-1}$)	0.011	0.002			
Regulation (r_{t-1})	0.122	0.036			
Prod.*Reg. ($\omega_{t-1} \times r_{t-1}$)	-0.026	0.011			
Dem. sh.*Reg. ($\mu_{t-1} \times r_{t-1}$)	-0.028	0.006			
Year fixed effects	Yes		Year fixed effects	Yes	
Subsector fixed effects	Yes		Year fixed effects	Yes	
Adjusted R squared	0.873		Adjusted R squared	0.686	
Coef. of ω_{t-1} terms are zero	F test	p value			
	1749.183	0.000			
Coef. of μ_{t-1} terms are zero	F test	p value			
	23.601	0.000			
Coef. of r_{t-1} terms are zero	F test	p value			
	7.599	0.000			
Persistence ($d\omega_t/d\omega_{t-1}$)	0.869		Persistence ($d\mu_t/d\mu_{t-1}$)	0.943	
Effect of demand ($d\omega_t/d\mu_{t-1}$)	0.018				
Effect of reg. ($d\omega_t/dr_{t-1}$)	0.077				

NOTE: Productivity and demand shocks and their evolution are estimated using the two-step estimation method in Section 3.1. Regulation is measured as the number of PBL approvals divided by population density. Municipalities are used as local markets. The mean values are presented for the marginal effects.

Figure 2 presents boxplots of the empirical distributions of revenue productivity and demand shocks for stores in different product-category quartiles in restrictive and liberal markets. First, the median revenue productivity ω_{jt} and demand shocks μ_{jt} are higher in liberal markets than in restrictive markets. Second, stores with higher productivity offer more product categories. Third, the interquartile range of demand shocks is lower in liberal than in restrictive markets for stores below the 75th percentile of the product category. Taken together, the results indicate that there is substantial heterogeneity in store-level primitives across stores and market types.

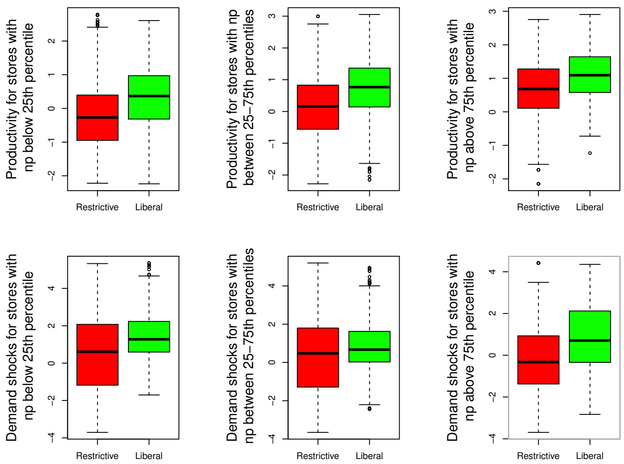

FIGURE 2: Relationship between number of product categories, productivity, and demand shocks

Entry regulations and product variety. Table 6 shows reduced-form evidence of the effect of productivity, demand shocks, investment and capital on the number of product categories and product-category competition for store sale space (the product Herfindahl–Hirschman index (HHI)) in restrictive and liberal markets.

An increase in productivity intensifies competition for product space inside a store (corresponding to a lower HHI). The effect of productivity on product-category competition is decreasing in productivity and demand shocks in restrictive markets, whereas the reverse holds in liberal markets. The magnitudes are larger in liberal than in restrictive markets. Stores with high demand shocks μ_{jt} have less intense competition between product categories (a higher HHI) in both types of markets. However, the impact of demand shocks is increasing (decreasing) in productivity in restrictive (liberal) markets. This implies that product-category competition is less fierce if stores with high demand shocks in restrictive markets have high productivity.

TABLE 6: Determinants of product categories at the store level

	HHI product categories				No. of product categories			
	Restrictive		Liberal		Restrictive		Liberal	
	Est.	Std.	Est.	Std.	Est.	Std.	Est.	Std.
Productivity (ω_t)	-0.1131	0.0225	-0.1870	0.0429	0.3810	0.0468	0.6226	0.0693
Productivity sq. (ω_t^2)	0.0039	0.0040	0.0125	0.0102	-0.0406	0.0148	-0.0616	0.0136
Demand shocks (μ_t)	0.1024	0.0127	0.1098	0.0223	-0.3574	0.0236	-0.3993	0.0390
Dem. shocks sq. (μ_t^2)	-0.0041	0.0005	-0.0057	0.0011	0.0165	0.0016	0.0208	0.0020
Prod.× Dem.($\omega_t \times \mu_t$)	0.0081	0.0025	-0.0020	0.0032	-0.0064	0.0094	0.0087	0.0046
Log of capital (k_{t-1})	-0.0404	0.0089	-0.0494	0.0151	0.0926	0.0263	0.1259	0.0294
Log of invest. (i_{t-1})	-0.0043	0.0052	0.0075	0.0058	0.0388	0.0109	0.0052	0.0135
Other controls	Yes		Yes		Yes		Yes	
Sector fixed effects	Yes		Yes		Yes		Yes	
Year fixed effects	Yes		Yes		Yes		Yes	
Market fixed effects	Yes		Yes		Yes		Yes	
Adj. R^2	0.5404		0.6040					

NOTE: All regressions include an intercept. The OLS estimator is used for HHI regressions, where the dependent variable, i.e., HHI, is computed based on sales product categories. A quasi-Poisson estimator is used for the number-of-product-category regressions. Additional store and market controls include inventories, wages, population, population density, and income. Standard errors are clustered at the sector level.

Productivity gains increase product categories by larger magnitudes in liberal than in restrictive markets. Stores in restrictive markets thus require larger productivity gains to obtain the same product-category increase as in liberal markets. Stores with high demand shocks offer fewer product categories, suggesting that stores reallocate resources from providing variety to decrease purchasing costs (e.g., providing shopping quality), in line with Bronnenberg (2015). We also find that markets with high median productivity have a larger unique number of product categories, with the magnitudes being larger in liberal than in restrictive markets.

Our findings suggest substantial heterogeneity in the impact of productivity and demand shocks in restrictive and liberal markets. This heterogeneity enables us to understand the drivers behind differences in variety across market types and is useful for designing policies aimed at leveling out regional discrepancies.

Dynamic model estimates. The results in Table 7 (Panel A) show that more liberal regulation decreases the marginal adjustment costs of product categories (the coefficient of the terms $np_{jt} \times r_{mt}$ is negative). Thus, stores with many product categories benefit more from the marginal cost reduction following liberalization of regulation. The coefficient of the term np_{jt}^2 is positive, implying decreasing returns to scale in the number of product categories, which is in line with findings in previous literature (Draganska and Jain, 2005). Last, stores with high demand for inventory before sales have higher marginal product-category adjustment costs. These findings show that stores trade off the marginal adjustment cost of product categories with the long-run benefits.

TABLE 7: Estimation of dynamic parameters

Panel A: Estimation of product variety adjustment cost	Estimate		Std.	
No. of product categories (np_{jt})	0.0104		0.0032	
No. of product categories squared (np_{jt}^2)	0.0245		0.0063	
Inventory before sales squared ($exp(a_{jt})^2$)	0.0019		0.0012	
No. of product categ. × Inv. before sales ($np_{jt} \times exp(a_{jt})$)	0.0384		0.0091	
No. of product categ. × Regulation ($np_{jt} \times r_{mt}$)	-0.1192		0.0343	
Inv. before sales × Regulation ($exp(a_{jt}) \times r_{mt}$)	0.0088		0.0009	
Panel B: Model prediction	Observed		Predicted	
	Mean	Std.	Mean	Std.
No. product categories	3.8748	1.7494	3.9337	1.7652
Log of inventory before sales	2.2143	0.9656	2.2505	0.9921
Panel C: Value function approximation	Median		$Q_{75} - Q_{25}$	
Value function	174.416		129.898	
Approximation error	3.551E-6			

NOTE: Panel A shows estimation results on the product variety adjustment cost specified in Section 3.2. Local market regulation is measured as the number of PBL approvals divided by the population density. Panel B shows the model prediction, whereas Panel C show the value function approximation (Section 3.2). The value function is in million SEK. Standard errors are computed using subsampling.

The estimated dynamic model accurately predicts the number of product categories and inventory (Table 7, Panel B). This is because we allow for high heterogeneity in the adjustment cost of product categories. The median of the value function (long-run profits) is 180.3 M SEK (Table 7, Panel C). In addition, the results show low errors in approximating the value function (median 2.39E-6), which ensures consistency of the estimation of the dynamic model.

Long-run profits and the benefits of variety. Table 8 shows incumbents' long-run profits, adjustment costs, and benefits from adding one more product category. We present the results for market types relevant for entry regulation and regional program policies: restrictive, liberal, rural, and urban markets. First, the median long-run profits in restrictive markets are approximately 30 percent higher than those in liberal markets, emphasizing that competition drives profitability differences between markets with contrasting regulations. The median long-run profits in urban markets are approximately two times higher than those in rural markets. The difference in long-run profits between a store in the 90th percentile and one in the 10th percentile is over 190 M SEK in restrictive and urban markets, which is larger than the difference in liberal and rural markets. Second, the median product category adjustment cost is 15 percent higher in restrictive than in liberal markets. Urban markets have a 46 percent higher product variety adjustment cost than rural markets. A store in the 90th percentile has 3.3–5.1 M SEK higher adjustment cost than a store in the 10th percentile. Restrictive markets have the largest dispersion in adjustment costs of product variety.

TABLE 8: Stores' long-run profits and the benefits of increasing product variety

	Type of market							
	Rural		Urban		Restrictive		Liberal	
	Q_{50}	IQR	Q_{50}	IQR	Q_{50}	IQR	Q_{50}	IQR
Value function	95.613	160.971	191.559	189.371	207.728	208.100	158.541	178.104
Adjustment cost	1.109	3.205	1.634	4.755	1.633	5.073	1.390	4.120
Benef. of variety	0.191	0.265	0.195	0.227	0.210	0.221	0.174	0.233

NOTE: Estimation of value function and adjustment cost in Section 3.2. All figures are in million SEK. Local market regulation is measured as the number of PBL approvals divided by population density. Liberal (restrictive) markets are defined as municipalities with a regulation measure above (below) the median. Rural (urban) markets are defined as municipalities with below-median (above-median) population. The incumbent's long-run benefit from offering an additional product category for sale is computed by solving the first-order condition of the store's dynamic optimization problem. $IQR = Q_{90} - Q_{10}$. Q_{10}, Q_{50}, and Q_{90} are 10th, 50th, and 90th percentiles.

Third, by solving the store's dynamic optimization problem, we compute the increase in long-run profits from offering one more product category, i.e., the incumbent's long-run marginal benefit from offering an additional product category for sale. Table 8 shows that the median marginal benefit of increasing product categories varies from 0.174 to 0.210 M SEK. The positive signs and magnitudes of the marginal benefits show that the expected future profits in our dynamic model are crucial for decisions of product categories. Such long-run benefits would not be captured by a static model. The median marginal benefit of adding variety is 20% higher in restrictive markets than in liberal markets. The median marginal benefit of adding variety is two percent lower in rural than in urban markets, reflecting less variety for consumers in rural areas. Stores in rural markets have the highest dispersion in the long-run marginal benefit of adding one more product category. For example, the long-run marginal benefits are approximately 0.3 M SEK higher for a store in the 90th percentile than for one in the 10th percentile. The variation in the benefit of adding variety across incumbents in markets of different types is a crucial component when we examine our counterfactual policy experiments.

5 Policy Evaluation

We perform four counterfactual policy experiments. The first two counterfactuals increase competitive pressure through more liberal entry regulation: approving one additional PBL application in all markets (CF_1) and doubling the number of PBL approvals in only markets with observed approvals (CF_2).[45] Understanding the consequences of these regulatory regimes is highly relevant for policymakers who decide on entry regulations in local markets. The regulatory policies that we consider use all channels through which entry regulations impact stores in our model. That is, more liberal regulation reduces the adjustment cost of variety and improves future productivity, which changes the benefit from repositioning. The store's optimal product repositioning balances changes in the marginal adjustment sales (short-run benefits) and cost and changes in the expected discounted future benefits from repositioning given the productivity improvements (long-run benefits).

The counterfactuals CF_3 and CF_4 evaluate the role of the productivity- and cost channels in the dynamic model for variety and store performance. In CF_3, we remove future productivity changes due to the liberalization of entry in CF_1. This turns off the productivity channel, removing the incentives to increase the number of product categories associated with future productivity improvements. In CF_4, we evaluate a cost reduction that yields a constant marginal cost of adding product categories under the modest liberalization of entry in CF_1. This last experiment assesses the importance of the cost channel, enabling stores to reduce costs for advertising, distribution and other operational costs related to product categories. We implement such a cost reduction at present without incentivizing stores to improve their future productivity.[46]

We compare store-level outcomes before and after the hypothetical changes to entry regulations in local markets, exploring the importance of the productivity- and cost channels. To

[45]In CF_1, the markets without observed approved PBL applications in the data also receive one approved PBL application.

[46]The incumbents' short- and long-run profits are affected through changes in adjustment costs, which also impact sales because stores reposition in product categories and inventory. An alternative policy would be to change the price equilibrium, which affects stores' optimal decisions (product variety and inventory).

compute the outcomes of a hypothetical change, we use the underlying primitives of the dynamic model and the estimated evolution of the state variables (productivity and demand shocks) to solve for the incumbents' number of product categories, sales per product category, and value function (long-run profits) using the Bellman equation. We report the share of stores that adjust their number of product categories, aggregate category adjustments, and mean product-category entry and exit rates at the store level (Panel A in tables). We also report the median and interquartile range of changes in the incumbent's adjustment cost with product categories, the value function, the market share, and long-run marginal benefits (i.e., marginal changes in expected value function) from an additional product category (panel B).[47] The results are presented for the four market types: rural, urban, restrictive and liberal markets.[48] We particularly focus on markets with restrictive regulation and those in rural locations, as a goal of policymakers is to equalize conditions across geographic regions (Section 2).

More liberal entry regulation. The CF_1 results presented in Table 9 show that one more PBL application approval leads to an increase between 3-7% in product category repositioning among incumbents. There is more entry than exit of product categories. Mean product entry rates are twice as high as exit rates for incumbents in all markets. Rural markets have the highest share of stores that reposition (7%) and the largest net entry of product categories (6 categories). Incumbents in urban and liberal markets enter and exit many products, i.e., product entry and exit rates are double than in rural and restrictive markets. Consumers thus experience lots of repositioning in urban and liberal markets, although net entry is lower than in rural and restrictive markets. The median decrease in value function is 2% in rural markets and about 1% in the other market types. Despite the profit decrease, consumers benefit from increased access to product variety and a refreshed product mix. These findings suggest that higher productivity and changes in adjustment costs induce new product offerings when

[47] Although data limitations prevent us from directly computing consumer welfare, the number of product categories np_{jt} and part of the demand shocks μ_{jt} associated with quality of the shopping experience drive consumer surplus. We also do not compute the change in total welfare although total welfare gains come from the changes in store surplus (profits) and consumer benefits from the ability to access a wider product variety that are provided by the dynamic model.

[48] Rural and urban markets are defined based on the total population. Restrictive markets are those with below-median PBL approvals per population density and liberal otherwise.

competition increases. The increased competition leads to a drop in median long-run marginal benefits of an additional category (i.e., marginal expected value function) by 1% in rural, urban, and liberal markets. It is important to reiterate that a store's reaction to changes in regulation depends on changes in rivals' productivity and demand shocks (i.e., oligopolistic competition). In addition, regulatory changes affect a store's sales and sales in the outside market, which includes possible new entrants. Stores that reposition their product variety obtain a slight increase in their market share.

TABLE 9: Counterfactual experiments ($CF_1 - CF_2$): More liberal entry regulations

CF_1 – One more approved PBL application in all markets								
Panel A: Prod. categories (np_{jt})	Rural		Urban		Restrictive		Liberal	
Share stores increasing categ. ($\Delta np_{jt} > 0$)	0.066		0.035		0.028		0.053	
Share stores decreasing categ. ($\Delta np_{jt} < 0$)	0.027		0.034		0.020		0.045	
Entry prod. category adjust.	12		31		16		27	
Exit prod. category adjust.	5		31		11		25	
Share prod. categ. entry	0.023		0.014		0.012		0.020	
Share prod. categ. exit	0.010		0.014		0.008		0.019	
Store prod. categ. entry rate ($\Delta np_{jt} \neq 0$)	0.288		0.496		0.453		0.423	
Store prod. categ. exit rate ($\Delta np_{jt} \neq 0$)	0.160		0.248		0.237		0.233	
Panel B: Changes in outcomes	Q_{50}	IQR	Q_{50}	IQR	Q_{50}	IQR	Q_{50}	IQR
Adjustment cost	-0.009	0.140	-0.001	0.016	-0.001	0.008	-0.004	0.096
Value function	-0.017	0.098	-0.006	0.025	-0.005	0.033	-0.008	0.039
Market share ($\Delta np_{jt} \neq 0$)	0.003	0.000	0.002	0.000	0.004	0.001	0.002	0.000
Long-run marginal benefits ($\zeta\partial\mathbb{E}[V(\cdot)]/\partial np_{jt}$)	-0.010	0.267	-0.009	0.114	0.000	0.173	-0.016	0.123
CF_2 – Double the number of approved PBL applications								
Panel A: Prod. categories (np_{jt})								
Share stores increasing categ. ($\Delta np_{jt} > 0$)	0.060		0.054		0.051		0.059	
Share stores decreasing categ. ($\Delta np_{jt} < 0$)	0.024		0.033		0.015		0.048	
Entry prod. category adjust.	11		45		25		31	
Exit prod. category adjust.	4		26		7		23	
Share prod. categ. entry	0.022		0.020		0.018		0.024	
Share prod. categ. exit	0.008		0.012		0.005		0.017	
Store prod. categ. entry rate ($\Delta np_{jt} \neq 0$)	0.037		0.049		0.022		0.071	
Store prod. categ. exit rate ($\Delta np_{jt} \neq 0$)	0.066		0.105		0.045		0.150	
Panel B: Changes in outcomes	Q_{50}	IQR	Q_{50}	IQR	Q_{50}	IQR	Q_{50}	IQR
Adjustment cost	-0.021	0.200	-0.014	0.155	-0.006	0.034	-0.051	0.277
Value function	-0.238	0.803	-0.092	0.580	-0.109	0.523	-0.100	0.700
Market share ($\Delta np_{jt} \neq 0$)	0.001	0.021	0.002	0.082	0.004	0.065	0.000	0.051
Long-run marginal benefits ($\zeta\partial\mathbb{E}[V(\cdot)]/\partial np_{jt}$)	-0.249	1.833	-0.234	1.253	-0.109	1.477	-0.341	1.241

NOTE: Figures in Panel B represent growth changes. Product category entry and exit rates are defined per store and year. Local market regulation is measured as the number of PBL approvals divided by population density. Liberal (restrictive) markets are defined as municipalities with a regulation measure above (below) the median. Rural (urban) markets are defined as municipalities with below-median (above-median) population.

The findings on the doubling of approved PBL applications in CF_2 are consistent with those in CF_1 (Table 9). The competitive pressure on incumbents from a stronger liberalization in CF_2 than in CF_1 leads to net entry of product categories across all market types. More stores adjust their product offerings. Over 6% of all stores in restrictive markets adjust their product-category mix, a slightly higher percentage than in CF_1. There is net entry in all market types, reflecting less exit in all markets and more entry in all markets except rural. Net entry is particularly high in restrictive markets (18 categories) but lower in rural markets with limited demand (7 categories). Incumbents with a wide range of products tend to alter their selection more than in CF_1, as indicated by lower category entry and exit rates of products within stores. This suggests that economies of scope are key when incumbents' face intensified competition. As a result of more intense competition in CF_2, consumers thus benefit from more products.

The doubling of the number of PBL approvals in CF_2 makes incumbents worse off. Although such generous liberalization promotes productivity and decreases the adjustment costs of variety, it does not compensate for the loss in future sales to rival stores.[49] Intense competition decreases incumbents' long-run profits across all market types, with the reduction being ten times larger than that in CF_1. Incumbents in rural and restrictive markets are harmed the most under this policy design, where the median decrease in long-run profits is 24% and 11%, respectively. Nevertheless, stores that reposition their category offerings gain market share. Stores in restrictive markets increase their market share the most (4 percentage points), reflecting high net entry of product categories and a relatively small decrease in marginal long-run benefits. Under more intense competition in both CF_1 and CF_2, the long-run marginal benefits vary significantly across firms and market types, and the benefits increase for firms in the 75th percentile. While consumers access more variety and long-run profits fall under strong competitive pressure in all markets, there is particularly room for liberalizations in restrictively regulated markets.

Productivity and cost channels. In the experiment CF_3, we shut down the future productivity improvements associated with a more liberal regulation in CF_1. The results provide evidence that the productivity mechanism matters for store performance and product offerings to consumers. Stores enter fewer product categories in CF_3 than in CF_1. In fact, there is net exit of product categories in urban markets and entry of categories exactly offsets the number of exits in liberal markets. Fewer stores adjust their categories in rural markets, with those making changes either adding or removing multiple categories. The absence of the productivity improvement channel leads to a more substantial reduction in long-run profits compared to CF_1. The largest reduction in value function is in restrictive markets (8%), about double than

[49]There is a small decrease in demand for inventory in all markets (not reported in table), which can be related to better inventory management due to improved productivity.

in the other three market types.

Incentives for stores to expand their product offerings are reduced in CF_3 relative to CF_1. Lower future productivity implies that the marginal long-run benefit of adding a new product category decreases by 5-7%. The more substantial reduction in long-run profits, compared to CF_1, suggests the importance of productivity improvements for driving future sales, long-run profits and product category offerings.

TABLE 10: Counterfactual experiments ($CF_3 - CF_4$): Role of productivity- and cost channels under more liberal entry regulations

CF_3 – No impact of regulation on productivity and one more approved PBL application in all markets								
Panel A: Prod. categories (np_{jt})	Rural		Urban		Restrictive		Liberal	
Share stores increasing categ. ($\Delta np_{jt} > 0$)	0.050		0.036		0.031		0.046	
Share stores decreasing categ. ($\Delta np_{jt} < 0$)	0.022		0.036		0.022		0.044	
Entry prod. category adjust.	11		29		16		24	
Exit prod. category adjust.	4		32		12		24	
Share prod. categ. entry	0.021		0.013		0.012		0.018	
Share prod. categ. exit	0.008		0.014		0.009		0.018	
Store prod. categ. entry rate ($\Delta np_{jt} \neq 0$)	0.349		0.353		0.367		0.342	
Store prod. categ. exit rate ($\Delta np_{jt} \neq 0$)	0.204		0.234		0.217		0.236	
Panel B: Changes in outcomes	Q_{50}	IQR	Q_{50}	IQR	Q_{50}	IQR	Q_{50}	IQR
Adjustment cost	-0.007	0.097	-0.001	0.015	-0.001	0.007	-0.004	0.049
Value function	-0.043	0.463	-0.054	0.142	-0.083	0.166	-0.036	0.202
Market share ($\Delta np_{jt} \neq 0$)	0.001	0.007	-0.000	0.059	0.001	0.048	0.001	0.048
Long-run marginal benefits ($\zeta \partial \mathbb{E}[V(\cdot)]/\partial np_{jt}$)	-0.002	1.276	-0.067	0.528	-0.074	0.640	-0.052	0.589
CF_4 – Cost reduction per product-category and one more approved PBL application in all markets								
Panel A: Prod. categories (np_{jt})								
Share stores increasing categ. ($\Delta np_{jt} > 0$)	0.080		0.064		0.037		0.096	
Share stores decreasing categ. ($\Delta np_{jt} < 0$)	0.023		0.031		0.018		0.041	
Entry prod. category adjust.	15		54		20		49	
Exit prod. category adjust.	4		27		10		21	
Share prod. categ. entry	0.030		0.024		0.014		0.036	
Share prod. categ. exit	0.008		0.012		0.007		0.015	
Store prod. categ. entry rate ($\Delta np_{jt} \neq 0$)	0.307		0.332		0.406		0.296	
Store prod. categ. exit rate ($\Delta np_{jt} \neq 0$)	0.190		0.234		0.233		0.226	
Panel B: Changes in outcomes	Q_{50}	IQR	Q_{50}	IQR	Q_{50}	IQR	Q_{50}	IQR
Adjustment cost	-0.174	0.571	-0.115	0.496	-0.112	0.468	-0.140	0.550
Value function	0.024	0.275	-0.005	0.060	-0.010	0.097	-0.001	0.097
Market share ($\Delta np_{jt} \neq 0$)	0.001	0.010	0.004	0.060	0.001	0.068	0.004	0.041
Long-run marginal benefits ($\zeta \partial \mathbb{E}[V(\cdot)]/\partial np_{jt}$)	-0.070	0.628	-0.095	0.467	-0.109	0.503	-0.077	0.393

NOTE: Figures in Panel B represent growth changes. Product category entry and exit rates are defined per store and year. Local market regulation is measured as the number of PBL approvals divided by population density. Liberal (restrictive) markets are defined as municipalities with a regulation measure above (below) the median. Rural (urban) markets are defined as municipalities with below-median (above-median) population.

The last experiment, CF_4, which reduces the marginal adjustment cost of product categories (e.g., lower costs of operation or advertising), sets the coefficient of the squared adjustment costs φ_2 to zero in the CF_1 setting. Unsurprisingly, Table 10 shows that more stores adjust their product categories in all markets, especially in liberal markets (approximately three percentage points more than in CF_1). Because of cost reductions, net entry of product categories is larger in all market types, i.e., consumers benefit from accessing more variety. The increased competition due to liberalization and cost reduction for all stores lowers the long-run profits in urban, restrictive, and liberal markets, but the magnitudes are similar to CF_1. In rural markets, the cost reductions help stores increase the long-run profits by 2%. Cost reductions are crucial for stores to add new product categories without being forced to withdraw existing categories in the store. An important implication of lower costs is thus less exit in CF_4 than in CF_1. The findings suggest that reducing "diseconomies of scope" is particularly favourable in markets where variety is sparser to begin with. The net entry in product categories and the increasing competition reduce long-run marginal benefits of an additional product category. To enhance the long-run marginal benefits of an additional product category, stores must improve productivity and utilize economies of scale and scope. Our results indicate that reducing costs is crucial for boosting the range of product categories available to consumers, and promotes profitability of incumbents in rural markets.

6 Conclusion

This paper assesses the impact of regulations on product variety and long-run performance. An essential goal for policymakers is to ensure that consumers enjoy access to products and services regardless of where they live. Despite that firms offer multiple products that are crucial for consumer welfare and growth in the long run, remarkably little attention has been given to the impact of entry regulations on the stores' dynamic incentives to reposition product variety and inputs.

We use a dynamic model with multiproduct technology and store adjustment of product categories and rich data to evaluate the long-run impact of different regulatory regimes in Swedish retail. This research takes a first step toward understanding the role of entry regulations in shifting stores' incentives to reposition their product variety, focusing on a new channel of adjustment costs and on productivity and accounting for local market competition. We pay attention to rural and restrictively regulated markets that raise policy concerns about equalizing living conditions across geographic regions.

The empirical findings show that more liberal regulation decreases the variety adjustment cost, increases productivity and spurs product-category repositioning. Stores in restrictive markets have the largest benefit from adding variety. Stores in rural markets have the largest dispersion in the long-run benefits, reflecting sparse variety in some rural markets.

Counterfactual policy experiments show that more liberal regulation of entry increases the number of product categories to offer. More product categories enter as competitive pressure intensifies. Net entry is largest in rural markets under modest liberalization, whereas most entry but also exit occur in urban and liberal markets. Strong liberalization induces even larger increases in product adjustments and variety offerings for all markets except rural. In addition, strong liberalization implies incumbents that already offer many products adjust to a larger extent than under modest liberalization, highlighting the importance of economies of scope as competition intensifies. We find evidence of a new channel behind product repositioning whereby efficiency gains and cost reductions incentivize stores to offer more variety to consumers. Without productivity gains, firms offer less variety to consumers and gain lower long-run profits. Cost reductions of product adjustments, related to for instance advertising, logistics, and other operational costs, mitigate products to exit and result in the largest increase in product category entry.

While consumers gain in terms of wider product offerings, incumbents obtain lower profits when competition intensifies. Incumbents' long-run profits decrease as a result of more intense competition but stores that reposition their product categories gain market share. Long-run profits decrease the least and market shares of repositioning stores increase the most in restrictive markets, suggesting room for liberalizations in restrictively regulated markets. Incumbents in rural markets experience the largest drop in long-run profits but also benefit the most from cost reductions, reflecting lack of demand in rural areas.

We provide a tractable and broadly applicable framework to assess the dynamic impact of regulations on product variety and long-run performance. The empirical results clearly reveal that future benefits are important for incentives to offer variety, suggesting that a dynamic approach is required. Our findings also provide new insights into product offerings, emphasizing a new mechanism whereby efficient gains and adjustment costs drive product repositioning in a competitive context.

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Online Appendix: Entry Regulations and Product Variety in Retail

Florin Maican and Matilda Orth[1]

Appendix A: General properties of the multiproduct service function

To simplify the notation, we omit the index of the firm and period and group the inputs into the vector $\boldsymbol{\Psi}$. For example, in our empirical implementation, $\boldsymbol{\Psi} = (L, K, A)$. We consider the general service generating function:

$$F(\mathbf{Q}, \boldsymbol{\Psi}) = G(\mathbf{Q}) - H(\boldsymbol{\Psi}) = 0, \tag{1-a}$$

where $G(\mathbf{Q}) = Q_1^{\tilde{\alpha}_1} \times \cdots \times Q_{np}^{\tilde{\alpha}_{np}} exp(\tilde{\gamma}_1 Q_1 + \cdots + \tilde{\gamma}_{np} Q_{np})$; $H(\boldsymbol{\Psi}) = \Psi_1^{\tilde{\beta}_1} \times \cdots \times \Psi_d^{\tilde{\beta}_d} exp(\tilde{\omega})$; $\mathbf{Q}$ is the vector of service output; Q_i is the i-th service output of the store, $(i = \overline{1, np})$; and Ψ_j is the j-th service input of the store, $(j = \overline{1, d})$. In what follows, we use i to index the service outputs and j to index the inputs.

Assuming that the prices are given, the Lagrangian function of the profit maximization problem at the store level is given by

$$\max_{\boldsymbol{\Psi}} \mathcal{L} = \mathbf{P}'\mathbf{Q} - \mathbf{W}'\boldsymbol{\Psi} - \lambda F(\mathbf{Q}, \boldsymbol{\Psi}), \tag{2-a}$$

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where **P** and **W** are the vectors of output and input prices, respectively. The first-order conditions (FOCs) under competition are

$$\begin{aligned} & P_i - \lambda F_i = 0, \quad i = \overline{1, np} \\ & W_j + \lambda F_j = 0, \quad j = \overline{1, d} \end{aligned} \tag{3-a}$$

where $F_i = \partial F / \partial Q_i$ and $F_j = \partial F / \partial \Psi_j$. The FOCs (3-a) imply that $sign(\lambda) = sign(F_i)$ and $sign(\lambda) = -sign(F_j)$. The derivatives of the implicit function with respect to inputs and outputs, i.e., F_i and F_j, are

$$\begin{aligned} & F_i = G(\mathbf{Q}) \left(\frac{\tilde{\alpha}_i}{Q_i} + \tilde{\gamma}_i \right), \quad i = \overline{1, np} \\ & F_j = -H(\mathbf{\Psi}) \frac{\tilde{\beta}_j}{\Psi_j}, \quad j = \overline{1, d} \end{aligned} \tag{4-a}$$

The cross derivatives of the Lagrangian are the following: $\partial^2 \mathcal{L} / \partial^2 \lambda = 0$; $\partial^2 \mathcal{L} / \partial \lambda \partial Q_i = -F_i$; $\partial^2 \mathcal{L} / \partial \lambda \partial \Psi_j = -F_j$; $\partial^2 \mathcal{L} / \partial Q_i \partial Q_{i'} = -\lambda F_{ii'}$; $\partial^2 \mathcal{L} / \partial \Psi_j \partial \Psi_{j'} = -\lambda F_{jj'}$; and $\partial^2 \mathcal{L} / \partial Q_i \partial \Psi_j = -\lambda F_{ij}$. The determinant of the bordered Hessian matrix $D_{\mathcal{L}}$ is given by

$$D_{\mathcal{L}} = \begin{vmatrix} \frac{\partial^2 L}{\partial \lambda \partial \lambda} & \frac{\partial^2 L}{\partial \lambda \partial Q_i} & \frac{\partial^2 L}{\partial \lambda \partial \Psi_j} \\ \frac{\partial^2 L}{\partial Q_i \partial \lambda} & \frac{\partial^2 L}{\partial Q_i \partial Q_i} & \frac{\partial^2 L}{\partial Q_i \partial \Psi_j} \\ \frac{\partial^2 L}{\partial \Psi_j \partial \lambda} & \frac{\partial^2 L}{\partial \Psi_j \partial Q_i} & \frac{\partial^2 L}{\partial \Psi_j \partial \Psi_{j'}} \end{vmatrix} = \begin{vmatrix} 0 & -\mathbf{F}_i & -\mathbf{F}_j \\ -\mathbf{F}_i & -\lambda \mathbf{F}_{ii'} & -\lambda \mathbf{F}_{ij} \\ -\mathbf{F}_j & -\lambda \mathbf{F}_{ji'} & -\lambda \mathbf{F}_{jj'} \end{vmatrix}, \tag{5-a}$$

where the cross derivatives of the elements of the block matrices of The determinants of the Hessian matrix are as follows:

$$\begin{aligned} & \text{Product-product: } F_{ii} = \frac{F_i^2}{G(\mathbf{Q})} - G(\mathbf{Q}) \frac{\tilde{\alpha}_i}{Q_i^2}, \quad i = \overline{1, np} \\ & \text{Product-product: } F_{ii'} = \frac{F_i F_{i'}}{G(\mathbf{Q})}, \quad i \neq i' \quad i, i' = \overline{1, np} \\ & \text{Input-input: } F_{jj} = -\frac{F_j^2}{H(\mathbf{\Psi})} + H(\mathbf{\Psi}) \frac{\tilde{\beta}_j}{\Psi_j^2}, \quad j = \overline{1, d} \\ & \text{Input-input: } F_{jj'} = -\frac{F_j F_{j'}}{H(\mathbf{\Psi})}, \quad j \neq j' \quad j, j' = \overline{1, d} \\ & \text{Product-input: } F_{ij} = 0, \quad i = \overline{1, np}, \quad j = \overline{1, d}. \end{aligned} \tag{6-a}$$

The second-order condition for profit maximization requires that the sign of the determinant of the bordered Hessian matrix $D_{\mathcal{L}}$ be $(-1)^{np+d}$. To prove this, we rewrite the determinant $D_{\mathcal{L}}$ as

$$D_{\mathcal{L}} = \begin{vmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{vmatrix}, \tag{7-a}$$

where $\mathbf{A} = 0$ $(1 \times 1$ matrix); $\mathbf{B} = [-\mathbf{F}_i, -\mathbf{F}_j]^T$ $(1 \times (np + d))$; $\mathbf{C} = [-\mathbf{F}_i, -\mathbf{F}_j]$ $((np + d) \times 1)$; and

$$\mathbf{D} = \begin{bmatrix} -\lambda \mathbf{F}_{ii'} & \mathbf{0} \\ \mathbf{0} & -\lambda \mathbf{F}_{jj'} \end{bmatrix}.$$

Using Schur complement decomposition, we have that

$$D_{\mathcal{L}} = det(\mathbf{D}) det(\mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{C}). \tag{8-a}$$

Because the matrix $\mathbf{D}$ is diagonal, its inverse is given by

$$\mathbf{D}^{-1} = (-\lambda)^{-1} \begin{bmatrix} \mathbf{F}_{ii'}^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{F}_{jj'}^{-1} \end{bmatrix}, \tag{9-a}$$

and the determinant of $\mathbf{D}$ is

$$det(\mathbf{D}) = (-1)^{-(np+d)} (\lambda)^{-(np+d)} det(\mathbf{F}_{ii'}) det(\mathbf{F}_{jj'}). \tag{10-a}$$

The product $\mathbf{B}\mathbf{D}^{-1}\mathbf{C}$ can be rewritten as as

$$\mathbf{B}\mathbf{D}^{-1}\mathbf{C} = -\lambda^{-1} [\mathbf{F}_i^T \mathbf{F}_{ii'}^{-1} \mathbf{F}_i + \mathbf{F}_j^T \mathbf{F}_{jj'}^{-1} \mathbf{F}_j]. \tag{11-a}$$

Therefore,

$$det(A - \mathbf{B}\mathbf{D}^{-1}\mathbf{C}) = (\lambda)^{-1} [\mathbf{F}_i^T \mathbf{F}_{ii'}^{-1} \mathbf{F}_i + \mathbf{F}_j^T \mathbf{F}_{jj'}^{-1} \mathbf{F}_j]. \tag{12-a}$$

Thus, the determinant of the bordered Hessian matrix is given by

$$D_{\mathcal{L}} = (-1)^{-(np+d)} (\lambda)^{-(np+d+1)} det(\mathbf{F}_{ii'}) det(\mathbf{F}_{jj'}) [\mathbf{F}_i^T \mathbf{F}_{ii'}^{-1} \mathbf{F}_i + \mathbf{F}_j^T \mathbf{F}_{jj'}^{-1} \mathbf{F}_j]. \tag{13-a}$$

The block matrices $\mathbf{F}_{ii'}$ and $\mathbf{F}_{jj'}$ have important properties that can be used to compute their inverse and the determinant. The matrices $\mathbf{F}_{ii'}$ and $\mathbf{F}_{jj'}$ can be written as

$$\mathbf{F}_{ii'} = \begin{bmatrix} \frac{F_1 F_1}{G(\mathbf{Q})} & \cdots & \frac{F_1 F_{np}}{G(\mathbf{Q})} \\ \vdots & \ddots & \vdots \\ \frac{F_{np} F_1}{G(\mathbf{Q})} & \cdots & \frac{F_{np} F_{np}}{G(\mathbf{Q})} \end{bmatrix} + \begin{bmatrix} -\frac{\tilde{\alpha}_1}{Q_1^2} G(\mathbf{Q}) & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & -\frac{\tilde{\alpha}_{np}}{Q_{np}^2} G(\mathbf{Q}) \end{bmatrix}$$

and

$$\mathbf{F}_{jj'} = \begin{bmatrix} -\frac{F_1 F_1}{H(\mathbf{\Psi})} & \cdots & -\frac{F_1 F_d}{H(\mathbf{\Psi})} \\ \vdots & \ddots & \vdots \\ -\frac{F_d F_1}{H(\mathbf{\Psi})} & \cdots & -\frac{F_d F_{np}}{H(\mathbf{\Psi})} \end{bmatrix} + \begin{bmatrix} \frac{\tilde{\beta}_1}{\Psi_1^2} H(\mathbf{\Psi}) & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \frac{\tilde{\beta}_d}{\Psi_d^2} H(\mathbf{\Psi}) \end{bmatrix}.$$

We introduce new notations for the vectors of derivatives in outputs and inputs, i.e.,

$$\mathbf{u}_i^T = \left[\frac{-F_1}{G(\mathbf{Q})^{\frac{1}{2}}}, \cdots, \frac{-F_{np}}{G(\mathbf{Q})^{\frac{1}{2}}} \right]$$

$$\mathbf{u}_j^T = \left[\frac{-F_1}{H(\mathbf{\Psi})^{\frac{1}{2}}}, \cdots, \frac{-F_d}{H(\mathbf{\Psi})^{\frac{1}{2}}} \right]$$

$$\mathbf{v}_j^T = \left[\frac{F_1}{H(\mathbf{\Psi})^{\frac{1}{2}}}, \cdots, \frac{F_d}{H(\boldsymbol{Psi})^{\frac{1}{2}}} \right]$$

$$\tilde{\mathbf{F}}_{ii'} = \begin{bmatrix} -\frac{\tilde{\alpha}_1}{Q_1^2} G(\mathbf{Q}) & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & -\frac{\tilde{\alpha}_{np}}{Q_{np}^2} G(\mathbf{Q}) \end{bmatrix}$$

$$\tilde{\mathbf{F}}_{jj'} = \begin{bmatrix} \frac{\tilde{\beta}_1}{\Psi_1^2} H(\mathbf{\Psi}) & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \frac{\tilde{\beta}_d}{\Psi_d^2} H(\mathbf{\Psi}) \end{bmatrix}.$$

The cross-derivative matrices $\mathbf{F}_{ii'}$ and $\mathbf{F}_{jj'}$ can be decomposed as

$$\begin{aligned} \mathbf{F}_{ii'} &= \tilde{\mathbf{F}}_{ii'} + \mathbf{u}_i \mathbf{u}_i^T \\ \mathbf{F}_{jj'} &= \tilde{\mathbf{F}}_{jj'} + \mathbf{u}_j \mathbf{v}_j^T. \end{aligned} \tag{14-a}$$

Based on these decompositions, we can compute the inverses and determinants of $\mathbf{F}_{ii'}$ and $\mathbf{F}_{jj'}$ using Sherman-Morrison formula, i.e.,

$$(\tilde{\mathbf{F}}_{ii'} + \mathbf{u}_i\mathbf{u}_i^T)^{-1} = \tilde{\mathbf{F}}_{ii'}^{-1} - \frac{\tilde{\mathbf{F}}_{ii'}^{-1}\mathbf{u}_i\mathbf{u}_i^T\tilde{\mathbf{F}}_{ii'}^{-1}}{1 + \mathbf{u}_i^T\tilde{\mathbf{F}}_{ii'}^{-1}\mathbf{u}_i} \tag{15-a}$$

$$(\tilde{\mathbf{F}}_{jj'} + \mathbf{u}_j\mathbf{v}_i^T)^{-1} = \tilde{\mathbf{F}}_{jj'}^{-1} - \frac{\tilde{\mathbf{F}}_{jj'}^{-1}\mathbf{u}_j\mathbf{v}_j^T\tilde{\mathbf{F}}_{jj'}^{-1}}{1 + \mathbf{v}_j^T\tilde{\mathbf{F}}_{jj'}^{-1}\mathbf{u}_j} \tag{16-a}$$

$$det(\tilde{\mathbf{F}}_{ii'} + \mathbf{u}_i\mathbf{u}_i^T) = (1 + \mathbf{u}_i^T\tilde{\mathbf{F}}_{ii'}^{-1}\mathbf{u}_i)det(\tilde{\mathbf{F}}_{ii'}) \tag{17-a}$$

$$det(\tilde{\mathbf{F}}_{jj'} + \mathbf{u}_j\mathbf{v}_j^T) = (1 + \mathbf{u}_j^T\tilde{\mathbf{F}}_{jj'}^{-1}\mathbf{u}_j)det(\tilde{\mathbf{F}}_{jj'}). \tag{18-a}$$

The inverses of the diagonal matrices $\tilde{\mathbf{F}}_{ii'}$ and $\tilde{\mathbf{F}}_{jj'}$ are given by

$$\tilde{\mathbf{F}}_{ii'}^{-1} = \begin{bmatrix} -\frac{Q_1^2}{\tilde{\alpha}_1 G(\mathbf{Q})} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & -\frac{Q_{np}^2}{\tilde{\alpha}_{np} G(\mathbf{Q})} \end{bmatrix} \tag{19-a}$$

$$\tilde{\mathbf{F}}_{jj'}^{-1} = \begin{bmatrix} \frac{\Psi_1^2}{\tilde{\beta}_1 H(\mathbf{\Psi})} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \frac{\Psi_d^2}{\tilde{\beta}_d H(\mathbf{\Psi})} \end{bmatrix}. \tag{20-a}$$

We use the Sherman-Morrison formula to evaluate the terms $\mathbf{F}_i^T\mathbf{F}_{ii'}^{-1}\mathbf{F}_i$ and $\mathbf{F}_j^T\mathbf{F}_{jj'}^{-1}\mathbf{F}_j$, i.e.,

$$\begin{aligned} \mathbf{F}_i^T\mathbf{F}_{ii'}^{-1}\mathbf{F}_i = \quad & G(\mathbf{Q})^{\frac{1}{2}}\mathbf{u}_i^T\left(\tilde{\mathbf{F}}_{ii'}^{-1} - \frac{\tilde{\mathbf{F}}_{ii'}^{-1}\mathbf{u}_i\mathbf{u}_i^T\tilde{\mathbf{F}}_{ii'}^{-1}}{1+\mathbf{u}_i^T\tilde{\mathbf{F}}_{ii'}^{-1}\mathbf{u}_i}\right)\mathbf{u}_i G(\mathbf{Q})^{\frac{1}{2}} \\ & = G(\mathbf{Q})\frac{\mathbf{u}_i^T\tilde{\mathbf{F}}_{ii'}^{-1}\mathbf{u}_i}{1+\mathbf{u}_i^T\tilde{\mathbf{F}}_{ii'}^{-1}\mathbf{u}_i} \end{aligned} \tag{21-a}$$

$$\begin{aligned} \mathbf{F}_j^T\mathbf{F}_{jj'}^{-1}\mathbf{F}_j = \quad & -H(\mathbf{\Psi})^{\frac{1}{2}}\mathbf{v}_j^T\left(\tilde{\mathbf{F}}_{jj'}^{-1} - \frac{\tilde{\mathbf{F}}_{jj'}^{-1}\mathbf{u}_j\mathbf{v}_j^T\tilde{\mathbf{F}}_{jj'}^{-1}}{1+\mathbf{v}_j^T\tilde{\mathbf{F}}_{jj'}^{-1}\mathbf{u}_j}\right)\mathbf{u}_j H(\mathbf{\Psi})^{\frac{1}{2}} \\ & = -H(\mathbf{\Psi})\frac{\mathbf{v}_j^T\tilde{\mathbf{F}}_{jj'}^{-1}\mathbf{u}_j}{1+\mathbf{v}_j^T\tilde{\mathbf{F}}_{jj'}^{-1}\mathbf{u}_j}. \end{aligned} \tag{22-a}$$

The terms $\mathbf{u}_i^T \tilde{\mathbf{F}}_{ii'}^{-1} \mathbf{u}_i$ and $\mathbf{v}_j^T \tilde{\mathbf{F}}_{jj'}^{-1} \mathbf{u}_j$ can be computed as follows:

$$\begin{aligned}
\mathbf{u}_i^T \tilde{\mathbf{F}}_{ii'}^{-1} \mathbf{u}_i &= \left[\frac{-F_1}{G(\mathbf{Q})^{\frac{1}{2}}}, \cdots, \frac{-F_{np}}{G(\mathbf{Q})^{\frac{1}{2}}}\right] \begin{bmatrix} -\frac{Q_1^2}{\tilde{\alpha}_1 G(\mathbf{Q})} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & -\frac{Q_{np}^2}{\tilde{\alpha}_{np} G(\mathbf{Q})} \end{bmatrix} \begin{bmatrix} \frac{-F_1}{G(\mathbf{Q})^{\frac{1}{2}}} \\ \vdots \\ \frac{-F_{np}}{G(\mathbf{Q})^{\frac{1}{2}}} \end{bmatrix} \\
&= -\sum_{i=1}^{np} \frac{1}{\tilde{\alpha}_i} \frac{F_i^2 Q_i^2}{G(\mathbf{Q})^2}
\end{aligned}$$

$$\begin{aligned}
\mathbf{v}_j^T \tilde{\mathbf{F}}_{jj'}^{-1} \mathbf{u}_j &= \left[\frac{F_1}{H(\mathbf{\Psi})^{\frac{1}{2}}}, \cdots, \frac{F_d}{H(\mathbf{\Psi})^{\frac{1}{2}}}\right] \begin{bmatrix} \frac{\Psi_1^2}{\tilde{\beta}_1 H(\mathbf{\Psi})} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \frac{\Psi_d^2}{\tilde{\beta}_{np} H(\mathbf{\Psi})} \end{bmatrix} \begin{bmatrix} \frac{-F_1}{H(\mathbf{\Psi})^{\frac{1}{2}}} \\ \vdots \\ \frac{-F_d}{H(\mathbf{\Psi})^{\frac{1}{2}}} \end{bmatrix} \\
&= -\sum_{j=1}^{d} \frac{1}{\tilde{\beta}_j} \frac{F_j^2 \Psi_j^2}{H(\mathbf{\Psi})^2}.
\end{aligned}$$

Therefore, we have

$$\mathbf{F}_i^T \mathbf{F}_{ii'}^{-1} \mathbf{F}_i = G(\mathbf{Q}) \frac{-\sum_{i=1}^{np} \frac{1}{\tilde{\alpha}_i} \frac{F_i^2 Q_i^2}{G(\mathbf{Q})^2}}{1-\sum_{i=1}^{np} \frac{1}{\tilde{\alpha}_i} \frac{F_i^2 Q_i^2}{G(\mathbf{Q})^2}}$$

$$\mathbf{F}_j^T \mathbf{F}_{jj'}^{-1} \mathbf{F}_j = H(\boldsymbol{Psi}) \frac{-\sum_{j=1}^{d} \frac{1}{\tilde{\beta}_j} \frac{F_j^2 \Psi_j^2}{H(\mathbf{\Psi})^2}}{1-\sum_{j=1}^{d} \frac{1}{\tilde{\beta}_j} \frac{F_j^2 \Psi_j^2}{H(\mathbf{\Psi})^2}}. \tag{23-a}$$

The next step is to compute the determinants of $\mathbf{F}_{ii'}$ and $\mathbf{F}_{jj'}$, i.e.,

$$\begin{aligned}
det(\mathbf{F}_{ii'}) &= (1 + \mathbf{u}_i^T \tilde{\mathbf{F}}_{ii'}^{-1} \mathbf{u}_i) det(\tilde{\mathbf{F}}_{ii'}) \\
&= \left(1 - \sum_{i=1}^{np} \frac{1}{\tilde{\alpha}_i} \frac{F_i^2 Q_i^2}{G(\mathbf{Q})^2}\right) \prod_{i=1}^{np} \frac{-\tilde{\alpha}_i}{Q_i^2} G(\mathbf{Q})
\end{aligned} \tag{24-a}$$

$$\begin{aligned}
det(\mathbf{F}_{jj'}) &= (1 + \mathbf{v}_j^T \tilde{\mathbf{F}}_{jj'}^{-1} \mathbf{u}_j) det(\tilde{\mathbf{F}}_{ii'}) \\
&= \left(1 - \sum_{j=1}^{d} \frac{1}{\tilde{\beta}_j} \frac{F_j^2 \Psi_j^2}{H(\mathbf{\Psi})^2}\right) \prod_{j=1}^{d} \frac{\tilde{\beta}_j}{\Psi_j^2} H(\mathbf{\Psi}).
\end{aligned} \tag{25-a}$$

Replacing expressions (10-a), (12-a), (23-a), (24-a), and (25-a) in (8-a), we have

$$\begin{aligned}
D_{\mathcal{L}} = \; & \lambda(-\lambda)^{-(np+d+2)} \left(1 - \sum_{i=1}^{np} \frac{1}{\tilde{\alpha}_i} \frac{F_i^2 Q_i^2}{G(\mathbf{Q})^2}\right) \left(\prod_{i=1}^{np} \frac{-\tilde{\alpha}_i}{Q_i^2} G(\mathbf{Q})\right) \\
\times \; & \left(1 - \sum_{j=1}^{d} \frac{1}{\tilde{\beta}_j} \frac{F_j^2 \Psi_j^2}{H(\mathbf{\Psi})^2}\right) \left(\prod_{j=1}^{d} \frac{\tilde{\beta}_j}{\Psi_j^2} H(\mathbf{\Psi})\right) \\
\times \; & \left[G(\mathbf{Q}) \frac{-\sum_{i=1}^{np} \frac{1}{\tilde{\alpha}_i} \frac{F_i^2 Q_i^2}{G(\mathbf{Q})^2}}{1-\sum_{i=1}^{np} \frac{1}{\tilde{\alpha}_i} \frac{F_i^2 Q_i^2}{G(\mathbf{Q})^2}} - H(\mathbf{\Psi}) \frac{-\sum_{j=1}^{d} \frac{1}{\tilde{\beta}_j} \frac{F_j^2 \Psi_j^2}{H(\mathbf{\Psi})^2}}{1-\sum_{j=1}^{d} \frac{1}{\tilde{\beta}_j} \frac{F_j^2 \Psi_j^2}{H(\mathbf{\Psi})^2}} \right],
\end{aligned} \tag{26-a}$$

where $\frac{F_i^2 Q_i^2}{G(\mathbf{Q})^2} = (\tilde{\alpha}_i + \tilde{\gamma}_i Q_i)^2$ and $\frac{F_j^2 \Psi_j^2}{H(\mathbf{\Psi})^2} = \tilde{\beta}_j^2$. We simplify the expression for $D_{\mathcal{L}}$ by introducing new notations for each term, i.e.,

$$
\begin{aligned}
T_1 &= \left(1 - \sum_{i=1}^{np} \frac{1}{\tilde{\alpha}_i} \frac{F_i^2 Q_i^2}{G(\mathbf{Q})^2}\right) \\
T_2 &= \left(\prod_{i=1}^{np} \frac{-\tilde{\alpha}_i}{Q_i^2} G(\mathbf{Q})\right) \\
T_3 &= \left(1 - \sum_{j=1}^{d} \frac{1}{\tilde{\beta}_j} \frac{F_j^2 \Psi_j^2}{H(\mathbf{\Psi})^2}\right) \\
T_4 &= \left(\prod_{j=1}^{d} \frac{\tilde{\beta}_j}{\Psi_j^2} H(\mathbf{\Psi})\right) \\
T_5 &= \left[G(\mathbf{Q}) \frac{-\sum_{i=1}^{np} \frac{1}{\tilde{\alpha}_i} \frac{F_i^2 Q_i^2}{G(\mathbf{Q})^2}}{1 - \sum_{i=1}^{np} \frac{1}{\tilde{\alpha}_i} \frac{F_i^2 Q_i^2}{G(\mathbf{Q})^2}} - H(\mathbf{\Psi}) \frac{-\sum_{j=1}^{d} \frac{1}{\tilde{\beta}_j} \frac{F_j^2 \Psi_j^2}{H(\mathbf{\Psi})^2}}{1 - \sum_{j=1}^{d} \frac{1}{\tilde{\beta}_j} \frac{F_j^2 \Psi_j^2}{H(\mathbf{\Psi})^2}} \right] \\
&= \left[- \frac{G(\mathbf{Q})}{1 - \sum_{i=1}^{np} \frac{1}{\tilde{\alpha}_i} \frac{F_i^2 Q_i^2}{G(\mathbf{Q})^2}} + \frac{H(\mathbf{\Psi})}{1 - \sum_{j=1}^{d} \frac{1}{\tilde{\beta}_j} \frac{F_j^2 \Psi_j^2}{H(\mathbf{\Psi})^2}} \right].
\end{aligned}
$$

Lemma 1: *In the general case of the transcendental service production function with np outputs and d inputs, the determinant of the bordered Hessian matrix of the profit maximization problem is given by*

$$D_{\mathcal{L}} = (-1)^{(np+d)} (\lambda)^{-(np+d+1)} T_1 T_2 T_3 T_4 T_5. \tag{27-a}$$

PROOF:

This finding results directly from equation (26-a).■

In what follows, we provide a general result on the restrictions of the coefficients of transcendental multiproduct functions that are required to satisfy the profit maximization conditions. This result is a generalization of Mundlak's (1964) result in the case of two outputs and two factor inputs.

Theorem 1: *Consider a general service generating function*

$$F(\mathbf{Q}, \mathbf{\Psi}) = G(\mathbf{Q}) - H(\mathbf{\Psi}) = 0 \tag{28-a}$$

where $G(\mathbf{Q}) = Q_1^{\tilde{\alpha}_1} \times \cdots \times Q_{np}^{\tilde{\alpha}_{np}} exp(\tilde{\gamma}_1 Q_1 + \cdots + \tilde{\gamma}_{np} Q_{np})$; $H(\mathbf{\Psi}) = \Psi_1^{\tilde{\beta}_1} \times \cdots \times \Psi_d^{\tilde{\beta}_d} exp(\tilde{\omega})$; Q_i *is the i-th service output of the store,* $(i = \overline{1, np})$*; and* Ψ_j *is the j-th service input of the store,* $(j = \overline{1, d})$*. If the parameters satisfy the following conditions:*

(a) $\tilde{\alpha}_i < 0$ *for all* $i = \overline{1, np}$*;*

(b) $\tilde{\beta}_j > 0$ *for all* $j = \overline{1, d}$

Then, the condition for profit maximization is satisfied.

PROOF:

We consider $\lambda > 0$ and an increasing returns to scale industry, i.e., $\sum_{i=1}^{np} \tilde{\beta} j \geq 1$. We assume that $\lambda > 0$. The FOCs for maximizing profit imply that $F_i > 0$ and $F_j < 0$, i.e.,

$$\left(\frac{\tilde{\alpha}_i}{Q_i} + \tilde{\gamma}_i\right) > 0, \quad i = \overline{1, np} \tag{29-a}$$

$$\frac{\tilde{\beta}_j}{\Psi_j} > 0, \quad j = \overline{1, d}. \tag{30-a}$$

In other words, we have

$$\tilde{\gamma}_i > \left|\frac{\tilde{\alpha}_i}{Q_i}\right|, \quad i = \overline{1, np} \tag{31-a}$$

$$\tilde{\beta}_j > 0, \quad j = \overline{1, d}. \tag{32-a}$$

The FOC (31-a) excludes the possibility that $\tilde{\gamma}_i = 0$ for all i.[2] This implies that $T_1 > 0$, $T_2 > 0$, and $T_4 > 0$. The term $T_5 < 0$ because $T_3 < 0$ and $T_2 > 0$, i.e., T_5 is a sum of two negative numbers. Therefore, the sign of the determinant of the bordered Hessian matrix $D_{\mathcal{L}}$ is $(-1)^{np+d}$, which is the second-order requirement for profit maximization.■

Proposition 1: *If the service function is simple Cobb-Douglas in outputs ($\tilde{\gamma}_i = 0$ for all i) and inputs and the first-order conditions are satisfied, then the optimal service quantity* $\mathbf{Q}^*$ *is sold at the minimum cost, and any inputs* $\mathbf{\Psi}^*$ *yield minimum revenues. The profit* $\pi(Q^*, \Psi^*)$ *at point* (Q^*, Ψ^*) *is a saddle point*

$$\pi(Q^*, \Psi) \leq \pi(Q^*, \Psi^*) \leq \pi(Q, \Psi^*).$$

PROOF:

If $\tilde{\gamma}_i = 0$ for all i, then from the FOC (29-a) we have that $\tilde{\alpha}_i > 0$ for all i. In this case, $sign(T_2) =$

[2]If $\tilde{\gamma}_i = 0$ for all i then $\tilde{\alpha}_i > 0$ for all i (see Proposition 1).

$(-1)^{np}$, and $sign(D_{\mathcal{L}})$ is different from $(-1)^1$ (condition for minimum) and $(-1)^{(np+d)}$ (condition for maximum).■

A direct consequence of Proposition 1 is that when the inputs $\mathbf{\Psi}$ produce minimum revenues and the first-order conditions are satisfied, then the profit can be maximized by a selection of products, i.e., a corner solution. This problem does not exist in the case of a single product.

Proposition 2: *The conditions $\tilde{\alpha}_i < 0$ and $\tilde{\gamma}_i > 0$ for all i are not the only second-order conditions for profit maximization.*

PROOF:

This result is also a direct consequence of Theorem 1. Note that the result in Theorem 1 holds that some $\tilde{\alpha}_i$ can be positive and, in this case, the corresponding $\tilde{\gamma}_i$ can be set to zero, which can be useful to reduce the number of parameters.■

Product (factor) substitution. Using the total differentiation of the service-generating function, we obtain the marginal rate of product (factor) substitution, i.e.,

$$\begin{aligned}
&\text{Product-factor:} && \tfrac{dQ_i}{d\Psi_e} = -\tfrac{F_e}{F_i} > 0 \\
&\text{Factor-factor:} && \tfrac{d\Psi_{e'}}{d\Psi_e} = -\tfrac{F_e}{F_{e'}} < 0 \\
&\text{Product-Product :} && \tfrac{dQ_i}{dQ_{e'}} = -\tfrac{F_{i'}}{F_i} < 0.
\end{aligned} \tag{33-a}$$

To evaluate the convexity of the different marginal rates of substitution, we compute the second derivatives, i.e.,

$$\begin{aligned}
&\text{Product-factor:} && \tfrac{d^2Q_i}{d\Psi_j^2} = -\tfrac{F_{jj}}{F_i} \\
&\text{Factor-factor:} && \tfrac{d^2\Psi_{j'}}{d\Psi_j^2} = -\tfrac{F_{jj}}{F_{j'}} + \tfrac{F_jF_{j'j}}{F_{j'}^2} \\
&\text{Product-Product :} && \tfrac{d^2Q_i}{dQ_{i'}^2} = -\tfrac{F_{i'i'}}{F_i^2} + \tfrac{F_{ii'}}{F_{i'}},
\end{aligned} \tag{34-a}$$

where

$$
\begin{aligned}
F_{jj} &= \tfrac{H(\Psi)}{\Psi_j^2}\tilde{\beta}_j(1-\tilde{\beta}_j) \\
F_i &= G(\mathbf{Q})\left(\tfrac{\tilde{\alpha}_i}{Q_i}+\tilde{\gamma}_i\right) \\
\tfrac{F_{jj}}{F_{j'}} &= \tfrac{\Psi_{j'}}{\Psi_j}\tfrac{\tilde{\beta}_j}{\tilde{\beta}_{j'}}(\tilde{\beta}_j-1) \\
\tfrac{F_j F_{j'j}}{F_{j'}^2} &= \tfrac{\tilde{\beta}_j^2}{\Psi_j^2}\tfrac{1}{\tilde{\beta}_{j'}} \\
-\tfrac{F_{i'i'}}{F_i^2}+\tfrac{F_{ii'}}{F_{i'}} &= G(\mathbf{Q})\tfrac{\tilde{\alpha}_{i'}}{Q_{i'}}\tfrac{1}{\left(\frac{\tilde{\alpha}_i}{Q_i}+\tilde{\gamma}_i\right)}.
\end{aligned}
\tag{35-a}
$$

In the case of Cobb-Douglas in inputs $0<\tilde{\beta}_j<1$,

$$
\begin{aligned}
&\text{Product-factor:} \quad \tfrac{d^2Q_i}{d\Psi_j^2}<0 \\
&\text{Factor-factor:} \quad \tfrac{d^2\Psi_{j'}}{d\Psi_j^2}>0,
\end{aligned}
\tag{36-a}
$$

which implies that the product-factor rate of substitution is a concave function (Figure A.1. (a)), and the factor-factor rate of substitution is convex (Figure A.1. (b)). The properties of the product-product rate of substitution depend on $\tilde{\gamma}_i$, i.e.,

$$
\frac{d^2Q_i}{dQ_{i'}^2}=G(\mathbf{Q})\frac{\tilde{\alpha}_{i'}}{Q_{i'}}\frac{1}{\left(\frac{\tilde{\alpha}_i}{Q_i}+\tilde{\gamma}_i\right)}. \tag{37-a}
$$

If $\tilde{\gamma}_i=0$ then from the first-order condition, we have $\tilde{\alpha}_i>0$, which yields $d^2Q_i/dQ_{i'}^2>0$. Therefore, $\tilde{\gamma}_i=0$ implies that the product-product rate of substitution is a convex function (Figure A.1. (c)). If $\tilde{\gamma}_i>0$, then from the first-order condition, we have $\tilde{\alpha}_i<0$, which yields $d^2Q_i/dQ_{i'}^2<0$. In this case, the product-product rate of substitution is a concave function (AB curve in Figure A.1. (c)).

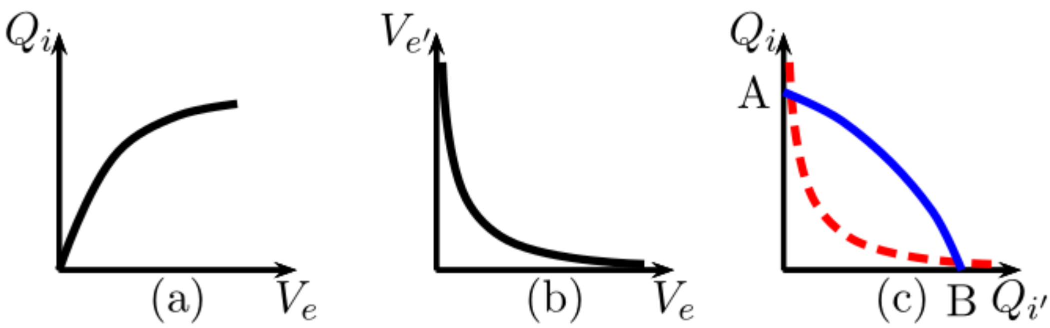

FIGURE A.1: Marginal rate of product (factor) substitution

Appendix B: Sales-generating function

This appendix presents the derivation of the sales-generating function using multiproduct service technology and a demand system. The main aim is to develop a multiproduct sales function and identify its parameters. Separately identifying the coefficients of production technology and demand without price data is beyond the scope of this paper.

The multiproduct service technology is given by

$$\sum_{i=1}^{np_{jt}} \tilde{\alpha}_i q_{ijt} + \tilde{\alpha}_y Y_{jt} = \tilde{\beta}_l l_{jt} + \tilde{\beta}_k k_{jt} + \tilde{\beta}_a a_{jt} + \tilde{\omega}_{jt} + \tilde{u}^p_{jt}, \tag{38-a}$$

where q_{ijt} is the logarithm of the quantity of product category i sold by store j in period t, Y_{jt} denotes the total sales of store j in period t, l_{jt} is the logarithm of the number of employees, k_{jt} is the logarithm of the capital stock, a_{jt} is the logarithm of the sum of the inventory level at the beginning of period t (n_{jt}) and the products bought during period t, and $\tilde{u}^p_{jt}$ are i.i.d. remaining service output shocks. Variable np_{jt} denotes the number of products (categories) of store j.[3]

We use a CES demand system to obtain an expression for the logarithm of the price of product category i (p_{ijt}), i.e., $p_{ijt} = -\frac{1}{\sigma}(q_{ijt} - q_{0t}) + \mathbf{x}'_{ijt}\frac{\tilde{\beta}_x}{\sigma} + \frac{\sigma_a}{\sigma} a_{jt} + \frac{1}{\sigma}\tilde{\mu}_{ijt}$. Multiplying the logarithm of price by $\tilde{\alpha}_i$ and summing over store j's product categories, we obtain the following expression:

$$\sum_{i=1}^{np_{jt}} \tilde{\alpha}_i p_{ijt} = -\frac{1}{\sigma}\sum_{i=1}^{np_{jt}} \tilde{\alpha}_i q_{ijt} + \frac{1}{\sigma}\sum_{i=1}^{np_{jt}} \tilde{\alpha}_i q_{0t} + \sum_{i=1}^{np_{jt}} \tilde{\alpha}_i \mathbf{x}'_{ijt}\frac{\tilde{\beta}_x}{\sigma} + \frac{\sigma_a}{\sigma}\sum_{i=1}^{np_{jt}} \tilde{\alpha}_i a_{jt} + \frac{1}{\sigma}\sum_{i=1}^{np_{jt}} \tilde{\alpha}_i \tilde{\mu}_{ijt}. \tag{39-a}$$

The logarithm of sales per product category is $y_{ijt} = q_{ijt} + p_{ijt}$. To obtain an expression for sales per product category, we sum up the expressions (38-a) and (39-a): $\sum_{i=1}^{np_{jt}}[\tilde{\alpha}_i y_{ijt} + \left(1-\frac{1}{\sigma}\right)\tilde{\alpha}_y Y_{jt}] = \left(1-\frac{1}{\sigma}\right)[\tilde{\beta}_l l_{jt} + \tilde{\beta}_k k_{jt} + \tilde{\beta}_a a_{jt}] + \frac{1}{\sigma}\sum_{i=1}^{np_{jt}}[\tilde{\alpha}_i q_{0t}] + \sum_{i=1}^{np_{jt}}[\tilde{\alpha}_i \mathbf{x}'_{ijt}\frac{\tilde{\beta}_x}{\sigma}] + \frac{\sigma_a}{\sigma}\sum_{i=1}^{np_{jt}}[\tilde{\alpha}_i] a_{jt} + \frac{1}{\sigma}\sum_{i=1}^{np_{jt}} \tilde{\alpha}_i \tilde{\mu}_{ijt} + (1-\frac{1}{\sigma})\tilde{\omega}_{jt} + (1-\frac{1}{\sigma})\tilde{u}^p_{jt}$. The logarithm of the aggregate quantity of the outside option q_{0t} can be written as $q_{0t} = \tilde{c}_{ij} q_{i0t}$, where $\tilde{c}_{ij} > 1$ and q_{i0t} is the logarithm of the quantity of product category i that is sold by stores in the outside

[3] As we mention in the main text, we have information only on product categories in the empirical application.

option.[4] Thus, we can write $\sum_{i=1}^{np_{jt}}[\tilde{\alpha}_i q_{0t}] = \sum_{i=1}^{np_{jt}}[\tilde{\alpha}_i \tilde{c}_{ij} q_{i0t}]$. Using $q_{i0t} = y_{i0t} - p_{i0t}$, we obtain $\sum_{i=1}^{np_{jt}}[\tilde{\alpha}_i q_{0t}] = \sum_{i=1}^{np_{jt}}[\tilde{\alpha}_i \tilde{c}_{ij}(y_{i0t} - p_{i0t})]$. Because $\tilde{c}_{ij} > 1$, there exist $s_{ij} < 1$ and $c_j > 1$ such that $\sum_{i=1}^{np_{jt}} \tilde{\alpha}_i \tilde{c}_{ij} = c_j$ and $\sum_{i=1}^{np_{jt}} s_{ij} = 1$. Therefore, we obtain $\sum_{i=1}^{np_{jt}}[\tilde{\alpha}_i q_{0t}] = c_j(\sum_{i=1}^{np_{jt}} s_{ij} y_{i0t} - \sum_{i=1}^{np_{jt}} s_{ij} p_{i0t}) = c_j(\tilde{y}_{0jt} - \tilde{p}_{0jt}) = c_j y_{0jt} \equiv y_{ot}$, where $\tilde{y}_{0jt}$ are the weighted sales of the product categories of store j that are sold in the outside option; $\tilde{p}_{0jt}$ is a weighted price index; y_{0jt} are the deflated sales of the product categories of store j that are sold in the outside option; and y_{ot} denotes outside option sales in the local market. We measure y_{ot} by the total sales of stores in the outside option. Most importantly, for any store j, we can write the term of the outside option as in terms of the total sales of the outside option in the multiproduct sales function. If there are no stores in the outside option, y_{ot} represents total sales in the market.

The next step is to regroup the remaining coefficients and determine how they are affected by σ. We denote $\beta_q \equiv 1/\sigma$, $\beta_l \equiv (1 - \frac{1}{\sigma})\tilde{\beta}_l$, and $\beta_k \equiv (1 - \frac{1}{\sigma})\tilde{\beta}_k$. As we mention in the main text, we are unable to identify the impact of inventory separately on demand and supply without additional assumptions. Therefore, we sum the net impact of inventory on sales under parameter $\overline{\beta}_a$, i.e., we denote $(1 - \frac{1}{\sigma})\overline{\beta}_a \equiv (1 - \frac{1}{\sigma})\tilde{\beta}_a + \frac{\sigma_a}{\sigma} \sum_{i=1}^{np_{jt}} \tilde{\alpha}_i$. Furthermore, to shorten the notation, we denote $\beta_a \equiv (1 - \frac{1}{\sigma})\overline{\beta}_a$. Because a_{jt} is part of both the supply and demand equations, we are unable to separately identify $\tilde{\beta}_a$ and σ_a. In other words, we can identify only the net effect $\overline{\beta}_a$. In our case, $\mathbf{x}_{ijt}$ includes only market variables, and therefore, we denote $\overline{\boldsymbol{\beta}}_x \equiv \sum_{i=1}^{np_{jt}} \tilde{\alpha}_i \tilde{\boldsymbol{\beta}}_x$ and $\boldsymbol{\beta}_x \equiv \overline{\boldsymbol{\beta}}_x/\sigma$. We also denote by $\omega_{jt} \equiv (1 - 1/\sigma)\tilde{\omega}_{jt}$ a measure of revenue (sales) productivity and refer to it as simple store productivity in what follows. Additionally, μ_{jt} is a weighted sum of all unobserved product demand shocks at the store level, determined as $\mu_{jt} \equiv (1/\sigma) \sum_{i=1}^{np_{jt}} \tilde{\alpha}_i \mu_{ijt}$ and measures store j's specific demand shocks in period t, and u_{ijt}^p are i.i.d. remaining shocks to sales that are mean-independent of all control variables and store inputs. Using this notation, we can write the multiproduct sales function as $\sum_{i=1}^{np_{jt}}[\tilde{\alpha}_i y_{ijt} + \left(1 - \frac{1}{\sigma}\right) \tilde{\alpha}_y Y_{jt}] = \beta_l l_{jt} + \beta_k k_{jt} + \beta_a a_{jt} + \beta_q y_{ot} + \mathbf{x}'_{jt}\boldsymbol{\beta}_x + \omega_{jt} + \mu_{jt} + u_{ijt}^p$.

The combination of the service technology and simple CES demand yields an expression for the sales technology where the left-hand-side is a linear combination of sales per product category and the right-hand side is a linear combination of store inputs, local demand shifters, store revenue productivity, and demand shocks. This relationship solves the aggregation problem across different products. How many output parameters $\tilde{\alpha}_i$ we can identify depends on

[4] Note that store j sells few product categories, and therefore $c_{ij} > 1$.

the available data on products (categories) and the variation across stores. If there is large heterogeneity in products offered for sale across stores, we need to reduce the number of parameters $\tilde{\alpha}_i$ such that can be identified. By choosing only stores that sell similar products, we induce a selection problem. As a result, even if we estimate many technology parameters, the overall inference of the empirical exercise might be biased. In our Swedish data, there is substantial heterogeneity in the product categories that stores offer for sale. Thus, since we solve the multiproduct aggregation problem across product categories using sales instead of quantity, we rewrite the linear expression for product sales to reduce the number of parameters. In other words, we focus on sales of product category i and sales of other product categories. To obtain an estimable product sales equation that includes the logarithm of sales of product category i, y_{ijt}, and the logarithm of sales of other product categories inside the store y_{-ijt}, we rewrite the linear sum of product category sales $\sum_{i=1}^{np_{jt}} \left[\tilde{\alpha}_i y_{ijt} + \left(1 - \frac{1}{\sigma}\right) \tilde{\alpha}_y Y_{ijt}\right] \equiv \alpha_i y_{ijt} + \alpha_y y_{-ijt}$. Using new transformations, we can rewrite the sales of product category i as[5]

$$y_{ijt} = -\alpha_y y_{-ijt} + \beta_l l_{jt} + \beta_k k_{jt} + \beta_a a_{jt} + \beta_q y_{ot} + \mathbf{x}'_{jt}\boldsymbol{\beta}_x + \omega_{jt} + \mu_{jt} + u^p_{ijt}, \tag{40-a}$$

which is the equation we estimate in the main text.

In summary, it is important to discuss several aspects of the identification of the multiproduct technology. First, we focus on developing a simple multiproduct setting that does not require detailed product data and that can be used to analyze trends and the impact of policies in local markets. Second, we need product prices to identify the initial quantity weights $\tilde{\alpha}_i$ and variation in other product characteristics. Most important, in empirical settings, even if we have access to detailed product data and prices, we need data over a long period to consistently identify $\tilde{\alpha}_i$ (solving a system of equations at the firm/store level). In our setting, the scope parameter α_y in the multiproduct sales-generating function (40-a) includes the sum of weights $\tilde{\alpha}_i$. In other words, α_y provides information on the economies of scope in the store based on supply-side information (the multiproduct service frontier) and demand (elasticity of substitution).

Logit demand for homogeneous consumers. For a better understanding of the demand specification, we show the derivation of the well-known equation of logit demand for homoge-

[5] We normalize $\alpha_i = 1$.

neous consumers from a CES demand system (see, e.g., Anderson et al., 1987; Verboven, 1996; Anderson and De Palma, 2006; and Dube et al., 2020). This derivation helps provide an understanding of the form of the price equation (39-a). We assume that consumers are homogeneous and have CES preferences over differentiated products and services $i \in \{1, \cdots, np_j\}$ of store j, and the utility function is given by

$$U(\{Q_{ijt}, \mathbf{x}_{ijt}, \mu_{ijt}, \mathbf{z}_{ijt}, \eta_{ijt}\}_{i=\overline{1,np_j}}) := \left(\sum_{i=1}^{np_j} \kappa(\mathbf{x}_{ijt}, \mu_{ijt}, \mathbf{z}_{ijt}, \eta_{ijt})^{\frac{1}{\sigma}} Q_{ijt}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \tag{41-a}$$

where $\kappa(\mathbf{x}_{ijt}, \mu_{ijt}, \mathbf{z}_{ijt}, \eta_{ijt})$ is the kernel quality function (Dube et al., 2020). The terms $\mathbf{x}_{ijt}$ and $\mathbf{z}_{ijt}$ are the observed determinants of the intensive and extensive margins of the utility function when consumers buy product i. They might include common variables, and $\mathbf{z}_{ijt}$ includes at least one component that is not part of $\mathbf{x}_{ijt}$. Variables μ_{ijt} and η_{ijt} are determinants of the intensive and extensive margins of utility and are unobservable by the researcher. The quality function $\kappa(\cdot)$ allows us to separate intensive and extensive margins and to accommodate a zero market share (Dube et al., 2020).

The optimization problem for the representative consumer is given by

$$\begin{aligned} \max_{Q_{ijt,i=\overline{1,np_j}}} & \left(\sum_{i=1}^{np_j} \kappa(\mathbf{x}_{ijt}, \mu_{ijt}, \mathbf{z}_{ijt}, \eta_{ijt})^{\frac{1}{\sigma}} Q_{ijt}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \\ s.t. \quad & \sum_{i=1}^{np_j} P_{ijt} Q_{ijt} = b_t \end{aligned} \tag{42-a}$$

The solution of this optimization problem gives us the demand function (Q_{ijt}) and the individual choice probability (π_{ijt}), which is the CES demand system with observed and unobserved product characteristics, i.e.,

$$\begin{aligned} Q_{ijt} &= \frac{\kappa(\mathbf{x}_{ijt}, \mu_{ijt}, \mathbf{z}_{ijt}, \eta_{ijt}) P_{ijt}^{-\sigma}}{\sum_{h=1}^{np_j} \kappa(\mathbf{x}_{hjt}, \mu_{hjt}, \mathbf{z}_{hjt}, \eta_{hjt}) P_{hjt}^{1-\sigma}} b_t \\ \pi_{ijt} &= \frac{\kappa(\mathbf{x}_{ijt}, \mu_{ijt}, \mathbf{z}_{ijt}, \eta_{ijt}) P_{ijt}^{-\sigma}}{\sum_{h=1}^{np_j} \kappa(\mathbf{x}_{hjt}, \mu_{hjt}, \mathbf{z}_{hjt}, \eta_{hjt}) P_{hjt}^{-\sigma}} \end{aligned} \tag{43-a}$$

The elasticity of substitution σ is globally identified for the set of products with positive individual choice probabilities, i. e., $\pi_{ijt} > 0$. The reason is that system $\{\pi_{ijt}\}$ satisfies the connected substitutes condition provided by Berry et al. (2013), i.e., it is invertible.

The choice of the exponential kernel quality has key implications for the identification of the demand system. For $\mathbf{x}_{ijt} = \mathbf{z}_{ijt}$ and $\mathbf{z}_{ijt}$ being exogenous for all i, $\mu_{ijt} = \eta_{ijt}$, $\pi_{ijt>0}$ for all i, and we do not need any exclusion restriction to identify the demand system. In this case, the logarithm of the ratio of individual choice probabilities of product j and the outside option (or numeraire) if we normalize $\mathbf{x}_{0jt} = 0$, $\mu_{0jt} = 0$, and $\kappa(\mathbf{x}_{ijt}, \mu_{ijt}) = exp(\mathbf{x}'_{ijt}\boldsymbol{\beta}_x + \mu_{ijt})$ is given by[6]

$$ln(\pi_{ijt}) - ln(\pi_{0jt}) = -\sigma ln(P_{ijt}) + \mathbf{x}'_{ijt}\boldsymbol{\beta}_x + \mu_{ijt}. \tag{44-a}$$

Equation (44-a) is a logit demand system for homogeneous consumers, which can be written in terms of quantity

$$q_{ijt} - q_{0jt} = -\sigma p_{ijt} + \mathbf{x}'_{ijt}\boldsymbol{\beta}_x + \mu_{ijt}. \tag{45-a}$$

B.1: Monte Carlo simulation

The multiproduct technology is estimated at the product level assuming the same production technology across products. We focus on a simple specification and assume perfect competition; Therefore, we choose $y_{ijt} = \alpha_y y_{-ijt} + \beta_l l_{jt} + \beta_k k_{jt} + \omega_{jt} + u_{jt}$. We consider 1,000 stores and set $\beta_l = 0.6$, $\beta_k = 0.4$, and $\alpha_y = -0.85$. Most of our simulation settings are similar to those used by the previous literature on production functions (Ackerberg et al., 2015). Productivity follows an $AR(1)$ process ($\omega_{jt} = \rho\omega_{jt-1} + \xi_{jt}$) with persistence $\rho = 0.7$. Productivity is simulated to have constant variance over time (standard deviation 0.3). Wages w_{jt} follow an $AR(1)$ process with persistence $\rho^w = 0.3$ and are simulated to have constant variance over time (standard deviation 0.3). Labor is simulated using the first-order condition of static profit maximization. Capital stock is constructed using the perpetual inventory method $K_{jt} = (1-0.2)K_{jt-1} + I_{jt-1}$.[7] The number of years (periods) is 10, and all variables are used in the steady state.[8]

To estimate α_y, β_l and β_k, we use a two-step estimator with labor demand as a proxy for store productivity. The identification of $(\alpha_y, \beta_l, \beta_k)$ is based on the moment conditions $\mathbb{E}[\xi_{jt}|y_{-ijt-1}, l_{jt-1}, k_{jt}] = 0$ and the GMM estimator. Table B.1 shows the estimates of the

[6]The reason is that $ln(\pi_{ijt}) - ln(\pi_{0jt}) = -\sigma ln(P_{ijt}) + ln(\kappa(\mathbf{x}_{ijt}, \mu_{ijt})) - ln(\kappa(\mathbf{x}_{0jt}, \mu_{0jt}))$.

[7]Investment is simulated based on a policy function that is increasing the in-store's state variables, i.e., $i_{jt} = 0.2 + 0.3\omega_{jt} + 0.1k_{jt}$. For robustness, we used a nonlinear specification $i_{jt} = 0.2 + 0.3\omega_{jt} + 0.1k_{jt} + 0.01\omega_{jt}^2 + 0.01k_{jt}^2 - 0.004\omega_{jt}^3 - 0.006k_{jt}^3$. However, because there are no substantial changes in the main findings, we show the results with the linear specification.

[8]We consider 100 warm-up simulations before simulating the data sets.

single- and multioutput technology based on 1,000 Monte Carlo simulations. For multiproduct technology, each store has three products, and their outputs are obtained by solving the nonlinear system of equations for each store in each period. The findings in Table B.1 show that we identify the parameters without bias when the DGPs are the true ones (single- and multiproduct DGPs), even if the estimation uses nonparametric labor demand and the data are generated using parametric labor demand.

Table B.2 shows the bias in the labor and capital coefficients of a single-output technology when the true DGP is a multioutput technology with three products. The results show a downward-biased labor coefficient (a decrease from 0.6 to 0.49) and an upward-biased capital coefficient (an increase from 0.4 to 0.542). These biases that translate into a large productivity bias are generated by the omission of the tradeoff between producing one product or different products with the same resources that affect the aggregate output. In a multiproduct setting, the productivity difference between two stores using the same inputs is generated by the choice of product mix. 3500

TABLE B.1: Estimation of single and multiproduct production function using two-step estimator

	DGP: Single-product		DGP: Multi-product	
	Estim.	Std.	Estim.	Std.
Log of labor (β_l)	0.599	0.008	0.601	0.026
Log of capital (β_k)	0.400	0.005	0.401	0.031
Log of other products (α_y)			-0.854	0.078

NOTE: Source: Maican and Orth (2019). The two-step estimator uses non-parametric labor demand function to proxy for productivity. Reported standard errors are computed based on 1000 simulations. Monte Carlo simulations use $\beta_l = 0.6$, $\beta_k = 0.4$, $\alpha_y = -0.85$. Single-product function is estimated at the firm level. Multi-product function is estimated at the product level assuming the same production technology across products. The number of firms is 1000. For the multiproduct DGP, the number of products for each firm is 3. Labor is simulated using first-order condition profit maximization. Investment is simulated based on policy function that is increasing in the state variables. Capital stock is constructed using perpetual inventory method $K_{jt} = (1 - 0.2)K_{jt-1} + I_{jt-1}$. Productivity follows an AR(1) process with the persistence $\rho = 0.7$ and it is simulated to have constant variance over time (standard deviation 0.3). Wages follow an AR(1) process with the persistence $\rho = 0.3$ and it is simulated to have constant variance over time (standard deviation 0.3). The number of years is 10 (all variables are used in steady state).

B.2: Identification and Estimation of the Multiproduct Sales Function

The identification and estimation of the sales-generating function, including the Markov processes for ω_{jt} and μ_{jt}, are based on the well-established two-step methods in the production

TABLE B.2: Estimation of single output production when DGP is a multiproduct production function

		Estim.	Std.
Log of labor (β_l)		0.490	0.004
Log of capital (β_k)		0.542	0.002
Distribution of productivity bias			
	Q_{25}	Q_{50}	Q_{75}
	-0.085	0.116	0.322

NOTE: Source: Maican and Orth (2019). The two-step estimator uses non-parametric labor demand function to proxy for productivity. Reported standard errors are computed based on 1000 simulations. Monte Carlo simulations use $\beta_l = 0.6$, $\beta_k = 0.4$, $\alpha_y = -0.85$. Single-product function is estimated at the firm level. Multi-product function is estimated at the product level assuming the same production technology across products. The number of firms is 1000. For the multiproduct DGP, the number of products for each firm is 3. Labor is simulated using first-order condition profit maximization. Investment is simulated based on policy function that is increasing in the state variables. Capital stock is constructed using perpetual inventory method $K_{jt} = (1-0.2)K_{jt-1} + I_{jt-1}$. Productivity follows an AR(1) process with the persistence $\rho = 0.7$ and it is simulated to have constant variance over time (standard deviation 0.3). Wages follow an AR(1) process with the persistence $\rho = 0.3$ and it is simulated to have constant variance over time (standard deviation 0.3). The number of years is 10 (all variables are used in steady state).

function literature. Identification comes from a system of equations (multiproduct sales and market share) and two unobservables (productivity and demand shocks), where one of the unobservables is part of only one equation. Two control functions based on the store's optimal policy functions are used to proxy for ω_{jt} and μ_{jt}.[9]

We estimate β_l, β_k, β_a, α_y, σ, ρ_{np}, $\rho_{inc,1}$, $\rho_{inc,2}$, γ^{ω}, and γ^{μ} together using a modified Olley and Pakes (1996) (OP) two-step estimator that includes product information (Olley and Pakes, 1996; Levinsohn and Petrin, 2003; Ackerberg et al., 2015; Gandhi et al., 2020). Compared to OP, we have two unobservables to recover, and we show how the market share index function helps to recover demand shocks μ_{jt} separate from productivity ω_{jt} and ensures the identification of the model. In our retail setting, the dynamics of store productivity are more complex, since productivity is affected by both demand shocks and local entry regulations.

To back out ω_{jt} and μ_{jt} from the policy functions $l_{jt} = \tilde{l}_t(\omega_{jt}, \mu_{jt}, k_{jt}, n_{jt}, w_{jt}, y_{ot}, \mathbf{x}_{mt}, r_{mt})$ and $a_{jt} = \tilde{a}_t(\omega_{jt}, \mu_{jt}, k_{jt}, n_{jt}, w_{jt}, y_{ot}, \mathbf{x}_{mt}, r_{mt})$, Assumption 5 must hold (Appendix C). By inverting these policy functions to solve for ω_{jt} and μ_{jt}, we obtain $\omega_{jt} = f_t^1(l_{jt}, k_{jt}, n_{jt}, w_{jt}, a_{jt}, y_{ot}, \mathbf{x}_{mt}, r_{mt})$ and $\mu_{jt} = f_t^2(l_{jt}, k_{jt}, n_{jt}, w_{jt}, a_{jt}, y_{ot}, \mathbf{x}_{mt}, r_{mt})$, which are nonparametric functions of the observed variables in the state space and the controls. In the first step, we construct

[9]Ackerberg et al. (2007) (Section 2.4) and Matzkin (2008) discuss the core of identification of such systems of equations.

measures of productivity ω_{jt} and demand shocks μ_{jt} as functions of the structural parameters that do not include the remaining shocks u^p_{ijt} and ν_{jt}.

By substituting the nonparametric inversion $f^2_t(l_{jt}, k_{jt}, n_{jt}, w_{jt}, a_{jt}, y_{ot}, \mathbf{x}_{mt}, r_{mt})$ for μ_{jt} in (8) and considering that the number of product categories np_{jt} is also a function of the store state variables (a policy function of the store optimization problem), the market share equation can be written as $ln(ms_{jt}/ms_{0t}) = b_t(l_{jt}, k_{jt}, n_{jt}, w_{jt}, a_{jt}, y_{ot}, \mathbf{x}_{mt}, r_{mt}) + \nu_{jt}$, which can be estimated using ordinary least squares (OLS) and a polynomial expansion of order two in l_{jt}, k_{jt}, n_{jt}, w_{jt}, a_{jt}, y_{ot}, $\mathbf{x}_{mt}$, r_{mt} to approximate function $b_t(\cdot)$.[10] Therefore, we obtain an estimate of $b_t(\cdot)$, denoted $\hat{b}_t$, which is the predicted $ln(ms_{jt}/ms_{0t})$. We can write demand shocks μ_{jt} as a parametric function: $\mu_{jt} = \hat{b}_{jt} - \rho_{np} np_{jt} - \rho_{inc,1} inc_{mt} - \rho_{inc,2} inc^2_{mt}$, which will be treated as an input in the multioutput sales-generating function (5). In the second step, when we substitute μ_{jt} (predicted) and ω_{jt} into (5), the sales-generating function becomes

$$y_{ijt} = -\alpha_y y_{-ijt} + \phi_t(l_{jt}, k_{jt}, n_{jt}, w_{jt}, a_{jt}, y_{ot}, \mathbf{x}_{mt}, r_{mt}) + u^p_{ijt}, \tag{46-a}$$

where $\phi_t(\cdot) = \beta_l l_{jt} + \beta_k k_{jt} + \beta_a a_{jt} + \beta_q y_{ot} + \mathbf{x}'_{mt}\boldsymbol{\beta}_x + \omega_{jt} + \mu_{jt}$. The function $\phi_t(\cdot)$ can be approximated using a polynomial expansion of order two in its arguments. The estimation of (46-a) yields an estimate of service output without service output shocks u^p_{ijt}, i.e., $\hat{\phi}_t$, which is used to obtain store productivity ω_{jt} as a function of the parameters, $\omega_{jt} = \hat{\phi}_{jt} - \beta_l l_{jt} - \beta_k k_{jt} - \beta_a a_{jt} - \beta_q y_{ot} - \mathbf{x}'_{mt}\boldsymbol{\beta}_x - \hat{b}_{jt} + \rho_{np} np_{jt} + \rho_{inc,1} inc_{mt} + \rho_{inc,2} inc^2_{mt}$. Then, we rewrite the sales and market share equations using parametric forms of productivity ω_{jt} and demand shocks μ_{jt} and Markov processes

$$\begin{aligned} y_{ijt} = \; & -\alpha_y y_{-ijt} + \beta_l l_{jt} + \beta_k k_{jt} + \beta_a a_{jt} + \beta_q y_{ot} + \mathbf{x}'_{mt}\boldsymbol{\beta}_x + \hat{b}_{jt} - \rho_{np} np_{jt} \\ & -\rho_{inc,1} inc_{mt} - \rho_{inc,2} inc^2_{mt} + h^{\omega}(\hat{\phi}_{jt-1} - \beta_l l_{jt-1} - \beta_k k_{jt-1} - \beta_a a_{jt-1} \\ & -\beta_q y_{ot-1} - \mathbf{x}'_{mt-1}\boldsymbol{\beta}_x - \hat{b}_{jt-1} + \rho_{np} np_{jt-1} + \rho_{inc,1} inc_{mt-1} \\ & +\rho_{inc,2} inc^2_{mt-1}, \hat{b}_{jt-1} - \rho_{np} np_{jt-1} - \rho_{inc,1} inc_{mt-1} - \rho_{inc,2} inc^2_{mt-1}, \\ & r_{mt-1}) + \xi_{jt} + u^p_{ijt} \end{aligned} \tag{47-a}$$

[10] A polynomial expansion of order three shows no improvement in the estimation of the first stage. Other approximations can be used, e.g., b-splines.

$$
\begin{aligned}
ln(ms_{jt}/ms_{0t}) = \;& \rho_{np}np_{jt} + \rho_{inc,1}inc_{mt} + \rho_{inc,2}inc_{mt}^2 + h^{\mu}(\hat{b}_{jt-1} \\
& -\rho_{np}np_{jt-1} - \rho_{inc,1}inc_{mt-1} - \rho_{inc,2}inc_{mt-1}^2) + \eta_{jt} + \nu_{jt}.
\end{aligned} \tag{48-a}
$$

The parameters of the multiproduct sales function (47-a) and market share equation (48-a) are identified using moment conditions on the remaining shocks in these equations, $\xi_{jt} + u_{ijt}^p$ and $\eta_{jt} + \nu_{jt}$.

Estimation. In the empirical implementation, we approximate the functions $h^{\omega}(\cdot)$ and $h^{\mu}(\cdot)$ in the Markov processes of ω_{jt} and μ_{jt} by polynomials. The estimated Markov processes are:

$$
\begin{aligned}
\omega_{jt} = \;& \gamma_0^{\omega} + \gamma_1^{\omega}\omega_{jt-1} + \gamma_2^{\omega}(\omega_{jt-1})^2 + \gamma_3^{\omega}(\omega_{jt-1})^3 + \gamma_4^{\omega}\mu_{jt-1} + \gamma_5^{\omega}r_{mt-1} \\
& +\gamma_6^{\omega}\omega_{jt-1} \times \mu_{jt-1} + \gamma_7^{\omega}r_{mt-1} \times \omega_{jt-1} + \gamma_8^{\omega}r_{mt-1} \times \mu_{jt-1} + \xi_{jt}
\end{aligned} \tag{49-a}
$$

$$
\mu_{jt} = \gamma_0^{\mu} + \gamma_1^{\mu}\mu_{jt-1} + \gamma_2^{\mu}(\mu_{jt-1})^2 + \gamma_3^{\mu}(\mu_{jt-1})^3 + \eta_{jt} \tag{50-a}
$$

The vector of parameters to be estimated is $\boldsymbol{\theta} = (\beta_l,\ \beta_k,\ \beta_a,\ \alpha_y,\ \sigma,\ \boldsymbol{\beta}_x,\ \rho_{np},\ \rho_{inc,1},\ \rho_{inc,2},\ \boldsymbol{\gamma}^{\omega}$, and $\boldsymbol{\gamma}^{\mu})$. Productivity ω_{jt} and μ_{jt} are functions of $\boldsymbol{\theta}$. We can identify the $\boldsymbol{\theta}$ coefficients using moment conditions based on $(\xi_{jt} + u_{ijt}^p)$ and $(\eta_{jt} + \nu_{jt})$ and the generalized method of moments (GMM) estimator. The identification uses the fact that the current shocks are conditionally independent of information in $t-1$, $\mathcal{F}_{jt-1}$.[11] To identify $\boldsymbol{\theta}$, we use the moment conditions $E[\xi_{jt} + u_{ijt}^p|\ y_{-ijt-1},\ l_{jt-1},\ k_{jt-1},\ a_{jt-1},\ \mathbf{x}_{mt-1}] = 0$ and $E[\eta_{jt} + \nu_{jt}|np_{jt-1}, inc_{mt-1}, inc_{jt-1}^2] = 0$. The parameters β_l, β_k, and β_a are identified using l_{jt-1}, k_{jt-1}, and a_{jt-1} as instruments. Thus, we exploit the fact that the current remaining productivity and sales shocks are not correlated with previous inputs to form moment conditions. To identify the economies of scope parameter α_y, we use y_{-ijt-1} as an instrument, which requires that the previous output is not correlated with current remaining sales and productivity shocks. The Monte Carlo experiments discussed below show the robustness of the identification of the scope parameter α_y using

[11] Ackerberg et al. (2007) discuss the use of previous variables as instruments in a two-step control function approach in the estimation of production technologies. Ackerberg et al. (2015) discuss in Section IV(i) different ways to estimate an OP framework based on second-step moments. Most importantly, stronger assumptions can lead to more precise estimates. Our results remain robust when we use moment conditions based on ξ_{jt} and η_{jt} to identify β_l, β_k, β_a, $\boldsymbol{\beta}_x$, σ, ρ_{np}, $\rho_{inc,1}$, and $\rho_{inc,2}$.

previous output.[12] The fact that previous local market characteristics $\mathbf{x}_{mt-1}$ are not correlated with current remaining sales and productivity shocks allows us to identify $\boldsymbol{\beta}_x$.[13] To identify the coefficients of the market share equation, we use the fact that $(\eta_{jt} + \nu_{jt})$ are not correlated with the previous number of product categories and income. The Markov process parameters $\boldsymbol{\gamma}^\omega$ and $\boldsymbol{\gamma}^\mu$ are identified using the corresponding polynomial terms in equations (49-a) and (50-a) as instruments.

The $\boldsymbol{\theta}$ parameters are estimated by minimizing the GMM objective function

$$\min_{\beta} Q_N = \left[\frac{1}{N} W' v(\boldsymbol{\theta})\right]' A \left[\frac{1}{N} W' v(\boldsymbol{\theta})\right], \tag{51-a}$$

where $v_{jt} = (u_{ijt} + \xi_{jt}, \nu_{jt} + \eta_{jt})'$, W is the matrix of instruments, and A is the weighting matrix defined as $A = \left[\frac{1}{N} W' v(\boldsymbol{\beta}) v'(\boldsymbol{\beta}) W\right]^{-1}$. [14]

Appendix C: Additional discussion on identification and extensions

Invertibility conditions with two unobservables. The general labor demand and inventory functions that arise from the stores' dynamic optimization problem are

$$\begin{aligned} l_{jt} &= \tilde{l}_t(\omega_{jt}, \mu_{jt}, k_{jt}, n_{jt}, w_{jt}) \\ a_{jt} &= \tilde{a}_t(\omega_{jt}, \mu_{jt}, k_{jt}, n_{jt}, w_{jt}). \end{aligned} \tag{52-a}$$

The main aim is to recover ω_{jt} and μ_{jt} using this system of equations. The conditions required for identification can be grouped as follows: (i) general conditions that the policy functions of the dynamic programming problem have to satisfy; (ii) conditions that the system of equations should satisfy to have a unique solution. In what follows, we discuss these conditions.

First, strict monotonicity guarantees inversion in the case of a single policy function and unobservable factor (Olley and Pakes, 1996). To back out ω_{jt} and μ_{jt}, i. e., the policy functions $\tilde{l}_t(\cdot)$ and $\tilde{a}_t(\cdot)$ must be strictly monotonic in ω_{jt} and μ_{jt}, which holds under mild regularity

[12] Appendix C discusses a more computationally demanding estimator.

[13] $\mathbf{x}_{mt}$ are also valid instruments because market characteristics are exogenous.

[14] Standard errors are computed according to Ackerberg et al. (2012).

conditions on the dynamic programming problem (Pakes, 1994). The static profits are assumed to be strictly increasing in ω_{jt}, μ_{jt}, and k_{jt} and continuously differentiable in these variables. Another condition is the supermodularity of the static profits with respect to ω_{jt} and μ_{jt}, i.e., the impact of productivity on profits is increasing in μ_{jt}. In other words, stores with large demand shocks experience larger increase in profits due to productivity. This assumption is not restrictive since stores that experience large demand shocks to increase their productivity to satisfy demand. Furthermore, static profits are assumed to be supermodular with respect to ω_{jt} (μ_{jt}) and k_{jt}, i.e., the marginal product of capital is increasing in productivity and demand shocks. This condition can also be interpreted as follows: stores with larger capital stock have higher profits due to an increase in productivity or demand shocks. All these conditions (strict monotonicity and supermodularity) on static profits yield that value and policy functions are strictly increasing in ω_{jt}, μ_{jt}, and k_{jt} (Pakes, 1994).

Second, we discuss the general properties that must be satisfied by the labor demand ($\tilde{l}(\cdot)$) and inventory ($\tilde{a}(\cdot)$) functions such that the system (52-a) has a unique solution. This system can be solved for ω_{jt} and μ_{jt} in terms of k_{jt}, n_{jt}, l_{jt}, w_{jt}, and a_{jt} when certain partial derivatives are continuous, and the 2×2 Jacobian determinant $\partial(\tilde{l}, \tilde{a})/\partial(\omega, \mu)$ is not zero. In other words, the ratios between the impact of ω and μ on the investment and inventories should not be the same, i.e., $(\partial\tilde{l}/\partial\omega)/(\partial\tilde{l}/\partial\mu) \neq (\partial\tilde{a}/\partial\omega)/(\partial\tilde{a}/\partial\mu)$. Therefore, this condition requires that productivity and demand shocks have a different impact on investment and inventory, and the relative impact is not the same.

We apply the implicit function theorem to prove the invertibility of the system (52-a). In our case, points in $(2 + 5)$-dimensional space $\mathbb{R}^{2+5}$ can be written in the form of $(\mathbf{x}; \mathbf{b})$, where $\mathbf{x} = (\omega, \mu)$ and $\mathbf{b} = (k, n, l, a, w)$. We can rewrite the system as $f_1(\mathbf{x}; \mathbf{b}) = 0$ and $f_2(\mathbf{x}; \mathbf{b}) = 0$ or simply as an equation $F(\mathbf{x}; \mathbf{b}2) = 0$. To understand the invertibility of the policy functions, we need to know when the relation $F(\mathbf{x}; \mathbf{b}) = 0$ is also a function. In other words, what the conditions are such that $F(\mathbf{x}; \mathbf{b}) = 0$ can be solved explicitly for $\mathbf{b}$ in terms of $\mathbf{x}$, obtaining a unique solution. Theorem C.1 (the implicit function theorem) provides the conditions that for a given point $(\mathbf{x}_0, \mathbf{b}_0)$ such that $F(\mathbf{x}_0, \mathbf{b}_0) = 0$ there exists a neighborhood of $(\mathbf{x}_0, \mathbf{b}_0)$ where the relation $F(\mathbf{x}; \mathbf{b}) = 0$ is a function.

Theorem C.1. *Let* $\mathbf{f} = (f_1, f_2)$ *be a vector of functions defined on the open set* S *in* $\mathbb{R}^{2+5}$ *with* *values in* $\mathbb{R}^2$. *Suppose that* $\mathbf{f} \in C'$ *on* S. *Let* $(\mathbf{x}_0; \mathbf{b}_0)$ *be a point in* S *for which* $\mathbf{f}(\mathbf{x}_0, \mathbf{b}_0) = \mathbf{0}$ *and for which the* 2×2 *Jacobian determinant* $\partial(f_1, f_2)/\partial(\omega, \mu)$ *is not zero at* $(\mathbf{x}_0, \mathbf{b}_0)$. *Then, there exists a 5-dimensional open set* B_0 *such that includes* $\mathbf{b}_0$ *and one and only one vector-based function* $\mathbf{g}$ *defined on* B_0 *and having values in* $\mathbb{R}^2$ *such that*

(i) $\mathbf{g} \in C'$ *on* B_0

(ii) $\mathbf{g}(\mathbf{b}_0) = \mathbf{x}_0$

(iii) $\mathbf{f}(\mathbf{g}(\mathbf{b}); \mathbf{b}) = \mathbf{0}$ *for every* $\mathbf{b}$ *in* B_0.

PROOF: This theorem is, in fact, the implicit function theorem applied in our case. The general proof of the theorem can be found in Apostol (1974).

Alternative identification for economies of scope parameter. There is an alternative identification strategy for the scope parameter α_y that fully endogenizes product-category sales in the estimation. That is, we can solve the system of output equations for each store instead of using the previous output of other product categories as an instrument. This is similar to the counterfactual experiments where we solve the system of output equations for each store. However, this estimator is computationally demanding because it requires solving the system of equations for each store-year observation and a new set of model parameters using fixed-point iteration. Monte Carlo experiments show no main advantages of this alternative estimator over the above IV identification strategy when stores use the same sales technology for their product categories.

Endogeneity of regulation. Because stores cannot influence or form expectations about the future stringency of regulation, we follow a two-step estimation procedure to alleviate endogeneity concerns regarding regulation. Our estimation accounts for the possible endogeneity of the regulation measure. We model the structure of supply and demand shocks and use many exogenous local market characteristics as controls in the first stage. Entry regulation is exogenous in the productivity process such that individual stores do not affect the outcome of regulation or form expectations about the stringency of future regulation. The nature of the semiparametric model helps address the possible concerns related to endogeneity of regulation when evaluating its impact on productivity. Removing the effect of local market characteristics from the sum of demand and production shocks in the first step reduces endogeneity concerns when estimating the productivity process. If productivity shocks ξ_{jt} are correlated with the previous stringency of regulation, we can identify the coefficient of r_{mt-1} by using an instrument. Our instrument needs to be correlated with regulatory stringency but unrelated to shocks in productivity ξ_{jt}. In the data section, we discuss the endogeneity of entry regulation using the IV approach and three instruments (Table 3).

Alternative demand specification. Our main empirical results are not driven by demand. assumption (the general form of the sales-generating function remains the same when allowing for nests) and are supported by various simple descriptions and reduced-form specifications (see Section 2).

The simple demand approach in Section 3.1 has a key benefit: CES preferences generate the same demands as would be obtained from aggregating many consumers who make discrete choices regarding in what store to shop. However, CES preferences impose a specific structure on demand, which is restrictive. Nevertheless, our model is rich on the supply side, and the form of our multiproduct sales-generating function (5) is also consistent with a demand specification that allows for rich substitution patterns, e.g., a constant expenditure specification in an aggregate nested logit model where price enters in log form. This is because in a constant expenditure specification, we use the volume of sales for each product category, which allows us to aggregate products when using the multiproduct function (3).[15] In a nested demand model, consumers choose stores and then products within a store. In this case, the output and input parameters depend on the nest parameter(s), and the scope parameter α_y includes information about product correlation in the nests at the store level. We use the simple CES specification in the estimation because we do not focus on a specific product category in the empirical application (e.g., yogurt), and we have high heterogeneity on the supply side in the data.

The relationship with other multiproduct estimators. Our model uses product output shares and store inputs from the data. There are also alternative estimators that estimate and use input shares to study the multiproduct case. In contrast to many alternative multiproduct estimators, we explicitly model economies of scope in the technology and endogenize the number of products. Our model is closely related to De Loecker et al. (2016), even if their method estimates input shares to construct a single-product technology. As in De Loecker et al. (2016), we have separability in inputs and outputs in the production technology and model

[15]All technical derivations are available from the authors upon request. A constant expenditure specification allows consumers to buy more than one product (Verboven, 1996).

firm/store productivity and not product-firm productivity.[16] In the retail context, it is difficult to define a meaningful measure of product-store productivity. Using the aggregation over inputs and outputs, we can shows that there is a direct relationship between the input shares from a Cobb-Douglas technology at the product level and output shares of transcendental technology. The relationship exists because both technologies use firm/store productivity, and there is no need to aggregate product productivity to the firm level.

Separating input allocations per product can be difficult in service industries. For example, different machinery and equipment are used to carry or store different product categories at the same time to increase efficiency. In the multiproduct case, a service technology function consistent with profit maximization implies aggregation over physical products, and this is restrictive for many data sets due to large heterogeneity (especially in retailing). The service sector is characterized only by multiple products, and In many cases, it is also difficult to measure physical products. Splitting all inputs is not entirely consistent with economies of scale and scope in retail. Since our focus is on entry regulations and economies of scope and not recovering product markups, transcendental technology that uses observed output shares is preferable; it does not require additional assumptions to recover input shares (not observed in the data).

Appendix D: Entry regulation: Plan and Building Act (PBL)

The majority of OECD countries have entry regulations that empower local authorities to decide on store entry. However, the regulations differ substantially across countries. (Boylaud and Nicoletti, 2001; Griffith and Harmgart, 2005; Schivardi and Viviano, 2011). While some countries strictly regulate large entrants, more flexible zoning laws exist, for instance, in the U.S. (Pilat, 1997).

The Swedish Plan and Building Act (PBL) regulates the use of land and water and buildings. The PBL consists of the planning requirements for land and water areas as well as buildings. The ultimate goal of PBL is to promote equal and adequate living conditions and a lasting sustainable environment for today and future generations. The regulation contains two documents/plans: (i) the comprehensive plan and (ii) the detailed development plan. Municipalities

[16] See also Valmari (2016), Orr (2022), and Dhyne et al. (2023).

are required to have a comprehensive plan that covers the entire municipality and that guides decisions regarding the use of land, water areas and the built environment. The comprehensive plan records public interests and national interests. Municipalities also have to provide detailed development plans that cover only a fraction of the municipality. Municipalities are divided into smaller areas. These plans indicate and set limits on the use and design of public spaces, land and water areas.

The purpose of the comprehensive plan is to provide an attractive public environment that is sustainable. It is the basis for decisions regarding the use of land and water and the development and preservation of buildings. It reflects the public interest and addresses important environmental and risk factors that must be accounted for in the planning of any endeavor. Necessary features include the housing needs of the municipal inhabitants, the protection of valuable natural and cultural environments, and providing inhabitants with access to services.

The detailed development plan consists of a map with text that indicates what, where and how one is allowed to build, as well as appropriate uses for the area. For instance, it indicates the appropriate design and use of housing, nature and water areas. Other examples include construction rights for real estate including the size and form of structures, the possibility of opening a restaurant, workplaces and businesses, housing, hotels, housing (villas or apartments), preschools, elementary schools, health care, energy and water services, parks, streets, squares, etc.

The detailed development plan indicates whether retail stores are allowed. The right to open and operate a retail food store is addressed in the detailed development plan. Each store seeking to enter the market is required to file a formal application with the local government. For the entry to occur, the municipality can accept a new detailed development plan or make changes in an existing plan. First, in the application, the store must state the purpose of the activity: retail, housing, offices, manufacturing, or other. Second, the store must describe the main purpose of its activity and what it is to contain, e.g., a new building of a certain size, wholesale provision with trucks, parking spaces and is obligated to be as detailed as much as possible. Before the new detailed development plan is approved, it must be made publicly available. Inhabitants of the municipality are allowed to express their opinions and views on the proposed changes. If some do not agree with the proposed plan, they can appeal. The municipality must then perform a new evaluation and seek alternative solutions to the question at hand.

When a retail store seeks to enter a local market, the municipality evaluates the consequences for exit, prices, local employment, availability of store types and product assortments for different types of consumers, purchasing patterns and purchasing trips, consumer travel behavior, traffic (e.g., generated traffic per square meter of the new sales space), including its effect on noise and air pollution for nearby consumers as well as the number of individuals who will be affected – probable health effects, risk evaluations, broader environmental issues, increased distance to the store, parking, water, energy supply, etc.

In addition, the municipal council must evaluate the positive and negative consequences of the new entrant for different inhabitants, the environment, traffic, public transport, safety, etc. The municipality must consider whether new bus lines are necessary, as well as walking and biking paths. This is to ensure that each consumer in the municipality has access to different types of stores, a broad product assortment and reasonable prices. A store entrant is prohibited from hindering real estate developments that will be useful for the public interest, i.e., housing, places of work, traffic infrastructure and leisure environments. The municipal council evaluates and gives an overall assessment of the trade-offs between the public interest and private retail interests. This assessment is based on contingency analysis, an investigation of alternative solutions and developments, and strategic judgment. It is important to evaluate the effects of accepting a new detailed development plan and changing an existing plan on the public interest.

All stores are regulated by the PBL in Sweden, in contrast with for example, the U.K., which explicitly focuses on regulating large stores (Maican and Orth, 2015; Sadun, 2015). PBL is considered one of the major barriers to entry and is the cause of a diverse array of outcomes, e.g., price levels across municipalities. Several reports stress the need to better analyze how entry regulation affects market outcomes (Pilat, 1997; Swedish Competition Authority, 2001:4; Swedish Competition Authority, 2004:2).

Appendix E: The short and long-run effects of an additional product category in main estimation and counterfactuals

The evaluation of each term of the first-order condition (FOC) of the Bellman equation provides useful information about the trade-off between the demand adjusted scale and scope effect (short-run) measured by the marginal sales (i.e., $\partial Y_{jt}/\partial np_{jt}$) and the marginal future gain from adding a new product category (long-run or dynamic effect) measured by $\zeta \partial \mathbb{E}[V(\cdot)]/\partial np_{jt}$.[17] While the marginal cost with respect to the number of product categories is always positive, the terms $\partial Y_{jt}/\partial np_{jt}$ and $\zeta \partial \mathbb{E}[V(\cdot)]/\partial np_{jt}$ can be positive or negative but their sum is positive.

Table E.3 shows the share of store-year observations in each of the three possible bins defined by the sign of each term of the FOC. The findings show that 83% of the stores have future gains by adding a new product category, and 45% have positive gains in per-period sales (i.e., positive the demand adjusted scale and scope effect or short-run effect). This result shows the importance of allowing for dynamic effects when studying endogenizing variety. Also, 28% of stores have positive gains in both the short- and long-run from increasing the number of product categories. In the counterfactuals, the share of stores in this bin with both positive scale-scope and dynamic effects increases with a less restrictive regulation (by one and 15 percentage points in CF_1 and CF_2, respectively). Nevertheless, cost subsidies (CF_4) decrease the shares of stores with scope and dynamic effect by eight percentage points.

TABLE E.3: Estimation: The decomposition of the impact of increasing the number of product categories

	Negative $\zeta \partial \mathbb{E}[V(\cdot)]/\partial np_{jt}$	Positive $\zeta \partial \mathbb{E}[V(\cdot)]/\partial np_{jt}$	Sum of columns
Negative $\partial Y_{jt}/\partial np_{jt}$		0.549	0.549
Positive $\partial Y_{jt}/\partial np_{jt}$	0.166	0.284	0.450
Sum of rows	0.166	0.833	1.000

NOTE: Figures represent share of store-year in a bin.

TABLE E.4: Counterfactual CF_1: The decomposition of the impact of increasing the number of product categories

	Negative $\zeta \partial \mathbb{E}[V(\cdot)]/\partial np_{jt}$	Positive $\zeta \partial \mathbb{E}[V(\cdot)]/\partial np_{jt}$	Sum of columns
Negative $\partial Y_{jt}/\partial np_{jt}$		0.0017	0.0017
Positive $\partial Y_{jt}/\partial np_{jt}$	-0.008	0.0063	-0.0017
Sum of rows	-0.008	0.0080	0.0

NOTE: Figures represent changes in share of store-year in a bin.

[17] The FOC is given by $\partial c/\partial np_{jt} = \partial Y_{jt}/\partial np_{jt} + \zeta \partial \mathbb{E}[V(\cdot)]/\partial np_{jt}$.

TABLE E.5: Counterfactual CF_2: The decomposition of the impact of increasing the number of product categories

	Negative $\zeta\partial\mathbb{E}[V(\cdot)]/\partial np_{jt}$	Positive $\zeta\partial\mathbb{E}[V(\cdot)]/\partial np_{jt}$	Sum of columns
Negative $\partial Y_{jt}/\partial np_{jt}$	0.0	-0.071	-0.071
Positive $\partial Y_{jt}/\partial np_{jt}$	-0.082	0.154	0.071
Sum of rows	-0.082	0.082	0.0

NOTE: Figures represent changes in share of store-year in a bin.

TABLE E.6: Counterfactual CF_3: The decomposition of the impact of increasing the number of product categories

	Negative $\zeta\partial\mathbb{E}[V(\cdot)]/\partial np_{jt}$	Positive $\zeta\partial\mathbb{E}[V(\cdot)]/\partial np_{jt}$	Sum of columns
Negative $\partial Y_{jt}/\partial np_{jt}$	0.0	-0.002	-0.002
Positive $\partial Y_{jt}/\partial np_{jt}$	0.007	-0.005	0.002
Sum of rows	0.007	-0.007	0.0

NOTE: Figures represent changes in share of store-year in a bin.

TABLE E.7: Counterfactual CF_4: The decomposition of the impact of increasing the number of product categories

	Negative $\zeta\partial\mathbb{E}[V(\cdot)]/\partial np_{jt}$	Positive $\zeta\partial\mathbb{E}[V(\cdot)]/\partial np_{jt}$	Sum of columns
Negative $\partial Y_{jt}/\partial np_{jt}$	0.0	0.054	0.054
Positive $\partial Y_{jt}/\partial np_{jt}$	0.025	-0.079	-0.054
Sum of rows	0.025	-0.025	0.0

NOTE: Figures represent changes in share of store-year in a bin.

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