Accessing U.S. Dollar Swap Lines: Macroeconomic Implications for a Small Open Economy*

Begoña Domínguez[†] Pedro Gomis-Porqueras[‡]

University of Queensland Queensland University of Technology

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Abstract

This paper proposes a framework to examine the effects of U.S. dollar swap arrangements between the Federal Reserve System and a small open economy's central bank. The framework is a dynamic general equilibrium model where domestic and foreign currencies are valued and where domestic short and long-term bonds have differential pledgeability. Then we investigate how U.S. dollar swap lines affect inflation and debt dynamics in the small open economy, particularly when combined with domestic quantitative easing policies and standard interest rate management policies. We also explore the circumstances under which an equivalence result for stationary equilibria is obtained under different combinations of U.S. dollar swap lines, and domestic quantitative easing as well as interest rate management policies. When calibrated to Australia during the pandemic and under some conditions, we find that a more favorable swap line would have allowed to cut back on long-term bond purchases from 35 % to 24 % of GDP. While those policies deliver the same state state, local dynamics are not. We find that swaps and quantitative easing dampen the fiscal eigenvalue. Finally, we identify combinations of international and domestic unconventional monetary policies that yield desirable equilibria.

JEL Codes: E440, F330, G150.

Keywords: swaps, quantitative easing, repos, interest rate management.

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[†]University of Queensland, School of Economics, Brisbane, Australia. E-mail: b.dominguez@uq.edu.au

[‡]Queensland University of Technology, School of Economics and Finance, Brisbane, Australia. E-mail: peregomis@gmail.com

1 Introduction

The global financial crises and the COVID-19 pandemic led to a sharp decrease in the availability of U.S. dollars and a rapid increase in the cost of acquiring them outside the United States. These circumstances hindered international trade, disrupted exchange rates and put undue stress on otherwise stable financial institutions in other countries. To alleviate this situation, the Federal Reserve setup swap lines arrangements with other central banks to provide U.S. dollar liquidity.[1] With the newly acquired U.S. dollars, the recipient central bank conducted repo arrangements with local banks for which it required high-quality domestic assets as collateral. This type of policy can be considered as an international unconventional monetary policy as it is conducted in cooperation with a foreign central bank.

Given how swaps lines inject U.S. dollars in local economies, we conjecture that swap policies can also help/hinder the conduct of domestic monetary policies by the recipient central bank. This is the case as swap lines can affect the demand for assets, denominated in another currency. This aspect is even more relevant when the other central bank engages in quantitative easing policies. This type of domestic unconventional monetary policy increases the money supply and lowers some interest rates on public debt, by directly affecting the demand for certain nominal public assets. Thus, by accessing U.S. dollar swap lines, the Federal Reserve can indirectly support the other central bank's quantitative easing policies by increasing liquidity in the local economy. This is the focus of this paper.

To explore such links, we propose a framework to study the macroeconomic implications of the U.S. dollar swap arrangements between a central bank of a small open economy and the Federal Reserve System. In particular, we incorporate the salient features of such agreements between central banks into a dynamic general equilibrium framework where domestic and foreign fiat money is valued. Within this environment, we then study how the U.S. dollar swap lines interact with domestic quantitative easing policies and ultimately impact inflation and debt dynamics in a small open economy. In our framework, swap lines are essential in expanding the private consumption possibilities when

[1]Bahaj and Reis (2022a) provide a detailed account of these arrangements, including their history, institutional features, and lessons for future research, policymakers, and practitioners.

market disruptions prevent private access to repo agreements in exchange for foreign currency or, when there is access, but such agreements provide low collateral value.

The characterization of the monetary equilibrium of our economy depends on whether households are satiated or not and in which states of the world. Analytically, we find that when domestic bonds do not exhibit premia and the consumption of foreign goods is satiated, different terms of trade in the swap lines (different values of δ) do not change the resulting stationary nor dynamic equilibria. Standard monetary and fiscal policy prescriptions deliver locally determinate equilibria. Moreover, we also find that quantitative easing policies that change the real value of total bonds in the hands of the public have an effect on allocations, but not on inflation.

This is sharp contrast when domestic bonds carry premia and/or the consumption of foreign goods is not satiated. In those cases, inflation depends on the details of the swap arrangements and bond and inflation dynamics are affected by both swaps and quantitative easing policies. Within this framework, we characterize the circumstances under which different combinations of U.S. dollar swap lines and domestic quantitative easing as well as interest rate management policies lead to observationally equivalent stationary equilibria in the small open economy. But even though the unique steady state is the same, those policies lead to different local dynamics and stability properties. In particular, we find that swaps and quantitative easing dampen the fiscal eigenvalue. Moreover, monetary policies that lead to a more favorable swap line, require that the interest policy setting responds more (less) aggressively to inflation in order to deliver the same steady state whenever monetary policy is active (passive). Thus, traditional prescriptions for monetary and fiscal policy to stabilize the economy are not operative in this environment.

To quantify the macroeconomic impact of swaps on conventional and unconventional monetary policies and their interaction, we resort to numerical analysis. To provide some discipline when deciding the parameter values describing the small open economy, we consider Australia's macroeconomic data as well as issuance of public debt and Reserve Bank of Australia (RBA) asset holdings for the period of 1993-2019. In addition, monetary and fiscal policies are then adjusted to reflect the economic conditions both at the beginning of and during the covid-19 pandemic.

We then explore how conventional and domestic and international unconventional monetary policies may be adjusted to deliver same stationary equilibria. At the beginning of the pandemic, with an active monetary policy, we obtain that a more favorable swap line would have required the RBA to make interest rates respond stronger to inflation and to provide less liquidity through quantitative easing policies. Indeed, we find that to deliver the same steady state, bond purchases programs were not required. During the pandemic, with a passive monetary policy, with find that a more favorable swap line would make the RBA adjust its policy setting by making interest rates respond less to inflation but, as before, to provide less liquidity through balance sheet policies. In particular, it would have allowed the RBA to cut back on long-term bond purchases from 35 % to 24 % of GDP.

Finally, we uncover international and domestic unconventional monetary policies that deliver equilibria. Analytically, we find that swaps and quantitative easing policies dampen the fiscal eigenvalue, while they don't affect the monetary one. With an active monetary policy, such a reduction in the fiscal eigenvalue supports determinacy of equilibria. However, our numerical results show that, with a passive monetary policy, swaps and quantitative can lead to indeterminacy. Further numerical results also show that swaps also lead to a portfolio re-adjustment that can reduce the efficacy of quantitative easing policies.

This paper is organized as follows. Section 2 reviews the literature on U.S. dollar swaps. Section 3 presents the theoretical framework and optimal decisions. Section 4 shows and characterizes the dynamic equilibria. Section 5 provides a numerical exercise. Finally, Section 6 concludes.

2 Literature Review

This paper makes a contribution to the emerging literature on U.S. dollar swaps by developing a theoretical framework to study the macroeconomic consequences of U.S. dollar swap lines and how they interact with other operating procedures for monetary policy. Most existing papers in the literature have empirically estimated the impact of U.S. dollar swap lines on the economies that received foreign liquidity. In contrast, so far, there is limited theoretical analysis to examine the broader macroeconomic implications of these policies. Therefore, this paper fills a gap in the literature by providing a comprehensive framework for analyzing the effects of U.S. dollar swap lines on inflation and debt dynamics for a small open economy, as well as their interactions with domestic quantitative easing policies.

Scholars have traced the origins of U.S. dollar swaps to the late 1960s, during the breakdown of the Bretton Woods system of fixed exchange rates. However, it was in the 1980s and 1990s that U.S. dollar swaps became more prevalent, as global financial markets became increasingly interconnected. McCauley et al. (2020) provide more details on the history of swaps, including their evolution over time and their role in facilitating international trade and investment.

Within the empirical literature, Rose and Spiegel (2012) conduct a study on the auctions of dollar assets by foreign central banks. The authors find robust evidence that these auctions disproportionately benefited countries that were more exposed to the United States through trade linkages or asset exposure. However, the authors find weaker results for differences in asset transparency or illiquidity. Interestingly, the study also finds that several important announcements regarding the international swap programs disproportionately benefited countries with greater asset opaqueness. These findings suggest that U.S. dollar swap lines can have differential effects on countries depending on their exposure to the United States and the transparency of their assets. Within the same spirit, Cetorelli and Goldberg (2012) study the role of global banks in transmitting financial shocks across borders during the global financial crisis, with a focus on the use of U.S. dollar swap lines. The authors find that policy interventions played a crucial role in influencing the lending channel effects on emerging markets of head office balance sheet shocks. Additionally, the study finds that exposure to international funding was not the main vehicle of propagation. Instead, it was the exposure to international funding from source country banking systems that were more likely to suffer from the liquidity shock.

After covid-19 there has been renewed interest in U.S. dollar swap lines. In a series of seminal papers, Bahaj and Reis (2022a, 2022b) argue that swap lines provide an alternative to discount-window lending by the source central bank to recipient-country banks, with the recipient central bank acting as an agent that assumes the credit risk. Thus, swap lines are consistent with controlling inflation and the lender-of-last-resort role, and are not directly linked to intervening in exchange rates, bailing out or transferring wealth to foreigners, or nationalizing private risk. The authors explain why these arrangements were necessary in addition to the traditional discount window or the purchase of securities in private markets. In a related study, Bahaj and Reis (2022b) demonstrate that with global banks and integrated private financial markets, the lending-of-last-resort function can be achieved through swap lines. The authors show that swap lines establish a cap on deviations from covered interest parity, reduce average ex-post bank borrowing costs, and increase ex-ante inflows from recipient-country banks into privately issued assets denominated in the source country's currency. Empirically, the authors find that the international lender-of-last-resort function is highly effective.

The paper closest to ours is that of Cesa-Bianchi et al. (2023) who provide a theoretical framework to understand the macroeconomic consequences of swap line policies. To do so, the authors consider a two-country New Keynesian model with financial frictions, where the foreign country is the United States and the home country is a small open economy. The model incorporates frictions between depositors and banks, as in the Gertler-Karadi-Kiyotaki framework, and assumes that domestic banks borrow both at home and abroad to finance capital purchases. The agency frictions are more severe when borrowing abroad in U.S. dollars, leading to endogenous deviations from uncovered interest rate parity related to the tightness of domestic banks' balance sheets. When introducing a swap policy rule as in Del Negro et al. (2017), the authors find that U.S. dollar swap lines mitigate the adverse macro-financial effects of dollar shortage shocks.

3 Economic Environment

To study the macroeconomic implications of U.S. dollar swap lines, we incorporate the salient features of these arrangements into a dynamic general equilibrium framework. Before doing so, we provide a detailed description on how swap lines work.

U.S. Dollar Swap Lines

U.S. dollar swap lines are arrangements between the Federal Reserve and other central banks. When a swap line is active, the central bank in the United States provides U.S. dollars to another central bank in exchange for an equivalent amount in the other central bank's currency at the current spot exchange rate. The two central banks agree to re-sell their respective currencies back to each other after a fixed period of time, usually one week or three months, at the same spot exchange rate as the initial exchange.[2]

Once the recipient central bank receives the U.S. dollars it then conducts repo arrangements with its domestic counter-parties, typically local banks. When doing so, the local central bank usually requires local currency-denominated collateral that is eligible in its usual domestic liquidity operations. By accessing these repo arrangements, private banks are able to obtain U.S. dollars at a more favorable exchange rate when compared to what they would typically get in the foreign exchange market. This arrangement provides additional liquidity to the recipient central bank helping support the stability of the local financial system.

Model

The small open economy builds on the frictional and incomplete-market framework of Gomis-Porqueras et al. (2013), which is based on Lagos and Wright (2005) and Rocheteau and Wright (2005).[3] Time is discrete and agents discount future time periods at a rate $\beta \in (0, 1)$. Each period is divided into two sub-periods. Agents trade sequentially in various markets within each sub-period. These markets are characterized by different frictions and trading protocols. The first one corresponds to a decentralized goods market, denoted DM, where agents face limited commitment, asset recognizability problems and limited access to foreign markets. The second one is a competitive goods and financial markets, which we refer as CM, where agents can consume, adjust their portfolio and trade with the rest of the world. From now on, we refer the small open economy as home or domestic and the rest of the world as foreign.

At the beginning of the first sub-period, agents face stochastic trading opportunities. As in Rocheteau and Wright (2005), agents in the small open economy are of fixed types: buyers and sellers. In particular, we consider a unit measure of buyers and a measure ψ of sellers. Domestic buyers receive a trading shock that determines who they will trade with in DM; i.e., domestic or

[2]The Federal Reserve charges an interest rate on the U.S. dollars it provides, which is set as a spread relative to its policy rate, paid at the fixed term later, and settled in U.S. dollars.

[3]Other open search theoretic models of money include Zhang (2014), Geromichalos et al. (2014, 2018) among others.

foreign sellers. After shocks are realized, DM domestic buyers are bilaterally matched with DM domestic or foreign producers. As in Rocheteau and Wright (2005), DM buyers are able to derive utility from consuming the DM good, but can not produce it. Sellers, on the other hand, can produce but do not obtain utility from consuming DM goods.

Other than search frictions, agents also face limited commitment when trading in DM. As a result, domestic DM producers do not provide unsecured credit. To obtain DM credit, domestic buyers are required to post collateral.[4] However, not all domestic DM producers accept the same assets as collateral. Moreover, not all assets have the same pledgeability properties. More precisely, DM foreign producers do not accept any domestic assets as collateral. In contrast, some domestic DM sellers accept all assets as collateral. Moreover, domestic (foreign) DM goods can always be bought with domestic (foreign) currency.

In the last sub-period both domestic buyers and sellers can produce an homogeneous good using labor as the only input. These agents derive utility from consuming the homogeneous good and disutility from CM effort. Domestic buyers and sellers trade such good in competitive in domestic and international markets. Agents can also rebalance their portfolios and have access to the foreign exchange market.[5]

Preferences: The representative buyer derives utility from DM and CM consumption and disutility from CM labor. Let q_t (q_t^F) denote DM domestic (foreign) output, while $X_{b,t}$ and $H_{b,t}$ represent consumption of the final good and labor in CM, respectively. Their expected utility is then given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[u(q_t, q_t^F) + \ln(X_{b,t}) - H_{b,t} \right], \tag{1}$$

where $\mathbb{E}_0$ denotes the linear expectation operator with respect to an equilibrium distribution of trading shocks. The utility function $u(.,.)$ is assumed to satisfy standard assumptions and to be

[4]We refer to Kiyotaki and Moore (1997) for more on the need to collateralize loans.

[5]Implicit in our environment is that in CM agents can access the foreign exchange market through intermediaries that have access to a perfectly competitive inter-bank market. We refer to Geromichalos et al. (2018) for an environment where agents have direct access to an over-the-counter foreign exchange market and face matching and bargaining frictions.

separable across both DM domestic and foreign goods, i.e. $u_{12} = u_{21} = 0$.[6] Similarly, the domestic seller derives disutility from DM and CM labor, while obtaining positive payoffs from consuming CM output. Their expected utility is then given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \, \left[-q_t + X_{s,t} - H_{s,t}\right], \tag{2}$$

where $X_{s,t}$ and $H_{s,t}$ represent the seller's CM consumption and effort, respectively.

Technologies: In DM only sellers are able to produce domestic DM goods with a linear technology where labor is the only input. The production function is such that one unit of labor yields one unit of output. Similarly, the CM final good is given by the linear technology $Y_t = H_t$, where H_t is total CM labor from both DM buyers and sellers. Implicitly, this normalizes labor productivity to one.

Assets: Agents in the small open economy have access to domestic fiat money M_t, domestic nominal one period bonds B_t^S, and domestic long-term bonds B_t^L. Agents have also access to foreign fiat money M_t^F. All these assets are issued by their respective central and fiscal authorities in CM. It is in this market that agents can rebalance their portfolio.

Frictions: In the first sub-period, agents face stochastic trading opportunities, limited commitment and asset recognizability problems. In particular, with probability $\sigma_F \in [0, 1]$ domestic DM consumers (buyers) trade in a competitive goods market with foreign sellers. With complementary probability, DM domestic buyers and domestic DM sellers are bilaterally matched. From now on, we assume $1 - \sigma_F = \psi$. Thus, given these assumptions, there is always a DM trading opportunity.

When trading with foreign DM sellers, foreign fiat money is the only acceptable means of payment in exchange for the foreign DM good, q_t^F. When trading domestically, to consume the DM local good, q_t, buyers can promise the DM seller its payment in the next CM. However, due to limited commitment, the DM buyer can renege on his promise. This possibility allows assets to have a role as collateral. The usual interpretation of such arrangement is that if a borrower reneges on his promise,

[6] Note that u_1 denotes the derivative of u with respect to the first element q, other derivatives follow a similar notation.

his assets are seized. This contingency dissuades opportunistic default. Note, however that as in Rocheteau et al. (2018), one can also describe DM trade as a repurchase agreement, where a buyer that obtains q_t gives assets to a seller, who gives them back at prearranged terms in the next CM.[7] In this paper we take such view. Moreover, we assume that not all assets are equally pledgeable when trading in DM.[8] In particular, we assume that only a fraction η^S, η^L, and η^F of short-term bonds, long-term bonds and foreign currency is pledgeable, respectively. Furthermore, not all DM producers accept collateral. With probability $\tilde{\mu}_M$, a DM consumer is in a meeting where only domestic fiat money can be used as a medium of exchange and no collateral is accepted. Finally, with probability $\tilde{\mu}_C = 1 - \tilde{\mu}_M$, a DM consumer is in a meeting with a seller where all nominal bonds and foreign currency can be used as collateral. To simplify notation, from now on we denote $\mu_j \equiv (1 - \sigma_F)\tilde{\mu}_j$ for $j \in \{M, C\}$.

Fiscal Authority: The government needs to finance an exogenous, and strictly positive constant stream of expenditures, which we denote by G, and outstanding debt interest payments. To finance them, the fiscal authority has access to lump-sum CM taxes, τ_t, the transfer from the central bank to the fiscal authority T_t^C, and the issuance of short B_t^S and long-term nominal bonds B_t^L. The corresponding budget constraint for the fiscal authority is then given by

$$\tau_t + \phi_t B_t^S + Q_t \phi_t B_t^L + T_t^C = G + R_{t-1} \phi_t B_{t-1}^S + (1 + \rho Q_t)\, \phi_t B_{t-1}^L, \tag{3}$$

where $\phi_t \equiv \frac{1}{P_t}$ is the real price of the CM good. R_t represents the nominal interest rate corresponding to short-term (one-period) public debt purchased at time t.[9] Following Woodford (2001), we model the long-term bond B_t^L as having a nominal payment structure equal to $\rho^{T-(t+1)}$, where $T > t$ and $0 < \rho < 1$, which can be interpreted as a portfolio of infinitely many nominal bonds, with weights

[7] As in Rocheteau et al. (2018), we do not propose a deep theory of repurchase agreements (repos). We refer to Antinolfi et al. (2015) or Gottardi et al. (2015) for more on repos.

[8] We refer to as in Rocheteau et al. (2018) and Dong et al. (2019), Domínguez and Gomis-Porqueras (2022), among others, for more on this type of assumption. This is consistent with the United States repo market where the majority of trades involve three months or less. We refer the reader to the Repurchase Agreement Operational Details for additional information, which can be found at Federal Reserve Bank of New York.

[9] It is worth noting that while the initial portfolio is the same across agents, secured loans, $\tilde{L}_{t-1}^j$ and domestic fiat money, may be different across agents. This is the case as agents in DM face different trading partners that may or may not accept collateral.

along the maturity structure given by $\rho^{T-(t+1)}$.[10] The price of these bonds is denoted by Q_t. The real value of all bond issuance is denoted by $\phi_t B_t = \phi_t \left(B_t^S + Q_t B_t^L\right)$ and we assume that it is bounded above by a sufficiently large constant to avoid Ponzi schemes. Furthermore, to describe the specific operating procedures for fiscal policy, we assume that taxes respond to previously issued public debt as follows

$$\tau_t^{CM} = \gamma_0 + \gamma^S \left(\phi_{t-1} B_{t-1}^S - b^{S*}\right) + \gamma^L \left(\phi_{t-1} Q_{t-1} B_{t-1}^L - b^{L*}\right), \tag{4}$$

where γ_0 determines how taxes are set regardless of the economy's debt structure and γ^S $\left(\gamma^L\right)$ captures how taxes respond to the level of short-term (long-term) real government bonds.[11] In addition, b^{S*} and b^{L*} represent the real target levels for short and long-term real public debt, respectively. From now on, an asterisk on a variable will denote the corresponding policy target. Finally, in terms of debt composition, we assume that the debt issuance of the fiscal authority is such that there is a constant ratio between the nominal value of short and long-term bonds, that is $\Omega \equiv \frac{B_t^S}{Q_t B_t^L}$.

Central Bank: This institution manages interest rates through a Taylor rule. In particular, the central bank implements monetary policy as follows

$$R_t = \alpha_0 + \alpha_1 \left(\Pi_t - \Pi^*\right), \tag{5}$$

where $\Pi_{t+1} = \frac{\phi_t}{\phi_{t+1}}$ denotes the gross domestic inflation rate, α_0 is a constant that determines how interest rates are set regardless of the economy's inflation rate, while α_1 captures how interest rates respond to inflation rate departures from its target $\Pi^* \equiv \beta\alpha_0$. When managing interest rates, we might assume that the central bank satisfies (does not satisfy) the Taylor principle $\alpha_1 > \frac{1}{\beta}$ ($\alpha_1 < \frac{1}{\beta}$). Note that the in order to implement the Taylor rule, the central bank is conducting open market operations.

When the domestic central bank has access to a swap line, it can borrow foreign currency from a foreign central bank. These additional units of foreign currency can then be lent out to DM

[10] In this case, one-period debt corresponds to $\rho = 0$, while a consol bond is consistent with $\rho = 1$.
[11] This specification is as in Leeper (1991), Woodford (1994), Sims (1994), among others.

consumers. For instance, the relevant borrowing constraint for DM buyers is given by

$$\tilde{S}_{t-1} \leq \frac{e_t^s}{e_t}\eta^{CB}\ \tilde{B}^S_{b,t-1},$$

where $\tilde{S}_{t-1}$ is the domestic central bank loan to DM consumers in domestic currency, e_t^s is the exchange rate that the local central bank offers to agents, which is the same exchange rate at which the local central bank accesses the swap line, and e_t is the market exchange rate (which measures the value of one unit of foreign currency (F) in units of the home currency). Finally, η^{CB} is the pledgeability required by the central bank, in terms of domestic short-term bonds, when accessing the swap lines. The swap exchange rate is given by

$$e_t^s = \delta\ e_t,$$

where $\delta \geq 1$ reflects the ability of the central bank to access cheaper foreign currency. This is the case as it has direct access to a swap line with the foreign central bank. By doing so it expands the DM consumption possibilities when foreign currency is accepted as payment or as collateral.[12] For short, we refer to δ as the swap line rate.

The central bank buys both short and long-term government bonds in CM. In particular, the monetary authority holds a fraction $1-\theta^S$ $(1-\theta^L)$ of the short-term (long-term) bonds issued by the fiscal authority.[13] Note that we can define the composition of bonds held by the monetary authority as follows $\Omega^M = \frac{\left(1-\theta^S\right)b_t^S}{(1-\theta^L)Q_t b_t^L} = \frac{\left(1-\theta^S\right)}{(1-\theta^L)}\Omega$. By changing the central bank holdings of domestic government bonds, the monetary authority can affect the amount and liquidity of the bonds available to households.

[12] The amount accessible through the swap line is limited at various amounts and for a number of countries. In the case of the Australia, the swap line allows the Reserve Bank of Australia to access up to US$60 billion from the Federal Reserve System in exchange for Australian dollars. The U.S. dollars are made available to Reserve Bank Information and Transfer System members via repurchase agreements (repos) contracted with the Reserve Bank of Australia. These U.S. dollar repos are against Australian-dollar denominated securities. In our model, we don't assume an upper limit on the amount of U.S. dollars that the domestic central bank can access through the swap line.

[13] Note then that θ^S (θ^L) correspond to the proportions that are held by households. These fractions are assumed constant for ease of exposition, but we will consider changes to those to reflect the implementation of QE policies.

The corresponding budget constraint for the monetary authority is then given by

$$T_t^C + \left(1-\theta^S\right)\phi_t B_t^S + \left(1-\theta^S\right) Q_t \phi_t B_t^L + \phi_t M_{t-1} + T_t^S = \phi_t M_t + R_{t-1}\left(1-\theta^S\right)\phi_t B_{t-1}^S + \tag{6}$$

$$+\left(1-\theta^L\right)\left(1+\rho Q_t\right)\phi_t B_{t-1}^L + T_t^S$$

where T_t^C is the transfer that the central bank provides to the fiscal authority and T_t^S is both the cost and the revenue from using the swap line. As we can see from this formulation, no profits or losses are made by the central bank when accessing the swap line. In the event that households can benefit from the swap lines up to their individual borrowing limit in all states of the world, we then have $T_t^S = \phi_t(\sigma_F + \mu^C)\tilde{S}_t^F$.

Note that DM buyers hold a fraction θ^S (θ^L) of short-term (long-term) government bonds. That is, after CM, $\tilde{B}_{b,t}^S = \theta^S B_t^S$ and $\tilde{B}_{b,t}^L = \theta^L B_t^L$. The total real value of bonds held by the public is given by $b_t^H = \theta^S b_t^S + \theta^L Q_t b_t^L$. Let us define $\theta \equiv \left[\frac{\Omega\theta^S+\theta^L}{1+\Omega}\right]$. It is easy to check then that $b_t^H = \theta b_t$, while the holdings of the central bank are $b_t^M = (1-\theta)b_t$. Thus, the composition of bonds held by households is given by $\Omega^H = \frac{\theta^S b_t^S}{\theta^L Q_t b_t^L} = \frac{\theta^S}{\theta^L}\Omega$, while that of the central bank is $\Omega^M = \frac{1-\theta^S}{1-\theta^L}\Omega$.

It is important to highlight that, for a given composition of the debt issuance by the fiscal authority, Ω, the central bank can implement domestic unconventional monetary policies such as quantitative easing (QE). These policies can change the level and the composition of public debt in the hands of the public. In what follows we explore two types or effects of QE policies.

1. QE-level: The central bank changes its holdings of short-term bonds, $1-\theta^S$, and of long-term bonds, $1-\theta^L$, such that $\frac{\theta^S}{\theta^L}$ remains constant. This action changes the fraction of the total holdings of public bonds in the hands of the household, denoted by $\theta = \frac{\Omega\theta^S+\theta^L}{1+\Omega}$, while it maintains constant its liquidity composition $\Omega^H = \frac{\theta^S}{\theta^L}\Omega$ by not changing the maturity structure.

2. QE-composition: The central bank decreases its holdings of short-term bonds, $1-\theta^S$, and increases its holdings of long-term bonds, $1-\theta^L$, such that $\theta = \frac{\Omega\theta^S+\theta^L}{1+\Omega}$ remains constant. This action leaves the fraction of the total holdings of public bonds in the hands of the household, θ, constant while it increases its liquidity composition Ω^H by reducing its maturity.

Studying these two forms of QE allows us to isolate the different effects implied by this domestic unconventional monetary policy. In addition, one can design a comprehensive QE policy by allowing for both changes in the fraction as well as the liquidity of the total holdings of public bonds in the hands of the household, that is, θ and Ω^H.

Optimal Decisions

Given the sequential nature of the environment, we solve the representative agent's problem backwards. Thus, we first solve the CM and then the DM problems, respectively.

CM Problem

In this market, buyers and sellers can produce and consume the CM good, while trading in competitive domestic and foreign markets. At the beginning of CM and given the holdings of domestic fiat money, foreign currency, domestic nominal government bonds, and outstanding secured loans or repos, which we denote by $\tilde{A}_{t-1} \equiv \left(\tilde{M}_{t-1}, \tilde{M}^F_{t-1}, \tilde{B}^S_{t-1}, \tilde{B}^L_{t-1}, \tilde{L}^C_{t-1}, \tilde{S}^F_{t-1}, \tilde{E}^F_{t-1}, \tilde{S}^C_{t-1}\right)$, the problem of a representative buyer is as follows

$$W_{b,t}\left(\tilde{A}_{t-1}\right) = \max_{X_{b,t}, H_{b,t}, \tilde{M}_{b,t}, \tilde{M}^F_{b,t}, \tilde{B}^S_{b,t}, \tilde{B}^L_{b,t}} \left\{ \ln(X_{b,t}) - H_{b,t} + \beta\, V^{DM}_{b,t+1}\left(\tilde{\mathcal{A}}_{b,t}\right)\right\} \quad \text{s.t.} \tag{7}$$

$$X_{b,t} + \phi_t\left(\tilde{M}_{b,t} + e_t\tilde{M}^F_{b,t} + \tilde{B}^S_{b,t} + Q_t\tilde{B}^L_{b,t}\right) + \phi_t\left(\tilde{L}^C_{t-1} + \tilde{S}^F_{t-1} + \tilde{E}^F_{t-1} + \tilde{S}^C_{t-1}\right) = H_{b,t} - \tau_t$$
$$+\phi_t\left(\tilde{M}_{b,t-1} + e_t\tilde{M}^F_{b,t-1}\right) + \phi_t R_{t-1}\tilde{B}^S_{b,t-1} + \phi_t(1+\rho Q_t)\tilde{B}^L_{b,t-1},$$

where $V^{DM}_b(\cdot)$ is the agent's expected DM value function, $\tilde{\mathcal{A}}_{b,t} = (\tilde{M}_{b,t}, \tilde{M}^F_{b,t}, \tilde{B}^S_{b,t}, \tilde{B}^L_{b,t})$ are the nominal assets when entering in DM, τ_t are CM lump sum taxes, $\tilde{L}^C_{t-1}$ represents the nominal payment of an agent that was granted a DM secured loan in state C where all assets (all bonds and foreign assets) are accepted as collateral. Note that in this state it also allows foreign currency received through a swap line to be used as collateral. We assume the CM good is homogeneous and can be traded in domestic and international competitive markets. Then, the law of one price holds, i.e. $P_t = e_t P^F_t$, where P^F_t is the foreign currency price of the CM good. We can rewrite this as $\phi^F_t = e_t\phi_t$.

The corresponding first-order conditions are given by

$$\frac{1}{X_{b,t}} - 1 = 0, \tag{8}$$

$$-\phi_t + \beta \, \frac{\partial V_b^{DM}\left(\tilde{A}_{b,t}\right)}{\partial \tilde{M}_t} = 0, \tag{9}$$

$$-\phi_t e_t + \beta \, \frac{\partial V_b^{DM}\left(\tilde{A}_{b,t}\right)}{\partial \tilde{M}_t^F} = 0, \tag{10}$$

$$-\phi_t + \beta \, \frac{\partial V_b^{DM}\left(\tilde{A}_{b,t}\right)}{\partial \tilde{B}_t^S} = 0, \tag{11}$$

$$-\phi_t Q_t + \beta \, \frac{\partial V_b^{DM}\left(\tilde{A}_{b,t}\right)}{\partial \tilde{B}_t^L} = 0. \tag{12}$$

The associated envelope conditions are $\frac{\partial W_b}{\partial \tilde{B}_{t-1}^S} = \phi_t R_{t-1}$, $\frac{\partial W_b}{\partial \tilde{B}_{t-1}^L} = \phi_t \left(1 + \rho Q_t\right)$, $\frac{\partial W_b}{\partial \tilde{L}_{t-1}^C} = -\phi_t$, and $\frac{\partial W_b}{\partial \tilde{S}_{t-1}^F} = -\phi_t$.

Similarly, for the domestic seller, we have that the CM problem is given by

$$W_{s,t}\left(\tilde{A}_{s,t-1}\right) = \max_{X_{s,t}, H_{s,t}, \tilde{M}_{s,t}, \tilde{M}_{s,t}^F, \tilde{B}_{s,t}^S, \tilde{B}_{s,t}^L} \left\{ X_{s,t} - H_{s,t} + \ \beta \, V_{s,t+1}^{DM}\left(\tilde{\mathcal{A}}_{s,t}\right)\right\} \quad \text{s.t.} \tag{13}$$

$$X_{s,t} + \phi_t\left(\tilde{M}_{s,t} + e_t \tilde{M}_{s,t}^F + \tilde{B}_{s,t}^S + Q_t \tilde{B}_t^L\right) - \phi_t\left(\tilde{L}_{t-1}^C + \tilde{S}_{t-1}^F + \tilde{E}_{t-1}^F + \tilde{S}_{t-1}^C\right) = H_{s,t} +$$

$$+ \phi_t\left(\tilde{M}_{s,t-1} + e_t \tilde{M}_{s,t-1}^F\right) + \phi_t R_{t-1} \tilde{B}_{s,t-1}^S + \phi_t (1 + \rho Q_t) \tilde{B}_{s,t-1}^L,$$

where the subscript s refers to the domestic seller. Since sellers do not need fiat money to consume, they will not hold any currency whenever is costly to hold. In addition, given sellers' linear preferences, when assets are priced fundamentally, they are indifferent between purchasing them or not. From now on, we assume that sellers do not hold any assets. Note that when assets have a premia they are priced above their fundamental value. In such circumstances, sellers optimally decide not to hold any assets as for DM sellers they are only useful as a savings instrument.

DM Problem

At the beginning of DM, domestic buyers receive a preference shock that determines whether they will trade with domestic or foreign DM sellers. After preference shocks are realized, DM domestic buyers are matched with either DM domestic or foreign sellers. Matches with DM domestic sellers are bilateral, while with DM foreign sellers are multilateral. Thus, before shocks are realized, the corresponding DM value function of a buyer with portfolio $\tilde{\mathcal{A}_{b,t}}$ is given by

$$V_b^{DM}\left(\tilde{\mathcal{A}}_{b,t}\right) = \sigma_F\left[u(0, q_{t+1}^F) + W_b\left(\tilde{M}_{b,t}, \tilde{M}_{b,t}^F + \frac{\tilde{S}_{b,t}^F + \tilde{E}_{b,t}^F}{e_{t+1}} - D_{t+1}^F, \tilde{B}_{b,t}^S, \tilde{B}_{b,t}^L, 0, \tilde{S}_{b,t}^F, \tilde{E}_{b,t}^F, 0\right)\right]$$

$$+\mu_M\left[u(q_{t+1}^M, 0) + W_b\left(\tilde{M}_{b,t} - D_{t+1}^M, \tilde{M}_{b,t}^F, \tilde{B}_{b,t}^S, \tilde{B}_{b,t}^L, 0, 0, 0, 0\right)\right]$$

$$+\mu_C\left[u(q_{t+1}^C, 0) + W_b\left(\tilde{M}_{b,t} - D_{t+1}^C, \tilde{M}_{b,t}^F, \tilde{B}_{b,t}^S, \tilde{B}_{b,t}^L, \tilde{L}_t^C, 0, 0, \tilde{S}_{b,t}^C\right)\right],$$

where q^j represents the quantity of DM goods purchased in the j state of the world, where $j = \{M, C\}$, while D^j denotes the corresponding domestic currency payment. Similarly, q^F and D^F correspond to the foreign good and the corresponding payment of foreign currency. Because of limited commitment (agents can renege on their future payments), in order to trade, DM consumers use their assets as collateral.[14] For instance, the amount of credit extended in state C is such that $\tilde{L}_t^C \leq \eta^S \tilde{B}_{b,t}^S + \eta^L Q_t \tilde{B}_{b,t}^L + \eta^F e_t \tilde{M}_{b,t}^F$. In addition, swap lines can expand the DM consumption possibilities through $\tilde{S}_{b,t}^j \leq \frac{e_t^s}{e_t}\eta^{CB}\tilde{B}_{b,t}^S$, in $j \equiv \{F, C\}$. Here F refers to trades with foreign DM sellers while C to trades with domestic DM sellers that accept collateral. This captures the idea that access to a swap line expands the U.S. dollars available, but does not restrict how individuals will use those U.S. dollars. They might be used to conduct international purchases and/or as collateral in domestic exchanges.

Similarly, at the beginning of DM, the corresponding DM value function of a seller with portfolio

[14] As in Kiyotaki and Moore (1997), Bernanke et al. (1999), Iacoviello (2005), Andolfatto and Martín (2018) and Berentsen and Waller (2018), among others, because of limited commitment, the loan extended to DM consumers has to be collaterized.

$\tilde{\mathcal{A}}_{s,t}$ is given by

$$V_{s,t}^{DM}\left(\tilde{\mathcal{A}}_{s,t}\right) = \mu_M\left[-q_{t+1}^M + W_{s,t}\left(\tilde{M}_{s,t} + D_{t+1}^M, \tilde{M}_{s,t}^F, \tilde{B}_{s,t}^S, \tilde{B}_{s,t}^L, 0, 0, 0, 0\right)\right]$$

$$+\mu_C\left[-q_{t+1}^C + W_{s,t}\left(\tilde{M}_{s,t} + D_{t+1}^C, \tilde{M}_{s,t}^F, \tilde{B}_{s,t}^S, \tilde{B}_{s,t}^L, 0, -\tilde{L}_t^C, 0, 0, -\tilde{S}_{b,t}^C\right)\right].$$

Terms of Trade

The trading protocol in the domestic frictional market is determined ex-post by a DM buyer take-it-or-leave-it offer with threat point of no trade. Depending in which state of the world the domestic buyer and the seller trade, the DM buyer will be able to use some assets as collateral. In order to induce trade, a domestic DM buyer needs to propose terms of trade that satisfy the DM producer's participation constraint and be consistent with his borrowing constraint.

Formally, when a DM buyer and domestic producer meet in a state of the world where no collateral is acceptable, the terms of trade solve the following problem

$$\max_{q_{t+1}^M, D_{t+1}^M}\left\{u(q_{t+1}^M, 0) + W_b\left(\tilde{M}_{b,t} - D_{t+1}^M, \tilde{M}_{b,t}^F, \tilde{B}_{b,t}^S, \tilde{B}_{b,t}^L, 0, 0, 0, 0\right) - W_b\left(\tilde{M}_{b,t}, \tilde{M}_{b,t}^F, \tilde{B}_{b,t}^S, \tilde{B}_{b,t}^L, 0, 0, 0, 0\right)\right\} \text{ s.t.}$$
$$-q_{t+1}^M + W_s\left(\tilde{M}_{s,t} + D_{t+1}^M, \tilde{M}_{s,t}^F, \tilde{B}_{s,t}^S, \tilde{B}_{s,t}^L, 0, 0, 0, 0\right) \geq W_s\left(\tilde{M}_{s,t}, \tilde{M}_{s,t}^F, \tilde{B}_{s,t}^S, \tilde{B}_{s,t}^L, 0, 0, 0, 0\right).$$

The problem can be further simplified as follows

$$\max_{q_{t+1}^M}\quad\left\{u(q_{t+1}^M, 0) - q_{t+1}^M\right\} \text{ s.t. } q_{t+1}^M \leq \phi_{t+1}\tilde{M}_{b,t},$$

delivering the following first-order conditions

$$u_1(q_{t+1}^M, 0) - 1 - \lambda_{t+1}^M = 0,$$
$$\lambda_{t+1}^M\left(\phi_{t+1}\tilde{M}_{b,t} - q_{t+1}^M\right) = 0.$$

When all bonds and foreign currency can be used as collateral, the terms of trade solve the following problem

$$\max_{q^C_{t+1}, D^C_{t+1}, \tilde{L}^C_t, \omega_{t+1}} \left\{ u(q^C_{t+1}, 0) + W_b \left(\tilde{M}_{b,t} - D^C_{t+1}, \tilde{M}^F_{b,t}, \tilde{B}^S_{b,t}, \tilde{B}^L_{b,t}, \tilde{L}^C_t, 0, 0, \tilde{S}^C_{b,t} \right) - W_b \left(\tilde{M}_{b,t}, \tilde{M}^F_{b,t}, \tilde{B}^S_{b,t}, \tilde{B}^L_{b,t}, 0, 0, 0, 0 \right) \right\} \text{ s.t.}$$

$$\tilde{L}^C_t \leq \eta^S \omega_{t+1} \tilde{B}^S_{b,t} + \eta^L Q_t \tilde{B}^L_{b,t} + \eta^F e_{t+1} \tilde{M}^F_{b,t},$$

$$\tilde{S}^F_{b,t} \leq \eta^F \frac{e^s_{t+1}}{e_{t+1}} \eta^{CB} (1 - \omega_{t+1}) \tilde{B}^S_{b,t},$$

$$-q^C_{t+1} + W_s \left(\tilde{M}_{s,t} + D^C_{t+1}, \tilde{M}^F_{s,t}, \tilde{B}^S_{s,t}, \tilde{B}^L_{b,t}, -\tilde{L}^C_t, 0, 0, -\tilde{S}^C_{b,t} \right) \geq W_s \left(\tilde{M}_{s,t}, \tilde{M}^F_{s,t}, \tilde{B}^S_{s,t}, \tilde{B}^S_{b,t}, 0, 0, 0, 0 \right),$$

which can be further simplified as follows

$$\max_{q^C_{t+1}, \omega_{t+1}} \left\{ u(q^C_{t+1}, 0) - q^C_{t+1} \right\} \text{ s.t. } q^C_{t+1} \leq \phi_{t+1} \left(\tilde{M}_{b,t} + \eta^S \omega_{t+1} \tilde{B}^S_{b,t} + \eta^L Q_t \tilde{B}^L_{b,t} \right) +$$

$$+ \phi_{t+1} \eta^F \left(e_{t+1} \tilde{M}^F_{b,t} + \ \eta^{CB} \ \frac{e^s_{t+1}}{e_{t+1}} (1 - \omega_{t+1}) \tilde{B}^S_{b,t} \right),$$

where ω_{t+1} is the fraction of domestic short-term domestic bonds used as collateral to obtain q^C, $1 - \omega_{t+1}$ is the fraction of short-term domestic bonds used to secure the swap, and $\mathbb{1}_C$ is an indicator function that takes the value of one when the proceeds of swaps can be used as collateral, zero otherwise. This problem then delivers the following first-order conditions

$$u_1(q^C_{t+1}, 0) - 1 - \lambda^C_{t+1} = 0,$$

$$\lambda^C_{t+1} \left[\phi_{t+1} \left(\tilde{M}_{b,t} + \eta^S \omega_{t+1} \tilde{B}^S_{b,t} + \eta^L Q_t \tilde{B}^L_{b,t} + \eta^F \left(e_{t+1} \tilde{M}^F_{b,t} + \eta^{CB} \ (1 - \omega_{t+1}) \tilde{B}^S_{b,t} \frac{e^s_{t+1}}{e_{t+1}} \eta^{CB} \ \tilde{B}^S_{b,t} \right) \right) - q^C_{t+1} \right] = 0,$$

and for an interior solution ω_{t+1} the following condition would need to be satisfied

$$\eta^S = \eta^F \ \eta^{CB} \ \frac{e^S_{t+1}}{e_{t+1}}.$$

Finally, when a domestic DM buyer is able to trade with DM foreign producers, they do so under perfect competition. Moreover, foreign sellers only accept payment in foreign currency. Then, the domestic buyer in DM solves the following problem

$$\max_{q^F_{t+1},D^F_{t+1},\omega^F_{t+1}} \left\{u(0,q^F_{t+1}) + W_b\left(\tilde{M}_{b,t}, \tilde{M}^F_{b,t} + \frac{\tilde{S}^F_{b,t}+\tilde{E}^F_{b,t}}{e_{t+1}} - D^F_{t+1}, \tilde{B}^S_{b,t}, \tilde{B}^L_{b,t}, 0, \tilde{S}^F_{b,t}, \tilde{E}^F_{b,t}, 0\right)\right\} \text{ s.t.}$$

$$\nu^F_{t+1} q^F_{t+1} = \phi_{t+1} e_{t+1} D^F_{t+1} \leq \phi_{t+1} e_{t+1} \tilde{M}^F_{b,t} + \phi_{t+1}(1-\omega^F_{t+1})\tilde{S}^F_{b,t} + \phi_{t+1}\omega^F_{t+1}\tilde{E}^F_{b,t},$$

$$\tilde{S}^F_{b,t} \leq \mathbb{1}_F \, \frac{e^s_{t+1}}{e_{t+1}} \eta^{CB} \, \tilde{B}^S_{b,t},$$

$$\tilde{E}^F_{b,t} \leq \mathbb{1}_E \, \eta^{SF} \tilde{B}^S_{b,t},$$

where ν^F_{t+1} is the relative price of the DM foreign good in terms of the CM good, $\mathbb{1}_F$ is an indicator function that takes the value of one when the proceeds of swaps can be used to pay foreign sellers, zero otherwise and $\mathbb{1}_E$ is another indicator function that takes the value of one when domestic DM consumers willing to buy foreign goods can enter in repurchase agreements using short-term domestic public as collateral in exchange for foreign currency at the market exchange rate, otherwise zero. In the previous formulation, swap lines are essential when market disruptions lead to no private access to repo agreements in exchange for foreign currency $\mathbb{1}_E = 0$ or, when there is access $\mathbb{1}_E = 1$, but such agreements provide low collateral value $\eta^{SF} \approx 0$ in those exchanges. When $\mathbb{1}_F = 1$, $\mathbb{1}_E = 1$, we find that if $\delta\eta^{CB} > \eta^{SF}$ then $\omega^F_{t+1} = 0$ and if $\delta\eta^{CB} < \eta^{SF}$, then $\omega^F_{t+1} = 1$. Thus, swap lines need to be advantageous $\delta\eta^{CB} > \eta^{SF}$ to be used in equilibrium. Note that since we are considering a small open economy, ν^F_{t+1} is exogenous.[15]

When agents do not have access to the foreign exchange market in DM, that is $\mathbb{1}_E = 0$, consumption of the foreign good in DM is limited by the agent's direct holdings of foreign currency. The opening of a swap line, i.e. $\mathbb{1}_F = 1$, allows agents to partly undo their portfolio decision and acquire additional foreign currency with a foreign currency repo with the domestic central bank. This is the case as it allows for addition consumption of foreign goods in DM that can increase welfare beyond what QE policies (through changes in $1-\theta^S$ and $1-\theta^L$, which affect θ and Ω^H) can achieve.

The terms of trade imply the following DM consumer's envelope condition for domestic currency

$$\frac{\partial V^{DM}_b}{\partial \tilde{M}_{b,t-1}} = \mu_M u_{1,t}\left(q^M_t, 0\right)\frac{\partial q^M_t}{\partial \tilde{M}_{b,t-1}} + \mu_C u_{1,t}\left(q^C_t, 0\right)\frac{\partial q^C_t}{\partial \tilde{M}_{b,t-1}} + \sigma_F \phi_t.$$

[15]This relative price can be different from unity whenever CM and DM foreign goods differ in their production technologies.

Since $\sigma_F + \mu_M + \mu_C = 1$, the above can be written as

$$\frac{\partial V_b^{DM}}{\partial \tilde{M}_{b,t-1}} = \mu_M \left[u_{1,t}\left(q_t^M, 0\right) \frac{\partial q_t^M}{\partial \tilde{M}_{b,t-1}} - 1 \right] + \mu_C \left[u_{1,t}\left(q_t^C, 0\right) \frac{\partial q_t^C}{\partial \tilde{M}_{b,t-1}} - 1 \right] + \phi_t.$$

The consumer's envelope condition for short-term debt is then given by

$$\frac{\partial V_b^{DM}}{\partial \tilde{B}_{b,t-1}^S} = \sigma_F \left[u_{2,t}\left(0, q_t^F\right) \frac{\partial q_t^F}{\partial \tilde{S}_{t-1}^F} \frac{\partial \tilde{S}_{t-1}^F}{\partial \tilde{B}_{b,t-1}^S} - \phi_t \frac{\partial \tilde{S}_{t-1}^F}{\partial \tilde{B}_{b,t-1}^S} + u_{2,t}\left(0, q_t^F\right) \frac{\partial q_t^F}{\partial \tilde{E}_{t-1}^F} \frac{\partial \tilde{E}_{t-1}^F}{\partial \tilde{B}_{b,t-1}^S} - \phi_t \frac{\partial \tilde{E}_{t-1}^F}{\partial \tilde{B}_{b,t-1}^S} \right]$$

$$+ \mu_C \left[u_{1,t}\left(q_t^C, 0\right) \frac{\partial q_t^C}{\partial \tilde{L}_{t-1}^C} \frac{\partial \tilde{L}_{t-1}^C}{\partial \tilde{B}_{b,t-1}^S} - \phi_t \frac{\partial \tilde{L}_{t-1}^C}{\partial \tilde{B}_{b,t-1}^S} + u_{1,t}\left(q_t^C, 0\right) \frac{\partial q_t^C}{\partial \tilde{S}_{t-1}^C} \frac{\partial \tilde{S}_{t-1}^C}{\partial \tilde{B}_{b,t-1}^S} - \phi_t \frac{\partial \tilde{S}_{t-1}^C}{\partial \tilde{B}_{b,t-1}^S} \right] + \phi_t R_{t-1}.$$

Similarly, for long-term nominal public debt we have that

$$\frac{\partial V_b^{DM}}{\partial \tilde{B}_{b,t-1}^L} = \mu_C \left[u_{1,t}\left(q_t^C, 0\right) \frac{\partial q_t^C}{\partial \tilde{L}_{t-1}^C} \frac{\partial \tilde{L}_{t-1}^C}{\partial \tilde{B}_{b,t-1}^L} - \phi_t \frac{\partial \tilde{L}_{t-1}^C}{\partial \tilde{B}_{b,t-1}^L} \right] + \phi_t \left(1 + \rho Q_t\right).$$

Finally, the envelope condition for the foreign currency is given by

$$\frac{\partial V_b^{DM}}{\partial \tilde{M}_{b,t-1}^F} = \sigma_F u_{2,t}\left(0, q_t^F\right) \frac{\partial q_t^F}{\partial \tilde{M}_{b,t-1}^F} + \mu_C \left[u_{1,t}\left(q_t^C, 0\right) \frac{\partial q_t^C}{\partial \tilde{L}_{t-1}^C} \frac{\partial \tilde{L}_{t-1}^C}{\partial \tilde{M}_{b,t-1}^F} - \phi_t \frac{\partial \tilde{L}_{t-1}^C}{\partial \tilde{M}_{b,t-1}^F} \right] + (1 - \sigma_F)\phi_t e_t.$$

Given the DM optimal terms of trade, the envelope conditions for the domestic seller are given by

$$\frac{\partial V_s^{DM}}{\partial \tilde{M}_{s,t-1}} = \phi_t, \quad \frac{\partial V_s^{DM}}{\partial \tilde{M}_{s,t-1}^F} = \phi_t e_t, \quad \frac{\partial V_s^{DM}}{\partial \tilde{B}_{s,t-1}^S} = \phi_t R_{t-1}, \quad \frac{\partial V_s^{DM}}{\partial \tilde{B}_{s,t-1}^L} = \phi_t \left(1 + \rho Q_t\right).$$

The marginal effects of bringing additional units of domestic currency, foreign currency, short and long-term bonds into DM imply the following intertemporal Euler equations

$$\phi_{t-1} = \beta \phi_t \{ \mu_M \left[u_{1,t}\left(q_t^M, 0\right) - 1 \right] + \mu_C \left[u_{1,t}\left(q_t^C, 0\right) - 1 \right] + 1 \},$$

$$e_{t-1}\phi_{t-1} = \beta \phi_t e_t \{ \sigma_F \left[\frac{1}{\nu_t^F} u_{2,t}\left(0, q_t^F\right) - 1 \right] + \mu_C \eta^F \left[u_{1,t}\left(q_t^C, 0\right) - 1 \right] + 1 \},$$

$$\phi_{t-1} = \beta \phi_t \{ \sigma_F \left[\eta^{SF} \omega_t^F + \delta \eta^{CB} (1 - \omega_t^F) \right] \left[\frac{1}{\nu_t^F} u_{2,t}\left(0, q_t^F\right) - 1 \right] + \mu_C \left(\eta^S \omega_t + \eta^F \delta \eta^{CB} (1 - \omega_t) \right) \left[u_{1,t}\left(q_t^C, 0\right) - 1 \right] + R_{t-1} \}$$

$$\phi_{t-1} Q_{t-1} = \beta \phi_t \{ \mu_C \eta^L Q_{t-1} \left[u_{1,t}\left(q_t^C, 0\right) - 1 \right] + (1 + \rho Q_t) \}.$$

From now on, we assume that it is costly to carry domestic currency across periods. As a result, buyers will economize the holdings of domestic currency.

4 Dynamic Equilibrium

Let us denote $\mathcal{E}_t$ as the gross exchange growth rate so that $\mathcal{E}_t = \frac{e_t}{e_{t-1}}$. Given the domestic operating procedures for monetary and fiscal policy as well as an exogenous prices p^F_{t+1} and ν^F_{t+1}, the dynamic equilibrium describing our small open economy is characterized by $X_{b,t} = 1$, $Y_t = H_t$, $P_t = e_t P^*_t$, $b_t = b^S_t + Q_t b^L_t$, $\Omega = \frac{b^S_t}{Q_t b^L_t}$, then $b^S_t = \frac{\Omega}{1+\Omega} b_t$, $Q_t b^L_t = \frac{b_t}{1+\Omega}$, $b^H_t = \theta b_t$, with $\theta = \left[\frac{\Omega\theta^S + \theta^L}{1+\Omega}\right]$, $\Omega^H = \frac{\theta^S}{\theta^L}\Omega$, $b^M_t = (1-\theta) b_t$, $\Omega^M = \frac{(1-\theta^S)}{(1-\theta^L)}\Omega$, $\gamma_1 = \left(\frac{\gamma^S + \gamma^L \Omega}{1+\Omega}\right)$, and the following equilibrium conditions:[16]

$$q^M_{t+1} = \frac{m_t}{\Pi_{t+1}}, \tag{14}$$

$$\nu^F_{t+1} q^F_{t+1} \leq \mathcal{E}_{t+1} \frac{m^F_t}{\Pi_{t+1}} + \left[\mathbb{1}_E \, \omega^F_{t+1} \eta^{SF} + \mathbb{1}_F \, (1-\omega^F_{t+1}) \delta \eta^{CB}\right] \frac{\Omega^H \theta}{1+\Omega^H} \frac{b_t}{\Pi_{t+1}}, \tag{15}$$

$$\omega^F_{t+1} = 1, \text{ or } \omega^F_{t+1} = 0, \text{ or } \eta^{SF} = \eta^{CB}\delta, \tag{16}$$

$$q^C_{t+1} \leq \frac{1}{\Pi_{t+1}} \left(m_t + \eta^S \omega_{t+1} \frac{\Omega^H}{1+\Omega^H} \theta b_t + \eta^L \frac{1}{1+\Omega^H} \theta b_t + \eta^F \mathcal{E}_{t+1} m^F_t + \eta^F \mathbb{1}_C \, \eta^{CB} \, \delta (1-\omega_{t+1}) \frac{\Omega^H}{1+\Omega^H} \theta b_t \right), \tag{17}$$

$$\omega_{t+1} = 1, \text{ or } \omega_{t+1} = 0, \text{ or } \eta^S = \eta^{CB}\delta, \tag{18}$$

$$\tau_t = \gamma_0 + \gamma_1 \left(b_{t-1} - b^*\right), \tag{19}$$

$$\tau_t + \theta b_t + m_t - \frac{m_{t-1}}{\Pi_t} = G + R_{t-1} \frac{\Omega^H}{1+\Omega^H} \frac{\theta b_{t-1}}{\Pi_t} + \left(\frac{1+\rho Q_t}{Q_{t-1}}\right) \frac{1}{1+\Omega^H} \frac{\theta b_{t-1}}{\Pi_t}, \tag{20}$$

$$R_t = \alpha_0 + \alpha_1 \left(\Pi_t - \Pi^*\right), \tag{21}$$

$$\Pi_t = \beta \left(1 + s^M_t\right), \tag{22}$$

$$\Pi_t = \beta \mathcal{E}_t \left(1 + s^F_t\right), \tag{23}$$

$$\Pi_t = \beta \left(R_{t-1} + s^S_t\right) \tag{24}$$

$$\Pi_t Q_{t-1} = \beta \left[(1+\rho Q_t) + s^L_t\right], \tag{25}$$

[16]Buyer's CM effort, H_t, is determined by his CM budget constraint. In addition, notice that in equilibrium we have that $\frac{R_{t-1}}{\Pi_t} = \frac{1}{\beta} - \frac{s^S_t}{\Pi_t}$ and $\frac{(1+\rho Q_t)}{\Pi_t} = \frac{Q_{t-1}}{\beta} - \frac{s^L_t}{\Pi_t}$.

$$s_t^M = \mu_M \left[u_{1,t}\left(q_t^M, 0\right) - 1\right] + \mu_C \left[u_{1,t}\left(q_t^C, 0\right) - 1\right], \tag{26}$$

$$s_t^F = \sigma_F \left[\frac{1}{\nu_t^F} u_{2,t}\left(0, q_t^F\right) - 1\right] + \mu_C \eta^F \left[u_{1,t}\left(q_t^C, 0\right) - 1\right] \tag{27}$$

$$s_t^S = \sigma_F \left[\mathbb{1}_E \eta^{SF} \omega_t^F + \mathbb{1}_F \delta \eta^{CB}(1-\omega_t^F)\right] \left[\frac{1}{\nu_t^F} u_{2,t}\left(0, q_t^F\right) - 1\right] + \mu_C \left[\eta^S \omega_t + \mathbb{1}_C \eta^F \delta \eta^{CB}(1-\omega_t)\right] \left[u_{1,t}\left(q_t^C, 0\right) - 1\right], \tag{28}$$

$$s_t^L = \mu_C \eta^L Q_{t-1} \left[u_{1,t}\left(q_t^C, 0\right) - 1\right]. \tag{29}$$

From now on, we assume that swap lines are open and that buyers have access to foreign currency market in DM through repos where short-term domestic debt is used collateral. That is, $\mathbb{1}_F = 1$, $\mathbb{1}_C = 1$ and $\mathbb{1}_E = 1$. In addition, we further assume that $\eta^S < \delta \eta^F \eta^{CB}$, which implies that $\omega_{t+1} = 0$ at the optimum. Similarly, we assume $\eta^{SF} < \delta \eta^{CB}$ so that $\omega_{t+1}^F = 0$ at the optimum. These imply that agents prefer to use the swap line to expand their collateral in both foreign and domestic DM trades. It is easy to show that the dynamic equilibrium can be reduce to a non-linear dynamical system for $(\Pi_{t,} b_t, m_t, m_t^F)$, which is given by

$$\Pi_t = \beta\alpha_0 + \beta\alpha_1\left(\Pi_{t-1} - \Pi^*\right) + \beta s_t^S, \tag{DS.1}$$

$$\frac{1}{\theta}\left\{G - \gamma_0 + \gamma_1 b^* - m_t + \frac{m_{t-1}}{\Pi_t}\right\} + \left\{\frac{1}{\beta} - \frac{\gamma_1}{\theta} - \left(s_t^S \Omega^H + \frac{s_t^L}{Q_{t-1}}\right)\frac{1}{1+\Omega^H}\frac{1}{\Pi_t}\right\} b_{t-1} = b_t, \tag{DS.2}$$

$$\frac{\Pi_t}{\beta} - 1 = s_t^M, \tag{DS.3}$$

$$\frac{\Pi_t^F}{\beta} - 1 = s_t^F, \tag{DS.4}$$

where the different spreads are given by

$$s_t^M = \mu_M \left[u_{1,t}\left(q_t^M, 0\right) - 1\right] + \mu_C \left[u_{1,t}\left(q_t^C, 0\right) - 1\right], \tag{30}$$

$$s_t^F = \sigma_F \left[\frac{1}{\nu_t^F} u_{2,t}\left(0, q_t^F\right) - 1\right] + \mu_C \eta^F \left[u_{1,t}\left(q_t^C, 0\right) - 1\right], \tag{31}$$

$$s_t^S = \delta \eta^{CB} \left\{\sigma_F \left[\frac{1}{\nu_t^F} u_{2,t}\left(0, q_t^F\right) - 1\right] + \mu_C \eta^F \left[u_{1,t}\left(q_t^C, 0\right) - 1\right]\right\}, \tag{32}$$

$$\frac{s_t^L}{Q_{t-1}} = \mu_C \eta^L \left[u_{1,t}\left(q_t^C, 0\right) - 1\right], \tag{33}$$

and the different DM consumption associated with the various states are given by

$$q_t^M = \frac{m_{t-1}}{\Pi_t}, \tag{34}$$

$$\nu_t^F q_t^F \leq \frac{m_{t-1}^F}{\Pi_t^F} + \delta\eta^{CB}\frac{\Omega^H\theta}{1+\Omega^H}\frac{b_{t-1}}{\Pi_t}, \tag{35}$$

$$q_t^C \leq \frac{m_{t-1}}{\Pi_t} + \eta^F\frac{m_{t-1}^F}{\Pi_t^F} + \left(\eta^L + \eta^F\eta^{CB}\delta\Omega^H\right)\frac{1}{1+\Omega^H}\theta\frac{b_{t-1}}{\Pi_t}. \tag{36}$$

Note that plugging (**??**) on to the gross exchange growth rate, we have that $\mathcal{E}_t = \frac{\Pi_t}{\Pi_t^F}$.

It is important to note that we observe different equilibria depending whether the DM liquidity constraint binds or not and for which state. These various scenarios will result in different stationary equilibria as well as different inflation, bond, and nominal exchange rate dynamics. Next, we explore these different cases. In what follows we assume that $\Pi_t^F = \Pi^F$ and $\nu_t^F = \nu^F \; \forall t$.

Case 0: Foreign and Domestic Good (with Collateral) Satiation

In this scenario we characterize an economy where consumption of foreign and domestic DM goods when collateral can be used are satiated. This implies that $q_t^F = \hat{q}^F$ and $q_t^C = \hat{q}^C \; \forall t$, where these quantities satisfy the following conditions $u_{2,t}\left(0, \hat{q}_t^F\right) = \nu^F$ and $u_{1,t}\left(\hat{q}_t^C, 0\right) = 1$; i.e., their first-best allocations. As a result, short and long-term public debt are priced fundamentally. The dynamic equilibrium and steady state corresponding to this economy (and for all other cases) can be found in the Appendix.[17] Below, we characterize the steady state, local dynamics and investigate the effect of swap lines and QE.

Proposition 1 *Under Case 0, we find that the monetary equilibrium has the following properties:*

(i) There exists a unique steady state, where domestic inflation is given by $\Pi = \Pi^*$*.*

(ii) Standard policy prescriptions deliver locally determinate equilibria.

(iii) Everything else equal, different terms of trade in the swap lines (different values of δ*) do not change the resulting stationary nor dynamic equilibria.*

(iv) Everything else equal, different degrees of QE-level (different values of θ*) imply different steady state levels and dynamics for total public debt* b_t*, but the same steady state inflation rate and inflation*

[17] As $\Pi = \Pi^*$, $q^F = \hat{q}^F$ and $q^C = \hat{q}^C$, Case 0 is used to determine b^* and other targets as the steady state values.

dynamics. However, different degrees of QE-composition (different values of Ω^H) do not affect the stationary nor the dynamic equilibrium.

From now on, all the proofs can be found in the Appendix.

As we can see, under Case 0, DM buyers have access to assets that are plentiful when financing their foreign DM consumption and their domestic DM consumption with collateral. Under those circumstances, steady state inflation equals target inflation. Furthermore, when assets are priced fundamentally, as in Leeper (1991) among others, local inflation and debt dynamics deliver standard prescriptions. As shown in the proof, the monetary eigenvalue is $\beta\alpha_1$, while the fiscal eigenvalue is $\frac{1}{\beta} - \frac{\gamma_1}{\theta}$. Note that the fiscal eigenvalue is adjusted by θ. This is in line with the closed economy findings of Domínguez and Gomis-Porqueras (2023), which show that, when normalizing the balance sheet of the central bank, the relevant quantities of bonds are those that are held by the public.

As we can see, provided the economy is in Case 0, swap lines do not affect the steady state nor local dynamics. It is worth pointing out that the satiation of foreign DM goods can be achieved either because the foreign central bank follows the Friedman rule or short-term domestic bonds are plentiful.[18] In this latter case, the issuance of short-term public debt is such that the collateral constraint corresponding to accessing the swap line is not binding. Furthermore, note that even when the swap lines are not required for purchasing foreign goods, they can help expand the domestic DM consumption possibilities whenever $\eta^S < \delta\eta^F\eta^{CB}$. Proposition **??** also highlights that QE policies that change the real value of total bonds in the hands of the public have an effect on allocations, but not on inflation. However, QE policies that only change the maturity composition, but not the total value of bond holdings, do not alter agent's behavior neither at steady state nor during the transition. Similarly, the satiation of foreign DM goods implies that there exists a combination of assets (domestic public debt of different maturities and foreign currency) such that the collateral constraint does not bind. In other words, while swap lines and QE-composition policies have no impact on the steady state and dynamics, they could have had a role in enabling the economy to reach the satiation level consistent with Case 0.

Cases 1, 2 and 3: No Satiation

We now examine the resulting equilibria when foreign and/or domestic DM consumption is not satiated. In our environment there are several scenarios.

[18] Note that satiation of both q^C and q^F requires that the foreign central bank follows the Friedman rule.

We define Case 1 as one where the consumption of DM foreign goods is satiated so that $q_t^F = \hat{q}^F$ $\forall t$, and the consumption of DM domestic good with collateral is not satiated. This implies $s_t^F = 0$ and $s_t^C > 0$. Case 2, on the other hand, contemplates a situation where consumption of the DM domestic good with collateral is satiated so that $q_t^C = \hat{q}^C$ $\forall t$, but the consumption of DM foreign good is not. This implies $s_t^C = 0$ and $s_t^F > 0$. Finally, Case 3 captures the circumstances when no DM goods are satiated. Then $s_t^C > 0$ and $s_t^F > 0$. For all these cases, short-term public debt is no longer priced fundamentally. In addition for Cases 1 and 3, long-term bonds also exhibit a premium. These various features on the spreads of domestic bonds imply different equilibrium properties relative to Case 0.

Proposition 2 *Under Cases 1, 2 and 3, we find the monetary equilibrium has the following properties:*

(i) There exists a unique steady state, where domestic inflation is given by $\Pi = \Pi^* - \frac{\delta\eta^{CB}}{(\beta\alpha_1 - 1)}\left(\Pi^F - \beta\right).$

(ii) Standard policy prescriptions do not necessarily deliver locally determinate equilibria.

(iii) Everything else equal, different terms of trade in the swap lines (different values of δ) change the stationary equilibria and its underlying dynamic properties.

(iv) Everything else equal, different degrees of QE-level (different values of θ) imply different public debt and inflation rate steady state levels and local dynamics. Similarly, different degrees of QE-composition (different values of Ω^H) affect the steady state and its local dynamic properties.

In this new environment, even though the steady state remains unique, steady state inflation now depends on the swap line rate δ and foreign inflation Π^F. With an active (passive) monetary policy i.e. $\beta\alpha_1 > 1$ ($\beta\alpha_1 < 1$), steady state inflation is below (above) target Π^*. Now inflation and debt dynamics depend on debt premia, which modify the necessary prescriptions for stability. These local dynamics results are consistent with the closed economy environments of Domínguez and Gomis-Porqueras (2019, 2023), among others, where government bonds exhibit a premium, while monetary policy is implemented through a Taylor rule and fiscal policy has a tax rule that links revenue with nominal debt issuance. One difference though is that now domestic inflation dynamics are also affected by foreign inflation.

Lemma 1 *For Case 1 and Case 2, the monetary eigenvalue is the same and equal to $\beta\alpha_1$. In contrast, the the fiscal eigenvalue differs across cases. For Case 1, it equals* $\frac{1}{\beta} - \frac{\gamma_1}{\theta} - \left(\delta\eta^{CB}\Omega^H + \frac{\eta^L}{\eta^F}\right)\left(\frac{\Pi_t^F}{\beta} - 1\right)\frac{1}{1+\Omega^H}\frac{1}{\Pi}$, *while for Case 2 is given by* $\frac{1}{\beta} - \frac{\gamma_1}{\theta} - \delta\eta^{CB}\left(\frac{\Pi_t^F}{\beta} - 1\right)\frac{\Omega^H}{1+\Omega^H}\frac{1}{\Pi}$.

As we can see, relative to Case 0, it is clear that swaps and QE dampen the fiscal eigenvalue for Case 1 and Case 2. Moreover, it is apparent, for all these cases, we find that both swaps and QE affect the stationary equilibria and the dynamics. Next, we explore if exists different combinations of conventional and unconventional monetary policies that deliver the same steady state.

Proposition 3 *For Cases 1 and 2, we find the following:*

1. *Everything else equal, different combinations of swap lines, QE-composition and aggressiveness in the Taylor rule (different values of δ, $\Omega^H$and α_1) can deliver the same steady state. Such combinations differ across Cases 1 and 2.*

2. *However, those different policy combinations imply different premia on short-term public debt as well as different local dynamics for inflation, debt and nominal exchange rates.*

3. *Monetary policies that lead to a more favorable swap line, i.e. a larger δ, require that the interest policy setting responds more (less) aggressively to inflation, i.e. larger (lower) α_1, in order to deliver the same steady state whenever monetary policy is active (passive).*

As we can see, to achieve the same steady state, a central bank would require a simultaneous adjustment to the swap rate δ, to QE-composition to change the liquidity of bonds owned by households Ω^H, and to the interest-rate aggressiveness towards inflation, i.e. α_1. The required change in α_1 is the same across Cases 1 and 2. For both Cases 1 and 2, a more favorable swap line, i.e. a larger δ, requires the central bank to change the composition of its balance sheet for QE to provide less liquidity to households, i.e. lower Ω^H, in order to deliver the same steady state. The required change in Ω^H is however different across Cases 1 and 2.

In order to quantify these required changes and further explore how swaps and QE interact and affect macroeconomic variables, we next turn to numerical exercises.

5 Numerical Exercise

In this Section, we resort to numerical analysis to investigate further properties of the monetary equilibria and explore how domestic and international unconventional monetary policies together with conventional ones and fiscal responses deliver equilibria that is locally determinate and desirable. From now on, we use the following utility

$$u(q, q^F) = \chi \left(\frac{q^{(1-\xi)}}{1-\xi} + \chi^F \frac{(q^F)^{(1-\xi)}}{1-\xi} \right).$$

Parametrization

In our calibration exercise, we consider Australia as our SOE and U.S. as our rest of the world. To provide some discipline when deciding the parameter values describing the small open economy, we consider Australia's macroeconomic data as well as issuance of public debt and Reserve Bank of Australia (RBA) asset holdings for the period of 1993-2013. We fix Π^F to the average U.S. inflation during the corresponding period. Moreover, we fix the following parameters values $v_F = 1$, $\eta^F = 1$, and $\eta^{SF} = 0$. We also let $\gamma_1 = 0.0235$ to ensure that fiscal policy is passive. For the rest of parameters, we apply a three-step procedure.

1. For assumed values for the preference parameters ξ and χ, we compute a steady state of our economy (prior to the commencement of the Swaps Program) and calibrate the parameters β, χ^F, η^S, η^L, σ^F, μ^C, μ^M, G and γ_0 to match long-run averages from 1993 to 2019.

2. We construct a time series for the model-implied money demand and calibrate the parameters ξ and χ to minimize the difference between the model-implied money demand and the quarterly M0 to GDP ratio data for Australia from 1993 to 2019.

3. We iterate steps 1 and 2 until the calibrated values for ξ and χ coincide with the assumed values.

This procedure delivers the model parameter values, which are reported in Table 1.

Parameter	**Target**
$\beta = 0.9743$	Annual real interest rate of 2.6%
$\xi = 0.1115$	Time-Series of M0 to GDP
$\chi = 0.9319$	from 1993 to 2019
$\chi_F = 0.8628$	U.S. Imports of 3.20% of GDP
$\sigma_F = 0.5002$	Deposits in U.S. Dollars to GDP of 6.5%
$\mu_M = 0.2890$	M0 to GDP of 14.4%
$\mu_C = 0.2108$	Model-implied as $\mu_C = 1 - \mu_M - \sigma_F$
$\eta^S = 0.8890$	Short-term bond premia of 0.8825%
$\eta^L = 0.3965$	Long-term bond premia of 0.3936%
$G = 0.2664$	Public demand of 23.8 % of GDP
$\gamma_0 = 0.2930$	Public debt of 30.2% of GDP (dom. held)

Table 1: Calibration: Parameters and Targets

The Reserve Bank of Australia started to implement domestic and international unconventional monetary policies at the beginning of the covid-19 pandemic and continued during the pandemic.[19] To adjust our calibration to the liquidity needs during the pandemic, we consider a shock to η^F, η^S, and η^L that decreases their values proportionally and uniformly. To get similar inflation experiences, this multiplicative shock needs to be around 50 %. Then we open the swaps program by assuming $\mathbb{1}_F = \mathbb{1}_C = 1$, $\eta^{CB} = \eta^S$ and $\delta \in [1, 1.25]$. Additionally, we consider different fiscal and monetary policy parameters that match debt-to-GDP ratios, RBA's balance sheet and fiscal/monetary policy stands and different U.S. inflation rates at two different dates: (i) at the beginning of the covid-19 pandemic and (ii) during the pandemic. To get us closer to the beginning of the pandemic, we re-calibrate monetary and fiscal parameters using the 2015-2019 averages. To consider the period during the pandemic, we re-calibrate those to the average values during 2020-2022. Tables 2 and 3 describe the two different monetary policies and fiscal policies respectively.

[19]Specifically, the RBA started the U.S. dollar swaps line program and QE in March 2020, and U.S. dollar swaps closed in December 2021.

Parameter	Target
$\alpha_0 = 1.0520$	Annual Inflation Rate Target of 2.5 %
$\alpha_1^{'} = 2.0000$	Active MP (2015-2019)
$\alpha_1^{''} = 0.5000$	Passive MP (2020-2022)
$1 - \theta^S = 0.0000$	RBA Short-term Bond Holdings of 0.0 % (2015-2022)
$1 - \theta^{L'} = 0.1869$	RBA Long-term Bond Holdings of 18.7 % (2015-2019)
$1 - \theta^{L''} = 0.4663$	RBA Long-term Bond Holdings of 46.7 % (2020-2022)
$\delta \in [1, 1.25]$	Swap Lines Program, from March 2020

Table 2: Monetary Policy Parameters

Note that the inflation target is kept at 2.5 % (middle of the range between 2 and 3 %) for both periods. Similarly, before and during the covid-19 pandemic, the short-term debt holdings of the RBA were zero; i.e., $(1 - \theta^S) = 0$. For the monetary policy stance α_1, we assume that the Taylor principle was followed at the beginning of the pandemic but not during the pandemic. This is consistent with below target inflation experiences at the beginning of the pandemic and above target inflation during the pandemic. For the swap line rate, we consider a range of values for δ. For QE, it is worth mentioning that the holdings of long-term debt relative total domestically held debt by the RBA increased from 19 to 47 %.

Within the same spirit, we consider the fiscal policy in Australia at the beginning and during the covid-19 pandemic. Table 3 describes the two different fiscal policies.

Parameter	Target
$\Omega^{'} = 0.0179$	Treasury Short to Long-Term Bond Issuance of 1.8% (2015-2019)
$\Omega^{''} = 0.0621$	Treasury Short to Long-Term Bond Issuance of 6.2% (2020-2022)
$G^{'} = 0.2762$	Public demand of 24.7% of GDP (2015-2019)
$G^{''} = 0.3022$	Public demand of 27.0% of GDP (2020-2022)
$\gamma_0^{'} = 0.3052$	Public debt of 47.1% of GDP (dom. held) (2015-2019)
$\gamma_0^{''} = 0.3351$	Public debt of 80.8 % of GDP (dom. held) (2020-2022)
$\gamma_1^{'} = 0.0235$	Passive FP (2015-2019)
$\gamma_1^{''} = 0.0129$	Active FP (2020-2022)

Table 3: Fiscal Policy Parameters

As is the case for monetary policy, we adjust the fiscal parameters to better reflect the economic conditions at the beginning and during the pandemic. The values in Table 3 reflect the increased public debt and liquidity provided by Treasury, the increased government spending and the difference in fiscal policy stance. Specifically, we consider that fiscal policy was passive in the lead up to the pandemic, while it became active during the pandemic. Such stance is calculated under the veil of the standard prescriptions (i.e. for an economy in Case 0). However, as the economy shifts to a different case (Case 1, 2 or 3), the actual fiscal stance (either passive or active) may change.

Given these different policy responses to covid-19, we can study the equivalence between conventional and unconventional monetary policies at the beginning and during the pandemic. We can also analyze the consequences for inflation and debt dynamics when domestic and international unconventional monetary policies are enacted.

5.1 Equivalence Results

Proposition **??** states that there are different combinations of swap line rates, QE-composition and aggressiveness in the Taylor rule (different values of δ, Ω^H and α_1) that can deliver the same steady state. Next we analyze by how much international and domestic unconventional as well as conventional monetary policies need to adjust. We do so by considering the type of responses one would have observed at the beginning and during the pandemic.

At the Beginning of the Pandemic

Here we focus on the period at the beginning of the pandemic. To do so we consider the monetary and fiscal regimes calibrated to 2015-2019 as well as the local and U.S. average inflation rates during this period. The RBA started its U.S. swap line program in March 2020. Below Figure 1 illustrates the different combinations of monetary policies that could deliver the same steady state.

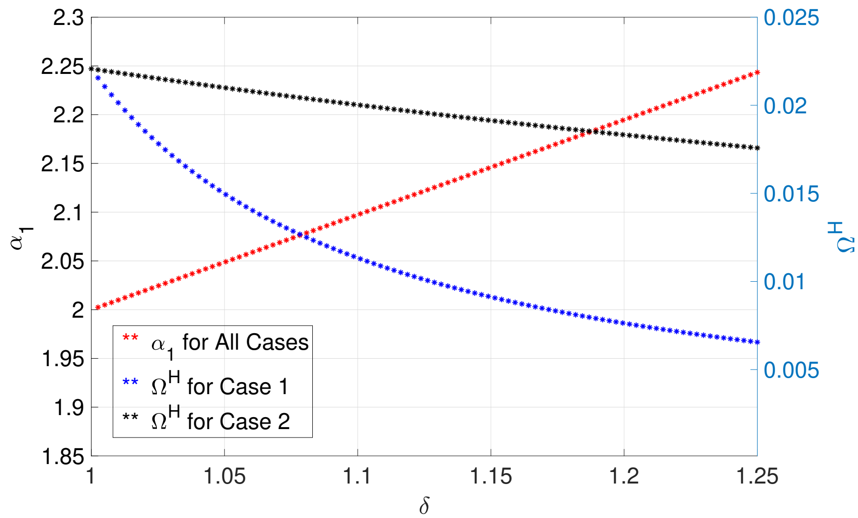

Figure 1: Different Monetary Policy Combinations that Deliver Same Steady State.

The horizontal axis shows changes in the swap line rate δ. The left (right) vertical axis displays changes in the response of interest rates to inflation α_1 (in QE-composition that lead to changes in the liquidity of bonds Ω^H in the hands of households). As the U.S. dollar swap terms of trade improve so that δ increases from 1 to 1.25, we find that in order to deliver the same steady state allocation the central bank implements an interest rate policy that responds stronger to inflation. In particular, we have that α_1 increases from 2.0 to 2.25. We also find that there is no need to provide so much liquidity as the short-term bond composition of households, Ω^H, decreases from 0.022 to 0.007 when the economy is described by Case 1 or to 0.0175 for Case 2. As those are similar or below to liquidity provided by Treasury ($\Omega = 0.0179$), we conjecture that with a more favorable swap line, there would have been no need for QE (to deliver the same steady state).

During Pandemic

We now explore the period during the pandemic. To do so we consider the monetary and fiscal regimes calibrated to 2020-2022 and the local and U.S. average inflation rates for the same period. Figure 2 illustrates the combination of monetary policies that could deliver the same steady state.

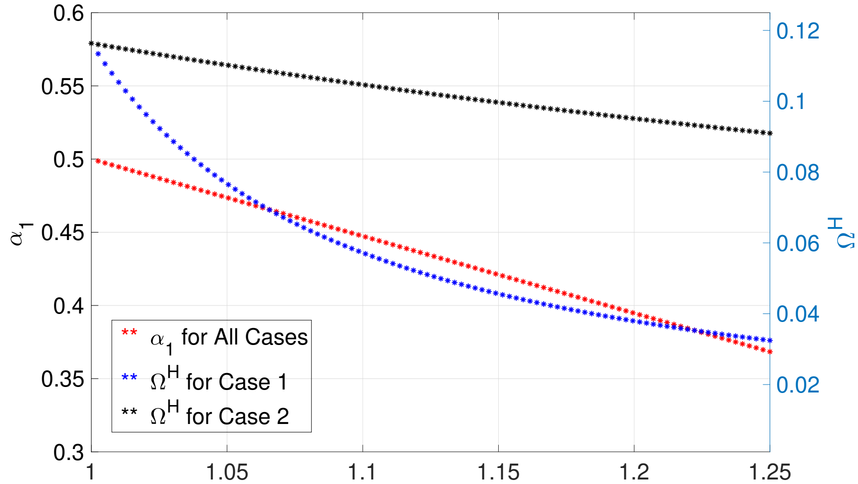

Figure 2: Different Monetary Policy Combinations that Deliver Same Steady State.

As the U.S. dollar swap terms of trade improve so that δ increases from 1 to 1.25, we find that in order to deliver the same steady state allocation the central bank implements an interest rate policy that responds less to inflation. In particular, we have that α_1 decreases from 0.5 to 0.37, which is the opposite response compared to the economy at the beginning of the pandemic. The direction of the response is the opposite because the monetary stance is now passive instead of active. We also find that there is no need to provide so much liquidity as the short-term bond composition of households, Ω^H, decreases from 0.116 to 0.09 when the economy is described by Case 2 or to 0.032 for Case 1. For Case 1, there is a dramatic fall in the required QE. The reason for this is that as q^F is satiated, the additional liquidity is very effective. For Case 2, QE is needed but requires less purchases, specifically $(1-\theta^L) = 0.31$ (24% of GDP) instead of the implemented $(1-\theta^L) = 0.47$ (35% of GDP).

5.2 Non-Equivalence Results: Stability

We now explore the implications of different monetary policy combinations that deliver same steady state on local dynamics. As we found in the previous propositions, there is going to be an impact on the local dynamics of the stationary equilibrium. Now we quantify these changes at the beginning and during the pandemic, which are depicted on Figures 3 and 4. In these figures, the horizontal axis shows changes in the swap line rate δ (and implicitly we are still changing α_1 and Ω^H to remain in the same steady state). The left (right) vertical axis displays changes in the monetary policy eigenvalue (fiscal policy eigenvalue) for the different monetary policy combinations.

At the Beginning of the Pandemic

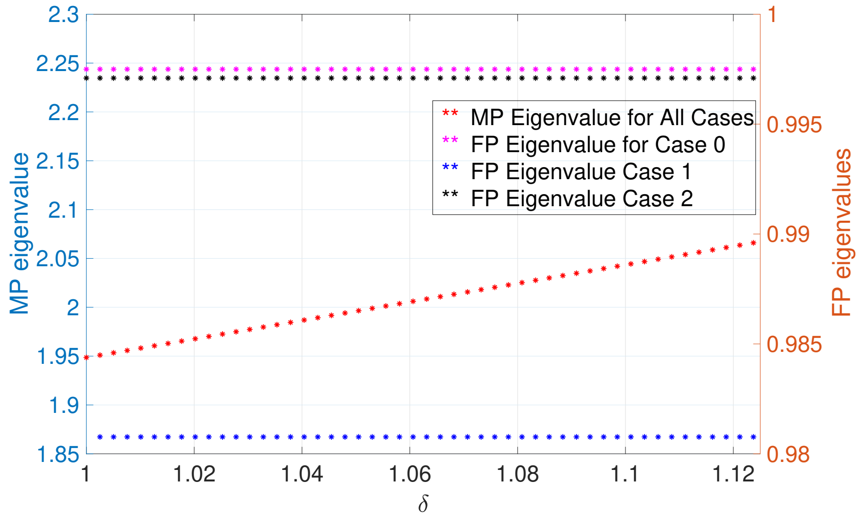

Figure 3: Local Dynamics at the Beginning of the Pandemic.

Even if we have the same steady state, increases in δ lead to different dynamics. Once swaps are enacted and used, traditional monetary and fiscal policy stabilization policies are not operative anymore. Specifically, relative to Case 0, QE and swaps dampened the (passive) fiscal eigenvalue. This is inconsequential for determinacy as monetary policy is considered active.

During the Pandemic

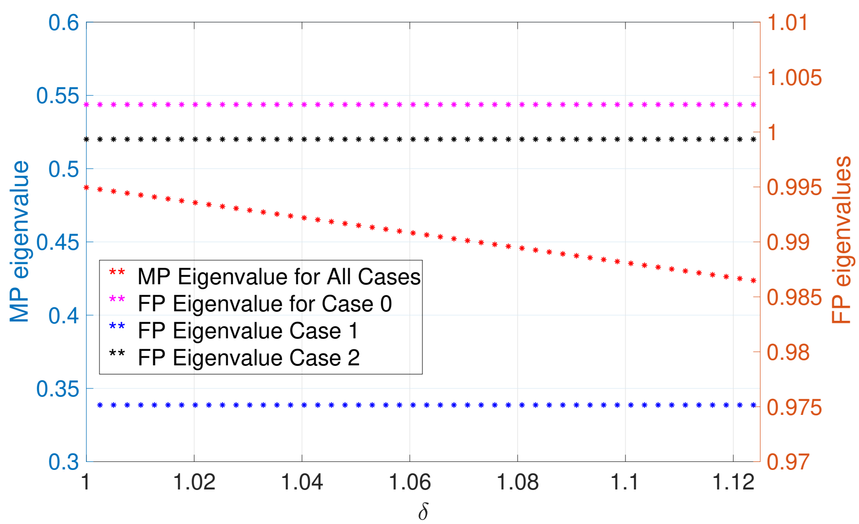

Figure 4: Local Dynamics During the Pandemic.

As at the beginning of the pandemic, increases in δ affect the dynamics and QE and swaps reduce the fiscal eigenvalue. However, now with a passive monetary policy, this reduced fiscal eigenvalue can lead to indeterminacy as it does in Figure 4 above for Cases 1 and 2. Additionally, and consistent with the analytical results, the fiscal eigenvalue for Case 1 is lower than for Case 2.

5.3 Additional Results

Next, we move away from policy equivalence and consider a range of δ and Ω^H (all other parameters constant, except for γ_0 which is re-calibrated to the same debt to GDP ratio). Figure 5 shows the stability properties of the resulting equilibria.

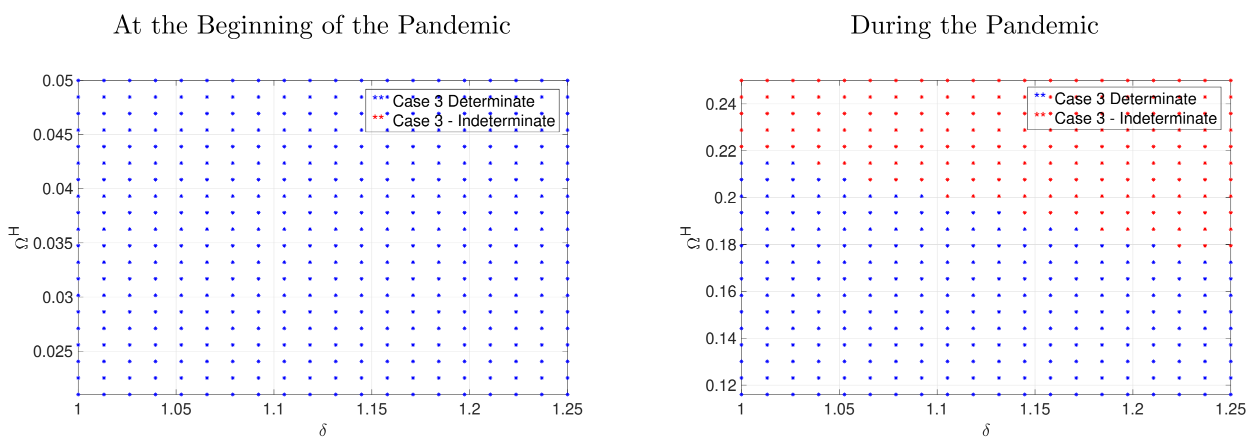

Figure 5: Stability.

At the beginning of and during the pandemic, we find that all these different unconventional policies deliver an economy consistent with Case 3. That is, the pandemic shock is so large that domestic and foreign DM consumption are not satiated even if QE and/or swaps had been more favorable. Both swaps and QE dampen the fiscal multiplier. At the beginning of the pandemic with an active monetary policy, this implies that the equilibrium is determinate. During the pandemic with a passive monetary policy, the equilibrium becomes indeterminate for a range of unconventional monetary policies. In particular, the region of indeterminacy is enlarged for more liquid QE and more favorable swaps. Next, we look at the effect of swaps and QE on inflation, portfolio and consumption at the beginning and during the pandemic.

At the Beginning of the Pandemic

We find that long run domestic inflation, Π, decreases with the swap line rates, δ. Note that as the gross growth rate of the nominal exchange rate, $\mathcal{E}$, moves with inflation, it also goes down. In addition, the different combinations of the swap line rates, δ, and composition of the central bank's balance sheet, Ω^H, make short-term debt more liquid and induce a portfolio re-allocation. In particular, domestic money, m, to GDP increases with both the swap line rates, δ, and composition of the central bank's balance sheet, Ω^H, while the demand of foreign currency in the domestic economy, m^F, to GDP decreases with both the swap line rates, δ, and the composition of the central bank's balance sheet, Ω^H.

This portfolio re-adjustment affects consumption levels: q^M increases with both the swap line rates δ and with Ω^H; q^C increases with the swap line rates δ but decreases slightly with Ω^H; and q^F decreases slightly with both the swap line rates δ and Ω^H. This is shown in Figure 6 below.

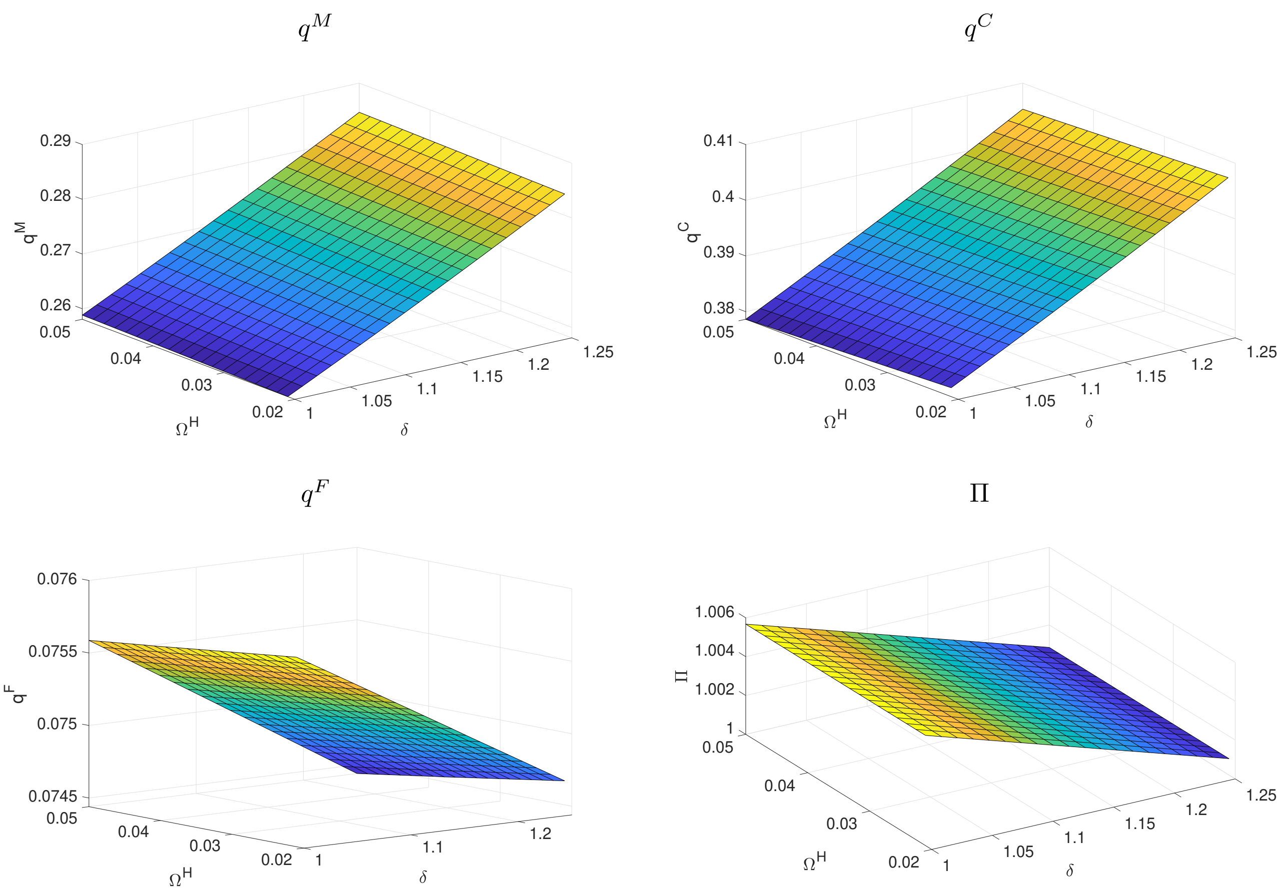

Figure 6: Consumption and Inflation - at the Beginning of the Pandemic.

During the Pandemic

Now, with a passive monetary policy, long-run domestic inflation Π increases with the swap line rates, δ. Likewise, the exchange growth rate, $\mathcal{E}$, increases. As before, the different combinations of the swap line rates, δ, and composition of the central bank's balance sheet, Ω^H, make short-term debt more liquid and induce a portfolio re-allocation. However, such portfolio re-allocation is different. Now domestic money, m, to GDP decreases with the swap line rates, δ, but increases with Ω^H, while the demand of foreign currency in the domestic economy, m^F, to GDP decreases with both δ and Ω^H.

This portfolio re-adjustment affects consumption levels in a different way as in the previous section. In particular, q^M decreases with δ but increases with Ω^H; q^C decreases with both δ and Ω^H; and q^F increases with δ and decreases with Ω^H. This is presented in Figure 7 below.

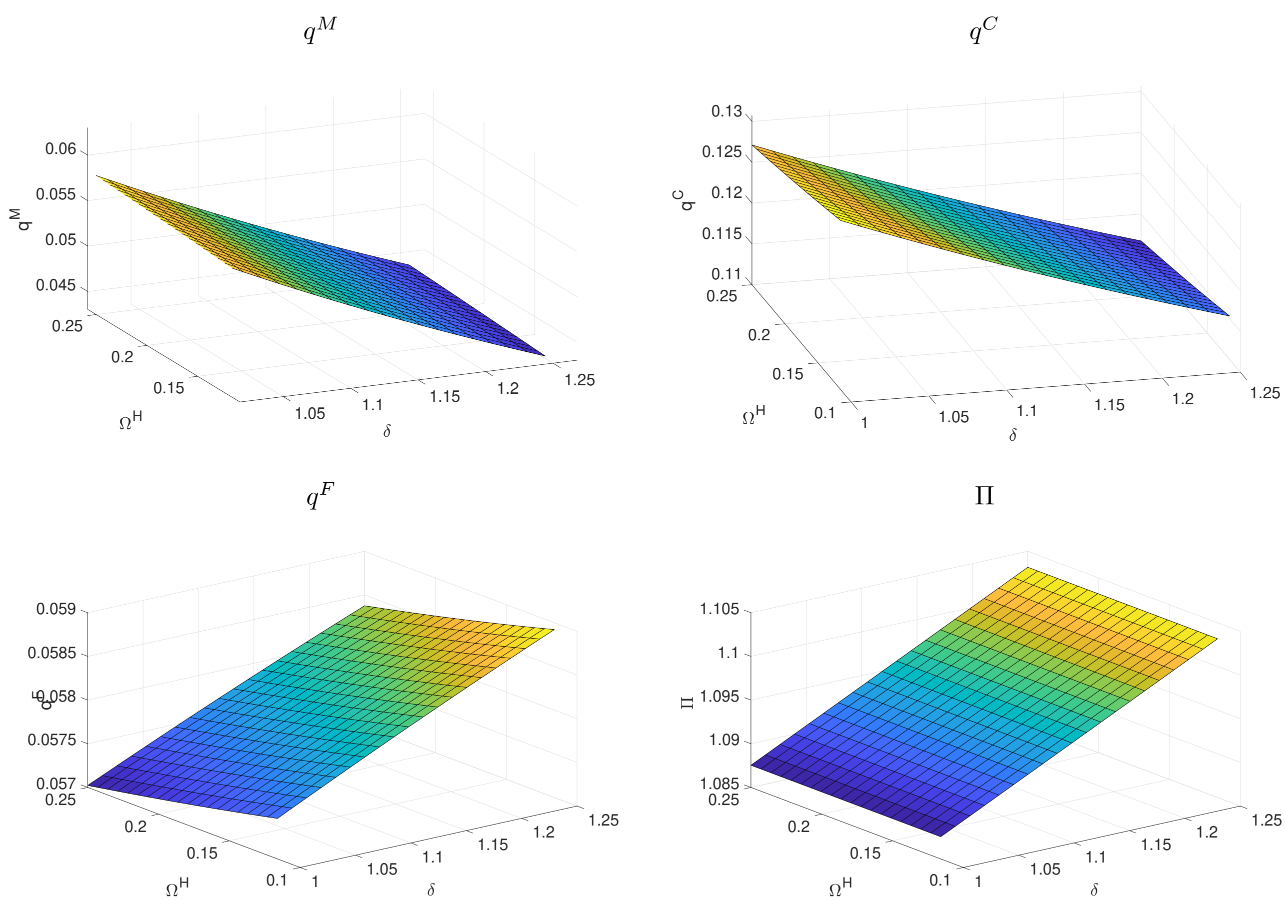

Figure 7: Consumption and Inflation - During the Pandemic.

6 Conclusions

This paper has proposed a theoretical framework to study U.S. dollar swap lines in a small open economy. We have shown that U.S. dollar swaps interact with other conventional and unconventional monetary policies of a small open economy. Swaps increase the collateral value of (already) high-quality domestic nominal public assets. As a result of these policies, we observe some degree of complementarity with QE policies, which also inject liquidity in the small open economy. Swaps can help expand consumption of both foreign and domestic goods. However, swaps also induce a portfolio re-allocation in the small open economy. Therefore, swaps also hinder the effectiveness of QE policies. Once swaps are enacted and used in the economy, traditional monetary and fiscal policy stabilization policies are not operative anymore. Specifically, swaps and QE tend to dampen the fiscal eigenvalue. This effect can lead to indeterminancy of equilibria in environments with passive monetary policy.

Through these findings, our paper provides new insights on how international and domestic unconventional monetary policies interact between each other and with fiscal policies. We conjecture that such interaction should be also present in other environments. This is the case as U.S. dollar swaps are designed as repo arrangements that require high-quality collateral, short-term nominal public debt, which is a key asset that central banks manage in their balance sheet when implementing QE policies.

Other than providing U.S. dollar liquidity, swaps also signal cooperation between the Federal Reserve and the local central bank. In future research, we aim to explore the role of U.S. dollar swaps as a cooperation device between central banks.

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Appendix: Derivation of the Monetary Equilibria and Proofs

Derivations

The dynamic system can be reduced to 4 equations in (Π_t, b_t, m_t, m_t^F) :

$$\Pi_t = \beta\alpha_0 + \beta\alpha_1 (\Pi_{t-1} - \Pi^*) + \beta s_t^S, \tag{DS.1}$$

$$\frac{1}{\theta}\left\{G - \gamma_0 + \gamma_1 b^* - m_t + \frac{m_{t-1}}{\Pi_t}\right\} + \left\{\frac{1}{\beta} - \frac{\gamma_1}{\theta} - \left(s_t^S \Omega^H + \frac{s_t^L}{Q_{t-1}}\right)\frac{1}{1+\Omega^H}\frac{1}{\Pi_t}\right\} b_{t-1} = b_t, \tag{DS.2}$$

$$\frac{\Pi_t}{\beta} - 1 = s_t^M, \tag{DS.3}$$

$$\frac{\Pi_t^F}{\beta} - 1 = s_t^F, \tag{DS.4}$$

where

$$s_t^M = \mu_M \left[u_{1,t}\left(q_t^M, 0\right) - 1\right] + \mu_C \left[u_{1,t}\left(q_t^C, 0\right) - 1\right],$$

$$s_t^F = \sigma_F \left[\frac{1}{\nu_t^F} u_{2,t}\left(0, q_t^F\right) - 1\right] + \mu_C \eta^F \left[u_{1,t}\left(q_t^C, 0\right) - 1\right]$$

$$s_t^S = \delta\eta^{CB} \left\{\sigma_F \left[\frac{1}{\nu_t^F} u_{2,t}\left(0, q_t^F\right) - 1\right] + \mu_C \eta^F \left[u_{1,t}\left(q_t^C, 0\right) - 1\right]\right\},$$

$$\frac{s_t^L}{Q_{t-1}} = \mu_C \eta^L \left[u_{1,t}\left(q_t^C, 0\right) - 1\right].$$

$$q_t^M = \frac{m_{t-1}}{\Pi_t},$$

$$\nu_t^F q_t^F \leq \frac{m_{t-1}^F}{\Pi_t^F} + \delta\eta^{CB} \frac{\Omega^H \theta}{1+\Omega^H} \frac{b_{t-1}}{\Pi_t},$$

$$q_t^C \leq \frac{m_{t-1}}{\Pi_t} + \eta^F \frac{m_{t-1}^F}{\Pi_t^F} + \left(\eta^L + \eta^F \eta^{CB} \delta\Omega^H\right) \frac{1}{1+\Omega^H} \theta \frac{b_{t-1}}{\Pi_t}.$$

Case 0: Reduced Dynamic System

Let $q^C = q^{C*}$, and $q^F = q^{F*}$, then all premia (except for money) disappears. Then the dynamic system can be reduced to 4 equations in (Π_t, b_t, m_t, m_t^F) :

$$\Pi_t = \beta\alpha_0 + \beta\alpha_1 (\Pi_{t-1} - \Pi^*), \tag{DS.1}$$

$$\frac{1}{\theta}\left\{G-\gamma_0+\gamma_1 b^*-m_t+\frac{m_{t-1}}{\Pi_t}\right\}+\left\{\frac{1}{\beta}-\frac{\gamma_1}{\theta}\right\}b_{t-1}=b_t, \tag{DS.2}$$

$$\frac{\Pi_t}{\beta}-1=\mu_M\left[u_{1,t}\left(\frac{m_{t-1}}{\Pi_t},0\right)-1\right], \tag{DS.3}$$

$$\frac{\Pi_t^F}{\beta}-1=0, \tag{DS.4}$$

Note that the last equation requires that the Fed is at the Friedman rule and that the third equation defines m_t and m_{t-1} as a function of Π_t (once we substitute the first one in). Thus, we the economy can be described as a system of two dynamic equations

$$\Pi_t=\beta\alpha_0+\beta\alpha_1\left(\Pi_{t-1}-\Pi^*\right), \tag{DS.1}$$

$$\frac{1}{\theta}\left\{G-\gamma_0+\gamma_1 b^*-m_t(\Pi_t)+\frac{m_{t-1}(\Pi_t)}{\Pi_t}\right\}+\left\{\frac{1}{\beta}-\frac{\gamma_1}{\theta}\right\}b_{t-1}=b_t, \tag{DS.2}$$

Inflation at steady state is Π^*, the monetary eigenvalue is $\beta\alpha_1$, while the fiscal eigenvalue is $\frac{1}{\beta}-\frac{\gamma_1}{\theta}$.

Case 2: q^C satiated

In this equilibrium we have that q^C is satiated. Then $s_t^L=0$ so that the dynamic system can be reduced to 4 equations in $(\Pi_{t,}b_t,m_t,m_t^F)$:

$$\Pi_t=\beta\alpha_0+\beta\alpha_1\left(\Pi_{t-1}-\Pi^*\right)+\beta\delta\eta^{CB}\left(\frac{\Pi_t^F}{\beta}-1\right), \tag{DS.1}$$

$$\frac{1}{\theta}\left\{G-\gamma_0+\gamma_1 b^*-m_t+\frac{m_{t-1}}{\Pi_t}\right\}+\left\{\frac{1}{\beta}-\frac{\gamma_1}{\theta}-\delta\eta^{CB}\left(\frac{\Pi_t^F}{\beta}-1\right)\frac{\Omega^H}{1+\Omega^H}\frac{1}{\Pi_t}\right\}b_{t-1}=b_t, \tag{DS.2}$$

$$\frac{\Pi_t}{\beta}-1=\mu_M\left[u_{1,t}\left(\frac{m_{t-1}}{\Pi_t},0\right)-1\right], \tag{DS.3}$$

$$\frac{\Pi_t^F}{\beta}-1=\sigma_F\left[\frac{1}{\nu_t^F}u_{2,t}\left(0,\frac{1}{\upsilon_t^F}\left(\frac{m_{t-1}^F}{\Pi_t^F}+\delta\eta^{CB}\frac{\Omega^H\theta}{1+\Omega^H}\frac{b_{t-1}}{\Pi_t}\right)\right)-1\right]. \tag{DS.4}$$

Note that the last equation is independent from the top 3, this implies that we do not need it to determine inflation and debt dynamics. However, inflation and debt (swaps and QE as well) determines foreign money holdings. The third equation defines m_t and m_{t-1} as a function of Π_t (once we substitute the first one in).

This reduces the system to be of 2 equations given by

$$\Pi_t = \beta\alpha_0 + \beta\alpha_1 (\Pi_{t-1} - \Pi^*) + \beta\delta\eta^{CB}\left(\frac{\Pi_t^F}{\beta} - 1\right), \quad \text{(DS.1)}$$

$$\frac{1}{\theta}\left\{G - \gamma_0 + \gamma_1 b^* - m_t(\Pi_t) + \frac{m_{t-1}(\Pi_t)}{\Pi_t}\right\} + \left\{\frac{1}{\beta} - \frac{\gamma_1}{\theta} - \delta\eta^{CB}\left(\frac{\Pi_t^F}{\beta} - 1\right)\frac{\Omega^H}{1+\Omega^H}\frac{1}{\Pi_t}\right\} b_{t-1} = b_t, \quad \text{(DS.2)}$$

where the monetary eigenvalue is $\beta\alpha_1$, while the fiscal eigenvalue is $\frac{1}{\beta} - \frac{\gamma_1}{\theta} - \delta\eta^{CB}\left(\frac{\Pi_t^F}{\beta} - 1\right)\frac{\Omega^H}{1+\Omega^H}\frac{1}{\Pi_t}$ and is affected by both swaps and QE. For equivalence in Case 2, we need policies that deliver

$$\delta\frac{\Omega^H}{1+\Omega^H} = \hat{Z}_1$$

$$\frac{\delta}{1-\beta\alpha_1} = Z_2.$$

This implies $\Omega^{H'} = \frac{\hat{Z}_1}{\delta' - \hat{Z}_1}$ and $\alpha_1' = \frac{Z_2 - \delta'}{\beta Z_2}$.

Case 1: q^F satiated

Then the dynamic system can be reduced to 4 equations in (Π_t, b_t, m_t, m_t^F) :

$$\Pi_t = \beta\alpha_0 + \beta\alpha_1 (\Pi_{t-1} - \Pi^*) + \beta\delta\eta^{CB}\left(\frac{\Pi_t^F}{\beta} - 1\right), \quad \text{(DS.1)}$$

$$\frac{1}{\theta}\left\{G - \gamma_0 + \gamma_1 b^* - m_t + \frac{m_{t-1}}{\Pi_t}\right\} + \left\{\frac{1}{\beta} - \frac{\gamma_1}{\theta} - \left(\delta\eta^{CB}\left(\frac{\Pi_t^F}{\beta} - 1\right)\Omega^H + \mu_C\eta^L\left[u_{1,t}\left(q_t^C, 0\right) - 1\right]\right)\frac{1}{1+\Omega^H}\frac{1}{\Pi_t}\right\} b_{t-1} = b_t \quad \text{(DS.2)}$$

$$\frac{\Pi_t}{\beta} - 1 = \mu_M\left[u_{1,t}\left(\frac{m_{t-1}}{\Pi_t}, 0\right) - 1\right] + \mu_C\left[u_1\left(q^C, 0\right) - 1\right], \quad \text{(DS.3)}$$

$$\frac{\Pi_t^F}{\beta} - 1 = \mu_C\eta^F\left[u_{1,t}\left(q_t^C, 0\right) - 1\right], \quad \text{(DS.4)}$$

$$q_t^C = \frac{m_{t-1}}{\Pi_t} + \eta^F\frac{m_{t-1}^F}{\Pi_t^F} + \left(\eta^L + \eta^F\eta^{CB}\delta\Omega^H\right)\frac{1}{1+\Omega^H}\theta\frac{b_{t-1}}{\Pi_t}.$$

Note that from the fourth equation, we can write

$$\mu_C\left[u_{1,t}\left(q_t^C, 0\right) - 1\right] = \frac{1}{\eta^F}\left(\frac{\Pi_t^F}{\beta} - 1\right).$$

That allow us to re-write the top three equations as

$$\Pi_t = \beta\alpha_0 + \beta\alpha_1 \left(\Pi_{t-1} - \Pi^*\right) + \beta\delta\eta^{CB}\left(\frac{\Pi_t^F}{\beta} - 1\right), \tag{DS.1}$$

$$\frac{1}{\theta}\left\{G - \gamma_0 + \gamma_1 b^* - m_t + \frac{m_{t-1}}{\Pi_t}\right\} + \left\{\frac{1}{\beta} - \frac{\gamma_1}{\theta} - \left(\delta\eta^{CB}\left(\frac{\Pi_t^F}{\beta} - 1\right)\Omega^H + \frac{\eta^L}{\eta^F}\left(\frac{\Pi_t^F}{\beta} - 1\right)\right)\frac{1}{1+\Omega^H}\frac{1}{\Pi_t}\right\} b_{t-1} = b_t, \tag{DS.2}$$

$$\frac{\Pi_t}{\beta} - 1 = \mu_M\left[u_{1,t}\left(\frac{m_{t-1}}{\Pi_t}, 0\right) - 1\right] + \frac{1}{\eta^F}\left(\frac{\Pi_t^F}{\beta} - 1\right), \tag{DS.3}$$

The third equation allows us to write m as a function of inflation. We get a 2-equation system

$$\Pi_t = \beta\alpha_0 + \beta\alpha_1 \left(\Pi_{t-1} - \Pi^*\right) + \beta\delta\eta^{CB}\left(\frac{\Pi_t^F}{\beta} - 1\right), \tag{DS.1}$$

$$\frac{1}{\theta}\left\{G - \gamma_0 + \gamma_1 b^* m_t(\Pi_t) + \frac{m_{t-1}(\Pi_t)}{\Pi_t}\right\} + \left\{\frac{1}{\beta} - \frac{\gamma_1}{\theta} - \left(\delta\eta^{CB}\left(\frac{\Pi_t^F}{\beta} - 1\right)\Omega^H + \frac{\eta^L}{\eta^F}\left(\frac{\Pi_t^F}{\beta} - 1\right)\right)\frac{1}{1+\Omega^H}\frac{1}{\Pi_t}\right\} b_{t-1} = b_t, \tag{DS.2}$$

The monetary eigenvalue is $\beta\alpha_1$, while the fiscal eigenvalue is $\frac{1}{\beta} - \frac{\gamma_1}{\theta} - \left(\delta\eta^{CB}\Omega^H + \frac{\eta^L}{\eta^F}\right)\left(\frac{\Pi_t^F}{\beta} - 1\right)\frac{1}{1+\Omega^H}\frac{1}{\Pi_t}$ and is affected by both swaps and QE.

Case 3: no satiation at all

Then the dynamic system can be reduced to 4 equations in (Π_t, b_t, m_t, m_t^F) :

$$\Pi_t = \beta\alpha_0 + \beta\alpha_1 \left(\Pi_{t-1} - \Pi^*\right) + \beta\delta\eta^{CB}\left(\frac{\Pi_t^F}{\beta} - 1\right), \tag{DS.1}$$

$$\frac{1}{\theta}\left\{G - \gamma_0 + \gamma_1 b^* - m_t + \frac{m_{t-1}}{\Pi_t}\right\} + \left\{\frac{1}{\beta} - \frac{\gamma_1}{\theta} - \left(\delta\eta^{CB}\left(\frac{\Pi_t^F}{\beta} - 1\right)\Omega^H + \mu_C\eta^L\left[u_{1,t}\left(q_t^C, 0\right) - 1\right]\right)\frac{1}{1+\Omega^H}\frac{1}{\Pi_t}\right\} b_{t-1} = b_t \tag{DS.2}$$

$$\frac{\Pi_t}{\beta} - 1 = \mu_M\left[u_{1,t}\left(\frac{m_{t-1}}{\Pi_t}, 0\right) - 1\right] + \mu_C\left[u_{1,t}\left(q_t^C, 0\right) - 1\right], \tag{DS.3}$$

$$\frac{\Pi_t^F}{\beta} - 1 = \sigma_F\left[\frac{1}{\nu_t^F}u_{2,t}\left(0, q_t^F\right) - 1\right] + \mu_C\eta^F\left[u_{1,t}\left(q_t^C, 0\right) - 1\right], \tag{DS.4}$$

where

$$\nu_t^F q_t^F = \frac{m_{t-1}^F}{\Pi_t^F} + \delta\eta^{CB}\frac{\Omega^H\theta}{1+\Omega^H}\frac{b_{t-1}}{\Pi_t},$$

$$q_t^C = \frac{m_{t-1}}{\Pi_t} + \eta^F\frac{m_{t-1}^F}{\Pi_t^F} + \left(\eta^L + \eta^F\eta^{CB}\delta\Omega^H\right)\frac{1}{1+\Omega^H}\theta\frac{b_{t-1}}{\Pi_t}.$$

Now q^C through the premia on long bonds affects debt dynamics. Also m seems a function of bonds through q^C. We can write

$$q_t^F = \Psi^F \left(m_{t-1}, m_{t-1}^F, b_{t-1}, \Pi_t\right),$$

$$q_t^C = \Psi \left(m_{t-1}, m_{t-1}^F, b_{t-1}, \Pi_t\right).$$

Thus we can have the following

$$\mu_C \left[u_{1,t}\left(q_t^C, 0\right) - 1\right] = \Upsilon \left(m_{t-1}, m_{t-1}^F, b_{t-1}, \Pi_t\right)$$

$$\sigma_F \left[\frac{1}{\nu_t^F} u_{2,t}\left(0, q_t^F\right) - 1\right] = \Upsilon^F \left(m_{t-1}, m_{t-1}^F, b_{t-1}, \Pi_t\right).$$

Then the system becomes

$$\Pi_t = \beta\alpha_0 + \beta\alpha_1 \left(\Pi_{t-1} - \Pi^*\right) + \beta\delta\eta^{CB}\left(\frac{\Pi_t^F}{\beta} - 1\right), \tag{DS.1}$$

$$\frac{1}{\theta}\left\{G - \gamma_0 + \gamma_1 b^* - m_t + \frac{m_{t-1}}{\Pi_t}\right\} + \left\{\frac{1}{\beta} - \frac{\gamma_1}{\theta} - \left(\delta\eta^{CB}\left(\frac{\Pi_t^F}{\beta} - 1\right)\Omega^H + \eta^L \Upsilon\left(m_{t-1}, m_{t-1}^F, b_{t-1}, \Pi_t\right)\right)\frac{1}{1+\Omega^H}\frac{1}{\Pi_t}\right\} b_{t-1} = \tag{DS.2}$$

$$\frac{\Pi_t}{\beta} - 1 = \mu_M \left[u_{1,t}\left(\frac{m_{t-1}}{\Pi_t}, 0\right) - 1\right] + \Upsilon\left(m_{t-1}, m_{t-1}^F, b_{t-1}, \Pi_t\right), \tag{DS.3}$$

$$\frac{\Pi_t^F}{\beta} - 1 = \Upsilon^F\left(m_{t-1}, m_{t-1}^F, b_{t-1}, \Pi_t\right) + \eta^F \Upsilon\left(m_{t-1}, m_{t-1}^F, b_{t-1}, \Pi_t\right), \tag{DS.4}$$

Note we we could use the third equation to solve for m_{t-1} as a function of $m_{t-1}^F, b_{t-1}, \Pi_t$. Then we can use the fourth equation to solve for m_{t-1}^F as a function of b_{t-1}, Π_t. Then we can write both m_{t-1} and m_{t-1}^F as a function of b_{t-1}, Π_{t-1}, once we have also substituted the first equation. We then have the following

$$\Pi_t = \beta\alpha_0 + \beta\alpha_1 \left(\Pi_{t-1} - \Pi^*\right) + \beta\delta\eta^{CB}\left(\frac{\Pi_t^F}{\beta} - 1\right), \tag{DS.1}$$

$$\frac{1}{\theta}\left\{G - \gamma_0 + \gamma_1 b^* - m_t(b_t, \Pi_t) + \frac{m_{t-1}\left(b_{t-1}, \Pi_t\right)}{\Pi_t}\right\} + \left\{\frac{1}{\beta} - \frac{\gamma_1}{\theta} - \left(\delta\eta^{CB}\left(\frac{\Pi_t^F}{\beta} - 1\right)\Omega^H + \eta^L \Upsilon\left(b_{t-1}, \Pi_t\right)\right)\frac{1}{1+\Omega^H}\frac{1}{\Pi_t}\right\} b \tag{DS.2}$$

Proofs

Proof of Proposition ??

With short and long-term public debt priced fundamentally, the dynamic equilibrium can be summarized into the following system of 2-equations:

$$\Pi_t = \beta\alpha_0 + \beta\alpha_1 \left(\Pi_{t-1} - \Pi^*\right), \tag{37}$$

$$\frac{1}{\theta}\left\{G - \gamma_0 + \gamma_1 b^* - m_t(\Pi_t) + \frac{m_{t-1}(\Pi_t)}{\Pi_t}\right\} + \left\{\frac{1}{\beta} - \frac{\gamma_1}{\theta}\right\} b_{t-1} = b_t, \tag{38}$$

where (??) (and (??)) has been used to write money holdings as a function of inflation. From equation (??), we obtain that inflation at steady state is unique and given by $\Pi = \Pi^* = \beta\alpha_0$. Substituting in Π into (??), we can uniquely determine total real debt balances b. The corresponding Jacobian delivers $\beta\alpha_1$ and $\frac{1}{\beta} - \frac{\gamma_1}{\theta}$ as the eigenvalues and, therefore, standard policy prescriptions yield locally determinate equilibria. It is clear that the system of equations is independent of δ and Ω^H. This makes both changes in the swap line rates and on QE-composition irrelevant. However, they do depend on θ. Therefore, QE-level has an effect on allocations both at steady state and in the dynamics.

Proof of Proposition ?? and Lemma 1

From the system described by (??)-(??), it is obvious that steady state inflation is unique and given by $\Pi = \Pi^* - \frac{\delta\eta^{CB}}{(\beta\alpha_1 - 1)}\left(\Pi^F - \beta\right)$ whenever q^C or/and q^F are not satiated. Equations (??) and (??) respectively pin down m and m^F as a function of inflation (inflation and bonds) for Cases 1 and 2 (Case 3). Plugging these and inflation back into (DS.2) yields a unique level of steady state debt b. For Case 1, the dynamic system can be summarized into the following 2 equations:

$$\Pi_t = \beta\alpha_0 + \beta\alpha_1 \left(\Pi_{t-1} - \Pi^*\right) + \beta\delta\eta^{CB}\left(\frac{\Pi_t^F}{\beta} - 1\right), \tag{39}$$

$$\frac{1}{\theta}\left\{G - \gamma_0 + \gamma_1 b^* m_t(\Pi_t) + \frac{m_{t-1}(\Pi_t)}{\Pi_t}\right\} + \left\{\frac{1}{\beta} - \frac{\gamma_1}{\theta} - \left(\delta\eta^F\eta^{CB}\Omega^H + \eta^L\right)\frac{1}{1+\Omega^H}\frac{1}{\Pi_t}\frac{1}{\eta^F}\left(\frac{\Pi_t^F}{\beta} - 1\right)\right\} b_{t-1} = b_t. \tag{40}$$

While for Case 2, the dynamic system becomes

$$\Pi_t = \beta\alpha_0 + \beta\alpha_1 \left(\Pi_{t-1} - \Pi^*\right) + \beta\delta\eta^{CB}\left(\frac{\Pi_t^F}{\beta} - 1\right), \tag{41}$$

$$\frac{1}{\theta}\left\{G-\gamma_0+\gamma_1 b^*-m_t(\Pi_t)+\frac{m_{t-1}(\Pi_t)}{\Pi_t}\right\}+\left\{\frac{1}{\beta}-\frac{\gamma_1}{\theta}-\delta\eta^{CB}\left(\frac{\Pi_t^F}{\beta}-1\right)\frac{\Omega^H}{1+\Omega^H}\frac{1}{\Pi_t}\right\}b_{t-1}=b_t. \tag{42}$$

From the above, it is clear that standard policy prescriptions do not necessarily deliver locally determinate equilibria for Cases 1 and 2. For both cases, the monetary eigenvalue is $\beta\alpha_1$. For Case 1, the fiscal eigenvalue equals $\frac{1}{\beta}-\frac{\gamma_1}{\theta}-\left(\delta\eta^F\eta^{CB}\Omega^H+\eta^L\right)\frac{1}{\eta^F}\left(\frac{\Pi_t^F}{\beta}-1\right)\frac{1}{1+\Omega^H}\frac{1}{\Pi}$, while for Case 2 is $\frac{1}{\beta}-\frac{\gamma_1}{\theta}-\delta\eta^{CB}\left(\frac{\Pi_t^F}{\beta}-1\right)\frac{\Omega^H}{1+\Omega^H}\frac{1}{\Pi}$. Inspection of (**??**)-(**??**) also shows that standard policy prescriptions may not necessarily yield determinacy for Case 3 either. For all cases, we see that swap lines and QE policies affect the steady state and dynamics.

Proof of Proposition ??

For both cases, we assume that the central bank is adjusting its balance sheet, through changes in $(1-\theta^S)$ and $(1-\theta^L)$, to induce a given level of bond liquidity in the hands of the households, i.e. $\Omega^H=\frac{\theta^S}{\theta^L}\frac{1-\theta^L}{1-\theta^S}\Omega^M$.

For Case 1, consider a policy regime defined by $(\delta,\Omega^H,\alpha_1)$ such that $\left[\delta\eta^F\eta^{CB}\,\Omega^H+\eta^L\right]\frac{1}{1+\Omega^H}\equiv Z_1$ and $\frac{\beta\delta}{1-\beta\alpha_1}\equiv Z_2$, where Z_1 and Z_2 are constant. Assume a change in policy $(\delta^{'},\Omega^{H'},\alpha_1^{'})$, this regime delivers the same steady state allocation as the previous policy combination $(\delta,\Omega^H,\alpha_1)$ as long as for every new $\delta^{'}$ the following restrictions on policy are satisfied: (i) $\Omega^{H'}=\frac{Z_1-\eta^L}{\delta'\eta^F\eta^{CB}-Z_1}$ and (ii) $\alpha_1^{'}=\frac{Z_2-\beta\delta^{'}}{\beta Z_2}$. Note that without a change in α_1, different combinations of unconventional monetary policies (δ and Ω^H) can not deliver the same steady state DM consumption when collateral can be used q^C as that would imply a different steady state inflation Π.

For Case 2, replace Z_1 with $\hat{Z}_1\equiv\frac{\delta\Omega^H}{1+\Omega^H}$ but consider the same Z_2. For the alternative policy combinations replace $\Omega^{H'}$ with $\hat{\Omega}^{H'}=\frac{\hat{Z}_1}{\delta'-\hat{Z}_1}$, but consider the same $\alpha_1^{'}$. Define as $\Omega^{\hat{M}'}$ the QE policies that yield $\Omega^{\hat{H}'}$. The new monetary policy combinations $(\hat{\delta}^{'},\Omega^{\hat{H}'},\alpha_1^{'})$ deliver the same steady state.