# Bargaining Competition and Vertical Mergers: The Problem of Model Selection*

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**Abstract**

The assumptions that distinguish bargaining models from one another are rarely observed, but can predetermine their predictions. In this paper, we map various popular bargaining models into vertical merger predictions across two competitive landscapes consisting of either one upstream firm and two downstream firms, or vice-versa. Models' assumptions vary with respect to (i) how parties bargain (Derived Demand vs. Nash-in-Nash vs. Nash-in-Shapley), and (ii) over what (linear wholesale prices vs. two-part prices vs. quantity). The paper and accompanying online vertical merger simulator are designed to help economists and enforcers select the bargaining model that best characterizes observed pre-merger competition, and the loss of such competition following vertical merger, in a given setting. The paper is designed to supplement merger simulators developed by Agency economists.

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# 1 Introduction

Contemporary vertical merger models are complex systems built on:

(i) a network of, e.g., upstream suppliers and downstream retailers
(ii) who bargain bilaterally in the presence of
(iii) externalities created by competition between downstream retailers,
(iv) the intensity of which is determined by a consumer demand surface.

Trying to understand how such a "complex system" works is difficult (Wolfram, 2002). Indeed, in the 2016 AT&T-Time-Warner merger challenge, the first litigated vertical merger case in forty years, the judge called the government's bargaining model a "Rube Goldberg machine" before ruling for the defendants (Caffarra et al., 2018). The trial implicitly raised the question of proper model selection, which in turn motivated a host of academic studies on bargaining (e.g., Sheu and Taragin, 2017; Crawford et al., 2018; Yu and Waehrer, 2018; Froeb et al., 2019; Rogerson, 2020; Slade, 2020), and spurred the Federal Trade Commission and Antitrust Division of the Department of Justice's recent request for public comment regarding bargaining in merger enforcement. Specifically, it asks whether their guidelines' approaches to markets characterized by bargaining is adequate.[1] In what follows, we try to answer this question.

How and over what parties bargain are rarely observed. Accordingly, bargaining model selection is difficult, because the assumptions that distinguish models from one another–threat points, bargaining externalities, and tactics–are also unobserved. Thus, we can select a model based only on its predictions. For vertical merger application, this means selecting a model based on how well it characterizes and predicts observed competition, and the loss thereof following a merger.

The problem of bilateral bargaining model selection has been addressed empirically by Villas-Boas (2007) who infers that manufacturers and retailers (i) bargain efficiently using nonlinear pricing (including slotting allowances), and (ii) that retailers have substantial bargaining power over yogurt sold in a large urban area in the U.S. Bonnet and Dubois (2010) and Bonnet and Réquillart (2013) use a similar methodology to show that manufacturers and retailers adopt two-part pricing with resale price maintenance in the French bottled water and soft drink markets. In these studies, non-nested tests are used to select the best vertical supply model in a given setting, based on how well predictions match observed data. The tests are similar to those of Vuong (1989), Smith (1992), and Rivers and Vuong (2002) who select among models based on how supply responds to various cost shifters.[2]

Bargaining model predictions are, however, quite sensitive to the assumed, but unobserved, features of bargaining. In other words, naive bargaining model selection may inadvertently predetermine our predictions about a merger's effects. In this paper, we characterize this sensitivity so that researchers and enforces can better understand how their modeling choices affect merger predictions. Our approach differs to that of previous studies to the extent that we explicitly allow for bargaining externalities; that is, where one bilateral

[1] See Question 12.a. in the Request for Information on Merger Enforcement (DOJ and FTC, 2022).

[2] Testing different supply models in this fashion does not guarantee identification of a single preferred model, as the condition of transitivity does not necessarily hold (Bonnet and Dubois, 2010).

agreement affects the profitability of others. To this end, we map bargaining unobservables into model predictions by running a series of computational experiments.

We consider three classes of models that describe how parties bargain (Derived Demand vs. Nash-in-Nash vs. Nash-in-Shapley). In Derived Demand, parties undertake take-it-or-leave-it bilateral bargaining over linear prices. In Nash-in-Nash bargaining, parties bargain to split the gains from trade under the assumption that other agreements–hence, threat points–remain fixed. In contrast, threat points are determined recursively and profit is split according to the Shapley value in Nash-in-Shapley bargaining.

For each model, we compare vertical merger predictions when parties bargain over three instruments (linear wholesale prices vs. two-part prices vs. quantity) in two different network structures, $1 \times 2$ (one upstream and two downstream firms) and $2 \times 1$ (two upstream and one downstream firm). These simple structures are not only sufficient to demonstrate the significant differences between bargaining models, but also share the same post-merger equilibrium.[3] Therefore, different models can be calibrated to a common equilibrium, and importantly, different merger effects can be attributed to differences in assumed pre-merger competition. Understanding how these simple structures work is a necessary first step to understanding the effects of vertical mergers in more complex $N \times M$ structures.

To isolate the effects of different bargaining assumptions, we adapt the familiar logit demand to vertical structures.[4] With the so-called "rectangular logit" demand, each upstream product sold through each downstream outlet is a separate logit choice. We calibrate the demand model to the monopoly equilibrium which yields a common set of model primitives. This is equivalent to a traditional "comparative statics exercise," which holds marginal costs and demand constant across models.

Below, we briefly summarize our major findings:

- In all models, vertical mergers give the merged firm a better outside option when bargaining with the non-merging firms, resulting in a bigger profit share. The size of this effect differs across models.
- The Derived Demand model leaves little scope for anti-competitive output reductions after a merger, because the elimination of double marginalization (EDM) is so big.
- When parties bargain over linear prices, Nash-in-Nash and Nash-in-Shapley yield similar results, as there is only one device with which to split profit, and to maximize joint gains. Where aggregate elasticity in the market is low, these models predict anti-competitive mergers.
- When parties bargain over two-part prices, Nash-in-Nash models predict a bigger post-merger profit shift than Nash-in-Shapley. In the $1 \times 2$ setting, the selection between Nash-in-Nash and Nash-in-Shapley bargaining entirely predetermines the predicted competitiveness of mergers.

In sum, we characterize six models and compare their predicted outcomes to show which bargaining assumptions matter, why they matter, and how much they matter. This paper can also be viewed as the users' guide to the accompanying online vertical merger simulator

[3] The one exception is the $1 \times 2$ case with two-part pricing as the merged firm loses the ability to commit without a wholesale price.

[4] In logit, cross-price elasticities are proportional to shares, so that products with big shares are good substitutes to rivals, and small ones are poor substitutes.

tool. It is intended to supplement the merger simulators developed by Agency economists to simulate the loss in quantity, price, and bidding competition.

## 2 Bargaining Models

In this section, we characterize the six pre-merger models of interest, as well as two benchmark cases, Competition and Monopoly. Concise descriptions of these models, in addition to the results of our computational experiments, are given in Tables 2 and 3. We consider three classes of bargaining behavior, as reviewed below:

**Derived Demand**: The earliest vertical merger models were chosen mostly for their tractability. Take-it-or-leave-it bargaining over linear prices, the oligopoly successor to the successive-monopoly model (Church, 2008), leads to prices above that of monopoly (O'Brien and Shaffer, 1992; Moresi et al., 2007). Consequently, the elimination of double marginalization (EDM) following a merger is relatively big, so the scope for anti-competitive outcomes is relatively small.

**Nash-in-Nash**: When parties bargain to split the gains from trade (Horn and Wolinsky, 1988), the size of EDM is smaller which increases the scope for anti-competitive outcomes, e.g., Sheu and Taragin (2017). The popular Nash-equilibrium-in-Nash-bargains has the virtue that it provides easily computable outcomes for complicated bargaining environments which has made it an empirical "workhorse" (Collard-Wexler et al., 2019). However, this tractability comes at a cost: Nash-in-Nash outcomes depend on who earns operating profit, a violation of the Coase Theorem (Froeb et al., 2019) and the Nash assumption that all other agreements are held constant implies intense pre-merger competition (Rey and Vergé, 2020), and big merger effects when such competition is eliminated by vertical merger (Froeb et al., 2019).

**Nash-in-Shapley**: In contrast, when parties bargain in anticipation of how one agreement affects other potential agreements, with the ability to renegotiate, profit is split according to the Shapley value (Stole and Zwiebel, 1996; Inderst and Wey, 2003). de Fontenay and Gans (2014), Yu and Waehrer (2018), and Froeb et al. (2019) have used variants of this approach to characterize vertical mergers, alternatively called "Nash-in-Nash with Recursive Threat Points" (Yu and Waehrer, 2018) or "Nash-in-Shapley" (Froeb et al., 2019) to indicate that the outcome is a Shapley division of surplus determined by noncooperative threat points.

We map these distinct modes of assumed bargaining across different network configurations of bilateral trade. Specifically, we consider two pre-merger industry settings:

- The $1 \times 2$ industry setting of one upstream firm, designated $A$, supplying two downstream firms, designated 1 and 2, with downstream firms competing for final consumers.
- The $2 \times 1$ industry setting of two upstream firms, designated $A$ and $B$, supplying one downstream firm, designated 1, with the downstream firm selling two products to final consumers.

We envision the upstream firm(s) as producing a product at some cost, agreeing to transfer the product to the downstream firm(s), having additional costs in selling to the final consumers. For each industry structure we consider a vertical merger between $A$ and 1. Before the merger, each firm acts to maximize its own final profit. After the merger, the merged firm acts with respect to the merged firm's total final profit.

In both settings, parties bargain over one of three instruments, namely linear (one-part) pricing, assigning a marginal wholesale price; two-part pricing, specifying a marginal wholesale price and a fixed fee; and a specified quantity at a fixed price. For one- or two-part pricing, we assume that marginal wholesale price determines Nash equilibrium consumer prices, and thus product demands, and that parties observe demand. For specified quantity, we assume that consumer prices are set to sell the specified quantities with zero waste. For most cases, specifying quantity is equivalent to two-part pricing. The exception is Nash-in-Nash, which assumes others agreements are held constant in the alternatives to each agreement, so that the form of agreement matters. As noted before, this is a a violation of the Coase Theorem (Froeb et al., 2019).

In order to evaluate agreements between upstream and downstream firms, we need to consider what happens when agreements fail. We assume that a particular product is unavailable to the final consumer when an upstream firm fails to agree with the corresponding downstream firm. For example, if A and 1 fail to agree, then the product A is not available through retailer 1. For the two different market structures we consider the following sets of agreements:

- $1 \times 2$:
  - No agreements: $\{\emptyset\}$
  - $A$ and 1 agree: $\{A1\}$
  - $A$ and 2 agree: $\{A2\}$
  - Both agreements: $\{A1, A2\}$
- $2 \times 1$:
  - No agreements: $\{\emptyset\}$
  - $A$ and 1 agree: $\{A1\}$
  - $B$ and 1 agree: $\{B1\}$
  - Both agreements: $\{A1, B1\}$

If no agreements are reached $\{\emptyset\}$ and no products sold, we assume zero profit for all parties. In the post-merger equilibrium, we assume that the merged parties, e.g., $A$ and 1, are automatically in agreement, so that the terms of agreement are irrelevant to the profit calculation of the merged firm.

In agreements associated with the Derived Demand model, the upstream firm simply dictates terms. For this specification, we do not consider two-part pricing because the upstream firm would set marginal wholesale price to realize a desired downstream price and a fixed fee in order to extract all profit from the downstream firm–an unrealistic scenario. Instead, we consider only the case of linear pricing dictated by the upstream firm(s) maximizing their own profit(s), resulting in the familiar phenomenon of double marginalization.

In other cases, we will assume that each agreement results from a Nash bargaining solution with respect to the parties' total profits over some threat point; that is, the payoff should an agreement not be reached. For two-part pricing, solutions amount to firms maximizing their combined total profit and setting the fixed fee to split equally all profit over the threat point. For linear pricing, the firm(s) sets marginal wholesale price(s) so as to maximize a product of the form $(\pi_A - \pi_A^0)(\pi_1 - \pi_1^0)$, where $\pi$ is the agreement payoff and $\pi^0$ is the threat point for $A$ and 1.[5] For Nash-in-Nash, we assume the threat point is given by profits when all other agreements are deemed fixed (e.g., Sheu and Taragin, 2017). For Nash-in-Shapley, we assume the threat point is determined by profits with all other agreements adjusted for the new set of agreements, each of these determined recursively from cases with fewer agreements (e.g., Froeb et al., 2020; Yu and Waehrer, 2018).

Finally, we consider two benchmark models, Competition and Monopoly. For perfect competition, or "Competition" or "Competition", in the $1 \times 2$ case, downstream firms acquire product at the upstream firm's marginal cost with downstream competition for consumers resulting in Nash equilibrium pricing. In the $2 \times 1$ case, the two upstream firms compete for final consumers through a "transparent" downstream sector, with upstream wholesale prices reflected fully in downstream prices (Froeb et al., 2017). In contrast, for the joint maximizing outcome, or "Monopoly", in both the $1 \times 2$ and $2 \times 1$ cases, prices to consumers are set to maximize the total profit of all firms. When only one of the two products are taken as available to consumers, we consider for comparison a monopoly for only that product.

# 3 Nested Rectangular Logit Demand

Once agreements are made, firms compete in a downstream market. Such downstream competition creates bargaining externalities. This implies that the profit of one agreement is affected by the presence of other agreements, whether it is the same product sold through an alternative retailer (the $1 \times 2$ case), or a different product sold through the same retailer (the $2 \times 1$ case).

To compute the size of the externalities, we assume that each product sold through each downstream firm is a separate choice in a (nested) logit demand, which includes a "no purchase" or outside option. For example, in the $1 \times 2$ case, if both agreements are made, $\{A1, A2\}$, then consumers face a choice of product from $A$ sold through 1, denoted $A@1$, or else the same product sold through 2, denoted $A@2$. When only one agreement is reached, consumers are restricted to a single option and the demand model is suitably adjusted.

In general, there are $n$ inside products, indexed 1 to $n$, together with an outside, no purchase, alternative indexed as 0. If $(V_0, V_1, \ldots, V_n)$ is the $n+1$-tuple of values of a random consumer, we suppose that the consumer chooses alternative $i$ when, for all $j \neq i$, $V_i - p_i > V_j - p_j$. The extreme value distribution has the form:

$$F_i(t) = \Pr(X_i \leq t) = \exp\left(-\exp\left(-\frac{t - (\eta_i - p_i)}{\lambda}\right)\right)$$

For a nested logit model with a nest around inside products, demands and total consumer

[5] It is possible to generalize this product to explicitly consider the effect of differing bargaining powers, as in Crawford et al. (2018).

surplus (up to a constant), are given by:

$$q_i = M \frac{\exp\left(\frac{\eta_i - p_i}{\lambda(1-\tau)}\right)}{S + S^\tau}, \quad q_0 = M \frac{S^\tau}{S + S^\tau},$$

and

$$CS = M\eta^\dagger_{\text{max all}} = M\lambda \log\left(1 + S^{1-\tau}\right),$$

where

$$S = \sum_{i=1}^{n} \exp\left(\frac{\eta_i - p_i}{\lambda(1-\tau)}\right)$$

and $S$ is the quantity share of the inside goods and $CS$ is consumer surplus. The derivation of the system of equations for the nested rectangular logit demand of concern are detailed in Appendix A.1.

# 4 Calibration

To compare pre- to post-merger outcomes across models, we recover a single set of parameters from the monopoly equilibrium, and then use the parameters in all of the computational experiments. Specifically, we fix the nest strength parameter ($\tau$), scaling parameter ($\lambda$), and recover the location parameters from the initial prices and quantities of the inside goods. We compare predictions of the models over different levels of aggregate elasticity of demand, determined by the quantity share of the inside goods.

The location parameters ($\eta_i$) of the logit demand function are determined as follows:

$$\log\left(\frac{q_i}{\sum_{j=1}^{n} q_j}\right) = \frac{\eta_i - p_i}{\lambda(1-\tau)} - \log(S)$$

so

$$\eta_i = p_i + \lambda(1-\tau)\left(\log\left(\frac{q_i}{\sum_{j=1}^{n} q_j}\right) + \log(S)\right)$$

with

$$S = \left(\frac{q_0}{\sum_{i=1}^{n} q_i}\right)^{1/(\tau-1)}$$

The demand model is calibrated to prices and elasticities appropriate for the monopolist. In the $1 \times 2$ and $2 \times 1$ case, it is assumed that the upstream firm(s) have zero marginal cost and that the marginal cost of the downstream firm(s) are inferred from the monopoly equilibrium, assuming that prices are set optimally. Specifically, for two products with total marginal costs $mc_{tot1}, mc_{tot2}$, the monopolist maximizes

$$\text{profit}_M = (p_1 - mc_{tot1})q_1 + (p_2 - mc_{tot2})q_2$$

and chooses prices that satisfy the first-order conditions:

$$0 = q_1 + (p_1 - mc_{tot1})\frac{\partial q_1}{\partial p_1} + (p_2 - mc_{tot2})\frac{\partial q_2}{\partial p_1}$$
$$0 = q_2 + (p_1 - mc_{tot1})\frac{\partial q_1}{\partial p_2} + (p_2 - mc_{tot2})\frac{\partial q_2}{\partial p_2}$$

This provides a system of two linear equations that can easily be solved for the total marginal costs. For the nested logit model, the following holds:

$$0 = q_1 + (p_1 - mc_{tot1})\left(q_1 + q_1 s_1\frac{f'(S)}{f(S)}\right) + (p_2 - mc_{tot2})\left(q_1 s_2\frac{f'(S)}{f(S)}\right)$$
$$0 = q_2 + (p_1 - mc_{tot1})\left(q_2 s_1\frac{f'(S)}{f(S)}\right) + (p_2 - mc_{tot2})\left(q_2 + q_2 s_2\frac{f'(S)}{f(S)}\right)$$

Dividing by quantities and subtracting shows $p_1 - mc_{tot1} = p_2 - mc_{tot2}$, i.e., any difference in the monopolist's pricing reflects differences in total marginal cost

Results are invariant to the units of price and we set the quantity weighted average price $\bar{p} = 1$. Similarly, we set the total quantity of inside products to $q_{tot} = 100$. We further assume that the prices, quantities and marginal cost of the two products are equal (i.e. they are balanced). This list of parameters: the initial prices, inside quantities, outside quantities, nest parameter, aggregate elasticity, scaling parameter and location parameters along with the conventions for $\bar{p} = 1$, $q_{tot} = 100$ and marginal costs inferred from monopoly pricing allow for the calibration of the demand model.

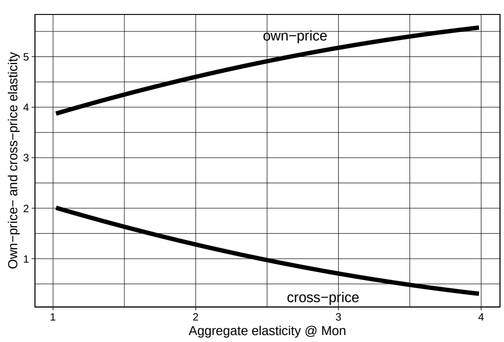



Figure 1: Calibration of elasticities vs ae

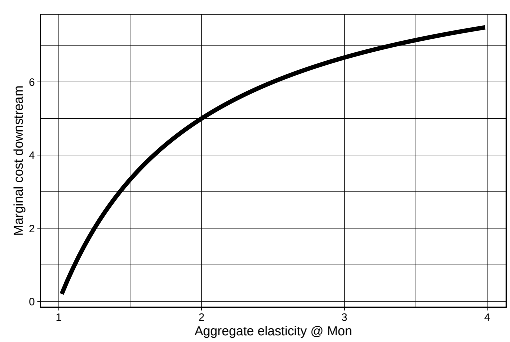



Figure 2: Calibration of marginal cost vs ae

We vary aggregate elasticity by increasing the quantity of the outside good and then recalibrate the the monopoly equilibrium after each exogenous increase of the outside quantity. In Figure 1, we see that this causes own- and cross-price elasticity to move in opposite directions. As initial prices and quantities are held fixed, an increase in aggregate elasticity ($ae$) increases the own-price sensitivity of demand for- and decreases the substitutability between inside goods. In other words, an exogenous increase in the quantity of the outside goods is emblematic of an increase in its relative attractiveness, and therefore greater diversion from the inside goods. This is coincident with a decrease in the location parameter of the gross valuations of inside goods ($\eta_i$).

In Figure 2, as aggregate demand becomes more elastic, individual demand also becomes more elastic, and hence margins decrease. Because cost is fixed, inferred marginal costs increase in both the $1 \times 2$ and $2 \times 1$ settings.

# 5 Results

Based on our calibrated models, we now study how alternative assumptions affect pre- and post-merger predictions within different contractual and industry settings. First, we show that the results for the non-bargaining models are on par with the traditional vertical integration literature. Second, we consider whether alternative assumptions about threat points matter, both for linear and nonlinear pricing. As expected, a linear pricing setup renders assumptions about the type of threat point less important. Importantly, under nonlinear pricing, different assumptions about threat points can predetermine predicted merger outcomes.

In this discussion, we focus on the simulated effects on total industry output, as it closely tracks the corresponding consumer welfare outcome. Figures 3 and 4 show how total quantity relates to aggregate elasticity for all of the pre- and post-merger models of interest.[6] The main results are presented in three separate panels. In the first panel, non-bargaining models are presented together; that is, Competition (COMP), Monopoly (MONOP), and Derived Demand (DD). In the second panel, Nash-in-Nash (NiN1), and Nash-in-Shapley (NiS1) are presented together, as these models assume bargaining over linear wholesale prices. In the third panel, Nash-in-Nash (NiN2), Nash-in-Shapley (NiS2), and Nash-in-Nash quantity (NiNQ) all assume bargaining over two components, namely wholesale price and -fixed fee, or fixed quantity and wholesale fixed fee. In these figures, arrows are drawn from pre-merger equilibria to a single post-merger equilibrium (VMDD, VM1, or VM2). If the arrows point up, quantity increases and the merger is deemed pro-competitive. Contrarily, if arrows point down, the merger is anti-competitive.

[6] The simulated results for the various individual outcome variables, i.e., firms' quantities, prices, wholesale prices and fees, and profits, are illustrated in Appendix A.2.

Figure 3: Total quantity vs aggregate elasticity $1 \times 2$ setting

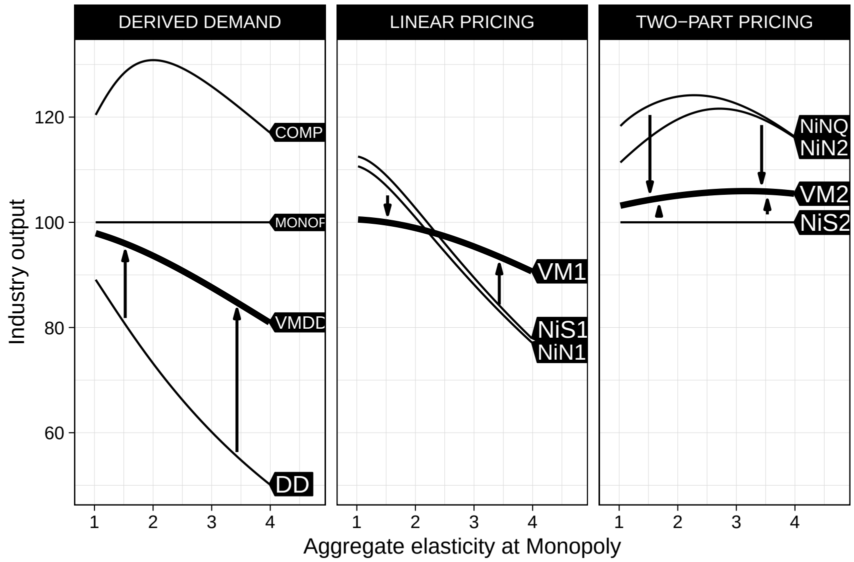
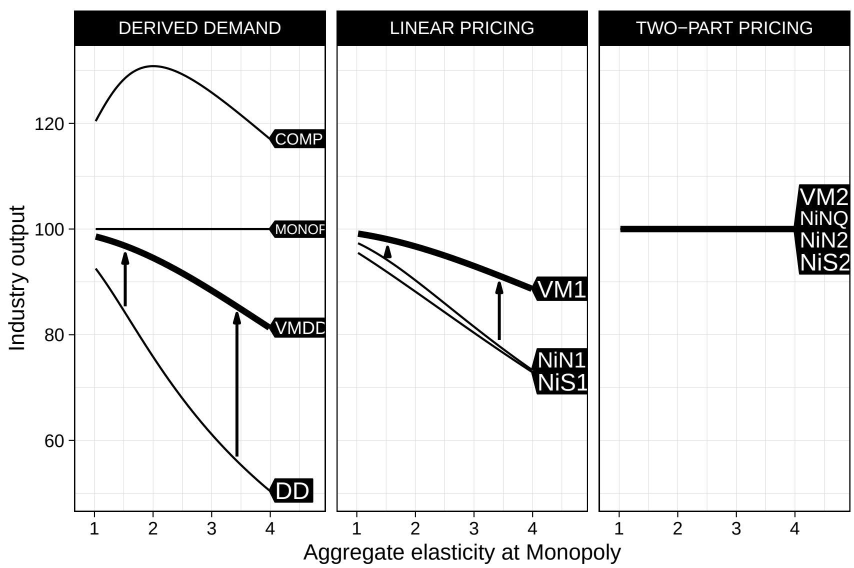



Figure 4: Total quantity vs aggregate elasticity $2 \times 1$ setting




These graphs demonstrate that vertical mergers evoke two opposing competitive effects. For the vertically integrating firms, an elimination of double marginalization (EDM) occurs. This pro-competitive effect sees the pre-merger wholesale price charged to the downstream merging firm erased post-merger.[7] Contrarily, the anti-competitive effect of raising of rival's cost (RRC) occurs in the $1 \times 2$ setting. This sees the vertically integrated upstream firm increasing the wholesale price to its downstream rival. Alternatively in the $2 \times 1$ setting, the anti-competitive effect of reducing of rival's revenue (RRR) takes place. This occurs

[7] Post-merger wholesale price for the merging downstream firm is absent in Figures 6(a) and 9(a).

when the vertically integrated downstream firm decreases the wholesale price it pays to its upstream rival.[8]

The EDM resulting from a vertical merger increases the quantity and decreases the downstream price of the vertically integrated firm's good in both $1 \times 2$ and $2 \times 1$ settings.[9] On the other hand, the RRC or RRR resulting from a vertical merger decreases the quantity and increases the downstream price of the rival firm.[10] The simultaneous occurrence of these contending effects affirms our focus on a vertical merger's net effects in terms of total industry output, as illustrated in Figures 3 and 4.

In what follows, we systematically evaluate the simulated results of models in their respective contractual setting. In addition to the variable-specific graphs illustrating the results of all models in all settings, Appendix A.2 also contains the tabulated summaries of our main findings vis-à-vis total industry output in Figures 3 and 4. Therein, Table 2 presents the results for derived demand and linear pricing models. Table 3 presents the results for two-part pricing models. Finally, Table 1 summarizes the grounds for our argument that bargaining model selection can predetermine the predicted competitiveness of vertical mergers.

## 5.1 Derived Demand: Maximum Double Marginalization

In the first panel of Figures 3 and 4, we present the traditional Derived Demand model for a vertical relationship,in addition to our benchmark models of perfect competition and monopoly. Benchmarks serve as useful reference points when comparing predicted merger effects, particularly the monopoly outcome which exhibits the joint maximizing outcome.[11] Since a vertical merger typically implies that firms are able to jointly maximize profits, predicted effects can be evaluated against this benchmark.

Along the DD locus, the upstream firm makes a take-it-or-leave-it linear wholesale price offer to the downstream firm(s). The wholesale price contributes to the downstream firm's total marginal cost, who in turn marks it up as well. In short, double marginalization is maximized in DD. Consequently, total industry output is substantially lower than that of the monopoly outcome.

Along the VMDD locus, the outcomes of post-merger derived demand are presented. The upstream firm still makes a take-it-or-leave-it offer to the unintegrated downstream firm, but the downstream equilibrium is no longer symmetric. The captive downstream firm gains share because the perceived margin on sales to its product is bigger than the margin on sales to its rival. As a result, it can decrease the price of its own product, which increases its output, but reduces the sales of its rivals. The substantial increase in the vertically integrated entity's output outweighs the decrease in rival's quantity leading to an increase in total output.

The upward shift from the DD to VMDD loci indicates what is already well-documented: A vertical merger in a derived demand setting moves the market closer to the monopoly

[8]See the RRC and RRR in Figures 6(b) and 9(b), respectively.

[9]See Figures 5(a) and 5(c) for the $1 \times 2$ setting, and Figures 8(a) and 8(c) for $2 \times 1$.

[10]See Figures 5(b) and 5(d) for the $1 \times 2$ setting, and Figures 8(b) and 8(d) for $2 \times 1$.

[11]Throughout this article, we refer interchangeably to the joint maximizing outcome and monopoly outcome.

outcome by raising total quantity, which is echoed by the change in consumer surplus. In accordance with previous findings, e.g., Moresi et al. (2007) and Church (2008), our Derived Demand model will strictly predict a pro-competitive vertical merger, because the EDM will always outweighs the potential anti-competitive effect; that is, RRC in a $1 \times 2$ setting or RRR in a $2 \times 1$ setting.

## 5.2 Linear Pricing: One Device, Two Goals

Now, consider the impact of bargaining over linear prices in the second panel of Figures 3 and 4. Along the NiN1 and NiS1 loci we assume bargaining over linear wholesale prices of goods. Each bilateral bargain is assumed to be reached on the basis of a Nash bargaining solution in relation to threat points specified in accordance with either NiN1 or NiS1. For the NiN1 model, a party's threat point for a particular agreement is taken, whilst assuming the continuation of the other agreement at the same wholesale price. The NiS1 model allows for renegotiation, or alternatively, bilateral contracts contingent on which other agreements are made. Thus, threat points are specified for an other agreement at a new wholesale price, also satisfying the Nash bargaining solution for a single good. After establishing their respective wholesale prices, downstream firms set retail prices in Nash equilibrium.

NiN1 and NiS1 models produce virtually identical predictions. This is because the distinction between NiN and NiS–the treatment of threat points–is almost completely eliminated in linear pricing negotiations. In linear pricing, there exists only one device–the wholesale price–with which to achieve two conflicting goals. First, the wholesale price determines downstream prices, quantities, and ultimately, profits. In turn, the downstream equilibrium determines the wholesale quantities that the upstream firm will sell. Therefore, the wholesale price is the only device the upstream firm can use to increase total industry profits. Secondly, the wholesale price is the only mechanism by which the upstream firm determines its profits. These goals are in conflict: a lower wholesale price is better for the former goal, but a higher wholesale price is better for the latter. This tension is present in both NiN and NiS bargains, as well as the wholesale prices determining their respective threat points, rendering them almost indistinguishable.

As opposed to a take-it-or-leave-it scenario, the introduction of bilateral bargaining enables downstream firms to also exert their influence on wholesale prices. These firms are indeed able to negotiate for lower wholesale prices than in the Derived Demand model.[12] Thus, the introduction of introduction of bilateral bargaining leads to higher total output for both industry structures.

The VM1 locus presents the post-merger outcomes for both NiN1 and NiS1 models. The merged firm negotiates a wholesale price with the now-rival firm in a Nash bargaining setting, after which asymmetric retail prices are determined in a Nash equilibrium. Again, the pro-competitive EDM increases output for the vertically integrated firm. The anti-competitive

[12] This effect is evident when comparing wholesale prices in the first and second panels of Figures 6(a) and 6(b) for the $1 \times 2$ setting, and Figures 6(a) and 6(b) for $2 \times 1$. It is illustrative of so-called "bargaining effects" (Horn and Wolinsky, 1988); that is, differences in predictions that arise when accounting for the externalities associated with bargaining competition.

RRC ($1 \times 2$ setting) or RRR ($2 \times 1$ setting) reduces the output of the rival. The relative magnitudes of these effects determine the ultimate predicted effects of a vertical merger.

Linear pricing in the $1 \times 2$ setting is our only simulated case in which the competitiveness of a vertical merger depends on the level of aggregate elasticity. When aggregate elasticity is low, a merger decreases the total industry output, as exhibited in Figure 3. In this setting, pre-merger wholesale prices are high. While this leads to substantial post-merger EDM, the vertically integrated firm also manages to affect RRC.[13] The latter outweighs the former when aggregate elasticity is low, and thus yields a welfare-decreasing vertical merger.[14] At high aggregate elasticity, the opposite holds.

As evidenced in Figure 4, linear pricing in the $2 \times 1$ setting consistently yields a pro-competitive merger for all levels of aggregate elasticity. It is similar to the $1 \times 2$ case insofar mergers are more pro-competitive at higher aggregate elasticities. However, the size of the pro-competitive effect of a merger, i.e., EDM, is smaller for every level of aggregate elasticity compared to that of its Derived Demand counterpart.

### 5.3 Two-part Pricing: Threat Points Matter

The particulars of bargaining, specifically the treatment of threat points, matter greatly once we consider two-part pricing. We deal with this contractual setting in the third panel of Figures 3 and 4, where we assume bargaining over wholesale prices and fees. Each bargain is assumed to be reached on the basis of a Nash bargaining solution relative to either a NiN2, NiNQ or NiS2 specified threat point.[15]

For all industry structures and bargaining types, parties wish to increase industry profits when negotiating over the wholesale price, hence the incentives of bargaining players are aligned. For NiN2 and NiS2, wholesale prices are set to maximize surpluses from agreement and fees are set such that profits over threat points are split equally. For the NiNQ model, the quantity is set to maximize surplus from agreement and the total price splits the profit over the threat point equally. As before, downstream firms subsequently set retail prices in Nash equilibrium.

In the $1 \times 2$ setting, downstream firms earn profit

$$\pi_i = (p_i - mc_i - w_i) \times q_i - f_i, i = 1, 2$$

and pay upstream firm(s) $w_i \times q_i + f_i$, where $w_i$ is the marginal wholesale price and $f_i$ is a fixed fee. The upstream firm earns profit

$$\pi_A = ((w_1 - mc_{A1}) \times q_1 + f_1) + ((w_2 - mc_{A2}) \times q_2 + f_2)$$

$mc_{A1}$ and $mc_{A2}$ are the potentially different upstream marginal costs of supplying goods to the respective downstream firms. For convenience, we assume that $mc_{A1} = mc_{A2} = 0$. $A$ and

[13] The second panels of Figures 6(a) and 6(b) illustrate said EDM and RRC, respectively.

[14] See the second panel of Figure 7(b).

[15] The threat points for NiN and NiS are as described in Section 5.2, being that NiS allows for renegotiation and NiN assumes continuation at the same wholesale price or quantity.

1 negotiate agreement $A1 = (w_1, f_1)$ while taking into consideration agreement $A2$. Similarly, $A$ and 2 negotiate agreement $A2 = (w_2, f_2)$ while taking into consideration agreement $A1$. In these negotiations, the assumed status of the other agreement characterizes the type of bargaining.

In the $2 \times 1$ setting, we invert the bargaining setup by assuming that the downstream monopolist (1) contracts with upstream firms ($A$ and $B$) to produce goods for sale by firm 1. 1 and $A$ bargain over contract $A1 = (w_1, f_1)$, while taking into consideration agreement $B1$. Similarly, 1 and $B$ negotiate agreement $B1 = (w_2, f_2)$, while taking into consideration agreement $A1$. Again, the assumptions we make about the status of the other agreement characterize either NiN or NiS bargaining.

With two-part pricing, there are two different devices with which to achieve the two conflicting goals set out in Section 5.2. Thus, when determining the wholesale price, there is now an incentive for the pivotal player (the firm involved in both bilateral bargains) to internalize competition in the opposing market.[16] The differences in the treatment of threat points–which distinguish types of bargaining–now matter greatly, because this incentive gives rise to different outcomes depending on the assumed bargaining type. These differences are detailed below.

### 5.3.1 Nash-in-Nash: Industry Setting Matters

For NiN2 in the $1 \times 2$ setting, 1 and $A$ negotiate agreement $A1 = (w_1, f_1)$ by assuming that agreement $A2 = (w_2, f_2)$ is fixed. To compute the equilibrium, we determine the conditions under which $A$ and 1 can increase joint profit by reaching a different agreement. This occurs only if a change leads to an increase in their joint profit

$$\Delta(\pi_A + \pi_1) = \Delta(q_1 \times (p_1 - mc_1) + w_2 \times q_2) > 0.$$

Wholesale payments are cancelled out as they serve as revenue to $A$, but costs to 1.

Intuitively, $A$ and 1 promote their joint profit at the expense of 2. Yet, when $A$ and 2 bargain, they attempt the same. NiN equilibrium occurs at a point where it is no longer profitable for either of the pairs $(A, 1)$ or $(A, 2)$ to deviate from the agreement such that, e.g., for $A$ and 1

$$w_1^* = \underset{mc_1}{\operatorname{argmax}}(\pi_A + \pi_1).$$

The fixed fee $f_1$ is chosen to split surplus by maximizing the product of the surpluses so that

$$f_1^* = \underset{f_1}{\operatorname{argmax}}(\pi_A - \pi_A^*)(\pi_1 - \pi_1^*).$$

In the case of transferable utility this reduces to

$$\pi_A - \pi_A^* = \pi_1 - \pi_1^*,$$

[16] The opposing market is the the upstream market in a $2 \times 1$ setting, and the downstream market in a $1 \times 2$ setting.

which is exactly why both $A$ and 1 want to maximize $\pi_A + \pi_1$ at the previous step. Here $\pi_1^* = 0$, but $\pi_A^* = (w_2^* - mc_2) \times q_2^* + f_2$ with only $q_2^*$ manufactured by 2.

What differentiates NiN from NiS is the suboptimal form of the former's contracts, as they would remain fixed even if the other agreement fails. In the NiN equilibrium $\{A1^*, A2^*\}$ for the $1 \times 2$ industry, the sub-optimality of this contractual form manifests as joint output above $(q_1^* + q_2^*)$ and joint profit $(\pi_1^* + \pi_2^*)$ below monopoly levels.[17] NiN leads the pair $(A, 1)$ to compete with $(A, 2)$, thereby lowering wholesale prices to maximize (almost) independent profits in Nash equilibrium. This has the ambiguous consequence of $A$ competing with- or bargaining against itself in the two negotiations.

In the $2 \times 1$ setting, we consider NiN bargaining between 1 and $A$ who negotiate by assuming that agreement $B1 = (w_2, f_2)$ is fixed. In $B1$, 1 pays $w_2 \times q_2 + f_2$ to $B$, depending on the quantity $q_2$ that 1 chooses to sell in order to maximize its final retail profit. In contrast to the $1 \times 2$ setting, the equilibrium is the joint profit maximizing outcome.

To illustrate, we show that neither firm has an incentive to change wholesale prices from marginal costs, $w_i = mc_i,\ i = A, B$. If $w_1$ was lower than $mc_A$, 1's total operating profit would increase but the total joint profit $\pi_1 + \pi_A = (p_1 - mc_A) \times q_1 + (p_2 - mc_B) \times q_2$ would decrease, as 1 sets retail prices to maximize $\pi_1 = (p_1 - w_1) \times q_1 + (p_2 - mc_B) \times q_2$. This would result in a price lower than the monopoly price for $p_1$, and correspondingly, if 1 and $A$ were to raise $w_1$ above $mc_A$. The pair $(1, A)$ has no incentive to deviate from the $w_1 = mc_A$ marginal wholesale price, and likewise $(1, B)$ will not deviate from $w_2 = mc_B$. The downstream firm 1 takes these wholesale prices as given and finds the monopoly retail prices and quantities maximizing joint surplus $(\pi_1 + f_1 + f_2)$ given that $f_1$ and $f_2$ are fixed.

Thus, NiN bargaining leads to results that depend on the industry structure given that wholesale prices are cost-based. This accords with the findings of, among others, O'Brien and Shaffer (1992) and Rey and Vergé (2020). While the pivotal player manages to completely internalize competition in the opposing market in the $2 \times 1$ setting, it is unable to do so in the $1 \times 2$ setting. In the latter, NiN models produce considerably more competitive pre-merger industry outcomes.[18]

Fixing the quantity and total price instead of the wholesale price and fee, as in NiNQ, produces outcomes even closer to perfect competition. NiN2 and NiNQ are distinct insofar NiN2's joint profit function for $(A, 1)$ includes $q_2$, and $(A, 2)$ includes $q_1$. To some extent, these terms internalize some of the "schizophrenia" exhibited by the pivotal player bargaining against itself in the $1 \times 2$ setting (Collard-Wexler et al., 2019). In NiNQ, the joint profit of $(A, 1)$ is not a function of $q_2$, and $(A, 2)$ is not a function of $q_1$. Hence, bargaining pairs in NiN2 will not compete against each other as vigorously as in NiNQ, where there is no such dependence on these terms.

### 5.3.2 Nash-in-Shapley: Internalizing Competition

In NiS2, the pivotal player takes full cognizance of the externality that the other agreement imposes on the bargain concerned, thereby enabling behavior that completely internalizes

[17] See Figures 3 and 7(c), respectively.
[18] See the third panel of Figure 3.

competition. For example, in the $1\times2$ setting, we assert that firms $A$ and 2 would anticipate a change in conditions if agreement $A1$ fails, and would set $(w_2, f_2)$ for exactly this contingency. This would differ from the contract should agreement $A1$ be made. In turn, this alters the threat point in negotiations with 1. Moreover, it is asserted that firms $A$ and 2 would anticipate how the split of profits determined by $f_2$ would change (through renegotiation) as $w_1$ varies. This leads to higher wholesale prices that signal the downstream firms to price at the joint profit maximizing level, thereby achieving the joint profit maximizing outcome. In the $2\times1$ NiS2 case, the pair $(1, A)$ anticipates the split in profits that 1 will realize with $B$, and so will set $w_1$ to maximize the total surplus $\pi_1+\pi_A+\pi_B = q_1\times(p_1-mc_A)+q_2\times(p_2-mc_B)$. Similarly, $(1, B)$ will maximize the same total surplus. Since 1 will set retail prices to maximize $q_1 \times (p_1 - w_1) + q_2 \times (p_2 - w_2)$, both $A$ and $B$ are satisfied to set $w_1 = mc_A$ and $w_2 = mc_B$, thereby leading to monopoly retail prices. Upstream firms will subsequently collect their share of the maximum possible total surplus.

The NiS2 model consistently predicts the joint profit maximization outcome. As opposed to NiN2, the prediction is not a consequence of industry structure. Instead, the monopoly outcome is a direct result of how NiS2 characterizes bargaining. NiS2 assumes that the total surplus from both agreements is maximized and that the effects of the other agreement are reflected in surpluses. When determining the wholesale price, it is exactly the monopoly outcome that maximizes total surplus in both industry configurations.[19] In other words, the monopoly outcome is robust to a change in the pivotal player, where the operating profit is earned, and where the marginal costs between the pivotal- and other players balances. Thus, the NiS2 model does not suffer from the opportunism problem as is the case in the NiN2 models.

### 5.3.3 Merger Effects and Drivers in Two-part Pricing

The VM2 locus in the third panel of Figures 3 and 4 shows post-merger outcome under two-part pricing. Along this locus, the merged firm negotiates a wholesale price and fee with the now-rival firm in a Nash bargaining setting. Nash equilibrium retail prices, therefore, are asymmetric.[20] As with vertical mergers in other contractual settings, the contending effects of the pro-competitive EDM and anti-competitive RRC in the $1 \times 2$ setting, or RRR in the $2 \times 1$ setting, are evoked. The net predicted competitiveness of the merger depends on their relative magnitudes. Below, we characterize two categories of net merger effects observed in our two-part pricing results, namely a) moving away from joint profit maximization, and 2) moving towards joint profit maximization.

A vertical merger usually implies that firms are able to achieve the joint profit maximizing outcome. However, in the $2 \times 1$ setting, all bargaining models under two-part pricing are already at the pre-merger joint profit maximization level, as is the NiS2 model in the $1 \times 2$ setting. Therefore, it is only the latter that moves away from joint profit maximization towards a more competitive post-merger outcome. In this case, the EDM of the vertically integrated firm does occur, but the now-integrated pivotal player upstream also inherits the

[19] See the third panels of Figures 3 and 4.

[20] Compare VM2 in Figures 5(c) and 5(d) for the $1 \times 2$ setting, and Figures 8(c) and 8(d) for $2 \times 1$.

marginal cost from its downstream firm.[21] It now acts as if this, and not the transfer price (which is zero by assumption), is its true marginal cost.

The rival downstream firm's total marginal cost is, however, partly determined by negotiation with the vertically integrated firm. In this negotiation, the vertically integrated firm cannot effect a commitment to raise retail prices to monopoly prices, as we assume firms are prohibited from setting retail prices as part of their negotiations. The vertically integrated firm also cannot credibly commit to the price at the monopoly level because of the change in its marginal cost. Therefore, the vertically integrated firm reduces its price and increases quantity.

In addition, given that NiS2 pre-merger wholesale prices are higher than that of NiN2 in $1 \times 2$, the scope for RRC for NiS2 is relatively diminished. As such, we observe some of the smallest increases in the downstream rival's wholesale price, and smallest decreases in their output.[22] Coupled with a greater increase in quantity of the vertically integrated firm, the total quantity increases following a vertical merger in a NiS2 setting.[23]

In the $2 \times 1$ setting, the pivotal player is downstream and does not inherit a marginal cost post-merger. Therefore, the vertical merger does not affect total quantity in this setting. All three models remain at the joint profit maximization equilibrium, hence there arises no pro-competitive EDM or anti-competitive RRR in this setting.[24]

In contrast, the post-merger equilibrium for NiN2 and NiNQ models in a $1 \times 2$ setting renders a move towards joint profit maximization. These are the only models that we consider in two-part pricing that gives rise to an anti-competitive merger. In this setting, the pivotal player in NiN models bargains against itself, thereby considerably heightening pre-merger competitiveness, as detailed Section 5.3.2. Indeed, NiN2 and NiNQ exhibit the lowest total marginal costs for rivals of all the models.[25]

A vertical merger in the $1 \times 2$ setting eliminates the incentive of the upstream firm–the pivotal player–to internalize competition between rival downstream firms. However, it cannot reach the joint profit maximization outcome as in the $2 \times 1$ case for two reasons. First, the pivotal player is now upstream, and thus cannot impose monopoly retail prices as a result of the order of profit maximization.[26] Secondly, the vertically integrated firm has an incentive to RRC such that we observe a substantial increase in the post-merger wholesale price.[27] Therefore, the $1 \times 2$ two-part pricing setting is unique in that merely the specification of the type of bargaining, NiN or NiS, entirely predetermines the predicted competitiveness of a vertical merger.

[21] Recall that we fixed the pre-merger marginal cost of the upstream firm to zero and it was able to induce monopoly prices and quantities by setting the wholesale prices to both downstream firms.

[22] See Figures 5(b) and 6(b). In fact, VM2 renders the lowest post-merger total marginal cost for a rival downstream firm of all the post-merger models in the $1 \times 2$ setting, as illustrated in Figure 6(c).

[23] (See the third panel of Figure 3.

[24] (See the third panel of Figure 4.

[25] See Figure 6(c).

[26] In the $2 \times 1$ case, the pivotal player is downstream and retail prices are determined after wholesale prices. Therefore, it is possible to end up at the monopoly outcome post-merger as well.

[27] See Figure 6(b).

Table 1: Bargaining assumptions can predetermine vertical merger effects

| **Model:** Comment | **Linear pricing** | **Two-part pricing** | **Quantity** |
|---|---|---|---|
| **Derived Demand:** Upstream firm(s) set wholesale linear prices and downstream firm(s) play a noncooperative game taking wholesale prices as given. | Pre-merger output is substantially lower than monopoly output due to large double marginalization. Vertical mergers always expand output as a result of the elimination of double marginalization (EDM). The Derived Demand model leaves no scope for anti-competitive mergers. | *NA* | *NA* |
| **Nash-in-Nash:** Upstream and downstream players bargain bilaterally over linear wholesale, two-part prices or quantities taking other agreements as fixed. Threat points for one agreement are profits in the existing remaining agreements | $1 \times 2$**:** Pre-merger output is above monopoly levels for low aggregate elasticity or high nest strength. Where aggregate elasticity is low or nest strength is high, vertical mergers have anti-competitive effects.<br><br>$2 \times 1$**:** Pre-merger output is consistently below monopoly levels. Vertical mergers will have pro-competitive effects, even more so when aggregate elasticity is high. | $1 \times 2$**:** Pre-merger output above monopoly levels points to a single upstream firm that appears to bargain against itself. Relatively competitive pre-merger conditions imply that vertical mergers will have a considerable anti-competitive effect.<br><br>$2 \times 1$**:** * | $1 \times 2$**:** Pre-merger output above monopoly levels points to a single upstream firm that appears to bargain against itself. Highly competitive pre-merger conditions imply that vertical mergers will have the largest anti-competitive effect.<br><br>$2 \times 1$**:** * |
| **Nash-in-Shapley:** Upstream and downstream players bargain bilaterally over linear wholesale price, but expect prices to change if agreements are not secured. Threat points are determined by renegotiating remaining agreements. | $1 \times 2$**:** Pre-merger output is above monopoly levels for low aggregate elasticity or high nest strength. Where aggregate elasticity is low or nest strength is high, vertical mergers have anti-competitive effects.<br><br>$2 \times 1$**:** Pre-merger output is consistently below monopoly levels. Vertical mergers will have pro-competitive effects, even more so when aggregate elasticity is high. | $1 \times 2$**:** Pre-merger output equalling monopoly levels reflect sophisticated bilateral bargaining between parties that maximize the joint surplus of their coalition. Vertical mergers will consistently produce small pro-competitive effects.<br><br>$2 \times 1$**:** * | *NA* |

* Pre-merger output equals monopoly levels because the downstream monopoly retailer internalizes upstream competition. Vertical mergers will have no effect.

## 6 Conclusion

How parties are assumed to bargain bilaterally–and over what—in the presence of externalities, where one agreement affects the profitability of others, affects vertical merger predictions. This is affirmed in our comparative statics exercise, wherein we compare six vertical relationship models, pre- and post-merger and in different industry settings. Indeed, we show that sometimes these assumptions can entirely predetermine the predicted competitiveness of a vertical merger.

Our chief finding is that the Nash-in-Nash bargaining structure results in intense pre-merger competition and correspondingly bigger merger effects in the simple network structures we examine. With more complex structures, merger effects will depend on the number of non-merging competitors as well, as they do in Cournot, Bertrand, and bidding models. The issue stems from the Nash assumption which computes equilibrium while holding other agreements constant. Though this assumption works well in horizontal models, its appropriateness for bargaining in vertically related markets is not as clear. As a descriptive model, it appears unrealistic to assume that parties would not renegotiate and anticipate how one agreement affects the other agreements they may reach. If they do, Nash-in-Shapley is the result.

We also show that vertical mergers nearly always give rise to two opposing effects, the pro-competitive elimination of double marginalization (EDM), and the raising of rival's cost (RRC) in a $1 \times 2$ industry setting or the reducing of rival's revenue (RRR) in a $2 \times 2$ setting. Our Derived Demand model leaves no scope for anti-competitive output reductions after a merger, because these will always be eclipsed by the EDM. When parties bargain over linear prices, the difference between Nash-in-Nash and Nash-in-Shapley is eroded, given that wholesale price is the only tool with which to maximize industry profits and determine profit shares. Linear pricing in a $1 \times 2$ industry is the only setting in which the predicted competitiveness of a merger is entirely a function of the industry's aggregate elasticity. In the two-part pricing in a $1 \times 2$ industry, the selection between Nash-in-Nash and Nash-in-Shapley bargaining also predetermines the predicted competitiveness of mergers.

In conclusion, ass a model that characterizes behavior, more work needs to be done. The results of this paper can potentially help identify the characteristics of observed pre-merger behavior that can serve as criteria for future model selection, e.g., Werden et al. (2004). Moreover, this paper may prove useful as a guide to identify which bargaining model best characterizes the observed pre-merger competition, or retrospectively, the effects of past mergers in a given setting.

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# A Appendix

## A.1 Derivation of Nested Rectangular Logit Demand

Suppose there are $n$ inside products, indexed 1 to $n$, together with an outside, no purchase, alternative indexed as 0. In our two industry structures, $n$ will be at most 2, with products $A@1$ and $A@2$ in the $1 \times 2$ case or $A@1$ and $B@1$ in the $2 \times 1$ case. Let $p_i$ be the price of the $i$-th inside goods, for each $i$, fixing $p_0 = 0$. Suppose that a consumer sees these products and prices and chooses one, with some total number of choices per specified time period, allowing for some scaling. If $(V_0, V_1, \ldots, V_n)$ is the $n+1$-tuple of values of a random consumer (nominated in the same units as prices), we suppose that the consumer will choose alternative $i$ when, for all $j \neq i$, $V_i - p_i > V_j - p_j$ (ignoring possible ties). Let $X_i = V_i - p_i$ be the net value for alternative $i$, then the total demand for alternative $i$ is $q_i = M \Pr(X_i > X_j, \text{all } j \neq i)$ where $M$ is the total number of consumer choices (during some period). The outside quantity $q_0$ represents those consumers not choosing any of the inside products, usually not observable.

It is convenient to take the $V_i$ to have marginal distributions that are extreme value with the same scale parameter $\lambda$ and various location parameters $\eta_i$. Then the marginal distribution of $X_i$ is also extreme value with scale parameter $\lambda$ and location parameters $\eta_i^\dagger = \eta_i - p_i$, that is, with cumulative distribution function

$$F_i(t) = \Pr(X_i \leq t) = \exp\left(-\exp\left(-\frac{t - \eta_i^\dagger}{\lambda}\right)\right)$$

In fact, these distributions are power-related (Froeb et al., 2001). Anticipating the application below, write $F_i(t) = (F_{\max}(t))^{s_i^{1/\theta}}$, for a parameter $\theta \geq 1$, where

$$F_{\max}(t) = \exp\left(-\exp\left(-\frac{t - \eta_{\max}^\dagger}{\lambda}\right)\right)$$

is the extreme value distribution function with scale parameter $\lambda$ and location parameter $\eta_{\max}^\dagger$, where $s_i = \exp(\theta\eta_i^\dagger/\lambda)/\exp(\theta\eta_{\max}^\dagger/\lambda)$ and $\eta_{\max}^\dagger$ is taken so $\sum_i s_i = 1$, i.e.,

$$\eta_{\max}^\dagger = \frac{\lambda}{\theta}\log\left(\sum_{i=1}^{n}\exp\left(\frac{\theta\eta_i^\dagger}{\lambda}\right)\right)$$

A flat logit demand model results if the $V_i$ (so $X_i$) are taken as independent, but a more general model is only slightly more complex. A Gumbel copula combines well with power-related distributed marginals to give a model with $V_i$ correlated, and these can be simply combined in nests of smaller nests of increasing strengths. For our purposes it suffices to assume that $X_0$ is independent of $X_1 \ldots X_n$, but take these inside goods as a nest in a nested logit demand model. The Gumbel copula is the Archimedean copula given by generator $\psi_\theta(t) = (-\log(t))\theta$ for parameter $\theta \geq 1$ reflecting the strength of the correlation, with $\theta = 1$ the limiting case of independence. For a nest of $n$ variables, the copula is $C(u_1, \ldots, u_n; \theta) = \psi_\theta^{-1}(\psi_\theta(u_1) + \ldots + \psi_\theta(u_n))$, where $\psi_\theta^{-1}(t) = \exp(-t^{1/\theta})$, That is, $C$ is a joint cumulative distribution function with uniform marginals and the joint distribution function of the inside $X_i$ is taken to be

$$\begin{aligned}
F_{1\ldots n}(t_1,\ldots,t_n) &= \Pr(X_i \le t_i, \text{for all } i > 0)\\
&= C(F_1(t_1),\ldots,F_n(t_n);\theta)\\
&= C(F_{\max}(t_1)^{s_1/\theta},\ldots,F_{\max}(t_n)^{s_n/\theta};\theta)\\
&= \psi_\theta^{-1}(\psi_\theta(F_{\max}(t_1)^{s_1^{1/\theta}}) + \ldots + \psi_\theta(F_{\max}(t_n)^{s_n^{1/\theta}}))\\
&= \psi_\theta^{-1}(s_1\psi_\theta(F_{\max}(t_1)) + \ldots + s_n\psi_\theta(F_{\max}(t_n))) \quad \text{and so}
\end{aligned}$$

$$\begin{aligned}
F_{1\ldots n}(t,\ldots,t) &= \Pr(\max_{i>0} X_i \le t)\\
&= \psi_\theta^{-1}((s_1 + \ldots + s_n)\psi_\theta(F_{\max}(t)))\\
&= F_{\max}(t)
\end{aligned}$$

In general, the distribution of the maximum of $n$ random variables having joint distribution given by the Gumbel copula applied to power-related marginal distributions is also power-related. Taking this maximum distribution as the base distribution, the marginal distributions are $F_{\max}(t_i)^{s_i/\theta}$ where the $s_i$ are shares that sum to unity. Moreover, under mild conditions,

$$\begin{aligned}
\Pr(X_i > X_j, \text{all } 0 < j \ne i) &= \int_{\substack{t_i > t_j,\\ \text{all } j \ne i}} dF_{1\ldots n}(t_1,\ldots,t_n)\\
&= \int_{t_i} \frac{\partial}{\partial t_i} F_{1\ldots n}(t_1,\ldots,t_n)\bigg|_{t_1=t_i,\ldots,t_n=t_i} dt_i\\
&= \int_{t_i} (\psi_\theta^{-1})'(s_1\psi_\theta(F_{\max}(t_i)) + \ldots + s_n\psi_\theta(F_{\max}(t_i)))\\
&\qquad\qquad \cdot s_i\psi_\theta'(F_{\max}(t_i))F_{\max}'(t_i)\, dt_i\\
&= s_i \int_{t_i} \frac{\partial}{\partial t_i}(\psi_\theta^{-1}(\psi_\theta(F_{\max}(t_i))))\, dt_i\\
&= s_i
\end{aligned}$$

so that $s_i$ reflects the probability that $X_i$ is the maximum of $X_1,\ldots,X_n$. In fact, the distribution of the maximum of $X_1,\ldots,X_n$ is independent of the identity $X_i$ that realizes that maximum, such that

$$\begin{aligned}
\Pr(X_i \le t | X_i > X_j, \text{all } 0 < j \ne i) &= \frac{1}{s_i}\Pr(t \ge X_i > X_j, \text{all } 0 < j \ne i)\\
&= \frac{1}{s_i}\int_{\substack{t \ge t_i > t_j,\\ \text{all } j \ne i}} dF_{1\ldots n}(t_1,\ldots,t_n)\\
&= \frac{1}{s_i} s_i \int_{t \ge t_i} \frac{\partial}{\partial t_i}(\psi_\theta^{-1}(\psi_\theta(F_{\max}(t_i))))\, dt_i\\
&= F_{\max}(t).
\end{aligned}$$

Taking $X_0$ as independent of the inside $X_i$, $X_0$ combines with the maximum of the $X_i$ in an outside nest with $\theta = 1$. Thus, the maximum of all $X_i$ is extreme value distributed with scale parameter $\lambda$ and location parameter,

$$\eta^{\dagger}_{\text{max all}} = \lambda \log\left(\exp\left(\frac{\eta^{\dagger}_0}{\lambda}\right) + \exp\left(\frac{\eta^{\dagger}_{\text{max}}}{\lambda}\right)\right).$$

Therefore, the probability that $X_0$ is greater than any other $X_i$ is

$$\begin{aligned}\pi_0 = \Pr(X_0 > X_i, \text{all } i > 0) &= \frac{\exp\left(\frac{\eta^{\dagger}_0}{\lambda}\right)}{\exp\left(\frac{\eta^{\dagger}_{\text{max all}}}{\lambda}\right)}\\ &= \frac{\exp\left(\frac{\eta^{\dagger}_0}{\lambda}\right)}{\exp\left(\frac{\eta^{\dagger}_0}{\lambda}\right) + \exp\left(\frac{\eta^{\dagger}_{\text{max}}}{\lambda}\right)}\end{aligned}$$

The probability that $X_i$ is greater than any $X_j$, $j \neq i$, is the probability that $X_i$ is the maximum of $X_j$, $j > 0$, times the probability that the maximum of $X_1, \ldots, X_n$ exceeds $X_0$,

$$\begin{aligned}\pi_i = \Pr(X_i > X_j, \text{all } j \neq i) &= s_i \cdot \frac{\exp\left(\frac{\eta^{\dagger}_{\text{max}}}{\lambda}\right)}{\exp\left(\frac{\eta^{\dagger}_{\text{max all}}}{\lambda}\right)}\\ &= s_i \cdot \frac{\exp\left(\frac{\eta^{\dagger}_{\text{max}}}{\lambda}\right)}{\exp\left(\frac{\eta^{\dagger}_0}{\lambda}\right) + \exp\left(\frac{\eta^{\dagger}_{\text{max}}}{\lambda}\right)}\end{aligned}$$

Adding a constant to all of the $\eta_i$ adjusts $\eta^{\dagger}_{\text{max}}$ and $\eta^{\dagger}_{\text{max all}}$ by the same constant and hence leaves all of the choice probabilities $\pi_i$ unchanged. We conventionally take $\eta_0 = 0$. The expected value of the maximum $X^{\dagger}_i$, the expected difference between the value of the product chosen by a random consumer and the price paid for that choice, is not determined without reference to an actual $\eta_i$, but the change in this quantity between two prices represents the change in consumer surplus in this model.

Sampling from this joint distribution of consumer values is not trivial when $\theta > 1$. A method for sampling from the Gumbel copula follows from work of Marshall and Olkin (1967). Sample $V$ from the Type 1 stable distribution with stability parameter $\alpha = 1/\theta$, skewness parameter $\beta = 1$, scale parameter $\sigma = \cos(\pi/2/\theta)^{\theta}$ and location parameter $\mu = 0$. Take $W_i$ independent uniform $[0, 1]$. Then $U_i = \phi^{-1}_{\theta}(-\log(W_i)/V)$ are jointly distributed with distribution function $C(u_1, \ldots, u_n; \theta)$. From there we can take $X_i = F^{-1}_i(U_i)$ and $V_i = X_i + p_i$, with $V_0$ taken independent extreme value with scale parameter $\lambda$ and location parameter $\eta_0 = 0$.

The correlation between variates defined by a copula is not independent of the marginal distributions, but the Kendall rank correlation coefficient $\tau$ is, as it depends on rank orderings only. For two-particular products, if $p$ is the probability that between two random consumers the one that values one product more highly is also the one that values the other product more highly, then $\tau = 2p - 1$. For the Gumbel copula, $\tau = 1 - 1/\theta$ is between 0 (independent) and 1 (in the limit). Put otherwise, for a specified Kendall $\tau \in [0, 1)$ we may take $\theta = 1/(1-\tau)$ as the nest parameter.

## A.2 Detailed Results

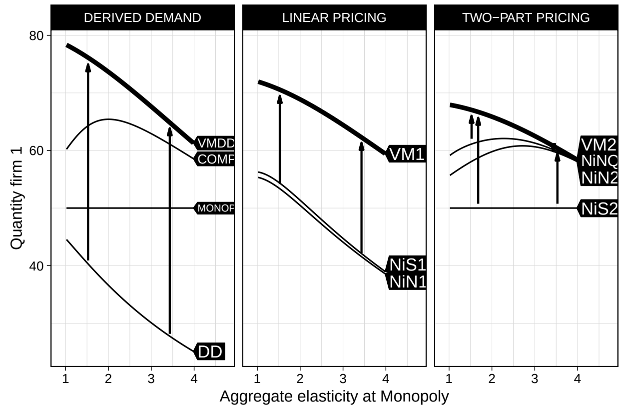
Tables 2 and 3 summarize models' outcomes in terms of total industry output, as observed in Figures 3 and 4. All possible outcome variables, namely retail quantities and prices, wholesale prices and fixed fees, total marginal costs, total industry output and profit, total consumer surplus, and downstream profits, for all models are presented in Figures 5(a) to 7(d) for the $1 \times 2$ industry setting, and Figures 8(a) to 10(d) for the $2 \times 1$ industry setting.

Table 2: Derived Demand and Linear Pricing in Figures 3 and 4

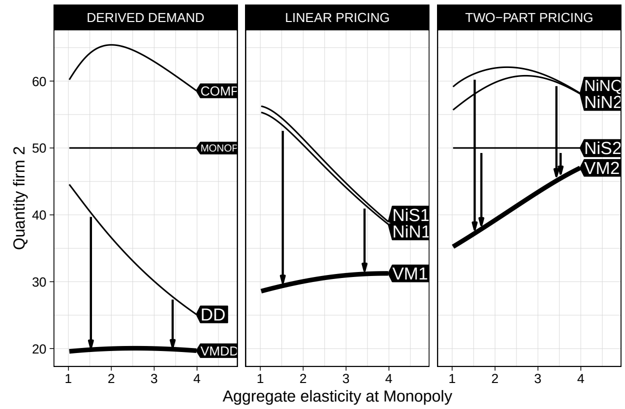
| **Model:** Comment | **Rules** | $1 \times 2$: Results | $2 \times 1$: Results |
|---|---|---|---|
| | **Panel 1** | | |
| **DD:** Derived Demand models generalize the old "successive monopoly" models, by allowing a more general downstream game, e.g. Bertrand or Cournot. | Upstream firm(s) set wholesale linear prices and downstream firm(s) play a noncooperative game taking wholesale prices as given. | Pre-merger output substantially below monopoly output due to large double marginalization. | Pre-merger output substantially below monopoly output due to large double marginalization. |
| **VMDD:** The merged firm eliminates double marginalization (EDM); raises rival's cost (RRC) or reduces rival's revenue (RRR). | Vertically integrated firm sets a wholesale linear price to unintegrated firm, then plays noncooperative game with same in downstream (or upstream) market. | Post-merger output slightly below monopoly output.<br><br>Vertical merger raises output because EDM > RRC. | Post-merger output slightly below monopoly output.<br><br>Vertical merger raises output because EDM > RRR. |
| | **Panel 2** | | |
| **NiN1:** One instrument (linear wholesale price) performs two tasks: determines the size of total profit and how profit is split. Therefore, NiN1 and NiN2 are very similar. | Upstream and downstream players bargain bilaterally over linear wholesale price, taking other agreements as fixed. Threat point for one agreement are profits in the existing remaining agreements. | Output above monopoly levels for low aggregate elasticity. | Output consistently below monopoly levels. |
| **NiS1:** As above, only one instrument (linear wholesale price) performs two tasks, but the alternatives to agreement can vary. | Upstream and downstream players bargain bilaterally over linear wholesale price, but expect prices to change if agreements are not made. Threat point determined by re-negotiating remaining agreements. | Output above monopoly levels for low aggregate elasticity. | Output consistently below monopoly levels. |
| **VM1:** The merged firm eliminates double marginalization. | Vertically integrated firm bargains over linear wholesale prices to unintegrated firm. | When aggregate elasticity is low, vertical mergers have anti-competitive effects (RRC>EDM). This results in fewer lost sales to "no purchase" alternative. | Output somewhat lower than monopoly levels. Vertical mergers always have beneficial effects, as EDM>RRR. |

Table 3: Two-part Pricing in Figures 3 and 4

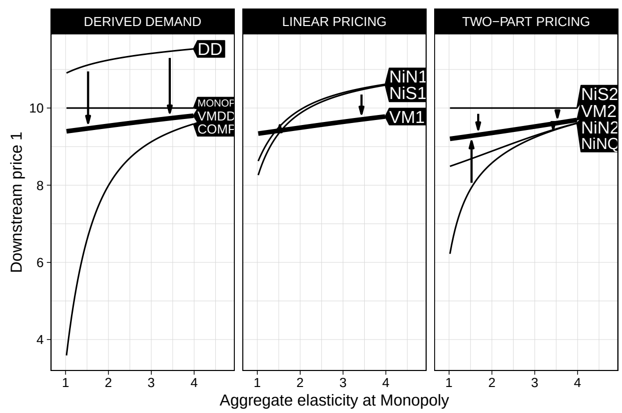
| **Model:** Comment | **Rules** | $1 \times 2$**:** Results | $2 \times 1$**:** Results |
|---|---|---|---|
| | | **Panel 3** | |
| **NiN2:** The NiN assumption that the other agreement is fixed makes parties appear to bargain against themselves. | Upstream and downstream players bargain bilaterally over two part prices, taking other agreements as fixed. Threat point for one agreement are profits in the existing remaining agreements. | Output well-above monopoly levels, as the NiN assumption results in very competitive outcomes. | Pre-merger output equals monopoly levels because downstream monopoly retailer internalizes upstream competition. |
| **NiNQ:** The NiN assumption that the other agreement is fixed makes parties appear to bargain against themselves. | Upstream and downstream players bargain bilaterally over fixed wholesale price and quantity, taking other agreements as fixed. Threat point for one agreement are profits in the existing remaining agreements. | Output well-above monopoly levels, as the NiN assumption results in the most competitive outcome. | Pre-merger output equals monopoly levels because downstream monopoly retailer internalizes upstream competition. |
| **NiS2:** Parties bargain while recognizing that they will share in any improvement of the grand coalition's profits (both agreements made). They are willing to *inter alia* reduce wholesale price if that leads to higher total profit. | Upstream and downstream players bargain bilaterally over two-part prices, but expect prices to change if agreements are not made. Threat point determined recursively by re-negotiating remaining agreements. | Output equals monopoly levels, the joint surplus maximizing outcome. | Pre-merger output equals monopoly levels because downstream monopoly retailer internalizes upstream competition. |
| **VM2:** The merged firm eliminates double marginalization and favours its captive downstream retailer in the $1 \times 2$ case, but not in $2 \times 1$. | Upstream and downstream players bargain bilaterally over two-part prices, but expect prices to change if agreements are not made. Threat points determined recursively by profits in set of agreements without current agreement. | Output above monopoly levels because of "inefficient contracting" Church (2008), i.e., the increased margin on the integrated product due to EDM gives the integrated firm an incentive to increase its sales.<br><br>NiN2: Vertical mergers have big anti-competitive effect.<br>NiNQ: Vertical mergers have the biggest anti-competitive effect.<br>NiS2: Vertical mergers have small pro-competitive effect. | Post-merger output equals monopoly levels because downstream monopoly retailer internalizes upstream competition.<br><br>Vertical Mergers have no effect. |



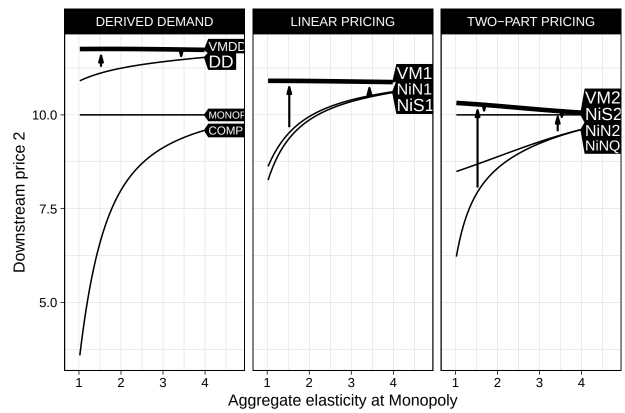

(a) Downstream quantity firm 1 vs. ae




(b) Downstream quantity firm 2 vs. ae

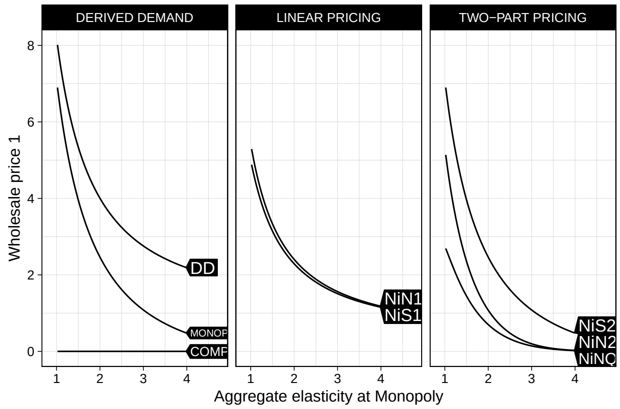



(c) Downstream price firm 1 vs. ae

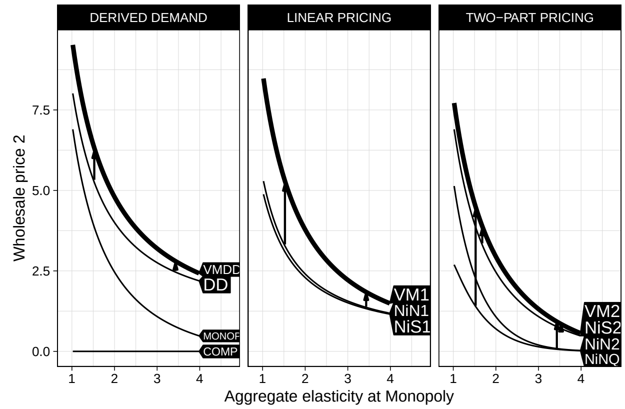



(d) Downstream price firm 2 vs. ae

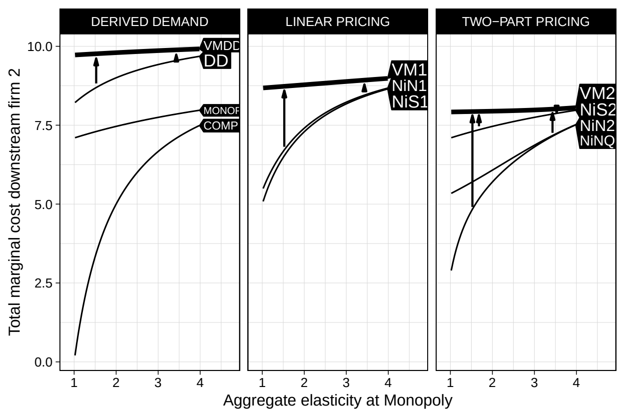
Figure 5: Retail prices and quantities for a 1× setting react in the anticipated manner:
(a) shows an increase in the vertically integrated firm's quantity;
(b) shows a decrease in the rival firm's quantity;
(c) shows an increase in the vertically integrated firm's price;
(d) shows a decrease in the rival firm's price.



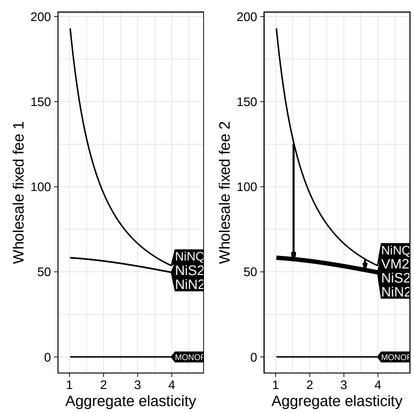

(a) Wholesale price firm 1 vs. ae




(b) Wholesale price firm 2 vs.ae

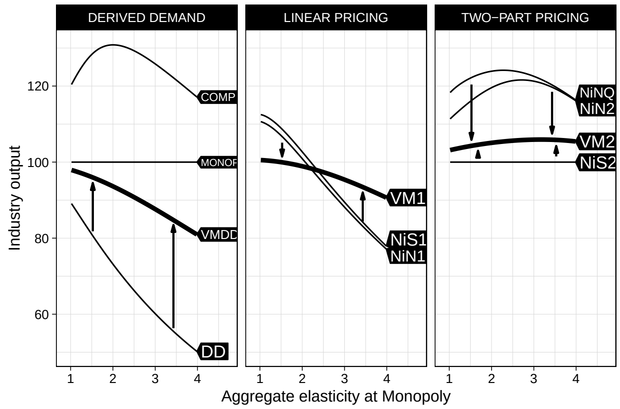



(c) Total marginal cost downstream firm 2 vs.ae

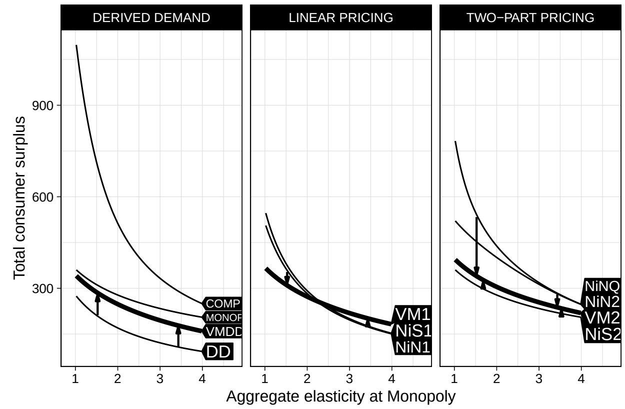



(d) Wholesale fees for firm 1 and 2

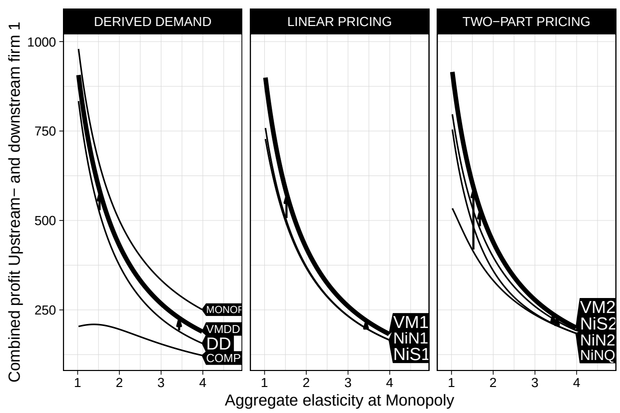
Figure 6: Wholesale prices and fees for a 1× setting:
(a) shows the elimination of double marginalization for the vertically integrated firm;
(b) shows the raising of rival's cost for the rival firm;
(c) shows the increase in total marginal cost for the rival firm;
(d) shows the wholesale fees for firm 1 and 2 in the two-part pricing setting.



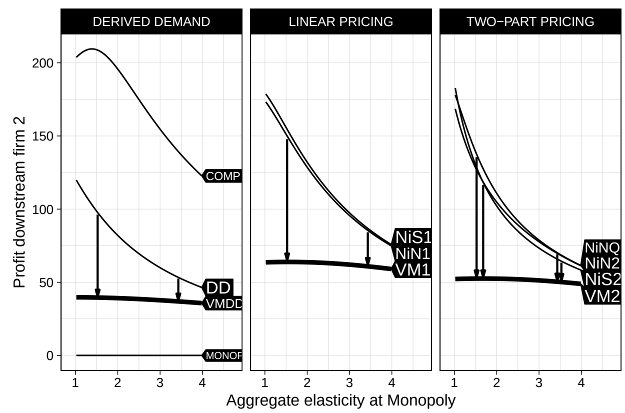

(a) Total quantity vs.ae




(b) Consumer surpluae

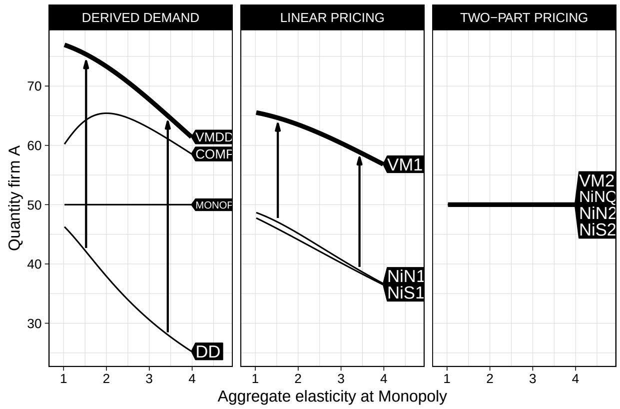



(c) Combined profits firm A and 1 vs. ae

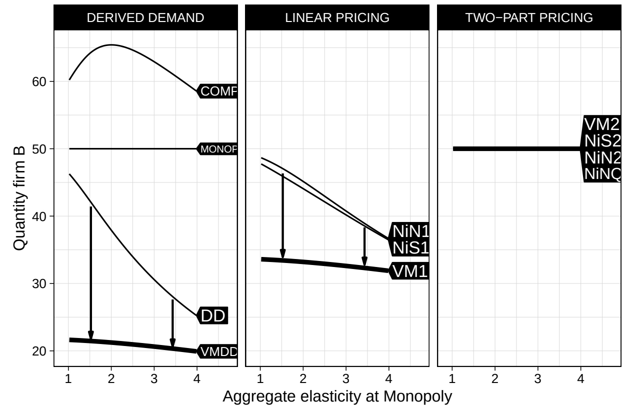



(d) Profit downstream firm 2 vs. ae

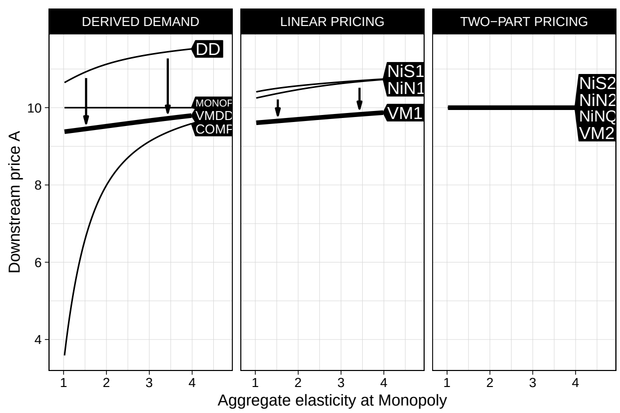
Figure 7: Profit and welfare in a 1× setting:
(a) shows total quantity;
(b) shows that consumer surplus closely follows total quantity;
(c) shows the combined profits of the upstream firm and downstream firm 1 - post-merger profit is always higher than pre-merger combined profit;
(d) shows the profit of firm 2 - post-merger profit is always lower than pre-merger profit.



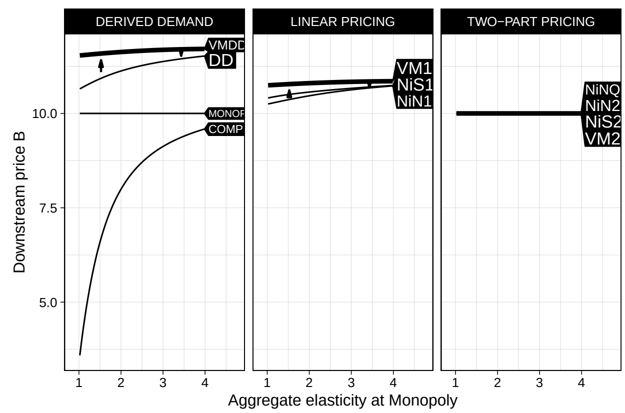

(a) Downstream quantity firm 1 vs. ae




(b) Downstream quantity firm 2 vs. ae

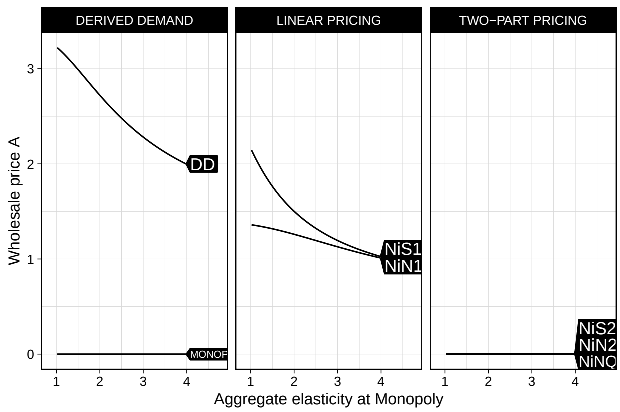



(c) Downstream price firm 1 vs. ae

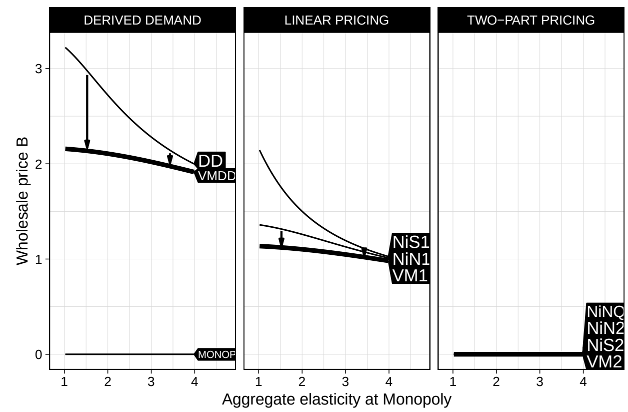



(d) Downstream price firm 2 vs. ae

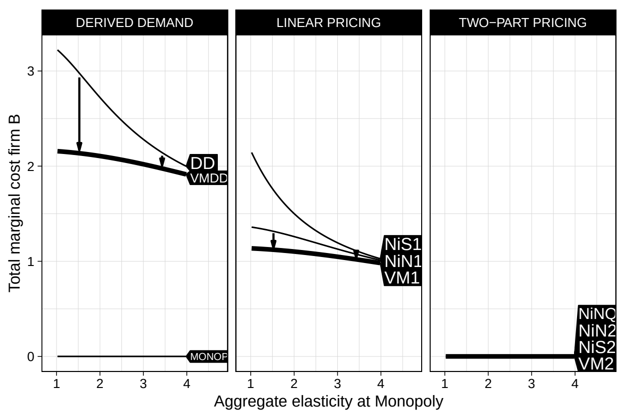
Figure 8: Retail prices and quantities for a $2 \times 1$ setting react in the anticipated manner:
(a) shows an increase in the vertically integrated firm's quantity;
(b) shows a decrease in the rival firm's quantity;
(c) shows an increase in the vertically integrated firm's price;
(d) shows a decrease in the rival firm's price.



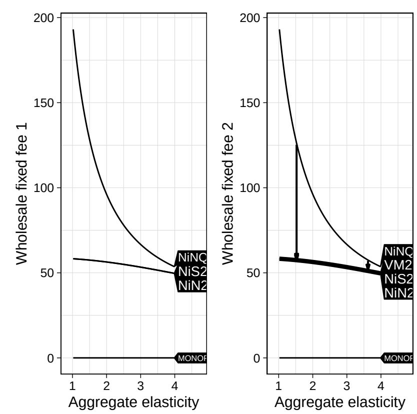

(a) Wholesale price firm 1 vs. ae




(b) Wholesale price firm 2 vs. ae

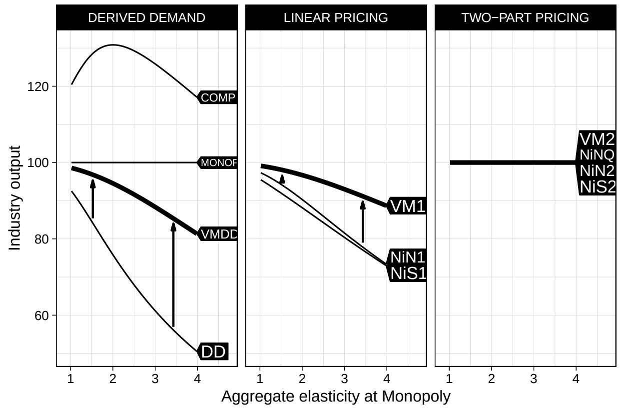



(c) Total marginal cost downstream firm 2 vs. ae

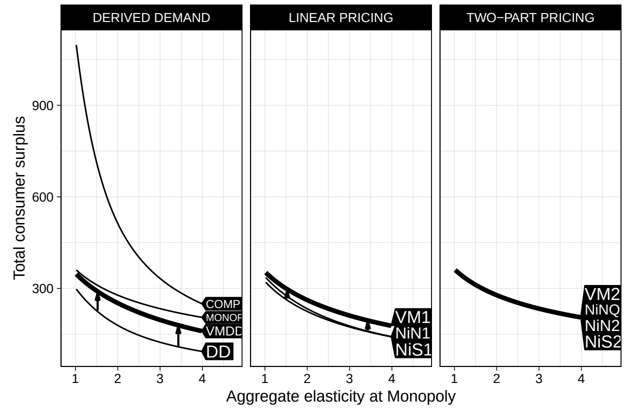



(d) Wholesale fees for firm A and B

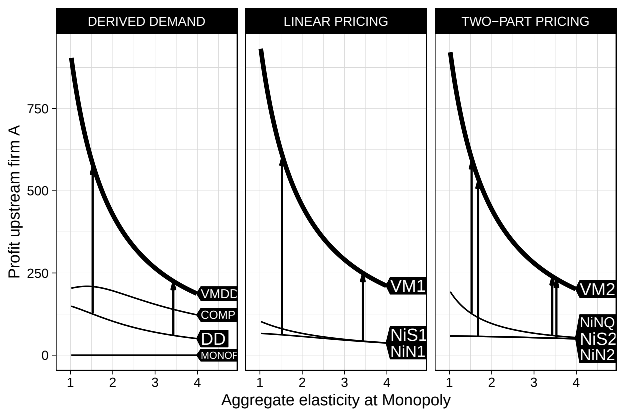
Figure 9: Wholesale prices and fees for a $2 \times 1$ setting:
(a) shows the elimination of double marginalization for the vertically integrated firm;
(b) shows the reducing of rival's revenue for the rival firm;
(c) shows the decrease in total marginal cost for the rival firm as a result of reducing of rival's revenue;
(d) shows the wholesale fees for firm 1 and 2 in the two-part pricing setting.



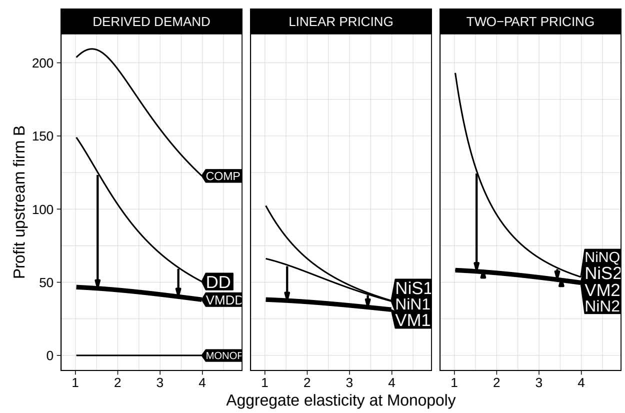

(a) Total quantity vs. ae




(b) Consumer surplus vs. ae




(c) Profit firm A and 1 vs. ae




(d) Profit firm B vs. ae

Figure 10: Profit and welfare in a $2 \times 1$ setting:
(a) shows total quantity;
(b) shows that consumer surplus closely follows total quantity;
(c) shows the profit of firm A - post-merger profit is always higher than pre-merger profit;
(d) shows the profit of firm B - post-merger profit is always lower than pre-merger profit.